MAT320 Quiz #5

10/3/2023

Please answer the following questions, and write your name on top of the quiz.

Question 1. Write the definition of a measurable set. Explain why, using the definition, if E_1 and E_2 are measurable, then

$$\mu^*(E_1 \cup E_2) + \mu^*(E_1 \cap E_2) = \mu^*(E_1) + \mu^*(E_2).$$

Question 2.

- a) Write down the axioms of a σ -algebra Σ on a set S.
- b) Give an example of a pair of σ -algebras Σ_1, Σ_2 on some set S such that $\Sigma_1 \cup \Sigma_2$ is not a σ -algebra on S. (It may help to play around with very simple examples for some finite sets S.)

Question 3. Write down any one of the equivalent forms of the axiom of choice.