VII Conice :

O conică este curba care se obține prin intersectarea unui plan cu un con

Secțiune conică	ecuație
cerc	$x^2+y^2=r^2$
elipsă	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$
parabolă	$y^2=4ax$
hiperbolă	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

Caz general :

$$\Gamma = \{(x_1,\dots,x_n)\in E_n \mid f(x_1,\dots,x_n)=0\}$$

unde $f: E_n \to \mathbb{R}$, $f(x_1,\dots,x_n)=\sum_{i,j=1}^{n} a_{ij}x_ix_j +$

$$+\,2\sum_{i=1}^{n} b_i x_i + c$$

$a = (a_{ij})_{i,j=\overline{1,n}} \in \mathcal{M}_{n,n}(\mathbb{R})$

$a_{ij}=a_{ji}$, $\forall\, i,j=\overline{1,n}$

(matrice simetrică)

$rg\, a \geq 1$

Matriceal avem: $f(x) = {}^t x\, a\, x + 2bx + c$

unde $x=\begin{pmatrix} x_1\\ \vdots \\ x_n\end{pmatrix}$, $b=(b_1,\dots b_n)$

$A = \begin{pmatrix} a & {}^t b \\ b & c \end{pmatrix} \in M_{n+1}(\mathbb{R})$

$r = rg\, a$

$r' = rg\, A \in \{r, r+1, r+2\}$

$\delta = \det a$

$\Delta = \det A$

$P(\lambda) = \det(a - \lambda I_n)$ = polinomul caracteristic

$Spec(a) = \{\lambda_1, \dots, \lambda_n\}$

invarianți absoluți: $r, r', \frac{\Delta}{\delta}, \frac{\lambda_i}{\lambda_j}, \frac{\delta}{\lambda_i}, \frac{\Delta}{\lambda_j}$

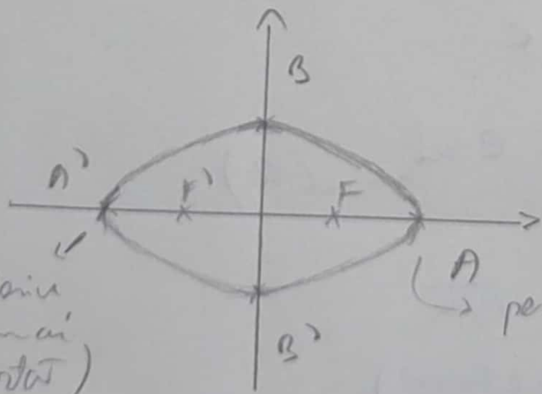
invarianți relativi: $\delta, \Delta, \lambda_i \to \lambda_j$

$\delta \to \alpha^n \delta$

$\Delta \to \alpha^{n+1} \Delta$

$I = Tr\, a \to \alpha\, Tr\, a$

elipsă:

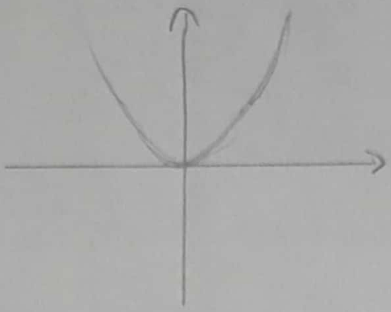


F, F' = puncte de focalizare

afeliu (cel mai depărtat)

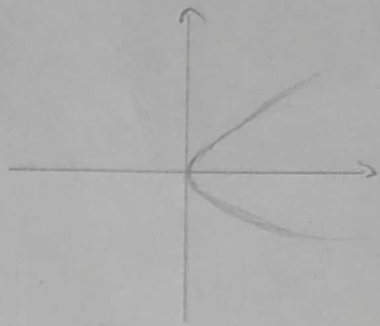
periheliu (punct apropiat)

Parabolă:



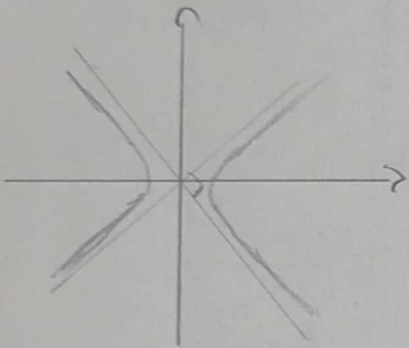
$y = ax^2 + bx + c$

sau



$y^2 = 2px, \; p > 0$

Hiperbolă:



sau