PH 434 Autumn 2025 – Programming Lab.

Practical Class 9 (Dated: 17. 10. 2025)

Question 1

Consider the circuit below:

1) Write a function that creates the 3-qubit initial state with i and j as input, where $i, j \in [0, 1]$.

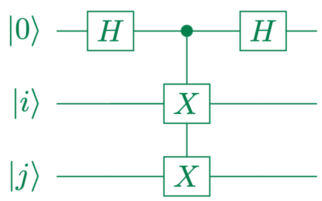
2) Write a function that implements the 3-qubit controlled-CNOT-CNOT gate.

3) Finally, write a function that combines the two above to implement the circuit. Display the output for input states: $\{i, j\} = [\{0, 0\}, \{0, 1\}, \{1, 1\}]$.

In [1]:

```
from IPython.display import Image
Image("HH.png",width=200)
```

Out[1]:



Hint: If gate U is applied on 1st qubit, then the combined three qubit gate is $U \otimes I \otimes I$, where I is applied to the other two qubits. See the command below:

```
import numpy as np
U_g = np.kron(np.kron(U,I),I)
```

Question 2

Define a function that implements a rotation by angle θ in the Bloch sphere, along the vector $\vec{n}$ = $\{n_x, n_y, n_z\}$. Note that $\vec{n}$ is real and has unit length.

$$R_n(\theta) = \cos(\theta/2)\,\mathrm{I} - i\,\sin(\theta/2)(n_x X + n_y Y + n_z Z)$$

```
def rotation(theta=45,n=[1,0,0]):
    ...
    ...
    return Rn
```

Apply the rotation to the state $|0\rangle$ and rotate along the Z axis. Compare with the $R_z(\theta)$ done earlier.

Question 3

Measurement and collapse of the wavefunction

Consider measuring one of the qubits in a two-qubit state: $\psi_{12} = \sum_{i,j} c_{i,j}|i, j\rangle$, where $i, j \in \{|0\rangle, |1\rangle\}$.

If the first qubit i is measured and collapses to the state $|i=0\rangle$, then the second qubit collapses to the state: $\phi_2 = \frac{1}{N}\sum_j c_{0,j}|j\rangle$.

Similarly, if i collapses to the state $|i=1\rangle$, then the second qubit is $\phi_2 = \frac{1}{N}\sum_j c_{1,j}|j\rangle$, where N is the normalization constant.

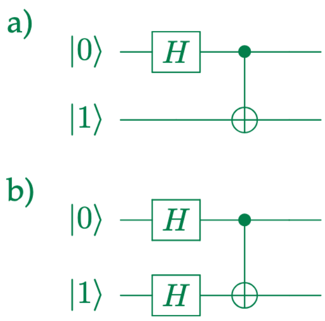
Write a function that finds the state of j when i is measured for an arbirary two-qubit state ψ_{12}. Note the measurement outcome needs to be mentioned.

```
def Meas(state=psi,mi = 0):
    ...
    ...
    return Rn
```

Perform the measurement on the output of the following circuits.

In [2]:
```
Image("Meas.png",width=200)
```

Out[2]:



Challenge

Write a function that finds the output of the first qubit in the circuit below.

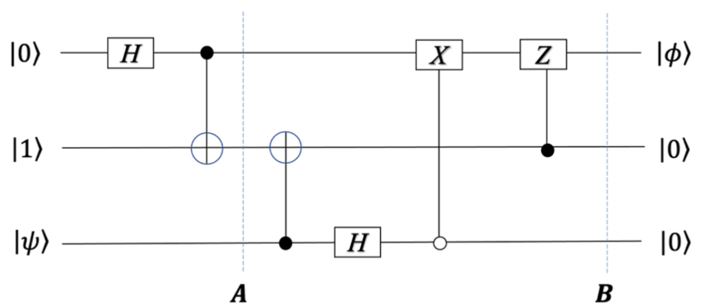
The states $|0\rangle$ at the end of second and third wires imply the measured or collapsed states.

Hint: For CNOT gate U_{CNOT} applied between qubits 1 and 2, the combined three qubit gate can be taken as:

```
import numpy as np
np.kron(U_{CNOT},I)
```

In [4]:
```
Image("Picture1.png",width=400)
```

Out[4]:



In [ ]:
```
```