$\boxed{\text{Ap1}}$ Să se aducă la o formă canonică următoarele forme pătratice Q, utilizând : a) metoda Gauss (sch. de coord.)
b) metoda Jacobi

a$_1$) $Q(x) = 4x_1^2 + x_2^2 + x_3^2 - 4x_1x_2 + 4x_1x_3 - 3x_2x_3$

$= (4x_1^2 - 4x_1x_2 + 4x_1x_3) + x_2^2 + x_3^2 - 3x_2x_3$

$= (2x_1 - x_2 + x_3)^2 - \cancel{x_2^2} - \cancel{x_3^2} + 2x_2x_3 + \cancel{x_2^2} + \cancel{x_3^2} - 3x_2x_3$

$= (2x_1 - x_2 + x_3)^2 - x_2x_3$

Efectuăm sch. de coord. $\begin{cases} y_1 = 2x_1 - x_2 + x_3 \\ y_2 = x_2 \\ y_3 = x_3 \end{cases}$ sau $\begin{cases} x_1 = \frac{1}{2}(y_1 + y_2 - y_3) \\ x_2 = y_2 \\ x_3 = y_3 \end{cases}$

$Q(x) = y_1^2 - y_2y_3$

$\begin{cases} y_1 = z_1 \\ y_2 = z_2 + z_3 \\ y_3 = z_2 - z_3 \end{cases}$

$\Rightarrow \underline{Q(x)} = z_1^2 - (z_2^2 - z_3^2) = \underline{z_1^2 - z_2^2 + z_3^2} \rightarrow$ forma canonică a f.p. Q

Compunând cele 2 sch. de coord. găsim:

$\begin{cases} x_1 = \frac{1}{2}(z_1 + 2z_3) \\ x_2 = z_2 + z_3 \\ x_3 = z_2 - z_3 \end{cases}$ $A = \begin{pmatrix} \frac{1}{2} & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & -1 \end{pmatrix}$

$\det A = \frac{1}{2}(-2) = -1 \neq 0$

$\rightarrow \begin{cases} p = 2 \\ q = 1 \text{ (indexul)} \\ s = 2 - 1 = 1 \text{ (signatura)} \end{cases}$

$a_2)\ Q(x) = x_1x_2 + x_2x_3 + x_3x_1$

Efectuăm sch. de coord. $\begin{cases} x_1 = y_1 + y_2 \\ x_2 = y_1 - y_2 \\ x_3 = y_3 \end{cases}$

$$\Rightarrow Q(x) = y_1^2 - y_2^2 + (y_1 - y_2)y_3 + (y_1 + y_2)y_3$$

$$= y_1^2 - y_2^2 + y_1y_3 - \cancel{y_2y_3} + y_1y_3 + \cancel{y_2y_3}$$

$$= y_1^2 - y_2^2 + 2y_1y_3 =$$

$$= (y_1^2 + 2y_1y_3) - y_2^2$$

$$= (y_1 + y_3)^2 - y_2^2 - y_3^2$$

Facem sch. de coord.: $\begin{cases} z_1 = y_1 + y_3 \\ z_2 = y_2 \\ z_3 = y_3 \end{cases}$ sau $\begin{cases} y_1 = z_1 - z_3 \\ y_2 = z_2 \\ y_3 = z_3 \end{cases}$

$$Q(x) = z_1^2 - z_2^2 - z_3^2$$

Compunând cele 2 sch. de coord. găsim:

$\begin{cases} x_1 = z_1 + z_2 - z_3 \\ x_2 = z_1 - z_2 - z_3 \\ x_3 = z_3 \end{cases}$ $A = \begin{pmatrix} 1 & 1 & -1 \\ 1 & -1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$

$\det A = 1 \cdot (-2) = -2 \neq 0$

$\rightarrow p = 1$

$q = 2$ (indexul)

$s = p - q = 1 - 2 = -1$ (signatura)

$Q : \mathbb{R}^4 \to \mathbb{R}$

$b_1)$ $Q(x) = x_1^2 + x_3^2 + x_1 x_2 + x_3 x_4$, $(\forall)\ x \in \mathbb{R}^4$

Matricea asoc. f.p. Q în raport cu baza canonică este:

$$G = \begin{pmatrix} 1 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 \end{pmatrix}$$

Minorii diagonali principali sunt:

$\Delta_1 = 1$

$$\Delta_2 = \begin{vmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{vmatrix} = -\frac{1}{4}$$

$$\Delta_3 = \begin{vmatrix} 1 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} = -\frac{1}{4}$$

$$\Delta_4 = \begin{vmatrix} 1 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 \end{vmatrix} = \frac{1}{2}(-1)^7 \begin{vmatrix} 1 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} \end{vmatrix} = -\frac{1}{2} \cdot \frac{1}{2}\left(-\frac{1}{4}\right) = \frac{1}{16}$$

$\Delta_i \neq 0$, $(\forall)\ i = \overline{1,4}$

$$Q(x) = \frac{1}{\Delta_1} y_1^2 + \frac{\Delta_1}{\Delta_2} y_2^2 + \frac{\Delta_2}{\Delta_3} y_3^2 + \frac{\Delta_3}{\Delta_4} y_4^2, \ (\forall)\ x \in \mathbb{R}^4$$

Deci: $Q(x) = y_1^2 - 4y_2^2 + y_3^2 - 4y_4^4$ $\to$ forma canonică a f.p. Q

$p = 2$

$q = 2$ (indexul)

$s = p - q = 0$ (signatura)

$b_2)$ $Q: \mathbb{R}^4 \to \mathbb{R}$,

$$Q(x) = 3x_1^2 + 2x_2^2 - x_3^2 - 2x_4^2 + 2x_1x_2 - 4x_2x_3 + 2x_2x_4,$$
$$(\forall)\, x \in \mathbb{R}^4$$

Matricea asoc. f. p. Q în raport cu baza canonică este:

$$G = \begin{pmatrix} 3 & 1 & 0 & 0 \\ 1 & 2 & -2 & 1 \\ 0 & -2 & -1 & 0 \\ 0 & 1 & 0 & -2 \end{pmatrix}$$

$$\Delta_1 = 3$$

$$\Delta_2 = \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} = 5$$

$$\Delta_3 = \begin{vmatrix} 3 & 1 & 0 \\ 1 & 2 & -2 \\ 0 & -2 & -1 \end{vmatrix} = \begin{vmatrix} 0 & -5 & 6 \\ \cancel{1} & \cancel{2} & \cancel{-2} \\ 0 & -2 & -1 \end{vmatrix} = 1\cdot(-1)^3 \begin{vmatrix} -5 & 6 \\ -2 & -1 \end{vmatrix}$$
$$= -17$$

$$\Delta_4 = \det G = \begin{vmatrix} 3 & 1 & 0 & 0 \\ 1 & 2 & -2 & 1 \\ 0 & -2 & -1 & 0 \\ 0 & 1 & 0 & -2 \end{vmatrix} = \begin{vmatrix} 0 & -5 & 6 & -3 \\ \cancel{1} & \cancel{2} & \cancel{-2} & \cancel{1} \\ 0 & -2 & -1 & 0 \\ 0 & 1 & 0 & -2 \end{vmatrix} =$$

$$= 1\cdot(-1)^3 \begin{vmatrix} -5 & 6 & -3 \\ -2 & -1 & 0 \\ 1 & 0 & -2 \end{vmatrix} = - \begin{vmatrix} \cancel{-5} & 6 & -13 \\ \cancel{-2} & -1 & -4 \\ \cancel{1} & \cancel{0} & \cancel{0} \end{vmatrix} = -(-1)^4 \begin{vmatrix} 6 & -13 \\ -1 & -4 \end{vmatrix}$$
$$= -(-37) = 37$$

$$\Delta_i \neq 0,\ (\forall)\, i = \overline{1,4}$$

$$Q(x) = \frac{1}{\Delta_1} y_1^2 + \frac{\Delta_1}{\Delta_2} y_2^2 + \frac{\Delta_2}{\Delta_3} y_3^2 + \frac{\Delta_3}{\Delta_4} y_4^2$$

Deci: $Q(x) = \frac{1}{3} y_1^2 + \frac{3}{5} y_2^2 - \frac{5}{17} y_3^2 - \frac{17}{37} y_4^2 \to$ forma canonică a f. p. Q

$p = 2$

$q = 2$ (indexul)

$s = p - q = 0$ (signatura)

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