§21, N6(4)

$$\frac{3x+4}{x^2+x-6} = \frac{3x+4}{(x-2)(x+3)} = \frac{A}{x-2} + \frac{B}{x+3} = \frac{Ax+3A+Bx-2B}{x^2+x-6}$$

$$\begin{cases} A+B=3 \\ 3A-2B=4 \end{cases} \qquad \begin{aligned} &5A=10 \\ &A=2 \\ &B=1 \end{aligned}$$

$$= -\left(1-\frac{x}{2}\right)^{-1} + \frac{1}{3}\left(1+\frac{x}{3}\right)^{-1} =$$

$$= -\sum_{k=0}^{\infty} \frac{x^k}{2^k} + \frac{1}{3}\sum_{k=0}^{\infty} \frac{(-1)^k x^k}{3^k} =$$

$$= \sum_{k=0}^{\infty} x^k \left(\frac{(-1)^k}{3^{k+1}} - \frac{1}{2^k}\right); \quad R=2$$

N9(3)

$$\ln\left(\frac{2+x^2}{\sqrt{1-2x^2}}\right) = \ln(2+x^2) - \frac{1}{2}\ln(1-2x^2) =$$

$$= \ln 2 + \ln\left(1+\frac{x^2}{2}\right) - \frac{1}{2}\ln(1-2x^2) =$$

$$= \ln 2 + \sum_{k=1}^{\infty} \frac{(-1)^{k-1} x^{2k}}{k \cdot 2^k} + \frac{1}{2}\sum_{k=1}^{\infty} \frac{2^k x^{2k}}{k} =$$

$$= \ln 2 + \sum_{k=1}^{\infty} \frac{x^{2k}}{k} \cdot \left(\frac{(-1)^{k-1}}{2^k} + 2^{k-1}\right)$$

$$R = \frac{1}{\sqrt{2}}$$

N11(4)

$$x \sin 2x \cos 3x = x \frac{1}{2} \left(\sin 5x - \sin x \right) =$$

$$= \frac{x}{2} \left(\sum_{k=0}^{\infty} \frac{(-1)^k \, 5^{2k+1} x^{2k+1}}{(2k+1)!} - \sum_{k=0}^{\infty} \frac{(-1)^k \, x^{2k+1}}{(2k+1)!} \right) =$$

$$= \frac{1}{2} \sum_{k=0}^{\infty} \frac{(-1)^k \, x^{2k+2}}{(2k+1)!} \left(5^{2k+1} - 1 \right) \quad ; \; R = \infty$$

N19(3)

$$\frac{1}{\sqrt{x^2-6x+18}}, \quad x_0 = 3 \; ; \; t = x-3 \; ; \; x = t+3$$

$$\frac{1}{\sqrt{t^2+6t+9-6t-\cancel{18}+\cancel{18}}} = \frac{1}{\sqrt{t^2+9}} = \frac{1}{3} \left(1 + \frac{t^2}{9} \right)^{-\frac{1}{2}} =$$

$$= \frac{1}{3} \sum_{k=0}^{\infty} C_{-\frac{1}{2}}^{k} \frac{t^{2k}}{9^k}, \quad \text{где } t = x-3 \; ; \; R = 3$$

N 30(4)

$$x \arccos\frac{x}{\sqrt{x^2+9}} = f(x)$$

$$\left(\arccos\frac{x}{\sqrt{x^2+9}}\right)' = \frac{-1}{\sqrt{1-\frac{x^2}{x^2+9}}} \cdot \frac{\sqrt{x^2+9} - x\cdot\frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+9}}}{(x^2+9)} =$$

$$= \frac{-\cancel{\sqrt{x^2+9}}}{3} \cdot \frac{(x^2+9)-x^2}{(x^2+9)\cancel{\sqrt{x^2+9}}} = \frac{-3}{(x^2+9)} = \frac{-1}{3}\left(1+\frac{x^2}{9}\right)^{-1} =$$

$$= -\sum_{k=0}^{\infty}\frac{(-1)^k x^{2k}}{9^k\cdot 3}$$

$$f(x) = -\sum_{k=0}^{\infty}\frac{(-1)^k x^{2k+2}}{3^{2k+1}\cdot(2k+1)} + \frac{\pi}{2}x \quad ; R=3$$

N56(2)

$$\sum_{n=0}^{\infty}\frac{(2n+1)x^{2n}}{n!}$$

$$\int \sum_{n=0}^{\infty} \frac{(2n+1)}{n!} x^{2n} dx = \sum_{n=0}^{\infty} \int \frac{(2n+1)x^{2n}}{n!} dx = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{n!} = xe^{x^2}$$

$$(xe^{x^2})' = e^{x^2} + x \cdot e^{x^2} \cdot 2x = \underline{e^{x^2}(1+2x^2)}$$

N31(2)

$$\int_0^x \frac{\sin t}{t} dt = f(x)$$

$$(\ )' = \frac{\sin x}{x} = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k+1)!}$$

$$f(x) = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!(2k+1)}, \quad R = \infty$$

N80

$$f(x) = \begin{cases} e^{-\frac{1}{x^2}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

$$f'(x) = \begin{cases} \frac{2}{x^3} e^{-\frac{1}{x^2}}, & x \neq 0 \end{cases}$$

$$\begin{cases} 0, & x = 0 \end{cases}$$

$$f''(x) = \begin{cases} \left(-\frac{6}{x^4} e^{-\frac{1}{x^2}} + \frac{4}{x^6} e^{-\frac{1}{x^2}}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

$$f^{n}(x) = \begin{cases} P_{3n}\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

Таким образом $f(x) = 0$ только при $x = 0$, при $x \neq 0$ $f(x) \neq 0$, но её ряд Тейлора $\equiv 0$ $\Rightarrow$ не для $\forall x \in U_\delta(x_0^{=0})$ представима в ряд.

T7

$$\sum_{n=0}^{\infty} \frac{x^{2^n}}{n^3}; \quad R = \frac{1}{\lim\limits_{n\to\infty} \sqrt[2^n]{\frac{1}{n^3}}} = \frac{1}{\lim\limits_{n\to\infty} \frac{1}{n^{\frac{3}{2^n}}}} = \underline{1}$$