BRADLEY'S MATHS

GCSE Higher Level Mathematics

NUMBER

1.3: Prime Numbers, HCF, LCM

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Abstract

This worksheet and answer sheet are, together, a stand alone teaching and learning document aimed directly at the GCSE syllabus item N1 4 Use the concepts and vocabulary of prime numbers, factors (divisors), multiples, common factors, common multiples, highest common factor, lowest common multiple, prime factorisation, including using product notation and the unique factorisation theorem

This booklet has been fully updated with our unique colour coded box system to facilitate teaching and learning:

- Violet for Deeper Insights into the subject matter
- Green for Methods and Examples
- Blue for Pro-Tips - how to approach questions
- Yellow for Caution - avoiding common pitfalls
- Cyan for Reminders

The worksheets have been designed with enough space for students to answer the questions directly in the booklet.

1.3 PRIME NUMBERS, HCF, LCM WORKSHEET AND ANSWER SHEET

Instructions

- Answer all questions.
- Show all your working clearly in the spaces provided.
- The number of marks for each question or part question is shown in brackets [ ].
- Do not use an electronic calculator for this worksheet.

Key Concepts: Prime Numbers, HCF and LCM

This worksheet covers the building blocks of numbers: prime factors. We will learn how to find these factors and then use them to solve problems involving the Highest Common Factor (HCF) and Lowest Common Multiple (LCM).

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1. Prime Numbers

Definition: Prime Number

A prime number is a natural number greater than 1 that has exactly two factors: 1 and itself.

Caution: Special Cases

- **1 is NOT a prime number.** It only has one factor (itself).
- **2 is the only even prime number.** All other even numbers can be divided by 2.

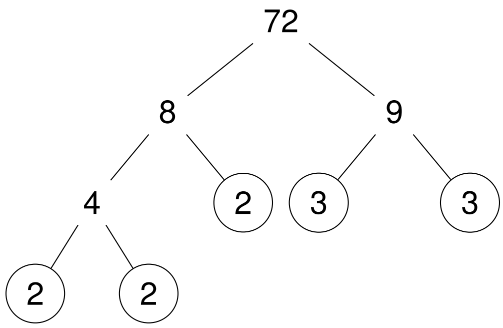
2. Prime Factorisation

This is the process of breaking a number down until it is written as a product of only prime numbers.

Method: The Factor Tree

1. Start with the number at the top.
2. Split the number into any two factors.
3. If a factor is a prime number, circle it.
4. If a factor is not prime, keep splitting it until you are left with only circled prime numbers.
5. Write down the final answer by multiplying all the circled primes together, using index notation for repeated factors.

Example: Find the prime factorisation of 72.



The prime factors are $2 \times 2 \times 2 \times 3 \times 3$. In index notation, this is $\mathbf{72 = 2^3 \times 3^2}$.

Deeper Insight: The Unique Factorisation Theorem

- Any natural number greater than 1 can be expressed as a product of prime factors in the way 72 has been expressed above.
- $\mathbf{72 = 2^3 \times 3^2}$
- This factorisation is unique, 72 can never be expressed as the product of any other prime factors.
- This is true for all natural numbers greater than one, they are either prime or their prime factorisation is unique.
- This theorem hold only for prime factorisation. 72 has 12 factors giving rise to 6 different 2 factor multiplicative factorisations.

This property is known as the Unique Factorisation Theorem.

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3. Highest Common Factor (HCF) and Lowest Common Multiple (LCM)

Key Definitions

- **Highest Common Factor (HCF):** The largest number that divides exactly into two or more numbers.
- **Lowest Common Multiple (LCM):** The smallest number that appears in the times tables of two or more numbers.

Method: Using Prime Factor Lists

This is the most reliable method for finding the HCF and LCM.

1. Find the **prime factorisation** of all the numbers and write them out.
2. **To find the HCF:** Look for the prime factors that are **common** to all the lists. Multiply one of each common factor together. If a factor is common multiple times (like two 2s), you include it multiple times.
3. **To find the LCM:** Start with the prime factorisation of the first number. Then, look at the other numbers and multiply in any prime factors that are "missing".

Example: Find the HCF and LCM of 48 and 180.

Step 1: Find the prime factorisations.

$$48 = 2 \times 2 \times 2 \times 2 \times 3$$
$$180 = 2 \times 2 \times 3 \times 3 \times 5$$

Step 2: Find the HCF (find the common factors).

$$48 = \mathbf{2} \times \mathbf{2} \times 2 \times 2 \times \mathbf{3}$$
$$180 = \mathbf{2} \times \mathbf{2} \times \mathbf{3} \times 3 \times 5$$

The factors they have in common are two 2s and one 3.

$$HCF = 2 \times 2 \times 3 = \mathbf{12}$$

Step 3: Find the LCM (start with 48's factors, then add the missing ones from 180).

- Start with all the factors of 48: $(2 \times 2 \times 2 \times 2 \times 3)$
- Look at 180's factors $(2 \times 2 \times 3 \times 3 \times 5)$. Compare them to 48's factors.
- We already have two 2s and a 3. The factors "missing" are another **3** and a **5**.
- Multiply these missing factors in:

$$LCM = (2 \times 2 \times 2 \times 2 \times 3) \times \mathbf{3} \times \mathbf{5} = 48 \times 15 = \mathbf{720}$$

Deeper Insight: The HCF/LCM Product Rule

For any two numbers A and B, there is a useful relationship:

$$\boldsymbol{A \times B = HCF(A, B) \times LCM(A, B)}$$

This can be a quick way to find the LCM if you already know the HCF, or vice-versa.

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1. (a) List all the prime numbers between 10 and 30. [2]

   (b) Explain why 1 is not a prime number. [1]

   (c) Explain why 39 is not a prime number. [1]

Total: [4]

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2. Express the following numbers as a product of their prime factors. Write your answers using index notation where appropriate.

   (a) 48 [2]

   (b) 90 [2]

   (c) 132 [2]

Total: [6]

3. Find the Highest Common Factor (HCF) of the following pairs of numbers:

   (a) 24 and 36 [2]

   (b) 50 and 75 [2]

   (c) 42 and 105 [2]

Total: [6]

4. Find the Lowest Common Multiple (LCM) of the following pairs of numbers:

   (a) 6 and 8 [2]

   (b) 10 and 15 [2]

   (c) 12 and 18 [2]

Total: [6]

5. Given that $A = 2^3 \times 3^2 \times 5$ and $B = 2^2 \times 3 \times 5^2$.

   (a) Find the HCF of A and B. Leave your answer as a product of prime factors. [2]

   (b) Find the LCM of A and B. Leave your answer as a product of prime factors. [2]

Total: [4]

6. Find the HCF and LCM of 12, 18 and 30.

(a) HCF [2]

(b) LCM [2]

Total: [4]

7. Two ribbons have lengths 84 cm and 140 cm. They are to be cut into smaller pieces of equal length, with no ribbon left over. What is the greatest possible length of each smaller piece? [2]

Total: [2]

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8. Two lighthouses flash their lights every 15 seconds and 18 seconds respectively. If they flash together at 9:00 pm, at what time will they next flash together? [3]

Total: [3]

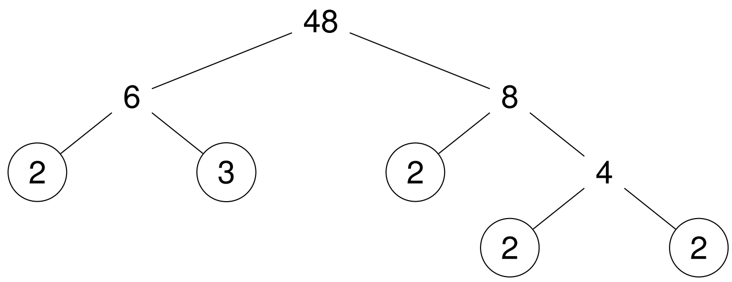
End of Worksheet

1) a) The prime numbers between 10 and 30 are: 11, 13, 17, 19, 23, 29.

b) The number 1 is not prime because it only has one divisor. All prime numbers have two divisors.

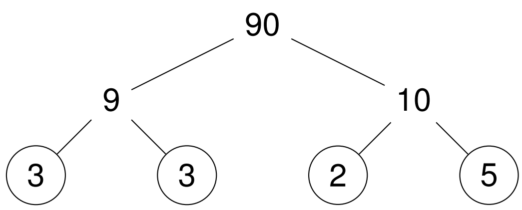
c) No, 39 is not a prime number. To check, we test for factors. Since $39 \div 3 = 13$, it has factors of 3 and 13 in addition to 1 and 39.

2) a) **Factor Tree for 48:**



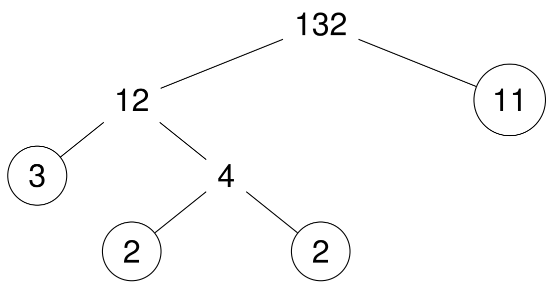
The prime factors are $2 \times 2 \times 2 \times 2 \times 3$. In index notation: $\mathbf{48 = 2^4 \times 3}$.

b) **Factor Tree for 90:**



The prime factors are $2 \times 3 \times 3 \times 5$. In index notation: $\mathbf{90 = 2 \times 3^2 \times 5}$.

c) **Factor Tree for 132:**



The prime factors are $2 \times 2 \times 3 \times 11$. In index notation: $\mathbf{132 = 2^2 \times 3 \times 11}$.

3)

Reminder: Finding the LCM from Prime Factors

(a) Express each number as a product of its prime factors (in index form).
(b) Identify the **highest power** of **all** the prime factors that appear in any of the numbers.
(c) Multiply these highest powers together to find the LCM.

a) $6 = 2 \times 3$ and $8 = 2^3$. The highest powers are 2^3 and 3.
LCM = $2^3 \times 3 =$ **24**.

b) $10 = 2 \times 5$ and $15 = 3 \times 5$. The highest powers are $2, 3, 5$.
LCM = $2 \times 3 \times 5 =$ **30**.

c) $12 = 2^2 \times 3$ and $18 = 2 \times 3^2$. The highest powers are 2^2 and 3^2.
LCM = $2^2 \times 3^2 = 4 \times 9 =$ **36**.

4) Given: $A = 2^3 \times 3^2 \times 5$ and $B = 2^2 \times 3 \times 5^2$.

a)

> **Pro-Tip: HCF from Index Form**
>
> To find the HCF, take the **lowest power** of each **common** prime factor.

Common factors are 2, 3, 5. Lowest powers are $2^2, 3^1, 5^1$.
HCF = $\mathbf{2^2 \times 3 \times 5}$.

b)

> **Pro-Tip: LCM from Index Form**
>
> To find the LCM, take the **highest power** of **every** prime factor present.

All factors are 2, 3, 5. Highest powers are $2^3, 3^2, 5^2$.
LCM = $\mathbf{2^3 \times 3^2 \times 5^2}$.

5) Prime factorisations: $12 = 2^2 \times 3,\ \ 18 = 2 \times 3^2,\ \ 30 = 2 \times 3 \times 5$.

a) The lowest powers of common factors (2, 3) are 2^1 and 3^1.
HCF = $2 \times 3 =$ **6**.

b) The highest powers of all factors (2, 3, 5) are $2^2, 3^2, 5^1$.
LCM = $2^2 \times 3^2 \times 5 = 4 \times 9 \times 5 =$ **180**.

6)

> **Pro-Tip: Identifying HCF Word Problems**
>
> Questions asking for the ”greatest”, ”largest”, or ”maximum” size of items that can be cut or arranged from larger groups without any remainder are HCF problems.

We need the HCF of 84 and 140.
$84 = 2^2 \times 3 \times 7$ and $140 = 2^2 \times 5 \times 7$.
The common factors are 2^2 and 7.
HCF = $2^2 \times 7 = 28$. The greatest possible length is **28 cm**.

7)

Pro-Tip: Identifying LCM Word Problems

Questions asking when two or more events with different cycles will happen ”together again” or ”simultaneously” are LCM problems.

We need the LCM of 15 and 18.
$15 = 3 \times 5$ and $18 = 2 \times 3^2$.
The highest powers of all factors are $2, 3^2, 5$.
LCM = $2 \times 3^2 \times 5 = 90$.
They will flash together after 90 seconds, which is 1 minute and 30 seconds. The time will be **9:01:30 pm**.