

# Structuri pentru mulțimi disjuncte



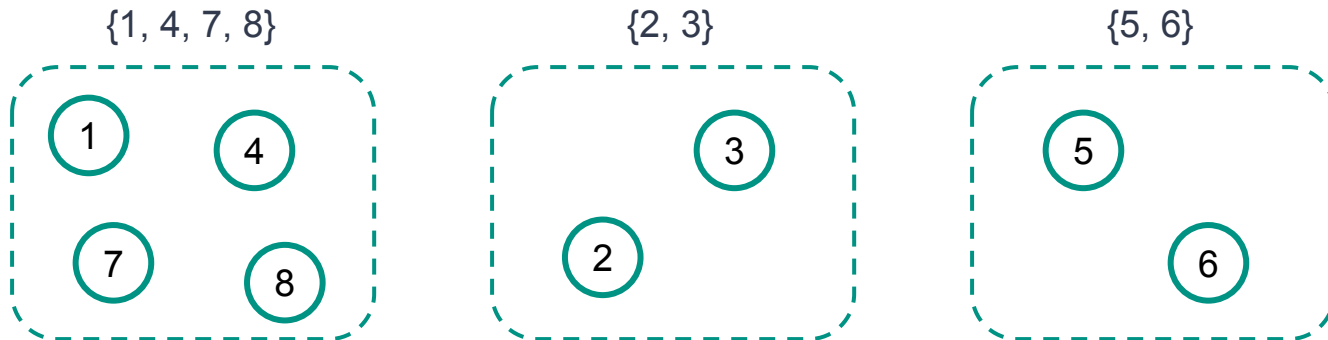
# Operații cu mulțimi disjuncte

## Problemă

Asupra unei partiții ale mulțimii  $\{1, 2, \dots, n\}$  (în submulțimi disjuncte) se efectuează o succesiune de operații de tip

- reuniune
- test de apartenență

Cum putem memora eficient submulțimile, astfel încât operațiile să se efectueze "eficient"?



# Operații cu mulțimi disjuncte

## Soluții

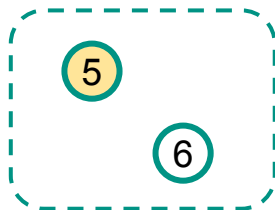
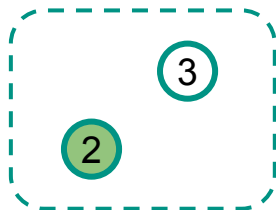
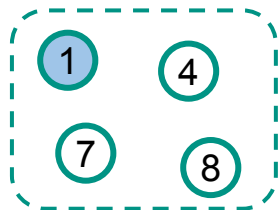
Asociem fiecărei submulțimi un reprezentant (o culoare).

Notăm operațiile:

- ☐ **Inițializare(u)** - creează o mulțime cu un singur element  $u$
- ☐ **Reprez(u)** - returnează reprezentantul mulțimii care conține pe  $u$
- ☐ **Reunește(u, v)** - unește mulțimea care conține  $u$  și cea care conține  $v$

# Vector de reprezentanți

**Varianta 1** - memorăm într-un vector  $r$ , pentru fiecare element  $x$ , reprezentantul mulțimii  $r[x]$  (v. Kruskal curs)

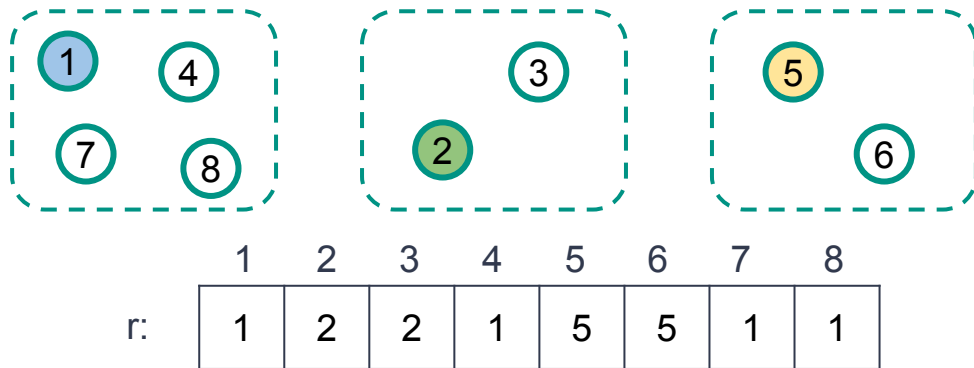


$r$ :

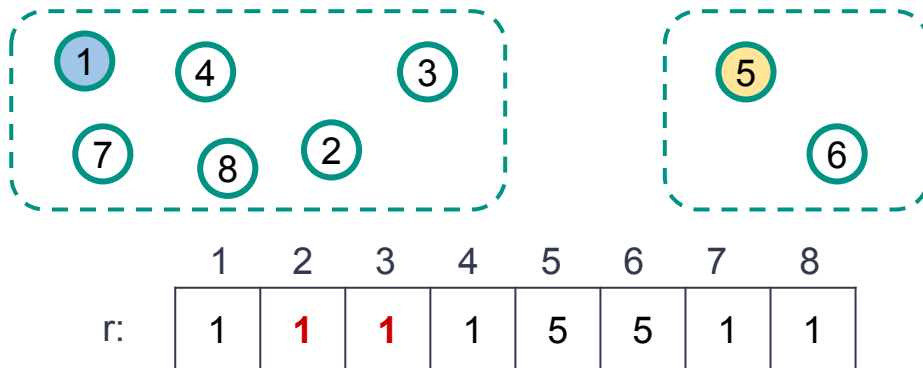
1	2	3	4	5	6	7	8
1	2	2	1	5	5	1	1

- **Inițializare(u)** -  $O(1)$       `void Inițializare(int u) { r[u] = u; }`
- **Reprez(u)** -  $O(1)$           `int Reprez(int u) { return r[u]; }`
- **Reuneste(u, v)** -  $O(n)$       `void Reuneste(int u, int v) {  
                                  r1 = Reprez(u);    // r1 = r[u]  
                                  r2 = Reprez(v);    // r2 = r[v]  
                                  for (k=1; k<=n; k++)  
                                      if (r[k] == r2)  
  r[k] = r1;  
                                  }`

# Vector de reprezentanți – Exemplu

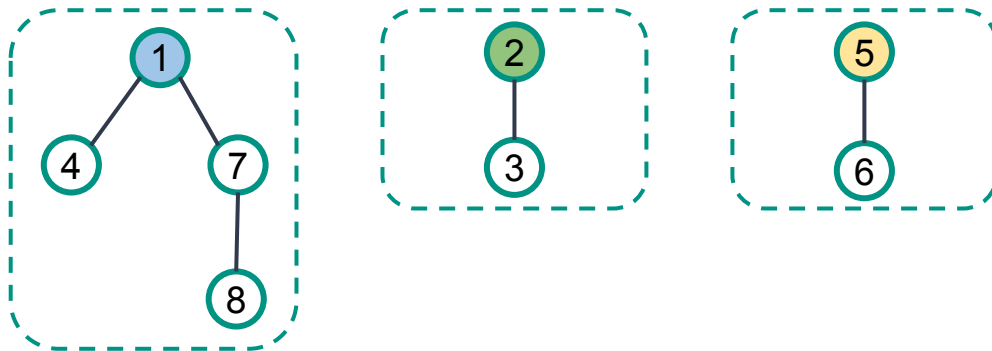


**Reuneste(4, 3)  $\Rightarrow$**



# Operații cu mulțimi disjuncte

**Varianta 2** - memorăm vârfurile fiecărei mulțimi ca un arbore (memorat cu tata), având ca reprezentant rădăcina



	1	2	3	4	5	6	7	8
tata:	0	0	2	1	0	5	1	7

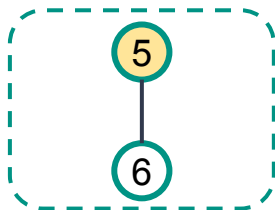
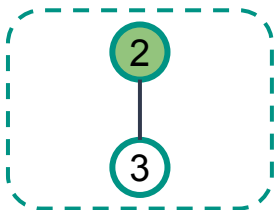
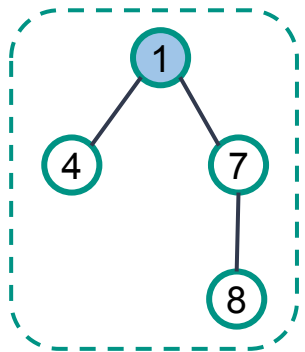
# Păduri de mulțimi disjuncte

**Varianta 2** - memorăm vârfurile fiecărei mulțimi ca un arbore (memorat cu tata), având ca reprezentant rădăcina

- **Inițializare(u) -  $O(1)$**  | `void Inițializare(int u) { tata[u] = h[u] = 0; }`
  - **Reprez(u)**
    - determinarea rădăcinii arborelui care conține u
    - **liniar în înălțimea arborelui**
  - **Reunește(u, v)**
    - reuniune ponderată
    - **în funcție de înălțimea arborilor**
    - $O(1)$  după determinarea reprezentanților lui u și v
  - **Arbori de înălțime logaritmică**
- ```
int Reprez(int u) {
    while (tata[u] != 0)
        u = tata[u];
    return u;
}

void Reunește(int u, int v) {
    int ru=Reprez(u), rv=Reprez(v);
    if (h[ru] > h[rv])
        tata[rv] = ru;
    else {
        tata[ru] = rv;
        if (h[ru] == h[rv])
            h[rv]++;
    }
}
```

# Păduri de mulțimi disjuncte – Exemplu

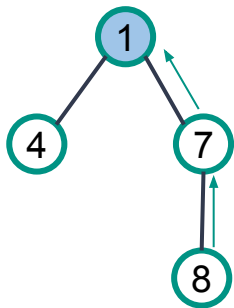


tata:

|       | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|-------|---|---|---|---|---|---|---|---|
| tata: | 0 | 0 | 2 | 1 | 0 | 5 | 1 | 7 |
| h:    | 2 | 1 | 0 | 0 | 1 | 0 | 1 | 0 |

h:

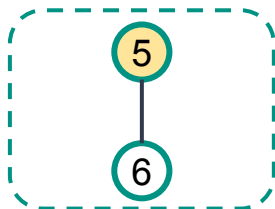
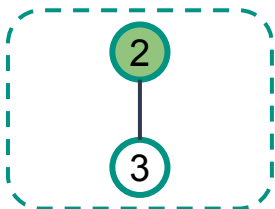
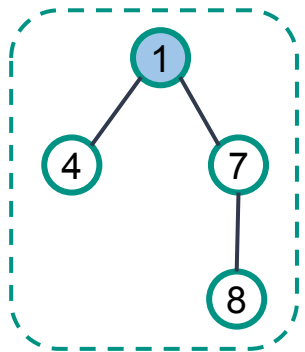
**Reprez(8) ⇒ returnează 1**



tata[8] = 7, tata[7] = 1, tata[1] = 0



# Păduri de mulțimi disjuncte – Exemplu

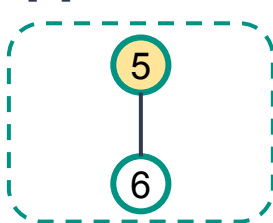
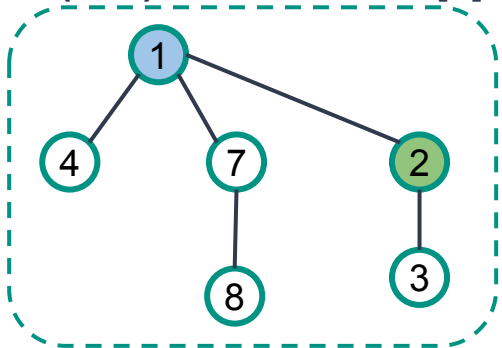


tata:

|       | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|-------|---|---|---|---|---|---|---|---|
| tata: | 0 | 0 | 2 | 1 | 0 | 5 | 1 | 7 |
| h:    | 2 | 1 | 0 | 0 | 1 | 0 | 1 | 0 |

h:

**Reuneste(4, 3)  $\Rightarrow$  deoarece  $h[1] > h[2]$ , se va seta  $tata[2] = 1$  (h nu se modifică)**



tata:

|       | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|-------|---|---|---|---|---|---|---|---|
| tata: | 0 | 1 | 2 | 1 | 0 | 5 | 1 | 7 |



# Păduri de mulțimi disjuncte

## Reprez(u) - Optimizare - **compresie de cale**

- tatăl vârfurilor de pe lanțul de la u la rădăcină se va seta ca fiind rădăcină

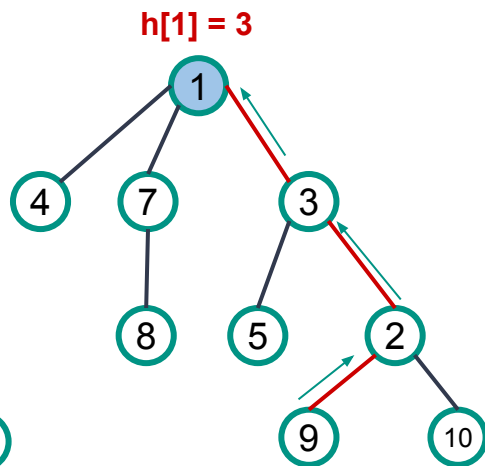
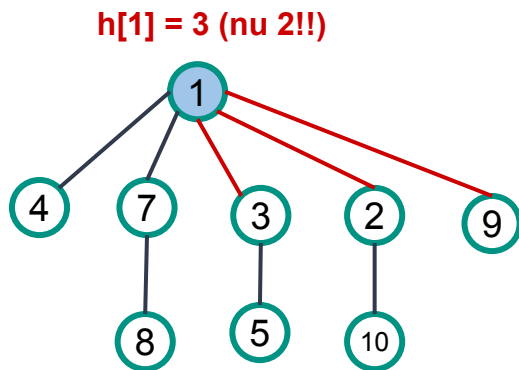
(vârfurile de pe acest lanț, parcurs pentru a găsi reprezentantul lui u, vor deveni fii ai rădăcinii, pentru ca reprezentantul lor să fie găsit mai ușor în căutările ulterioare)

## **!! h nu se actualizează**

De exemplu, după apelul **Reprez(9)** pentru arborele din dreapta,

**rezultatul va fi 1, iar arborele devine**

```
int Reprez(int u) {  
    if (tata[u] == 0)  
        return u;  
    tata[u] = Reprez(tata[u]);  
    return tata[u];  
}
```



# Algoritmul lui Kruskal

Implementare cu păduri disjuncte

# Kruskal – Pseudocod

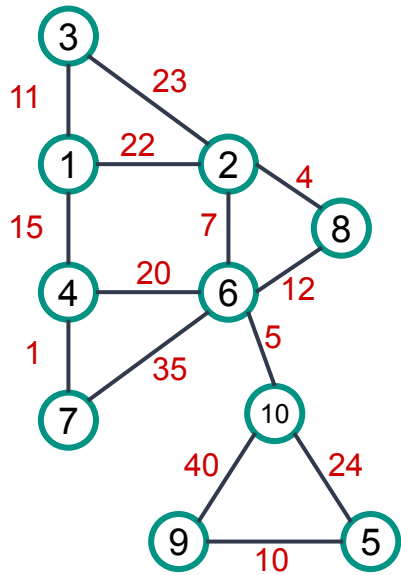
**sorteaza**(E)

**for**(v=1; v<=n; v++)  
    **Initializare**(v)

nrmsel=0

**for**(uv  $\in$  E)  
    **if** (**Reprez**(u)  $\neq$  **Reprez**(v)) {  
        E(T) = E(T)  $\cup$  {uv}  
        **Reuneste**(u,v)  
        nrmsel = nrmsel +1  
        **if** (nrmsel == n-1)  
            **STOP** // break  
    }



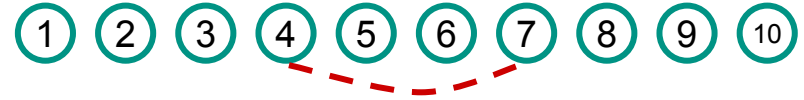


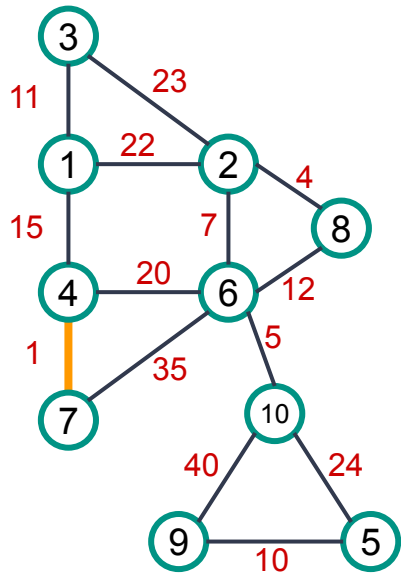
Ordine muchii

**(4, 7)**      (4, 6)  
 (2, 8)      (1, 2)  
 (6, 10)      (2, 3)  
 (2, 6)      (5, 10)  
 (5, 9)      (6, 7)  
 (1, 3)      (9, 10)  
 (6, 8)  
 (1, 4)

Muchia curentă  
**(4, 7):**

Pădurea de mulțimi disjuncte la pasul curent





Ordine muchii

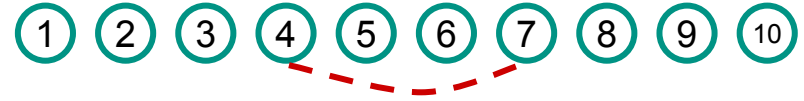
**(4, 7)** (4, 6)  
 (2, 8) (1, 2)  
 (6, 10) (2, 3)  
 (2, 6) (5, 10)  
 (5, 9) (6, 7)  
 (1, 3) (9, 10)  
 (6, 8)  
 (1, 4)

Muchia curentă

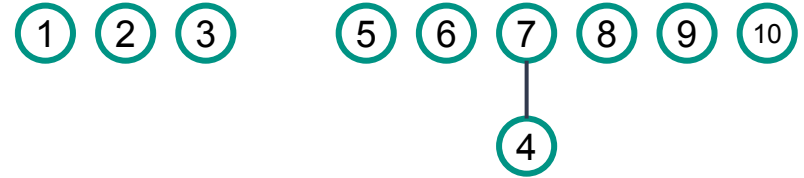
**(4, 7):**

**Reprez(4) ≠ Reprez(7)**

Pădurea de mulțimi disjuncte la pasul curent

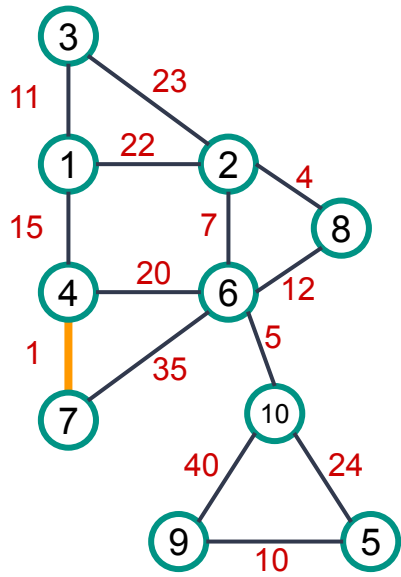


**Reunește(4, 7)**



|      | 1 | 2 | 3 | 4        | 5 | 6 | 7        | 8 | 9 | 10 |
|------|---|---|---|----------|---|---|----------|---|---|----|
| tata | 0 | 0 | 0 | <b>7</b> | 0 | 0 | 0        | 0 | 0 | 0  |
| h    | 0 | 0 | 0 | 0        | 0 | 0 | <b>1</b> | 0 | 0 | 0  |



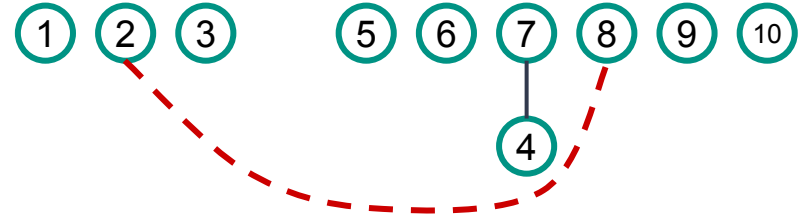


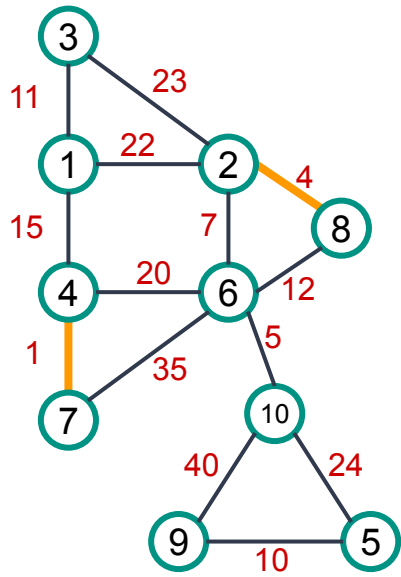
Ordine muchii

- |               |         |
|---------------|---------|
| (4, 7)        | (4, 6)  |
| <b>(2, 8)</b> | (1, 2)  |
| (6, 10)       | (2, 3)  |
| (2, 6)        | (5, 10) |
| (5, 9)        | (6, 7)  |
| (1, 3)        | (9, 10) |
| (6, 8)        |         |
| (1, 4)        |         |

Muchia curentă  
(2, 8):

Pădurea de mulțimi disjuncte la pasul curent





Ordine muchii

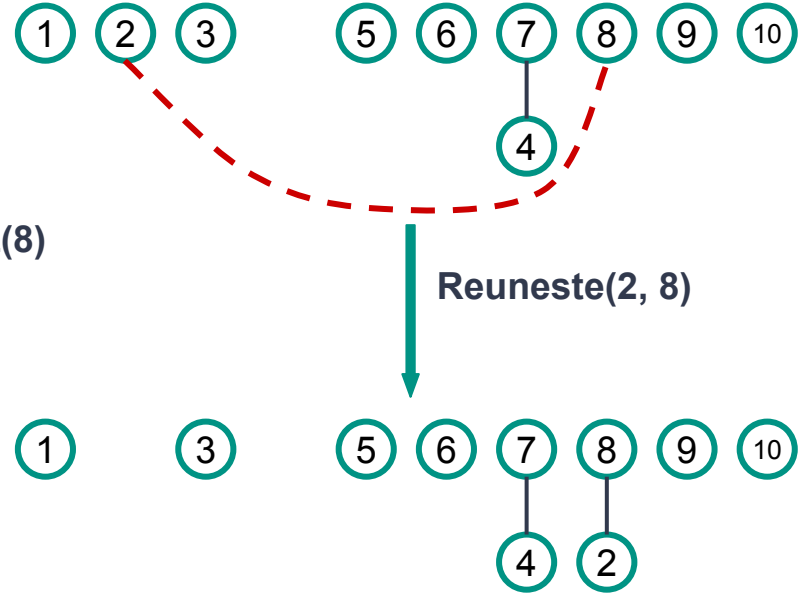
|               |         |
|---------------|---------|
| (4, 7)        | (4, 6)  |
| <b>(2, 8)</b> | (1, 2)  |
| (6, 10)       | (2, 3)  |
| (2, 6)        | (5, 10) |
| (5, 9)        | (6, 7)  |
| (1, 3)        | (9, 10) |
| (6, 8)        |         |
| (1, 4)        |         |

Muchia curentă

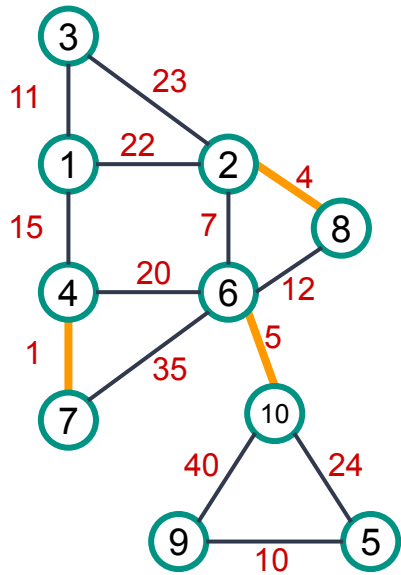
(2, 8):

**Reprez(2) ≠ Reprez(8)**

Pădurea de mulțimi disjuncte la pasul curent



|      | 1 | 2        | 3 | 4 | 5 | 6 | 7 | 8        | 9 | 10 |
|------|---|----------|---|---|---|---|---|----------|---|----|
| tata | 0 | <b>8</b> | 0 | 7 | 0 | 0 | 0 | 0        | 0 | 0  |
| h    | 0 | 0        | 0 | 0 | 0 | 0 | 1 | <b>1</b> | 0 | 0  |



Ordine muchii

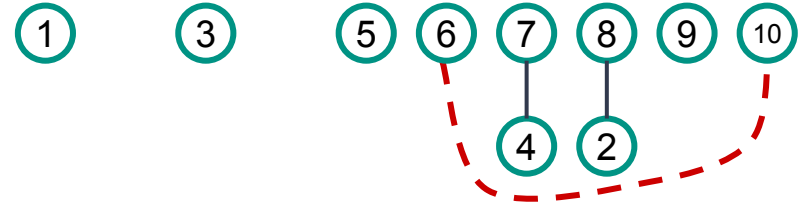
|                |         |
|----------------|---------|
| (4, 7)         | (4, 6)  |
| (2, 8)         | (1, 2)  |
| <b>(6, 10)</b> | (2, 3)  |
| (2, 6)         | (5, 10) |
| (5, 9)         | (6, 7)  |
| (1, 3)         | (9, 10) |
| (6, 8)         |         |
| (1, 4)         |         |

Muchia curentă

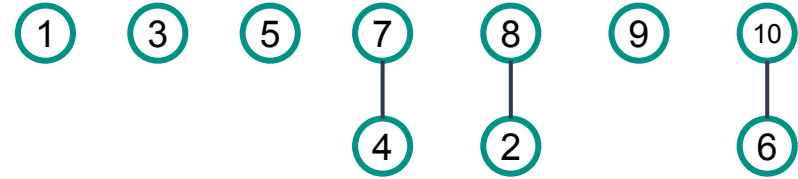
**(6, 10):**

**Reprez(6) ≠ Reprez(10)**

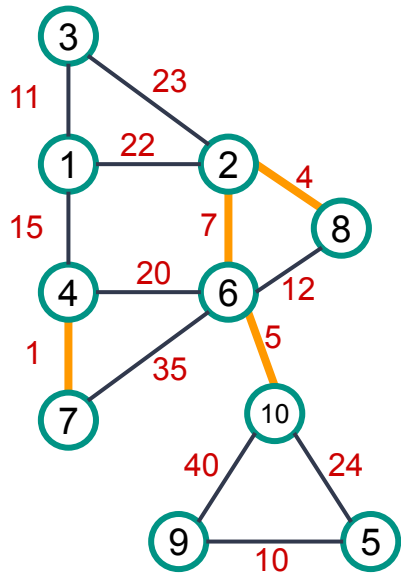
Pădurea de mulțimi disjuncte la pasul curent



**Reunește(6, 10)**



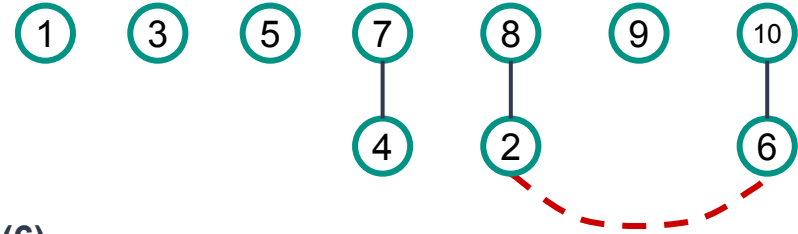
|      | 1 | 2 | 3 | 4 | 5 | 6         | 7 | 8 | 9 | 10       |
|------|---|---|---|---|---|-----------|---|---|---|----------|
| tata | 0 | 8 | 0 | 7 | 0 | <b>10</b> | 0 | 0 | 0 | 0        |
| h    | 0 | 0 | 0 | 0 | 0 | 0         | 1 | 1 | 0 | <b>1</b> |



Ordine muchii

|               |         |
|---------------|---------|
| (4, 7)        | (4, 6)  |
| (2, 8)        | (1, 2)  |
| (6, 10)       | (2, 3)  |
| <b>(2, 6)</b> | (5, 10) |
| (5, 9)        | (6, 7)  |
| (1, 3)        | (9, 10) |
| (6, 8)        |         |
| (1, 4)        |         |

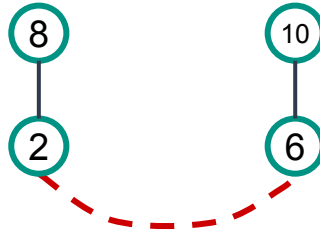
Pădurea de mulțimi disjuncte la pasul curent

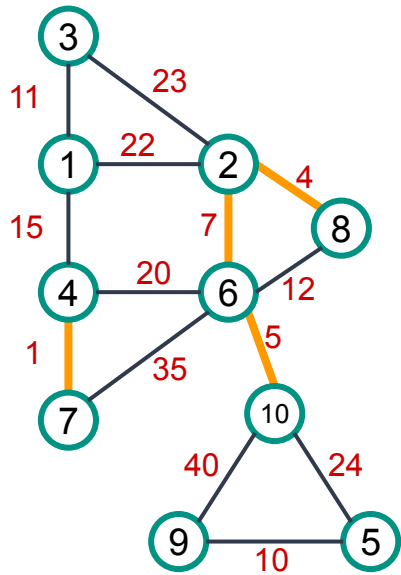


Muchia curentă

(2, 6):

**Reprez(2) ≠ Reprez(6)**

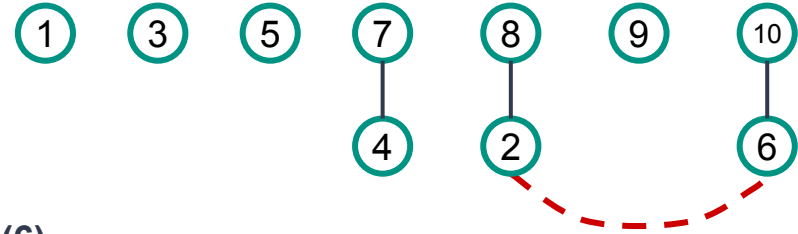




Ordine muchii

|               |         |
|---------------|---------|
| (4, 7)        | (4, 6)  |
| (2, 8)        | (1, 2)  |
| (6, 10)       | (2, 3)  |
| <b>(2, 6)</b> | (5, 10) |
| (5, 9)        | (6, 7)  |
| (1, 3)        | (9, 10) |
| (6, 8)        |         |
| (1, 4)        |         |

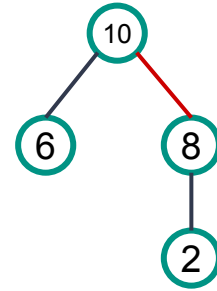
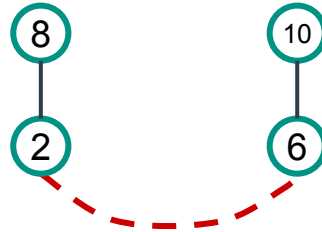
Pădurea de mulțimi disjuncte la pasul curent

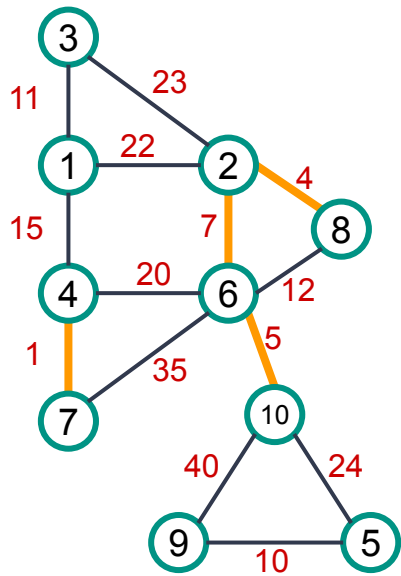


Muchia curentă

(2, 6):

$\text{Reprez}(2) \neq \text{Reprez}(6)$





Ordine muchii

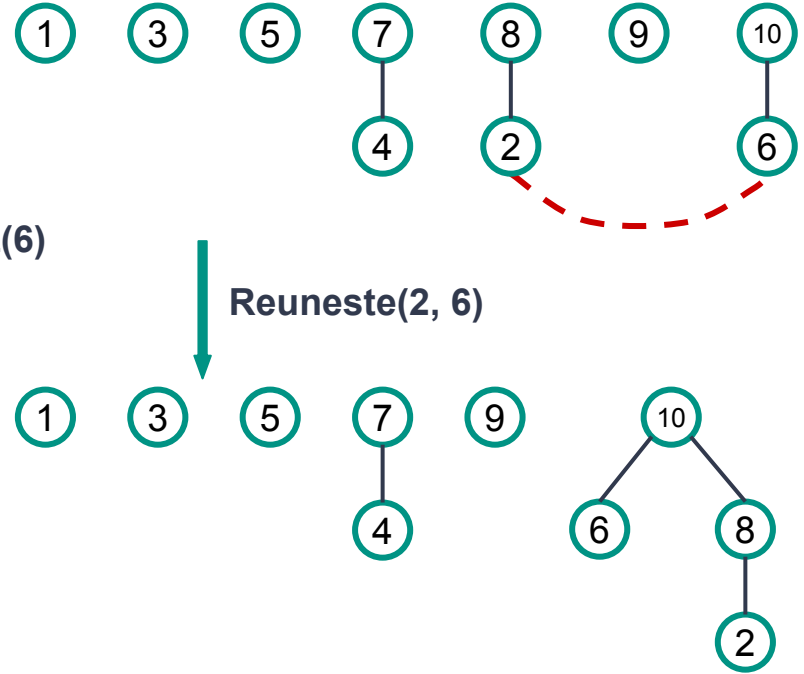
|               |         |
|---------------|---------|
| (4, 7)        | (4, 6)  |
| (2, 8)        | (1, 2)  |
| (6, 10)       | (2, 3)  |
| <b>(2, 6)</b> | (5, 10) |
| (5, 9)        | (6, 7)  |
| (1, 3)        | (9, 10) |
| (6, 8)        |         |
| (1, 4)        |         |

Muchia curentă

(2, 6):

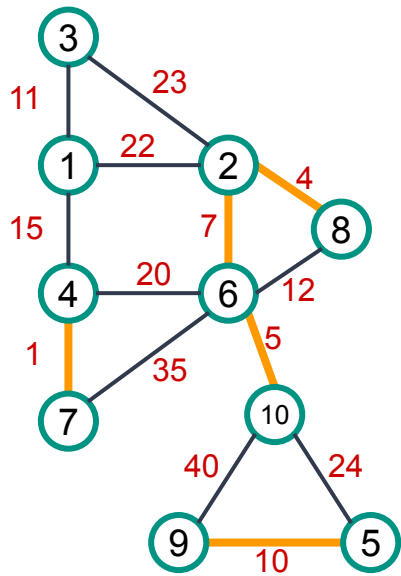
**Reprez(2) ≠ Reprez(6)**

Pădurea de mulțimi disjuncte la pasul curent



**Reunește(2, 6)**

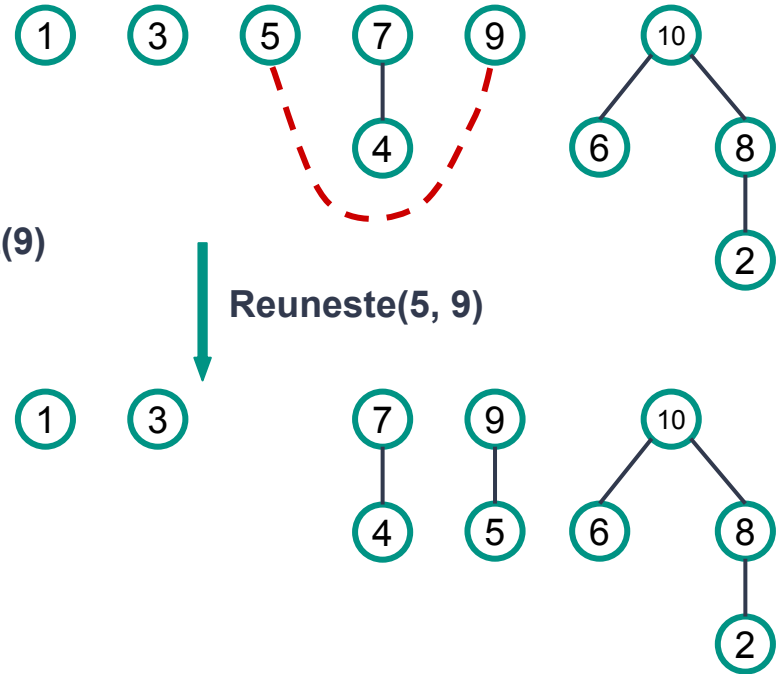
|      | 1 | 2 | 3 | 4 | 5 | 6  | 7 | 8         | 9 | 10       |
|------|---|---|---|---|---|----|---|-----------|---|----------|
| tata | 0 | 8 | 0 | 7 | 0 | 10 | 0 | <b>10</b> | 0 | 0        |
| h    | 0 | 0 | 0 | 0 | 0 | 0  | 1 | 1         | 0 | <b>1</b> |



Ordine muchii

|               |         |
|---------------|---------|
| (4, 7)        | (4, 6)  |
| (2, 8)        | (1, 2)  |
| (6, 10)       | (2, 3)  |
| (2, 6)        | (5, 10) |
| <b>(5, 9)</b> | (6, 7)  |
| (1, 3)        | (9, 10) |
| (6, 8)        |         |
| (1, 4)        |         |

Pădurea de mulțimi disjuncte la pasul curent



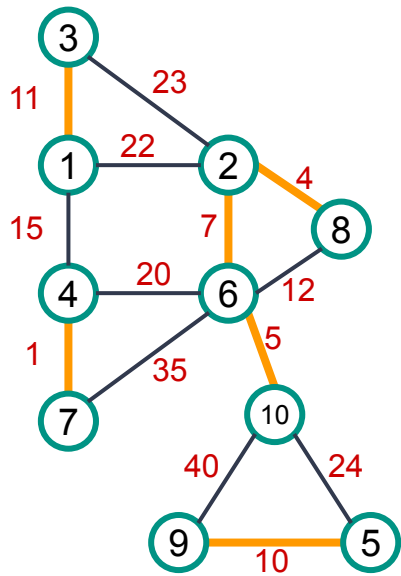
Muchia curentă

(5, 9):

$\text{Reprez}(5) \neq \text{Reprez}(9)$

Reunește(5, 9)

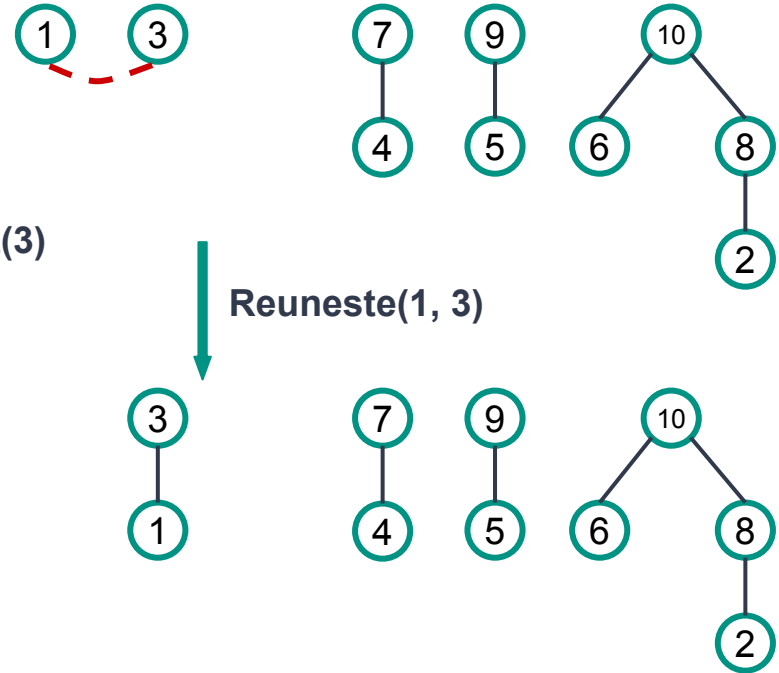
|      | 1 | 2 | 3 | 4 | 5        | 6  | 7 | 8  | 9        | 10 |
|------|---|---|---|---|----------|----|---|----|----------|----|
| tata | 0 | 8 | 0 | 7 | <b>9</b> | 10 | 0 | 10 | 0        | 0  |
| h    | 0 | 0 | 0 | 0 | 0        | 0  | 1 | 1  | <b>1</b> | 1  |



Ordine muchii

|               |         |
|---------------|---------|
| (4, 7)        | (4, 6)  |
| (2, 8)        | (1, 2)  |
| (6, 10)       | (2, 3)  |
| (2, 6)        | (5, 10) |
| (5, 9)        | (6, 7)  |
| <b>(1, 3)</b> | (9, 10) |
| (6, 8)        |         |
| (1, 4)        |         |

Pădurea de mulțimi disjuncte la pasul curent



Muchia curentă

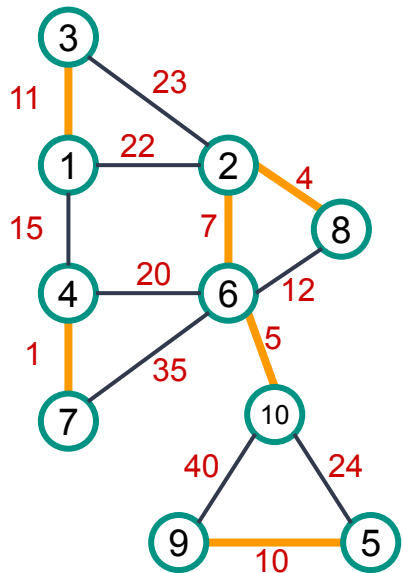
(1, 3):

$\text{Reprez}(1) \neq \text{Reprez}(3)$

Reunește(1, 3)

|      | 1        | 2 | 3        | 4 | 5 | 6  | 7 | 8  | 9 | 10 |
|------|----------|---|----------|---|---|----|---|----|---|----|
| tata | <b>3</b> | 8 | 0        | 7 | 9 | 10 | 0 | 10 | 0 | 0  |
| h    | 0        | 0 | <b>1</b> | 0 | 0 | 0  | 1 | 1  | 1 | 1  |

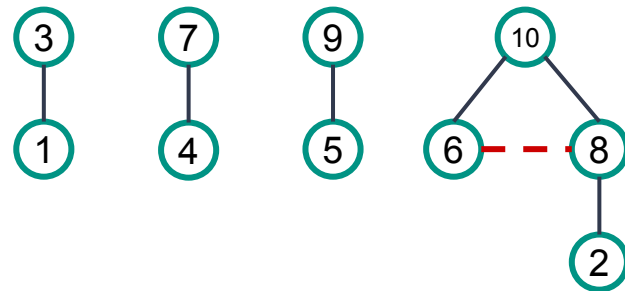




Ordine muchii

(4, 7)      (4, 6)  
 (2, 8)      (1, 2)  
 (6, 10)    (2, 3)  
 (2, 6)      (5, 10)  
 (5, 9)      (6, 7)  
 (1, 3)      (9, 10)  
**(6, 8)**  
 (1, 4)

Pădurea de mulțimi disjuncte la pasul curent



Muchia curentă

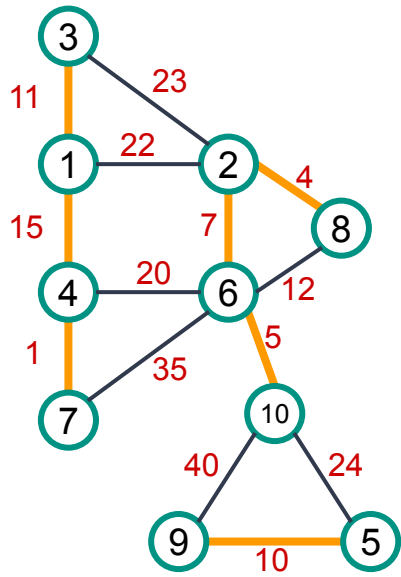
(6, 8):

**Reprez(6) = Reprez(8)**

⇒ nu este selectată

**Observație:** Până acum, în funcția Reprez nu a fost modificat vectorul tata prin compresie de cale, deoarece vârfurile erau la distanță cel mult 1 față de rădăcină.

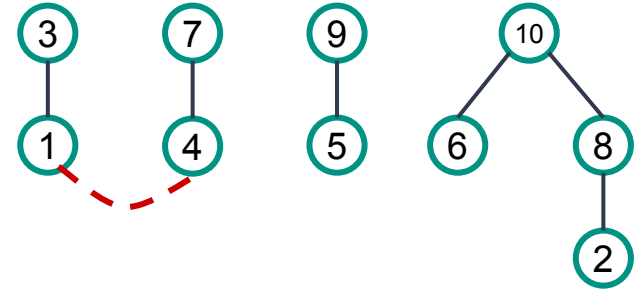
|      | 1 | 2 | 3 | 4 | 5 | 6  | 7 | 8  | 9 | 10 |
|------|---|---|---|---|---|----|---|----|---|----|
| tata | 3 | 8 | 0 | 7 | 9 | 10 | 0 | 10 | 0 | 0  |
| h    | 0 | 0 | 1 | 0 | 0 | 0  | 1 | 1  | 1 | 1  |



Ordine muchii

|               |         |
|---------------|---------|
| (4, 7)        | (4, 6)  |
| (2, 8)        | (1, 2)  |
| (6, 10)       | (2, 3)  |
| (2, 6)        | (5, 10) |
| (5, 9)        | (6, 7)  |
| (1, 3)        | (9, 10) |
| (6, 8)        |         |
| <b>(1, 4)</b> |         |

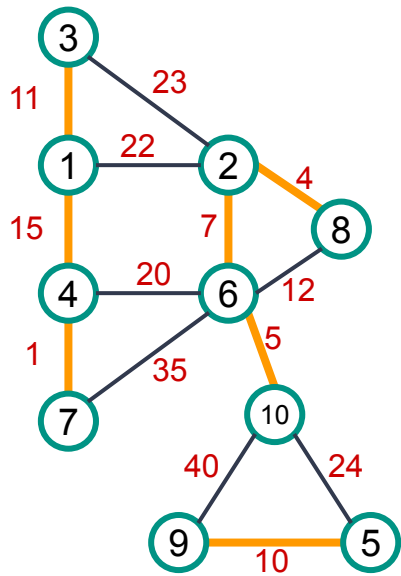
Pădurea de mulțimi disjuncte la pasul curent



Muchia curentă

(1, 4):

**Reprez(1) ≠ Reprez(4)**



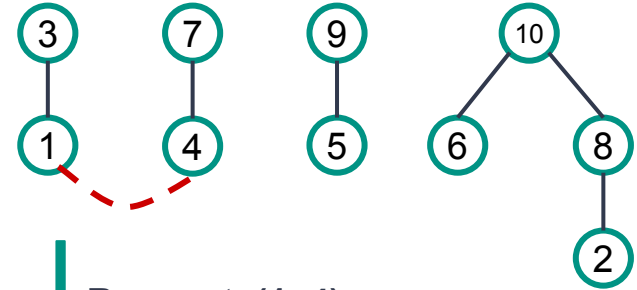
Ordine muchii

(4, 7)      (4, 6)  
 (2, 8)      (1, 2)  
 (6, 10)    (2, 3)  
 (2, 6)      (5, 10)  
 (5, 9)      (6, 7)  
 (1, 3)      (9, 10)  
 (6, 8)  
**(1, 4)**

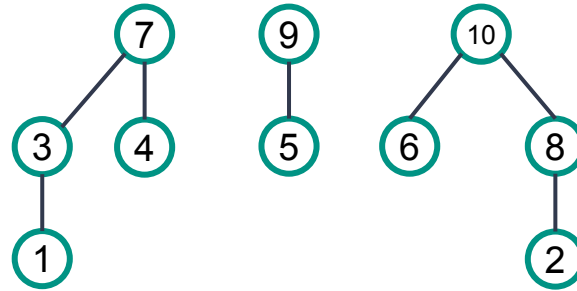
Pădurea de mulțimi disjuncte la pasul curent

Muchia curentă  
 (1, 4):

**Reprez(1) ≠ Reprez(4)**

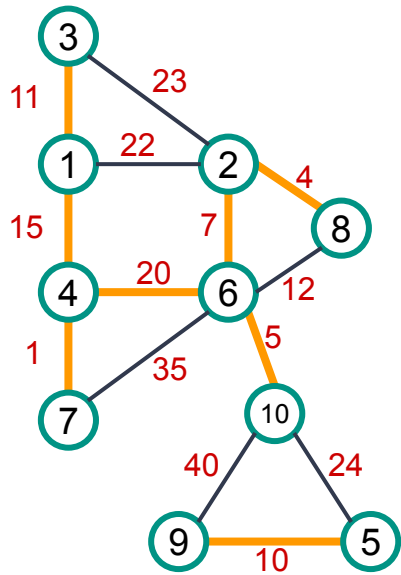


**Reuneste(1, 4)**



1   2   3   4   5   6   7   8   9   10

|      |   |   |          |   |   |    |          |    |   |   |
|------|---|---|----------|---|---|----|----------|----|---|---|
| tata | 3 | 8 | <b>7</b> | 7 | 9 | 10 | 0        | 10 | 0 | 0 |
| h    | 0 | 0 | 1        | 0 | 0 | 0  | <b>2</b> | 1  | 1 | 1 |



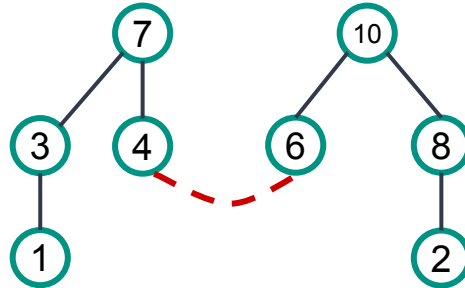
Ordine muchii

|         |               |
|---------|---------------|
| (4, 7)  | <b>(4, 6)</b> |
| (2, 8)  | (1, 2)        |
| (6, 10) | (2, 3)        |
| (2, 6)  | (5, 10)       |
| (5, 9)  | (6, 7)        |
| (1, 3)  | (9, 10)       |
| (6, 8)  |               |
| (1, 4)  |               |

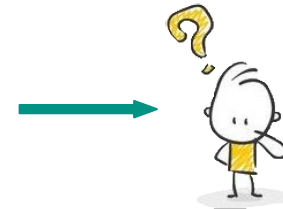
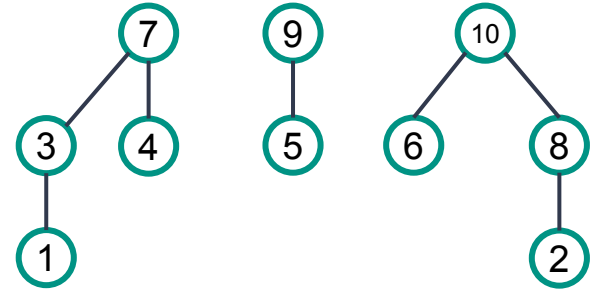
Muchia curentă

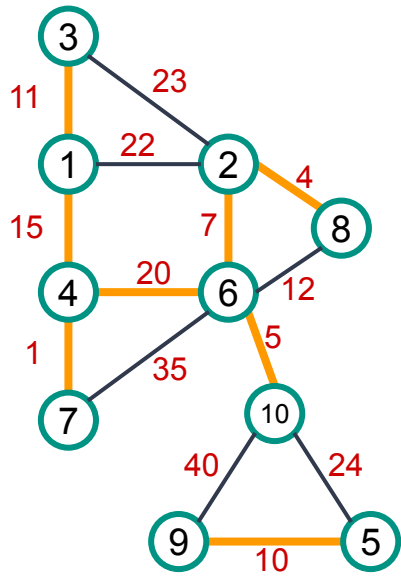
**(4, 6):**

**Reprez(4)  $\neq$  Reprez(6)**



Pădurea de mulțimi disjuncte la pasul curent





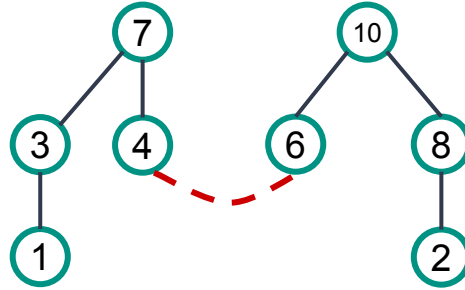
Ordine muchii

|         |               |
|---------|---------------|
| (4, 7)  | <b>(4, 6)</b> |
| (2, 8)  | (1, 2)        |
| (6, 10) | (2, 3)        |
| (2, 6)  | (5, 10)       |
| (5, 9)  | (6, 7)        |
| (1, 3)  | (9, 10)       |
| (6, 8)  |               |
| (1, 4)  |               |

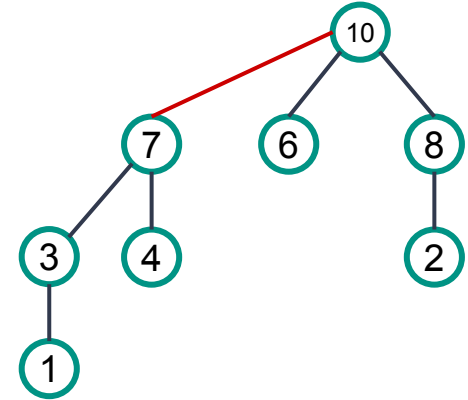
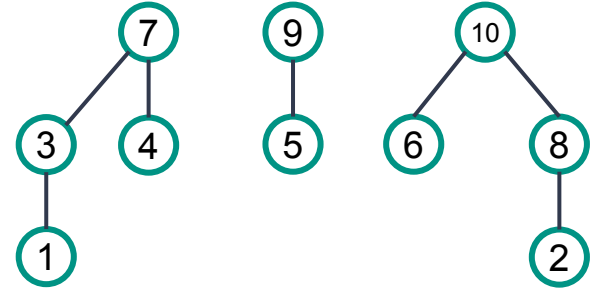
Muchia curentă

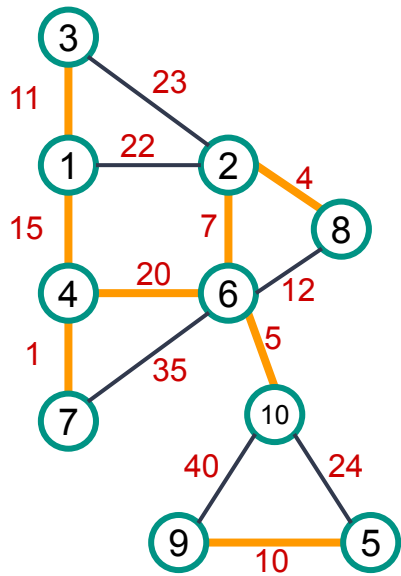
(4, 6):

**Reprez(4)  $\neq$  Reprez(6)**



Pădurea de mulțimi disjuncte la pasul curent





Ordine muchii

|         |               |
|---------|---------------|
| (4, 7)  | <b>(4, 6)</b> |
| (2, 8)  | (1, 2)        |
| (6, 10) | (2, 3)        |
| (2, 6)  | (5, 10)       |
| (5, 9)  | (6, 7)        |
| (1, 3)  | (9, 10)       |
| (6, 8)  |               |
| (1, 4)  |               |

Muchia curentă

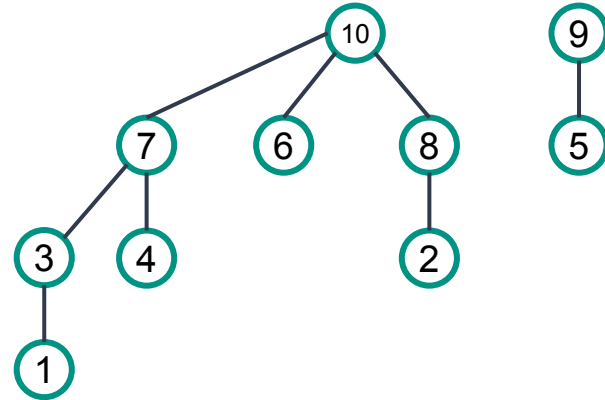
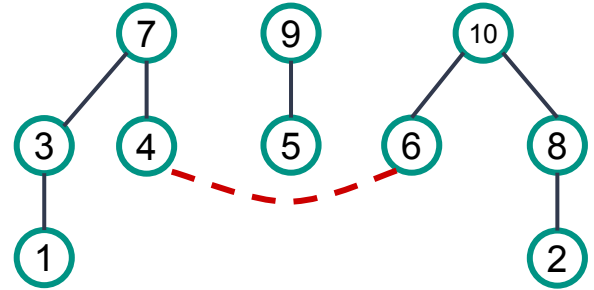
**(4, 6):**

**Reprez(4) ≠ Reprez(6)**

**Reunește(4, 6)**

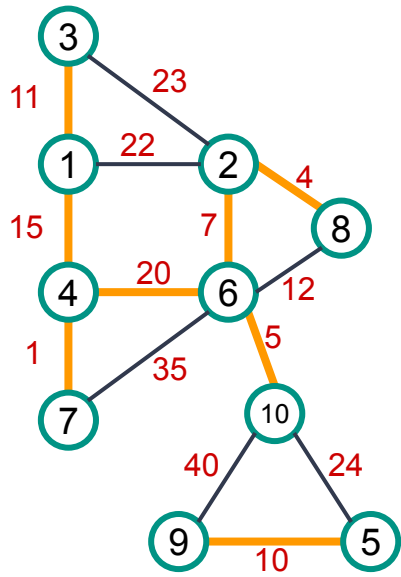


Pădurea de mulțimi disjuncte la pasul curent



1 2 3 4 5 6 7 8 9 10

|      |   |   |   |   |   |    |           |    |   |          |
|------|---|---|---|---|---|----|-----------|----|---|----------|
| tata | 3 | 8 | 7 | 7 | 9 | 10 | <b>10</b> | 10 | 0 | 0        |
| h    | 0 | 0 | 1 | 0 | 0 | 0  | 2         | 1  | 1 | <b>3</b> |



Ordine muchii

|         |               |
|---------|---------------|
| (4, 7)  | (4, 6)        |
| (2, 8)  | <b>(1, 2)</b> |
| (6, 10) | (2, 3)        |
| (2, 6)  | (5, 10)       |
| (5, 9)  | (6, 7)        |
| (1, 3)  | (9, 10)       |
| (6, 8)  |               |
| (1, 4)  |               |

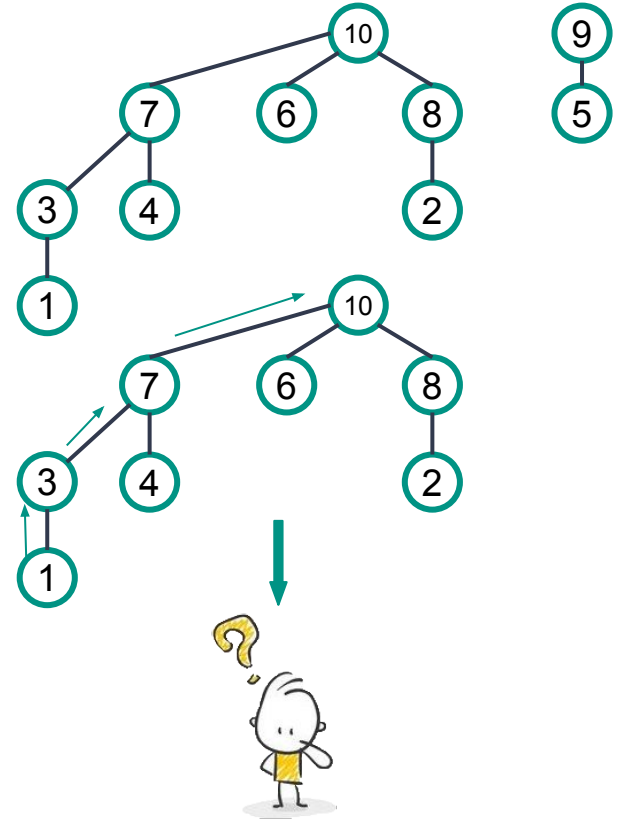
Muchia curentă

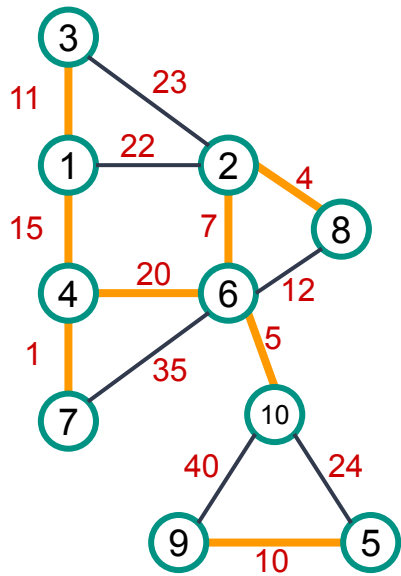
(1, 2):

**Reprez(1):  $\Rightarrow 10$  +  
compresie de cale**

**!! h nu se modifică  
(h[7] rămâne 2)**

Pădurea de mulțimi disjuncte la pasul curent





Ordine muchii

|         |               |
|---------|---------------|
| (4, 7)  | (4, 6)        |
| (2, 8)  | <b>(1, 2)</b> |
| (6, 10) | (2, 3)        |
| (2, 6)  | (5, 10)       |
| (5, 9)  | (6, 7)        |
| (1, 3)  | (9, 10)       |
| (6, 8)  |               |
| (1, 4)  |               |

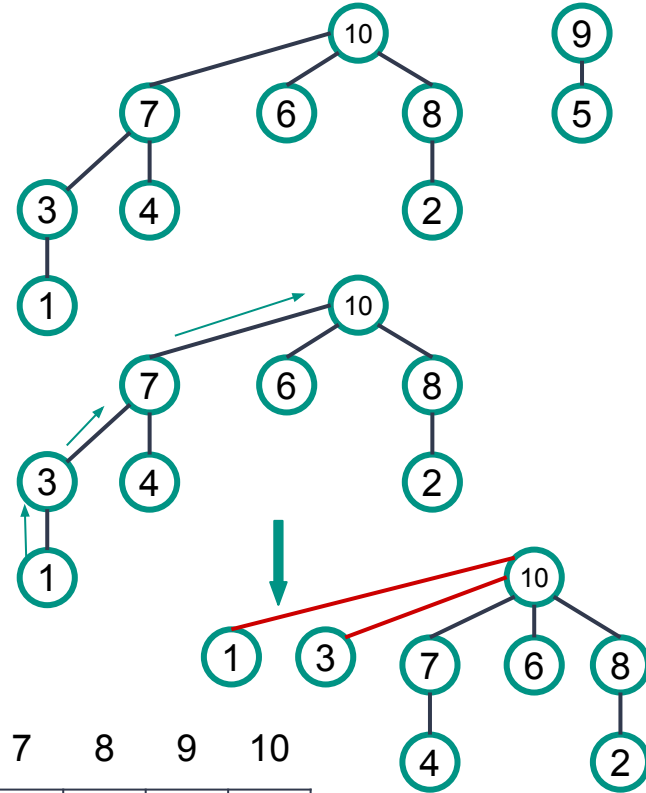
Muchia curentă

(1, 2):

Reprez(1):  $\Rightarrow 10 +$   
compresie de cale

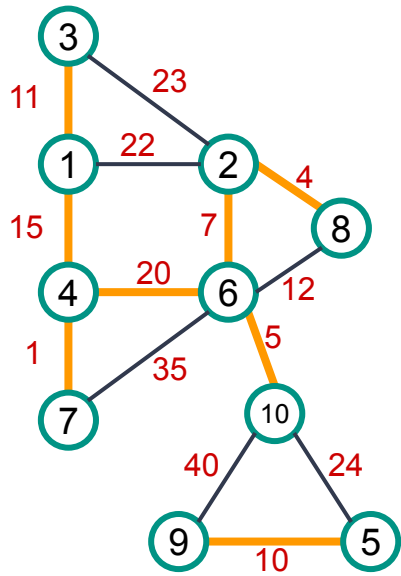
!! h nu se modifică  
(h[7] rămâne 2)

Pădurea de mulțimi disjuncte la pasul curent



|      | 1         | 2 | 3         | 4 | 5 | 6  | 7  | 8  | 9 | 10 |
|------|-----------|---|-----------|---|---|----|----|----|---|----|
| tata | <b>10</b> | 8 | <b>10</b> | 7 | 9 | 10 | 10 | 10 | 0 | 0  |
| h    | 0         | 0 | 1         | 0 | 0 | 0  | 2  | 1  | 1 | 3  |





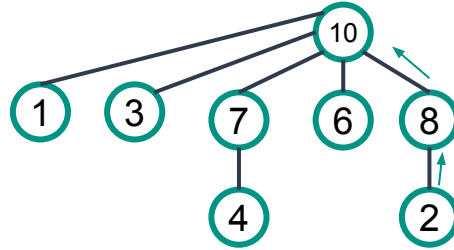
Ordine muchii

|         |               |
|---------|---------------|
| (4, 7)  | (4, 6)        |
| (2, 8)  | <b>(1, 2)</b> |
| (6, 10) | (2, 3)        |
| (2, 6)  | (5, 10)       |
| (5, 9)  | (6, 7)        |
| (1, 3)  | (9, 10)       |
| (6, 8)  |               |
| (1, 4)  |               |

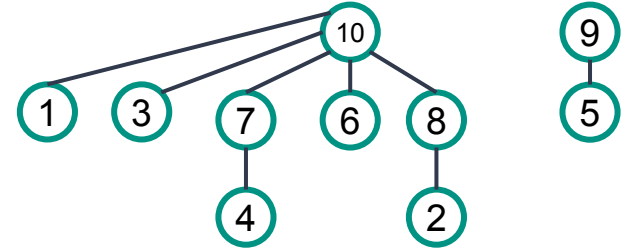
Muchia curentă

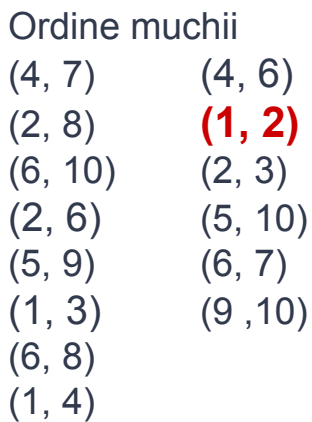
(1, 2):

**Reprez(2):  $\Rightarrow 10 +$   
compresie de cale**

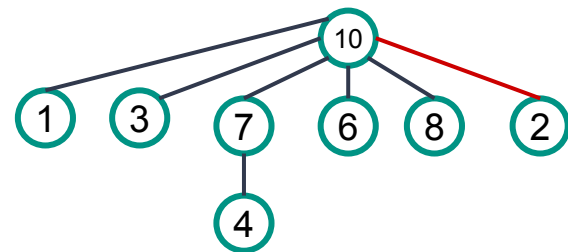
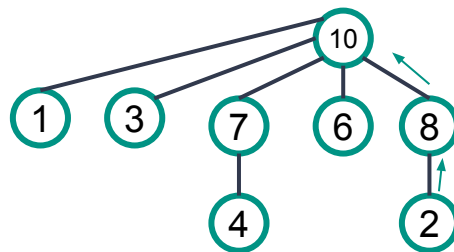


Pădurea de mulțimi disjuncte la pasul curent

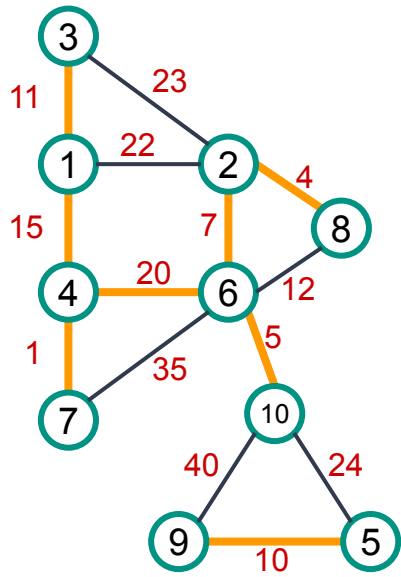




**Reprez(2):  $\Rightarrow 10 +$   
compresie de cale**



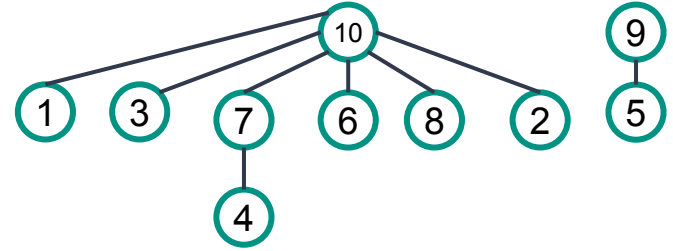
|      | 1  | 2  | 3  | 4 | 5 | 6  | 7  | 8  | 9 | 10 |
|------|----|----|----|---|---|----|----|----|---|----|
| tata | 10 | 10 | 10 | 7 | 9 | 10 | 10 | 10 | 0 | 0  |
| h    | 0  | 0  | 1  | 0 | 0 | 0  | 2  | 1  | 1 | 3  |



Ordine muchii

|         |               |
|---------|---------------|
| (4, 7)  | (4, 6)        |
| (2, 8)  | <b>(1, 2)</b> |
| (6, 10) | (2, 3)        |
| (2, 6)  | (5, 10)       |
| (5, 9)  | (6, 7)        |
| (1, 3)  | (9, 10)       |
| (6, 8)  |               |
| (1, 4)  |               |

Pădurea de mulțimi disjuncte la pasul curent



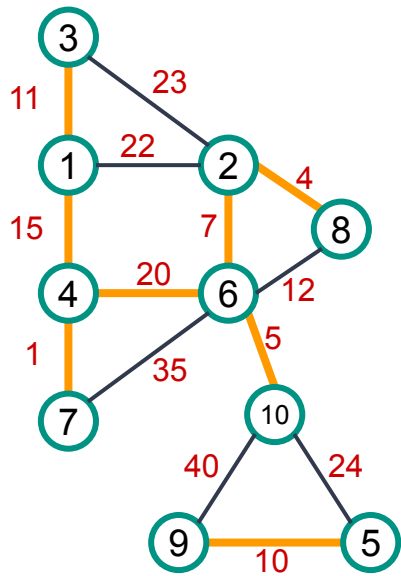
Muchia curentă

(1, 2):

**Reprez(1) = 10**

**Reprez(2) = 10  $\Rightarrow$  nu este selectată**

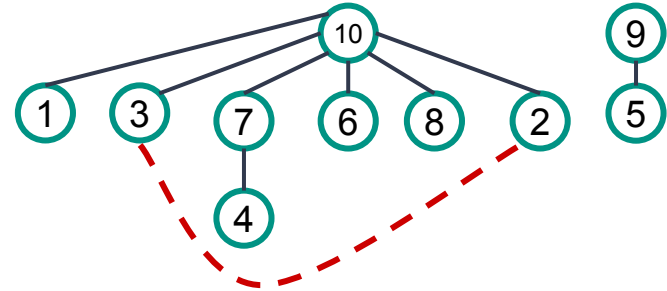
|      | 1  | 2  | 3  | 4 | 5 | 6  | 7  | 8  | 9 | 10 |
|------|----|----|----|---|---|----|----|----|---|----|
| tata | 10 | 10 | 10 | 7 | 9 | 10 | 10 | 10 | 0 | 0  |
| h    | 0  | 0  | 1  | 0 | 0 | 0  | 2  | 1  | 1 | 3  |



Ordine muchii

- (4, 7)      (4, 6)
- (2, 8)      (1, 2)
- (6, 10)    **(2, 3)**
- (2, 6)      (5, 10)
- (5, 9)      (6, 7)
- (1, 3)      (9, 10)
- (6, 8)
- (1, 4)

Pădurea de mulțimi disjuncte la pasul curent



Muchia curentă

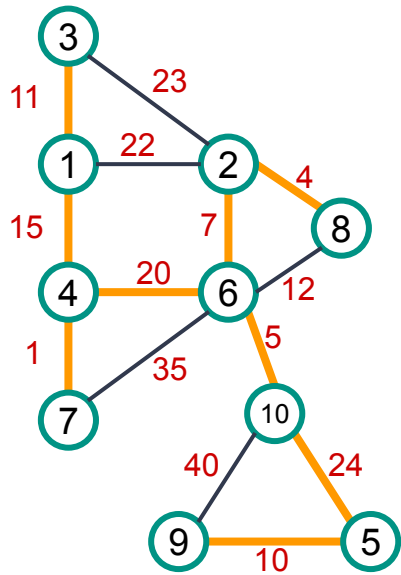
(2, 3):

**Reprez(2) = Reprez(3)**

**⇒ nu este selectată**

**2 și 3 sunt fii ai rădăcinii, compresia de cale nu modifică vectorul tata**

|      | 1  | 2  | 3  | 4 | 5 | 6  | 7  | 8  | 9 | 10 |
|------|----|----|----|---|---|----|----|----|---|----|
| tata | 10 | 10 | 10 | 7 | 9 | 10 | 10 | 10 | 0 | 0  |
| h    | 0  | 0  | 1  | 0 | 0 | 0  | 2  | 1  | 1 | 3  |



Ordine muchii

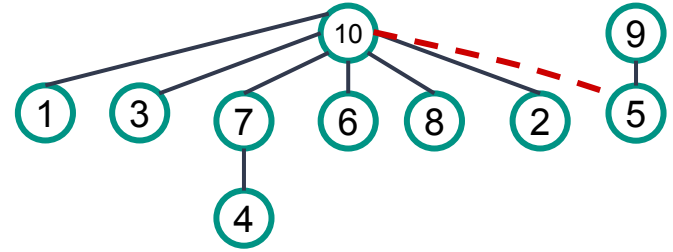
|         |                |
|---------|----------------|
| (4, 7)  | (4, 6)         |
| (2, 8)  | (1, 2)         |
| (6, 10) | (2, 3)         |
| (2, 6)  | <b>(5, 10)</b> |
| (5, 9)  | (6, 7)         |
| (1, 3)  | (9, 10)        |
| (6, 8)  |                |
| (1, 4)  |                |

Muchia curentă

**(5, 10):**

**Reprez(5) ≠ Reprez(10)**

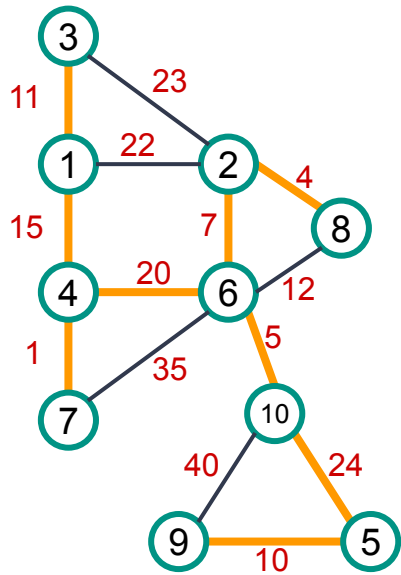
Pădurea de mulțimi disjuncte la pasul curent



**Reunește(5, 10)**  
**reuniune ponderată**



|      | 1  | 2  | 3  | 4 | 5 | 6  | 7  | 8  | 9 | 10 |
|------|----|----|----|---|---|----|----|----|---|----|
| tata | 10 | 10 | 10 | 7 | 9 | 10 | 10 | 10 | 0 | 0  |
| h    | 0  | 0  | 1  | 0 | 0 | 0  | 2  | 1  | 1 | 3  |



Ordine muchii

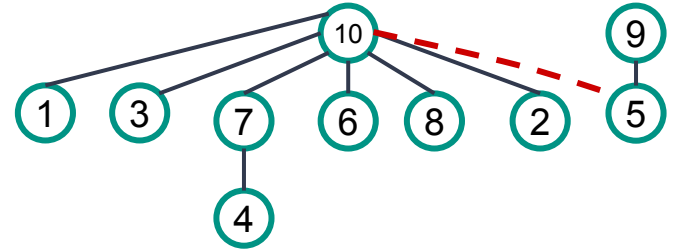
|         |                |
|---------|----------------|
| (4, 7)  | (4, 6)         |
| (2, 8)  | (1, 2)         |
| (6, 10) | (2, 3)         |
| (2, 6)  | <b>(5, 10)</b> |
| (5, 9)  | (6, 7)         |
| (1, 3)  | (9, 10)        |
| (6, 8)  |                |
| (1, 4)  |                |

Muchia curentă

(5, 10):

$\text{Reprez}(5) \neq \text{Reprez}(10)$

Pădurea de mulțimi disjuncte la pasul curent

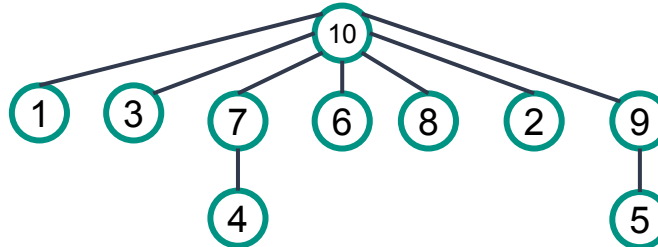


Reunește(5, 10)

reuniune ponderată

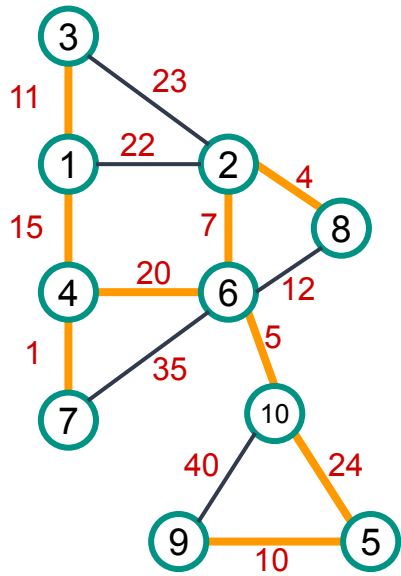
$h[\text{Reprez}(5)] = h[9] = 1$

$< h[\text{Reprez}(10)] = h[10] = 3$



1 2 3 4 5 6 7 8 9 10

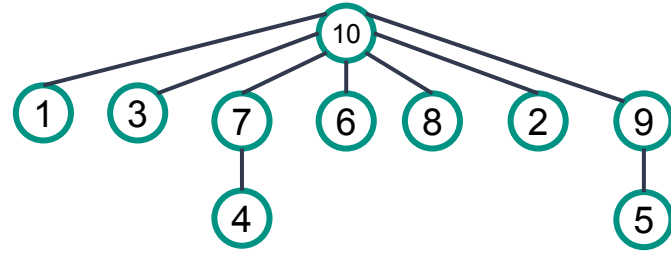
|      |    |    |    |   |   |    |    |    |           |   |
|------|----|----|----|---|---|----|----|----|-----------|---|
| tata | 10 | 10 | 10 | 7 | 9 | 10 | 10 | 10 | <b>10</b> | 0 |
| h    | 0  | 0  | 1  | 0 | 0 | 0  | 2  | 1  | 1         | 3 |



Ordine muchii

|         |         |
|---------|---------|
| (4, 7)  | (4, 6)  |
| (2, 8)  | (1, 2)  |
| (6, 10) | (2, 3)  |
| (2, 6)  | (5, 10) |
| (5, 9)  | (6, 7)  |
| (1, 3)  | (9, 10) |
| (6, 8)  |         |
| (1, 4)  |         |

Pădurea de mulțimi disjuncte la pasul curent



**STOP** - au fost selectate  $n-1$  muchii

**Muchii apcm  $\neq$  muchiile din pădurea de mulțimi disjuncte finală (formată dintr-un singur arbore)**



