The definite integral has a lot of uses. How do you evaluate it? This section will show many rules to help evaluate it. Doing so, we will learn what the *indefinite integral* is, and how it relates to the definite integral. Let's look at a few theorems to see how this is so.

EVALUATION AND NET CHANGE THEOREMS

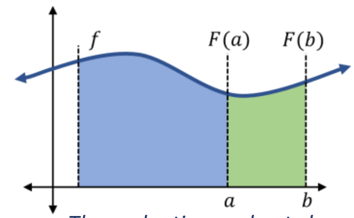
Evaluation Theorem *(theorem)* – a theorem that relates the definite integral of a function to its antiderivatives. It says $\int_a^b f(x)dx = F(b) - F(a)$.

Net Change Theorem *(theorem)* – a theorem that relates the definite integral of the derivative of an antiderivative with the differences of the antiderivative. It says that $\int_a^b F'(x)dx = F(b) - F(a)$.

WHAT DO THEY TELL?

Looking at any definite integral *(section 5.2)* such as $\int_a^b f(x)dx$, it equals $F(b) - F(a)$, where $F' = f$ (This is an antiderivative *[section 4.7]*.) Mathematically: $\int_a^b f(x)dx = F(b) - F(a)$, for $F' = f$.

Since $F' = f$, the evaluation theorem and the net change theorem do the same job; they're just expressed differently.



The evaluation and net change theorems subtract the antiderivatives between two points.

HOW WOULD YOU USE THEM?

For the definite integral $\int_a^b f(x)dx$,

1. Find the antiderivative *(section 4.7)* of $f(x)$. It is notated $F(x)$.
2. Evaluate $F(b) - F(a)$. That is the answer.

LOOK IN THE BOOK: for examples of evaluating definite integrals with the evaluation or net change theorems, see examples 1 and 2 on page 283 *(section 5.3)*.

WHERE WOULD YOU USE THEM?

The evaluation theorem helps find the value of a definite integral. For example:

- **PHYSICS**: If a train travels starting at a miles per hour but over the course of one hour accelerating linearly to b miles per hour, how many miles did it travel over that hour? Let's write the speed as $v(t) = (b - a)t + a$. The distance traveled over one hour is $s = \int_0^1 v(t)dt$. How do we evaluate it? The evaluation theorem tells us to find the antiderivative of $v(t) = (b - a)t + a$. The antiderivative $V(t) = \frac{1}{2}(b - a)t^2 + at$. The distance traveled is $V(1) - V(0)$, which is $\frac{1}{2}(a + b)$.
- **ELECTRICITY**: Suppose an electricity meter that tells how many joules of energy have been used and the current rate of energy usage of a house. Between two intervals of time, how many joules of energy will be used? The evaluation theorem tells us to simply record how much energy was used already at the start of the interval, measure it again at the end, and then subtract the start from the end.

WHY IS THE EVALUATIN THEOREM TRUE?

If and only if the evaluation theorem is true, then will $\int_a^b f(x)dx = F(b) - F(a)$,

1. Divide $[a, b]$ into n subintervals each with the width $\Delta x = (b - a)/n$.
2. Each subinterval $[x_{i-1}, x_i]$ means a rectangle with area equal to $f(x_i^*)\Delta x = F(x_{i-1}) - F(x)$.
3. The sum of rectangles' areas is $\sum_{i=1}^{n} F(x_{i-1}) - F(x_i)$
4. This series expands to be $F(x_0) - F(x_1) + F(x_1) - F(x_2) + F(x_2) \cdots - F(x_n)$.
5. That simplifies to $F(x_0) - F(x_n)$.

6. $\int_a^b f(x)dx = F(b) - F(a)$.

INDEFINITE INTEGRALS

Indefinite Integral *(object)* – the single antiderivative or family of antiderivative of a function.

NOTATION FOR THE INDEFINITE INTEGRAL: $\int f(x)dx$ means the antiderivative of $f(x)$; that is, $F(x)$.

WHAT IS SAME AND DIFFERENT ABOUT THE DEFINITE AND INDEFINITE INTEGRALS?

Both the definite and indefinite integrals:

- Use the integrand's antiderivative *(section 5.2)*.
- Signify area under the curve.

The difference is that:

- The definite integral is a number *(section 5.2)*.
- The indefinite integral is one or many function(s).

Definite Integral	Indefinite Integral	Both
Means a number.	Means one or more function(s).	Use the antiderivative.
Has a definite interval.	Works accross the domain.	Signify area under the curve.

The two are related:

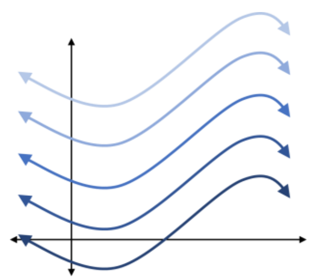
$$\int_a^b f(x)dx = \int f(x)dx\Big]_a^b$$

LOOK IN THE BOOK: for examples of converting a definite integral to an indefinite integral, see examples 4-6 on pages 285-6 *(section 5.3)*.

HOW WOULD YOU FIND THE INDEFINITE INTEGRAL?

Since a constant has a derivative of 0, the function (integrand) in the *indefinite* integral may have up to *infinity* antiderivatives *(section 4.7)*. To express this, we add a **constant of integration** C to the **general** antiderivative (if it isn't already there). You can plug in numbers to solve for C in a specific instance.

The function in the indefinite integral may have up to infinity anti-derivatives.



This table shows the family of functions that an indefinite integral can mean:

If it looks like this…	…then it can be this
$\int [f(x) + g(x)]dx$	$\int f(x)dx + \int g(x)dx$
$\int cf(x)dx$	$c \int f(x)dx$
$\int x^n dx$	$\frac{x^{n+1}}{n+1} + C$
$\int \frac{1}{x} dx$	$\ln\lvert x\rvert + C$
$\int e^x dx$	$e^x + C$

If it looks like this…	…then it can be this
$\int a^x dx$	$\frac{a^x}{\ln a} + C$
$\int \sin x \, dx$	$-\cos x + C$
$\int \cos x \, dx$	$\sin x + C$
$\int \sec^2 x \, dx$	$\tan x + C$
$\int \csc^2 x \, dx$	$-\cot x + C$
$\int \sec x \tan x \, dx$	$\sec x + C$
$\int \csc x \cot x \, dx$	$\csc x + C$
$\int \frac{1}{x^2+1} dx$	$\tan^{-1} x + C$
$\int \frac{1}{\sqrt{1-x^2}} dx$	$\sin^{-1} x + C$
$\int \sinh x \, dx$	$\cosh x + C$
$\int \cosh x \, dx$	$\sinh x + C$

CONNECT THE DOTS: this table is an expanded version of the table of antiderivatives *(section 4.7)*.

LOOK IN THE BOOK: for examples of finding the general indefinite integral, see examples 4-6 on pages 285-6 *(section 5.3)*.

WHERE WOULD YOU USE THE INDEFINITE INTEGRAL?

Anywhere you would use the definite integral, you can use the indefinite integral. It lets you save time by simplifying the math before plugging in actual numbers. Because the indefinite integral is more general than the definite integral, it's useful when the definite integral would be used again and again.

For more practical applications of the indefinite integral, consider that all the practical uses of definite integrals, such as calculating distance and displacement from speed and velocity, can also be used by indefinite integrals *(section 5.1)*.

WHAT DID YOU LEARN?

- What are the net change and evaluation theorems? What do they do? How do you use them?
- What is an indefinite integral? What does it mean? How do you find it? Why is it true?
- Where would you use the net change and evaluation theorems? the indefinite integral? Why?