MAT320 Practice Problems

10/5/2023

Problem. For every $\epsilon > 0$, given an example of an open set $U_\epsilon \subset \mathbb{R}$ such that the Lebesgue measure of U_ϵ is less than ϵ, and such that U_ϵ is dense in $\mathbb{R}$. Please justify your answer.

Problem. Review the Cantor set and the related material.

Problem. Show that the intersection of any collection of σ-algebras on X is a σ-algebra. Do the extra credit problem on problem set 4!

Problem. An F_δ set is a set that is a countable union of closed sets. Show that continuous functions $f : \mathbb{R} \to \mathbb{R}$ map F_δ sets to F_δ sets.

(Hint: show first that the image of a compact set under a continuous function is compact.)

Problem. Suppose that $\{x_n\}_{n=1}^\infty$ and $\{y_n\}_{n=1}^\infty$ are Cauchy sequences in a metric space (X, d). Show that the sequence of real numbers $d(x_n, y_n)$ converges as $n \to \infty$. (Note: the metric space X is not assumed to be complete, and one should not use the existence of the completion of the metric space in this argument.)

Problem. Problem 2.4.19.

Problem. Give examples of subsets of $\mathbb{R}^2$ that are

- Countable, compact,

- Uncountable, compact,

- Uncountable, compact, have countably many pairwise distinct subsets which are simulatenously closed and open (with respect to the subspace metric).