Electrical Circuit Analysis
Practical 3 - High Pass Filters
ENG223

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1 Objective

To develop an understanding of high-pass filter operation through the derivation of a theoretical transfer function and theoretical cut-off frequency. Theoretical estimates will be tested through the collection of experimental evidence.

1.1 Background

Transfer function for RC high pass filter Figure 1 shows an RC circuit with the output voltage taken over the resistor. This set-up is called a high-pass filter. This type of filter allows high frequency voltage signals to pass through from input to output, however, low frequency voltage signals are attenuated.

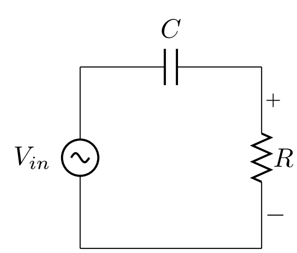

Figure 1: Series RC circuit with Sinusoidal Input Voltage

Consider the RC circuit in figure 1 with input voltage $V_{in} = V_m \cos(\omega t - \phi)$. In general, to calculate the output voltage, V_{out}, we exploit impedance models in the circuit's steady state. The impedance of the resistor is given by $Z_R = R$ and the impedance of the capacitor is given by $Z_C = \frac{1}{j\omega C}$, where ω is the frequency of the input voltage. Since the elements are in series, we can use the voltage divider arrangement:

$$\begin{aligned} V_{out} &= \frac{Z_R}{Z_C + Z_R} V_{in} \\ \frac{V_{out}}{V_{in}} &= \frac{Z_R}{Z_C + Z_R} \\ &= \frac{R}{R + \frac{1}{j\omega C}} \\ &= \frac{R}{R - j\frac{1}{\omega C}} \end{aligned}$$

This is the transfer function and is written as:

$$H(\omega) = \frac{R}{R - j\frac{1}{\omega C}} \tag{1}$$

This expression can be written in phasor form as follows:

$$\begin{aligned} H(\omega) &= \frac{R}{\sqrt{R^2 + (\frac{1}{\omega C})^2} \angle \arctan(-\frac{\frac{1}{\omega C}}{R})} \\ &= \frac{R}{\sqrt{R^2 + (\frac{1}{\omega C})^2} \angle \arctan(-\frac{1}{\omega RC})} \end{aligned}$$

Hence, the magnitude of the transfer function is given by:

$$|H(\omega)| = \frac{R}{\sqrt{R^2 + (\frac{1}{\omega C})^2}} \tag{2}$$

And the phase angle of the transfer function is given by:

$$\angle H(\omega) = -\arctan(-\frac{1}{\omega RC}) \tag{3}$$

A plot of both the magnitude of the transfer function and the phase angle can be seen in figure 2.

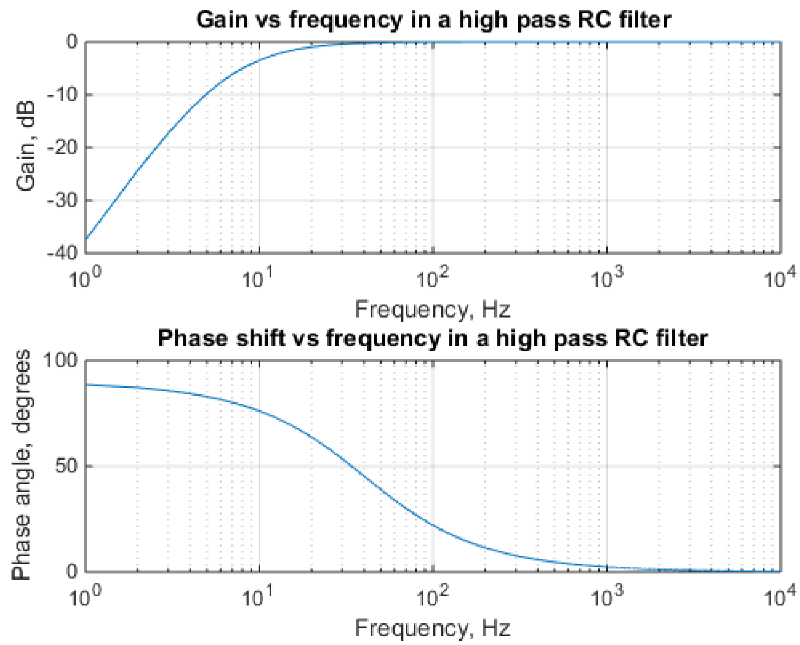

Figure 2: Theoretical bode plots for high pass RC filter

Cut-off frequency for RC high pass filter The definition of cut-off frequency that has been widely used by electrical engineers is the frequency for which the transfer function magnitude is decreased by the factor of $\frac{1}{\sqrt{2}}$ from its maximum value. That is:

$$|H(\omega_c)| = \frac{1}{\sqrt{2}} H_{max}$$

For a high pass RC circuit, the magnitude of the transfer function is at its maximum when $\omega \to \infty$, that is $H_{max} = 1$. Hence, setting $|H(\omega_c)|$ equal to $\frac{1}{\sqrt{2}}$ we get:

$$
\begin{aligned}
\frac{R}{\sqrt{R^2 + (\frac{1}{\omega C})^2}} &= \frac{1}{\sqrt{2}} \\
\frac{R^2}{R^2 + (\frac{1}{\omega C})^2} &= \frac{1}{2} \\
2R^2 &= R^2 + (\frac{1}{\omega C})^2 \\
R^2 &= (\frac{1}{\omega C})^2 \\
R &= \frac{1}{\omega C}
\end{aligned}
$$

Hence, for a high pass RC circuit, the cut off frequency is given by:

$$\omega_c = \frac{1}{RC} \tag{4}$$

Now, suppose the RC circuit was constructed using a 330 Ω resistor and a 0.47 µF capacitor. The cut off frequency in radians per second would be:

$$
\begin{aligned}
\omega_c &= \frac{1}{RC} \\
&= \frac{1}{330 \times 0.47e^{-6}} \\
&= 6447.45 \,{}^{\mathrm{rad}}\!/\!{}_{\mathrm{s}} \\
&= 1026.14\mathrm{Hz}
\end{aligned}
$$

Transfer function for RL high pass filter Figure 2 shows an RL circuit with the output voltage taken over the inductor. This set-up is called a high-pass filter. This type of filter allows high frequency voltage signals to pass through from input to output, however, low frequency voltage signals are attenuated.

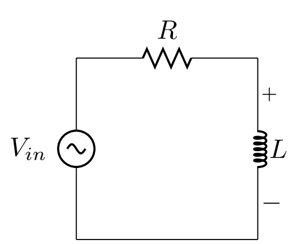

Figure 3: Series RL circuit with Sinusoidal Input Voltage

Consider the RL circuit in figure 1 with input voltage $V_{in} = V_m \cos(\omega t - \phi)$. In general, to calculate the output voltage, V_{out}, we exploit impedance models in the circuit's steady state. The impedance of the resistor is given by $Z_R = R$ and the impedance of the inductor is given by $Z_L = j\omega L$, where ω is the frequency of the input voltage. Since the elements are in series, we can use the voltage divider arrangement:

$$
\begin{aligned}
V_{out} &= \frac{Z_L}{Z_L + Z_R} V_{in} \\
\frac{V_{out}}{V_{in}} &= \frac{Z_L}{Z_L + Z_R} \\
&= \frac{j\omega L}{1 + j\omega L} \\
&= \frac{1}{1 + \frac{R}{j\omega L}}
\end{aligned}
$$

This is the transfer function and is written as:

$$H(\omega) = \frac{1}{1 - j\frac{R}{\omega L}} \tag{5}$$

This expression can be written in phasor form as follows:

$$H(\omega) = \frac{1}{\sqrt{1 + (\frac{R}{\omega L})^2} \angle \arctan(-\frac{R}{\omega L})}$$

Hence, the magnitude of the transfer function is given by:

$$|H(\omega)| = \frac{1}{\sqrt{1 + (\frac{R}{\omega L})^2}} \tag{6}$$

And the phase angle of the transfer function is given by:

$$\angle H(\omega) = -\arctan(-\frac{R}{\omega L}) \tag{7}$$

A plot of both the magnitude of the transfer function and the phase angle can be seen in figure 4.

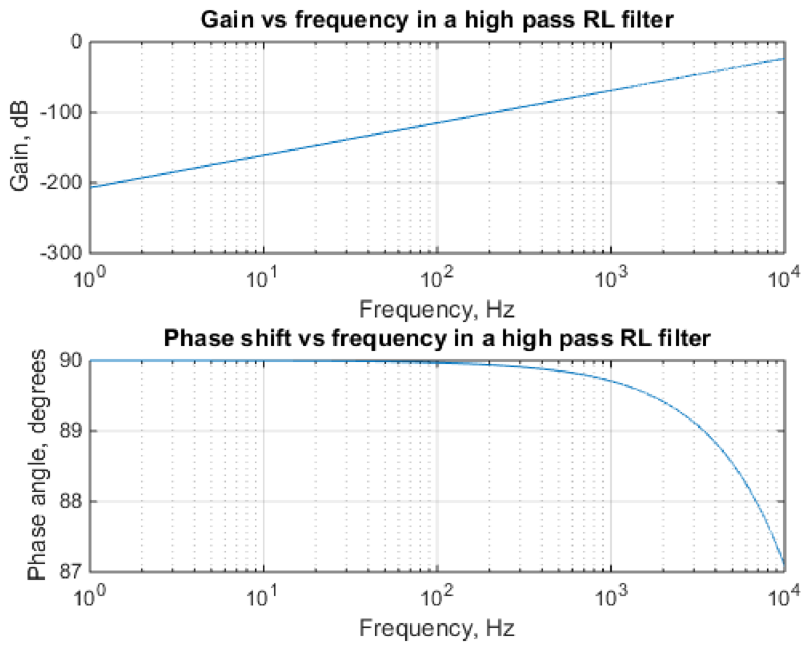

Figure 4: Theoretical bode plots for low pass RL filter

Cut-off frequency for RL high pass filter The definition of cut-off frequency that has been widely used by electrical engineers is the frequency for which the transfer function magnitude is decreased by the factor of $\frac{1}{\sqrt{2}}$ from its maximum value. That is:

$$|H(\omega_c)| = \frac{1}{\sqrt{2}} H_{max}$$

For a high pass RL circuit, the magnitude of the transfer function is at its maximum when $\omega \to \infty$, that is $H_{max} = 1$. Hence, setting $|H(\omega_c)|$ equal to $\frac{1}{\sqrt{2}}$ we get:

$$
\begin{aligned}
\frac{1}{\sqrt{1+(\frac{R}{\omega_c L})^2}} &= \frac{1}{\sqrt{2}} \\
1+(\frac{R}{\omega_c L})^2 &= 2 \\
(\frac{R}{\omega_c L})^2 &= 1 \\
\frac{R}{\omega_c L} &= 1
\end{aligned}
$$

Hence, for a high pass RL circuit, the cut off frequency is given by:

$$\omega_c = \frac{R}{L} \tag{8}$$

Now, suppose the RL circuit was constructed using a 75 Ω resistor and a 2.4 mH inductor. The cut off frequency in radians per second would be:

$$
\begin{aligned}
\omega_c &= \frac{75}{2.4e^{-3}} \\
&= 31250\,{}^{\mathrm{rad}}\!/\!_{\mathrm{s}}
\end{aligned}
$$

Hence, the cutoff frequency in Hertz is:

$$f_c = 4973.59\mathrm{Hz}$$

2 Experimental Circuit Set-up, Results and Calculations

The simple series RC circuit was set up as shown in figure 4. The input voltage, V_{in}, was a sinusoidal voltage source generated by the function generator. The output voltage, V_{out}, was measured across the capacitor.

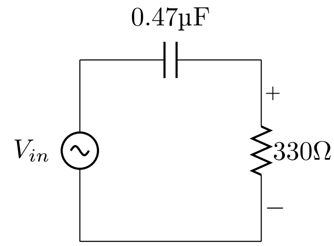

Figure 5: Series RC circuit with Sinusoidal Input Voltage

The input voltage, V_{in}, was set to 4V peak to peak. Both the output voltage, V_{out}, and the time difference between the input signal peak and output signal peak, $|\Delta X|$, were measured. The gain and phase shift were calculated using the formulas from practical 1. The empirical data and calculated values can be seen in table 1.

frequency (f)	V_{in}	V_{out}	$\lvert\Delta X\rvert$	Gain dB	Phase Shift ˚
10 Hz	4 V	44 mV	24.40 ms	-39.172 dB	87.84 ˚
20 Hz	4 V	88 mV	12.80 ms	-33.151 dB	92.16 ˚
50 Hz	4 V	224 mV	5.04 ms	-25.036 dB	90.72 ˚
100 Hz	4 V	400 mV	2.32 ms	-20.000 dB	83.52 ˚
200 Hz	4 V	920 mV	1.08 ms	-12.760 dB	77.76 ˚
500 Hz	4 V	1.92 V	3.68 µs	-6.375 dB	66.24 ˚
1000 Hz	4 V	2.80 V	136 µs	-3.098 dB	48.96 ˚
2000 Hz	4 V	3.68 V	44 µs	-0.724 dB	31.68 ˚
4000 Hz	4 V	4 V	0 µs	0 dB	0 ˚
5000 Hz	4 V	4 V	0 µs	0 dB	0 ˚
8000 Hz	4 V	4 V	0 µs	0 dB	0 ˚
10000 Hz	4 V	4 V	0 µs	0 dB	0 ˚

Table 1: Output characteristics for high pass RC circuit

By adjusting the frequency of the function generator until the output voltage had an amplitude of 2.828 V_{pp}, the cut off frequency was measured. The measured value was:

$$f_c = 1000\text{Hz}$$

A simple RL circuit was set up as shown in figure 5. The input voltage, V_{in}, was a sinusoidal voltage source generated by the function generator. The output voltage, V_{out}, was measured across the resistor.

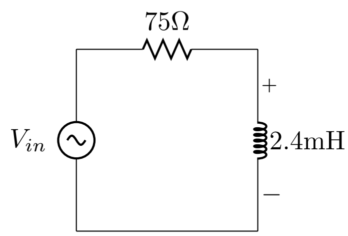

Figure 6: Series RL circuit with Sinusoidal Input Voltage

The input voltage, V_{in}, was set to 4V peak to peak. Both the output voltage, V_{out}, and the time difference between the input signal peak and output signal peak, $|\Delta X|$, were measured. The gain and phase shift were calculated using the formulas from practical 1. The empirical data and calculated values can be seen in table 2.

frequency (f)	V_{in}	V_{out}	$\|\Delta X\|$	Gain dB	Phase Shift °
10 Hz	4 V	32 mV	8 ms	-41.938 dB	28.80 °
20 Hz	4 V	37.6 mV	5.2 ms	-40.537 dB	37.44 °
50 Hz	4 V	61.6 mV	3.52 ms	-36.249 dB	63.18 °
100 Hz	4 V	108 mV	2.4 ms	-31.372 dB	86.40 °
200 Hz	4 V	196 mV	1.04 ms	-26.196 dB	74.88 °
500 Hz	4 V	472 mV	432 µs	-18.562 dB	77.76 °
1000 Hz	4 V	940 mV	204 µs	-12.578 dB	73.44 °
2000 Hz	4 V	1.8 V	90 µs	-6.93 dB	64.80 °
4000 Hz	4 V	2.76 V	36 µs	-3.223 dB	51.84 °
5000 Hz	4 V	3 V	24 µs	-2.498 dB	43.20 °
8000 Hz	4 V	3.48 V	12 µs	-1.209 dB	34.56 °
10000 Hz	4 V	3.60 V	8.8 µs	-0.915 dB	31.68 °

Table 2: Output characteristics for low pass RL circuit

By adjusting the frequency of the function generator until the output voltage had an amplitude of 2.828 V_{pp}, the cut off frequency was measured. The measured value was:

$$f_c = 4500\text{Hz}$$

3 Results and Conclusions

The gain and phase angle calculations made for the empirical data shown in tables 1 and 2 for both the RC and RL circuits, respectively, were plotted over the theoretical plots. This can be seen in figure 7 for the RC circuit and figure 8 for the RL circuit.

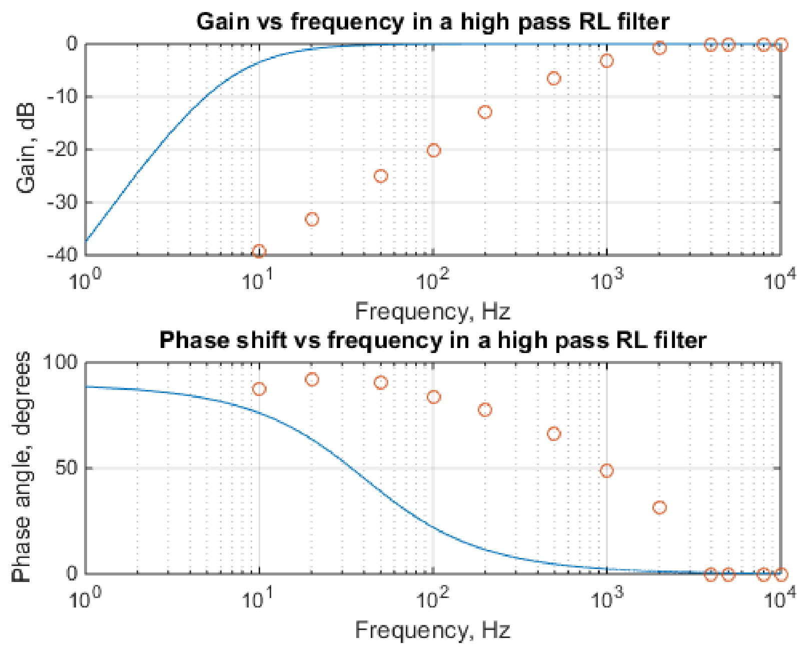

Figure 7: Empirical data super-imposed on theoretical data for low pass RC filter

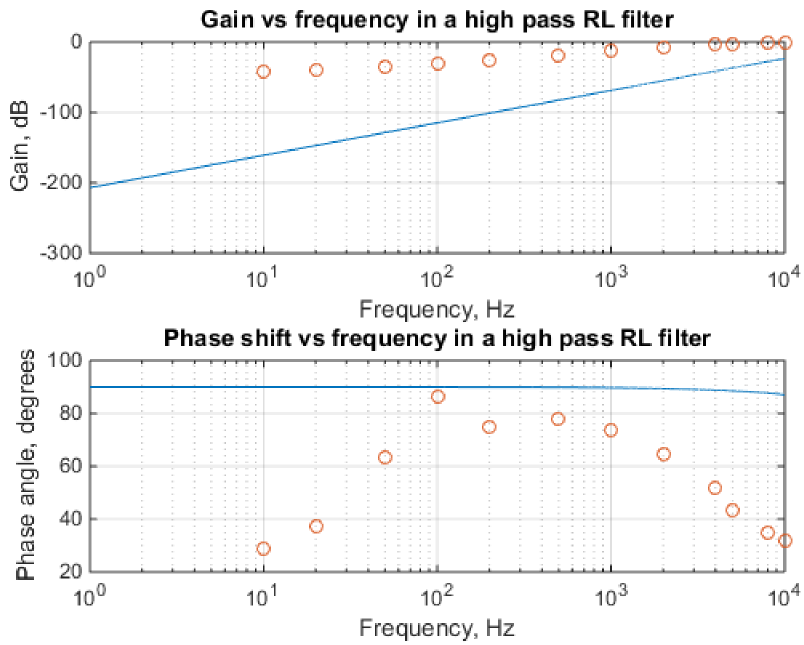

Figure 8: Empirical data super-imposed on theoretical data for low pass RL filter

From figures 7 and 8 we can see that the empirical phase angle data does not fit the theoretical data well. Additionally, the empirical data for the magnitude of the RC filter attenuates early, whereas the data for the RL filter has a lagging attenuation. This is most likely due to erroneous results obtained when the practical was undertaken. Evidence supporting this is provided from the cut off frequencies, which appear to have been measured correctly. Finally, the data on the phase plot for the RL filter appears to be significantly deviate from the theoretical plot.