1 Equations

$$\frac{\partial S}{\partial t} = P - ET - R - \nabla \cdot Q \tag{1}$$

$$R_{net} = LH + SH + G \tag{2}$$

$$\mathbf{F}_v = \rho q \mathbf{v} \tag{3}$$

where ρ is the density; q is the specific humidity; and $\mathbf{v}$ is the velocity vector.

$$\mathbf{F}_v = \mathbf{i}F_{vx} + \mathbf{j}F_{vy} + \mathbf{k}F_{vz} \qquad \mathbf{v} = \mathbf{i}u + \mathbf{j}v + \mathbf{k}w \tag{4}$$

where $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are the unit vectors.

$$\overline{F_{vx}} = \rho(\overline{u}\,\overline{q} + \overline{u'q'}) \qquad \overline{F_{vy}} = \rho(\overline{v}\,\overline{q} + \overline{v'q'}) \qquad \overline{F_{vz}} = \rho(\overline{w}\,\overline{q} + \overline{w'q'}) \tag{5}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \tag{6}$$

$$\frac{\overline{\partial q}}{\partial t} + \overline{u}\frac{\overline{\partial q}}{\partial x} + \overline{v}\frac{\overline{\partial q}}{\partial x} + \overline{w}\frac{\overline{\partial q}}{\partial z} = -[\frac{\partial}{\partial x}(\overline{u'q'}) + \frac{\partial}{\partial y}(\overline{v'q'}) + \frac{\partial}{\partial z}(\overline{w'q'})] \tag{7}$$

$$\frac{\partial \overline{q}}{\partial t} = 0 \qquad and \qquad (\frac{\partial}{\partial x}, \frac{\partial}{\partial y}) = 0 \qquad and \qquad \overline{w} = 0 \tag{8}$$

$$F_{vz} = \rho \overline{w'z'} \equiv LH \tag{9}$$

$$F_{mz} = \rho \overline{w'u'} \equiv \tau = \tau_0 \qquad F_{hz} = \rho c_p \overline{w'\theta'} \equiv SH \tag{10}$$

for the momentum τ_0 and sensible heat flux SH, respectively.

Shear stress

$$u_* = (\tau_0/\rho)^{1/2} \qquad u_*^2 = \rho\overline{w'u'} = F_{mz} \tag{11}$$

$$\begin{aligned} \overline{w'q'} &= -C_e(\overline{u_2} - \overline{u_1})(\overline{q_4} - \overline{q_3}) \\ \overline{w'\theta'} &= -C_h(\overline{u_2} - \overline{u_1})(\overline{\theta_4} - \overline{\theta_3}) \\ \overline{w'u'} &= -C_e(\overline{u_2} - \overline{u_1})^2 \end{aligned} \tag{12}$$

$$\frac{u*}{(z-d_0)(d\overline{u}/dz)} = k\overline{u_2} - \overline{u_1} = \frac{u_*}{k}ln(\frac{z_2 - d_0}{z_1 - d_0})\overline{u} = \frac{u_*}{k}ln(\frac{z - d_0}{z_0})C_d = k/ln[(z-d_0)/z_0] \tag{13}$$

Monin-Obukhov length

$$L = \frac{-u_*^3\rho}{kg[\frac{H}{T_a c_p} + 0.61E]}$$

Steady turbulent kinetic energy equation

$$\overline{u'w'}\frac{\partial\overline{u}}{\partial z} - \frac{g}{T_{VS}}[\overline{w'\theta'} + 0.61\overline{T}\ \overline{w'q'}] + \frac{\partial}{\partial z}(\overline{w'e_t} + \frac{\overline{w'p'}}{\rho}) = -\epsilon \tag{14}$$

where the first term on the left hand side is the mechanical production of turbulence; the second term is the contribution due to buoycany forces, $\frac{1}{\rho}\nabla p$; and the third is the divergence of the turbulent flux of the total turbulent energy. All the production terms are balanced by ϵ representing viscous effects that are dissipating the energy constituting the sink.

Rewriting L

$$Lkg[\frac{H}{T_a c_p} + 0.61E] = -u_*^3\rho \tag{15}$$

$$\overline{w'q'} = -C_e(\overline{u_2} - \overline{u_1})(\overline{q_4} - \overline{q_3})$$

$$\overline{w'\theta'} = -C_h(\overline{u_2} - \overline{u_1})(\overline{\theta_4} - \overline{\theta_3})$$

$$\overline{w'u'} = -C_e(\overline{u_2} - \overline{u_1})^2$$

$$\frac{u*}{(z-d_0)(d\overline{u}/dz)} = k$$

$$\overline{u_2} - \overline{u_1} = \frac{u_*}{k} ln(\frac{z_2 - d_0}{z_1 - d_0})$$

$$\overline{u} = \frac{u_*}{k} ln(\frac{z - d_0}{z_0})$$

$$C_d = k/ln[(z - d_0)/z_0]$$

Darcy's law

$$q = K\frac{\Delta h}{\Delta l} = \frac{dh}{dl} \tag{16}$$

$$Q = Aq \tag{17}$$

Linear velocity

$$\overline{v} = \frac{q}{\phi} = \frac{K}{\phi}\frac{dh}{dl} \tag{18}$$

$$A_{eff} = A\phi \tag{19}$$

Hydraulic conductivity

$$K = \frac{k\rho g}{\mu}[L/T] \tag{20}$$

Potential

$$\oint \mathbf{F}\, ds = 0 \tag{21}$$

$$\int_a^b \mathbf{F} ds = W \tag{22}$$

Elevation potential

$$\Phi_e = \int_0^{z'} mgds = mgz' \tag{23}$$

Pressure/matric potential

$$\Phi_p = V\int_0^{p'} dp = Vp' \tag{24}$$

$$p' = \frac{\mathbf{F}}{A} \tag{25}$$

Elevation head

$$z = mgz' \frac{1}{mg} = z' \tag{26}$$

Pressure/matric head

$$\psi = V \frac{\mathbf{F}}{A} \frac{1}{mg} = \psi A \frac{mg}{A} \frac{1}{mg} \tag{27}$$

Shallow water equation

$$\frac{\partial \psi_s}{\partial t} = \nabla \cdot (\mathbf{v} \psi_s) \tag{28}$$

Mannings equation

$$v = \frac{\sqrt{S_f}}{n_m} \psi_s^{2/3} \tag{29}$$

Overland flow boundary condition

$$K_{zz} k_r \frac{\partial (z + \psi_s)}{\partial z} = \frac{\partial \psi_s}{\partial t} - \nabla \cdot (\mathbf{v} \psi_s) \tag{30}$$

Continuity equation

$$\frac{\partial \theta}{\partial t} = \nabla \cdot \mathbf{q} + \Gamma \tag{31}$$

Specific storage

$$\frac{\partial \phi S_r}{\partial t} = \phi \frac{\partial S_r}{\partial t} + S_r \frac{\partial \phi}{\partial t} \tag{32}$$

$$S_r \frac{\partial \phi}{\partial \psi} \frac{\partial \psi}{\partial t} = S_r S_s \frac{\partial \psi}{\partial t} \tag{33}$$