

## An improved CP/MIP formulation for batch processing of non-identical jobs

Efficiency in manufacturing often boils down to managing limited resources in clever ways in order to perform the given jobs as quickly as possible. One such type of problem,  $1|p\text{-batch}; n < b; \text{non-identical}|L_{\max}$ , is concerned with jobs of non-identical sizes, processing times and due dates that are to be assigned to a resource (e.g. an oven) in batches. The following constraints then apply: batch sizes must not exceed the resource's capacity; each batch's processing time equals that of the greatest of all the processing times of its jobs; and each batch's lateness  $L$  is the difference between its completion time and the the earliest of all the due dates of its jobs.

The objective is to batch jobs together such as to minimize the greatest individual batch lateness  $L_{\max}$ ; the problem is NP-hard as shown in [1].

### Research goal

This problem was recently examined by Malapert [2], who suggested a global constraint approach implemented as three filtering rules applied each time a job is assigned to a batch. This technique was compared with a simple Mixed Integer Programming (MIP) model to show its superior performance.

However, the MIP model given in [2] can likely be improved. Using [7], I will first re-model the given MIP formulation and compare it to an equivalent Constraint Programming (CP) formulation. I will then experiment with additional constraints in both CP and MIP to speed up the search for feasible batch assignments, particularly bounds on  $L_{\max}$  and  $x_{jk}$ . Such constraints will be taken from considerations discussed in prior conversations, existing literature, and potentially from examinations of how the solver handles the given models. The goal is to tighten the CP/MIP models until their performance match or exceed that of the global constraint implementation in [2], if possible.

### Existing literature

An extensive overview over CP/MIP techniques in scheduling is given in [3]. General literature reviews are provided in [4] and [5]. Keha et al. [6] discuss various MIP formulations for single-machine scheduling problems. The same problem as in this paper, but with a different objective was examined in [8], and optimality criteria and a heuristic are proposed in [9].

## References

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