MAT320 Quiz #7

10/31/2023

Please answer the following questions, and write your name on top of the quiz.

Question 1. Let $f : E \to \mathbb{R}$ be a measurable function such that $|f(x)| < M$ for all $x \in E$ and for some $M \in \mathbb{R}$ independent of x, and such that $\mu(E) < \infty$.

Write the definition of Lebesgue integrability of f. (Added: This was a little bit of a trick question: every such function is Lebesgue integrable. Make sure you know how the Lebesgue integral of such a function is defined!)

Question 2. Write the definition of Lebesgue integrablity of a general measurable function $f : \mathbb{R} \to \mathbb{R}$.

Question 3. For $\alpha \in \mathbb{R}$ consider the function $f(x) : [0,1] \to \mathbb{R}$ defined by $f(x) = x^\alpha$ if $x \neq 0$, and otherwise $f(0) = 0$.

a) For what values of α is the range of f bounded?

b) For what values of α is f Lebesgue integrable over $[0,1]$? For partial credit, give some cases where you know the answer.

c) Let $f(x) = 1/x$ for $x \neq 0$ and $f(0) = 0$. Is f Lebesgue integrable over $[-1,1]$?