N119

$$\begin{cases} \dot{x} = -3x + y \\ \dot{y} = x - 3y \end{cases} \qquad A = \begin{pmatrix} -3 & 1 \\ 1 & -3 \end{pmatrix}$$

$$\begin{vmatrix} -3-\lambda & 1 \\ 1 & -3-\lambda \end{vmatrix} = (\lambda+3)^2 - 1 = \lambda^2 + 6\lambda + 8 = 0$$

$$D = 36 - 32 = 4$$

$$\lambda_1 = -2, \quad \lambda_2 = -4$$

$\lambda = -2$

$$\begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix} \Rightarrow \overline{h_1} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$\lambda = -4$

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \Rightarrow \overline{h_2} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = e^{At} \overline{C}$$

$$e^{At} = S e^{\Lambda t} S^{-1}$$

$$\Lambda = \begin{pmatrix} -2 & 0 \\ 0 & -4 \end{pmatrix} \Rightarrow e^{\Lambda t} = \begin{pmatrix} e^{-2t} & 0 \\ 0 & e^{-4t} \end{pmatrix}$$

$$e^{At} = -\frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} e^{-2t} & 0 \\ 0 & e^{-4t} \end{pmatrix}\begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix} =$$

$$= -\frac{1}{2}\begin{pmatrix} e^{-2t} & e^{-4t} \\ e^{-2t} & -e^{-4t} \end{pmatrix}\begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix} =$$

$$= -\frac{1}{2}\begin{pmatrix} -e^{-2t}-e^{-4t} & -e^{-2t}+e^{-4t} \\ -e^{-2t}+e^{-4t} & -e^{-2t}-e^{-4t} \end{pmatrix} =$$

$$= \frac{1}{2}\begin{pmatrix} e^{-4t}+e^{-2t} & e^{-2t}-e^{-4t} \\ e^{-2t}-e^{-4t} & e^{2t}+e^{4t} \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{2}\begin{pmatrix} e^{-4t}+e^{-2t} & e^{-2t}-e^{-4t} \\ e^{-2t}-e^{-4t} & e^{2t}+e^{4t} \end{pmatrix}\begin{pmatrix} C_1 \\ C_2 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} C_1 \\ C_2 \end{pmatrix} \Rightarrow$$

$$\Rightarrow C_1 = C_2 = 1$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 2e^{-2t} \\ 2e^{-2t} \end{pmatrix} = \begin{pmatrix} e^{-2t} \\ e^{-2t} \end{pmatrix}$$

N123

$$\begin{cases} \dot{x} = 3x - y \\ \dot{y} = x + y \end{cases} \qquad A = \begin{pmatrix} 3 & -1 \\ 1 & 1 \end{pmatrix}$$

$$\begin{vmatrix} 3-\lambda & -1 \\ 1 & 1-\lambda \end{vmatrix} = (\lambda - 1)(\lambda - 3) + 1 =$$

$$= \lambda^2 - 4\lambda + 4 = (\lambda - 2)^2 = 0$$

$$\lambda_1 = \lambda_2 = 2$$

$$\begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix} \Rightarrow \bar{h}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix} \bar{h}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \Rightarrow \bar{h}_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$e^{tA} = -\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} e^{2t} \begin{pmatrix} 1 & t \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ -1 & 1 \end{pmatrix} =$$

$$= e^{2t} \begin{pmatrix} 1 & t+1 \\ 1 & t \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix} = e^{2t} \begin{pmatrix} t+1 & -t \\ t & 1-t \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = e^{2t} \begin{pmatrix} t+1 & -t \\ t & 1-t \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} \Rightarrow C_1 = C_2 = \underline{1}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = e^{2t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

N128

$$\begin{cases} \dot{x} = 2x - 3y \\ \dot{y} = 3x + 2y \end{cases} \qquad A = \begin{pmatrix} 2 & -3 \\ 3 & 2 \end{pmatrix}$$

$$E = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \begin{matrix} S^{2n} = (-1)^n E \\ S^{2n+1} = (-1)^n S \end{matrix}$$

$$e^{tA} = e^{2tE - 3tS} = e^{2t} e^{-3tS}$$

$$e^{-3tS} = E + \frac{(-3tS)}{1} + \frac{(-3tS)^2}{2!} + \frac{(-3tS)^3}{3!} + \dots \pm$$

$$= E\left(1 - \frac{(-3t)^2}{2!} + \frac{(-3t)^4}{4!} + \dots\right) +$$

$$- S\left(3t - \frac{(3t)^3}{3!} + \dots\right) =$$

$$= e^{2t}(\cos 3t\, E - \sin 3t\, S) =$$

$$= e^{2t}\begin{pmatrix} \cos 3t & -\sin 3t \\ \sin 3t & \cos 3t \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = e^{2t}\begin{pmatrix} \cos 3t & -\sin 3t \\ \sin 3t & \cos 3t \end{pmatrix}\begin{pmatrix} C_1 \\ C_2 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} C_1 \\ C_2 \end{pmatrix} \Rightarrow C_1 = C_2 = 1$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = e^{2t}\begin{pmatrix} \cos 3t - \sin 3t \\ \cos 3t + \sin 3t \end{pmatrix}$$

N6.

a) $\bar{X} = C_4 e^{3t}\bar{h}_4 + C_5 e^{3t}\bar{h}_5 + C_1 e^{2t}\bar{h}_1 +$

$+ C_2 e^{2t}(t\bar{h}_1 + \bar{h}_2) + C_3 e^{3t}\left(\frac{t^2}{2}\bar{h}_1 + t\bar{h}_2 + \bar{h}_3\right)$

б) $e^{A'}$ - ?

$$A' = \left(\begin{array}{c|c} J_1 & 0 \\ \hline 0 & J_2 \end{array}\right), \quad J_1 = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}, \quad J_2 = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$$

$$e^{A'} = \left(\begin{array}{c|c} e^{J_1} & 0 \\ \hline 0 & e^{J_2} \end{array}\right); \quad e^{J_1} = e^{2t}\begin{pmatrix} 1 & t & \frac{t^2}{2} \\ 0 & 1 & t \\ 0 & 0 & 1 \end{pmatrix}$$

$$e^{J_2} = e^{3t}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$e^{A'} = \begin{pmatrix} e^{2t} & te^{2t} & \frac{t^2}{2}e^{2t} & 0 & 0 \\ 0 & e^{2t} & te^{2t} & 0 & 0 \\ 0 & 0 & e^{2t} & 0 & 0 \\ 0 & 0 & 0 & e^{3t} & 0 \\ 0 & 0 & 0 & 0 & e^{3t} \end{pmatrix}$$