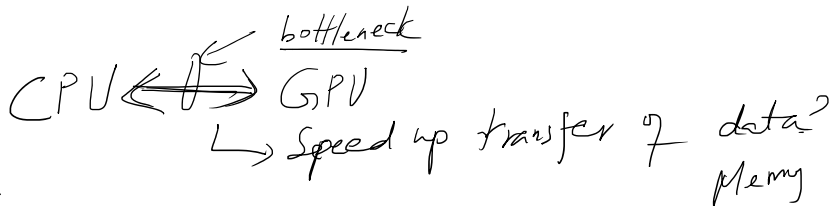


Pinned memory

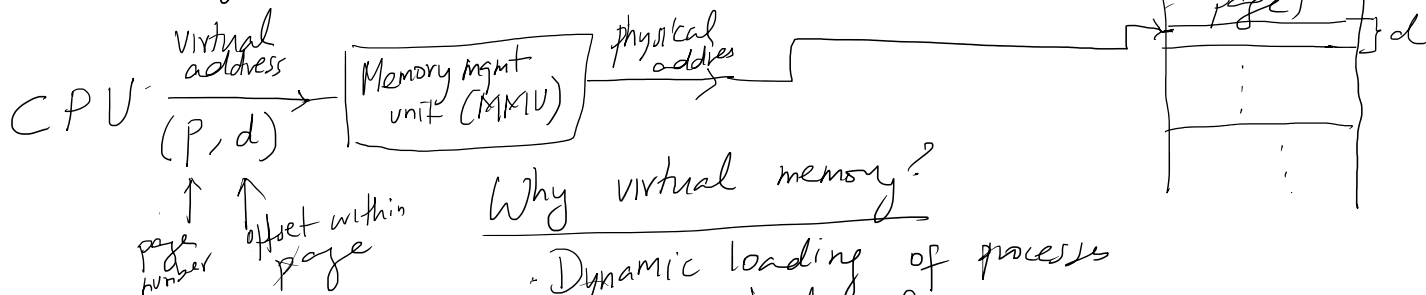
Tuesday, March 2, 2021 8:35 AM

- Pinned or page-locked memory improves achieved bandwidth between host and device
 - o Enables zero-copy transfers
- Use `cudaHostAlloc()` to request pinned pages from the Operating System
- Code example: `page_locked_memory`

Page locked memory / Pinned memory



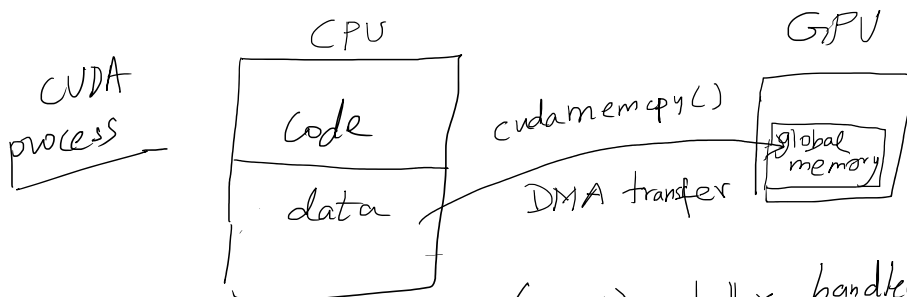
Virtual memory



Why virtual memory?

- Dynamic loading of processes
- demand paging
- Address space protection

Overhead due to address translation

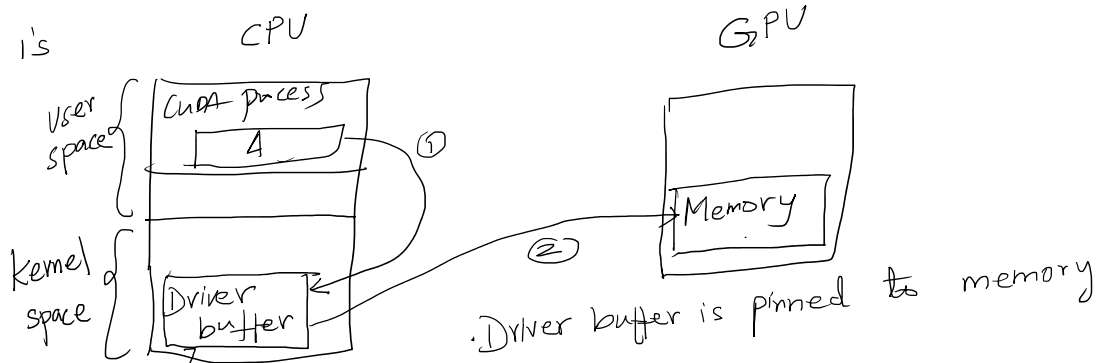


Direct memory access (DMA) controller handles data transfer

- src address (host)
- dest address (device)
- # of bytes to transfer

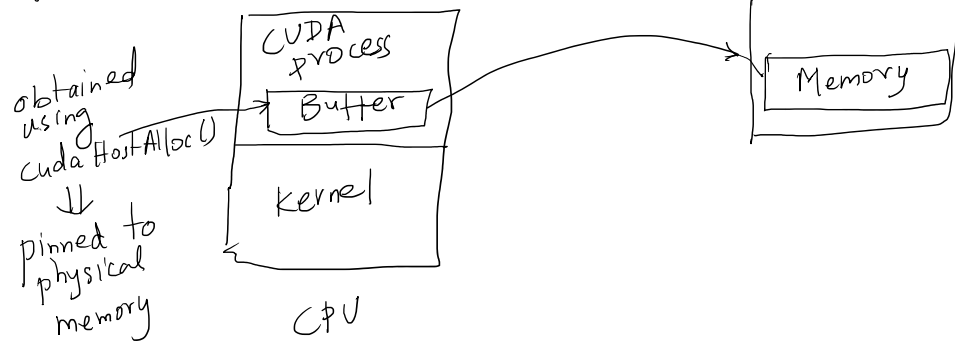
→ DMA happens in background
 → Frees up CPU for other jobs

DMA transfer is a two step process



Zero-copy transfer:

Use `cudaHostAlloc()` to request pined memory from operating system



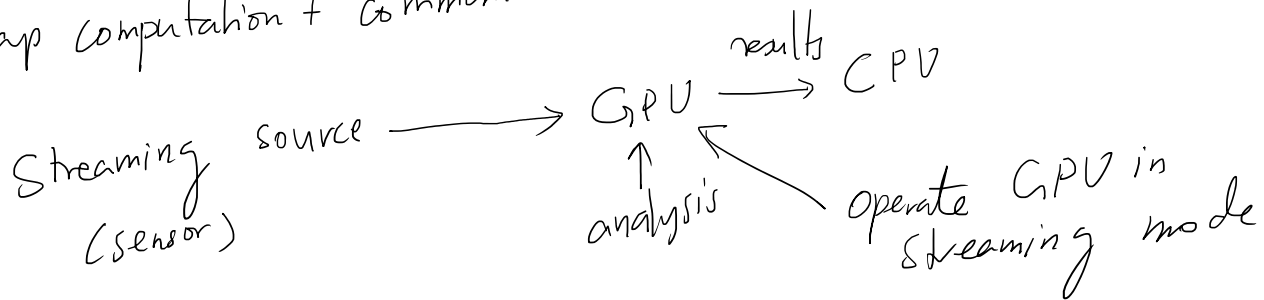
Upto 2x speedup of
CPU \leftrightarrow GPU
data transfer

So far: data is transferred in full before computation begins



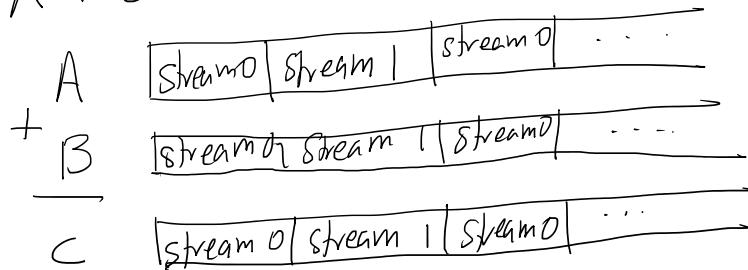
Streaming model:

- 1) Transfer data in smaller chunks
- 2) GPU can start processing chunk
- 3) Continue to transfer additional chunks to GPU
- 4) Overlap computation + communication

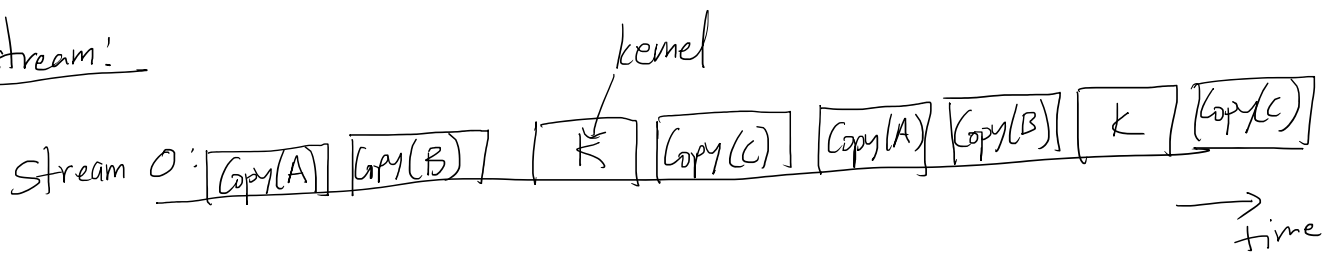


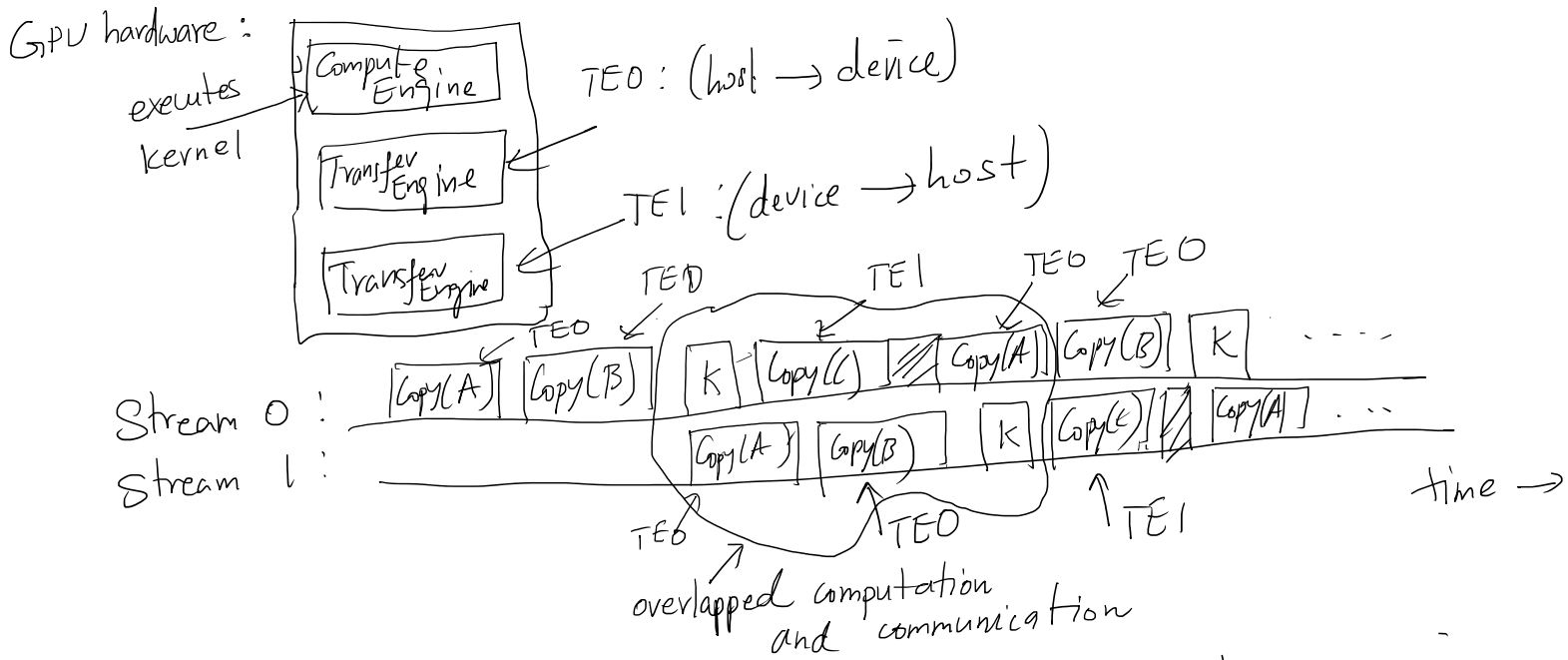
Vector addition:

$$C = A + B$$



Single stream:





- All operations are done in async fashion using pinned memory.

Unified memory

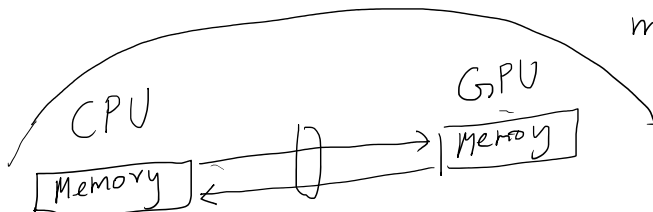
Tuesday, March 2, 2021 8:35 AM

<https://devblogs.nvidia.com/unified-memory-cuda-beginners/>

<https://devblogs.nvidia.com/maximizing-unified-memory-performance-cuda/>

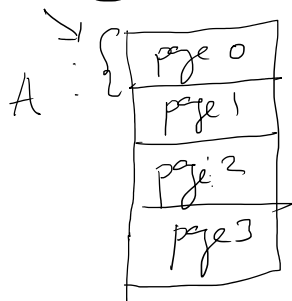
Code example: `unified_mem`

Data transfer between the memories is managed by the CUDA runtime



Page migration between CPU and GPU:

16 KB CPU



Page table
valid dirty

page 0	1	0
page 1	1	0
page 2	0	0
page 3	1	0

Launch kernel

GPU

page 0	0	0
page 1	0	0
page 2	1	0
page 3	0	0

Suppose kernel accesses data in page 2:

- 1) Page fault
- 2) Page 2 migrated from CPU to GPU
- 3) Page tables updated on GPU and CPU

Using CUDA with OpenMP

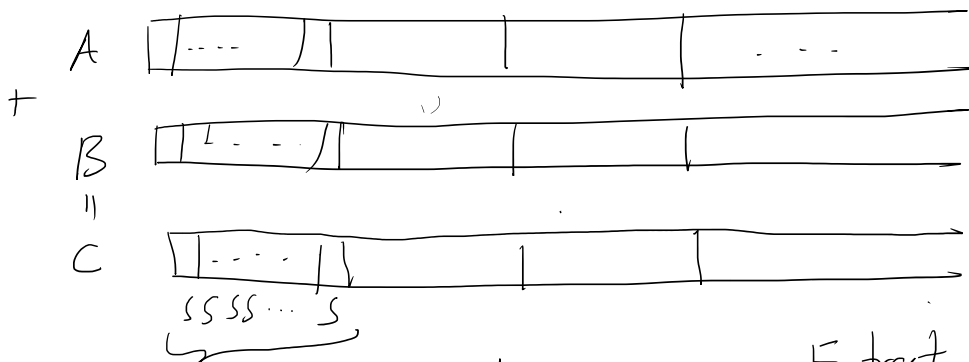
Tuesday, March 9, 2021 2:17 PM

Code example: `cuda_omp`

$$C = A + B$$

OpenMP threads on CPU

- Extract coarse-grained parallelism using OpenMP



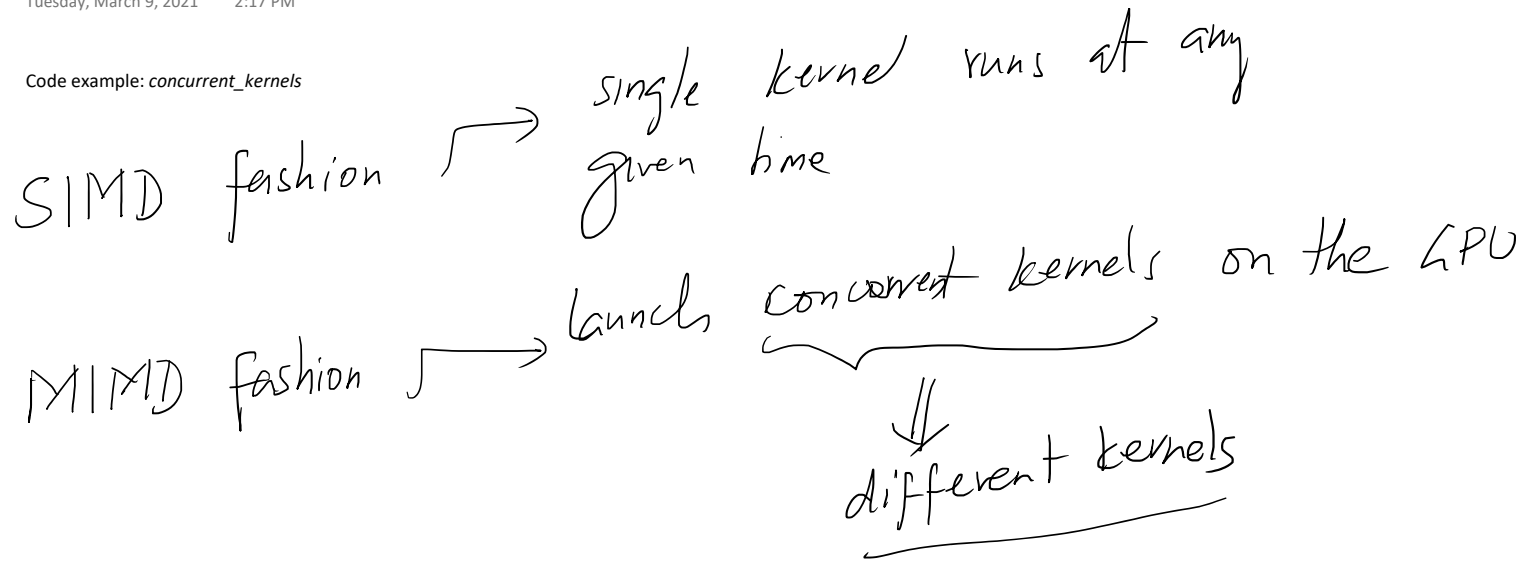
↑ CUDA threads per OpenMP thread

- Extract fine-grained parallelism using CUDA

Concurrent kernel execution

Tuesday, March 9, 2021 2:17 PM

Code example: `concurrent_kernels`



CUDA BLAS examples

Sunday, May 31, 2020 12:04 PM

<https://docs.nvidia.com/cuda/cublas/index.html>

Single precision AX plus Y (SAXPY)

Vector dot product (dot)

Single precision general matrix-matrix multiplication (sgemm)

Single precision general matrix-vector multiplication (sgemv)

See CUBLAS documentation on BBLearn

BLAS : Basic linear algebra subroutines

Vector processing on the CPU

Thursday, March 11, 2021 8:43 AM

- *Streaming SIMD Extensions (SSE)* is a single instruction, multiple data (SIMD) instruction set extension to the x86 architecture

- SSE2:

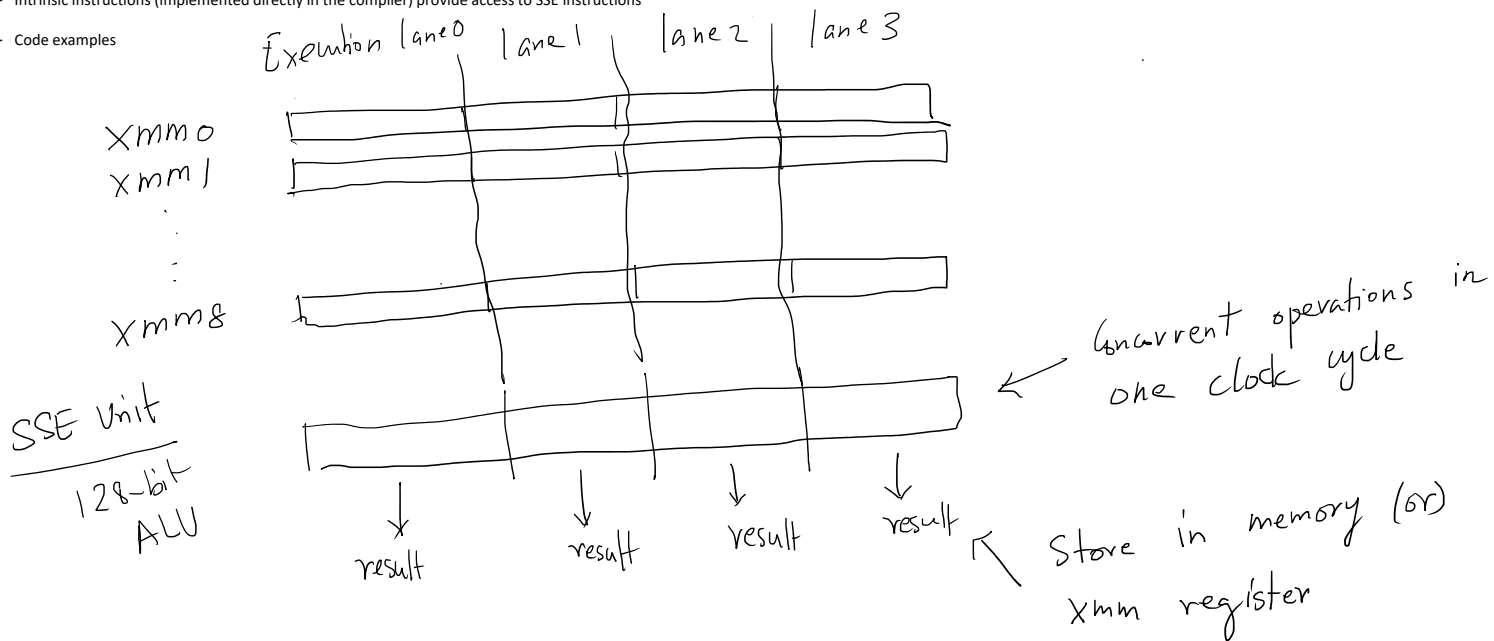
- o Eight new 128-bit registers known as XMM0 through XMM7
- o XMM registers can be configured to hold
 - Four 32-bit single-precision floating-point numbers or
 - Two 64-bit double-precision floating-point numbers or
 - Two 64-bit integers or
 - Four 32-bit integers or
 - Eight 16-bit short integers or
 - Sixteen 8-bit bytes or characters
- o The ISA supports both scalar and packed scalar (vector) instructions
 - Memory-to-register/register-to-memory/register-to-register data movement
 - Arithmetic operations (add, Subtract, multiply, divide)
 - Bit-wise logical operations
 - Compare and shuffle
 - ...

- More recent developments: *Advanced Vector Extensions (AVX)*

- AVX uses sixteen YMM registers, each of width 256 bits, to perform SIMD operations
 - Eight 32-bit single-precision floating point numbers or
 - Four 64-bit double-precision floating point numbers
- AVX-512 uses 32 512-bit registers (ZMM0-ZMM31) for SIMD operations

- Intrinsic instructions (implemented directly in the compiler) provide access to SSE instructions

- Code examples



$$\begin{array}{c}
 A_1 x = b \\
 \uparrow \quad \uparrow \quad \uparrow \\
 n \times n \quad n \times 1 \quad n \times 1
 \end{array}
 \Rightarrow
 \begin{array}{c}
 U x = b' \\
 \uparrow \\
 \text{upper} \\
 \text{triangular} \\
 \text{matrix}
 \end{array}$$

Gaussian elimination

$$\begin{aligned}
 3x + 5y + 2z &= 19 \\
 2x + 3y + z &= 11 \\
 x + 2y + 2z &= 11
 \end{aligned}$$

$$A \begin{bmatrix} 3 & 5 & 2 \\ 2 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 19 \\ 11 \\ 11 \end{bmatrix} \begin{matrix} R1 \\ R2 \\ R3 \end{matrix}$$

1) Division step: $R1/3$

$$A' = \begin{bmatrix} 1 & 5/3 & 2/3 \\ 2 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

$$b' = \begin{bmatrix} 19/3 \\ 11 \\ 11 \end{bmatrix}$$

2) Elimination step

$$\begin{aligned}
 R2 &= R2 - 2R1 \\
 R3 &= R3 - R1
 \end{aligned}
 \begin{bmatrix} 1 & 5/3 & 2/3 \\ 0 & -1/3 & -1/3 \\ 0 & 1/3 & 4/3 \end{bmatrix}$$

$$b' = \begin{bmatrix} 19/3 \\ -5/3 \\ 14/3 \end{bmatrix} \begin{matrix} R1 \\ R2 \\ R3 \end{matrix}$$

3) Division step: $R2/-1/3$

$$A' = \begin{bmatrix} 1 & 5/3 & 2/3 \\ 0 & 1 & 1 \\ 0 & 1/3 & 4/3 \end{bmatrix} \quad b' = \begin{bmatrix} 19/3 \\ 5 \\ 14/3 \end{bmatrix}$$

4) Elimination step:

$$R3 = R3 - \frac{1}{3}R2 \quad A' = \begin{bmatrix} 1 & 5/3 & 2/3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad b' = \begin{bmatrix} 19/5 \\ 5 \\ 3 \end{bmatrix}$$

5) Division:

$$U = \begin{bmatrix} 1 & 5/3 & 2/3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad b' = \begin{bmatrix} 19/5 \\ 5 \\ 3 \end{bmatrix}$$

$$U \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_{n-1} \end{bmatrix} = \begin{bmatrix} b_0 \\ b_1 \\ \vdots \\ b_{n-1} \end{bmatrix}$$

zeros \rightarrow a

Back substitution

$$x_{n-1} = b_{n-1}$$

$$x_{n-2} + a \cdot x_{n-1} = b_{n-2}$$

$$x_{n-2} = b_{n-2} - a \cdot x_{n-1}$$

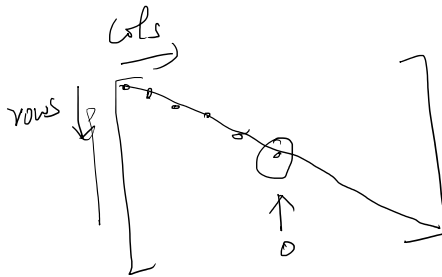
Back substitution :

$$\left. \begin{aligned} z &= 3 \\ y + z &= 5 \\ y &= 5 - 3 = 2 \\ x + 5/3 y + 2/3 z &= 19/5 \\ x &= 1 \end{aligned} \right\}$$

• Numerical stability

• Not stable

• Exceptions on GPU
- Division by zero



• Silent exception

$$a/0 \Rightarrow \text{NaN}$$

$$\text{NaN} + \text{NaN} = \text{NaN}$$

Host-side code :

```

Transfer A to device
for (k=0; k<n; k++) { // Process rows
    call division kernel(A, k, n)
    sync.
    Call elimination kernel(A, k, n)
} sync.
Read A from device.
    
```

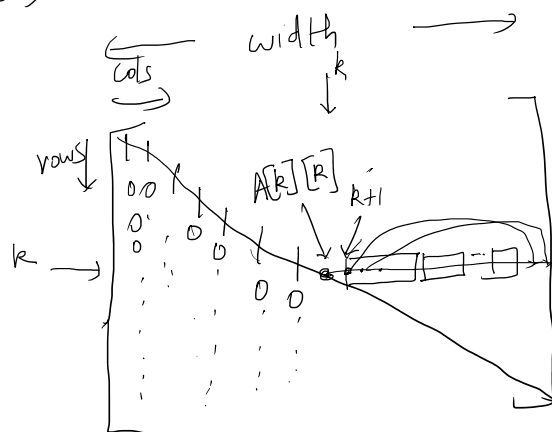
Device code:

• Design considerations:

- 1) Fewer number of thread blocks
- 2) Coalesced access to global memory

thread blocks: m
 thread block size: $(p, 1, 1)$
 $grid(m, 1)$

Division:

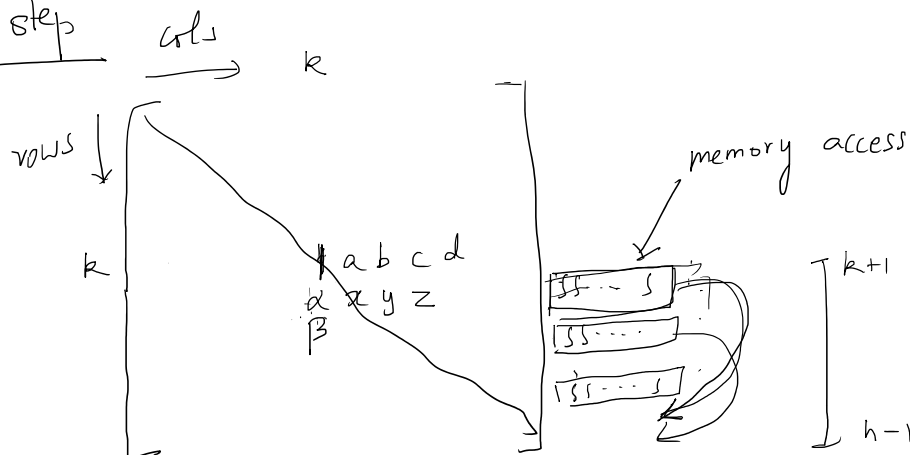


kernel code:

```

tid = blockIdx.x * blockDim.x + threadIdx.x
stride = gridDim.x * blockDim.x
pivot = A[k * width + k]
while (tid < n) {
    elem = A[k * width + k + 1 + tid]
    elem = elem / pivot
    tid = tid + stride
}
if (blockIdx.x == 0 &&
    threadIdx.x == 0)
    A[k * width + k] = 1
  
```

Elimination step



kernel :

$$\text{row} = k + 1 + \text{blockIdx} \cdot x$$

```
while (row < n) { // stride across rows
    tid = threadIdx.x
    while (tid < n) { // stride along row
        x = A[row * width + k + 1 + tid]
        x = x -  $\alpha$  * a ← A[k * width + k + 1 + tid]
                     ↑
                     A[row * width + k]
        tid = tid + blockDim.x
    }
    sync
    if (threadIdx.x == 0)
         $\alpha = 0$ 
    row = row + blockDim.x
}
```