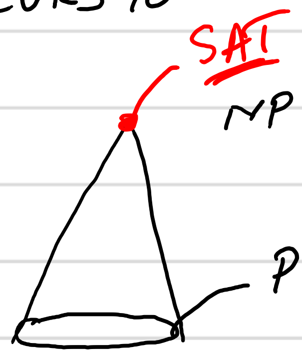
CURS 10

UNDE SUNTEM



NP ⤳ nedeterminist polynomial time

$A \leq_m^p B$: $\exists f : \Sigma^* \to \Sigma^*$ f calculabilă în timp polinomial

$\forall x \in \Sigma^* \quad x \in A \Leftrightarrow f(x) \in B$

A NP completă:
- $A \in NP$ (1)
- $\forall B \in NP \quad B \leq_m^p A$ (2)

A NP-hard: (2)

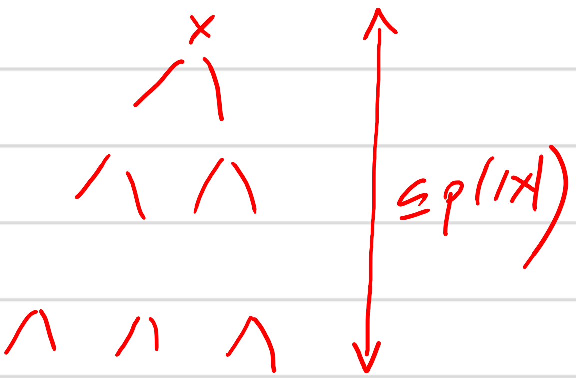
(T) SAT este NP-completă

NP

$C_1 \to C_2$, $C_1 \to C_2'$

$$\delta : \Sigma^k \times S \to \underline{\underline{\mathcal{P}}}(\Sigma^{k-1} \times S \times \{\leftarrow, \to, \downarrow\}^k)$$

multifuncție



$A \in NP \iff \exists\, g(\cdot,\cdot)$ relație $(g(x,y) \in \{0,1\})$

y martor pt x

- g calculabilă în timp polinomial
- $\exists$ polinom $p(x)$ a.î.

$\forall x \in \Sigma^* \quad x \in A \iff \exists y \in \Sigma^* \quad |y| \le p(|x|)$

$g(x,y) = \text{TRUE}$

Obs

$$\left.\begin{array}{l} A, B \in NP \\ A \text{ NP-completă} \\ A \le_m^P B \end{array}\right\} \Rightarrow B \text{ NP completă}$$

Data trecută $\quad SAT \le_m^P 3\text{-}SAT$

$\boxed{\text{1-IN-3 SAT}}$

INPUT $\quad \phi = \bigwedge_{i=1}^{m} C_i(x,y,z)$

DE DECIS $\quad \phi$ satisfiabilă sau NU.

$C_i(x,y,z) = \text{TRUE} \iff$ exact una din x, y, z TRUE

$x \vee y \vee z$

8

x	y	z	
0	0	0	0
			1
			1
			1
			1

1-IN-3 (x, y, z)

x	y	z	
			0
0	0	1	1
1	0	0	1
0	1	0	1
			0

(T) 1-IN-3 SAT este NP-completă

Dem

(1) 1-IN-3 SAT $\in NP$ ușor

(2) 3-SAT $\le_m^?$ 1-IN-3 SAT

$$\phi \longrightarrow \tilde{\phi}$$

$x \vee y \vee z$

1-IN-3(x, a, d)
1-IN-3(y, b, d)
1-IN-3(a, b, e)
1-IN-3(c, d, f)
1-IN-3(z, c, FALSE)

variabila care inițial este falsă

x	y	z
T	F	F

a	b	c	d	e	f
F	T	T	F	F	F

~~F F F~~ $\to$ ~~mă blochez~~

$C \longrightarrow \tilde{C}$ formulă

$$\phi = \bigwedge_{i=1}^{m} C_i \longrightarrow \tilde{\phi} = \bigwedge_{i=1}^{m} \tilde{C}_i$$

~~variabile~~ noi în fiecare $\tilde{C}_i$

Obs FALSE?

1-in-3 (x, a, b)

1-in-3 (x, b, c)

1-in-3 (x, a, c)

$x = $ TRUE

$a, b, c = $ FALSE

Obs ~~1-in-2 SAT~~ $\in P$

2-SAT $\in P$

HORN-SAT $\in P$

Prolog

Formule Horn

$\bar{x} \vee \bar{y} \vee z$

$\bar{x} \vee \bar{y} \vee \bar{z}$

$z : x, y$

Teorema lui Schefer (1977)

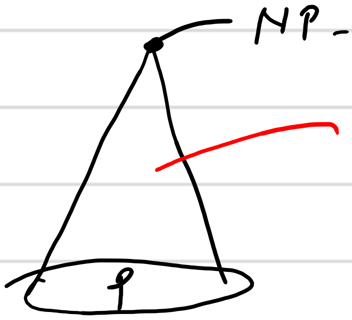
Informal toate problemele de tip SAT sunt fie în P fie NP-complete

- există patru clase maximale de probleme în P
  - 2-SAT
  - HORN-SAT
  - negated HORN-SAT
    $(\bar{x} \vee \bar{y} \vee \bar{z} \to x \vee y \vee z)$
    $(\bar{x} \vee \bar{y} \vee z \to x \vee y \vee \bar{z})$
  - XOR-SAT

sistem liniar ecuații peste $\mathbb{Z}_2$

1-in-2 SAT

$\text{1-in-2}(x, y) \iff x \oplus y = 1$



$P \neq NP$ există pb cu complexitate "intermediară"

artificiale

INDEPENDENT SET

Se dă $G=(V,E)$, k
$1 \le k \le n$

Să decid există k vârfuri $x_1, x_2 \dots x_k \in V$
oricare două sunt neadiacente.

Obs Alg brute face verificări $\binom{n}{k}$ perechi

$O\left(\binom{n}{k} k^2\right)$ $\binom{n}{k} \sim \Theta(n^k)$

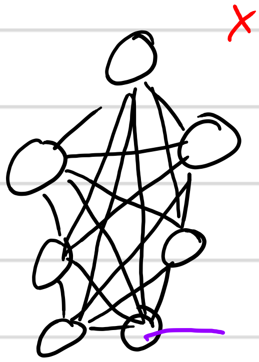
– **IS ∈ NP** ușor, ghicesc k vf
verific că nu sunt adiacente.

$\boxed{3\text{-SAT} \le_m^P IS}$

$\phi = \bigwedge_{i=1}^{m} C_i$ m clauze
n variabile $\longrightarrow$ G_ϕ graf
k_ϕ

$x \vee y \vee z$
$\downarrow$
7 din 8 comb
satisfac

$x=1, y=1, z=1$



K_7

$x=1, y=1, z=0$

DA/NU?

vf din clustere diferite

$x=1$, $y=1$, $z=1$ —— $x=0$, $a=1$, $b=1$

Pt vârfuri inconsistente pun o muchie.

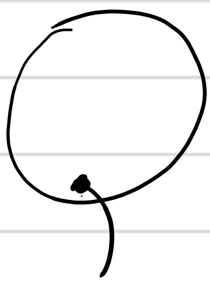
$\phi \longrightarrow G_\phi$

$k_\phi \overset{?}{=} m$ (#clauze)

<u>CLAIM</u> G_ϕ are IS cu k_ϕ vf. $\Leftrightarrow$ $\phi \in SAT$

$\Leftarrow$ Pp $A \models \phi$

exp $A = 0 \ldots 0$

$C \longrightarrow$ 

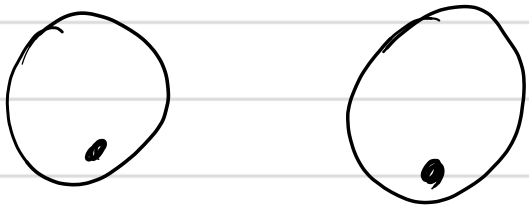
vf $A(x)\, A(y)\, A(z)$

din fiecare clustere exact 1 vf

$\Downarrow$

$\boxed{m \text{ vf}}$

IS



nu am
muchii din contracție
$\Downarrow$
IS cu n vf.
$\Downarrow$
G_ϕ are un IS cu k_ϕ vf.