N 1 (4)

$$\left(3+\frac{1}{2}\right)+\left(\frac{3}{2}-\frac{1}{6}\right)+\ldots+\left(\frac{3}{2^{n-1}}+\frac{(-1)^{n-1}}{2\cdot 3^{n-1}}\right)$$

$$\sum_{k=1}^{n}\left(\frac{3}{2^{n-1}}+\frac{(-1)^{n-1}}{2\cdot 3^{n-1}}\right)=S_n$$

$$S_n=\underbrace{\sum_{k=1}^{n}\frac{3}{2^{n-1}}}_{S_1}+\underbrace{\sum_{k=1}^{n}\frac{1}{2}\cdot\left(-\frac{1}{3}\right)^{n-1}}_{S_2}$$

$$S_1=-2\cdot\left(\frac{1}{2}\cdot\frac{3}{2^{n-1}}-3\right)=-6\left(\frac{1}{2^n}-1\right)=-6(\;)\to 6$$

$$S_2=-\frac{3}{4}\left(\left(-\frac{1}{3}\right)\cdot\frac{1}{2}\cdot\left(-\frac{1}{3}\right)^{n-1}-\frac{1}{2}\right)\to$$

$$\to\frac{3}{8}$$

$$\boxed{S_n\to\frac{51}{8}}$$

$$S_1=6-\frac{6}{2^n}=6-\frac{3}{2^{n-1}}$$

$$S_2=\frac{3}{8}-\frac{3}{8}\left(-\frac{1}{3}\right)^n=\frac{3}{8}+\frac{1}{8}\left(-\frac{1}{3}\right)^{n-1}$$

$$S_n = \frac{51}{8} - \frac{3}{2^{n-1}} + \frac{1}{8}\left(-\frac{1}{3}\right)^{n-1}$$

N 3(1)

$$\sum_{n=1}^{\infty} \frac{1}{16n^2-8n-3} = \sum_{n=1}^{\infty} \frac{1}{(4n-3)(4n+1)}$$

$$D = 64 + 12\cdot 16 = 64 + 192 = 256 \Rightarrow n_1 = \frac{8+16}{32} = \frac{3}{4};\ n_2 = -\frac{1}{4}$$

$$\frac{1}{(4n-3)(4n+1)} = \frac{A}{4n-3} + \frac{B}{4n+1}$$

$$4An + A + 4Bn - 3B = 1$$

$$\begin{cases} A + B = 0 \\ A - 3B = 1 \end{cases} \qquad 4B = -1 \Rightarrow B = -\frac{1}{4}$$

$$A = \frac{1}{4}$$

$$\sum_{n=1}^{\infty} \frac{1}{4(4n-3)} - \sum_{n=1}^{\infty} \frac{1}{4(4n+1)}$$

$$\frac{1}{4} - \frac{1}{4\cdot 5} + \frac{1}{4\cdot 5} - \frac{1}{4\cdot 9} + \frac{1}{4\cdot 9} - \frac{1}{4\cdot 13} =$$

$$= \boxed{\frac{1}{4} - \frac{1}{4(4n+1)} = S_n \longrightarrow \frac{1}{4}}$$

N11(6)

$$\sum_{n=1}^{\infty}\left(\frac{3n^3-2}{3n^3+4}\right)^{n^3}$$

$$a_n=\left(\frac{3n^3-2}{3n^3+4}\right)^{n^3}=\frac{\left(1-\frac{2}{3n^3}\right)^{n^3}}{\left(1+\frac{4}{3n^3}\right)^{n^3}}\to\frac{e^{-\frac{2}{3}}}{e^{\frac{4}{3}}}=$$

$$=\frac{1}{e^2}\not\to 0$$

N13(2)

$$a_n=\frac{\sin n\alpha}{n(n+1)}$$

$$\left|\sum_{k=n+1}^{n+p}\frac{\sin k\alpha}{k(k+1)}\right|\le\left|\sum_{k=n+1}^{n+p}\frac{1}{k(k+1)}\right|\le\left|\sum_{k=n+1}^{n+p}\frac{1}{k^2}\right|\le$$

$$\le\frac{1}{N^2}\sum_{k=n+1}^{n+p}1=\frac{p}{N^2}$$

$$\le\sum_{k=n+1}^{n+p}\cancel{\frac{1}{(n+p)^2}}$$

N14 (3)

$$a_n = \frac{1}{\sqrt{n(n+1)}}$$

$$\exists \varepsilon = \frac{1}{3} : \forall N > 0 \ \begin{cases} \exists n = N \\ \exists p = N \end{cases} : \left| \sum_{k=n+1}^{n+p} a_k \right| > \frac{1}{3}$$

$$\left| \sum_{k=N+1}^{2N} \frac{1}{\sqrt{k(k+1)}} \right| \geqslant \sum_{k=N+1}^{2N} \frac{1}{\sqrt{2N(2N+1)}} > \sum_{k=N+1}^{2N} \frac{1}{\sqrt{(2N+1)^2}} =$$

$$= \sum_{k=N+1}^{2N} \frac{1}{2N+1} = \frac{1}{2N+1} \cdot N = \frac{1}{2+\frac{1}{N}} \geqslant \frac{1}{3}$$