STAT 600 - HW 2

Kevin Korsurat

Question 1

(a)

First, consider the likelihood and the log-likelihood function.

$$L\left(\theta\right) = \prod_{i=1}^{n} \frac{1}{\pi\left(1 + (x_i - \theta)^2\right)}$$

$$l\left(\theta\right) = \log\left(L\left(\theta\right)\right)$$

$$= \log\left(\prod_{i=1}^{n} \frac{1}{\pi\left(1 + (x_i - \theta)^2\right)}\right)$$

$$= \sum_{i=1}^{n} \log\left(\frac{1}{\pi\left(1 + (x_i - \theta)^2\right)}\right)$$

$$= -\sum_{i=1}^{n} \log\left(\pi\left(1 + (x_i - \theta)^2\right)\right)$$

$$= -n\log\left(\pi\right) - \sum_{i=1}^{n} \log\left(1 + (x_i - \theta)^2\right)$$

Then, consider the derivative of the log-likelihood, $l'\left(\theta\right)$.

$$l'\left(\theta\right) = \frac{d}{d\theta} l\left(\theta\right)$$

$$= -\sum_{i=1}^{n} \frac{1}{1 + (x_i - \theta)^2} \left[\frac{d}{d\theta}(x_i - \theta)^2\right]$$

$$= 2\sum_{i=1}^{n} \frac{x_i - \theta}{1 + (x_i - \theta)^2}$$

```r
dat <- c(-8.86, -6.82, -4.03, -2.84, 0.14, 0.19, 0.24, 0.27, 0.49, 0.62, 0.76, 1.09,
         1.18, 1.32, 1.36, 1.58, 1.58, 1.78, 2.13, 2.15, 2.36, 4.05, 4.11, 4.12,
         6.83)
rangeTheta <- seq(-10, 10, 0.01)
data.frame(theta = rangeTheta, dll = sapply(rangeTheta, dloglik, x = dat)) %>%
  ggplot(aes(x = theta, y = dll)) +
  geom_line() +
  theme_bw() +
  labs(x = TeX("\\theta"), y = TeX("l'(\\theta)"))
```