

Muchii critice



Muchii critice

O muchie este **critică** \Leftrightarrow nu este conținută într-un ciclu.

Găsirea unui ciclu - parcurgere DF

- ☐ **muchii de avansare** - ale arborelui DF (memorat cu un vector de tați), prin care se descoperă vârfuri noi
- ☐ **muchii de întoarcere** - închid ciclu, nu pot fi critice

Muchii critice



Cum testăm dacă o muchie de avansare (i, j) este critică?

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Cum testăm dacă o muchie de avansare (i, j) este critică?

- nu este conținută într-un ciclu închis de o muchie de întoarcere



Muchii critice

O muchie de avansare (i, j) este critică

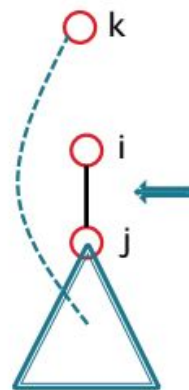


nu este conținută într-un ciclu închis de o muchie de întoarcere



nu există nicio muchie de întoarcere cu

- ☐ o extremitate în **j sau într-un descendent al lui j**
- ☐ cealaltă extremitate în **i sau într-un ascendent al lui i** (într-un vârf de pe **un nivel mai mic sau egal** cu nivelul lui i)



Muchii critice

Memorăm, pentru fiecare vârf i :

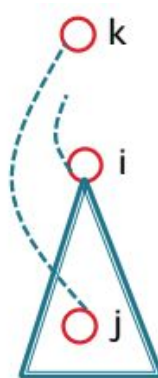
$niv_min[i]$ = nivelul minim al unui vârf care este extremitate a unei muchii de întoarcere din i sau dintr-un descendent al lui i

= nivelul minim la care se închide un ciclu elementar care conține vârful i (printr-o muchie de întoarcere)



Muchii critice

- **nivel[i]** = nivelul lui *i* în arborele DF
- **niv_min[i]** = $\min \{ \text{nivel}[i], A, B \}$
 - $A = \min \{ \text{nivel}[k] \mid ik \text{ muchie de întoarcere} \}$
 - $B = \min \{ \text{nivel}[k] \mid j \text{ descendent al lui } i, jk \text{ muchie de întoarcere} \}$



Muchii critice

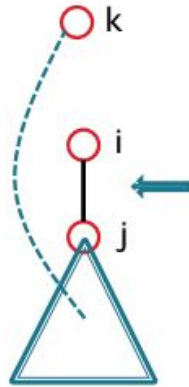
O muchie de avansare ij este critică

\Leftrightarrow



Muchii critice

O muchie de avansare ij este critică $\Leftrightarrow \text{niv_min}[j] > \text{nivel}[i]$



Muchii critice



Cum calculăm eficient $\text{niv_min}[i]$?

- $\text{niv_min}[i] = \min \{ \text{nivel}[i], A, B \}$
 - $A = \min \{ \text{nivel}[k] \mid ik \text{ muchie de întoarcere} \}$
 - $B = \min \{ \text{nivel}[k] \mid j \text{ descendent al lui } i, jk \text{ muchie de întoarcere} \}$

Muchii critice

Cum calculăm eficient $\text{niv_min}[i]$?

- $\text{niv_min}[i] = \min \{ \text{nivel}[i], A, B \}$
 - $A = \min \{ \text{nivel}[k] \mid i \text{ k muchie de întoarcere} \}$
 - $B = \min \{ \text{nivel}[k] \mid j \text{ descendent al lui } i, j \text{ k muchie de întoarcere} \}$



B se poate calcula recursiv

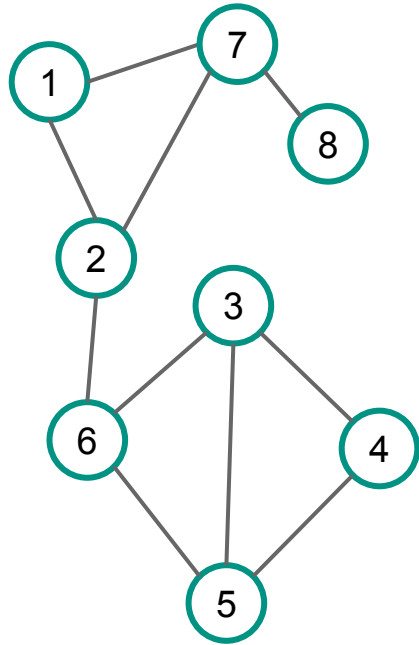
Muchii critice

Cum calculăm eficient $\text{niv_min}[i]$?

- $\text{niv_min}[i] = \min \{ \text{nivel}[i], A, B \}$
 - $A = \min \{ \text{nivel}[k] \mid ik \text{ muchie de întoarcere} \}$
 - $B = \min \{ \text{nivel}[k] \mid j \text{ descendent al lui } i, jk \text{ muchie de întoarcere} \}$

$$B = \min \{ \text{niv_min}[j] \mid j \text{ fiu al lui } i \}$$

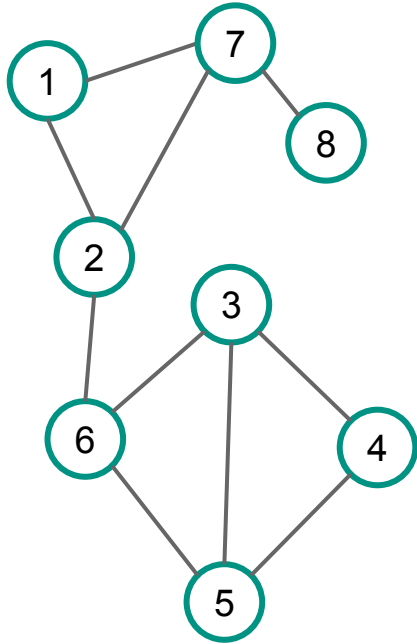
Muchii critice – Exemplu



Muchii critice – Exemplu

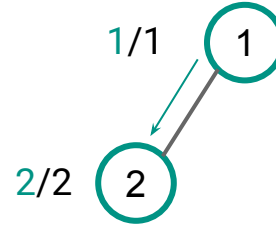
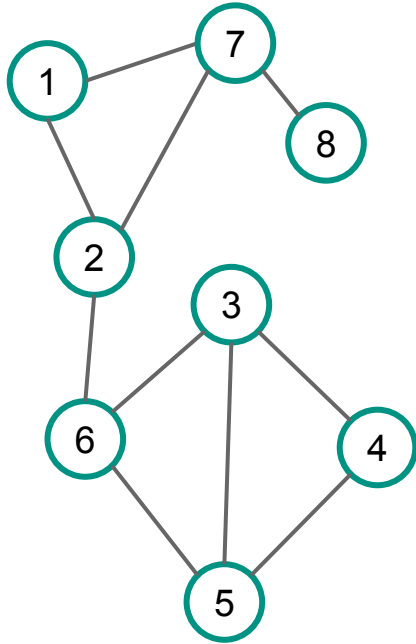
nivel/niv_min

1/1



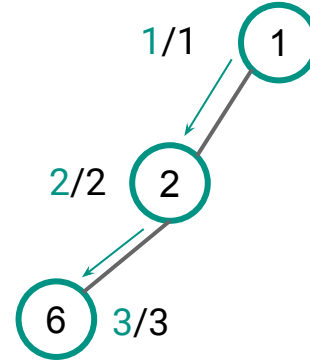
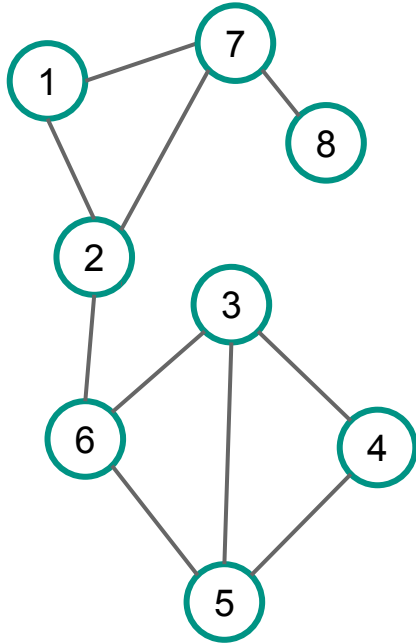
Muchii critice – Exemplu

nivel/niv_min



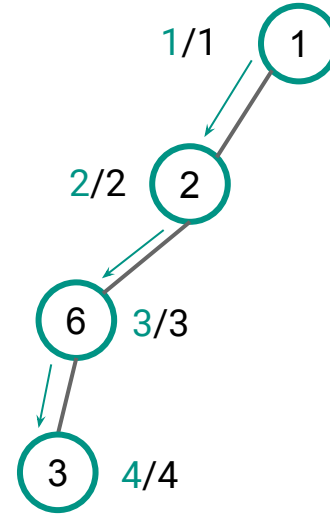
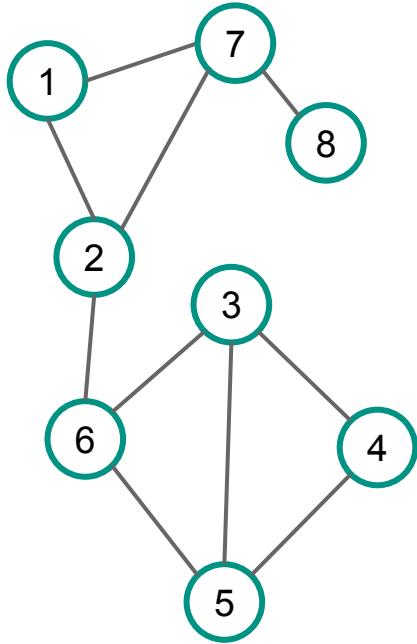
Muchii critice – Exemplu

nivel/niv_min



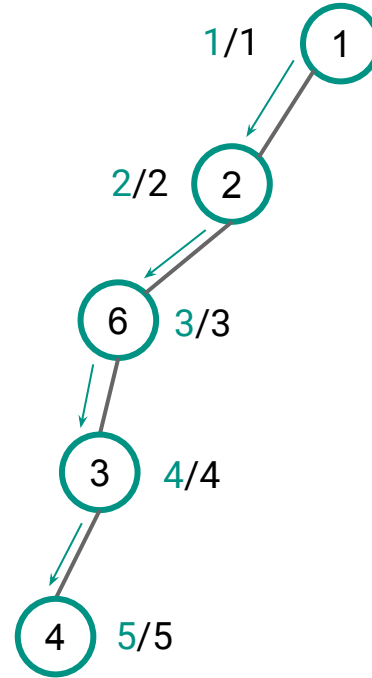
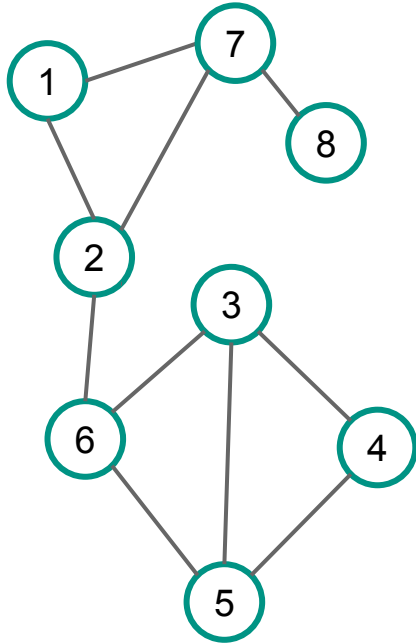
Muchii critice – Exemplu

nivel/niv_min



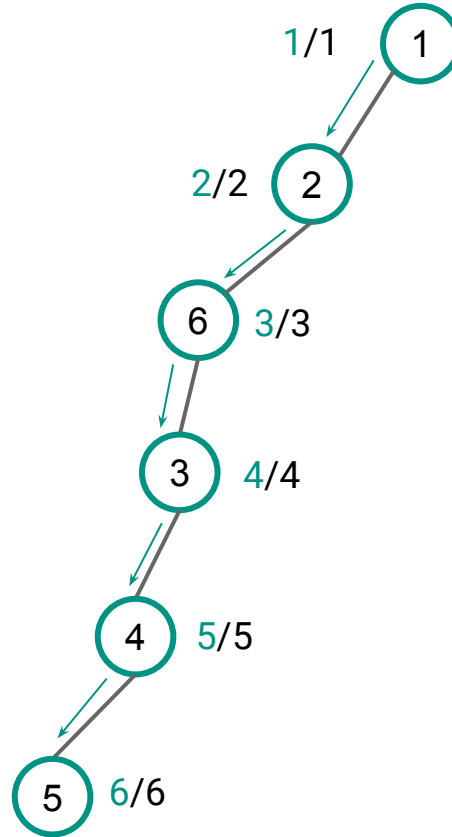
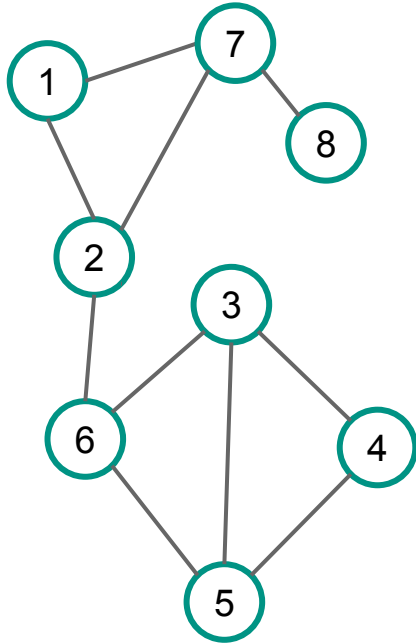
Muchii critice – Exemplu

nivel/niv_min



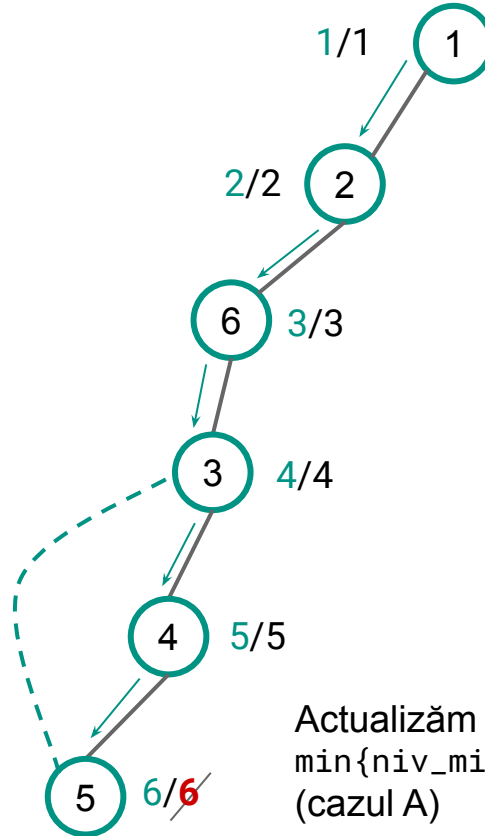
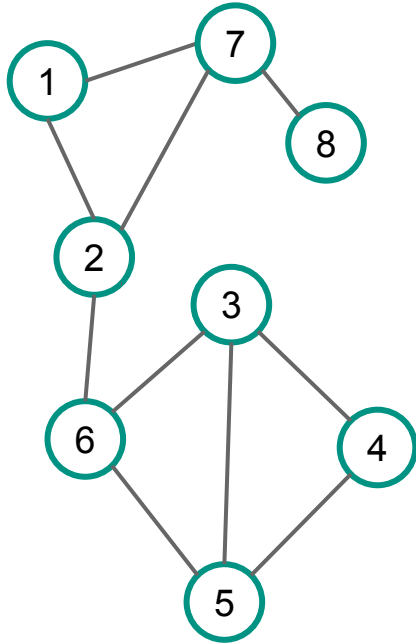
Muchii critice – Exemplu

nivel/niv_min



Muchii critice – Exemplu

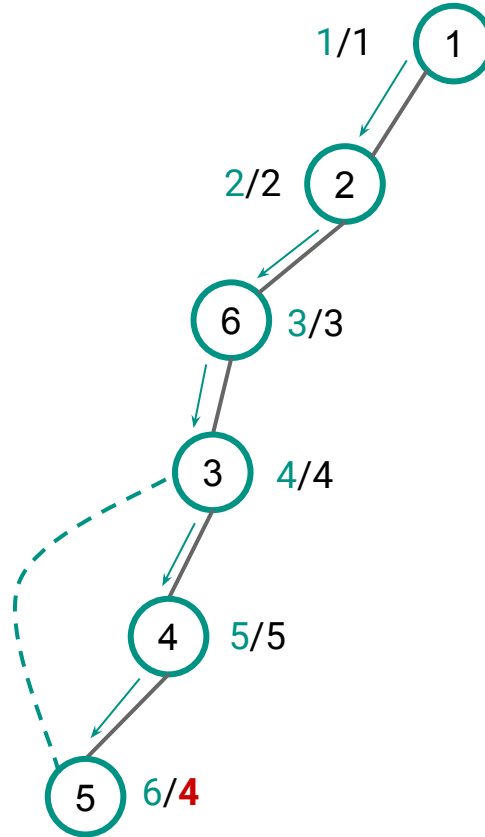
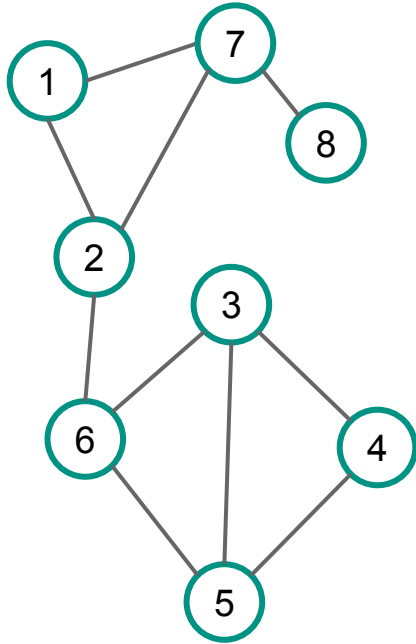
nivel/niv_min



Actualizăm $\text{niv_min}[5] = \min\{\text{niv_min}[5], \text{nivel}[3]\}$
(cazul A)

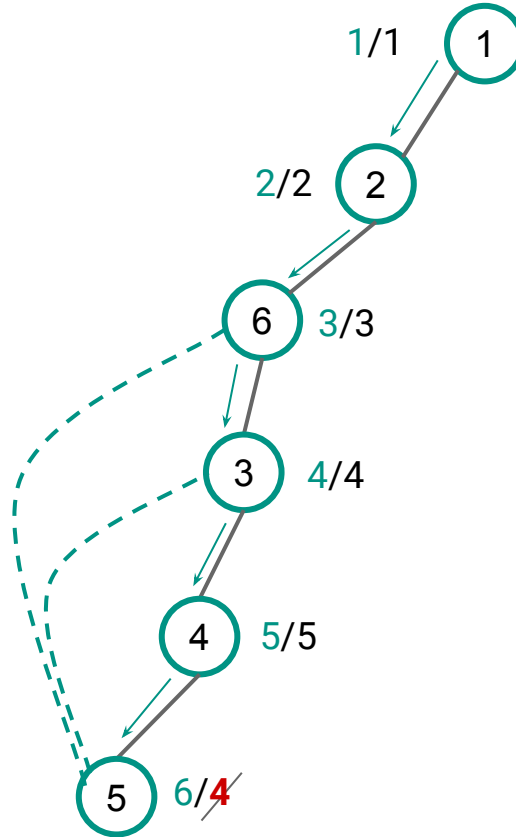
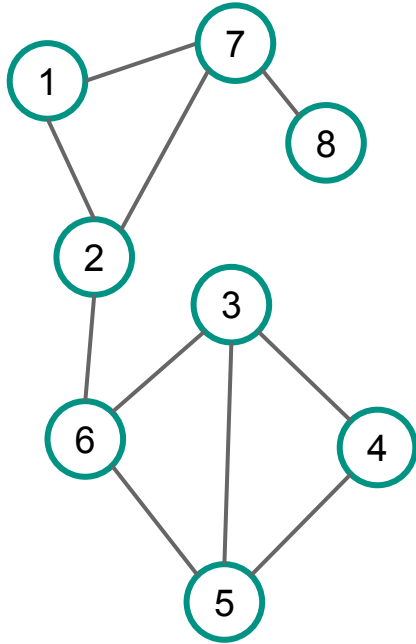
Muchii critice – Exemplu

nivel/niv_min



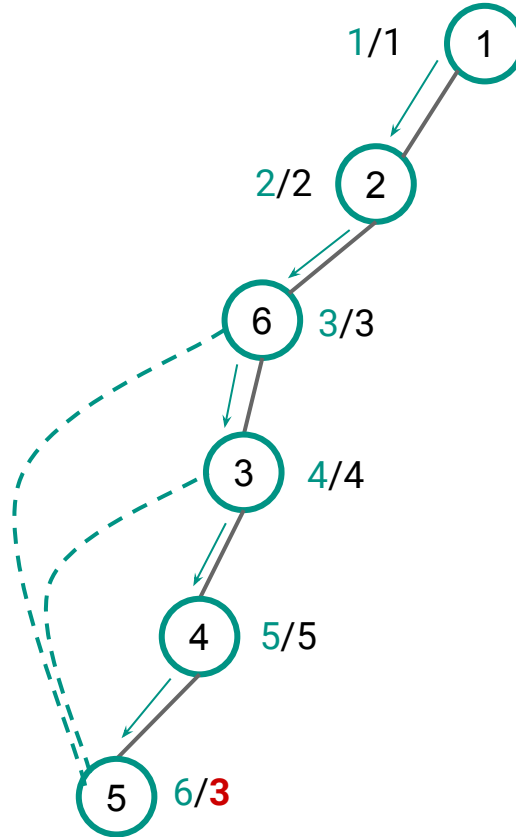
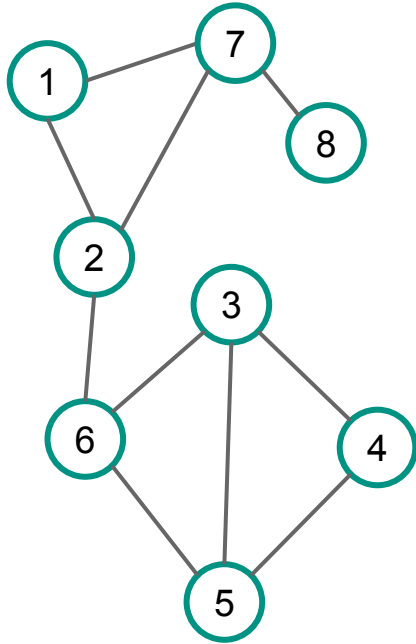
Muchii critice – Exemplu

nivel/niv_min

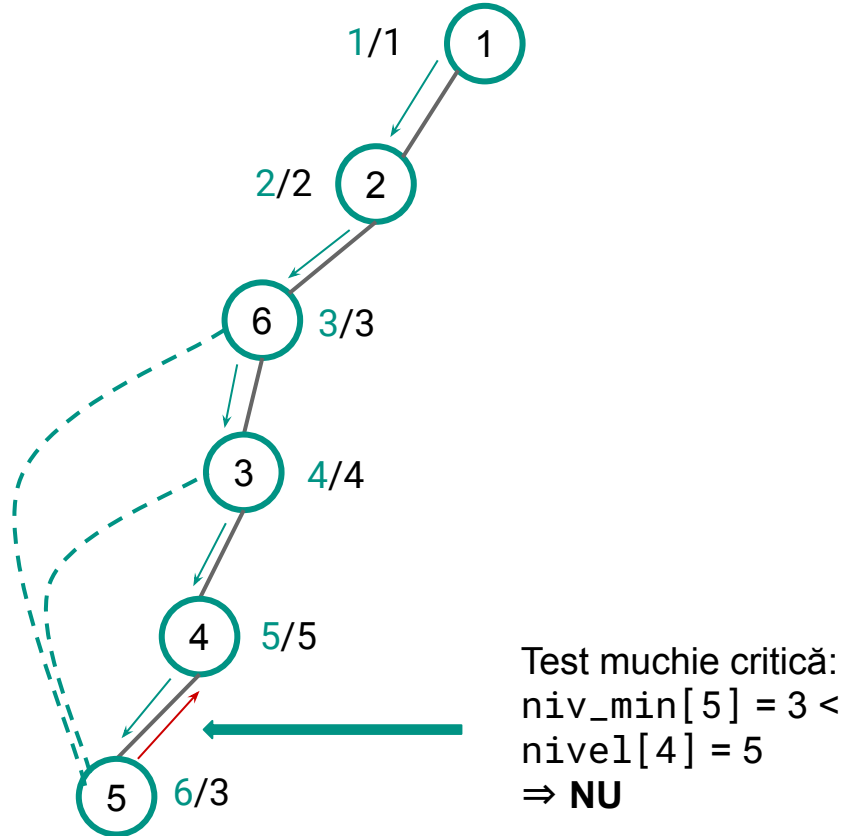
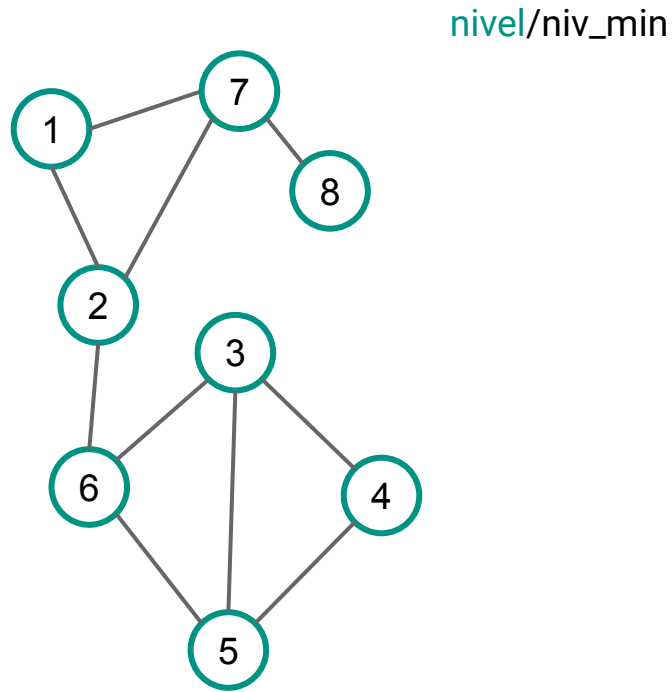


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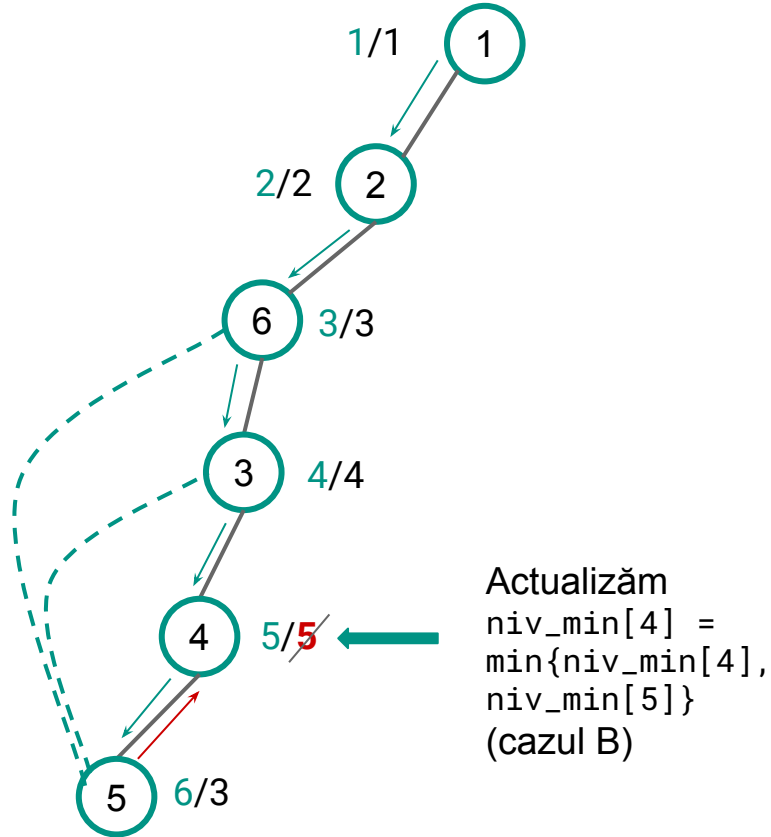
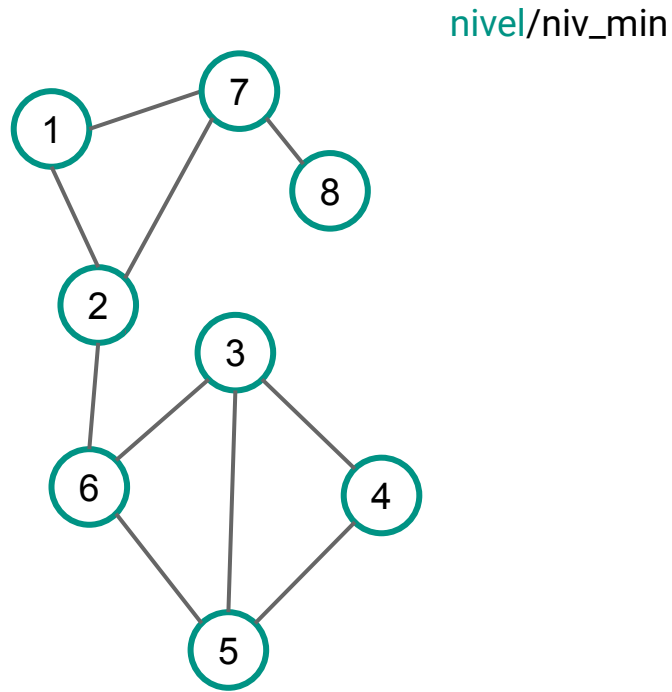
nivel/niv_min



Muchii critice – Exemplu

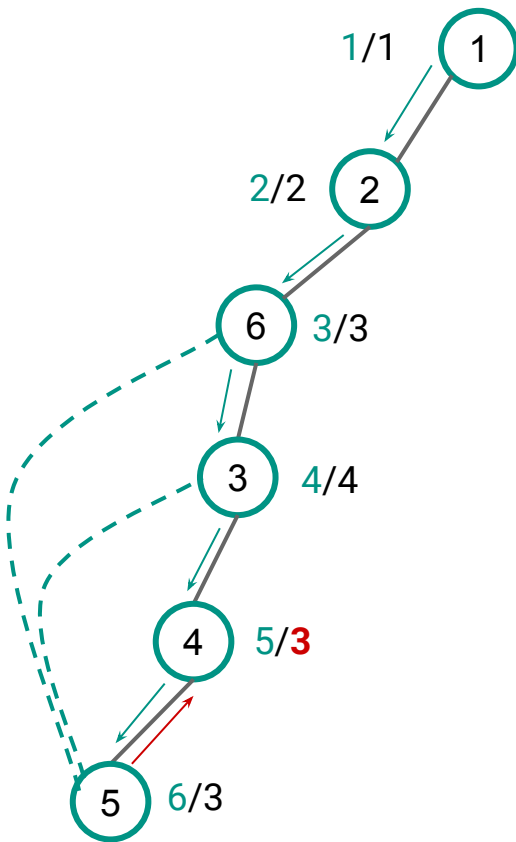
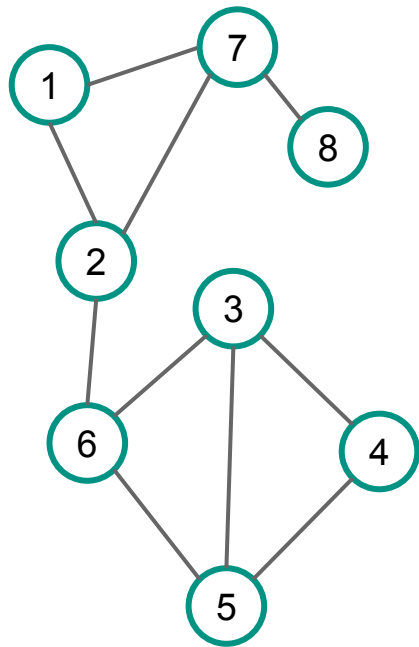


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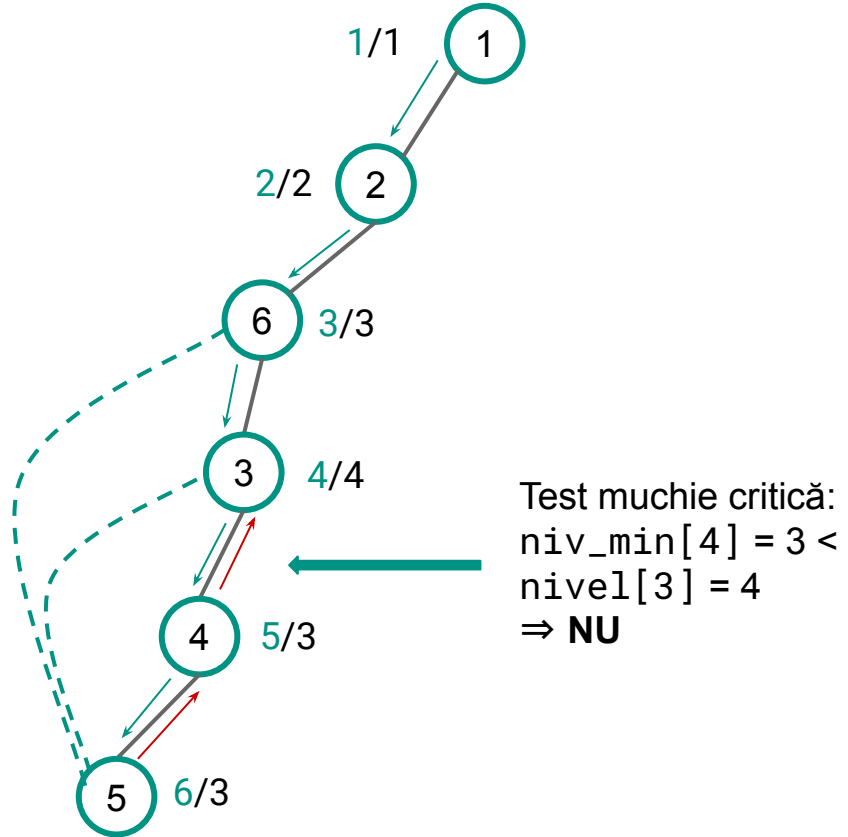
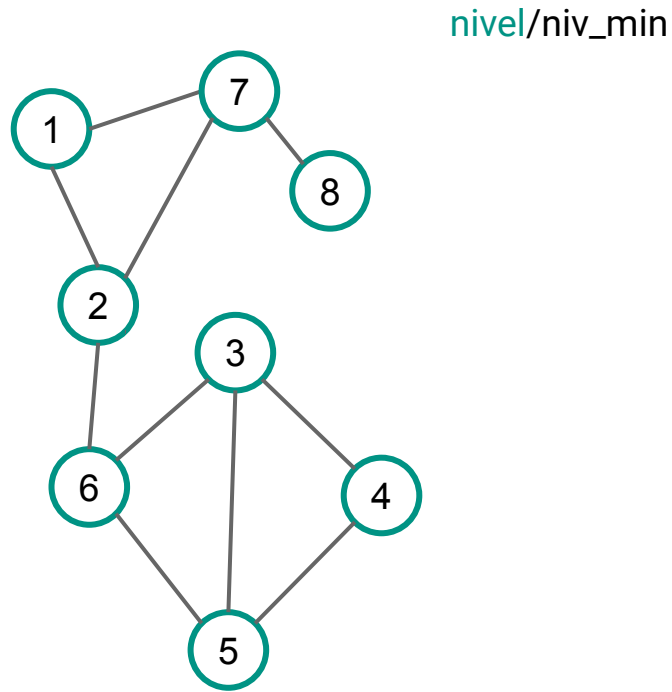


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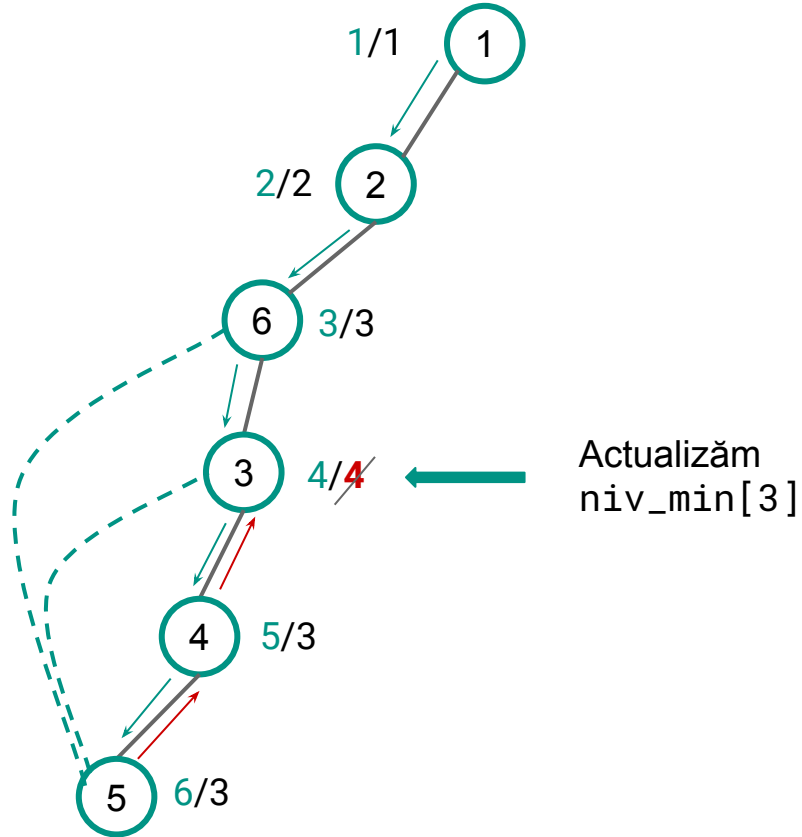
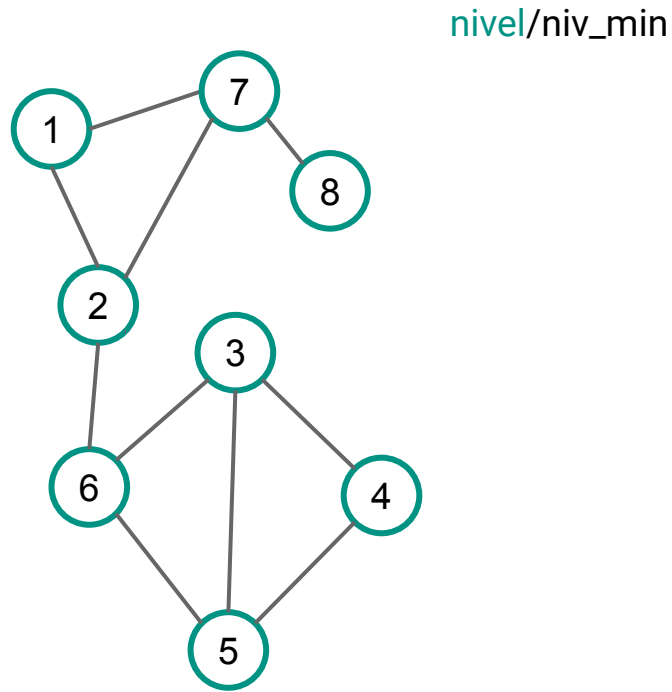
nivel/niv_min



Muchii critice – Exemplu

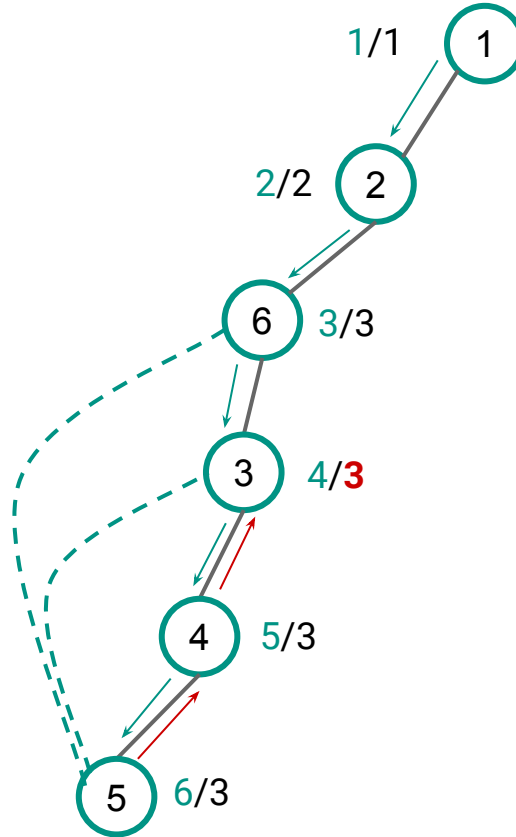
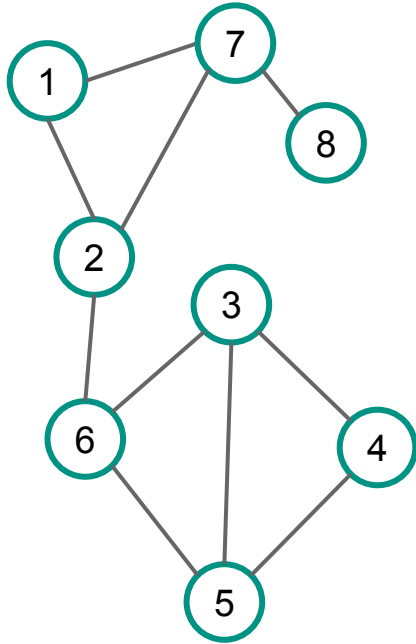


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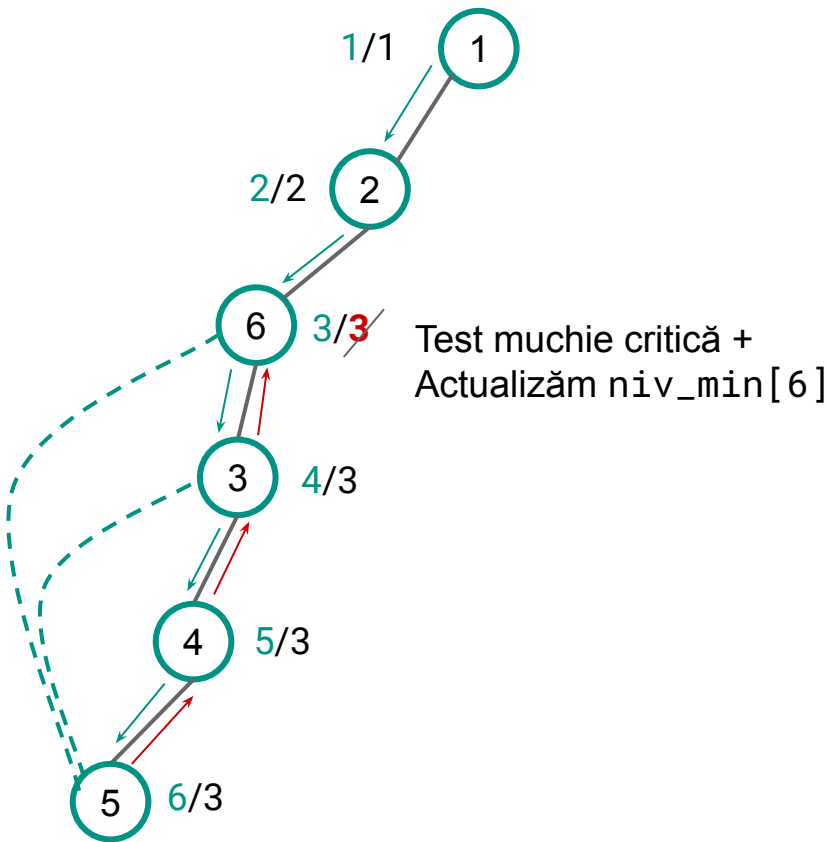
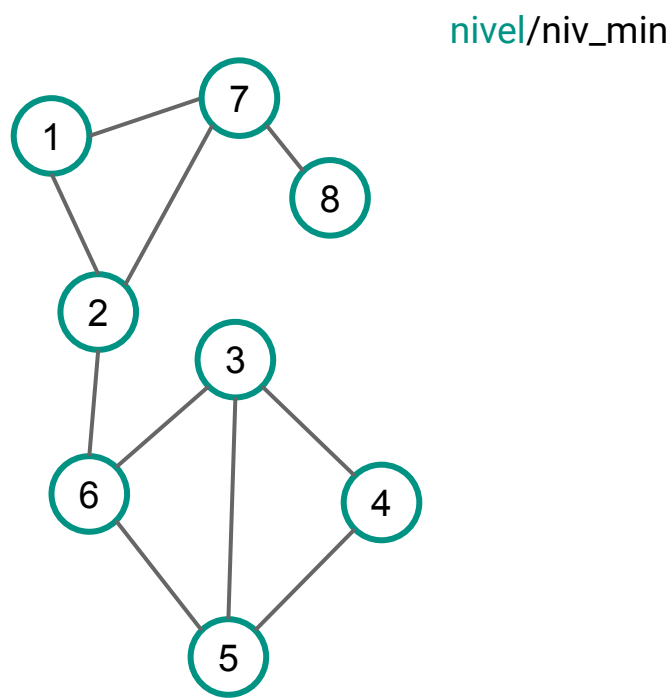


Muchii critice – Exemplu

nivel/niv_min

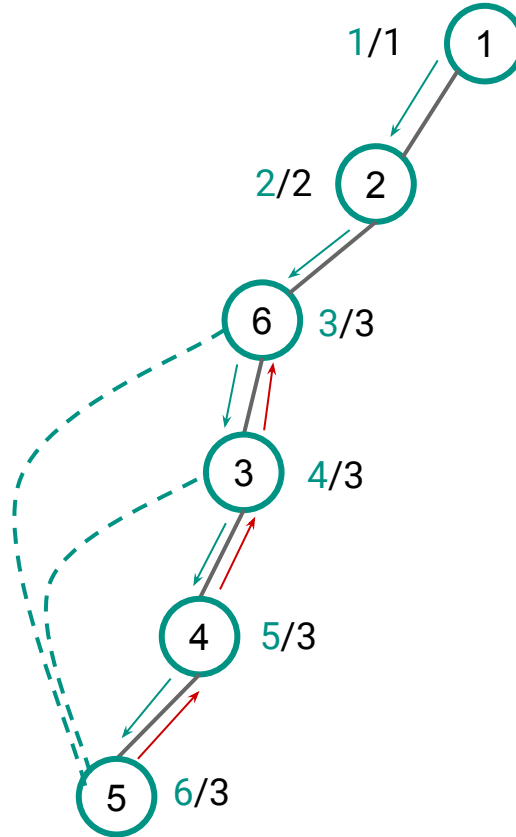
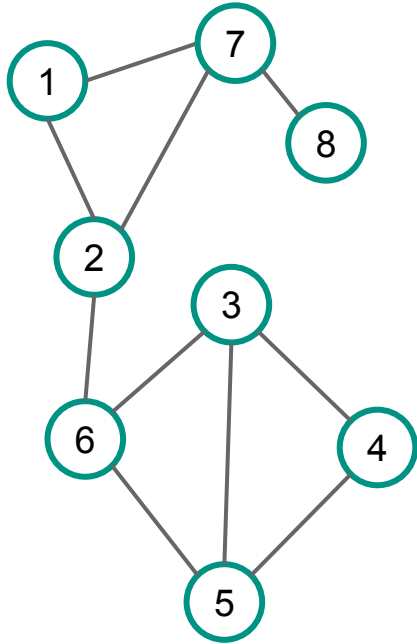


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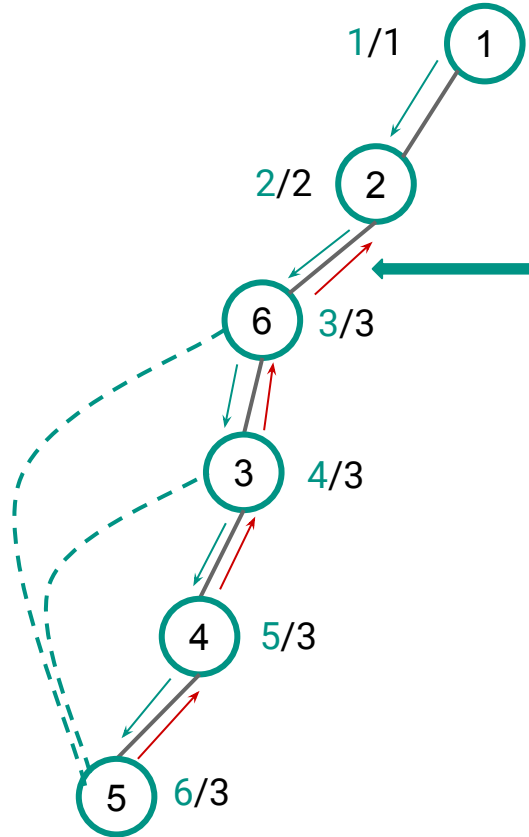
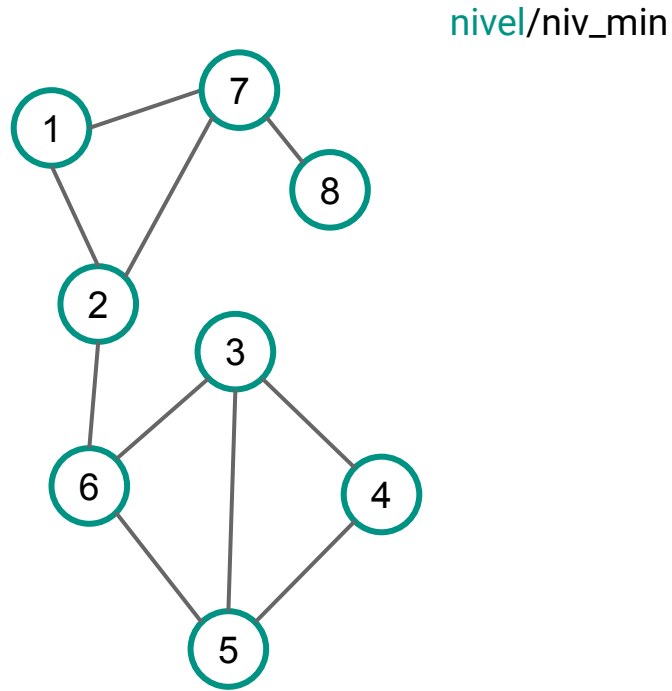


Muchii critice – Exemplu

nivel/niv_min



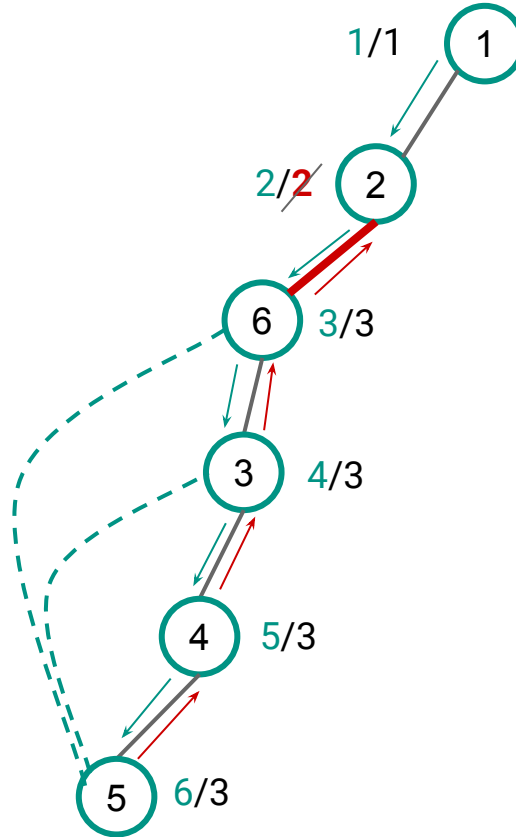
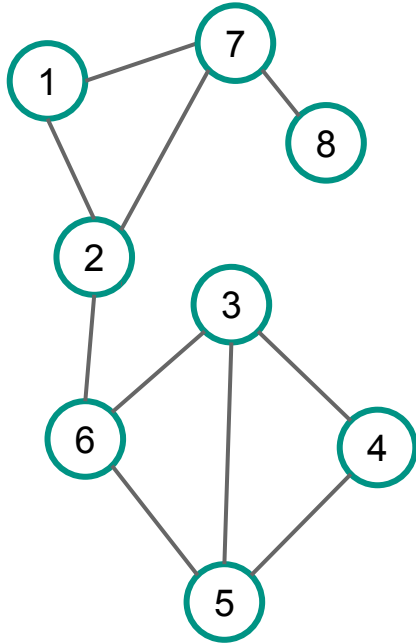
Muchii critice – Exemplu



Test muchie critică:
 $\text{niv_min}[6] = 3 >$
 $\text{nivel}[2] = 2$
 $\Rightarrow \mathbf{DA}$

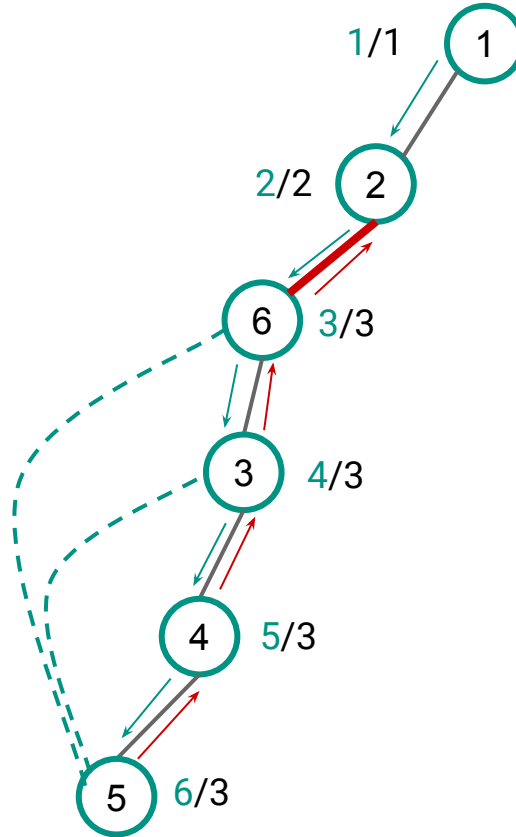
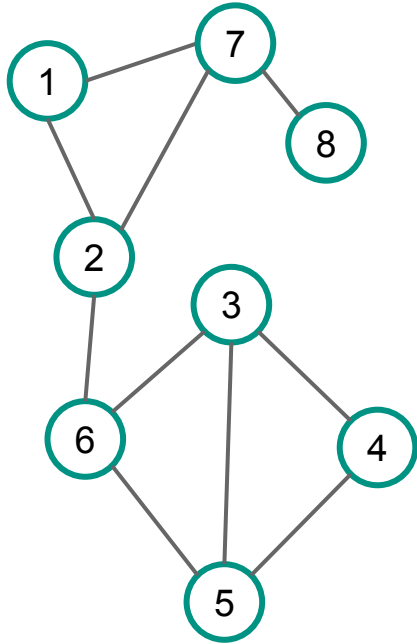
Muchii critice – Exemplu

nivel/niv_min



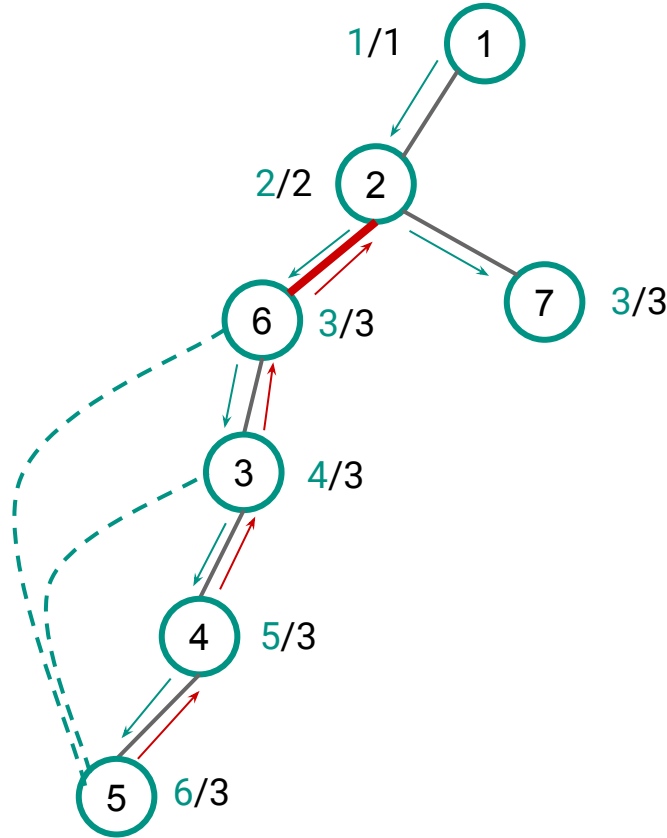
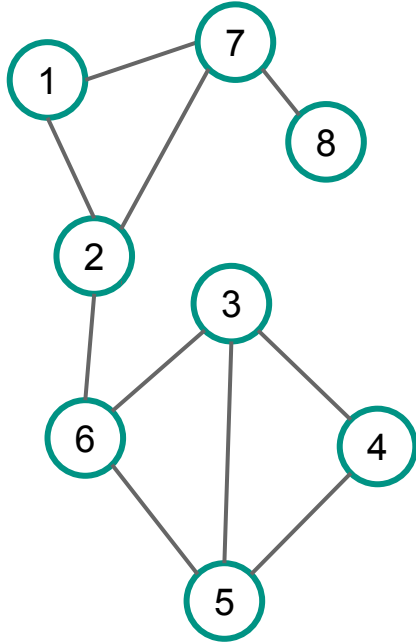
Muchii critice – Exemplu

nivel/niv_min



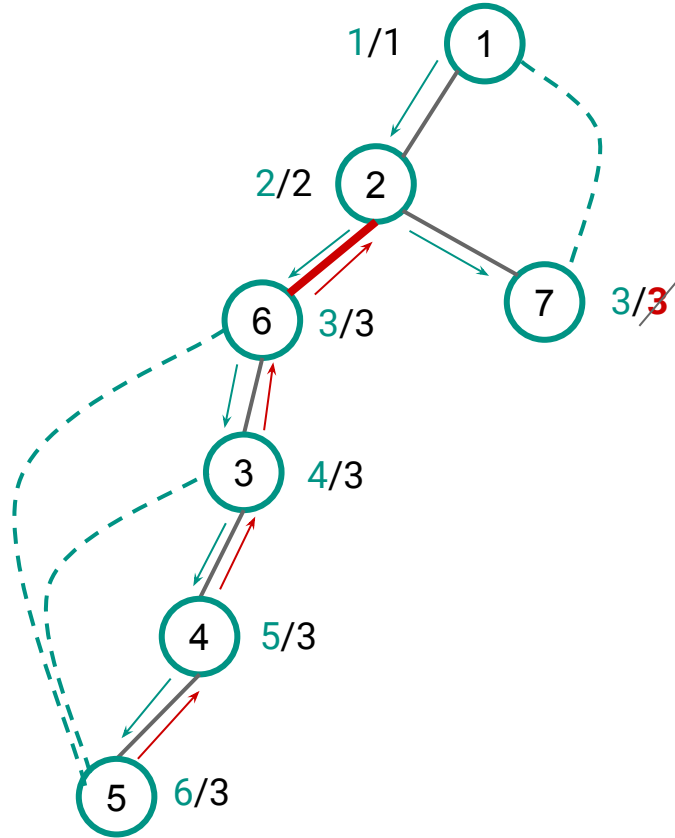
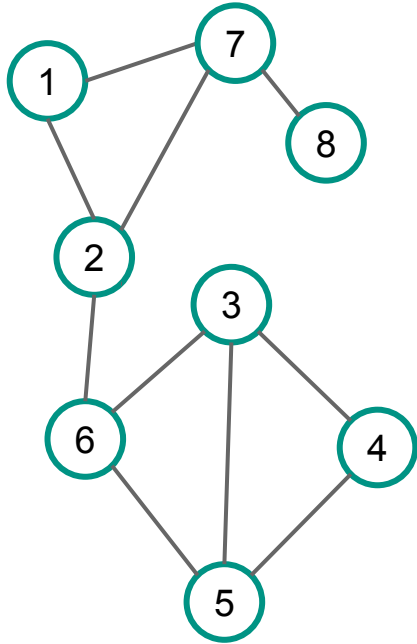
Muchii critice – Exemplu

nivel/niv_min



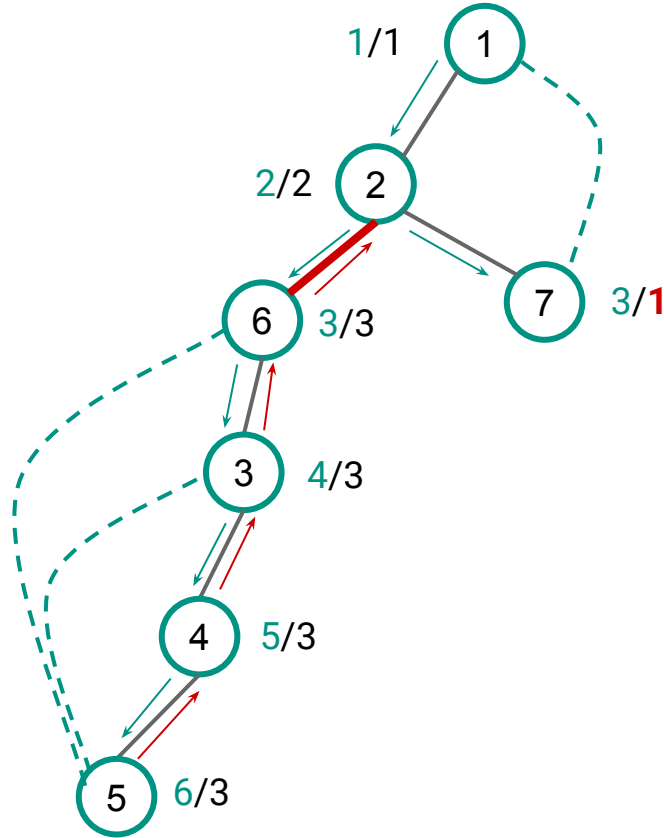
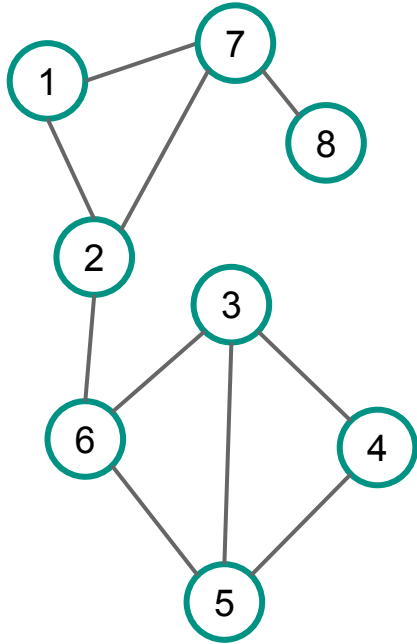
Muchii critice – Exemplu

nivel/niv_min



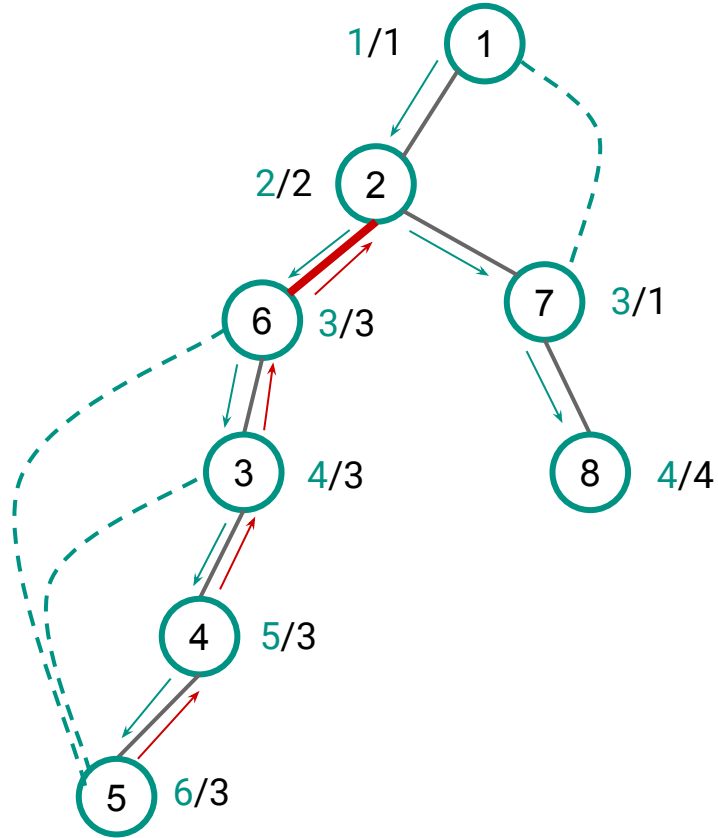
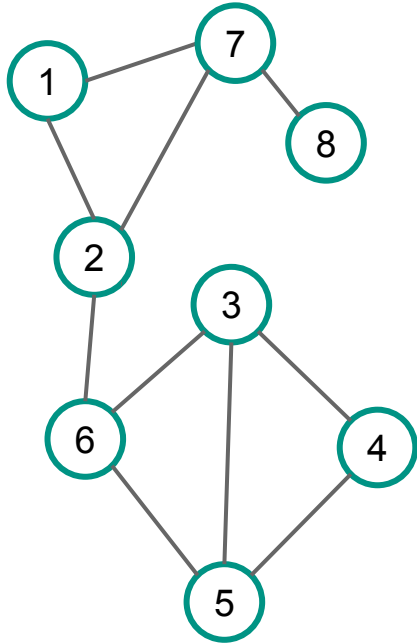
Muchii critice – Exemplu

nivel/niv_min

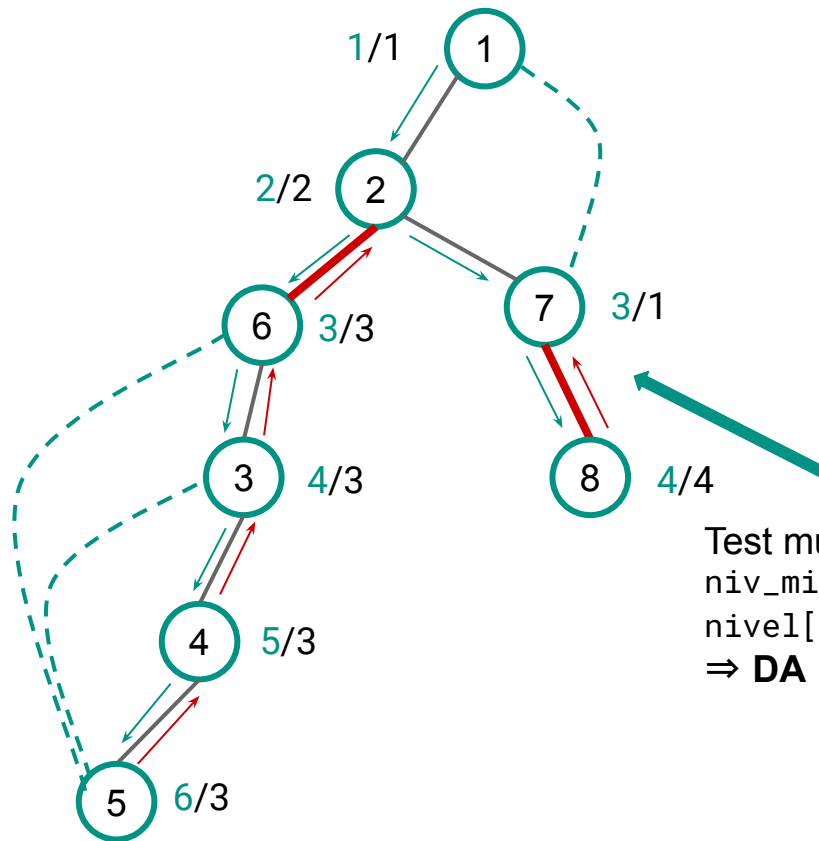
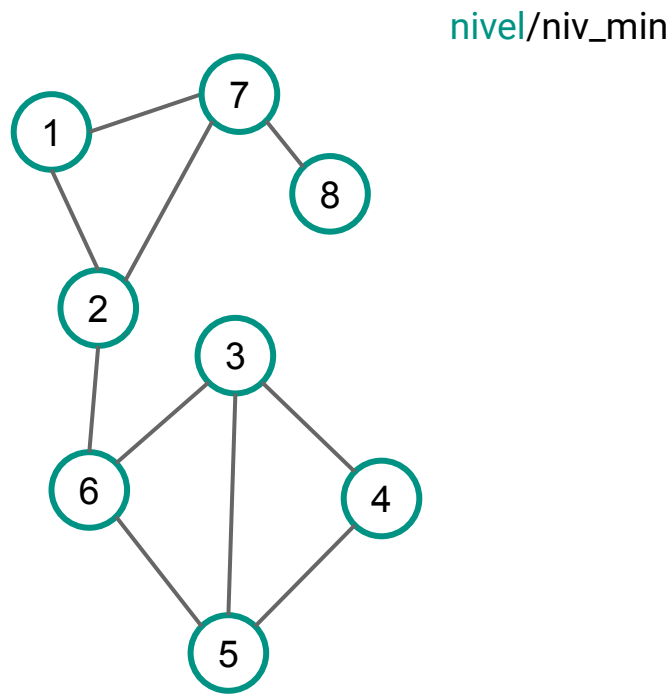


Muchii critice – Exemplu

nivel/niv_min



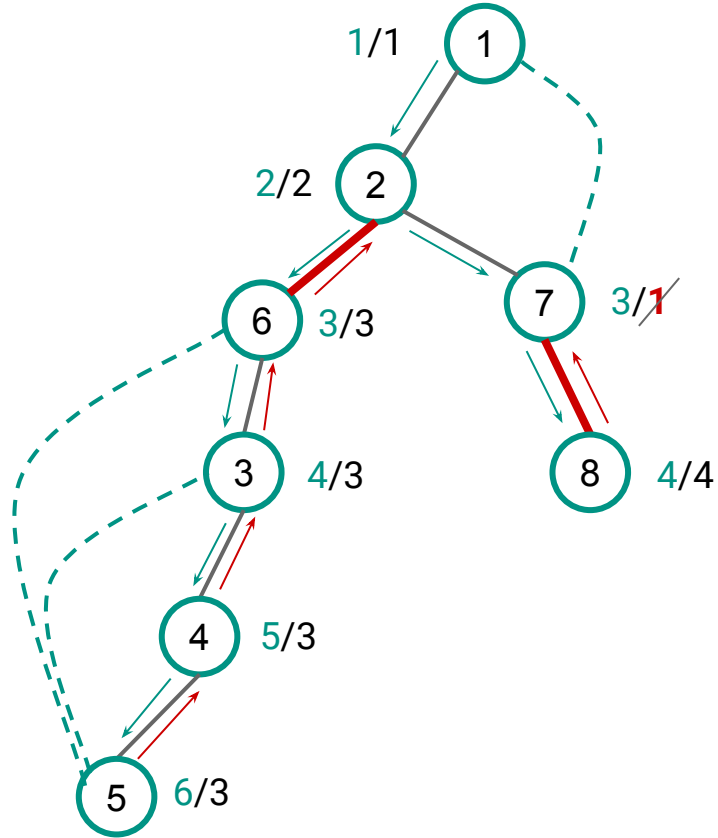
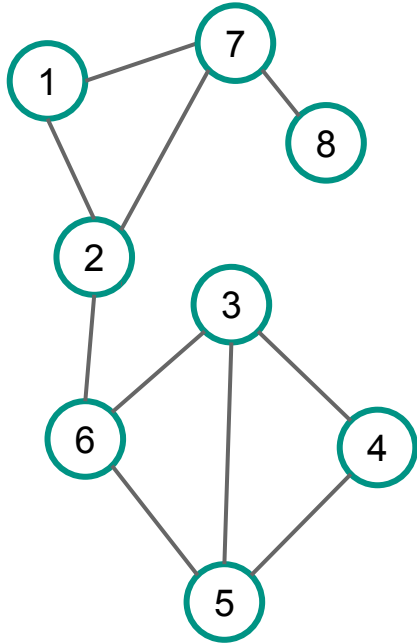
Muchii critice – Exemplu



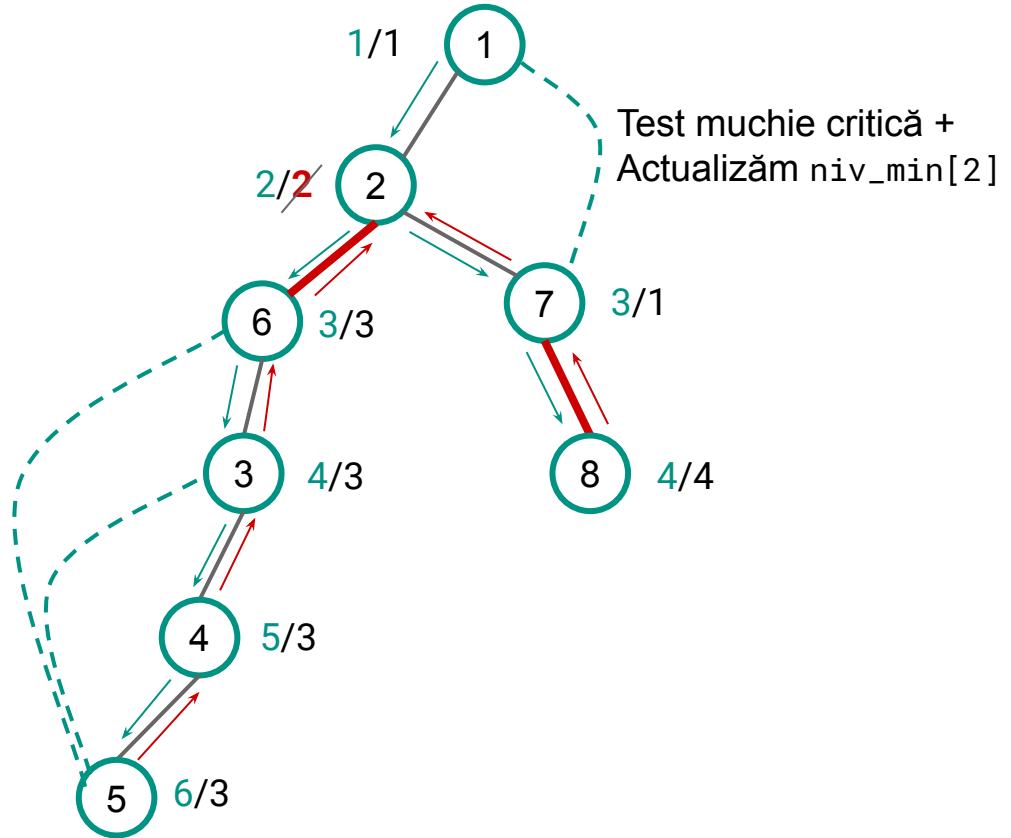
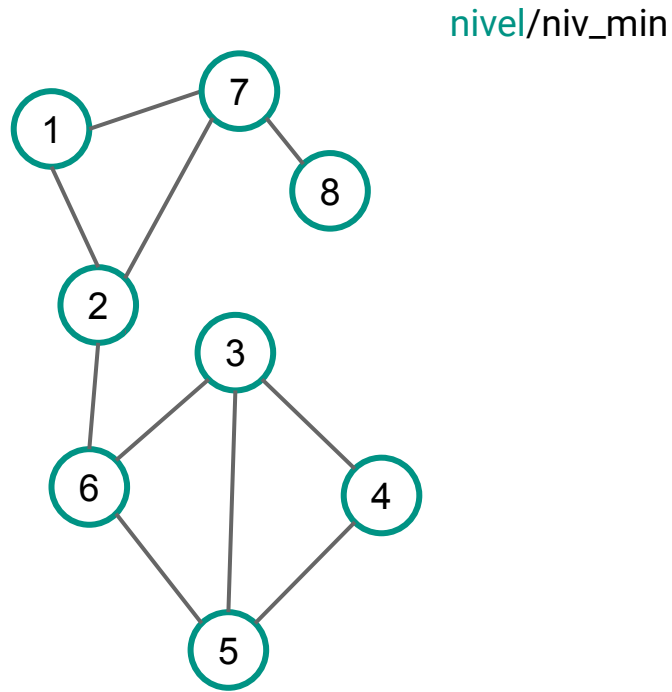
Test muchie critică:
 $niv_min[8] = 4 >$
 $nivel[7] = 3$
 \Rightarrow **DA**

Muchii critice – Exemplu

nivel/niv_min

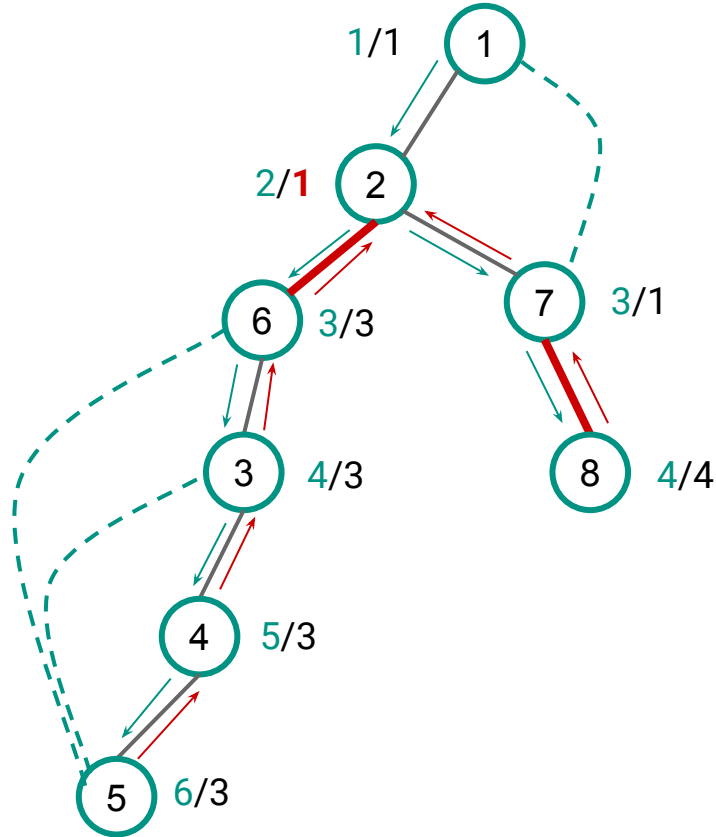
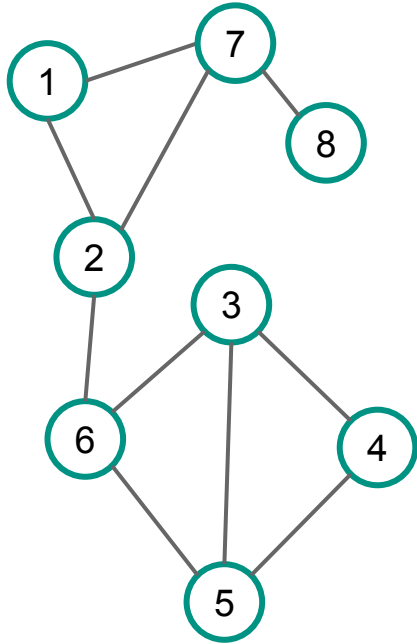


Muchii critice – Exemplu

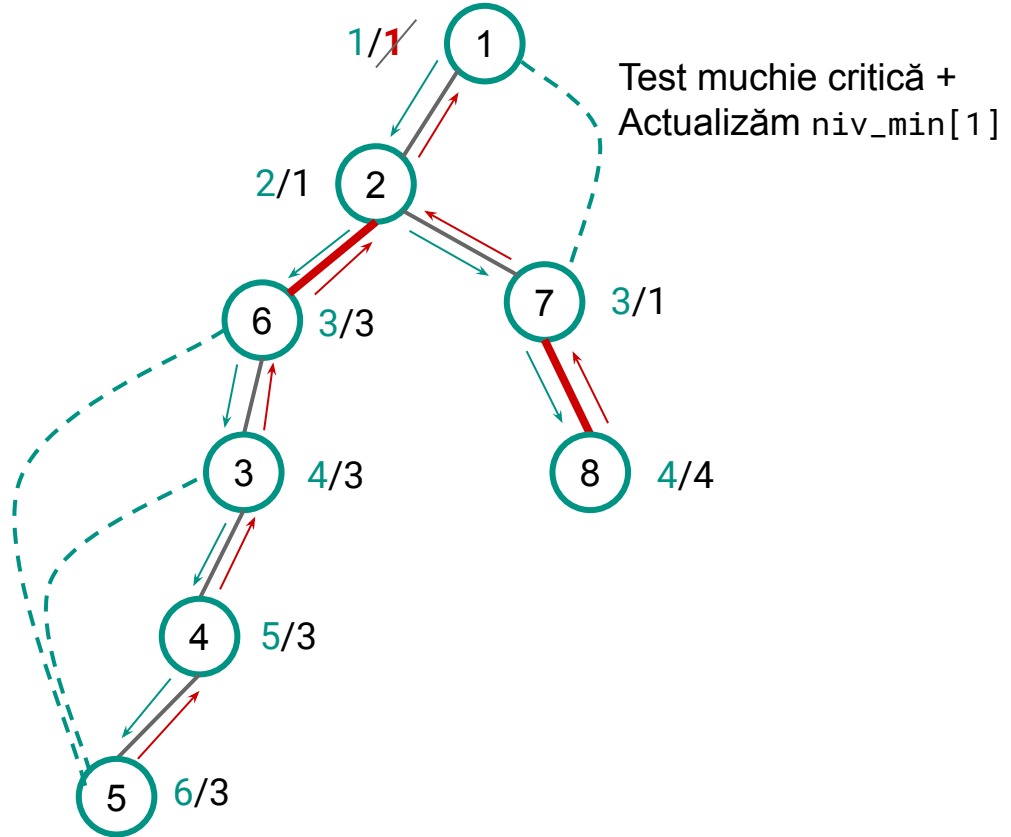
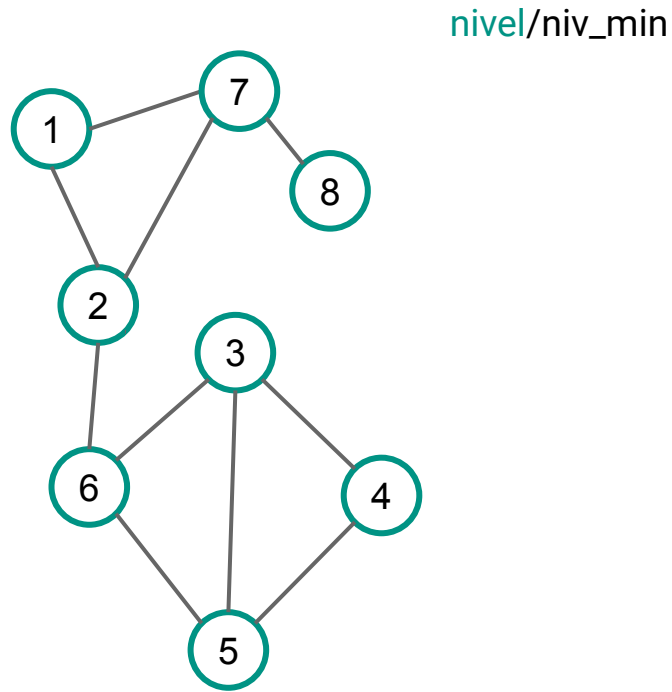


Muchii critice – Exemplu

nivel/niv_min

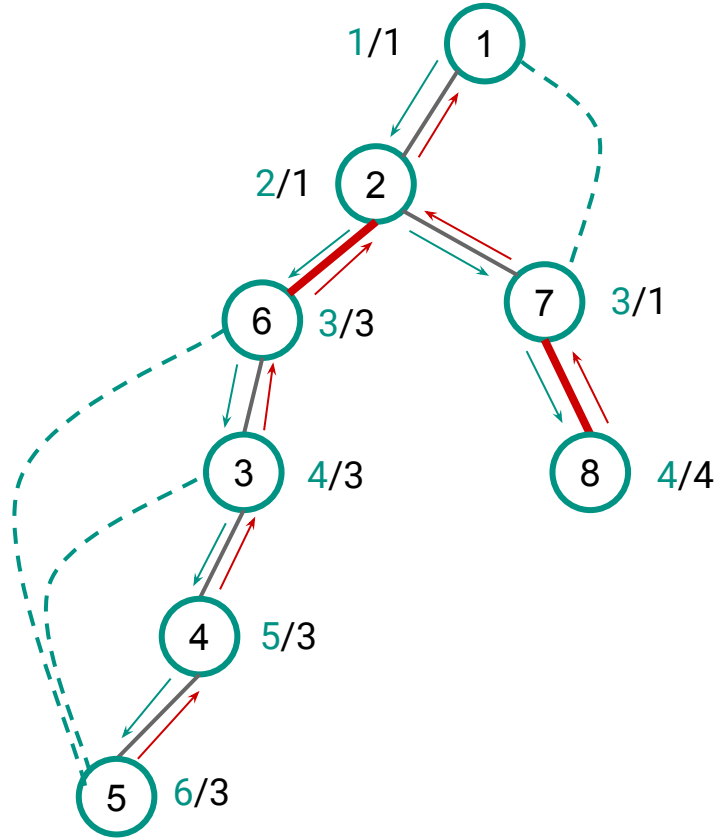
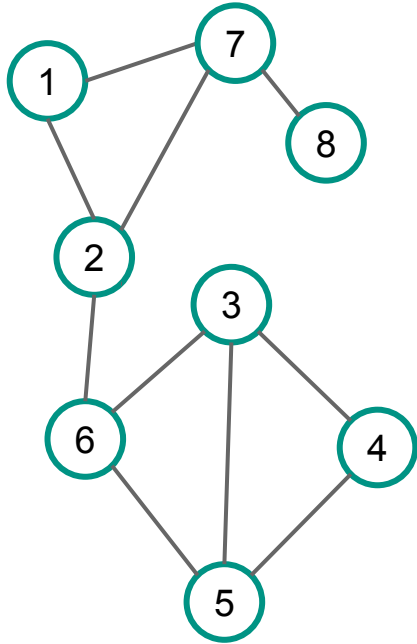


Muchii critice – Exemplu



Muchii critice – Exemplu

nivel/niv_min



Indicații de implementare

```
void df(int i) {
    viz[i] = 1;
    niv_min[i] = nivel[i];
    for (j vecin al lui i)
        if (viz[j] == 0) {           //ij muchie de avansare
            nivel[j] = nivel[i] + 1;
            df(j);
            //actualizare niv_min[i] - formula B
            niv_min[i] = min{ niv_min[i], niv_min[j] }
            //test ij este muchie critica
            ...
        }
    else
        if(nivel[j] < nivel[i] - 1)   //ij muchie de intoarcere
            //actualizare niv_min[i] - formula A
            ...
}
```

Indicații de implementare

```
void df(int i) {  
    viz[i] = 1;  
    niv_min[i] = nivel[i];  
    for (j vecin al lui i)  
        if (viz[j] == 0) { //ij muchie de avansare  
            nivel[j] = nivel[i] + 1;  
            df(j);  
            //actualizare niv_min[i] - formula B  
            niv_min[i] = min{ niv_min[i], niv_min[j] }  
            //test ij este muchie critica  
            if (niv_min[j] > nivel[i]) scrie muchia ij  
        }  
    else  
        if (nivel[j] < nivel[i] - 1) //ij muchie de intoarcere  
            //actualizare niv_min[i] - formula A  
            niv_min[i] = min{ niv_min[i], niv[j] }  
}
```

Puncte critice



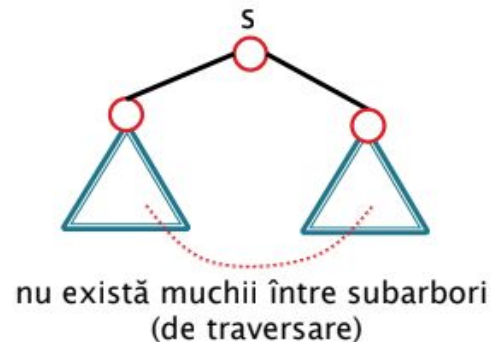
Puncte critice

Un vârf este **punct critic** \Leftrightarrow există două vârfuri $x, y \neq v$ astfel încât aparține oricărui x, y -lanț.

Arborele DF

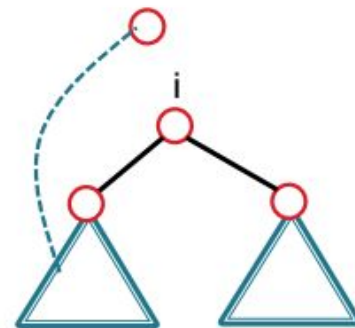
☐ **rădăcina** este punct critic

\Leftrightarrow



☐ **un alt vârf i** din arbore este critic

\Leftrightarrow



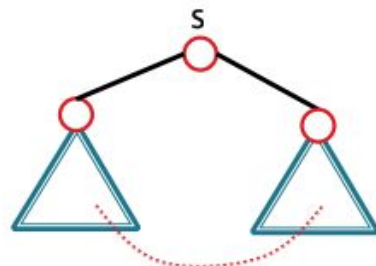
Puncte critice

Un vârf este **punct critic** \Leftrightarrow există două vârfuri $x, y \neq v$ astfel încât aparține oricărui x, y -lanț.

Arborele DF

- **rădăcina** este punct critic \Leftrightarrow

are cel puțin 2 fii în arborele DF



nu există muchii între subarbori
(de traversare)

- **un alt vârf i** din arbore este critic \Leftrightarrow

are cel puțin un fiu j cu $\text{niv_min}[j] \geq \text{nivel}[i]$

