EU2 - Extra Exercise 2

ID1018

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Algorithms

Sorting a set

A problem: sorting elements of a set

There exists a set U, whose elements can be compared with each other by the operator $<$. It is possible to determine the lesser of two arbitrary elements in the set. The set X is a finite, non-empty subset of U.

Sort elements in X in ascending order.

An algorithm to solve the problem — exchange sort

Algorithm: sort

Preconditions:
U is a set whose elements can be compared by the operator $<$.
N is the set of natural numbers,
$n \in N, n \geq 1, X = \{x_1, x_2, \ldots, x_n\} \subset U$,
for an arbitrary integer $i, 1 \leq i \leq n$, then x_i denotes the element on position i.
Postconditions:
$x_1 < x_2 < \cdots < x_n$

The steps in the algorithm:

```
sort(n, X)
{
  i = 1
  while i < n
  {
    j = i + 1
    while j ≤ n
    {
      if (x_j < x_i)
        swap x_j and x_i
      j++
    }
    i++
  }
}
```

Exercises on the problem and algorithm

1. Visualise the algorithm. Draw a series of images that shows how a set is sorted.

2. Determine the time complexity of the algorithm for element comparisons; determine the corresponding complexity function. To which set of θ does this complexity function belong?

3. Determine the time complexity of the algorithm for element exchanges. Determine the complexity for the best case, the worst case, and the average case. Assume that the probability of an exchange is 0.5.

Categorise the corresponding complexity functions: to which set of θ do they belong?

4. Compare the time complexity between exchange sort and selection sort. Both the number of comparisons and exchanges are to be considered.

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5. Prove the algorithm. The proof shall have the following structure:

A) INNER LOOP

A STATEMENT ABOUT THE INNER LOOP

When the inner loop is complete, the following holds:

$x_i = \text{minimum}\{x_i, x_{i+1}, \ldots, x_n\}$

PROOF

Here you present the proof. Establish a statement about the variables, and prove that this statement is a loop invariant in the inner loop. With the aid of this loop invariant the proof shall then be derived.

B) MAIN LOOP

A STATEMENT ABOUT THE MAIN LOOP

When the main loop is complete, the following holds:

$x_1 < x_2 < \cdots < x_n$

PROOF

Here you present the proof, using the statement about the inner loop. Establish a statement about the variables, and prove that this statement is a loop invariant in the main loop. With the aid of this loop invariant the proof shall then be derived.