



Optimization problems (section 4.5) and other methods of calculus require finding roots,  $x$ -intercepts, solutions. Algebra does this, but it cannot solve every equation. *Newton's method* lets us find the approximate roots of many equations.

## NEWTON'S METHOD



**Newton's method, Newton-Raphson method** (*method*) – an iterative method for solving roots of equations by successively approximating the value of  $x$ .



**NOTE:** Newton's method works best on a computer or a programmable calculator.

### WHAT NEWTON'S METHOD DOES

Newton's method gets closer and closer (*converges*) to the actual root of a function by following tangent lines from the function to the  $x$ -intercept line again and again. See Figure 1.

### HOW TO USE NEWTON'S METHOD

To approximate the roots of a function  $f$ ,

1. Start with  $n = 1$  and  $x_n$  being some number in  $f$ 's domain.
2. Take the derivative at  $x_n$ , which is  $f'(x_n)$ .
3. Take the value of  $f$  at  $x_n$ , which is  $f(x_n)$ .
4. The next  $x$ -number,  $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ .
5. Repeat steps 2-4 until enough precision of  $x_n$  is achieved.



**WARNING:** Newton's method is not guaranteed to converge to a value of  $x_n$ .

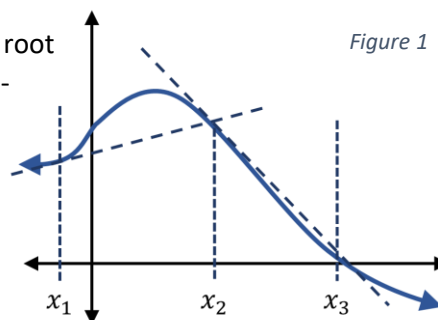


Figure 1

### HOW WOULD YOU ANSWER?

- ♦ What is Newton's method? How does it work?
- ♦ Why is Newton's method not guaranteed to approximate a function's root? Where might this happen?