§16 N 1(4)

$$\Gamma\left(\tfrac{1}{2}\right) \overset{?}{=} \sqrt{\pi}$$

$$\Gamma\left(\tfrac{1}{2}\right) = \int_0^{+\infty} x^{-\frac{1}{2}} e^{-x}\,dx = \int_0^{+\infty} \frac{e^{-x}}{x^{\frac{1}{2}}}\,dx \ominus$$

$$t = x^{\frac{1}{2}} \Rightarrow x = t^2 \Rightarrow dx = 2t\,dt$$

$$\ominus\ 2\int_0^{+\infty} \frac{e^{-t^2}\,t\,dt}{t} = 2\int_0^{+\infty} e^{-t^2}\,dt = \sqrt{\pi}$$

N 7(3)

$$\int_{-2}^{2} \frac{dx}{\sqrt[4]{(2+x)^3(2-x)}} \ominus$$

$$4t = 2 - x \Rightarrow x = 2 - 4t \Rightarrow dx = -4\,dt$$

$$\Downarrow$$

$$2 + x = 4(1-t)$$

$$\ominus \int_0^1 \frac{4\,dt}{4^{\frac{3}{4}}(1-t)^{\frac{3}{4}}\,4^{\frac{1}{4}}\,t^{\frac{1}{4}}} = \int_0^1 t^{-\frac{1}{4}}(1-t)^{-\frac{3}{4}}\,dt =$$

$$= B\left(\tfrac{3}{4}, \tfrac{1}{4}\right) = \frac{\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{1}{4}\right)}{\Gamma(1)} = \Gamma\left(\tfrac{3}{4}\right)\Gamma\left(\tfrac{1}{4}\right) =$$

$$= \frac{\pi}{\ldots} = \sqrt{2}\,\pi$$

$\sin \pi \cdot \frac{3}{4}$

N14(5)

$$\int_0^{\pi/2} \sin^{\alpha} x \cos^{\beta} x \, dx = \left\{ \begin{array}{l} \sin x = \sqrt{t}, \ \cos x = \sqrt{1-t}; \ x = \arcsin\sqrt{t} \\ dx = \frac{1}{\sqrt{1-t}} \cdot \frac{1}{2\sqrt{t}} \, dt \end{array} \right\}$$

$$= \frac{1}{2} \int_0^1 \frac{t^{\frac{\alpha}{2}}}{t^{\frac{1}{2}}} \frac{(1-t)^{\frac{\beta}{2}}}{(1-t)^{\frac{1}{2}}} \, dt = \frac{1}{2} \int_0^1 t^{\frac{(\alpha-1)}{2}} (1-t)^{\frac{(\beta-1)}{2}} \, dt =$$

$$= \frac{1}{2} B\left(\frac{\alpha+1}{2}, \frac{\beta+1}{2}\right)$$

N13(6)

$$I = \int_0^{+\infty} \frac{x^{\alpha}}{(a+bx^{\beta})^{p}} \, dx \ ; \ a > 0, \ b > 0, \ \beta > 0, \ 0 < \frac{\alpha+1}{\beta} < p$$

$$t = \frac{x}{a+bx}$$

$$\frac{a^{-p}}{\beta} \left(\frac{a}{b}\right)^{\frac{\alpha+1}{\beta}} \int_0^1 t^{\frac{\alpha+1-\beta}{\beta}} (1-t)^{p-1-\frac{\alpha+1}{\beta}}$$

$\alpha \qquad \alpha+1-\beta$

$$X = t \quad \frac{\quad}{1^3}$$