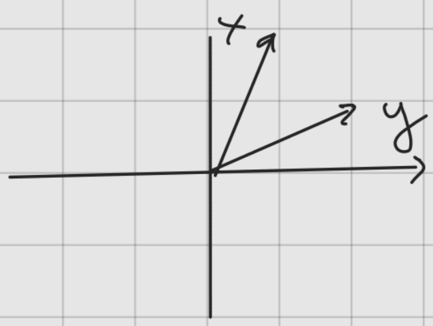
№28.5 (1)

$$\xi = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \quad \eta = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$



$$x + y = e \begin{pmatrix} 3 \\ 3 \end{pmatrix} \Rightarrow$$

$\Rightarrow$ Зеркало $\langle \begin{pmatrix} 1 \\ 1 \end{pmatrix} \rangle \Rightarrow$

$$e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \Rightarrow A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\text{Пр}\, x = \frac{\left(x, \begin{pmatrix} 1 \\ 1 \end{pmatrix}\right)}{\left(\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix}\right)} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{3}{2} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ 2 \end{pmatrix} - \begin{pmatrix} \frac{3}{2} \\ \frac{3}{2} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\frac{3}{2} \begin{pmatrix} 1 \\ 1 \end{pmatrix} - \frac{1}{2} \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$\begin{cases} A\xi = \eta \\ A\eta = \xi \end{cases} \qquad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$\begin{cases} a + 2b = 2 \\ c + 2d = 1 \\ 2a + b = 1 \\ 2c + d = 2 \end{cases} \qquad \begin{array}{l} a - b = -1 \\ 3a = 0 \\ \Rightarrow b = 1 \\ c - d = 1 \\ d = 0 \Rightarrow c = 1 \end{array}$$

№28.21

$$\varphi = \frac{d}{dt} \qquad (\varphi(x), y) = (x, \varphi^*(y))$$

$e_1 = 1$

$e_2 = t$

$e_3 = t^2$

$$\int_{-1}^{1} \frac{dx}{dt} \cdot y(t)\,dt = $$

$$= \underset{a_1t^2+b_1t+c_1}{\underset{\|}{x(t)}}\ \underset{a_2t^2+b_2t+c_2}{\underset{\|}{y(t)}}\Big|_{-1}^{1} - \underbrace{\int_{-1}^{1} x(t) \frac{dy}{dt}\,dt}_{= -(x, \varphi(y)) = -x^T \Gamma A y}$$

$$= 2(a_1 b_2 + b_1 a_2 + b_1 c_2 + c_1 b_2) = x^T \Gamma A_1 y$$

$$2 \cdot (c_1, b_1, a_1)\, X \begin{pmatrix} c_2 \\ b_2 \\ a_2 \end{pmatrix}$$

$$X = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = \frac{3}{4}\begin{pmatrix} 3 & 0 & -5 \\ 0 & 4 & 0 \\ -5 & 0 & 15 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} =$$

$$A_1 = \Gamma^{-1} X =$$

$$= \frac{3}{4}\begin{pmatrix} 0 & -2 & 0 \\ 4 & 0 & 4 \\ 0 & 10 & 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 0 & -3 & 0 \\ 6 & 0 & 6 \\ 0 & 15 & 0 \end{pmatrix}$$

$$x^T \Gamma \underbrace{(A_1 - A)}_{\psi} y = (x, \psi(y))$$

$$A_1 - A = \frac{1}{\ldots}\begin{pmatrix} 0 & -5 & 0 \\ \ldots \end{pmatrix}$$

$$A^T A = 2\begin{pmatrix} 6 & 0 & 2 \\ 0 & 15 & 0 \end{pmatrix}$$

$$\Gamma = \begin{pmatrix} & & 0 & \\ & & 1 & \\ 0 & 1 & 0 & 1 \\ & & 1 & a \end{pmatrix}; \quad X = \begin{pmatrix} 0 \\ 0 \\ b \\ c \end{pmatrix}$$

$$(0\ 0\ b\ c)\begin{pmatrix} & & 0 & \\ & & 1 & \\ 0 & 1 & 0 & 1 \\ e & d & 1 & a \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ b \\ c \end{pmatrix}$$

$$= (ce \quad b+cd$$

$$(b\ c)\begin{pmatrix} 0 & 1 \\ 1 & a \end{pmatrix}\begin{pmatrix} b \\ c \end{pmatrix} = (c \quad b+ca)\begin{pmatrix} b \\ c \end{pmatrix} =$$

$$= bc + cb + c^2 a < 0$$

$$2b + ac < 0$$

$$b < -\frac{ac}{2}$$

№29.14

Проверить на вещественные корни

№29.17

$(\Leftarrow)$ $(\varphi(\psi(x)), y) = (\psi(x), \varphi(y)) = (x, \psi(\varphi(y)))$

$= (x, \varphi(\psi(y))) \Rightarrow$ ЧТД.

№ 29.19

№ 29.40

1) $S^T S = E$ $\quad S_{\pm} = \begin{pmatrix} \cos\varphi & \mp\sin\varphi \\ \pm\sin\varphi & \cos\varphi \end{pmatrix}$

$|\det S| = 1$

$(\varphi(x), \varphi(y)) = x^T A^T A y = x^T y$

$S_+ + S_- = \mathrm{diag}(2\cos\varphi) \Rightarrow \det = 4\cos^2\varphi$

2) Нет

№ 29.41

Сохраняет, т.к. $\varphi(x) = y$

Зеркало

№ 29.42

Через Γ' и Γ

№ 29.45

$A^T \Gamma A = \Gamma$ – сохранение скалярного

$\det^2 A = 1$

произведение

№29.47

$\varphi : A \to B$

$A = \begin{pmatrix} 4 & 2 \\ 7 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 8 & 2 \\ 1 & -1 \end{pmatrix}$

$\forall x = \alpha \begin{pmatrix} 4 \\ 7 \end{pmatrix} + \beta \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$\forall y = \gamma \begin{pmatrix} 4 \\ 7 \end{pmatrix} + \delta \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$(\varphi(x), \varphi(y)) = (\alpha \begin{pmatrix} 8 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} 2 \\ -1 \end{pmatrix},$