Probabilistic inference for data science 1 Examination 15.12.2020

1. Assume that we have 3 cards that are identical in form, except that both sides of the first card are colored red, both sides of the second card are colored black, and one side of the third card is colored red and the other side black. The 3 cards are mixed up in a hat, and 1 card is randomly selected and put down on the ground. If the upper side of the chosen card is colored red, what is the probability that the other side is colored red?

2. Let X have cumulative distribution function

$$F(x) = \begin{cases} 0, & x \leq 0 \\ \frac{x}{5}, & 0 < x \leq 5 \\ 1, & 5 < x \end{cases}$$

   (a) Find $\mathbb{P}(X \leq 1.5), \mathbb{P}(1 < X \leq 3)$, and $\mathbb{P}(X > 4.5)$.

   (b) Find $f(x)$.

   (c) Compute $\mathbb{E}(X)$

   (d) Let Y have cumulative distribution function

$$F(y) = \begin{cases} 0, & y < 0 \\ \frac{1}{10}y, & 0 \leq y < 5 \\ \frac{2}{3}, & 5 \leq y < 8 \\ \frac{1}{10}y, & 8 \leq y < 10 \\ 1, & 10 \leq y \end{cases}$$

   Find $\mathbb{P}(8 \leq Y < 9)$ and $\mathbb{P}(1 < Y < 5)$.

3. Let random variable X describe hourly methan emissions to the atmosphere in a small sample plot from the green vegetation, and let Y describe corresponding emissions from the soil. There is a negative correlation among the two emission sources so that if emission from vegetation is high, then the emission from the soil is low, and vice versa. The total emission $Z = X + Y$. Assume that we know that $\mathbb{E}(X) = 5, \mathbb{E}(Z) = 15, var(X) = 1, var(Z) = 4.6$, and $Cov(X, Y) = -0.43$.

   (a) Find $\mathbb{E}(Y)$

   (b) Find $var(Y)$

4. a) The joint probability density function of random vector (X, Y) is given by

$$f(x, y) = \begin{cases} \frac{6}{7}\left(x^2 + \frac{xy}{2}\right), & 0 \leq x \leq 1, 0 < y < 2 \\ 0, & \text{otherwise} \end{cases}$$

   Give the probabilty density function of X.

   b) Random variable X has the probability density function

$$f_X(x) = \begin{cases} 3(1-x)^2, & 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

   Assume that $Y = 10 \cdot e^{5 \cdot X}$. Give f_Y the probability density function of Y.