MAT320 Problem Set 6

Due Nov 9, 2023

Please write your homework on paper neatly or type it up in LaTeX, and hand it in at the beginning of class next Thursday. For us, *integrable* always means *Lebesgue integrable* unless otherwise specified.

Royden $X.Y.Z$ refers Problem Z in Royden-Fitzpatrick, found in the collection of problems at the end of section $X.Y$.

Problem 1. Royden 4.4.30.

Problem 2. Royden 4.4.36. This justifies, to some extent, why the theorems that we have proven in our discussion on measure theory are useful for calculations.

Problem 3. Royden 4.3.21. Hint: one of the theorems in earlier measure theory section might be helpful for the second part of the problem.

Problem 4. Let

$$C^1([0,1]) = \{f : [0,1] \to \mathbb{R} \,:\, f \text{ is differentiable },f' \text{ is continuous }\}.$$

be the normed vector space equipped with the norm

$$\|f\|_{C^1} = \|f\|_{C^0} + \|f'\|_{C^0},$$

where

$$\|f\|_{C^0} = \sup_{x\in[0,1]} |f(x)|.$$

We call $\|f\|_{C^1}$ the C^1 *norm* of f. Show, by a similar argument to the one presented in class, that $C^1([0,1])$ is a Banach space. Hint: one of the theorems about uniform convergence and integration might be helpful.

Problem 5. Royden 13.1.6.

Extra credit.

Let $f : \mathbb{R} \to \mathbb{R}$ be an integrable function. Prove that

$$\lim_{n\to\infty} \int_a^b f\ \sin(nx)\, dx = 0.$$

(You should make sure to prove that $f \sin(nx)$ is integrable! We suggest that you first prove this for step functions and then extend the result of Problem 3 to all integrable functions in order to finish off the problem.)