CHARLES DARWIN UNIVERSITY, SCHOOL OF ENGINEERING

Electrical Machines and Power Systems
Assignment 1

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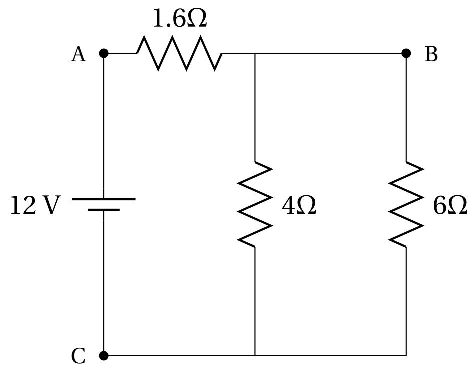
ASSIGNMENT 1.1

Assignment 1.2



Node C
$v_C = 0\text{V}$, since this is the ground node.

Node A
$v_A = 12\text{V}$

Node B
By KCL, we get the following equation:

$$
\begin{aligned}
\frac{v_B - 12}{1.6} + \frac{v_B}{4} + \frac{v_B}{6} &= 0 \\
v_C(\frac{1}{1.6} + \frac{1}{4} + \frac{1}{6}) &= \frac{12}{1.6} \\
v_C &= \frac{12}{1.6}(\frac{1}{1.6} + \frac{1}{4} + \frac{1}{6})^{-1} \\
v_C &= 7.2\text{V}
\end{aligned}
$$

Question 1.2.1

We can reduce the circuit down to a single equivalent resistor and the battery to obtain the current supplied from the battery to the load. The equivalent resistor is given by:

$$
\begin{aligned}
R_{eq} &= R_{1.6\Omega} + R_{4\Omega}||R_{6\Omega} \\
&= 4\Omega
\end{aligned}
$$

Now, using Ohm's law, we can solve for the current in the circuit. That is:

$$
\begin{aligned}
V &= iR_{eq} \\
i &= \frac{V}{R_{eq}} \\
&= \frac{12}{4} \\
&= 3\text{A}
\end{aligned}
$$

Hence the power supplied from the battery to the load is given by:

$$
\begin{aligned}
p &= Vi \\
p &= iR_{eq}i \\
p &= i^2 R_{eq} \\
&= 3^2 \times 4 \\
&= 36\text{W}
\end{aligned}
$$

Question 1.2.2

This question has been solved using the node voltages found earlier.
The current flowing from Node A to Node B through $R_{1.6\Omega}$ is given by:

$$
\begin{aligned}
i &= \frac{v_A - v_B}{R_{1.6\Omega}} \\
&= \frac{12 - 7.2}{1.6} \\
&= 3\text{A}
\end{aligned}
$$

The current flowing from Node B to Node C through $R_{4\Omega}$ is given by:

$$
\begin{aligned}
i &= \frac{v_B - v_C}{R_{4\Omega}} \\
&= \frac{7.2}{4} \\
&= 1.8\text{A}
\end{aligned}
$$

The current flowing from Node B to Node C through $R_{6\Omega}$ is given by:

$$
\begin{aligned}
i &= \frac{v_B - v_C}{R_{6\Omega}} \\
&= \frac{7.2}{6} \\
&= 1.2\text{A}
\end{aligned}
$$

QUESTION 1.2.2

This question has been solved using the node voltages found earlier.
The power consumed by resistor $R_{1.6\Omega}$ is given by:

$$\begin{aligned} p &= \frac{(v_A - v_B)^2}{R_{1.6\Omega}} \\ &= \frac{(12-7.2)^2}{1.6} \\ &= 14.4\text{W} \end{aligned}$$

The power consumed by resistor $R_{4\Omega}$ is given by:

$$\begin{aligned} p &= \frac{(v_B - v_C)^2}{R_{6\Omega}} \\ &= \frac{(7.2-0)^2}{4} \\ &= 12.96\text{W} \end{aligned}$$

The power consumed by resistor $R_{1.6\Omega}$ is given by:

$$\begin{aligned} p &= \frac{(v_B - v_C)^2}{R_{1.6\Omega}} \\ &= \frac{(7.2-0)^2}{6} \\ &= 8.64\text{W} \end{aligned}$$

QUESTION 1.2.4

This question has been solved using the node voltages found earlier. The voltages across each resistor are as follows:

$$\begin{aligned} v_{R_{1.6\Omega}} &= 12-7.2 \\ &= 4.8\text{V} \end{aligned}$$

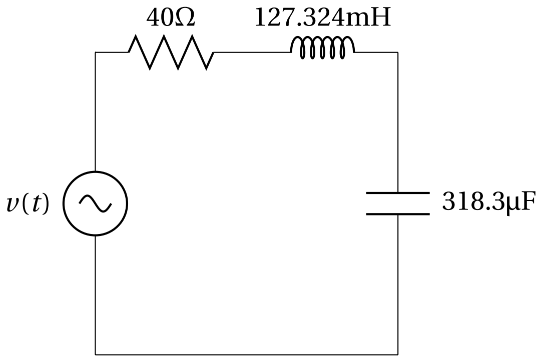
$$\begin{aligned} v_{R_{4\Omega}} &= 7.2-0 \\ &= 7.2\text{V} \end{aligned}$$

$$\begin{aligned} v_{R_{6\Omega}} &= 7.2-0 \\ &= 7.2\text{V} \end{aligned}$$

QUESTION 1.2.5

The aggregate power consumed from the load is equal to 36W which is equal to the power produced by the source.

ASSIGNMENT 1.3



The sinusoidal voltage source is 240 volts with a 50 Hz frequency. This means that:

$$f = 50\text{Hz}$$
$$\omega = 2\pi f = 100\pi$$

Further,

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$
$$V_m = V_{rms} \times \sqrt{2}$$
$$= 240\sqrt{2}$$

Hence,

$$v(t) = 240\sqrt{2}\cos(100\pi t)$$

In phasor form,

$$v(t) = 240\sqrt{2}\angle 0°$$

The impedance of the resistor is $Z_R = 40\Omega$. The impedance of the inductor is given by:

$$\begin{aligned} Z_L &= j\omega L \\ &= j \times 100\pi \times 127.324e-3\Omega \\ &= j40\Omega \\ &= 40\angle 90°\Omega \end{aligned}$$

The impedance of the capacitor is given by:

$$\begin{aligned} Z_C &= \frac{1}{j\omega C} \\ &= \frac{1}{j \times 100\pi \times 318.3e-6}\Omega \\ &= \frac{10}{j}\Omega \\ &= -j10\Omega \\ &= 10\angle -90°\Omega \end{aligned}$$

Question 1.3.1

Now to find the current we can sum the impedances (because they are in series) and apply Ohm's law to solve for the current.

$$\begin{aligned} Z_{eq} &= Z_R + Z_L + Z_C \\ &= 40 + j40 - j10 \\ &= 40 + j30 \\ &= 50\angle 36.86° \end{aligned}$$

Hence, the current is given by:

$$\begin{aligned} \mathbf{V} &= \mathbf{I}Z_{eq} \\ \mathbf{I} &= \frac{\mathbf{V}}{Z_{eq}} \\ &= \frac{240\sqrt{2}\angle 0°}{50\angle 36.86°} \\ &= 6.79\angle -36.86° \end{aligned}$$

Hence,

$$\begin{aligned} I_{rms} &= \frac{6.79}{\sqrt{2}} \\ &= 4.80\text{A} \end{aligned}$$

Now, we get the real power according to the following formula:

$$P = V_{rms} I_{rms} \cos(\theta),$$

where θ is the difference between the voltage and current phase angles.

Hence,

$$\begin{aligned} P &= 240 \times 4.8 \times \cos(-36.86) \\ &= 921\text{W} \end{aligned}$$

The average power consumed by the resistor is given by:

$$\begin{aligned} P &= I_{rms}^2 R \\ &= 4.8^2 \times 40 \\ &= 921.60\text{W} \end{aligned}$$

The two values align, indicating that the resistor consumes all of the real power delivered by the source.

QUESTION 1.3.2

The power factor at which the power is delivered is given by:

$$PF = \cos(\theta) \qquad = \cos(-36.86°) = 0.8$$

QUESTION 1.3.3

QUESTION 1.3.4

The voltage across the resistor is given by:

$$p$$

The voltage across the capacitor is given by: