

MAT320 Practice Problem

10/3/2023

Please answer the following questions, and write your name on top of the quiz.

Problem. Define the outer measure for subsets in \mathbb{R}^n to be ...

Prove, from the definition, that the outer measure of a ball has the right scaling.

Problem.

Find the measure of the size of all numbers in $[0, 1]$ which do not have a 4 in their decimal expansion.

Problem. Give an example of an uncountable set X and a function $d : X \times X \rightarrow \mathbb{R}$ such that (X, d) are a metric space and such that every subset of X is closed. Describe all compact subsets of your example.

Problem. Prove that if $B = \bigcup_{i=1}^{\infty} A_i$ then $\overline{B} \supset \bigcup_{i=1}^{\infty} \overline{A_i}$. Show that the latter inclusion does not have to be an equality.

Problem. Suppose that x_n and y_n are Cauchy sequences. Show that the sequence of real numbers $d(x_n, y_n)$ converges. (Note: the metric space is not assumed to be complete, and one should not use the existence of the completion of the metric space in this argument.)

Problem. Show that the set of points at which a function $f : \mathbb{R} \rightarrow \mathbb{R}$ is continuous is a Borel set.

Problem. Let K be compact and let C be closed sets. Show that $K + C$ is closed. (Walk through.)

Problem. Continuous bijections with compact domains are closed.

MORE ON MEASURE THEORY!

Problem. Show that continuous functions map F_δ sets to F_δ sets.

Problem. Problem 2.4.19.

Problem. Show that a function $f : \mathbb{R} \rightarrow \mathbb{R}$ that is continuous except at a finite number of points is measurable.

Problem. Problems from website

Problem. Give an example of an open set of arbitrarily small measure that is dense in \mathbb{R} .

Problem. Prove (without consulting the textbook) that an increasing function $f(x) \leq f(y)$ if $x < y$ is measurable.