

SIG N1(4)

$$\Gamma\left(\frac{1}{2}\right) = ? \sqrt{\pi}$$

$$\Gamma\left(\frac{1}{2}\right) = \int_0^{+\infty} x^{-\frac{1}{2}} e^{-x} dx = \int_0^{+\infty} \frac{e^{-x}}{x^{\frac{1}{2}}} dx \quad \textcircled{=}$$

$$t = x^{\frac{1}{2}} \Rightarrow x = t^2 \Rightarrow dx = 2t dt$$

$$t = \frac{1}{\sqrt{x}} \Rightarrow x = \frac{1}{t^2} \Rightarrow dx = -\frac{2}{t^3} dt$$

$$\textcircled{=} \int_0^{+\infty} \frac{e^{-\frac{1}{t^2}}}{\frac{1}{t^2}} \cdot \left(-\frac{2}{t^3}\right) dt = -2 \int_0^{+\infty} \frac{e^{-\frac{1}{t^2}}}{t} dt$$

$$u = \frac{1}{t} \Rightarrow t = \frac{1}{u}$$

$$\textcircled{=} 2 \int_0^{+\infty} \frac{e^{-t^2} t dt}{t} = 2 \int_0^{+\infty} e^{-t^2} dt = \sqrt{\pi}$$

N7(3)

$$\int_{-2}^2 \frac{dx}{\sqrt[4]{(2+x)^3(2-x)}} \quad \textcircled{=}$$

$$\begin{aligned} \text{1. } \int \frac{1}{\sqrt[4]{(2+x)^3(2-x)}} dx &= \int \frac{1}{\sqrt[4]{(2+x)^3(2-x)}} dx \\ \text{2. } \int \frac{1}{\sqrt[4]{(2+x)^3(2-x)}} dx &= \int \frac{1}{\sqrt[4]{(2+x)^3(2-x)}} dx \\ \text{3. } \int \frac{1}{\sqrt[4]{(2+x)^3(2-x)}} dx &= \int \frac{1}{\sqrt[4]{(2+x)^3(2-x)}} dx \\ \text{4. } \int \frac{1}{\sqrt[4]{(2+x)^3(2-x)}} dx &= \int \frac{1}{\sqrt[4]{(2+x)^3(2-x)}} dx \\ \text{5. } \int \frac{1}{\sqrt[4]{(2+x)^3(2-x)}} dx &= \int \frac{1}{\sqrt[4]{(2+x)^3(2-x)}} dx \\ \text{6. } \int \frac{1}{\sqrt[4]{(2+x)^3(2-x)}} dx &= \int \frac{1}{\sqrt[4]{(2+x)^3(2-x)}} dx \\ \text{7. } \int \frac{1}{\sqrt[4]{(2+x)^3(2-x)}} dx &= \int \frac{1}{\sqrt[4]{(2+x)^3(2-x)}} dx \\ \text{8. } \int \frac{1}{\sqrt[4]{(2+x)^3(2-x)}} dx &= \int \frac{1}{\sqrt[4]{(2+x)^3(2-x)}} dx \\ \text{9. } \int \frac{1}{\sqrt[4]{(2+x)^3(2-x)}} dx &= \int \frac{1}{\sqrt[4]{(2+x)^3(2-x)}} dx \\ \text{10. } \int \frac{1}{\sqrt[4]{(2+x)^3(2-x)}} dx &= \int \frac{1}{\sqrt[4]{(2+x)^3(2-x)}} dx \end{aligned}$$

$$4t = 2-x \Rightarrow x = 2-4t \Rightarrow dx = -4 dt$$

$$\Downarrow$$

$$2+x = 4(1-t)$$

$$\textcircled{=} \int_0^1 \frac{4 dt}{4^{\frac{3}{4}} (1-t)^{\frac{3}{4}} 4^{\frac{1}{4}} t^{\frac{1}{4}}} = \int_0^1 t^{-\frac{1}{4}} (1-t)^{-\frac{3}{4}} dt =$$

$$= B\left(\frac{3}{4}, \frac{1}{4}\right) = \frac{\Gamma\left(\frac{3}{4}\right) \Gamma\left(\frac{1}{4}\right)}{\Gamma(1)} = \Gamma\left(\frac{3}{4}\right) \Gamma\left(\frac{1}{4}\right) =$$

$$= \frac{\pi}{\sqrt{2}} = \sqrt{2}\pi$$

$$\sin \pi \cdot \frac{3}{4}$$

N14(5)

$$\int_0^{\pi/2} \sin^{\alpha} x \cos^{\beta} x \, dx = \int \left. \begin{array}{l} \sin x = \sqrt{t}, \cos x = \sqrt{1-t}; x = \arcsin \sqrt{t} \\ dx = \frac{1}{\sqrt{1-t}} \cdot \frac{1}{2\sqrt{t}} dt \end{array} \right\}$$

$$= \frac{1}{2} \int_0^1 \frac{t^{\frac{\alpha}{2}}}{t^{\frac{1}{2}}} \frac{(1-t)^{\frac{\beta}{2}}}{(1-t)^{\frac{1}{2}}} dt = \frac{1}{2} \int_0^1 t^{\frac{(\alpha-1)}{2}} (1-t)^{\frac{(\beta-1)}{2}} dt =$$

$$= \frac{1}{2} B\left(\frac{\alpha+1}{2}, \frac{\beta+1}{2}\right)$$

N13(6)

$$I = \int_0^{+\infty} \frac{x^{\alpha}}{(a+\beta x^{\beta})^p} dx; \quad a > 0, \beta > 0, \beta > 0, 0 < \frac{\alpha+1}{\beta} < p$$

$$t = \frac{x}{a+\beta x}$$

$$\frac{a^{-p}}{\beta} \left(\frac{a}{\beta}\right)^{\frac{\alpha+1}{\beta}} \int_0^1 t^{\frac{\alpha+1-\beta}{\beta}} (1-t)^{p-1-\frac{\alpha+1}{\beta}}$$

$$\alpha \quad \alpha+1-\beta$$

$$X = t \sqrt{\beta}$$