PH 434 Autumn 2025 – Programming Lab.

Practical Class 10 (Dated: 24. 10. 2025)

Transverse Field Ising Model

Consider the 1D Transverse Field Ising Model (TFIM) for a chain of N spins, described by the Hamiltonian:

$$H = -J \sum_{i=1}^{N-1} \sigma_x^{(i)} \sigma_x^{(i+1)} - h \sum_{i=1}^{N} \sigma_z^{(i)}, \tag{1}$$

where the Pauli matrices are given by:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{2}$$

The operators acting on site i are defined as:

$$\sigma_i^k = I_1 \otimes I_2 \otimes \cdots \otimes \sigma_i^k \otimes \cdots \otimes I_N, \tag{3}$$

and for nearest-neighbor interactions:

$$\sigma_i^k \sigma_i^{k+1} = I_1 \otimes I_2 \otimes \cdots \otimes \sigma_i^k \otimes \sigma_i^{k+1} \otimes \cdots \otimes I_N. \tag{4}$$

Question 1: Construct the Hamiltonian

1. Construct the full Hamiltonian matrix H for $N = 8$ spins using Kronecker products (`np.kron` in Python).
   (Take $J = 1.0$ and $h = 0.5$.)
2. Diagonalize the Hamiltonian numerically using np.linalg.eigh(H)

Question 2: Finding the Ground State Energy using the Power Method

In this question, you will estimate the **ground state energy** of the same Transverse Field Ising Hamiltonian constructed in **Question 1** using the **Power Method**.

Algorithm

Follow the steps below to implement the Power Method:

1. Initialize a random normalized state of size 2^L.
2. To find ground state (lowest energy), define:

   $$H' = \lambda I - H \tag{5}$$

   where (λ) is a constant slightly larger than the maximum eigenvalue of H. Repeating the above process on H' makes the ground state dominate.
3. Iterative update : Apply the Hamiltonian repeatedly:

   $$|\psi_{k+1}\rangle = H'|\psi_k\rangle \tag{6}$$

   After each step, normalize the new state.
4. Compute the energy expectation at each step:

   $$E_k = \langle \psi_k | H' | \psi_k \rangle \tag{7}$$

5. Iterate the above process and stop when $|E_{k+1} - E_k| < 10^{-7}$. Print the ground state energy.

Open Quantum System Dynamics

In quantum mechanics, the state of a system is described by a **density operator** $\rho(t)$.

For a **closed system** that evolves in isolation (no interaction with the environment), the time evolution of $\rho(t)$ is given by the **von Neumann–Liouville equation**:

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H, \rho]. \tag{8}$$

Here H is the **Hamiltonian** of the system, and $\rho(t) = U(t)\rho(0)U^\dagger(t)$ with $U(t) = e^{-iHt/\hbar}$.

In reality, most systems are **open**, meaning they interact with an external **environment** or **bath**.
The total system (system + bath) evolves unitarily, but when we trace out the environment's degrees of freedom, the system itself evolves **non-unitarily**.

The dynamics of such an open system can be written in the **Lindblad master equation** form:

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H, \rho] + \sum_k \gamma_k \left(L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\} \right). \tag{9}$$

In this equation, H is the **effective system Hamiltonian** and L_k are the **Lindblad operators** (or **jump operators**), representing different dissipative channels such as spontaneous emission, dephasing, or absorption. $\gamma_k \geq 0$ are the **decay rates** associated with each process, and $\{A, B\} = AB + BA$ denotes the **anticommutator**.

The first term describes the **coherent (unitary)** part of the evolution, while the second term (the **dissipator**) accounts for **irreversible processes** due to coupling with the environment.

Question 3: Solve the Lindblad master equation

Solve the Lindblad master equation for the Hamiltonian, $H = \Omega S_x$ and the jump operator $L_1 = S_-$ where $S_\alpha = \sum_i^N \sigma_\alpha^i$ for N = 5.

Note that solve_ivp only takes vectors and not matrices. Use A.flatten() to convert a matrix (A) to a vector and A.reshape((n, n)) to reshape it back to a matrix.

Question 4: Plot the expectation value

Plot the expectation value of S_z = Trace($S_z\rho$) as a function of time for $\Omega = 5$ and $\Omega = 0.5$. Set $\gamma_1 = 1$.