№ 1

$$f(x) = (x^2-\pi^2)\sin^2 x$$

Найти порядок убывания коэф. РФ

- $f^{(0)} = (x^2-\pi^2)\sin^2 x$

$f^{(0)}(-\pi) = f^{(0)}(\pi) = 0$

- $f'(x) = 2x\sin^2 x + 2(x^2-\pi^2)\sin x\cos x$

$f'(-\pi) = f'(\pi) = 0$

- $f''(x) = 2\sin^2 x + 4x\sin x\cos x + 4x\sin x\cos x +$

$+ 2(x^2-\pi^2)\cos^2 x - 2(x^2-\pi^2)\sin^2 x$

$f''(-\pi) = 0 = f(\pi)$

- $f'''(x) = 8\sin x\cos x + 12x\cos^2 x - 12x\sin^2 x + 4\sin x\cos x +$

$- 8(x^2-\pi^2)\sin x\cos x$

$f'''(-\pi) \neq f'''(\pi)$

$$\boxed{|a_k| + |b_k| = O\left(\frac{1}{k^3}\right)}$$

№2

$$x^2=\frac{\pi^2}{3}+\sum_{k=1}^{\infty}\frac{4}{k^2}(-1)^k\cos kx$$

Воспользуемся равенством парсеваля:

$$\frac{1}{\pi}\int_{-\pi}^{\pi}x^4dx=\frac{a_0^2}{2}+\sum_{n=1}^{\infty}a_n^2+b_n^2$$

$$a_0=\frac{2\pi^2}{3}; \quad a_n=\frac{4(-1)^n}{n^2}$$

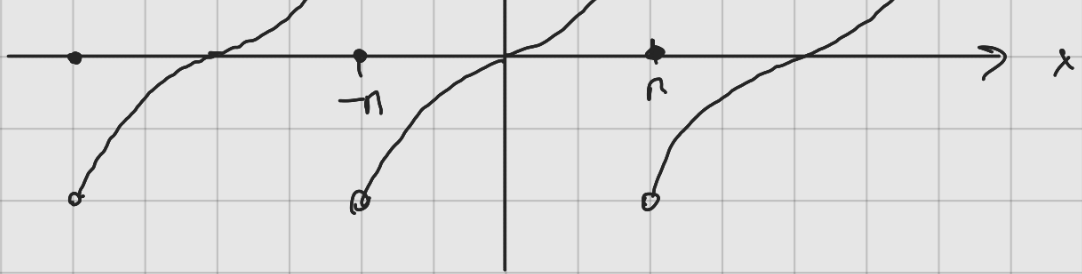
$$\frac{1}{\pi}\frac{2\pi^5}{5}=\frac{2\pi^4}{5}=\frac{2\pi^4}{9}+\sum_{k=1}^{\infty}\frac{16}{k^4}$$

$$\Rightarrow \sum_{k=1}^{\infty}\frac{1}{k^4}=\frac{\pi^4}{16}\left(\frac{18-10}{45}\right)=\frac{\pi^4}{2\cdot 45}=\frac{\pi^4}{90}$$

№3

$$\operatorname{sh}(\pi)=\frac{e^{\pi}-e^{-\pi}}{2}=\frac{e^{2\pi}-1}{2e^{\pi}}=a$$

$$f(x)=\operatorname{sh}x, \quad x\in(-\pi,\pi)$$



$$b_k = \frac{2}{\pi}\int_0^\pi \underbrace{sh x}_{dv}\, \underbrace{\sin kx}_{u}\, dx = J$$

$$= \frac{2}{\pi}\left(ch x\, \overset{0}{\sin kx}\Big|_0^\pi - k\int_0^\pi \underbrace{ch x}_{dv}\, \underbrace{\cos kx}_{u}\, dx \right) =$$

$$= \frac{2}{\pi}\left(-k\, sh x \cos kx\Big|_0^\pi - k^2\int_0^\pi sh x \sin kx\, dx \right)$$

$$= -\frac{2}{\pi} k\, sh\pi (-1)^k - k^2 J$$

$$J(1+k^2) = -\frac{2}{\pi} k\, sh\pi (-1)^k$$

$$J = \frac{-\frac{2}{\pi} k\, sh\pi (-1)^k}{1+k^2 \frac{\pi}{2}}$$

$$f(x) \sim \sum_{k=1}^{\infty} \frac{-\frac{2}{\pi} k\, sh\pi\, (-1)^k}{1+k^2} \sin kx$$

3) $f(x)$ – 2π-период., абс. инт-ма, все точки

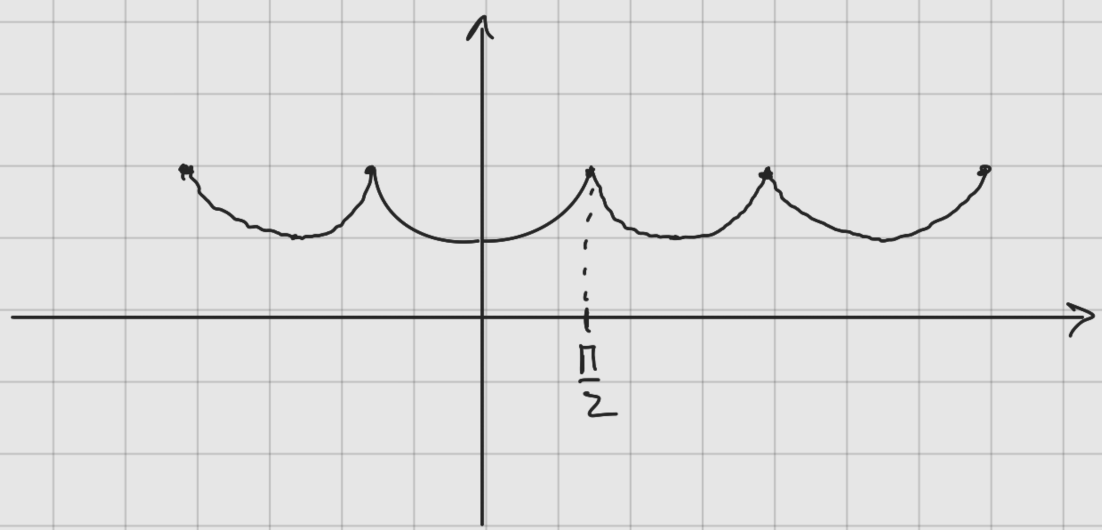
регулярное => РФ сх. поточечно к $f(x)$ =>

$$\Rightarrow f(x) = \sum_{k=1}^{\infty} \frac{-\frac{2}{\pi} \cdot k \cdot \operatorname{sh}\pi \cdot (-1)^k}{1+k^2} \sin kx$$

4) Сумма ряда Фурье, который состоит из непр. ф-ий и сх-ся равномерно – непр. ф-ия. Но у нашего РФ сумма разрывна => РФ $\not\rightrightarrows$

№4

$f(x) = x^2+1$; $x \in (0; \frac{\pi}{2}) \xrightarrow{\text{РФ}}$ по $\cos 2kx$



$$\bullet\ a_0 = \frac{4}{\pi}\int_0^{\frac{\pi}{2}} x^2+1\ dx = \frac{4}{\pi}\left(\frac{x^3}{3}+x\right)\Bigg|_0^{\frac{\pi}{2}} =$$

$$= \frac{4}{\pi}\left(\frac{\pi^3}{24}+\frac{\pi}{2}\right) = \frac{\pi^2}{6} + 2$$

$$\bullet\ a_{2k} = \frac{4}{\pi}\int_0^{\frac{\pi}{2}} (x^2+1)\cos 2kx\, dx =$$

$$= \frac{4}{\pi}\left(\cancelto{0}{\frac{1}{2k}(x^2+1)\sin 2kx\Big|_0^{\frac{\pi}{2}}} - \frac{1}{k}\int_0^{\frac{\pi}{2}} x\sin 2kx\, dx\right)$$

$$= \frac{4}{\pi}\left(\frac{1}{2k^2}x\cos 2kx\Big|_0^{\frac{\pi}{2}} - \cancelto{0}{\frac{1}{2k^2}\int_0^{\frac{\pi}{2}}\cos 2kx\, dx}\right) =$$

$$= \frac{\cancel{4}}{\cancel{\pi}k^2}\frac{\cancel{\pi}}{\cancel{4}}\cdot(-1)^k = \frac{(-1)^k}{k^2}$$

$$f(x) = \frac{\pi^2}{12} + 1 + \sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}\cos 2kx$$