Power Systems Analysis: Assignment 3

Due on June 3, 2017 at 3:00pm

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Consider the power system shown in Figure 1. Each three phase transformer is made up of three single phase transformers. The squares B_1, B_2, B_3,...,B_6 are circuit breakers which can be considered to have very low series impedance while closed and infinite impedance when open.

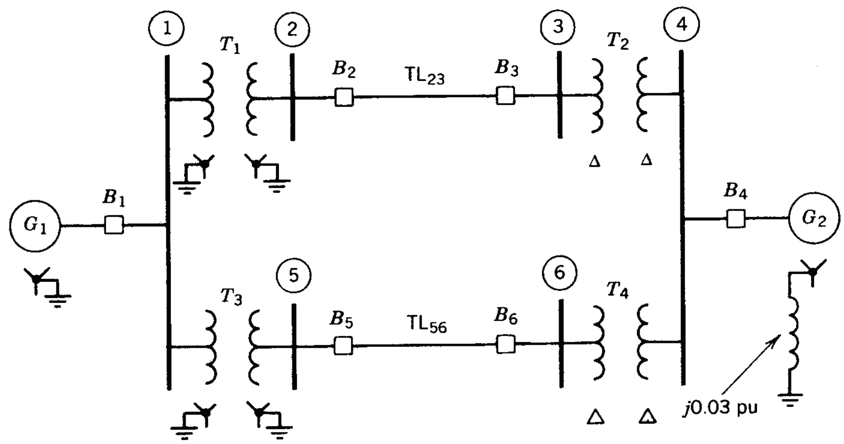


Figure 1: text

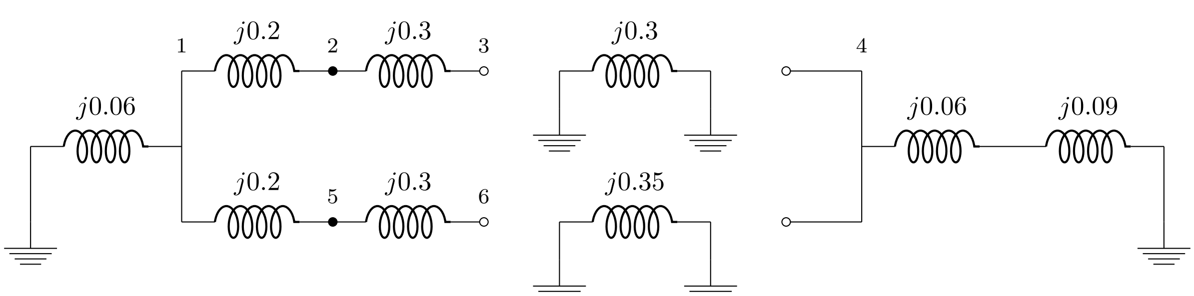
Figure 2: text

Component	**Power Rating** (MVA)	**Voltage Rating** (kV)	X_1 (pu)	X_2 (pu)	X_0 (pu)
G_1	200	20	0.2	0.14	0.06
G_2	200	13.2	0.2	0.14	0.06
T_1	200	20/230	0.2	0.2	0.2
T_2	200	13.2/230	0.3	0.3	0.3
T_3	200	20/230	0.25	0.25	0.25
T_4	200	13.2/230	0.35	0.35	0.35
TL_{23}	200	230	0.15	0.15	0.3
TL_{56}	200	230	0.22	0.22	0.5

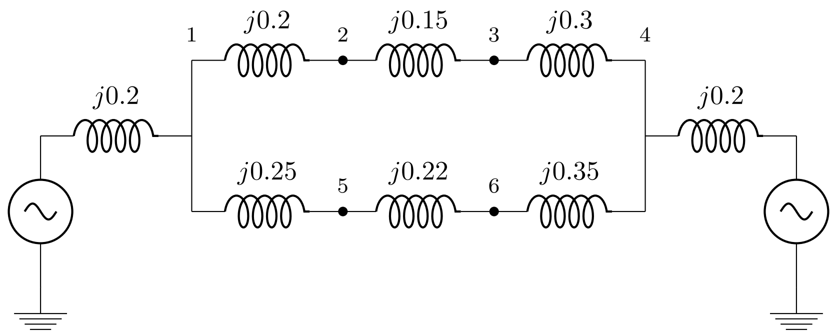
Assignment 3.1 - PART A

The following section details the three sequence networks, and the impedance vlued

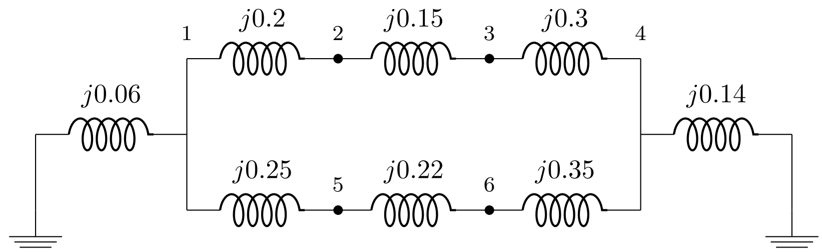
Zero Sequence Network



Positive Sequence Network



Negative Sequence Network



Assignment 3.1 - PART B

Assuming that there is a fault on bus 3, to analyse the response to this, the Thevenin equivalent circuits for the zero, positive, and negative sequences need to be found looking into bus 3. To find the Thevenin impedance, the Y_{BUS} matrix was first found for each sequence, and inverted to determine Z^0_{th}, Z^1_{th}, and Z^2_{th}.

Zero Sequence

We first determine all of the admittances for each of the links between the buses for all i, j combinations. The results can be seen in Table 2.

Figure 3: text

Admittance	Y_{ij}^{BUS}
$y_{12} = -j5$	$Y_{12} = Y_{21} = j5$
$y_{13} = j0$	$Y_{13} = Y_{31} = j0$
$y_{14} = j0$	$Y_{14} = Y_{41} = j0$
$y_{15} = -j4$	$Y_{15} = Y_{51} = j4$
$y_{16} = j0$	$Y_{16} = Y_{61} = j0$
$y_{23} = -j3.33$	$Y_{23} = Y_{32} = j3.33$
$y_{24} = j0$	$Y_{24} = Y_{42} = j0$
$y_{25} = j0$	$Y_{25} = Y_{52} = j0$

Figure 4: text

Admittance	Y_{ij}^{BUS}
$y_{26} = j0$	$Y_{26} = Y_{62} = j0$
$y_{34} = j0$	$Y_{34} = Y_{43} = j0$
$y_{35} = j0$	$Y_{35} = Y_{53} = j0$
$y_{36} = j0$	$Y_{36} = Y_{63} = j0$
$y_{45} = j0$	$Y_{45} = Y_{54} = j0$
$y_{46} = j0$	$Y_{46} = Y_{64} = j0$
$y_{56} = -j2$	$Y_{56} = Y_{65} = j2$

Now, we need to find the Y_{ii} elements, which are calculated as follows:

$$\begin{aligned}
Y_{11} &= y_{10} + y_{12} + y_{15} = -j25.66 \\
Y_{22} &= y_{12} + y_{23} = -j8.33 \\
Y_{33} &= y_{23} = -j3.33 \\
Y_{44} &= y_{04} = -j6.67 \\
Y_{55} &= y_{15} + y_{56} = -j6 \\
Y_{66} &= y_{56} = -j2
\end{aligned}$$

The Y_{BUS}^0 matrix is given as follows:

$$Y_{BUS}^0 = \begin{bmatrix} -j25.66 & +j5 & +j0 & +j0 & +j4 & +j0 \\ +j5 & -j8.33 & +j3.33 & +j0 & +j0 & +j0 \\ +j0 & +j3.33 & -j3.33 & +j0 & +j0 & +j0 \\ +j0 & +j0 & +j0 & -j6.67 & +j0 & +j0 \\ +j4 & +j0 & +j0 & +j0 & -j6 & +j2 \\ +j0 & +j0 & +j0 & +j0 & +j2 & -j2 \end{bmatrix}$$

Noting that $Z_{BUS}^0 = (Y_{BUS}^0)^{-1}$, we can find the Z_{BUS}^0 by inverting the Y_{BUS}^0 matrix. This operation was performed in MATLAB, which yielded the following output:

```
Z0 =

0.0000 + 0.0600i   0.0000 + 0.0600i   0.0000 + 0.0600i   0.0000 + 0.0000i   0.0000 + 0.0600i   0.0000 + 0.0600i
0.0000 + 0.0600i   0.0000 + 0.2600i   0.0000 + 0.2600i   0.0000 + 0.0000i   0.0000 + 0.0600i   0.0000 + 0.0600i
0.0000 + 0.0600i   0.0000 + 0.2600i   0.0000 + 0.5603i   0.0000 + 0.0000i   0.0000 + 0.0600i   0.0000 + 0.0600i
0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.1499i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0600i   0.0000 + 0.0600i   0.0000 + 0.0600i   0.0000 + 0.0000i   0.0000 + 0.3100i   0.0000 + 0.3100i
0.0000 + 0.0600i   0.0000 + 0.0600i   0.0000 + 0.0600i   0.0000 + 0.0000i   0.0000 + 0.3100i   0.0000 + 0.8100i
```

This allows us to easily extract the Thevenin impedance, noting that $Z_{th}^0 = (Z_{BUS}^0)_{3,3}$. Hence, we note that the zero sequence impedance is:

$$(Z_{BUS}^0)_{3,3} = j0.5603$$

The Thevenin equivalent for the Zero sequence circuit looking in from bus 3, is shown in Figure 4 below.

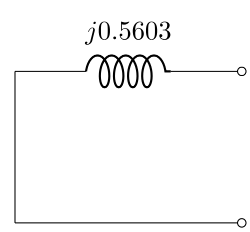


Figure 5: text

Positive Sequence

We first determine all of the admittances for each of the links between the buses for all i, j combinations. The results can be seen in Table 2.

Figure 6: text

Admittance	Y_{ij}^{BUS}
$y_{12} = -j5$	$Y_{12} = Y_{21} = j5$
$y_{13} = j0$	$Y_{13} = Y_{31} = j0$
$y_{14} = j0$	$Y_{14} = Y_{41} = j0$
$y_{15} = -j4$	$Y_{15} = Y_{51} = j4$
$y_{16} = j0$	$Y_{16} = Y_{61} = j0$
$y_{23} = -j6.67$	$Y_{23} = Y_{32} = j6.67$
$y_{24} = j0$	$Y_{24} = Y_{42} = j0$
$y_{25} = j0$	$Y_{25} = Y_{52} = j0$

Figure 7: text

Admittance	Y_{ij}^{BUS}
$y_{26} = j0$	$Y_{26} = Y_{62} = j0$
$y_{34} = -j3.33$	$Y_{34} = Y_{43} = j3.33$
$y_{35} = j0$	$Y_{35} = Y_{53} = j0$
$y_{36} = j0$	$Y_{36} = Y_{63} = j0$
$y_{45} = j0$	$Y_{45} = Y_{54} = j0$
$y_{46} = -j2.86$	$Y_{46} = Y_{64} = j2.86$
$y_{56} = -j4.54$	$Y_{56} = Y_{65} = j4.54$

Now, we need to find the Y_{ii} elements, which are calculated as follows:

$$\begin{aligned}
Y_{11} &= y_{10} + y_{12} + y_{15} = -j14 \\
Y_{22} &= y_{12} + y_{23} = -j11.67 \\
Y_{33} &= y_{23} = -j10 \\
Y_{44} &= y_{04} = -j11.19 \\
Y_{55} &= y_{15} + y_{56} = -j8.54 \\
Y_{66} &= y_{56} = -j7.4
\end{aligned}$$

The Y_{BUS}^{1} matrix is given as follows:

$$Y_{BUS}^{0} = \begin{bmatrix} -j14 & +j5 & +j0 & +j0 & +j4 & +j0 \\ +j5 & -j11.67 & +j6.67 & +j0 & +j0 & +j0 \\ +j0 & +j6.67 & -j10 & +j3.33 & +j0 & +j0 \\ +j0 & +j0 & +j3.33 & -j11.19 & +j0 & +j2.86 \\ +j4 & +j0 & +j0 & +j0 & -j8.54 & +j4.54 \\ +j0 & +j0 & +j0 & +j2.86 & +j4.54 & -j7.4 \end{bmatrix}$$

Noting that $Z_{BUS}^{1} = (Y_{BUS}^{1})^{-1}$, we can find the Z_{BUS}^{1} by inverting the Y_{BUS}^{1} matrix. This operation was performed in MATLAB, which yielded the following output:

```
Z1 =

0.0000 + 0.1476i   0.0000 + 0.1183i   0.0000 + 0.0964i   0.0000 + 0.0524i   0.0000 + 0.1186i   0.0000 + 0.0930i
0.0000 + 0.1183i   0.0000 + 0.2455i   0.0000 + 0.1910i   0.0000 + 0.0817i   0.0000 + 0.1071i   0.0000 + 0.0973i
0.0000 + 0.0964i   0.0000 + 0.1910i   0.0000 + 0.2619i   0.0000 + 0.1036i   0.0000 + 0.0986i   0.0000 + 0.1005i
0.0000 + 0.0524i   0.0000 + 0.0817i   0.0000 + 0.1036i   0.0000 + 0.1476i   0.0000 + 0.0814i   0.0000 + 0.1070i
0.0000 + 0.1186i   0.0000 + 0.1071i   0.0000 + 0.0986i   0.0000 + 0.0814i   0.0000 + 0.2810i   0.0000 + 0.2039i
0.0000 + 0.0930i   0.0000 + 0.0973i   0.0000 + 0.1005i   0.0000 + 0.1070i   0.0000 + 0.2039i   0.0000 + 0.3016i
```

This allows us to easily extract the Thevenin impedance, noting that $Z_{th}^{1} = (Z_{BUS}^{1})_{3,3}$. Hence, we note that the zero sequence impedance is:

$$(Z_{BUS}^{1})_{3,3} = j0.2619$$

The Thevenin equivalent for the Positive sequence circuit looking in from bus 3, is shown in Figure 4 below.

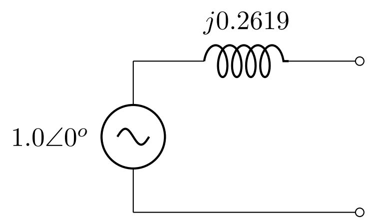


Figure 8: text

Negative Sequence

We first determine all of the admittances for each of the links between the buses for all i, j combinations. The results can be seen in Table 2.

Figure 9: text

Admittance	Y_{ij}^{BUS}
$y_{12} = -j5$	$Y_{12} = Y_{21} = j5$
$y_{13} = j0$	$Y_{13} = Y_{31} = j0$
$y_{14} = j0$	$Y_{14} = Y_{41} = j0$
$y_{15} = -j4$	$Y_{15} = Y_{51} = j4$
$y_{16} = j0$	$Y_{16} = Y_{61} = j0$
$y_{23} = -j6.67$	$Y_{23} = Y_{32} = j6.67$
$y_{24} = j0$	$Y_{24} = Y_{42} = j0$
$y_{25} = j0$	$Y_{25} = Y_{52} = j0$

Figure 10: text

Admittance	Y_{ij}^{BUS}
$y_{26} = j0$	$Y_{26} = Y_{62} = j0$
$y_{34} = -j3.33$	$Y_{34} = Y_{43} = j3.33$
$y_{35} = j0$	$Y_{35} = Y_{53} = j0$
$y_{36} = j0$	$Y_{36} = Y_{63} = j0$
$y_{45} = j0$	$Y_{45} = Y_{54} = j0$
$y_{46} = -j2.86$	$Y_{46} = Y_{64} = j2.86$
$y_{56} = -j4.54$	$Y_{56} = Y_{65} = j4.54$

Now, we need to find the Y_{ii} elements, which are calculated as follows:

$$\begin{aligned}
Y_{11} &= y_{10} + y_{12} + y_{15} = -j16.14 \\
Y_{22} &= y_{12} + y_{23} = -j11.67 \\
Y_{33} &= y_{23} = -j10 \\
Y_{44} &= y_{04} = -j13.33 \\
Y_{55} &= y_{15} + y_{56} = -j8.54 \\
Y_{66} &= y_{56} = -j7.41
\end{aligned}$$

The Y_{BUS}^2 matrix is given as follows:

$$Y_{BUS}^2 = \begin{bmatrix} -j16.14 & +j5 & +j0 & +j0 & +j4 & +j0 \\ +j5 & -j11.67 & +j6.67 & +j0 & +j0 & +j0 \\ +j0 & +j6.67 & -j10 & +j3.33 & +j0 & +j0 \\ +j0 & +j0 & +j3.33 & -j13.33 & +j0 & +j2.86 \\ +j4 & +j0 & +j0 & +j0 & -j8.54 & +j4.54 \\ +j0 & +j0 & +j0 & +j2.86 & +j4.54 & -j7.4 \end{bmatrix}$$

Noting that $Z_{BUS}^2 = (Y_{BUS}^2)^{-1}$, we can find the Z_{BUS}^2 by inverting the Y_{BUS}^2 matrix. This operation was performed in MATLAB, which yielded the following output:

```
Z2 =

0.0000 + 0.1095i   0.0000 + 0.0852i   0.0000 + 0.0670i   0.0000 + 0.0305i   0.0000 + 0.0854i   0.0000 + 0.0642i
0.0000 + 0.0852i   0.0000 + 0.2144i   0.0000 + 0.1612i   0.0000 + 0.0548i   0.0000 + 0.0760i   0.0000 + 0.0678i
0.0000 + 0.0670i   0.0000 + 0.1612i   0.0000 + 0.2319i   0.0000 + 0.0730i   0.0000 + 0.0689i   0.0000 + 0.0705i
0.0000 + 0.0305i   0.0000 + 0.0548i   0.0000 + 0.0730i   0.0000 + 0.1095i   0.0000 + 0.0546i   0.0000 + 0.0758i
0.0000 + 0.0854i   0.0000 + 0.0760i   0.0000 + 0.0689i   0.0000 + 0.0546i   0.0000 + 0.2498i   0.0000 + 0.1744i
0.0000 + 0.0642i   0.0000 + 0.0678i   0.0000 + 0.0705i   0.0000 + 0.0758i   0.0000 + 0.1744i   0.0000 + 0.2713i
```

This allows us to easily extract the Thevenin impedance, noting that $Z^2_{th} = (Z^2_{BUS})_{3,3}$. Hence, we note that the negative sequence impedance is:

$$(Z^2_{BUS})_{3,3} = j0.2319$$

The Thevenin equivalent for the Negative sequence circuit looking in from bus 3, is shown in Figure 4 below.

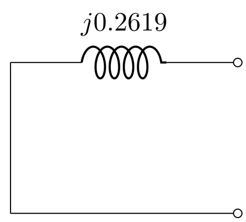


Figure 11: text

Assignment 3.2 - PART A

Assuming a power system which is similar, but not exactly the same as the one shown in Figure 1 we consider the impact of line to ground fault in the network at bus three, with a fault resistance, R_f, of 5Ω. Given that the Thevenin impedances of the zero, positive, and negative sequence networks looking in from bus 3 are $j0.56$, $j0.2618$, and $j0.3619$, and the per unit values are based on 200MV A and 230kV bases, we can determine the fault current and the line to line voltages given the fault event. The fault resistance first needs to be converted to a per unit basis. We note that:

$$Z_{base} = \frac{V^2_{base}}{S_{base}} = \frac{(230 \times 10^3)^2}{200 \times 10^6} = 264.50\text{pu}$$

Hence, we find that:

$$(Z_f)_{base} = \frac{5}{264} = 0.0189\text{pu}$$

In the line to ground fault instance, we note that the sequence currents are identical and given by the following equation:

$$\begin{aligned} I^0 = I^1 = I^2 &= \frac{V_f}{Z^0_{th} + Z^1_{th} + Z^2_{th} + 3Z_f} \\ &= \frac{1.0\angle 0^o}{j0.56 + j0.2618 + j0.3619 + 3 * 0.0189} \\ &= \frac{1.0\angle 0^o}{0.0567\angle 4.51^o} \\ &= 0.0664\angle - 4.51^o\text{pu} \end{aligned}$$

Hence, we find the fault current for the a, b, and c phase is given by the relationship $I_f^{abc} = A \cdot I_f^{012}$. Noting that the matrix A is defined as:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

The term a, seen in the A matrix, is defined as:

$$a = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$$

Hence, we get that:

$$\begin{bmatrix} I_f^a \\ I_f^b \\ I_f^c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \cdot \begin{bmatrix} 0.0664\angle - 4.51^o \\ 0.0664\angle - 4.51^o \\ 0.0664\angle - 4.51^o \end{bmatrix}$$

Hence, we find that:

$$\begin{bmatrix} I_f^a \\ I_f^b \\ I_f^c \end{bmatrix} = \begin{bmatrix} 0.2\angle - 4.5^o \\ 0 \\ 0 \end{bmatrix} \text{ pu}$$

Assignment 3.2 - PART B

Now, to find the line to line voltage, we first need to find the voltages for each of the sequences:

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ V_f \\ 0 \end{bmatrix} - \begin{bmatrix} Z_{th}^0 & 0 & 0 \\ 0 & Z_{th}^1 & 0 \\ 0 & 0 & Z_{th}^2 \end{bmatrix} \cdot \begin{bmatrix} I^0 \\ I^1 \\ I^2 \end{bmatrix}$$

Hence, we see that:

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1.0\angle 0^o \\ 0 \end{bmatrix} - \begin{bmatrix} j0.56 & 0 & 0 \\ 0 & j0.2618 & 0 \\ 0 & 0 & j0.3619 \end{bmatrix} \cdot \begin{bmatrix} 0.066\angle - 4.5^o \\ 0.066\angle - 4.5^o \\ 0.066\angle - 4.5^o \end{bmatrix}$$

Evaluating the matrix expression above, we find that:

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} -0.03696\angle 85.5^o \\ 0.9987\angle - 0.99^o \\ -0.0238\angle 85.5^o \end{bmatrix}$$

Similarly to the phase currents, we determine the phase voltages using the fact that $V_f^{abc} = A \cdot V_f^{012}$. Hence, the equation is as follows:

$$\begin{bmatrix} V_f^a \\ V_f^b \\ V_f^c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \cdot \begin{bmatrix} -0.03696\angle 85.5^o \\ 0.9987\angle - 0.99^o \\ -0.0238\angle 85.5^o \end{bmatrix}$$

Hence, we find that:

$$\begin{bmatrix} V_f^a \\ V_f^b \\ V_f^c \end{bmatrix} = \begin{bmatrix} 0.9929\angle - 4.44^o \\ 1\angle - 119.05^o \\ 0.9868\angle 120^o \end{bmatrix} \text{ pu}$$

Assignment 3.3 - PART A

Assignment 3.3 - PART B