

# **Basic Trigonometry**

## **Understanding Triangles and Ratios**

# What is Trigonometry?

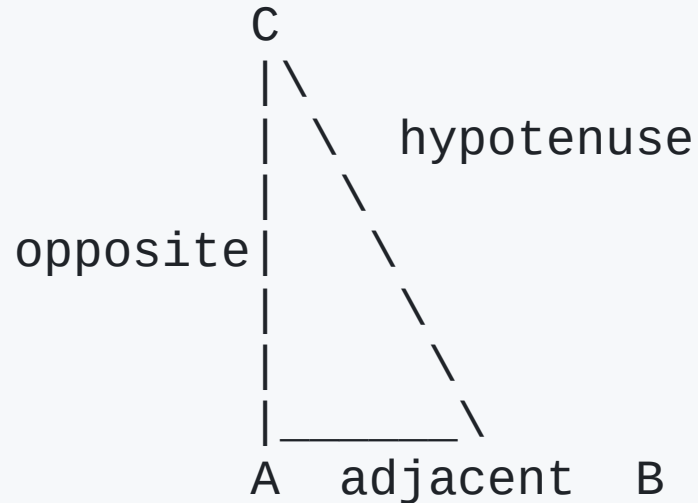
**Trigonometry** is the study of relationships between:

- Angles
- Side lengths of triangles

The word comes from Greek:

- *trigonon* = triangle
- *metron* = measure

# The Right Triangle



Key components:

- **Hypotenuse:** longest side (opposite the right angle)
- **Opposite:** side opposite to the angle of interest
- **Adjacent:** side next to the angle of interest

# The Three Basic Ratios

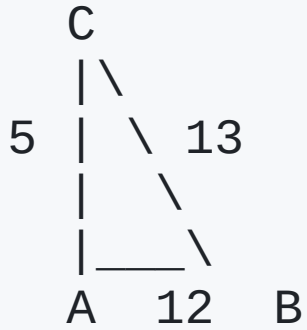
For angle  $\theta$  (theta) in a right triangle:

## SOH-CAH-TOA

- $\sin \theta = \text{Opposite} / \text{Hypotenuse}$
- $\cos \theta = \text{Adjacent} / \text{Hypotenuse}$
- $\tan \theta = \text{Opposite} / \text{Adjacent}$

This mnemonic helps you remember!

## Example: Finding sin, cos, tan



For angle A:

- $\sin A = 5/13 = 0.385$
- $\cos A = 12/13 = 0.923$
- $\tan A = 5/12 = 0.417$

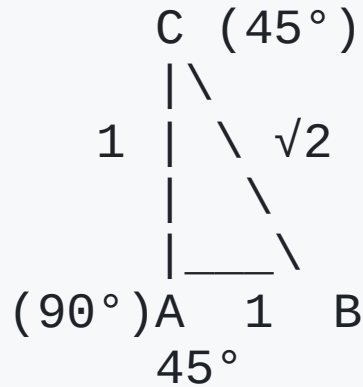
# The Reciprocal Functions

Three additional ratios:

- **$\csc \theta = 1/\sin \theta = \text{Hypotenuse} / \text{Opposite}$**
- **$\sec \theta = 1/\cos \theta = \text{Hypotenuse} / \text{Adjacent}$**
- **$\cot \theta = 1/\tan \theta = \text{Adjacent} / \text{Opposite}$**

These are less commonly used but important!

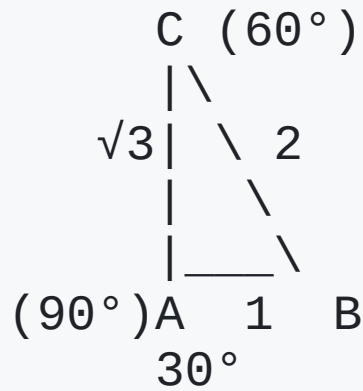
# Special Right Triangles: 45-45-90



In a 45-45-90 triangle:

- Legs are equal (ratio 1:1: $\sqrt{2}$ )
- Hypotenuse = leg  $\times \sqrt{2}$
- $\sin 45^\circ = \cos 45^\circ = \sqrt{2}/2 \approx 0.707$
- $\tan 45^\circ = 1$

# Special Right Triangles: 30-60-90



In a 30-60-90 triangle (ratio 1: $\sqrt{3}$ :2):

- $\sin 30^\circ = 1/2$ ,  $\cos 30^\circ = \sqrt{3}/2$ ,  $\tan 30^\circ = 1/\sqrt{3}$
- $\sin 60^\circ = \sqrt{3}/2$ ,  $\cos 60^\circ = 1/2$ ,  $\tan 60^\circ = \sqrt{3}$



# Finding Missing Sides

**Given:** angle and one side

**Find:** other sides

**Example:** If angle  $A = 30^\circ$  and hypotenuse = 10

- opposite =  $10 \times \sin 30^\circ = 10 \times 0.5 = 5$
- adjacent =  $10 \times \cos 30^\circ = 10 \times 0.866 = 8.66$

Use the appropriate ratio based on what you know!

# Finding Missing Angles

**Given:** two sides

**Find:** angle

Use inverse functions:

- $\theta = \sin^{-1}(\text{opposite}/\text{hypotenuse})$
- $\theta = \cos^{-1}(\text{adjacent}/\text{hypotenuse})$
- $\theta = \tan^{-1}(\text{opposite}/\text{adjacent})$

**Example:** If opposite = 3, adjacent = 4

- $\theta = \tan^{-1}(3/4) = 36.87^\circ$

# The Pythagorean Theorem

In any right triangle:

$$a^2 + b^2 = c^2$$

where  $c$  is the hypotenuse

**Example:** If legs are 3 and 4:

- $3^2 + 4^2 = c^2$
- $9 + 16 = c^2$
- $c = \sqrt{25} = 5$

# Trigonometric Identity #1

**Pythagorean Identity:**

$$\sin^2\theta + \cos^2\theta = 1$$

This is always true for any angle  $\theta$ !

Derivation from Pythagorean theorem:

- $\text{opposite}^2 + \text{adjacent}^2 = \text{hypotenuse}^2$
- Divide by  $\text{hypotenuse}^2$
- $(\text{opposite}/\text{hypotenuse})^2 + (\text{adjacent}/\text{hypotenuse})^2 = 1$
- $\sin^2\theta + \cos^2\theta = 1$

# Complementary Angles

In a right triangle, the two acute angles add to  $90^\circ$ .

## Key relationships:

- $\sin \theta = \cos(90^\circ - \theta)$
- $\cos \theta = \sin(90^\circ - \theta)$
- $\tan \theta = \cot(90^\circ - \theta)$

**Example:**  $\sin 30^\circ = \cos 60^\circ$

# Real-World Applications

Trigonometry is used in:

- **Architecture:** calculating roof slopes
- **Navigation:** finding distances and bearings
- **Physics:** analyzing forces and motion
- **Engineering:** designing structures
- **Astronomy:** measuring distances to stars

# Practice Problem

A ladder leans against a wall at a  $70^\circ$  angle with the ground. The ladder is 15 feet long.

## Questions:

1. How high up the wall does it reach?
2. How far is the base from the wall?

## Answers:

1. height =  $15 \times \sin 70^\circ = 14.1$  feet
2. distance =  $15 \times \cos 70^\circ = 5.1$  feet

# Summary

## Remember SOH-CAH-TOA:

- Sin = Opposite/Hypotenuse
- Cos = Adjacent/Hypotenuse
- Tan = Opposite/Adjacent

## Key Tools:

- Pythagorean theorem:  $a^2 + b^2 = c^2$
- Inverse functions for finding angles
- Special triangles: 30-60-90 and 45-45-90



# Questions?

Thank you for your attention!

Keep practicing and trigonometry will become second nature.