BRADLEY'S MATHS

GCSE Higher Level Mathematics

NUMBER

1.4: Rational and Irrational Numbers

William Bradley

19 November 2025

Abstract

This worksheet and answer sheet are, together, a stand alone teaching and learning document which introduce the concepts of rational and irrational numbers and reciprocals which are used in several sections of the syllabus.

This booklet has been fully updated with our unique colour coded box system to facilitate teaching and learning:

- Violet for Deeper Insights into the subject matter
- Green for Methods and Examples
- Blue for Pro-Tips - how to approach questions
- Yellow for Caution - avoiding common pitfalls
- Cyan for Reminders

The worksheets have been designed with enough space for students to answer the questions directly in the booklet.

1.4 RATIONAL AND IRRATIONAL NUMBERS WORKSHEET AND ANSWER SHEET

Instructions

- Answer all questions.
- Show all your working clearly in the spaces provided.
- The number of marks for each question or part question is shown in brackets [ ].
- Do not use an electronic calculator for this worksheet.

Key Concepts: Rational Numbers, Irrational Numbers, and Reciprocals

This worksheet explores the different types of numbers that make up the ”real” number system. Understanding the distinction between rational and irrational numbers is a key skill in mathematics.

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1. The Hierarchy of Real Numbers ($\mathbb{R}$)

All the numbers you use in IGCSE Maths are Real Numbers. They can be split into two distinct groups: Rational and Irrational.

The Family of Real Numbers

Real Numbers ($\mathbb{R}$)

Rational Numbers ($\mathbb{Q}$)
(Can be written as a fraction)
- Integers ($\mathbb{Z}$) {...,-2,-1,0,1,2,...}
- Terminating Decimals (e.g. 0.5)
- Recurring Decimals (e.g. 0.333...)

Irrational Numbers
(Cannot be written as a fraction)
- Famous Constants (e.g. π)
- Non-perfect Square Roots (e.g. $\sqrt{2}, \sqrt{3}, \sqrt{5}$)
- Non-recurring, non-terminating decimals

The Rational Number Test

A number is **rational** if it can be written as a fraction $\frac{a}{b}$, where a and b are integers and $b \neq 0$. Ask yourself:

- Is it an **integer**? (e.g. $7 = \frac{7}{1}$) $\rightarrow$ Rational.
- Is it a **terminating decimal**? (e.g. $0.5 = \frac{1}{2}$) $\rightarrow$ Rational.
- Is it a **recurring decimal**? (e.g. $0.\overline{3} = \frac{1}{3}$) $\rightarrow$ Rational.
- Is it a **square root of a square number**? (e.g. $\sqrt{9} = 3 = \frac{3}{1}$) $\rightarrow$ Rational.

If the answer to all of these is no, the number is **irrational**.

Common Error: The Value of Pi (π)

The fraction $\frac{22}{7}$ is a useful *approximation* for π, but it is not its exact value. π is an irrational number, meaning its decimal representation goes on forever without repeating. It **cannot** be written as an exact fraction.

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2. Reciprocals

Method: Finding the Reciprocal

The reciprocal of a number is simply 1 divided by that number. A simple way to think of it is "flipping the fraction". The product of a number and its reciprocal is always 1.

1. **Write the number as a fraction.**
   - An integer like 5 becomes $\frac{5}{1}$.
   - A mixed number like $1\frac{1}{2}$ becomes an improper fraction $\frac{3}{2}$.
2. **Flip the fraction upside down.**

Examples:

- The reciprocal of 5 (which is $\frac{5}{1}$) is $\mathbf{\frac{1}{5}}$.
- The reciprocal of $\frac{2}{3}$ is $\mathbf{\frac{3}{2}}$.
- The reciprocal of $1\frac{1}{2}$ (which is $\frac{3}{2}$) is $\mathbf{\frac{2}{3}}$.

Caution: Special Cases

- The reciprocal of a **negative number** is also negative. The sign does not change.
- The number **zero does not have a reciprocal**, as you cannot divide by zero.

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1. State whether each of the following numbers is Rational (R) or Irrational (I). [1 mark each]

   (a) 7

   (b) $\sqrt{9}$

   (c) 0.5

   (d) π

   (e) $\frac{3}{4}$

   (f) $\sqrt{5}$

   (g) $0.333...$ (recurring)

(h) -2

Total: [8]

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2. State whether each of the following statements is True (T) or False (F). If it is false, give an example which demonstrates that it is false. [1 mark each]

   (a) All integers are rational numbers.

   (b) All square roots are irrational numbers.

   (c) Terminating decimals are irrational numbers.

   (d) π can be written exactly as $\frac{22}{7}$.

   (e) The sum of two rational numbers is always rational.

   (f) The product of two irrational numbers is always irrational.

Total: [6]

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3. (a) Give an example of a rational number between 2 and 3. [1]

(b) Give an example of an irrational number. [1]

(c) Give an example of an integer that is not a natural number. [1]

Total: [3]

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4. Write down the reciprocal of each of the following numbers. Give fractional answers in their simplest form.

   (a) 4 [1]

   (b) $\frac{1}{3}$ [1]

   (c) -2 [1]

   (d) $\frac{5}{6}$ [1]

   (e) 1 [1]

   (f) $-\frac{3}{4}$ [1]

Total: [6]

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5. Write down the reciprocal of each of the following numbers. Give fractional answers in their simplest form where appropriate.

   (a) 0.5 [1]

   (b) 0.1 [1]

   (c) $1\frac{1}{2}$ [2]

   (d) 0.25 [1]

Total: [5]

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6. Work out the following:

   (a) The reciprocal of 5, added to the reciprocal of 2. [2]

   (b) The reciprocal of $\frac{3}{5}$, multiplied by 6. [2]

Total: [4]

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7. Place the following numbers in order of size, smallest first:

$$\sqrt{4}, \quad \pi, \quad 3, \quad \sqrt{10}, \quad \frac{7}{2}$$

(Hint: Estimate the value of the irrational numbers.) **Total: [3]**

End of Worksheet

1)

Reminder: Rational and Irrational Numbers

- A **Rational** number can be written as a fraction $\frac{a}{b}$, where a and b are integers. Integers, terminating decimals, and recurring decimals are all rational.
- An **Irrational** number cannot be written as a simple fraction. They are non-terminating, non-recurring decimals (e.g., π, $\sqrt{2}$).

a) R (Rational). Any integer can be written as a fraction with a denominator of 1, e.g., $\frac{7}{1}$.

b) R (Rational). $\sqrt{9} = 3$, which is an integer.

c) R (Rational). 0.5 can be written as the fraction $\frac{1}{2}$.

d) I (Irrational). π is a non-terminating, non-recurring decimal.

e) R (Rational). It is expressed as a fraction of two integers.

f) I (Irrational). The square root of a non-square number is irrational.

g) R (Rational). All recurring decimals are rational (in this case, $\frac{1}{3}$).

h) R (Rational). -2 is an integer.

2)

Caution: Common Misconceptions

- The square root of a number is not always irrational (e.g., $\sqrt{4} = 2$).
- $\frac{22}{7}$ is a rational **approximation** of π, not its exact value.
- The product of two irrational numbers can be rational (e.g., $\sqrt{2} \times \sqrt{2} = 2$).

a) **T (True)**.

b) **F (False)**.

c) **F (False)**.

d) **F (False)**.

e) **T (True)**.

f) **F (False)**.

3) a) For example: $\frac{5}{2}$ or 2.5.

b) For example: π or $\sqrt{2}$.

c) For example: –5 or 0.

4)

Reminder: Finding a Reciprocal

The reciprocal of a number is 1 divided by the number.

- For a simple fraction $\frac{a}{b}$, the reciprocal is $\frac{b}{a}$.
- For other numbers (integers, decimals, mixed numbers), you must first convert them into a simple fraction before finding the reciprocal.

a) $\frac{1}{4}$

b) 3

c) $-\frac{1}{2}$

d) $\frac{6}{5}$

e) 1

f) $-\frac{4}{3}$

5) a) First, $0.5 = \frac{1}{2}$. The reciprocal is 2.

b) First, $0.1 = \frac{1}{10}$. The reciprocal is 10.

c) First, $1\frac{1}{2} = \frac{3}{2}$. The reciprocal is $\frac{2}{3}$.

d) First, $0.25 = \frac{1}{4}$. The reciprocal is 4.

6) a) The reciprocals are $\frac{1}{5}$ and $\frac{1}{2}$. $\frac{1}{5} + \frac{1}{2} = \frac{2}{10} + \frac{5}{10} = \frac{7}{10}$

b) The reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$. $\frac{5}{3} \times 6 = \frac{30}{3} = 10$

7)

Reminder: Ordering Different Number Types

The most reliable method to compare and order different types of numbers (fractions, roots, etc.) is to convert them all into a common format. Decimals are usually the easiest.

Converting to decimals: $\sqrt{4} = 2$, $3 = 3$, $\pi \approx 3.142$, $\sqrt{10} \approx 3.162$, $\frac{7}{2} = 3.5$.

In order from smallest to largest: $\sqrt{4}$, $\mathbf{3}$, π, $\sqrt{\mathbf{10}}$, $\frac{\mathbf{7}}{\mathbf{2}}$