Basic Trigonometry

Understanding Triangles and Ratios

What is Trigonometry?

Trigonometry is the study of relationships between:

- Angles
- Side lengths of triangles

The word comes from Greek:

- *trigonon* = triangle
- *metron* = measure

The Right Triangle

```
         C
         |\
         | \  hypotenuse
         |  \
 opposite|   \
         |    \
         |     \
         |______\
         A  adjacent  B
```

Key components:

- **Hypotenuse**: longest side (opposite the right angle)
- **Opposite**: side opposite to the angle of interest
- **Adjacent**: side next to the angle of interest

The Three Basic Ratios

For angle θ (theta) in a right triangle:

SOH-CAH-TOA

- **sin θ = Opposite / Hypotenuse**
- **cos θ = Adjacent / Hypotenuse**
- **tan θ = Opposite / Adjacent**

This mnemonic helps you remember!

Example: Finding sin, cos, tan

```
      C
      |\
    5 | \ 13
      |  \
      |___\
      A  12  B
```

For angle A:

- sin A = 5/13 = 0.385
- cos A = 12/13 = 0.923
- tan A = 5/12 = 0.417

The Reciprocal Functions

Three additional ratios:

- **csc θ = 1/sin θ = Hypotenuse / Opposite**
- **sec θ = 1/cos θ = Hypotenuse / Adjacent**
- **cot θ = 1/tan θ = Adjacent / Opposite**

These are less commonly used but important!

Special Right Triangles: 45-45-90

```
         C (45°)
         |\
       1 | \ √2
         |  \
         |___\
    (90°)A  1  B
         45°
```

In a 45-45-90 triangle:

- Legs are equal (ratio 1:1:√2)
- Hypotenuse = leg × √2
- sin 45° = cos 45° = √2/2 ≈ 0.707
- tan 45° = 1

Special Right Triangles: 30-60-90

```
            C (60°)
            |\
          √3|  \ 2
            |   \
            |___\
     (90°)A  1   B
            30°
```

In a 30-60-90 triangle (ratio 1:√3:2):

- sin 30° = 1/2, cos 30° = √3/2, tan 30° = 1/√3
- sin 60° = √3/2, cos 60° = 1/2, tan 60° = √3

Finding Missing Sides

Given: angle and one side
Find: other sides

Example: If angle A = 30° and hypotenuse = 10

- opposite = $10 \times \sin 30° = 10 \times 0.5 = 5$
- adjacent = $10 \times \cos 30° = 10 \times 0.866 = 8.66$

Use the appropriate ratio based on what you know!

Finding Missing Angles

Given: two sides

Find: angle

Use inverse functions:

- $\theta = \sin^{-1}(\text{opposite}/\text{hypotenuse})$
- $\theta = \cos^{-1}(\text{adjacent}/\text{hypotenuse})$
- $\theta = \tan^{-1}(\text{opposite}/\text{adjacent})$

Example: If opposite = 3, adjacent = 4

- $\theta = \tan^{-1}(3/4) = 36.87°$

The Pythagorean Theorem

In any right triangle:

$a^2 + b^2 = c^2$

where c is the hypotenuse

Example: If legs are 3 and 4:

- $3^2 + 4^2 = c^2$
- $9 + 16 = c^2$
- $c = \sqrt{25} = 5$

Trigonometric Identity #1

Pythagorean Identity:

$\sin^2\theta + \cos^2\theta = 1$

This is always true for any angle θ!

Derivation from Pythagorean theorem:

- $\text{opposite}^2 + \text{adjacent}^2 = \text{hypotenuse}^2$
- Divide by hypotenuse^2
- $(\text{opposite}/\text{hypotenuse})^2 + (\text{adjacent}/\text{hypotenuse})^2 = 1$
- $\sin^2\theta + \cos^2\theta = 1$

Complementary Angles

In a right triangle, the two acute angles add to 90°.

Key relationships:

- $\sin \theta = \cos(90° - \theta)$
- $\cos \theta = \sin(90° - \theta)$
- $\tan \theta = \cot(90° - \theta)$

Example: $\sin 30° = \cos 60°$

Real-World Applications

Trigonometry is used in:

- **Architecture**: calculating roof slopes
- **Navigation**: finding distances and bearings
- **Physics**: analyzing forces and motion
- **Engineering**: designing structures
- **Astronomy**: measuring distances to stars

Practice Problem

A ladder leans against a wall at a 70° angle with the ground. The ladder is 15 feet long.

Questions:

1. How high up the wall does it reach?
2. How far is the base from the wall?

Answers:

1. height = $15 \times \sin 70° = 14.1$ feet
2. distance = $15 \times \cos 70° = 5.1$ feet

Summary

Remember SOH-CAH-TOA:

- Sin = Opposite/Hypotenuse
- Cos = Adjacent/Hypotenuse
- Tan = Opposite/Adjacent

Key Tools:

- Pythagorean theorem: $a^2 + b^2 = c^2$
- Inverse functions for finding angles
- Special triangles: 30-60-90 and 45-45-90

Questions?

Thank you for your attention!

Keep practicing and trigonometry will become second nature.