Electrical Machines & Power Systems: Assignment 2

Due on September 22, 2016 at 3:10pm

Dr. Kamal Debnath

S.Reynolds (262538)

Problem 1

Three identical single-phase transformers are each rated 5MV A, 66/13.2kV and each has 9% impedance and 0.65% resistance. They are connected Δ - Δ and deliver 10MW at 0.8 power factor lagging at an output voltage of 13.2kV. Compute the input voltage.

Now resistance and reactance are orthogonal to each other, as indicated in Figure 1, we can find reactance using Pythagoras' theorem:

$$\begin{aligned}
\%X^H &= \sqrt{(\%Z^H)^2 + (\%R^H)^2} \\
&= \sqrt{9^2 - 0.65^2} \\
&= 8.98\%
\end{aligned}$$

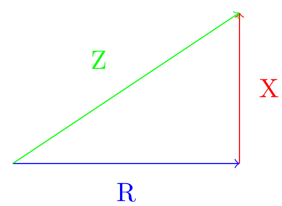

Figure 1: Reactance triangle used for finding the value of the reactance.

Now, we can find the current on the high voltage side of the transformer using the power formula $|S^H| = |I^H| \cdot |V^H|$. Rearranging we get:

$$\begin{aligned}
|I^H| &= \frac{|S^H|}{|V^H|} \\
&= \frac{5 \times 10^6}{66 \times 10^3} \\
&= 75.75\text{A}
\end{aligned}$$

Now, using a formula found in Lathi (2006), we see that:

$$\%R^H = \frac{R^H \cdot I^H}{V^H}$$

Rearranging, gives us:

$$\begin{aligned}
R^H &= \frac{\%R^H \cdot V^H}{I^H} \\
&= \frac{0.0065 \cdot 66 \times 10^3}{75.75} \\
&= 5.66\Omega
\end{aligned}$$

We can find the reactance similarly:

$$\begin{aligned}
\%X^H &= \frac{X^H \cdot I^H}{V^H} \\
&= \frac{0.0898 \cdot 66 \times 10^3}{75.75} \\
&= 78.24\Omega
\end{aligned}$$

Hence, we get that the impedence on the high voltage side of the transformer is:

$$Z^H = (5.66 + j78.24)\Omega$$

Now, since the transformers are Δ-Δ connected and deliver 10MW at a power factor of 0.8 (lagging) we consider the resistive power, which is given by:

$$\begin{aligned} P^L_{total} &= 3 \cdot V^L_\phi \cdot I^L_\phi \cdot \cos(\theta) \\ &= 3 \cdot V^L_{line} \cdot \frac{I^L_{line}}{\sqrt{3}} \cdot \cos(\theta) \\ &= \sqrt{3} \cdot V^L_{line} \cdot I^L_{line} \cdot \cos(\theta) \end{aligned}$$

Rearranging we get an expression for I^L_{line}, which we solve as follows:

$$\begin{aligned} I^L_{line} &= \frac{P^L_{total}}{\sqrt{3} \cdot V^L_{line} \cdot \cos(\theta)} \\ &= \frac{10 \times 10^6}{\sqrt{3} \cdot 13.2 \times 10^3 \cdot 0.8} \\ &= 546.73\angle -36.86^\circ\text{A} \end{aligned}$$

Now, we know that we can find I^L_ϕ as follows:

$$\begin{aligned} I^L_\phi &= \frac{I^L_{line}}{\sqrt{3}} \\ &= 315.65\angle -36.86^\circ\text{A} \end{aligned}$$

Referring I^L_ϕ to the high voltage side of the transformer, we first need to work out the turns ratio:

$$a = \frac{N_1}{N_2} = \frac{E_1}{E_2} = \frac{66}{13.2} = 5$$

Now, we find I^L_ϕ as follows:

$$\begin{aligned} I^L_\phi &= \frac{1}{a} \cdot I^H_\phi \\ &= \frac{1}{5} \cdot 315.65\angle -36.86^\circ\text{A} \\ &= 63.13\angle -36.86^\circ\text{A} \end{aligned}$$

Hence, using the equivalent circuit shown in Figure, the input voltage can be found by KVL as follows:

$$-V_{in} + I^L_\phi \cdot Z^H + 66\text{kV} = 0$$

Therefore, we get that:

$$\begin{aligned} V_{in} &= I^L_\phi \cdot Z^H + 66\text{kV} \\ &= 63.13\angle -36.86^\circ\text{A} \cdot (5.66 + j78.24)\Omega + 66\text{kV} \\ &= 69248.84 + j3737.56\text{kV} \end{aligned}$$

Hence, the input voltage is given by:

$$\boxed{V_{in} = 69.35\angle 3.08\text{kV}}$$

Problem 2

A 9kV A, 208V, star connected synchronous generator has a winding resistance of $5.6\Omega/\phi$. Determine its voltage regulation when the power factor of the load is 80% leading. What will is be when the power factor is 80% leading (*should this say lagging*)? Show the power angle δ in a phasor diagram in both cases. Consider the full load operation.

The equivalent circuit, for a single phase, for the synchronous generator can be seen in Figure:

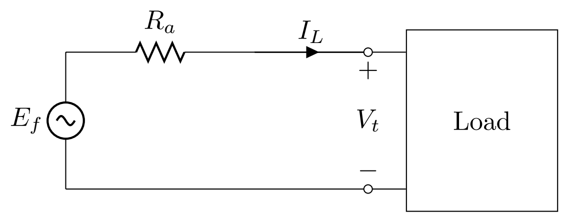

Figure 2: Synchronous generator equivalent circuit

The line voltage of the synchronous generator is given in the problem statement. That is $V_{line} = 208\angle 0^\circ$. Hence, since the synchronous generator is Y-configured, then the phase (or terminal voltage), V_t, is given by:

$$V_t = \frac{V_{line}}{\sqrt{3}} = \frac{208\angle 0^\circ}{\sqrt{3}} = 120.08\angle 0^\circ \text{V}$$

Now, if the power factor is 80% leading then, $pf = \cos(\theta) = 0.8$. If the generator is operating at full load, then we note that:

$$P_{out} = 3 \cdot V_\phi \cdot I_\phi \cdot \cos(\theta)$$

Rearranging, we obtain an expression for I_ϕ:

$$\begin{aligned} I_\phi &= \frac{P_{out}}{3 \cdot V_\phi \cdot \cos(\theta)} \\ &= \frac{|S_{out}| \cdot \cos(\theta)}{3 \cdot V_\phi \cdot \cos(\theta)} \\ &= \frac{9 \times 10^3}{\sqrt{3} \cdot 208} \\ &= 24.98\angle 36.86^\circ \text{A} \end{aligned}$$

Now, by KVL from Figure, we see that:

$$-E_f + I_L \cdot R_a + V_t = 0$$

Hence, the excitation voltage, E_f, is given by:

$$\begin{aligned} E_f &= I_L \cdot R_a + V_t \\ &= (24.98\angle 36.86) \cdot 5.6 + 120.08\angle 0^\circ \\ &= 139.55\angle 36.86^\circ + 120.08\angle 0^\circ \\ &= 231.91 + j83.91 \\ &= 246.62\angle 19.89^\circ \text{V} \end{aligned}$$

Problem 2 continued on next page. . .

Now, if there is no load attached to the generator, then there is no current that flows. Hence, we get that $E_f = V_t = 120.08\angle 0^{\circ}$V.

Finally, the voltage regulation can be calculated as:

$$\begin{aligned} V_{reg} &= \frac{|V_{FL}| - |V_{NL}|}{|V_{NL}|} \\ &= \frac{246.62 - 120}{120} \\ &= 105.51\% \end{aligned}$$

$$\boxed{\text{The voltage regulation with an 80\% leading power factor for the load is 105.51\%}}$$

If the power factor was 80% lagging, then the phase current would be $I_\phi = 24.98\angle -36.86^{\circ}$A. Hence, when we calculate the excitation voltage, we get:

$$\begin{aligned} E_f &= I_L \cdot R_a + V_t \\ &= (24.98\angle -36.86) \cdot 5.6 + 120.08\angle 0^{\circ} \\ &= 139.55\angle -36.86^{\circ} + 120.08\angle 0^{\circ} \\ &= 231.91 - j83.91 \\ &= 246.62\angle -19.89^{\circ}\text{V} \end{aligned}$$

Hence, we get the same voltage regulation irrespective if the 80% power factor is leading or lagging. That is:

$$\boxed{\text{The voltage regulation with an 80\% lagging power factor for the load is 105.51\%}}$$

The phasor diagrams for the leading case and the lagging case can be seen in Figure and Figure, respectively.

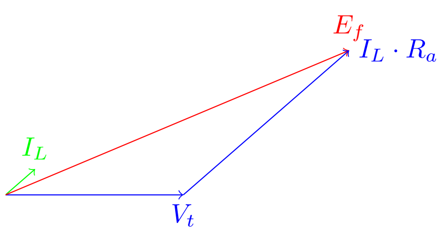

Figure 3: Phasor diagram for 80% leading

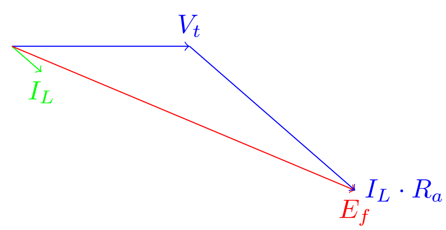

Figure 4: Phasor diagram for 80% lagging

Finally, we the power angle δ for the leading case is given by:

$$\boxed{\delta = 19.89^{\circ}}$$

The power angle δ for the lagging case is given by:

$$\boxed{\delta = -19.89^{\circ}}$$

Problem 3

Three identical 500 kV A, 2300/230 V,single phase transformers are connected in δ/Y. each transformer has Z = 0.2 + j0.6 ohms per phase equivalent impedance refereed to the HV side. The transformer bank supplies the following balanced three-phase loads at the rated low voltage:

- 750 kV A at 0.6 power factor lagging
- 500 kV A at 0.8 power factor lagging
- 350 kV A at unity power factor

Determine the primary line-to-line voltage.

The transformers are connected to three loads and are loaded up in series as shown in Figure

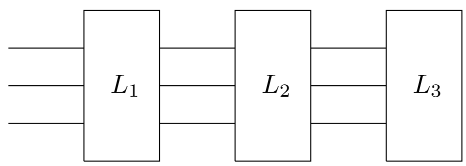

Figure 5: Three 3-ϕ loads connected in series.

The phase lag for load 1 is given by:

$$\begin{aligned} \theta_1 &= \arccos(0.6) \\ &= -53.13^\circ \end{aligned}$$

The phase lag for load 2 is given by:

$$\begin{aligned} \theta_2 &= \arccos(0.8) \\ &= -36.86^\circ \end{aligned}$$

The phase lag for load 3 is given by:

$$\theta_3 = 0^\circ$$

Now we can find the apparent power for each of the loads. The apparent power of the first load is given by:

$$\begin{aligned} S_1 &= 750\angle-53.13^\circ \text{kV A} \\ &= (450 - j600)\text{kV A} \end{aligned}$$

The apparent power of the second load is given by:

$$\begin{aligned} S_2 &= 500\angle-36.86^\circ \text{kV A} \\ &= (400 - j300)\text{kV A} \end{aligned}$$

The apparent power of the third load is given by:

$$S_3 = 350\angle^\circ \text{kV A}$$

The total apparent power absorbed by the loads is given by:

$$\begin{aligned}S_{total} &= S_1 + S_2 + S_3 \\ &= 450 - j600 + 400 - j300 + 350 \\ &= 1200 - j900 \\ &= 1500\angle -36.86^{\circ}\text{kV A}\end{aligned}$$

Now, we can find the phase current, I_ϕ^L, using the following relationship:

$$|S_{total}^L| = 3 \cdot |S_\phi^L| = 3 \cdot |V_\phi^L| \cdot |I_\phi^L|$$

rearranging, we get that:

$$\begin{aligned}|I_\phi^L| &= \frac{|S_{total}|}{3 \cdot |V_\phi|} \\ &= \frac{1500 \times 10^3}{3 \cdot 230} \\ &= 2173.91\text{A}\end{aligned}$$

Hence, the current is given by:

$$I_\phi^L = 2173.91\angle -36.86^{\circ}\text{A}$$

Now, we can refer the current, I_ϕ^L, to the high voltage side, using the turns ratio. The turns ratio is given by:

$$a = \frac{N_1}{N_2} = \frac{E_1}{E_2} = \frac{2300}{230} = 10$$

Hence, we find I_ϕ^H, as follows:

$$\begin{aligned}I_\phi^H &= \frac{1}{a} \cdot I_\phi^L \\ &= \frac{1}{10} \cdot 2173.91\angle -36.86^{\circ} \\ &= 217.39\angle -36.86^{\circ}\text{A}\end{aligned}$$

Finally, we can find the line to line voltage (which is the same as the phase voltage for a Δ-circuit) using a KVL on the equivalent circuit shown in Figure:

$$-V_{line}^H + I_{line}^H \cdot Z_{line}^H + 2300 = 0$$

Therefore, we get that:

$$\begin{aligned}V_{line}^H &= I_{line}^H \cdot Z_{line}^H + 2300\angle 0^{\circ} \\ &= 217.39\angle -36.86^{\circ}\text{A} \cdot (0.2 + j0.6) + 2300\angle 0^{\circ} \\ &= 137.39\angle 34.70^{\circ} + 2300\angle 0^{\circ} \\ &= 112.88 + j78.16 + 2300 \\ &= 2412.88 + j78.16\end{aligned}$$

Hence the line to line voltage of the transformer is given by:

$$\boxed{V_{line}^H = 2414.15\angle 1.85^{\circ}\text{V}}$$

Problem 4

A 6-pole, 230V, 60Hz, star connected, three phase induction motor has the following parameters on a per phase basis:

- $R_1 = 0.5\Omega$
- $R_2 = 0.25\Omega$
- $X_1 = 0.75\Omega$
- $X_2 = 0.5\Omega$
- $X_m = 100\Omega$
- $R_c = 500\Omega$

All the impedances are refereed to the stator side. The friction and windage loss is 150 watts. Determine the efficeincy of the motor at its rated slip of 2.5%.

The motor is Y-connected, so given that $V_{line} = 230$V, then:

$$V_\phi = \frac{V_{line}}{\sqrt{3}} = \frac{230\angle 0^\circ}{\sqrt{3}} = 132.79\angle 0^\circ\text{V}$$

The equivalent circuit for the induction motor, which includes both R_C and X_m, is shown in Figure:

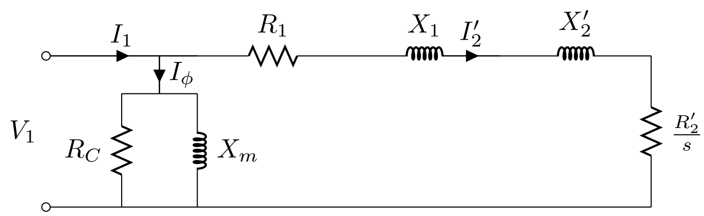

Figure 6: Equivalent circuit for the induction motor described in the problem.

Therefore, with reference to Figure, we see that:

$$\begin{aligned}
I_2' &= \frac{V_1}{Z^e q} \\
&= \frac{V_1}{R_1 + jX_1 + jX_2' + \frac{R_2'}{s}} \\
&= \frac{132.79\angle 0^\circ}{0.5 + \frac{0.25}{0.025} + j(0.75 + 0.5)} \\
&= \frac{132.79\angle 0^\circ}{10.57\angle 6.79^\circ} \\
&= 12.56\angle -6.79^\circ\text{A}
\end{aligned}$$

Further, we can find the current, I_ϕ, in the excitation branch using:

$$I_\phi = \frac{V_1}{Z_\phi^{eq}}$$

We have R_C and X_m are in parallel, hence, we can find that:

$$Z_\phi^{eq} = \left(\frac{1}{R_C} + \frac{1}{jX_m}\right)^{-1} = 98.06\angle 78.69^\circ$$

Hence, the current in the excitation branch is given by:

$$I_\phi = \frac{V_1}{Z_\phi^{eq}} = \frac{132.79\angle 0^\circ}{98.06\angle 78.69^\circ} = 1.35\angle -78.69^\circ\text{A}$$

Problem 4 continued on next page...

Finally, we can find the current I_1 as follows:

$$\begin{aligned} I_1 &= I_\phi + I_2 \\ &= 1.35\angle - 78.69^\circ + 12.56\angle - 6.79 \\ &= 0.2647 - j1.323 + 12.47 - j1.4849 \\ &= 12.73 - j2.80 \\ &= 13.03\angle - 12.40^\circ \mathrm{A} \end{aligned}$$

Now, we can find the apparent input power as follows:

$$\begin{aligned} S_{in} &= 3 \cdot V_1 \cdot I_1^* \\ &= 3 \cdot 132.79\angle 0^\circ \cdot 13.03\angle 12.40^\circ \\ &= 5190.76\angle 12.40^\circ \mathrm{V\,A} \end{aligned}$$

Hence the input power is given by:

$$P_{in} = 5190.76\cos(12.40^\circ) = 5069.67\mathrm{W}$$

Now, the air gap power, P_{ag}, is given by the formula:

$$\begin{aligned} P_{ag} &= 3 \cdot I_2^2 \cdot \frac{R_2}{s} \\ &= 3 \cdot (12.56)^2 \cdot \frac{0.25}{0.025} \\ &= 4732.61\mathrm{W} \end{aligned}$$

Then we can find the mechanical power, P_{mech}, as follows:

$$\begin{aligned} P_{mech} &= P_{ag} \cdot (1 - s) \\ &= 4732.61 \cdot (1 - 0.025) \\ &= 4614.29\mathrm{W} \end{aligned}$$

Finally, we find that the output power, P_{out}, is given by:

$$P_{out} = P_{mech} - P_{f+w} = 4614.29 - 150 = 4464.29\mathrm{W}$$

Finally, the effienicy is given by:

$$\boxed{\eta = \frac{P_{out}}{P_{in}} = \frac{4464.29}{5069.67} = 88.05\%}$$