ENG252 Dynamics: Practical 3

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1 Introduction

1.1 Moment of Inertia

Consider a body with mass m undergoing rotational motion, with angular acceleration α, around a fixed axis OO'. Graphically this scenario is depicted in Figure 1. If we want to calculate the Moment of this body around the axis OO', then we need to consider the moments of each and every infinitesimally small mass particle that make up the body. The moment of a single particle mass is given by:

$$M = \boldsymbol{r} \times \boldsymbol{F} \tag{1}$$

We note $\boldsymbol{r}$ is the position vector of the particle from point where the moment is to calculated, and $\boldsymbol{F}$ is the force acting on the particle. If our motion is constrained to a 2D plane, equation (1) simplifies to the well known equation:

$$M = F \times d \tag{2}$$

The quantity F is the scalar magnitude of the force acting orthogonal to the shortest line connecting the particle mass to the moment point of calculation; and d is the distance between between these two points. Using (2) we can calculate the moment M_O about axis OO' for a small mass element, dm, of the body in Figure 1:
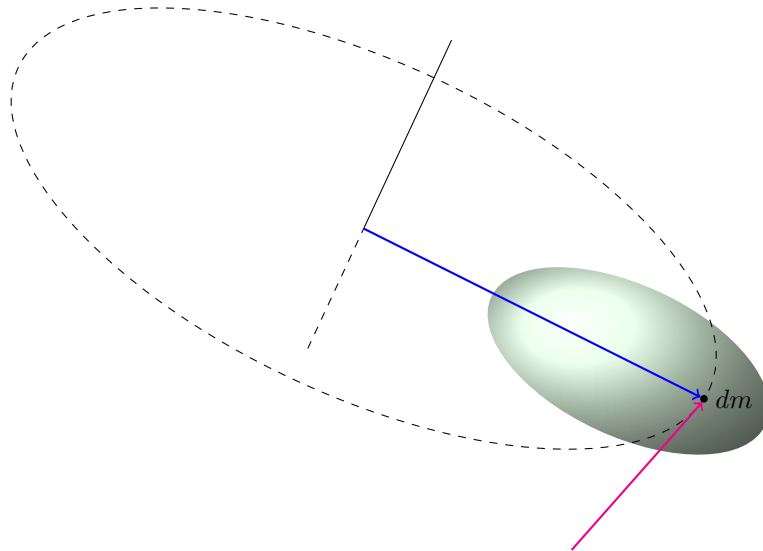
$$M_O = Fr = a_t \; dm \; r \tag{3}$$

Figure 1: A body of mass m rotating about a fixed axis OO' with some angular acceleration α.

According to Giancoli a particle undergoing fixed axis rotation can re-express tangential acceleration a_t, as αr, where r is the distance from the centre of rotation to the particle. We can now rite (3) as:

$$M_O = \alpha r \ dm \ r \tag{4}$$

This is convenient since α remains constant for all infinitesimally small particles in the body meaning that the only variable that needs consideration is r. In fact, to calculate the sum of the moments of the body around axis OO' we need to integrate the right hand side of equation (4), which yields:

$$\sum M_O = \alpha \int r^2 dm \tag{5}$$

Equation (5) is often thought of as somewhat analogous to $\sum F = ma$, but for rotational motion. In fact since α is the angular acceleration, the integral in equation (5) is often referred to as the resistance of a body to change it's state of rotation. In the literature this quantity is denoted I_O and referred to as the Moment of Inertia and is defined as:

$$I = \int r^2 dm \tag{6}$$

For a body with a uniform mass density ρ, we note that $dm = \rho dV$, where dV is an infinitesimally small volume located a distance of r from the centre of rotation. Equation (6) can be written as:

$$I = \rho \int r^2 dV \tag{7}$$

Equation (7) is deceptively simple, however, the evaluation of I can be fiendishly difficult for axes of rotation which do not pass through the body's centre of mass. In practice, (7) is typically only used to determine the moment of inertia through the body's mass centre, I_G. To find a moment of inertia around an axis that does not pass through the mass centre, the parallel axis theorem is often applied. The theorem derivation is beyond the scope of this paper, however, the result can be seen in equation (8).

$$I_O = I_G + md^2 \tag{8}$$

Equation (8) tells us if the moment of inertia around an axis passing through the mass centre of a body I_G is known, then the moment of inertia around any parallel axis I_O is calculated with an additive translation of I_G by the body mass m multiplied by the square of orthogonal Euclidean distance between the two axes d.

1.2 Determining Moments of Inertia with a Trifilar

consisting of a disk suspended by three wires. Objects are placed in the middle of the disk, and the disk is given a small initial perturbation resulting in small oscillations around the mass centre.
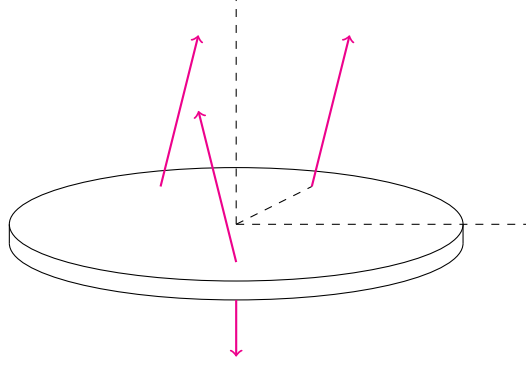
Figure 2: A Trifilar is a simple apparatus used to determine the moment of inertia for different objects.

1.3 Scope

2 Results

Experimental characteristics of the Trifilar were measured prior to undertaking the experiment. Measurements included disk weight m_d; disk radius r_d; filament length L; and the radius from centre of disk to middle set of holes R. These measurements are presented in Table 1 below. To better understand where the masses were mounted on the disk, the Trifilar disk geometries are shown in Figure 3.

Table 1: Trifilar parameter measurements

Description	Value	Units
Disk Mass (m_d)	0.20	kg
Disk Radius (r_d)	0.14	m
Filament Length (L)	0.45	m
Distance to Middle Holes (R)	0.09	m

Table 2: text

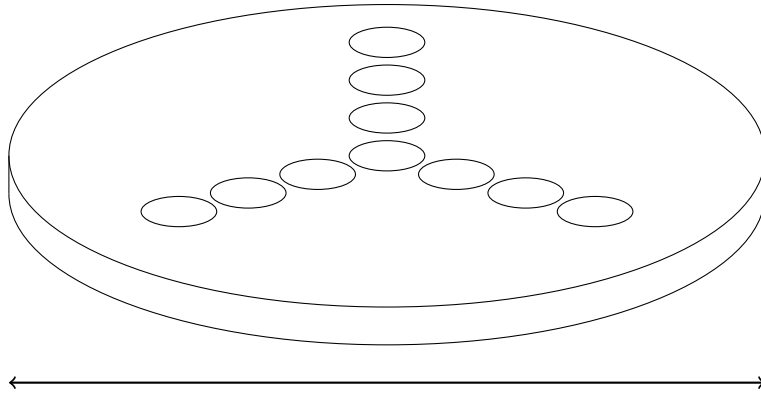| Description	Value	Units
Disk Mass (m_d)	0.20	kg
Disk Radius (r_d)	0.14	m

Figure 3: Geometries of the Trifilar disk and holes where small masses were mounted.

Measurement information was also taken about the small objects, which included their mass m_{obj}; radius of upper cylinder r_1; height of the upper cylinder h_1; radius of the lower cylinder r_2; and the height of the lower cylinder h_2. These measurements are presented in Table 2 above. The object geometries can be seen in Figure
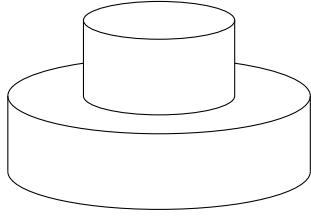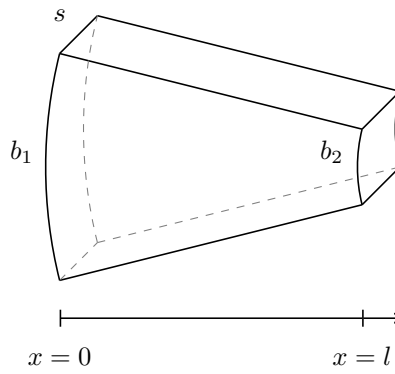
Figure 4: The small objects are essentially a small cylinder stacked on top of a larger cylinder

Table 3: text

Object	$(T_{10})_1$	$(T_{10})_2$	$(T_{10})_3$	$(T_{10})_4$	$(T_{10})_5$

3 Calculations

4 Discussion

5 Conclusion