

# Power Systems Analysis: Assignment 3

Due on June 3, 2017 at 3:00pm

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Consider the power system shown in Figure 1. Each three phase transformer is made up of three single phase transformers. The squares  $B_1, B_2, B_3, \dots, B_6$  are circuit breakers which can be considered to have very low series impedance while closed and infinite impedance when open.

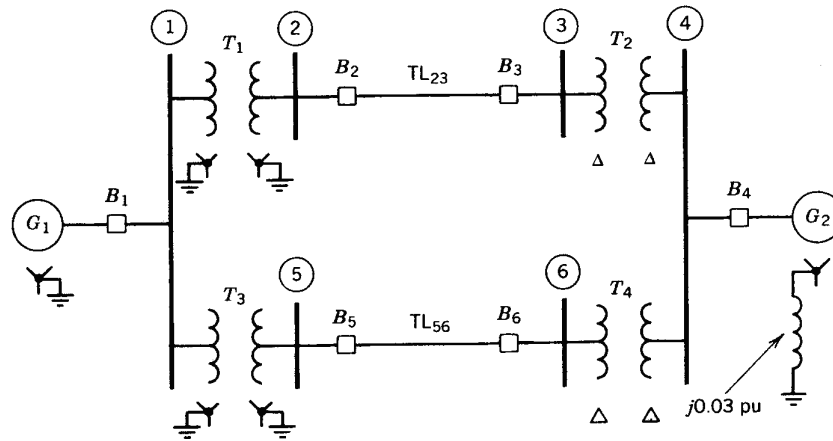


Figure 1: text

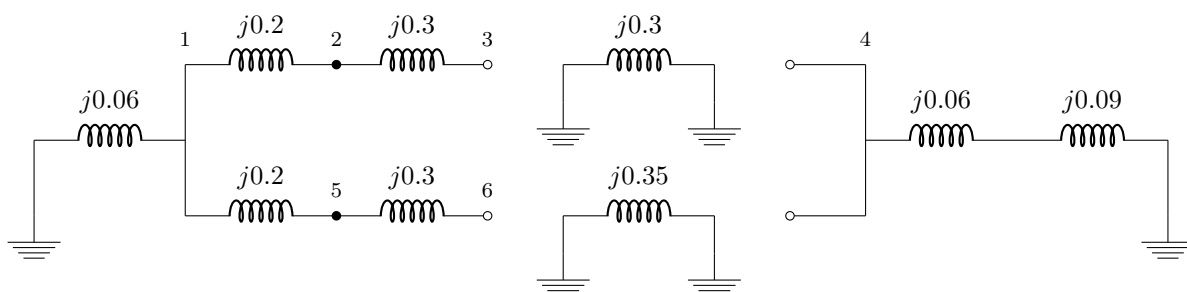
Figure 2: text

| Component | Power Rating<br>(MVA) | Voltage Rating<br>(kV) | $X_1$<br>(pu) | $X_2$<br>(pu) | $X_0$<br>(pu) |
|-----------|-----------------------|------------------------|---------------|---------------|---------------|
| $G_1$     | 200                   | 20                     | 0.2           | 0.14          | 0.06          |
| $G_2$     | 200                   | 13.2                   | 0.2           | 0.14          | 0.06          |
| $T_1$     | 200                   | 20/230                 | 0.2           | 0.2           | 0.2           |
| $T_2$     | 200                   | 13.2/230               | 0.3           | 0.3           | 0.3           |
| $T_3$     | 200                   | 20/230                 | 0.25          | 0.25          | 0.25          |
| $T_4$     | 200                   | 13.2/230               | 0.35          | 0.35          | 0.35          |
| $TL_{23}$ | 200                   | 230                    | 0.15          | 0.15          | 0.3           |
| $TL_{56}$ | 200                   | 230                    | 0.22          | 0.22          | 0.5           |

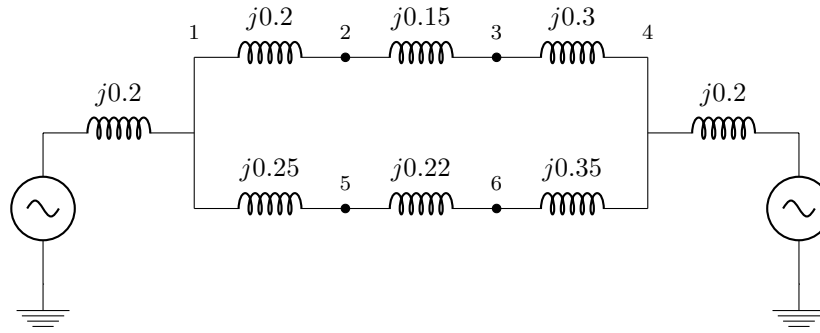
## Assignment 3.1 - PART A

The following section details the three sequence networks, and the impedance vlued

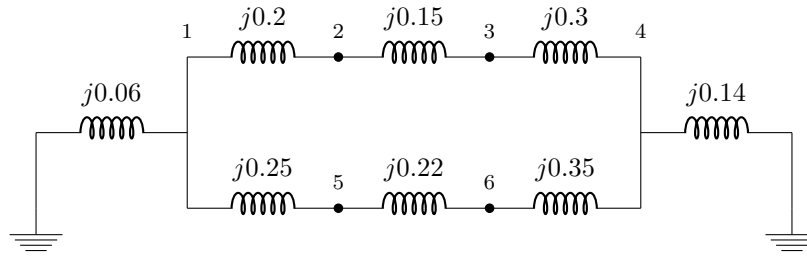
### Zero Sequence Network



### Positive Sequence Network



### Negative Sequence Network



## Assignment 3.1 - PART B

Assuming that there is a fault on bus 3, to analyse the response to this, the Thevenin equivalent circuits for the zero, positive, and negative sequences need to be found looking into bus 3. To find the Thevenin impedance, the  $Y_{BUS}$  matrix was first found for each sequence, and inverted to determine  $Z_{th}^0$ ,  $Z_{th}^1$ , and  $Z_{th}^2$ .

### Zero Sequence

We first determine all of the admittances for each of the links between the buses for all  $i, j$  combinations. The results can be seen in Table 2.

Figure 3: text

| Admittance        | $Y_{ij}^{BUS}$            |
|-------------------|---------------------------|
| $y_{12} = -j5$    | $Y_{12} = Y_{21} = j5$    |
| $y_{13} = j0$     | $Y_{13} = Y_{31} = j0$    |
| $y_{14} = j0$     | $Y_{14} = Y_{41} = j0$    |
| $y_{15} = -j4$    | $Y_{15} = Y_{51} = j4$    |
| $y_{16} = j0$     | $Y_{16} = Y_{61} = j0$    |
| $y_{23} = -j3.33$ | $Y_{23} = Y_{32} = j3.33$ |
| $y_{24} = j0$     | $Y_{24} = Y_{42} = j0$    |
| $y_{25} = j0$     | $Y_{25} = Y_{52} = j0$    |

Figure 4: text

| Admittance     | $Y_{ij}^{BUS}$         |
|----------------|------------------------|
| $y_{26} = j0$  | $Y_{26} = Y_{62} = j0$ |
| $y_{34} = j0$  | $Y_{34} = Y_{43} = j0$ |
| $y_{35} = j0$  | $Y_{35} = Y_{53} = j0$ |
| $y_{36} = j0$  | $Y_{36} = Y_{63} = j0$ |
| $y_{45} = j0$  | $Y_{45} = Y_{54} = j0$ |
| $y_{46} = j0$  | $Y_{46} = Y_{64} = j0$ |
| $y_{56} = -j2$ | $Y_{56} = Y_{65} = j2$ |

Now, we need to find the  $Y_{ii}$  elements, which are calculated as follows:

$$Y_{11} = y_{10} + y_{12} + y_{15} = -j25.66$$

$$Y_{22} = y_{12} + y_{23} = -j8.33$$

$$Y_{33} = y_{23} = -j3.33$$

$$Y_{44} = y_{04} = -j6.67$$

$$Y_{55} = y_{15} + y_{56} = -j6$$

$$Y_{66} = y_{56} = -j2$$

The  $Y_{BUS}^0$  matrix is given as follows:

$$Y_{BUS}^0 = \begin{bmatrix} -j25.66 & +j5 & +j0 & +j0 & +j4 & +j0 \\ +j5 & -j8.33 & +j3.33 & +j0 & +j0 & +j0 \\ +j0 & +j3.33 & -j3.33 & +j0 & +j0 & +j0 \\ +j0 & +j0 & +j0 & -j6.67 & +j0 & +j0 \\ +j4 & +j0 & +j0 & +j0 & -j6 & +j2 \\ +j0 & +j0 & +j0 & +j0 & +j2 & -j2 \end{bmatrix}$$

Noting that  $Z_{BUS}^0 = (Y_{BUS}^0)^{-1}$ , we can find the  $Z_{BUS}^0$  by inverting the  $Y_{BUS}^0$  matrix. This operation was performed in MATLAB, which yielded the following output:

z0 =

```
0.0000 + 0.0600i    0.0000 + 0.0600i    0.0000 + 0.0600i    0.0000 + 0.0000i    0.0000 + 0.0600i    0.0000 + 0.0600i
0.0000 + 0.0600i    0.0000 + 0.2600i    0.0000 + 0.2600i    0.0000 + 0.0000i    0.0000 + 0.0600i    0.0000 + 0.0600i
0.0000 + 0.0600i    0.0000 + 0.2600i    0.0000 + 0.5603i    0.0000 + 0.0000i    0.0000 + 0.0600i    0.0000 + 0.0600i
0.0000 + 0.0000i    0.0000 + 0.0000i    0.0000 + 0.0000i    0.0000 + 0.1499i    0.0000 + 0.0000i    0.0000 + 0.0000i
0.0000 + 0.0600i    0.0000 + 0.0600i    0.0000 + 0.0600i    0.0000 + 0.0000i    0.0000 + 0.3100i    0.0000 + 0.3100i
0.0000 + 0.0600i    0.0000 + 0.0600i    0.0000 + 0.0600i    0.0000 + 0.0000i    0.0000 + 0.3100i    0.0000 + 0.8100i
```

This allows us to easily extract the Thevenin impedance, noting that  $Z_{th}^0 = (Z_{BUS}^0)_{3,3}$ . Hence, we note that the zero sequence impedance is:

$$(Z_{BUS}^0)_{3,3} = j0.5603$$

The Thevenin equivalent for the Zero sequence circuit looking in from bus 3, is shown in Figure 4 below.

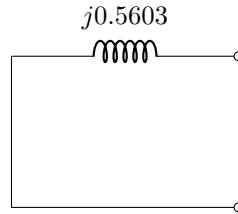


Figure 5: text

## Positive Sequence

We first determine all of the admittances for each of the links between the buses for all  $i, j$  combinations. The results can be seen in Table 2.

| Figure 6: text    |                           | Figure 7: text    |                           |
|-------------------|---------------------------|-------------------|---------------------------|
| Admittance        | $Y_{ij}^{BUS}$            | Admittance        | $Y_{ij}^{BUS}$            |
| $y_{12} = -j5$    | $Y_{12} = Y_{21} = j5$    | $y_{26} = j0$     | $Y_{26} = Y_{62} = j0$    |
| $y_{13} = j0$     | $Y_{13} = Y_{31} = j0$    | $y_{34} = -j3.33$ | $Y_{34} = Y_{43} = j3.33$ |
| $y_{14} = j0$     | $Y_{14} = Y_{41} = j0$    | $y_{35} = j0$     | $Y_{35} = Y_{53} = j0$    |
| $y_{15} = -j4$    | $Y_{15} = Y_{51} = j4$    | $y_{36} = j0$     | $Y_{36} = Y_{63} = j0$    |
| $y_{16} = j0$     | $Y_{16} = Y_{61} = j0$    | $y_{45} = j0$     | $Y_{45} = Y_{54} = j0$    |
| $y_{23} = -j6.67$ | $Y_{23} = Y_{32} = j6.67$ | $y_{46} = -j2.86$ | $Y_{46} = Y_{64} = j2.86$ |
| $y_{24} = j0$     | $Y_{24} = Y_{42} = j0$    | $y_{56} = -j4.54$ | $Y_{56} = Y_{65} = j4.54$ |
| $y_{25} = j0$     | $Y_{25} = Y_{52} = j0$    |                   |                           |

Now, we need to find the  $Y_{ii}$  elements, which are calculated as follows:

$$Y_{11} = y_{10} + y_{12} + y_{15} = -j14$$

$$Y_{22} = y_{12} + y_{23} = -j11.67$$

$$Y_{33} = y_{23} = -j10$$

$$Y_{44} = y_{04} = -j11.19$$

$$Y_{55} = y_{15} + y_{56} = -j8.54$$

$$Y_{66} = y_{56} = -j7.4$$

The  $Y_{BUS}^1$  matrix is given as follows:

$$Y_{BUS}^0 = \begin{bmatrix} -j14 & +j5 & +j0 & +j0 & +j4 & +j0 \\ +j5 & -j11.67 & +j6.67 & +j0 & +j0 & +j0 \\ +j0 & +j6.67 & -j10 & +j3.33 & +j0 & +j0 \\ +j0 & +j0 & +j3.33 & -j11.19 & +j0 & +j2.86 \\ +j4 & +j0 & +j0 & +j0 & -j8.54 & +j4.54 \\ +j0 & +j0 & +j0 & +j2.86 & +j4.54 & -j7.4 \end{bmatrix}$$

Noting that  $Z_{BUS}^1 = (Y_{BUS}^1)^{-1}$ , we can find the  $Z_{BUS}^1$  by inverting the  $Y_{BUS}^1$  matrix. This operation was performed in MATLAB, which yielded the following output:

z1 =

```
0.0000 + 0.1476i    0.0000 + 0.1183i    0.0000 + 0.0964i    0.0000 + 0.0524i    0.0000 + 0.1186i    0.0000 + 0.0930i
0.0000 + 0.1183i    0.0000 + 0.2455i    0.0000 + 0.1910i    0.0000 + 0.0817i    0.0000 + 0.1071i    0.0000 + 0.0973i
0.0000 + 0.0964i    0.0000 + 0.1910i    0.0000 + 0.2619i    0.0000 + 0.1036i    0.0000 + 0.0986i    0.0000 + 0.1005i
0.0000 + 0.0524i    0.0000 + 0.0817i    0.0000 + 0.1036i    0.0000 + 0.1476i    0.0000 + 0.0814i    0.0000 + 0.1070i
0.0000 + 0.1186i    0.0000 + 0.1071i    0.0000 + 0.0986i    0.0000 + 0.0814i    0.0000 + 0.2810i    0.0000 + 0.2039i
0.0000 + 0.0930i    0.0000 + 0.0973i    0.0000 + 0.1005i    0.0000 + 0.1070i    0.0000 + 0.2039i    0.0000 + 0.3016i
```

This allows us to easily extract the Thevenin impedance, noting that  $Z_{th}^1 = (Z_{BUS}^1)_{3,3}$ . Hence, we note that the zero sequence impedance is:

$$(Z_{BUS}^1)_{3,3} = j0.2619$$

The Thevenin equivalent for the Positive sequence circuit looking in from bus 3, is shown in Figure 4 below.

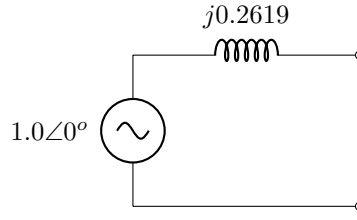


Figure 8: text

## Negative Sequence

We first determine all of the admittances for each of the links between the buses for all  $i, j$  combinations. The results can be seen in Table 2.

| Figure 9: text    |                           | Figure 10: text   |                           |
|-------------------|---------------------------|-------------------|---------------------------|
| Admittance        | $Y_{ij}^{BUS}$            | Admittance        | $Y_{ij}^{BUS}$            |
| $y_{12} = -j5$    | $Y_{12} = Y_{21} = j5$    | $y_{26} = j0$     | $Y_{26} = Y_{62} = j0$    |
| $y_{13} = j0$     | $Y_{13} = Y_{31} = j0$    | $y_{34} = -j3.33$ | $Y_{34} = Y_{43} = j3.33$ |
| $y_{14} = j0$     | $Y_{14} = Y_{41} = j0$    | $y_{35} = j0$     | $Y_{35} = Y_{53} = j0$    |
| $y_{15} = -j4$    | $Y_{15} = Y_{51} = j4$    | $y_{36} = j0$     | $Y_{36} = Y_{63} = j0$    |
| $y_{16} = j0$     | $Y_{16} = Y_{61} = j0$    | $y_{45} = j0$     | $Y_{45} = Y_{54} = j0$    |
| $y_{23} = -j6.67$ | $Y_{23} = Y_{32} = j6.67$ | $y_{46} = -j2.86$ | $Y_{46} = Y_{64} = j2.86$ |
| $y_{24} = j0$     | $Y_{24} = Y_{42} = j0$    | $y_{56} = -j4.54$ | $Y_{56} = Y_{65} = j4.54$ |
| $y_{25} = j0$     | $Y_{25} = Y_{52} = j0$    |                   |                           |

Now, we need to find the  $Y_{ii}$  elements, which are calculated as follows:

$$Y_{11} = y_{10} + y_{12} + y_{15} = -j16.14$$

$$Y_{22} = y_{12} + y_{23} = -j11.67$$

$$Y_{33} = y_{23} = -j10$$

$$Y_{44} = y_{04} = -j13.33$$

$$Y_{55} = y_{15} + y_{56} = -j8.54$$

$$Y_{66} = y_{56} = -j7.41$$

The  $Y_{BUS}^2$  matrix is given as follows:

$$Y_{BUS}^2 = \begin{bmatrix} -j16.14 & +j5 & +j0 & +j0 & +j4 & +j0 \\ +j5 & -j11.67 & +j6.67 & +j0 & +j0 & +j0 \\ +j0 & +j6.67 & -j10 & +j3.33 & +j0 & +j0 \\ +j0 & +j0 & +j3.33 & -j13.33 & +j0 & +j2.86 \\ +j4 & +j0 & +j0 & +j0 & -j8.54 & +j4.54 \\ +j0 & +j0 & +j0 & +j2.86 & +j4.54 & -j7.41 \end{bmatrix}$$

Noting that  $Z_{BUS}^2 = (Y_{BUS}^2)^{-1}$ , we can find the  $Z_{BUS}^2$  by inverting the  $Y_{BUS}^2$  matrix. This operation was performed in MATLAB, which yielded the following output:

$Z_2 =$

|                    |                    |                    |                    |                    |                    |
|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|
| $0.0000 + 0.1095i$ | $0.0000 + 0.0852i$ | $0.0000 + 0.0670i$ | $0.0000 + 0.0305i$ | $0.0000 + 0.0854i$ | $0.0000 + 0.0642i$ |
| $0.0000 + 0.0852i$ | $0.0000 + 0.2144i$ | $0.0000 + 0.1612i$ | $0.0000 + 0.0548i$ | $0.0000 + 0.0760i$ | $0.0000 + 0.0678i$ |
| $0.0000 + 0.0670i$ | $0.0000 + 0.1612i$ | $0.0000 + 0.2319i$ | $0.0000 + 0.0730i$ | $0.0000 + 0.0689i$ | $0.0000 + 0.0705i$ |
| $0.0000 + 0.0305i$ | $0.0000 + 0.0548i$ | $0.0000 + 0.0730i$ | $0.0000 + 0.1095i$ | $0.0000 + 0.0546i$ | $0.0000 + 0.0758i$ |
| $0.0000 + 0.0854i$ | $0.0000 + 0.0760i$ | $0.0000 + 0.0689i$ | $0.0000 + 0.0546i$ | $0.0000 + 0.2498i$ | $0.0000 + 0.1744i$ |
| $0.0000 + 0.0642i$ | $0.0000 + 0.0678i$ | $0.0000 + 0.0705i$ | $0.0000 + 0.0758i$ | $0.0000 + 0.1744i$ | $0.0000 + 0.2713i$ |

This allows us to easily extract the Thevenin impedance, noting that  $Z_{th}^2 = (Z_{BUS}^2)_{3,3}$ . Hence, we note that the negative sequence impedance is:

$$(Z_{BUS}^2)_{3,3} = j0.2319$$

The Thevenin equivalent for the Negative sequence circuit looking in from bus 3, is shown in Figure 4 below.

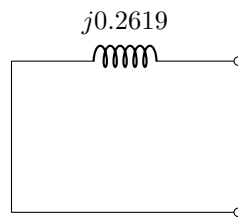


Figure 11: text