Calculus has real-world applications, for example, in the field of economics. Many such problems begin as word problems. This section will show how to convert word problems into optimization problems and solve them using calculus.

STEPS IN SOLVING OPTIMIZATION PROBLEMS

The book[1] recommends these steps when solving a word problem using calculus:

1. Understand the problem. What is known? unknown? How are the parts related to one another?
2. Draw a diagram. What do its dimensions mean?
3. Introduce notation. What variables exist? Which will be maximized or minimized (optimized)?
4. How would you define the optimized variable in terms of the others? (using mathematics)
5. Simplify the problem: if the optimized variable is expressed in terms of more than one unknown variable, define equations between the unknowns so that the optimized variable is expressed in terms of only one unknown variable.
6. Find the *absolute* maximum or minimum value of the optimized variable using methods from *sections 4.1* and *4.3*, possibly using the Closed Interval Method *(section 4.1)* if the unknown variable is in a closed interval.

THE FIRST DERIVATIVE TEST FOR ABSOLUTE EXTREME VALUES

The First Derivative Test for Absolute Extreme Values (test) – a variant of the first derivative test *(section 4.3)* that finds *absolute* extreme values.

i The first derivative test for absolute extreme values can be used to solve optimization problems.

WHAT THE FIRST DERIVATIVE TEST FOR ABSOLUTE EXTREME VALUES TELLS

For any critical number c defined in an interval of a continuous function f,

If $f'(x) > 0$ for all $x < c$ and $f'(x) < 0$ for all $x > c$ ➔ $f(c)$ is the absolute maximum value of f.

If $f'(x) > 0$ for all $x > c$ and $f'(x) < 0$ for all $x < c$ ➔ $f(c)$ is the absolute minimum value of f.

WHY THE FIRST DERIVATIVE TEST FOR ABSOLUTE EXTREME VALUES IS TRUE

The first derivative test for absolute extreme values is a special case of the first derivative test *(section 4.3)* which guarantees its truthfulness (provided the first derivative test is true).

HOW TO USE THE FIRST DERIVATIVE TEST FOR ABSOLUTE EXTREME VALUES

To use "the first derivative test for absolute extreme values" to find extreme values of a function f,

1. Identify the critical numbers of f.
2. For each critical number, identify which case it fits into in TBL.
3. Accept the consequences from the table.

APPLICATIONS TO BUSINESS AND ECONOMICS

Unit *(object)* – something that is bought and sold in the field of economics.

Cost Function *(function)* – a function $C(x)$ that measures the cost of producing x units.
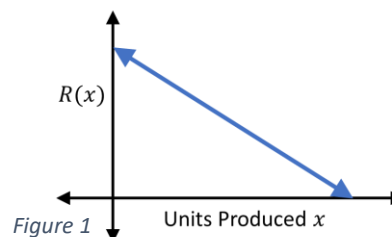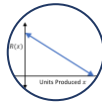

Figure 1

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[1] Essential Calculus – Early Transcendentals (Second Edition) by James Stewart, Page 232.

Marginal Cost *(value)* – the change in cost in producing one additional unit; the derivative, $C'(x)$.

Demand Function, Price Function *(function)* – a function $p(x)$ that measures the price for each unit when x units are sold. Likely a decreasing function. FIG shows a demand curve.

ⓘ **ECONOMICS**. The more an economic actor has demand for a good or service, the more they benefit from specializing for that good or service, so the demand function is likely a decreasing function.

Revenue Function *(function)* – a function $R(x)$ that measures the revenue for selling x units. $R(x) = xp(x)$.

Marginal Revenue Function *(function)* – the additional revenue from selling one additional unit; the derivative $R'(x)$.

Profit Function *(function)* – a function $P(x)$ that measures the profit in selling x units. $P(x) = R(x) - C(x)$.

Marginal Profit Function *(function)* – the change in profit from selling one additional unit; the derivative of the profit function, $P'(x)$.

Often, when optimizing economic problems, you will be working to maximize the profit function, $P(x)$.

WHAT DID YOU LEARN?
◆ What is an optimization problem? How do you solve one?
◆ What equations govern the price of a good or service (in economics)? Which theorems are used?
◆ Where do optimization problems apply beyond mathematics itself?