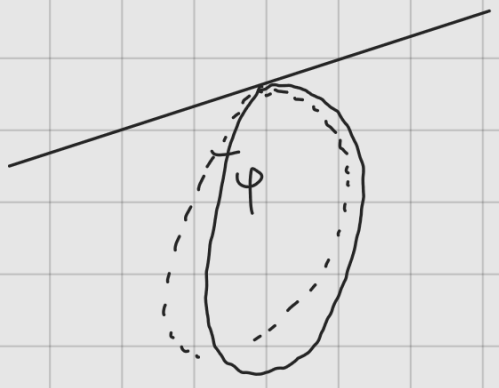
№69

Найти: $\frac{T_1}{T_2}$

Решение:

1)



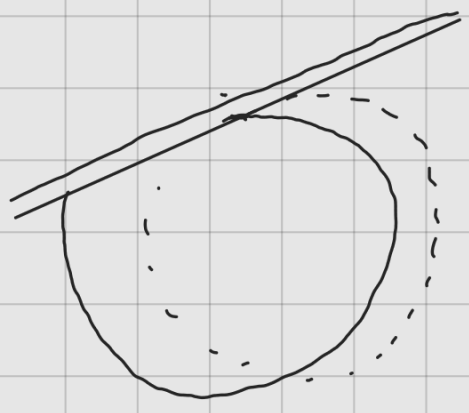
$$\Delta U = mgR(1-\cos\varphi) = mgR\frac{\varphi^2}{2} = \frac{mgR\varphi^2}{2}$$

$$\Delta K = \frac{\left(m\frac{R^2}{2} + mR^2\right)\cdot\dot{\varphi}^2}{2} = \frac{\frac{3mR^2}{2}}{2}\dot{\varphi}^2$$

$$\omega_1 = \sqrt{\frac{mgR}{\frac{3mR^2}{2}}} = \sqrt{\frac{2\cdot g}{3\cdot R}} = \sqrt{\frac{2}{3}\frac{g}{R}} \Rightarrow$$

$$\Rightarrow T_1 = 2\pi\sqrt{\frac{3R}{2g}}$$

2)



$$\Delta U = \frac{mgR}{2}\varphi^2$$

$$\Delta K = \frac{\left(mR^2 + mR^2\right)\dot{\varphi}^2}{2} = \frac{2mR^2\dot{\varphi}^2}{2}$$

$$\omega_2 = \sqrt{\frac{mgR}{2mR^2}} = \sqrt{\frac{g}{2R}}$$

3) $T_1 / T_2 = \sqrt{\frac{1}{3}\cdot 4} = \frac{\sqrt{3}}{2}$

$$\sqrt{\frac{3R}{2g}}\cdot\sqrt{\frac{g}{2R}}$$

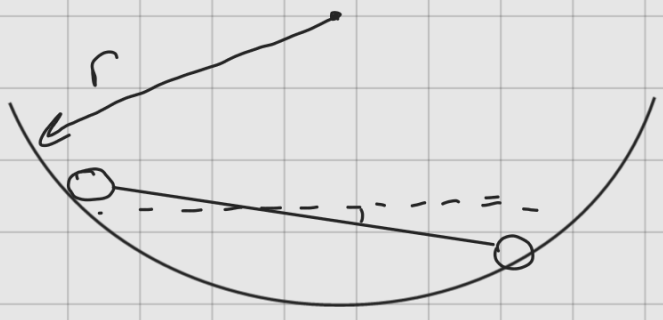
N70

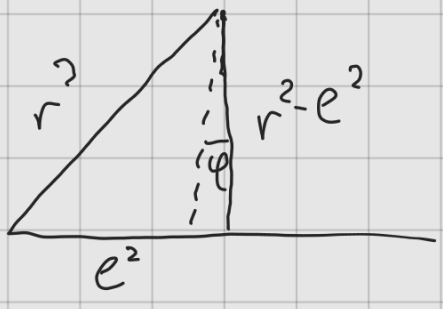
Дано: | Решение:

$2\ell, r$

Найти:

T_1, T_2





2) $\Delta U = 2mgR(1-\cos\varphi) =$

$= \frac{2mgR\varphi^2}{2} = 2mg\cdot\sqrt{r^2-e^2}\cdot\frac{\varphi^2}{2}$

$K = \frac{2m(r\dot{\varphi})^2}{2} = \frac{2mr^2\dot{\varphi}^2}{2}$

$\omega_2 = \sqrt{\frac{2mg\sqrt{r^2-e^2}}{2mr^2}} = \sqrt{\frac{g\sqrt{r^2-e^2}}{r^2}} \rightarrow T_2 = 2\pi\sqrt{\frac{r^2}{g\sqrt{r^2-e^2}}}$

1) $\Delta U = \frac{2mgR\varphi^2}{2} = 2mg\sqrt{r^2-e^2}\cdot\frac{\varphi^2}{2}$

$K = \frac{2mR^2\dot{\varphi}^2}{2} = \frac{2m\dot{\varphi}^2\cdot(r^2-e^2)}{2}$

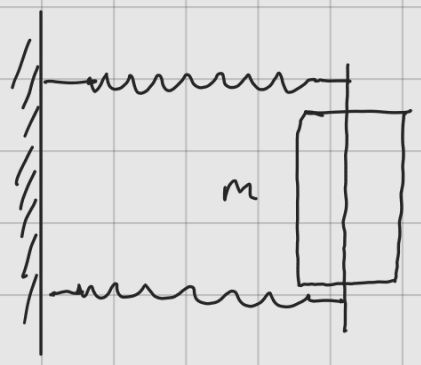
$\omega_1 = \sqrt{\frac{2mg\sqrt{r^2-e^2}}{2m(r^2-e^2)}} = \sqrt{\frac{g}{\sqrt{r^2-e^2}}} \rightarrow T_1 = 2\pi\sqrt{\frac{\sqrt{r^2-e^2}}{g}}$

№71

Дано: I, m, r, k

Найти: T

Решение:



$$\omega = \frac{\dot{x}}{R}$$

$$\dot{\omega} = \varepsilon = \frac{a}{R} = \frac{\ddot{x}}{R}$$

ЗСЭ: $\frac{2kx^2}{2} + \frac{m\dot{x}^2}{2} + \frac{I}{2}\cdot\omega^2 = const$

Продифференцируем по времени:

$$\frac{2kx\cdot\dot{x}\cdot 2}{2} + \frac{2m\dot{x}\ddot{x}}{2} + \frac{I}{2}\cdot\frac{2\dot{x}}{R}\cdot\frac{\ddot{x}}{R} = 0$$

$$2kx + m\ddot{x} + \frac{I\ddot{x}}{R^2} = 0$$

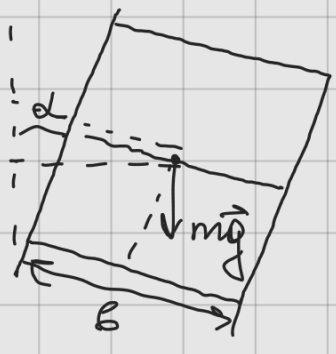
$$\ddot{x} = -\frac{2k}{m+\frac{I}{R^2}}x \Rightarrow \omega^2 = \frac{2k}{m+\frac{I}{R^2}} \Rightarrow T = 2\pi\sqrt{\frac{m+\frac{I}{R^2}}{2k}}$$

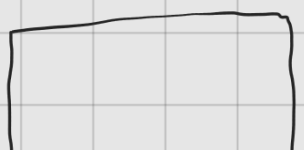
№72

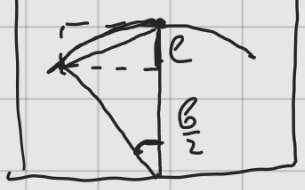
Дано: α, b

Найти: T

Решение:







→ мгновенная ось вращения

$$l = \frac{b}{2}(1-\cos\varphi) \; ; \; \Delta h = \frac{b}{2}(1-\cos\varphi)\sin\alpha$$

$$\Delta U = \frac{mgb}{2}(1-\cos\varphi)\sin\alpha = \frac{mgb}{2}\sin\alpha \cdot \frac{\varphi^2}{2}$$

$$\Rightarrow K = \frac{I\omega^2}{2} = \frac{\left(\frac{mb^2}{4} + \frac{mb^2}{12}\right)\dot{\varphi}^2}{2}$$

$$\omega^2 = \frac{mgb}{2}\sin\alpha \cdot \frac{3}{mb^2} = \frac{3}{2}\frac{g\sin\alpha}{b}$$

$$\Rightarrow T = 2\pi\sqrt{\frac{2b}{3g\sin\alpha}}$$