

MAT320 Problem Set 6

Due Nov 7, 2024

Please write your homework on paper neatly or type it up in LaTeX, and hand it in at the beginning of class next Thursday.

Royden $X.Y.Z$ refers Problem Z in Royden-Fitzpatrick, found in the collection of problems at the end of section $X.Y$.

Problem 1.

1. Show that there is some $\alpha > 0$ such that $f(x) = x^{-\alpha} \sin(1/x)$ is integrable over $(0, 1)$.
2. Compute, for this the value of α above,

$$\lim_{n \rightarrow \infty} \int_0^\infty x^{-\alpha} \frac{n \sin(x/n)}{x}.$$

(Hint: use some ideas from Calc I to compute the limit under the integral. Can you justify subsequently the interchange of limits?)

Problem 2. Royden 4.5.38. Doing one of the two parts is sufficient for full credit for this problem; the second part is sufficient for half of a point of extra credit.

Problem 3.

- (a) Royden 4.4.34.
- (b) (1/2 point of extra credit.) Please prove that the same statement holds whenever f is assumed to be measurable and integrable, but now no longer assumed to be nonnegative.