

Design and analysis of algorithms

Exam

4.11.2016

Instructions:

Answer using *Finnish* or *English* language

1. There are 10 algorithms and 10 claims. Connect each algorithm with one claim that is true for the algorithm. Do it by drawing a line between them. Points are given +1 for a correct and -1 for an incorrect link. Leave those unlinked that you do not know answer (don't guess).

Algorithm:

Brute force
Christofides
Dijkstra
Divide-and-Conquer MST
Insertion sort
Karatsuba-Ofman
Kruskal
Las Vegas
Quicksort
Satisfiability

Claim:

Dynamic programming
Works in $O(N^2)$ in the worst case
Greedy algorithm
Solves both MST and TSP
Works in $O(N^{1.58})$ time
Works in $O(N^{1.5})$ time
Does not always find solution
It is not an algorithm
Works in $O(N)$ in the best case
Works in $O(2^N)$ time

2. Company X published a new sorting algorithm that works as follows. It first calculates $k = \text{SQRT}(N)$, where N is the number of elements to be sorted. It then proceeds by sorting the first bunch containing elements 1 to k . Then it sorts the next bunch from $k+1$ to $2k$, and so on, until all elements have been sorted. Now solve the following tasks:

- Which type of algorithm is it?
- Is this algorithm correct? If yes, give proof. If not, fix it.
- Give pseudo code of the algorithm.
- Analyze its time complexity.
- Give five other existing sorting algorithms deal with during the course.
- List them from fastest to slowest in terms of time complexities.
- Compare the new sorting algorithms to the existing ones.



3. Are the following claims TRUE or FALSE? Answer only if you know. Don't guess. Correct answer gives +1 point and incorrect -1 point. If no answer then 0 points. No arguments needed but allowed if you think your answer requires clarification.

- a. $100 \cdot n^2 = O(n^{100})$
- b. $a^{\log n} = n^{\log a}$
- c. Heapsort is better than Mergesort in that it does not use extra memory.
- d. Heapsort takes $O(N)$ in the best case.
- e. To find a solution to 8-Queens problem is NP-hard.
- f. Christofides algorithm can provide result that is 40% longer than the optimal solution, but never solution that is longer than 60%.
- g. Algorithm Z finds the solution to a problem Q with the probability of 90%. It is therefore 10%-approximation algorithm.
- h. If there exists an NP-hard problem that can be solved polynomial time by non-deterministic Turing machine, then $P=NP$.
- i. Minimum Spanning Tree problem belongs to the class NP.
- j. There exists NP-hard problem solvable in polynomial time by Turing machine.