

# MAT320 Problem Set 1

Due Sept 14, 2023

Please write your homework on paper neatly or type it up in LaTeX, and hand it in at the beginning of class next Thursday.

Royden  $X.Y.Z$  refers Problem  $Z$  in Royden-Fitzpatrick, found in the collection of problems at the end of section  $X.Y$ .

**Problem 1.** Show that a composition of surjective maps is surjective, a composition of injective maps is injective, and a composition of bijective maps is bijective. Give an example of maps  $f$  and  $g$  such that

- $f$  is surjective,  $g$  is injective, and  $f \circ g$  is neither,
- $f$  is injective,  $g$  is surjective, and  $f \circ g$  is neither.

**Problem 2.** Let  $S$  and  $S'$  be subsets of the real number which are both bounded above. Show that if  $S \subset S'$  then  $\sup(S) \leq \sup(S')$ .

**Problem 3.** Royden 1.1.3.

**Problem 4.** From the axioms of the real numbers, show that between every pair of real numbers there is a rational number, and also between every pair of real numbers there is an irrational number. (Feel free to use the fact that there *is* an irrational number, like  $\sqrt{2}$ ). Hint: use the Archimedean property of the reals!

**Problem 5.** Royden 1.2.13. Feel free to use any theorem through section 1.3 of Royden.

(Hint: Problems 2 and 4 may be helpful!)

**Problem 6.** Royden 1.3.22.

**Extra credit.** Show that if  $S$  is countable and there is a surjective map from  $S$  to  $T$  then  $T$  is countable. Use this to show that the rational numbers are countable.