ℓ) $b_n \to b \neq 0 \Rightarrow \exists N: \forall n \geq N \ |b_n| > \frac{b}{2}$

$\varepsilon = \frac{b}{2} \ \exists N(\varepsilon): \forall n \geq N \mapsto b - \frac{b}{2} < b < b + \frac{b}{2}$

2) Th. арифм. операций

1) Дано: $\lim\limits_{n\to\infty} a_n = a$ $\quad \lim\limits_{n\to\infty} b_n = b$

Док-ть: $\lim(a_n + b_n) = a + b$

Док-во:

$\forall \varepsilon > 0 \ \exists N_1(\varepsilon): \forall n \geq N_1 \mapsto |a_n - a| < \frac{\varepsilon}{2}$

$\forall \varepsilon > 0 \ \exists N_2(\varepsilon): \forall n \geq N_2 \mapsto |b_n - b| < \frac{\varepsilon}{2}$

$|(a_n + b_n) - (a + b)| \leq |a_n - a| + |b_n - b| < \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon$

$\forall \varepsilon > 0 \ \exists N_3(\varepsilon) = \max\{N_1, N_2\}: \forall n \geq N_3 \mapsto |(a_n + b_n) - (a + b)| < \varepsilon$

2) Док-ть: $\lim\limits_{n\to\infty}(a_n b_n) = ab$ (a и b известны)

Док-во:

$\forall \varepsilon > 0 \ \exists N_1(\varepsilon): \forall n \geq N_1 \hookrightarrow |a_n - a| < \varepsilon$

$\forall \varepsilon > 0 \ \exists N_2(\varepsilon): \forall n \geq N_2 \hookrightarrow |b_n - b| < \varepsilon$

a_n - огр. $\Rightarrow \exists M > 0: \forall n \in \mathbb{N} \hookrightarrow |a_n| \leq M$

$|a_n b_n - ab| = |a_n b_n + a_n b - a_n b - ab|$

$= |a_n(b_n - b) + b(a_n - a)| \leq |a_n||b_n - b| + |b||a_n - a| =$

$< M\varepsilon + |b|\varepsilon = \varepsilon(M + |b|)$

$\forall \varepsilon > 0 \ \exists N_3 = \max\{N_1, N_2\}: \forall n \geq N_3 \hookrightarrow |a_n b_n - ab| < \varepsilon(M + |b|)$

$\Rightarrow \lim a_n b_n = ab$