Electrical Circuit Analysis
Assignment 2
ENG223

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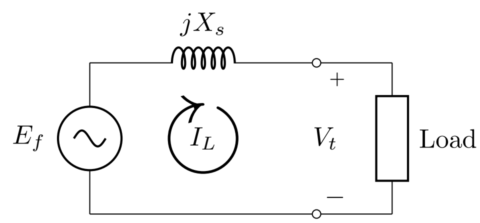
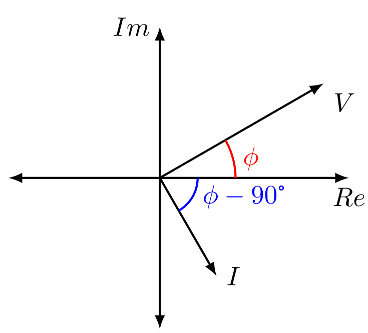
Assignment 2.1

The diagram of the circuit is as follows:



We are not told what the load is comprised of, only that the current I_L lags the voltage V_t by some unspecified angle. This leaves us unable to determine what the phasor diagram for the voltage and current look like. One possible example is as follows:



In this example, it is clear that the current lags the voltage by 90°.

Assignment 2.2

a. In any circuit system, power is conserved. Hence, it must be true that the power delivered by the source, S_{source}, is equal to the power dissipated by the Δ-load plus the power dissipated by the Y-load. That is:

$$S_{source} = S_{\Delta} + S_{Y}$$

Now, rearranging the above we get:

$$S_{\Delta} = S_{source} - S_{Y}$$

We are told that the power factor for the source is 0.75 lagging, hence the phase angle is given by:

$$\begin{aligned}\theta_{source} &= \arccos(powerfactor_{source})\\ &= \arccos(0.75)\\ &= 41.41^{\circ}\end{aligned}$$

The power factor for the Y-connected load is 0.6 lagging, hence the phase angle is given by:

$$\begin{aligned}\theta_{Y} &= \arccos(powerfactor_{Y})\\ &= \arccos(0.6)\\ &= 53.13^{\circ}\end{aligned}$$

Using the phase angles, we can now solve for the power dissipated from the Δ-load:

$$\begin{aligned}S_{\Delta} &= 14000\angle 41.41^{\circ} - 9000\angle 53.13^{\circ}\\ &= 14000\cos(41.41^{\circ}) + j14000\cos(41.41^{\circ}) - [9000\cos(53.13^{\circ}) + j9000\sin(53.13^{\circ})]\\ &= 5099.93 + j2060.21\\ &= 5500.34\angle 22^{\circ}\text{VA}\end{aligned}$$

Now, we know that the power per phase is one third of the total power dissipated by the Δ-connected load, hence:

$$
\begin{aligned}
S_{\Delta/\phi} &= \frac{S_{\Delta}}{3} \\
&= \frac{5500.34\angle 22^{\circ}}{3}
\end{aligned}
$$

Hence the complex power per phase for the Δ-connected load is:

$$S_{\Delta/\phi} = 1833.45\angle 22^{\circ}\text{VA}$$

b. For a positive sequenced, balanced three phase Y-connected load, we know that:

$$\boldsymbol{V_{AB}} = \sqrt{3}\angle 30^{\circ} \times \boldsymbol{V_{AN}}$$

Now, to find $\boldsymbol{V_{AN}}$ we use the power formula:

$$\boldsymbol{S_{\phi}} = \boldsymbol{V_{\phi}} \times \boldsymbol{I_{\phi}^{*}}$$

The complex power per phase in the Y-connected load is given by:

$$
\begin{aligned}
\boldsymbol{S_{\phi}} &= \frac{\boldsymbol{S_{Y}}}{3} \\
&= \frac{9000\angle 53.13^{\circ}}{3} \\
&= 3000\angle 53.13^{\circ}\text{VA}
\end{aligned}
$$

Hence, rearranging the above formula we find that the phase voltage at the Y-connected load is given by:

$$
\begin{aligned}
\boldsymbol{V_{\phi}} &= \frac{\boldsymbol{S_{\phi}}}{\boldsymbol{I_{\phi}^{*}}} \\
&= \frac{3000\angle 53.13^{\circ}}{10\angle 30^{\circ}} \\
&= 300\angle 23.13^{\circ}\text{V}
\end{aligned}
$$

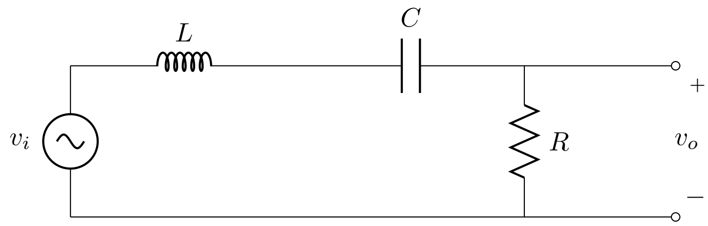
Finally, we find the line voltage:

$$\begin{aligned}\boldsymbol{V_{AB}} &= (\sqrt{3}\angle 30^{\circ}) \times (300\angle 23.13^{\circ}) \\ &= 519.62\angle 53.13^{\circ}\text{V}\end{aligned}$$

Hence, the magnitude of the line voltages are:

$$|\boldsymbol{V_{line}}| = |\boldsymbol{V_{AB}}| = 519.62\text{V}$$

Assignment 2.3



a. The circuit shown above is a series RLC circuit and it has a resonant frequency which is given by:

$$\omega_o = \sqrt{\frac{1}{LC}}$$

Hence the resonant frequency is:

$$\omega_o = \sqrt{\frac{1}{0.1 \times 10\text{e}^{-6}}} = 1000 \quad {}^{\text{rad}}\!/_{\text{sec}}$$

The lower half power frequency is given by the following:

$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

Hence the lower half frequency is:

$$\omega_1 = -\frac{10}{2 \times 0.1} + \sqrt{\left(\frac{10}{2 \times 0.1}\right)^2 + \frac{1}{0.1 \times 10e^{-6}}} = 951.25 \quad {}^{\text{rad}}\!/_{\text{sec}}$$

The upper half power frequency is given by the following:

$$\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

Hence the upper half frequency is:

$$\omega_2 = \frac{10}{2 \times 0.1} + \sqrt{\left(\frac{10}{2 \times 0.1}\right)^2 + \frac{1}{0.1 \times 10e^{-6}}} = 1051.25 \quad {}^{\text{rad}}/_{\text{sec}}$$

The bandwidth, β, is given by the following:

$$\begin{aligned}\beta &= \omega_2 - \omega_1 \\ &= 1051.25 - 951.25 \\ &= 100 \quad {}^{\text{rad}}/_{\text{sec}}\end{aligned}$$

Finally, the quality factor is given by the following:

$$\begin{aligned}Q &= \frac{\omega_o}{\beta} \\ &= \frac{1000}{100} \\ &= 10\end{aligned}$$

b. To find the transfer function, we first identify the impedance for each circuit element. The impedance for the resistor is $Z_R = R$. The impedance for the capacitor is $Z_C = \frac{1}{j\omega C}$. And the impedance for the inductor is $Z_L = j\omega L$.

Now, using the voltage divider law, the voltage taken across the resistor is given by:

$$v_o = \frac{Z_R}{Z_R + Z_C + Z_L} \times v_i$$

Therefore, the transfer function can be written as:

$$H(\omega) = \frac{v_o}{v_i} = \frac{R}{R + j\omega L + \frac{1}{j\omega C}}$$

Writing the denominator in phasor form, we get:

$$H(\omega) = \frac{R}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2} \angle \arctan(\frac{\omega L - \frac{1}{\omega C}}{R})}$$

Rearranging yields:

$$\begin{aligned} H(\omega) &= \frac{R}{R\sqrt{1 + (\omega \frac{L}{R} - \frac{1}{\omega RC})^2} \angle \arctan(\frac{\omega L - \frac{1}{\omega C}}{R})} \\ &= \frac{1}{\sqrt{1 + (\omega \frac{L}{R} - \frac{1}{\omega RC})^2} \angle \arctan(\frac{\omega L - \frac{1}{\omega C}}{R})} \end{aligned}$$

Hence, the transfer function is given by:

$$H(\omega) = \frac{1}{\sqrt{1 + (\omega \frac{L}{R} - \frac{1}{\omega RC})^2} \angle \arctan(\frac{\omega L - \frac{1}{\omega C}}{R})}$$

Where the magnitude of the transfer function is given by:

$$|H(\omega)| = \frac{1}{\sqrt{1 + (\omega \frac{L}{R} - \frac{1}{\omega RC})^2}}$$

And the magnitude of the phase angle is given by:

$$\theta(\omega) = -\arctan(\frac{\omega L - \frac{1}{\omega C}}{R})$$

c. The filter is called a bandpass filter and it allows frequencies that are within the bandwidth of the filter to pass with minimal attenuation. The passband is centered on the resonant frequency, ω_o. Frequencies below the lower half power cut off frequency and above the upper half power cut off frequency are attenuated.