Q7

$2x + 3y - 8 = 0$ — (I)

$2y + x - 5 = 0$ — (II)

Variance of $x \Rightarrow \sigma_x^2 = 4$

$\therefore \sigma_x = \sqrt{4} = 2 \Rightarrow \boxed{\sigma_x = 2}$

(a) Variance of y

Relation b/w correlation coefficient (or) b/w x & y in terms of standard deviation & Slope

$$\boxed{r = b\left(\frac{\sigma_y}{\sigma_x}\right)} \quad \text{— (III)}$$

r = correlation coeff. b/w x & y

b = slope of reg line

σ_x = std. of variable x

σ_y = std. of variable y

using eqn (I)

$2x + 3y - 8 = 0$

$\Rightarrow x = \frac{-3y}{2} + \frac{8}{2}$ — (IV) Regression Line of x on y

$b_{xy} = r\frac{\sigma_x}{\sigma_y} = \frac{-3}{2}$ → by comparing eq (II) & (IV)

— (a)

eqn (3)

using eqn 2

$2y + x - 5 = 0$

$y = \frac{-x}{2} + \frac{5}{2}$ — (V)

$b_{yx} = r\frac{\sigma_y}{\sigma_x} = \frac{-1}{2}$ → by comparing eqn (III) & (V)

— (b)

$\therefore r^2 = b_{xy} * b_{yx} = \frac{-3}{2} \times \frac{-1}{2} = \frac{3}{4}$

$r = \sqrt{\frac{3}{4}}$

$\boxed{r = \frac{-\sqrt{3}}{2}}$ both b_{xy} & b_{yx} -ve so r is -ve

from eqn (a)

$b_{xy} = r \frac{\sigma_x}{\sigma_y}$

as we know $\sigma_x = 2$

$\therefore \frac{-3}{2} = \frac{-\sqrt{3}}{2} \times \frac{2}{\sigma_y} \Rightarrow \frac{-3}{2} = \frac{-\sqrt{3}}{\sigma_y}$

$\Rightarrow \boxed{\sigma_y = 1.15}$

Variance of y is 1.15

(b) Coeffi of determination of x & y

Coeffi of determinatn = (Correlation Coeffi (r))^2

$\therefore$ Coeffi of Determinatn $= \left(\frac{-\sqrt{3}}{2}\right)^2 = \frac{3}{4}$

$\boxed{\text{Coeffi of Determinatn} = \frac{3}{4}}$

© Standard error of estimation of x on y & y on x

$$S_{xy} = \sigma_y \sqrt{1-r^2}$$ std. error of x on y

$$S_{yx} = \sigma_x \sqrt{1-r^2}$$ std. error of est. y on x

* standard error of estimation of x on y

$$S_{xy} = \sigma_y \sqrt{1-r^2}$$

$$= 1.15 \times \sqrt{1-\frac{3}{4}}$$

$$S_{xy} = 0.575$$

* standard error of estimation of y on x

$$S_{yx} = \sigma_x \sqrt{1-r^2}$$

$$= 2\sqrt{1-\frac{3}{4}}$$

$$S_{yx} = 1$$

Standard Error of Estimation

$$S_{xy}(x \text{ on } y) = 0.575$$ and,

$$S_{yx}(y \text{ on } x) = 1$$