MAT320 Quiz #9

11/14/2023

Please answer the following questions, and write your name on top of the quiz.

Question 1. Write

$$f_k(x) = a_k \sin(kx).$$

How can you choose $a_k \in \mathbb{R}$, $a_k \neq 0$ for all k, such that the sequence of C^1 norms $c_k = \|f_k\|_{C^1}$ is bounded by a constant independent of k? Show that for any such choice, $\lim_{k\to\infty} a_k = 0$.

Question 2. Let

$$V = \{f : [0,1] \to \mathbb{R} \,|\, f \text{ measurable },\ \text{exists } M \in \mathbb{R} \text{ such that } |f| < M.\}.$$

Consider the function

$$\ell : V \to \mathbb{R}, \ell(f) = \int_0^1 |f|.$$

Is ℓ a norm? If it is a norm, is V a Banach space with respect to this norm?

Question 3. The Cauchy-Schwartz inequality for vectors in $\mathbb{R}^n$ states that given a pair of vectors $\mathbf{x} = (x_i)_{i=1}^n$, $\mathbf{y} = (y_i)_{i=1}^n$ in $\mathbb{R}^n$, we have

$$\left| \sum_{i=1}^{n} x_i y_i \right| \leq \|\mathbf{x}\| \|\mathbf{y}\|$$

where

$$\|\mathbf{x}\|^2 = \sum_{i=1}^{n} x_i^2$$

is the square of the Euclidean distance.

Let

$$f(x) = \frac{(x+k)^2}{x^2+1}.$$

Show that $f(x) \leq k^2 + 1$ for all x.