

Inference for imputed latent classes using multiple imputation

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Abstract. I introduce a command to multiply impute latent classes following `gsem`, `lclass()` latent class analysis. This allows properly propagating uncertainty in class membership to downstream analysis that may characterize the demographic composition of the classes, or use the class as a predictor variable in statistical models.

Keywords: st0001, postlca_class_prep, latent class analysis, multiple imputation

1 Latent class analysis

Latent class analysis (LCA) is a commonly used statistical and quantitative social science technique of modeling counts in high dimensional contingency tables, or tables of associations of categorical variables Hagenaars and McCutcheon (2002); McCutcheon (1987). LCA is a form of loglinear modeling, so let us explain that first. If the researcher has several categorical variables X_1, X_2, \dots, X_p with categories 1 through $m_j, j = 1, \dots, p$, at their disposal, and can produce counts $n_{k_1 k_2 \dots k_p}$ in a complete p -dimensional table, the first step could be modeling in main effects:

$$\log \mathbb{E} n_{k_1 k_2 \dots k_p} = \text{offset} + \sum_{j=1}^p \sum_{k=1}^{m_j} \beta_{jk} \quad (1)$$

with applicable identification constraints (such as the sum of the coefficients of a single variable is zero, or the coefficient for the first category of a variable is zero). Parameter estimates can be obtained by maximum likelihood, as equation (1) is a Poisson regression model. This model can be denoted as $X_1 + X_2 + \dots + X_p$ main effects model. The fit of the model is assessed by the Pearson χ^2 test comparing the expected vs. observed cell counts, or the likelihood ratio test against a saturated model where each cell in the full p -dimensional table has its own coefficient. If the main effects model were to be found inadequate, the researcher can entertain adding interactions, e.g. the interaction of X_1 and X_2 would have $m_1 \times m_2$ terms for each pair of values of these variables, rather than $m_1 + m_2$ main effects, as well as remaining $p - 2$ variables in their main effects form:

$$\log \mathbb{E} n_{k_1 k_2 \dots k_p} = \text{offset} + \sum_{k_1=1}^{m_1} \sum_{k_2=1}^{m_2} \beta_{12, k_1 k_2} + \sum_{j=3}^p \sum_{k=1}^{m_j} \beta_{jk_j} \quad (2)$$

This model, in the notation that is closer to Stata interaction terms than the traditional exposition of LCA, can be denoted as $X_1 \# X_2 + X_3 + \dots + X_p$.

In the loglinear model notation, the latent class models are models of the form $C \# (X_1 + X_2 + \dots + X_p)$. Categorical latent variable C is the latent class. The model is now a mixture of Poisson regressions, and maximum likelihood estimation additionally involves estimating the prevalence of each class of C .

Further extensions of latent class analysis may include:

1. Analysis with interactions of the observed variables;
2. Analysis with complex survey data (in which case estimation proceeds with `svy` prefix, and the counts are the weighted estimates of the population totals in cells);
3. Constrained analyses with structural zeroes or ones (e.g. that every member of class $C = 1$ must have the value $X_1 = 1$, expressed as the corresponding coefficient $\beta_{11.} = -\infty$ for all categories except 1);
4. Constrained analyses (measurement invariance) where some variables or interactions have identical coefficients across classes.

1.1 Official Stata implementation

Official Stata `gsem`, `lclass()` implements the *main effects* LCA. The syntax is that of the SEM families, with the variables that the arrow points to interpreted as the outcome variables, and the latent class variable considered the source of the arrow:

```
. webuse gsem_lca1
. gsem (accident play insurance stock <- ), logit lclass(C 2)
```

The goodness of fit test against the free tabulation counts is provided by `estat gof` (not available after the complex survey data analysis.)

As LCA is implemented through `gsem`, all the link functions and generalized linear model families are supported, extending the “mainstream” LCA to include multinomial and ordinal variables. (When all outcomes are continuous, the model is referred to as *profile analysis*.)

1.2 Examples

LCA has found use in analyses of complicated economic concepts from survey data, and in assessment of measurement errors that arise in the process.

Biemer (2004) provided analysis of labor force classification status (employed, unemployed, and out of labor force) following changes in the survey instrument used in the Current Population Survey (CPS). The latent classes are the true LFS categories, and the observed variables are the corresponding survey measurements, demographics, and survey interview mode. The model is a variation of LCA that accounts for the survey methodology aspects of CPS: its panel nature (responses are collected over four consecutive months) and response mode (proxy reporting when a family member provides the survey responses rather than the target person.) He found that measurement of being employed and not being in labor force are highly accurate (98% and 97% accuracy) while measurement of being unemployed is much less accurate (between 74% and 81% depending on the analysis year.) LCA allowed to further attribute the drop in accuracy of the unemployment status measurement to proxy reporting, and to the problems with measuring the employment status when the worker is laid off.

Kolenikov and Daley (2017) analyzed the latent classes of employees using the U.S. Department of Labor Worker Classification Survey. The observed variables were (composite) self-report of the employment status (are you an employee at your job; do you refer to your work as your business, your client, your job, etc.); tax status (the forms that the worker receives from their firm: W-2, 1099, K-1, etc.); behavioral control (functions the worker performs and the degree of control over these functions, such as direct reporting to somebody, schedule, permission to leave, etc.); and non-control composite (hired for fixed time or specific project). They found the best fitting model to contain three classes: employees-and-they-know-it (59%), nonemployees-and-they-know-it (24%), and confused (17%) who classify themselves as employees but their tax documentation is unclear, and other variables tend to place them into non-employee status.

1.3 Scope for this package

Researchers are often interested in describing the latent classes or using these classes in analysis as predictors or as moderators. The official [SEM] **gsem postestimation** commands provide limited possibilities, namely reporting of the means of the dependent variables by class via **estat lcmean**. For nearly all meaningful applications of LCA, this is insufficient.

The program distributed with the current package, **postlca_class_prep**, provides a pathway for the appropriate statistical inference that would account for uncertainty in class prediction. This is achieved through the mechanics of multiple imputation (van Buuren 2018). The name is supposed to convey that

1. it is run after LCA as a post-estimation command;

2. it predicts / imputes the latent classes.

2 The new command

Imputation of latent classes, a gsem postestimation command:

```
postlca_class_prepdu, lcimpute(varname) addm(#) [ seed(#) ]
```

`lcimpute(varname)` specifies the name of the latent class variable to be imputed. This option is required.

`addm(#)` specifies the number of imputations to be created. This option is required.

`seed(#)` specifies the random number seed.

3 Examples

3.1 Stata manual data set example

The LCA capabilities of Stata are exemplified in [SEM] **Example 50g**:

```
. frame change default
. cap frame gsem_lca1: clear
. cap frame drop gsem_lca1
. frame create gsem_lca1
. frame change gsem_lca1

.
. webuse gsem_lca1.dta, clear
(Latent class analysis)

. describe
Contains data from https://www.stata-press.com/data/r18/gsem_lca1.dta
Observations: 216
Variables: 4
                17 Jan 2023 12:52
(_dta has notes)

Variable      Storage   Display    Value
          name       type    format   label
                                Variable label
accident      byte     %9.0g
play          byte     %9.0g
insurance     byte     %9.0g
stock          byte     %9.0g
                                Would testify against friend in accident case
                                Would give negative review of friend's play
                                Would disclose health concerns to friend's
                                insurance company
                                Would keep company secret from friend

Sorted by: accident play insurance stock
. gsem (accident play insurance stock <-, logit lclass(C 2)
        (output omitted)
Generalized structural equation model
Log likelihood = -504.46767
Number of obs = 216


```

	Coefficient	Std. err.	z	P> z	[95% conf. interval]
1.C	(base outcome)				

2.C						
	_cons	-.9482041	.2886333	-3.29	0.001	-1.513915 -.3824933

Class: 1
 Response: accident
 Family: Bernoulli
 Link: Logit
 Response: play
 Family: Bernoulli
 Link: Logit
 Response: insurance
 Family: Bernoulli
 Link: Logit
 Response: stock
 Family: Bernoulli
 Link: Logit

	Coefficient	Std. err.	z	P> z	[95% conf. interval]
accident					
_cons	.9128742	.1974695	4.62	0.000	.5258411 1.299907
play					
_cons	-.7099072	.2249096	-3.16	0.002	-1.150722 -.2690926
insurance					
_cons	-.6014307	.2123096	-2.83	0.005	-1.01755 -.1853115
stock					
_cons	-1.880142	.3337665	-5.63	0.000	-2.534312 -1.225972

Class: 2
 (output omitted)

	Coefficient	Std. err.	z	P> z	[95% conf. interval]
accident					
_cons	4.983017	3.745987	1.33	0.183	-2.358982 12.32502
play					
_cons	2.747366	1.165853	2.36	0.018	.4623372 5.032395
insurance					
_cons	2.534582	.9644841	2.63	0.009	.6442279 4.424936
stock					
_cons	1.203416	.5361735	2.24	0.025	.1525356 2.254297

The official post-estimation commands available after `gsem`, `lclass()` produce the estimated class probabilities and class-specific means of the outcome variables used in the model:

```
. set rmsg on
r; t=0.00 14:45:54
```

```
. estat lcprob
Latent class marginal probabilities                                         Number of obs = 216

```

	Delta-method		
	Margin	std. err.	[95% conf. interval]
C			
1	.7207539	.0580926	.5944743 .8196407
2	.2792461	.0580926	.1803593 .4055257

r; t=1.13 14:45:55

```
. estat lcmean
Latent class marginal means                                         Number of obs = 216

```

	Delta-method		
	Margin	std. err.	[95% conf. interval]
1			
accident	.7135879	.0403588	.6285126 .7858194
play	.3296193	.0496984	.2403573 .4331299
insurance	.3540164	.0485528	.2655049 .4538042
stock	.1323726	.0383331	.0734875 .2268872
2			
accident	.9931933	.0253243	.0863544 .9999956
play	.9397644	.0659957	.6135685 .9935191
insurance	.9265309	.0656538	.6557086 .9881667
stock	.769132	.0952072	.5380601 .9050206

r; t=4.48 14:46:00

```
. set rmsg off
```

The multiple imputation version of this estimation task could look as follows:

```
. set rmsg on
r; t=0.00 14:46:00
. postlca_class_preditpe, lcimpute(lclass) addm(10) seed(12345)
(216 missing values generated)
(10 imputations added; M = 10)
r; t=0.05 14:46:00
. mi estimate : prop lclass
Multiple-imputation estimates      Imputations = 10
Proportion estimation             Number of obs = 216
                                         Average RVI = 0.4594
                                         Largest FMI = 0.3319
                                         Complete DF = 215
DF adjustment: Small sample       DF:    min = 55.99
                                         avg = 55.99
Within VCE type: Analytic        max = 55.99

```

	Normal		
	Proportion	Std. err.	[95% conf. interval]
lclass			
1	.7236111	.0367281	.6500355 .7971867
2	.2763889	.0367281	.2028133 .3499645

		Mean	Std. err.	[95% conf. interval]
c.accident@lclass_modal	1	.6896552	.0385529	.6136651 .7656452
	2	1	0	.

3.2 NHANES complex survey data example

In many important and realistic applications of LCA, including the case that necessitated the development of this package (Rowan et al. 2024), the data come from complex survey designs that require setting the data up for the appropriate survey-design adjusted analyses. See [SVY] **svyset**, [MI] **mi svyset**, and Kolenikov and Pitblado (2014).

The standard data set for the [SVY] commands is an extract from the National Health and Nutrition Examination Survey, Round Two (NHANES II) data. I will use a handful of binary health outcomes and one ordinal outcome to demonstrate LCA; the ordinal outcome is arguably an extension that is not quite well covered in the “classical” social science LCA.

```

. frame change default
. cap frame nhanes2: clear
. cap frame drop nhanes2
. frame create nhanes2
. frame change nhanes2
.
. webuse nhanes2.dta, clear
. svyset
Sampling weights: finalwgt
VCE: linearized
Single unit: missing
Strata 1: strata
Sampling unit 1: psu
FPC 1: <zero>
. svy , subpop(if hlthstat<8) : ///
>      gsem ///
>          (heartatk diabetes highbp <-, logit) ///
>          (hlthstat <-, ologit) ///
>          , lclass(C 2) nolog startvalues(randomid, draws(5) seed(101))
(running gsem on estimation sample)
Survey: Generalized structural equation model
Number of strata = 31                                         Number of obs     =    10,351
Number of PSUs   = 62                                         Population size = 117,157,513
                                                               Subpop. no. obs =    10,335
                                                               Subpop. size   = 116,997,257
                                                               Design df       =        31

```

	Linearized	Coefficient	std. err.	t	P> t	[95% conf. interval]
1.C	(base outcome)					

2.C	<code>_cons</code>	1.330043	.1259401	10.56	0.000	1.073186	1.586899
-----	--------------------	----------	----------	-------	-------	----------	----------

Class: 1
 Response: heartatk Number of obs = 10,335
 Family: Bernoulli
 Link: Logit
 Response: diabetes Number of obs = 10,335
 Family: Bernoulli
 Link: Logit
 Response: highbp Number of obs = 10,335
 Family: Bernoulli
 Link: Logit
 Response: hlthstat Number of obs = 10,335
 Family: Ordinal
 Link: Logit

		Linearized				
		Coefficient	std. err.	t	P> t	[95% conf. interval]
heartatk	<code>_cons</code>	-1.874967	.1150791	-16.29	0.000	-2.109672 -1.640261
diabetes	<code>_cons</code>	-1.785271	.0805057	-22.18	0.000	-1.949463 -1.621078
highbp	<code>_cons</code>	.4244921	.076861	5.52	0.000	.2677332 .5812511
/hlthstat						
	<code>cut1</code>	-3.659014	.8903346		-5.474863	-1.843165
	<code>cut2</code>	-2.272516	.4402984		-3.17051	-1.374521
	<code>cut3</code>	-.2566588	.2032721		-.671235	.1579173
	<code>cut4</code>	1.229244	.1951641		.8312038	1.627283

Class: 2
 Response: heartatk Number of obs = 10,335
 Family: Bernoulli
 Link: Logit
 Response: diabetes Number of obs = 10,335
 Family: Bernoulli
 Link: Logit
 Response: highbp Number of obs = 10,335
 Family: Bernoulli
 Link: Logit
 Response: hlthstat Number of obs = 10,335
 Family: Ordinal
 Link: Logit

		Linearized				
		Coefficient	std. err.	t	P> t	[95% conf. interval]
heartatk	<code>_cons</code>	-6.081307	.6280801	-9.68	0.000	-7.362285 -4.800329
diabetes						

	<u>_cons</u>	-5.223215	.6044468	-8.64	0.000	-6.455993	-3.990438
highbp	<u>_cons</u>	-.8166105	.0750027	-10.89	0.000	-.9695795	-.6636415
/hlthstat							
	cut1	-.657824	.0483113			-.7563555	-.5592926
	cut2	.7123144	.0649814			.5797839	.8448448
	cut3	2.647239	.1192958			2.403934	2.890544
	cut4	24.64389	14.1421			-4.199113	53.48689

. set rmsg on
 r; t=0.00 14:46:16

. estat lcprob

Latent class marginal probabilities

Number of strata = 31 Number of obs = 10,351
 Number of PSUs = 62 Population size = 117,157,513
 Design df = 31

		Delta-method		
		Margin	std. err.	[95% conf. interval]
C				
1		.2091523	.0208315	.1698206 .2547976
2		.7908477	.0208315	.7452024 .8301794

r; t=10.03 14:46:26

. estat lcmean

Latent class marginal means

Number of strata = 31 Number of obs = 10,351
 Number of PSUs = 62 Population size = 117,157,513
 Design df = 31

		Delta-method		
		Margin	std. err.	[95% conf. interval]
1				
heartatk		.1329681	.0132672	.1081603 .1624295
diabetes		.1436535	.0099036	.1246119 .1650562
highbp		.6045577	.018375	.5665363 .6413552
hlthstat				
Excellent		.0251111	.0217959	.0041733 .1366775
Very good		.0683138	.021455	.0355579 .127263
Good		.3427603	.0254834	.2928195 .3964437
Fair		.3375009	.0210993	.2958981 .3817814
Poor		.2263139	.0341724	.1642029 .3033906
2				
heartatk		.00228	.0014287	.0006343 .0081599
diabetes		.0053611	.0032231	.0015686 .0181559
highbp		.3064836	.0159419	.2749643 .3399221
hlthstat				
Excellent		.3412286	.01086	.319438 .3637112
Very good		.3296838	.0082507	.3130796 .346724
Good		.2629283	.0094265	.2441597 .2826002

Fair	.0661594	.0073704	.052623	.0828732
Poor	1.98e-11	2.81e-10	5.68e-24	.9857545

r; t=80.58 14:47:47
 . set rmsg off

This analysis approximates breaking down the population into "generally healthy" and "unhealthy" groups, as e.g. the gradient of *hlthstat* variable between the classes shows. The official **gsem postestimation** commands take approximately forever to run. Some complicated interaction of **svy** and **gsem** made the default starting infeasible, hence I had to specify the initial random search. The use of the **postlca_class_preditpe** command makes it possible to run the analysis much faster (although that has never been the intent of the package), and to conduct complementary analyses, e.g. analysis of the racial composition of the two classes using a variable that is outside of the model.

```

  . set rmsg on
  r; t=0.00 14:47:47
  . postlca_class_preditpe, lcimpute(lclass) addm(10) seed(5678)
  (10,351 missing values generated)
  (10 imputations added; M = 10)
  Sampling weights: finalwgt
  VCE: linearized
  Single unit: missing
  Strata 1: strata
  Sampling unit 1: psu
  FPC 1: <zero>
  r; t=0.22 14:47:47
  . mi estimate : prop lclass
  Multiple-imputation estimates      Imputations      =      10
  Proportion estimation            Number of obs    =    10,351
  Average RVI                      =      0.5318
  Largest FMI                      =      0.3641
  Complete DF                       =     10350
  DF adjustment: Small sample      DF:      min    =      73.85
                                         avg    =      73.85
  Within VCE type: Analytic        max    =      73.85
  
```

	Normal		
	Proportion	Std. err.	[95% conf. interval]
lclass			
1	.2675394	.0053851	.256809 .2782698
2	.7324606	.0053851	.7217302 .743191

r; t=1.53 14:47:49
 . mi estimate : prop hlthstat if hlthstat < 8, over(lclass)
 Multiple-imputation estimates Imputations = 10
 Proportion estimation Number of obs = 10,335
 Average RVI = .
 Largest FMI = .
 Complete DF = 10334
 DF adjustment: Small sample DF: min = 36.31
 avg = .
 Within VCE type: Analytic max = .

	Proportion	Std. err.	Normal [95% conf. interval]	
hlthstat@lclass				
Excellent 1	.0196036	.0034636	.0126501	.0265572
Excellent 2	.3104144	.0055292	.2995676	.3212613
Very good 1	.0550625	.0061217	.0426506	.0674743
Very good 2	.3217978	.0056545	.3106989	.3328966
Good 1	.2971359	.0106787	.2758813	.3183905
Good 2	.2795891	.0056354	.2685025	.2906756
Fair 1	.3635286	.0106978	.3423564	.3847008
Fair 2	.0881987	.003944	.0803607	.0960367
Poor 1	.2646694	.0089821	.2470257	.282313
Poor 2	0	(no observations)		

Note: Numbers of observations in e(_N) vary among imputations.
r; t=3.01 14:47:52

```
. mi estimate : prop race, over(lclass)
Multiple-imputation estimates      Imputations      =      10
Proportion estimation               Number of obs    =   10,351
                                         Average RVI     =     0.4677
                                         Largest FMI    =     0.4671
                                         Complete DF    =    10350
DF adjustment: Small sample        DF:      min    =     45.27
                                         avg    =    119.43
Within VCE type:      Analytic      max    =    321.80
```

	Proportion	Std. err.	Normal [95% conf. interval]	
race@lclass				
White 1	.839782	.0093485	.8209563	.8586078
White 2	.8889046	.0042799	.8804196	.8973897
Black 1	.1434614	.0085447	.1263568	.160566
Black 2	.0908358	.0038489	.0832192	.0984524
Other 1	.0167566	.0031195	.0105146	.0229986
Other 2	.0202596	.0017697	.0167778	.0237413

Note: Numbers of observations in e(_N) vary among imputations.
r; t=2.98 14:47:55
. set rmsg off

The less healthy class has a higher concentration of ethnic minorities.

3.3 Choosing the number of imputations

One “researcher’s degrees of freedom” aspect of this analysis is the number of imputations M that need to be created. What this number affects the most is the stability of the standard errors obtained through the multiple imputation process. This stability is internally assessed with estimated degrees of freedom associated with the variance estimate (Barnard and Rubin 1999). With $M = 10$ imputations, the smaller “poor health” class have about 50 degrees of freedom:

```
. mi estimate : prop race, over(lclass)
Multiple-imputation estimates      Imputations      =      10
Proportion estimation            Number of obs    =   10,351
                                         Average RVI    =     0.4677
                                         Largest FMI    =     0.4671
                                         Complete DF    =    10350
DF adjustment: Small sample       DF:      min    =     45.27
                                         avg    =    119.43
Within VCE type:     Analytic      max    =    321.80
```

	Proportion	Std. err.	Normal [95% conf. interval]	
race@lclass				
White 1	.839782	.0093485	.8209563	.8586078
White 2	.8889046	.0042799	.8804196	.8973897
Black 1	.1434614	.0085447	.1263568	.160566
Black 2	.0908358	.0038489	.0832192	.0984524
Other 1	.0167566	.0031195	.0105146	.0229986
Other 2	.0202596	.0017697	.0167778	.0237413

Note: Numbers of observations in e(_N) vary among imputations.

```
. mi estimate, dftable
Multiple-imputation estimates      Imputations      =      10
Proportion estimation            Number of obs    =   10,351
                                         Average RVI    =     0.4677
                                         Largest FMI    =     0.4671
                                         Complete DF    =    10350
DF adjustment: Small sample       DF:      min    =     45.27
                                         avg    =    119.43
Within VCE type:     Analytic      max    =    321.80
```

	Proportion	Std. err.	df	std. err.
race@lclass				
White 1	.839782	.0093485	45.3	34.13
White 2	.8889046	.0042799	106.2	18.59
Black 1	.1434614	.0085447	57.9	28.28
Black 2	.0908358	.0038489	126.3	16.62
Other 1	.0167566	.0031195	59.0	27.89
Other 2	.0202596	.0017697	321.8	9.38

Note: Numbers of observations in e(_N) vary among imputations.

The public use version of the NHANES II data uses the approximate design that has 62 PSUs in 31 strata, resulting in 31 design degrees of freedom. The imputation degrees of freedom barely exceed that. Let us push the number of imputations up (I used $M = 62$ to match the number of PSUs):

```
. webuse nhanes2.dta, clear
. qui svy , subpop(if hlthstat<8) : ///
>           gsem //(
>           (heartatk diabetes highbp <-, logit) //(
>           (hlthstat <-, ologit) //(
>           , lclass(C 2) nolog startvalues(randomid, draws(5) seed(101))
```

```
. postlca_class_prelude, lcimpute(lclass) addm(62) seed(9752)
(10,351 missing values generated)
(62 imputations added; M = 62)

Sampling weights: finalwgt
      VCE: linearized
      Single unit: missing
      Strata 1: strata
      Sampling unit 1: psu
      FPC 1: <zero>

. mi estimate : prop race, over(lclass)

Multiple-imputation estimates      Imputations      =       62
Proportion estimation      Number of obs      =    10,351
                           Average RVI      =      0.2483
                           Largest FMI      =      0.2893
                           Complete DF      =     10350
DF adjustment: Small sample      DF:      min      =     671.70
                           avg      =   1,786.62
                           max      =   3,092.35
Within VCE type: Analytic
```

	Normal			
	Proportion	Std. err.	[95% conf. interval]	
race@lclass				
White 1	.8369206	.0079313	.8213584	.8524827
White 2	.8900061	.0038498	.8824571	.897555
Black 1	.1455849	.007628	.1306162	.1605536
Black 2	.0900031	.0035541	.0830334	.0969728
Other 1	.0174946	.0029465	.0117091	.02328
Other 2	.0199908	.001709	.01664	.0233417

Note: Numbers of observations in e(_N) vary among imputations.

```
. mi estimate, dftable

Multiple-imputation estimates      Imputations      =       62
Proportion estimation      Number of obs      =    10,351
                           Average RVI      =      0.2483
                           Largest FMI      =      0.2893
                           Complete DF      =     10350
DF adjustment: Small sample      DF:      min      =     671.70
                           avg      =   1,786.62
                           max      =   3,092.35
Within VCE type: Analytic
```

	Normal			
	Proportion	Std. err.	df	std. err.
race@lclass				
White 1	.8369206	.0079313	1100.3	13.14
White 2	.8900061	.0038498	2639.5	7.08
Black 1	.1455849	.007628	1004.8	13.98
Black 2	.0900031	.0035541	2211.1	8.08
Other 1	.0174946	.0029465	671.7	18.45
Other 2	.0199908	.001709	3092.4	6.26

Note: Numbers of observations in e(_N) vary among imputations.

The MI degrees of freedom are now comfortably above 600. In many i.i.d. data situations, increasing the number of imputations to several dozens can often send the MI degrees of freedom to approximate infinity (the reported numbers are in hundreds of

thousands). With complex survey designs that have limited degrees of freedom within each implicate, this may not materialize. Researchers are encouraged to adopt the workflow where, in parallel, they

1. start with a small number of imputations, like `addm(10)` in the example above, and develop the analysis code for all the substantive analyses, and
2. working with the key outcomes or analyses, experiment with several values of M to find a reasonable trade-off when degrees of freedom exceed the sample size for i.i.d. data, and/or exceed the design degrees of freedom for complex survey data by a factor of 3–5.

Then a chosen value of M can be used for all analyses in the paper.

An additional consideration for the choice of the number of imputations is ensuring diversity of the simulated classes. Even a large number of replications may not protect the researcher from classes that may have structural zeroes. These produce zero standard errors and missing degrees of freedom and variance increase statistics:

```
. mi estimate , dftable : prop hlthstat if hlthstat < 8, over(lclass)
Multiple-imputation estimates      Imputations      =       62
Proportion estimation             Number of obs   =    10,335
                                         Average RVI   =
                                         Largest FMI   =
                                         Complete DF   =     10334
DF adjustment: Small sample       DF:      min   =    234.37
                                         avg   =
                                         max   =
Within VCE type: Analytic          max   =     .
```

	Proportion	Std. err.	df	Normal std. err.
hlthstat@lclass				
Excellent 1	.0183993	.0033919	309.7	32.68
Excellent 2	.3111818	.0054853	6242.4	3.09
Very good 1	.0573944	.0062471	234.4	41.21
Very good 2	.3212476	.0056361	4001.1	5.03
Good 1	.2947403	.0104573	575.3	20.57
Good 2	.2804536	.0055871	2155.5	8.23
Fair 1	.3656239	.0104118	1033.8	13.71
Fair 2	.087117	.00393	549.1	21.27
Poor 1	.2638421	.0087535	4603.6	4.40
Poor 2	0	(no observations)		

Note: Numbers of observations in `e(_N)` vary among imputations.

4 Simulation

So how much of a problem is the modal imputation of the latent classes? I ran a small simulation to investigate. Samples were taken from a model with three classes,

five binary indicators and one additional continuous variable not used in the model as follows:

Variable	Class 1	Class 2	Class 3
Class probability	0.4	0.4	0.2
$\mathbb{P}[y_1 = 1 C]$	0.7	0.3	0.6
$\mathbb{P}[y_2 = 1 C]$	0.8	0.5	0.6
$\mathbb{P}[y_3 = 1 C]$	0.5	0.4	0.7
$\mathbb{P}[y_4 = 1 C]$	0.5	0.3	0.7
$\mathbb{P}[y_5 = 1 C]$	0.8	0.4	0.3
$y_6 \sim N(\mu_c, 1)$	1	$\sqrt{2} = 1.41$	$\sqrt{3} = 1.73$

The LCA model with outcomes y_1 through y_5 was estimated, and the classes were predicted using the posterior modal prediction, multiple imputation with $M = 5$ imputed data sets, and $M = 50$ imputed data sets. (To be precise, there was a single imputation with $M = 50$, but two versions were run: `mi estimate , imp(1/5)` for the limited application with $M = 5$, and without the `imp()` option for the full set of $M = 50$.) Inspecting the individual runs visually, a shorter set of imputations often results in insufficient detail, namely failing to capture realizations of (posterior) rare classes.

There were at least two complications with the simulation. First, the classes in any LCA model are only identified up to a permutation of the class labels. To wit, there are no distinguishable differences between estimates when say classes 1 and 2 are swapped in a model with 2+ classes. The likelihoods are the same, the point estimates are likely to be the same within numeric accuracy. In any particular run of the `gsem`, `lclass()` command, the classes depend first and foremost on the starting values. The help file `help gsem_estimation_options##startvalues()` outlines the possible options:

- In my own practical work, I typically use `startvalues(randomid)`, `draws(10)` or `draws(20)` to get this many random assignments of the starting classes, and having Stata run the EM algorithm to get some decent starting values for the gradient-based optimization methods.
- For the simulation purposes, you have to fix either the initial assignment of classes, or the the starting values of the estimates. The former is implementable with `startvalues(classid true_class)` since the latter is, of course, known. Strangely, in this simulation, this did not work out well as it resulted in convergence issues: it has been pushing the model into areas of the parameter space where the likelihood was too flat to climb out of.
- The resulting simulation specification I used in the simulation was a combination of `from(b0) startvalues(jitter 0.1, draws(10))`. The value of the starting matrix `b0` was obtained from a sample of size 10^6 computed once outside of the simulation. Jitter was added to allow some exploration of the sample optima near that point.

The second complication of the simulation was that in some runs, the `mi estimate` calls with imputed classes were returning errors. There may be some minor incompatibilities between this third-party implementation of multiple imputation, and the expectations of `mi estimate`. Also, the discrete nature of the imputed data may have played a role. (The specific error message stated that an imputation had missing data, and/or that sample sizes varied between imputations, although visual inspection of the offending simulation runs showed this was not the case; this may have been a somewhat generic error message produced by the `mi` engine when it encountered something unusual.)

. tabulate method			
method	Freq.	Percent	Cum.
modal	2,047	37.29	37.29
predpute_5	1,836	33.44	70.73
predpute_50	1,607	29.27	100.00
Total	5,490	100.00	

With these limitations in mind, here are the simulation results.

Class probabilities are biased for the multiple imputation class prediction methods (the population value for class 1 was 0.4.)

. mean class1pr if touse3, over(n_method)		Mean estimation Number of obs = 4,797		
		Mean	Std. err.	[95% conf. interval]
c.class1pr@n_method				
modal		.3938263	.004415	.3851708 .4024818
predpute_5		.3646841	.0038655	.3571059 .3722624
predpute_50		.364754	.0038643	.3571783 .3723298

The modal method exhibits greater variability in class probabilities than the imputation methods. The standard errors are severely biased down for all methods. Multiple imputation with more replicates has more stable standard errors.

. bysort method (rep): sum class1pr if touse3					
-> method = modal					
Variable	Obs	Mean	Std. dev.	Min	Max
class1pr	1,583	.3938263	.1756604	.039	.942
-> method = predpute_5					
Variable	Obs	Mean	Std. dev.	Min	Max
class1pr	1,607	.3646841	.1549593	.0418	.8014

-> method = predpute_50

Variable	Obs	Mean	Std. dev.	Min	Max
class1pr	1,607	.364754	.1549089	.04078	.80522
. mean class1se if touse3, over(n_method)					
Mean estimation Number of obs = 4,797					
	Mean	Std. err.	[95% conf. interval]		
c.class1se@n_method					
modal	.0143072	.0000464	.0142163	.0143981	
predpute_5	.0188952	.0000906	.0187177	.0190728	
predpute_50	.0184366	.0000579	.0183232	.0185501	

Turning to the outcomes in the model, I picked two random summaries out of 15 ($=5$ outcomes \times 3 classes). For the outcome y_1 in class 2, estimates are biased away from the true value of 0.3 for the modal method:

. mean y1cl2pr if touse3 & y1cl2pr>0.01 & y1cl2pr<0.99, over(n_method)					
Mean estimation Number of obs = 4,503					
	Mean	Std. err.	[95% conf. interval]		
c.y1cl2pr@n_method					
modal	.218858	.0027115	.2135422	.2241739	
predpute_5	.2953609	.0020341	.291373	.2993489	
predpute_50	.2954626	.002029	.2914848	.2994404	

The same observations concerning the standard errors apply: the variability of the modal method estimates is greater than the multiple imputation estimates, the standard errors are biased for all methods, and the standard errors are more stable with the greater number of replicates.

. bysort method (rep): sum y1cl2pr if touse3 & y1cl2pr>0.01 & y1cl2pr<0.99					
-> method = modal					
Variable	Obs	Mean	Std. dev.	Min	Max
y1cl2pr	1,355	.218858	.0998111	.0234987	.702381
-> method = predpute_5					
Variable	Obs	Mean	Std. dev.	Min	Max
y1cl2pr	1,574	.2953609	.0807021	.0179912	.5569573
-> method = predpute_50					
Variable	Obs	Mean	Std. dev.	Min	Max
y1cl2pr	1,574	.2954626	.0804971	.0193931	.5578867
. mean y1cl2se if touse3 & y1cl2pr>0.01 & y1cl2pr<0.99, over(n_method)					
Mean estimation Number of obs = 4,503					

	Mean	Std. err.	[95% conf. interval]	
c.y1cl2se@n_method				
modal	.0196673	.0001217	.0194288	.0199059
predpute_5	.0296163	.0002038	.0292168	.0300158
predpute_50	.029101	.0001752	.0287575	.0294444

Another outcome summarized, y_4 in class 1, has the target value of 0.5. Biases are in opposite directions for the modal method and multiple imputation methods.

Mean estimation				
	Mean	Std. err.	[95% conf. interval]	
c.y4cl1pr@n_method				
modal	.5134519	.0027162	.5081269	.5187769
predpute_5	.4861217	.0018979	.4824009	.4898424
predpute_50	.4862154	.001881	.4825277	.489903

Problems with the variability and its estimation with the standard errors persist.

. bysort method (rep): sum y4cl1pr if touse3 & y4cl1pr>0.01 & y4cl1pr<0.99, over(n_method)				
Mean estimation				
Number of obs = 4,687				
-> method = modal				
Variable Obs Mean Std. dev. Min Max				
y4cl1pr 1,487 .5134519 .1047403 .0546875 .8440678				
-> method = predpute_5				
Variable Obs Mean Std. dev. Min Max				
y4cl1pr 1,600 .4861217 .0759155 .0521549 .9703288				
-> method = predpute_50				
Variable Obs Mean Std. dev. Min Max				
y4cl1pr 1,600 .4862154 .0752401 .0491738 .9807849				
. mean y4cl1se if touse3 & y4cl1pr>0.01 & y4cl1pr<0.99, over(n_method)				
Mean estimation				
Number of obs = 4,687				
-> method = modal				
Variable Mean Std. err. [95% conf. interval]				
c.y4cl1se@n_method				
modal	.0257628	.0001536	.0254616	.0260639
predpute_5	.0342909	.0002728	.0337561	.0348258
predpute_50	.0338453	.0002555	.0333444	.0343461

Moving now to investigate the variable outside the model, y_6 , we observe that the estimates are severely biased towards the pooled mean.

Mean estimation				Number of obs = 4,797
	Mean	Std. err.	[95% conf. interval]	
c.y6cl1pr@n_method				
modal	1.178873	.0019722	1.175007	1.18274
predpute_5	1.208982	.0015596	1.205925	1.21204
predpute_50	1.209956	.0014737	1.207067	1.212845

Mean estimation				Number of obs = 4,728
	Mean	Std. err.	[95% conf. interval]	
c.y6cl3pr@n_method				
modal	1.453111	.0034072	1.446431	1.45979
predpute_5	1.40193	.0021874	1.397642	1.406219
predpute_50	1.401776	.0020405	1.397776	1.405777

The modal method continues to exhibit greater variability than the imputation methods. For this outcome, however, the standard errors are much closer to their targets. The standard errors of the modal method (mean of 0.058 for class 1) underestimate the true Monte Carlo variability (mean of 0.078), while the multiple imputation standard errors are biased upwards (0.071 vs. the targets of 0.060). Multiple imputation with more replicates has more stable standard errors.

. bysort method (rep): sum y6cl1pr if touse3					
-> method = modal					
Variable	Obs	Mean	Std. dev.	Min	Max
y6cl1pr	1,583	1.178873	.0784673	.8461757	1.417481
-> method = predpute_5					
Variable	Obs	Mean	Std. dev.	Min	Max
y6cl1pr	1,607	1.208982	.0625194	.897253	1.401546
-> method = predpute_50					
Variable	Obs	Mean	Std. dev.	Min	Max
y6cl1pr	1,607	1.209956	.0590762	.9668165	1.388266
. mean y6cl1se if touse3, over(n_method)					
Mean estimation	Number of obs = 4,797				
	Mean	Std. err.	[95% conf. interval]		
c.y6cl1se@n_method					
modal	.0575567	.0004523	.05667	.0584435	
predpute_5	.0723921	.0006426	.0711323	.073652	
predpute_50	.0712562	.0005766	.0701258	.0723867	

Same observations apply to the standard errors for class 3.

```
. bysort method (rep): sum y6cl3pr if touse3

-> method = modal
      Variable |       Obs        Mean      Std. dev.       Min       Max
    y6cl3pr |     1,514    1.453111     .132573    .9581373   2.053384

-> method = predpute_5
      Variable |       Obs        Mean      Std. dev.       Min       Max
    y6cl3pr |     1,607    1.40193     .0876854   1.136963   1.883443

-> method = predpute_50
      Variable |       Obs        Mean      Std. dev.       Min       Max
    y6cl3pr |     1,607    1.401776     .0817985   1.139892   1.755444

. mean y6cl3se if touse3, over(n_method)
Mean estimation                                         Number of obs = 4,728

```

	Mean	Std. err.	[95% conf. interval]
c.y6cl3se@n_method			
modal	.0923276	.0012236	.0899288 .0947263
predpute_5	.1099187	.0013831	.1072071 .1126303
predpute_50	.1078059	.0012745	.1053072 .1103046

Overall, it seems like the problem of the class indeterminacy (permutation of classes) has still been damaging the simulation. Further steps would need to be taken to better match the estimated classes with their population counterparts.

5 References

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Stas Kolenikov is Principal Statistician at NORC who has been using Stata and writing Stata programs for about 25 years. He had worked on economic welfare and inequality, spatiotemporal environmental statistics, mixture models, missing data, multiple imputation, structural equations with latent variables, resampling methods, complex sampling designs, survey weights, Bayesian mixed models, combining probability and non-probability samples, latent class analysis, and likely some other stuff, too.