Derivatives help us find the speed of a moving train and its acceleration if we know its position through time. What if we only have the speed? Only the acceleration? Speed over time is position, but how can calculus do this? To assemble the position of the train from its speed, we need the opposite of a derivative. We use an *antiderivative*.

ANTIDERIVATIVES

Antiderivative *(object)* – the inverse of the derivative in an interval. If F is the antiderivative of f on the interval I, then $F'(x) = f(x)$ for every x in the interval I.

CALC KEY **The antiderivative is the opposite, the inverse, of a derivative.**

NOTATION FOR ANITDERIVATIVES: The function F is the antiderivative of f, which means that $F' = f$.

HOW DO WE FIND THE ANTIDERIVATIVE?

Just like there are rules for finding the derivative of a function, there are rules for finding the antiderivative of a function. Although it doesn't list all the rules for finding an antiderivative, you can use the table below to see which case your function fits into to find its antiderivative.

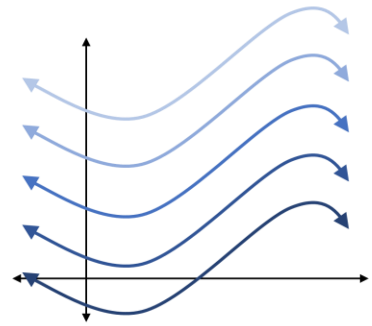
Function	Antiderivative
$cf(x)$	$cF(x)$
$f(x) + g(x)$	$F(x) + G(x)$
x^n (where $n \neq -1$)	$\frac{x^{n+1}}{n+1}$
$\frac{1}{x}$	$ln\lvert x\rvert$
e^x	e^x
$cos\, x$	$\sin x$
$sin\, x$	$-\cos x$
$sec^2 x$	$tan\, x$
$sec\, x\, tan\, x$	$sec\, x$
$\frac{1}{\sqrt{1-x^2}}$	$sin^{-1} x$
$\frac{1}{1+x^2}$	$tan^{-1} x$
$cosh\, x$	$sinh\, x$
$sinh\, x$	$cosh\, x$

CONNECT THE DOTS: this table seems like the table for trigonometric functions and their derivatives *(section 3.5)* or the derivatives rules scattered in chapter 2. The difference is that now we look at a derivative and try to find the function it came from.

LOOK IN THE BOOK: for examples of finding the antiderivative(s) of a function, see examples 1 through 4 in the textbook *(section 4.7)*.

HOW MANY ANTIDERIVATIVES ARE THERE?

Since the derivative of a constant is zero *(section 2.3)*, for some function f, there can be an infinite number of antiderivatives of f, each just varying by a constant factor from each other, but all having the same derivative and thus all being antiderivatives of the same function f.



The general antiderivative of a derivative f is stated this way: $F(x) + C$, where C is a constant. The figure to the right shows many antiderivatives of the same function, each with different values of C.

Some problems may give you another equation that will state a certain point defined on the antiderivative. This lets you calculate what C is equal to.

DIFFERENTIAL EQUATIONS

Differential equation *(object)* – an equation that involves the derivatives of a function.

WHERE DO WE USE DIFFERENTIAL EQUATIONS?

Differential equations let us find the relationship between how a number changes and how large (or small) a number is. For example:

- A worker works harder when there's more work to do, according to some proportion. How much more work will make him work 100% harder?
- A train accelerates by 3mi/hr every hour. It starts off going at 10mi/hr. How many hours will pass until it has passed 70mi?

LOOK IN THE BOOK: for examples of solving simple differential equations involving *rectilinear motion*, see examples 5 and 6 in the textbook *(section 4.7)*.

HOW WOULD YOU ANSWER?

- What is a derivative? What is an antiderivative? Where might you use them?
- How do you find an antiderivative? For any function, how many antiderivatives are there of it?
- What is a differential equation? Where are they? How do you solve thhem?