ENG252 Dynamics: Practical 3

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1 Introduction

Consider a body with mass m undergoing rotational motion, with angular acceleration α, around a fixed axis OO'. Graphically this scenario is depicted in Figure 1. If we want to calculate the Moment of this body around the axis OO', then we need to consider the moments of each and every infinitesimally small mass particle that make up the body. The moment of a single particle mass is given by:

$$M = \boldsymbol{r} \times \boldsymbol{F} \tag{1}$$

We note $\boldsymbol{r}$ is the position vector of the particle from point where the moment is to calculated, and $\boldsymbol{F}$ is the force acting on the particle. If our motion is constrained to a 2D plane, equation (1) simplifies to the well known equation:

$$M = F \times d \tag{2}$$

The quantity F is the scalar magnitude of the force acting orthogonal to the shortest line connecting the particle mass to the moment point of calculation; and d is the distance between between these two points. Using (2) we can calculate the moment M_O about axis OO' for a small mass element, dm, of the body in Figure 1:

$$M_O = Fr = a_t \ dm \ r \tag{3}$$

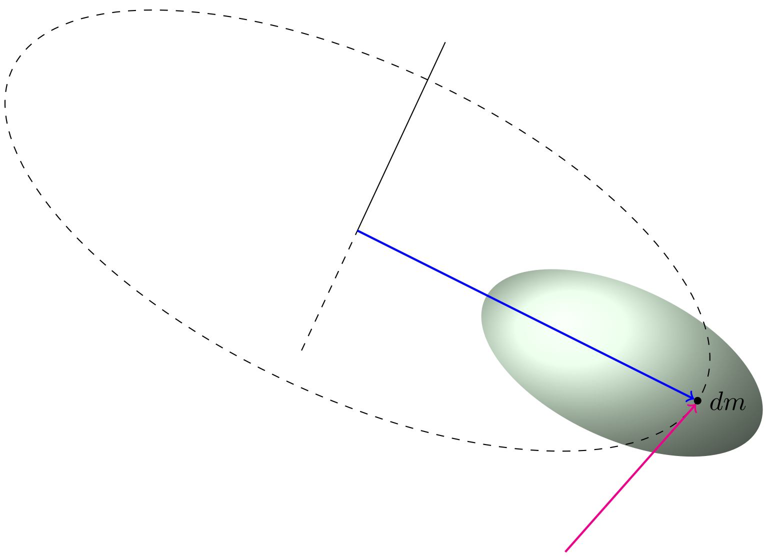


Figure 1: A body of mass m rotating about a fixed axis OO' with some angular acceleration α.

According to Giancoli a particle undergoing fixed axis rotation can re-express tangential acceleration a_t, as αr, where r is the distance from the centre of rotation to the particle. We can now rite (3) as:

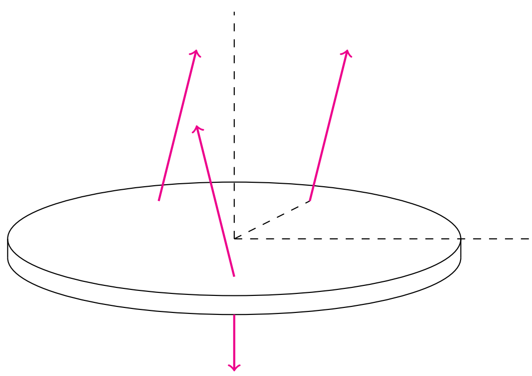
$$M_O = \alpha r \ dm \ r \tag{4}$$

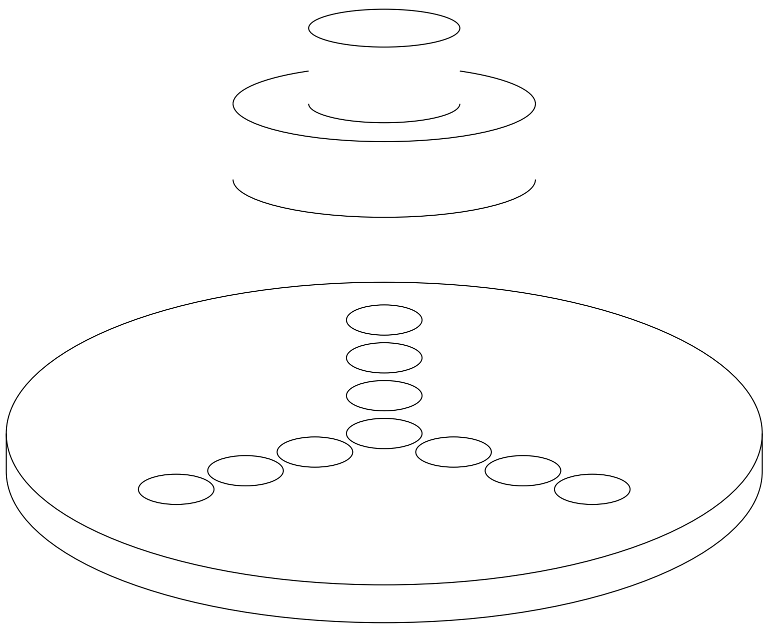
This is convenient since α remains constant for all infinitesimally small particles in the body meaning that the only variable that needs consideration is r. In fact, to calculate the sum of the moments of the body around axis OO' we need to integrate the right hand side of equation (4), which yields:

$$\sum M_O = \alpha \int r^2 dm \tag{5}$$

Equation (5) is often thought of as somewhat analogous to $\sum F = ma$, but for rotational motion. In fact since α is the angular acceleration, the integral in equation (5) is often referred to as the resistance of a body to change it's state of rotation. In the literature this quantity is denoted I_O and referred to as the Moment of Inertia and is defined as:

$$I = \int r^2 dm \tag{6}$$



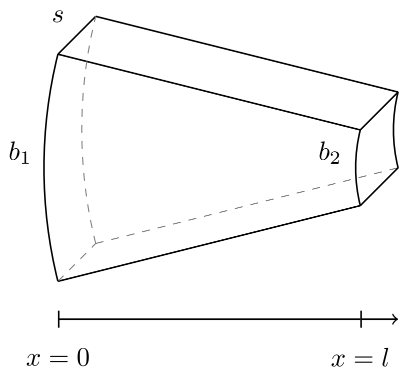


Talk about rotational inertia
Talk about derivation because atan = alpha times r
Talk about the analogous nature to f = ma
Talk about mass moments of inertia and how to calculate
Talk about radius of gyration

1.1 Scope

2 Results

3 Calculations



4 Discussion

5 Conclusion