Experiment 2

Hand Calculations

(262538)
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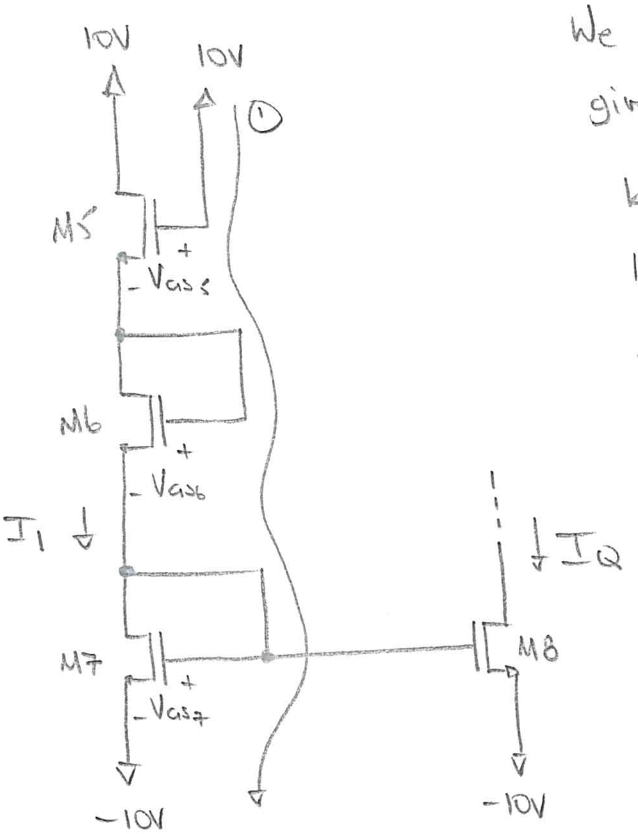
The circuit given in this practical can be divided into two main components:

- A current source in the form of a current mirror.
- An active loaded Differential Amp.

Consider the current source component shown below:



We want to find I_1 given:

$k_{n5} = k_{n6} = 50\mu A/V^2$

$k_{n7} = 200\mu A/V^2$

$\lambda = 0.01 V^{-1}$

If we take a KVL by loop ①:

$$-10 + V_{GS5} + V_{GS6} + V_{GS7} - 10 = 0.$$

Since M_5 + M_6 are identical

$$\therefore \quad 2V_{GS6} + V_{GS7} = 20. \qquad -1)$$

Now, assuming M5, M6 & M7 are operating in saturation:

$$I_{D5} = I_{D6} = \frac{1}{2} k_{n6} (V_{GS6} - V_T)^2. \qquad -2)$$

Similarly,

$$I_{D7} = \frac{1}{2} k_{n7} (V_{G7} - V_T)^2 \qquad -3)$$

(cont'd over)

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Dividing $-2)$ by $-3)$ gives us:

$$\frac{I_{D6}}{I_{D7}} = \frac{k_{n6}(V_{GS6} - V_T)^2}{k_{n7}(V_{GS7} - V_T)^2} \qquad -4).$$

Now, given M6 + M7 are in the same branch it must be that:

$$I_{D6} = I_{D7}.$$

So we can rearrang $-4)$ into:

$$\frac{k_{n7}}{k_{n6}} = \left[\frac{V_{GS6} - V_T}{V_{GS7} - V_T}\right]^2.$$

$$\pm\sqrt{\frac{k_{n7}}{k_{n6}}} = \frac{V_{GS6} - V_T}{V_{GS7} - V_T}$$

$$\therefore \pm\sqrt{\frac{k_{n7}}{k_{n6}}}\,(V_{GS7} - V_T) = V_{GS6} - V_T \qquad -5).$$

Rearranging eqn $-1)$ we see that:

$$V_{GS7} = 20 - 2V_{GS6}. \qquad -6)$$

Sub $-6)$ into $-5)$:

$$\pm\sqrt{\frac{k_{n7}}{k_{n6}}}\cdot(20 - 2V_{GS6} - V_T) = V_{GS6} - V_T \qquad -7)$$

If we let $\pm\eta = \sqrt{\frac{k_{n7}}{k_{n6}}}$ (cont'd over).

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We can rewrite -7) as:

$$\pm \eta (20 - 2V_{asb} - V_T) = V_{asb} - V_T$$

Case 1

$$\eta (20 - 2V_{asb} - V_T) = V_{asb} - V_T$$

$$20\eta - 2\eta V_{asb} - \eta V_T = V_{asb} - V_T$$

$$20\eta + V_T - \eta V_T = V_{asb} + 2\eta V_{asb}$$

$$20\eta + V_T - \eta V_T = V_{asb}(1 + 2\eta)$$

$$\therefore \; V_{asb} = \frac{20\eta + V_T - \eta V_T}{1 + 2\eta}$$

Case 2

$$-\eta (20 - 2V_{asb} - V_T) = V_{asb} - V_T$$

$$-20\eta + 2\eta V_{asb} + \eta V_T = V_{asb} - V_T$$

$$\eta V_T - 20\eta + V_T = V_{asb}(1 - 2\eta)$$

$$\therefore \; V_{asb} = \frac{\eta V_T - 20\eta + V_T}{1 - 2\eta}.$$

(cont'd over)

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Hence,

<u>Case 1</u>

$$V_{GS6} = \frac{V_T + 20\eta - \eta V_T}{1 + 2\eta}$$

$$= \frac{V_T + \eta(20 - V_T)}{1 + 2\eta}$$

$$= \frac{2 + \sqrt{4}\,(20 - 2)}{1 + 2\sqrt{4}}$$

$$= \frac{2 + 2.18}{1 + 4}$$

$$= 7.6V$$

<u>Case 2</u>

$$V_{GS6} = \frac{V_T - 20\eta + \eta V_T}{1 - 2\eta}$$

$$= \frac{V_T - \eta(20 - V_T)}{1 - 2\eta}.$$

$$= \frac{2 - \sqrt{4}\,(20 - 2)}{1 - 2\sqrt{4}}$$

$$= \frac{2 - 2.18}{1 - 4}$$

$$= 11.33V.$$

Now, Since $V_{GS7} = 20 - 2V_{GS6}$

if $V_{GS6} = 11.33V$, then

$$V_{GS7} = 20 - 2 \cdot 11.33$$

$$\Rightarrow V_{GS7} < 0$$

$$\Rightarrow V_{GS7} < V_T$$

and would be in pinch off mode.

Hence, $V_{GS6} = 7.6V$, $V_{GS6} \neq 11.33V$

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Hence, if $V_{GS6} = 7.6V$, then:

$$I_{D6} = I_1 = \frac{1}{2} k_{n6} \left(V_{GS6} - V_T\right)^2$$

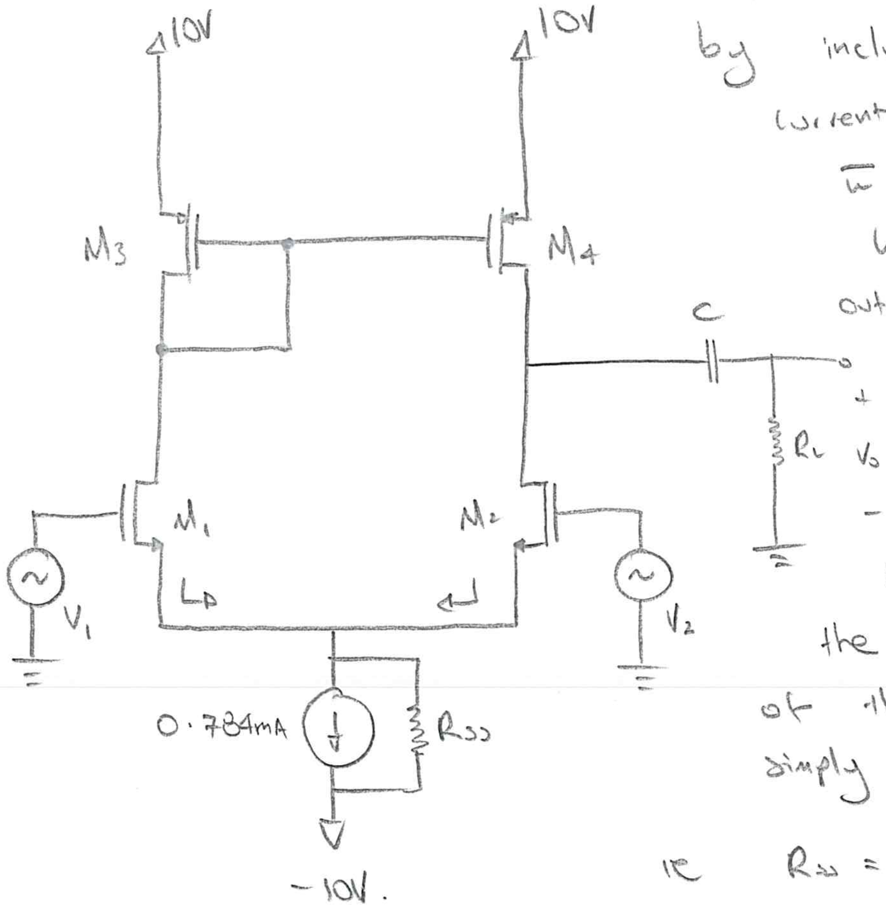
$$\therefore I_1 = \frac{1}{2} \cdot 50e\text{-}6 \left(7.6 - 2\right)^2$$

$$= 0.784mA.$$

Finally, M_7 + M_8 are identical transistors and are in the shape of a current mirror.

Hence, $I_1 = I_Q = 0.784mA.$

Now, we turn our attention to the differential amplifier component:



We can abstract away the detail of the current source by including a constant current source in parallel with the resistor R_{SS} which represents the output resistance of the current source.

ie $R_{SS} = (R_o)_{\text{current mirror}}$.

Now, we know that the finite output resistance of the current mirror is simply r_{o8}.

ie $R_{SS} = (R_o)_{\text{current mirror}} = r_{o8}.$

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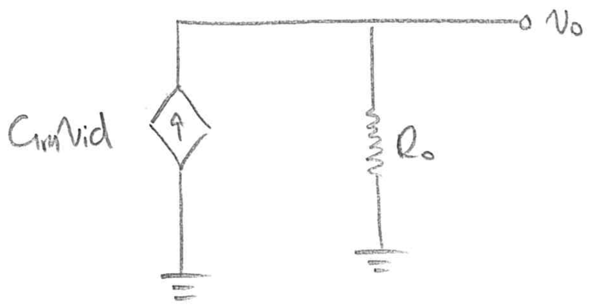
Now, for a standard Differential Amp $\bar{c}$ active load we have that the general form:



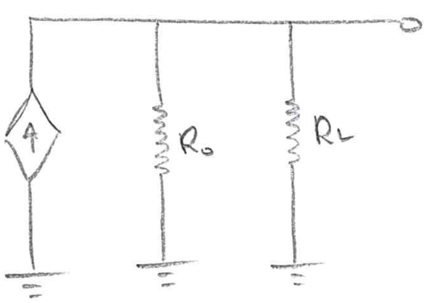
where

$$R_o = r_{o2} \| r_{o4}$$

$$A_d = g_m (r_{o2} \| r_{o4})$$

$$A_{cm} = -\frac{1}{2 g_{m3} R_{SS}}.$$

But in our case we have a load at the output, R_L : Hence, our circuit looks like



Hence,

$$A_d = G_m . (R_o \| R_L).$$

$$\therefore A_d = g_m (r_{o2} \| r_{o4} \| R_L)$$

if $r_{o2} = r_{o4}$ since transistors have identical parameters, then

$$A_d = g_m \left(\tfrac{1}{2} r_o \| R_L\right) = g_m \left(\frac{2}{r_o} + \frac{1}{R_L}\right)^{-1}$$

$$\boxed{A_d = \frac{g_m r_o R_L}{2R_L + r_o}.}$$

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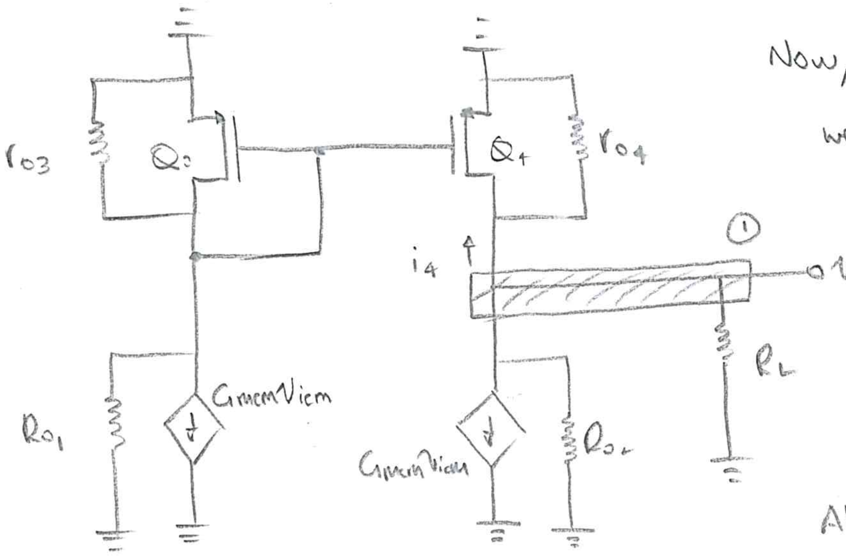
Finding A_{cm} is slightly more involved. Consider the small signal model of the active loaded Diff amp



Now, from Sedra & Smith we obtain the eqⁿ for i_4 from considering the half circuit.

$$i_4 = -g_m G_{mcm} v_{icm} \left(R_{o1} \| r_{o3} \| \frac{1}{g_m}\right)$$

Also $G_{mcm} = \frac{1}{2R_{ss}}$

By taking a node eqⁿ at 1 we get that:

$$i_4 + G_{mcm} v_{icm} + \frac{v_o}{R_{o2}} + \frac{v_o}{r_{o4}} + \frac{v_o}{R_L} = 0$$

Subbing for i_4 and G_{mcm}, we get that:

$$-g_m \frac{1}{2R_{ss}} v_{icm} \left(R_{o1} \| r_{o3} \| \frac{1}{g_m}\right) + \frac{1}{2R_{ss}} v_{icm} + v_o\left(\frac{1}{R_{o2}} + \frac{1}{r_{o4}} + \frac{1}{R_L}\right) = 0.$$

$$v_o\left(\frac{1}{R_{o2}} + \frac{1}{r_{o4}} + \frac{1}{R_L}\right) = v_{icm} \cdot \frac{1}{2R_{ss}} \cdot \left(g_m\left[R_{o1} \| r_{o3} \| \frac{1}{g_m}\right] - 1\right)$$

$$\frac{v_o}{v_{icm}} = \frac{(R_{o2} \| r_{o4} \| R_L)}{2R_{ss}} \cdot \left(g_m\left[R_{o1} \| r_{o3} \| \frac{1}{g_m}\right] - 1\right)$$

$$\boxed{A_{cm} = -\frac{(R_{o2} \| r_{o4} \| R_L)}{2R_{ss}} \left(1 - g_m\left[R_{o1} \| r_{o3} \| \frac{1}{g_m}\right]\right)}$$

(cont'd over)

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Now, we just need to calculate the value of the device parameters.

$$g_m = \sqrt{2 k_n I_D} \qquad I_{D_1} = \frac{0.784\,mA}{2} = 0.392\,mA$$

$$= \sqrt{2 . 100e{-6} . .392e{-3}} \qquad k_n = 100\,\mu A/V^2$$

$$= \underline{2.8e-4\ S}$$

$$r_o = \frac{1}{\lambda I_D} = \frac{1}{0.01 . 0.392e{-3}}$$

$$= \underline{255.1\,k\Omega}$$

$$R_{ss} = r_{o8} = \frac{1}{\lambda . I} = \frac{1}{0.01 . 0.784e{-3}}$$

$$= \underline{127.55\,k\Omega}$$

$$R_{o1} = R_{o2} = 2R_{ss} + r_o + (g_m r_o)(2R_{ss}).$$

$$= 2(127.55e3) + 255.1e3$$

$$+ 2.8e{-4} . 255.ke3 . 2 . 127.55e3$$

$$= \underline{18.71\,M\Omega}$$

(Cont'd over).

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Now,

- If $R_L = 10M\Omega$

$$A_d = \frac{g_m r_o R_L}{2R_L + r_o}$$

$$= \frac{2.8e-4 . 255.1e3 . 10e6}{2.(10e6) + 255.1e3}$$

$$= \underline{35.76 \ V/V}$$

$$A_{cm} = \frac{-1}{2r_{o3}}\left(R_{o2} \| r_{o4} \| R_L\right)\left(1 - g_m\left[R_{o1} \| r_{o3} \| \frac{1}{g_m}\right]\right).$$

if $\xi = R_{o2} \| r_{o4} \| R_L$

$\eta = R_{o1} \| r_{o2} \| \frac{1}{g_m}$.

$$\therefore A_{cm} = \frac{-1}{2r_{o3}} . \xi . (1 - g_m \eta)$$

Now, $$\xi = \left(\frac{1}{18.71e6} + \frac{1}{255.1e3} + \frac{1}{10e6}\right)^{-1}$$

$$= 245.5e3$$

$$\eta = \left(\frac{1}{18.71e6} + \frac{1}{255.1e3} + 2.8e-4\right)^{-1}$$

$$= 3521.45$$

$$\therefore A_{cm} = \frac{-1}{2.(127.55e3)}\left[245.5e3\right]\left(1 - 2.8e-4 . 3521.45\right)$$

$$= \underline{-0.0134 \ V/V}$$

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Finally, if $R_L = 400k\Omega$

$$A_d = \frac{g_m r_o R_L}{2R_L + r_o}$$

$$= \frac{2.8e-4 \,.\, 255.1e3 \,.\, 400e3}{2.400e3 + 255.1e3}$$

$$= \underline{27.07\ V/V}$$

$$A_{cm} = \frac{-1}{2r_{o3}} \cdot \xi \,.\, (1 - g_m \eta)$$

where $\xi = R_{o2} \| r_{o4} \| R_L$

$\eta = R_{o1} \| r_{o3} \| \frac{1}{g_m}$

$$\therefore \quad \xi = \left(\frac{1}{18.71e6} + \frac{1}{255.1e3} + \frac{1}{400e3}\right)^{-1}$$

$$= 154.45e3$$

$$\eta = \left(\frac{1}{18.71e6} + \frac{1}{255.1e3} + 2.8e-4\right)^{-1}$$

$$= 3521.45.$$

$$\therefore \quad A_{cm} = \frac{-1}{2.127.55e3} \cdot 154.45e3 \,.\, (1 - 2.8e-4 \,.\, 3521.45)$$

$$= -\underline{8.47e-3\ V/V}$$