# STAT 600 - HW 2

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## Question 1

### (a)

First, consider the likelihood and the log-likelihood function.

$$
\begin{aligned}
L(\theta) &= \prod_{i=1}^{n} \frac{1}{\pi\left(1+(x_i-\theta)^2\right)} \\
l(\theta) &= \log\left(L(\theta)\right) \\
&= \log\left(\prod_{i=1}^{n} \frac{1}{\pi\left(1+(x_i-\theta)^2\right)}\right) \\
&= \sum_{i=1}^{n} \log\left(\frac{1}{\pi\left(1+(x_i-\theta)^2\right)}\right) \\
&= -\sum_{i=1}^{n} \log\left(\pi\left(1+(x_i-\theta)^2\right)\right) \\
&= -n\log(\pi) - \sum_{i=1}^{n} \log\left(1+(x_i-\theta)^2\right)
\end{aligned}
$$

Then, consider the derivative of the log-likelihood, $l'(\theta)$.

$$
\begin{aligned}
l'(\theta) &= \frac{d}{d\theta} l(\theta) \\
&= -\sum_{i=1}^{n} \frac{1}{1+(x_i-\theta)^2}\left[\frac{d}{d\theta}(x_i-\theta)^2\right] \\
&= 2\sum_{i=1}^{n} \frac{x_i-\theta}{1+(x_i-\theta)^2}
\end{aligned}
$$

```
dat <- c(-8.86, -6.82, -4.03, -2.84, 0.14, 0.19, 0.24, 0.27, 0.49, 0.62, 0.76, 1.09,
         1.18, 1.32, 1.36, 1.58, 1.58, 1.78, 2.13, 2.15, 2.36, 4.05, 4.11, 4.12,
         6.83)
rangeTheta <- seq(-10, 10, 0.01)
data.frame(theta = rangeTheta, dll = sapply(rangeTheta, dloglik, x = dat)) %>%
  ggplot(aes(x = theta, y = dll)) +
  geom_line() +
  theme_bw() +
  labs(x = TeX("\\theta"), y = TeX("l'(\\theta)"),
       title = "The plot of the derivative of the log-likelihood based on 25 daat points")
```

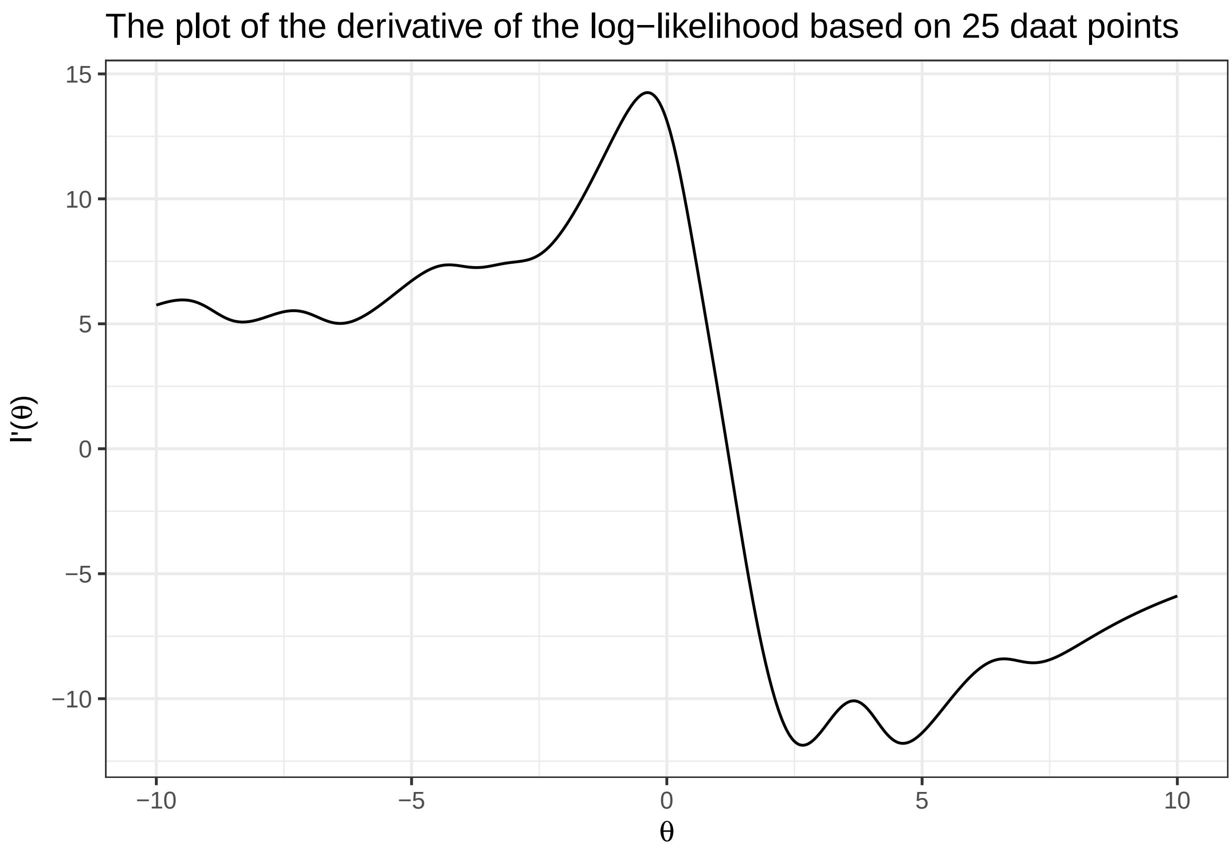


Figure 1: The plot of the derivative of the log-likelihood on the original dataset.

**(b)**

This is the second derivation for the log-likelihood function.

$$
\begin{aligned}
l''(\theta) &= \frac{d}{d\theta} l'(\theta) \\
&= \frac{d}{d\theta} 2 \sum_{i=1}^{n} \frac{x_i - \theta}{1 + (x_i - \theta)^2} \\
&= 2 \sum_{i=1}^{n} \frac{d}{d\theta} \frac{x_i - \theta}{1 + (x_i - \theta)^2} \\
&= 2 \sum_{i=1}^{n} \frac{-1 + (x_i - \theta)^2}{1 + (x_i - \theta)^2}
\end{aligned}
$$

**(d)**

For the convergence criteria used in this problem, I decided to employ the absolute convergence criterion, as the $x^{(t)}$ might be close to 0 in some iterations, as indicated by the plot shown in part (a). Additionally, the value of x is neither too tiny nor too huge compared to $\epsilon$.