1.1 Introduction and Linearity

Linearity

Equations of Motion

Dynamic Variables

Maxwell:

$(\vec{E}, \vec{B}, \rho, \vec{J})$

$(\alpha\vec{E}, \alpha\vec{B}, \alpha\rho, \alpha\vec{J})$ is solution

$\alpha \in \mathbb{R}$

$\left.\begin{array}{l}(\vec{E}_1, \vec{B}_1, \rho_1, \vec{J}_1) \\ (\vec{E}_2, \vec{B}_2, \rho_2, \vec{J}_2)\end{array}\right\}$ solutions

$(\vec{E}_1 + \vec{E}_2, \vec{B}_1 + \vec{B}_2, \rho_1 + \rho_2, \vec{J}_1 + \vec{J}_2)$ is also solution

Linear Equation

$$Lu = 0$$

u: unknown

L: Linear operator

$L_1 u = 0$

$L_2 u = 0$

$\vdots$

$L(u, v, w, \dots) = 0$

$L(au) = aLu$

$L(u_1 + u_2) = Lu_1 + Lu_2$

$L(\alpha u_1 + \beta u_2) = L(\alpha u_1) + L(\beta u_2) = \alpha L u_1 + \beta L u_2$

$\frac{du}{dt} + \frac{1}{\tau}u = 0 \Rightarrow Lu = 0 \Rightarrow Lu \equiv \frac{du}{dt} + \frac{1}{\tau}u \Rightarrow L = \frac{d}{dt} + \frac{1}{\tau}$

$L(au) = \frac{d}{dt}(au) + \frac{1}{\tau}au = a\left(\frac{du}{dt} + \frac{1}{\tau}u\right) = aLu$

1.2: Schrödinger Equation

Motion in 1-D

$V(x),\ x(t)$

$m=\frac{d^2x(t)}{dt^2}=-V'(x(t))$

Quantum is linear

Ψ: wavefunction

$i\hbar\frac{\partial\Psi}{\partial t}=\hat{H}\Psi$

$\hat{H}$ = Hamiltonian (linear operator)

$L\Psi=0 \Rightarrow L\Psi=i\hbar\frac{\partial\Psi}{\partial t}-\hat{H}\Psi$

$\Psi(x,t)$

1.3: Complex Numbers

$i=\sqrt{-1}$

$z=a+ib\in\mathbb{C}\quad a,b\in\mathbb{R}$

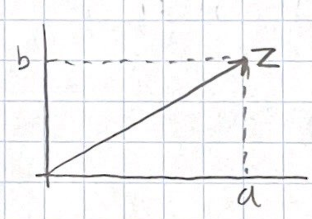
$Re(z)=a,\ Im(z)=b$

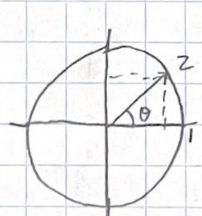
$z^*=z-ib$

$|z|=\sqrt{a^2+b^2}$

$|z|^2=a^2+b^2=zz^*$

$Norm\in\mathbb{R}$





$z=\cos\theta+i\sin\theta=e^{i\theta}$

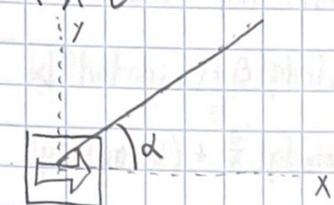
$\Psi\in\mathbb{C}\rightarrow|\Psi|^2$ = probabilities

1.4: Determinism

Wave/particle $\Rightarrow$ photons

$E = h\nu$

$\nu\lambda = c$



$\vec{E}_\alpha = E_0 \cos\alpha\,\hat{x} + E_0 \sin\alpha\,\hat{y}$

After polarizer $\rightarrow$ $\underline{E} = E_0 \cos\alpha\,\hat{x}$

Photon:
- either get through or not
- can only predict probabilities

$|photon; x\rangle$

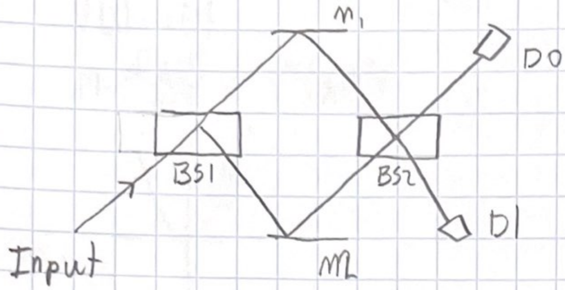
$|photon; y\rangle$

can exist $\cos\alpha|photon; x\rangle + \sin\alpha|photon; y\rangle = |photon; \alpha\rangle$

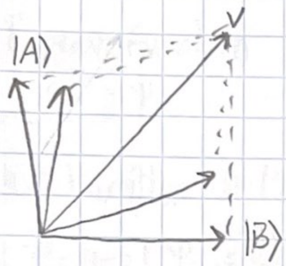
After Polarizer, $|photon; x\rangle$

1.5: Superposition

Mach-Zendor Interferometer



superposition in upper beam + photon in lower beam = superposition single photon



Superposition: $|A\rangle$ and $|B\rangle$
measure property on $|A\rangle$ get "a"
measure property on $|B\rangle$ get "b"

$\alpha|A\rangle + \beta|B\rangle$, $\alpha, \beta \in \mathbb{C}$

Probability (a) $\sim |\alpha|^2$
Probability (b) $\sim |\beta|^2$

If get "a", state becomes $|A\rangle$
If get "b", state becomes $|B\rangle$

2.1: Spin and Photon States

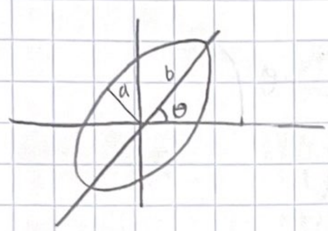
Physical assumption: superposing state to itself does not change the physics

$|A\rangle \cong 2|A\rangle \cong -|A\rangle \cong i|A\rangle \ldots$

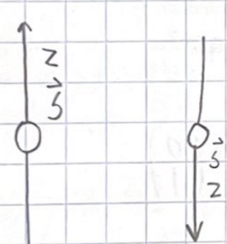
$\alpha|\text{Photon}; x\rangle + \beta|\text{photon}; y\rangle$

$|\text{photon}; x\rangle + \frac{\beta}{\alpha}|\text{photon}; y\rangle$

$\frac{\beta}{\alpha} = \gamma$ two parameters $\in \mathbb{C} \rightarrow$ one parameter $\in \mathbb{C}$



Parameters: $(\frac{a}{b}, \theta)$



$|\uparrow; z\rangle$ $|\downarrow; z\rangle$

$|\psi\rangle = |\uparrow; z\rangle + |\downarrow; z\rangle$

50% $|\uparrow; z\rangle$
50% $|\downarrow; z\rangle$

2.2: Entanglement

particle 1: $|u_1\rangle, |u_2\rangle$
particle 2: $|v_1\rangle, |v_2\rangle$

States of two particles

$$|u_1\rangle \otimes |v_1\rangle$$

$$(\alpha_1|u_1\rangle + \alpha_2|u_2\rangle) \otimes (\beta_1|v_1\rangle + \beta_2|v_2\rangle) = \alpha_1\beta_1|u_1\rangle\otimes|v_1\rangle + \alpha_1\beta_2|u_1\rangle\otimes|v_2\rangle + \alpha_2\beta_1|u_2\rangle\otimes|v_1\rangle + \alpha_2\beta_2|u_2\rangle\otimes|v_2\rangle$$

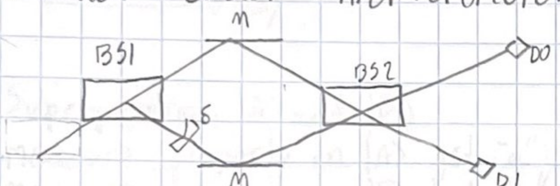
$$|u_1\rangle\otimes|v_1\rangle + |u_2\rangle\otimes|v_2\rangle \neq (\cdots)_1 \otimes (\cdots)_2$$

$\alpha_1\beta_1 = 1 \quad \alpha_2\beta_2 = 1$
$\alpha_1\beta_2 = 0 \quad \alpha_2\beta_1 = 0$

2 spin $\frac{1}{2}$ particles

$$|\uparrow; z\rangle_1 \otimes |\downarrow; z\rangle_2 + |\downarrow; z\rangle_1 \otimes |\uparrow; z\rangle_2$$

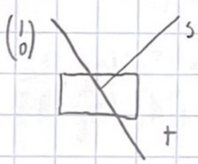
2.3: Mach-Zender Interferometers



$\begin{pmatrix}\alpha\\ \beta\end{pmatrix}$, $\alpha, \beta \in \mathbb{C}$
$|\alpha|^2 + |\beta|^2 = 1$

$\begin{pmatrix}1\\0\end{pmatrix}$ upper beam $\quad \begin{pmatrix}0\\1\end{pmatrix}$ lower beam $\quad \begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \alpha\begin{pmatrix}1\\0\end{pmatrix} + \beta\begin{pmatrix}0\\1\end{pmatrix}$

$\delta \in \mathbb{R} \qquad -\alpha-] -\alpha e^{i\delta}-$
$|\alpha| = |\alpha e^{i\delta}|$



$\begin{pmatrix}1\\0\end{pmatrix} \to \begin{pmatrix}s\\t\end{pmatrix}, \ |s|^2 + |t|^2 = 1$

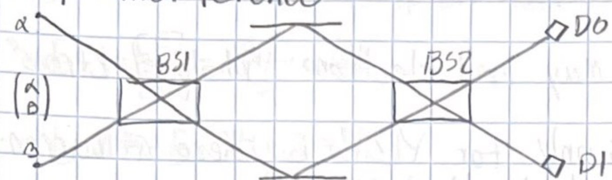
$\begin{pmatrix}0\\1\end{pmatrix} \to \begin{pmatrix}u\\v\end{pmatrix}, \ |u|^2 + |v|^2 = 1$

$$\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \alpha\begin{pmatrix}1\\0\end{pmatrix} + \beta\begin{pmatrix}0\\1\end{pmatrix} \to \alpha\begin{pmatrix}s\\t\end{pmatrix} + \beta\begin{pmatrix}u\\v\end{pmatrix} = \begin{pmatrix}\alpha s + \beta u\\ \alpha t + \beta v\end{pmatrix} = \begin{pmatrix}s & u\\ t & v\end{pmatrix}$$

$$BS = \begin{pmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}}\end{pmatrix}\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}\alpha+\beta\\ \alpha-\beta\end{pmatrix} = \frac{1}{2}|\alpha+\beta|^2 + \frac{1}{2}|\alpha-\beta|^2$$

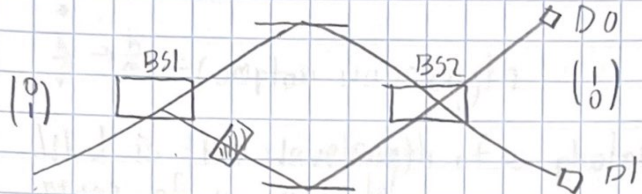
$$BS1 = \frac{1}{\sqrt{2}}\begin{pmatrix}-1 & 1\\ 1 & 1\end{pmatrix} \qquad BS2 = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$$

2.4: Interference



$$\text{Output} = (BS2)_{mat}(BS1)_{mat}\begin{pmatrix}\alpha\\ \beta\end{pmatrix}$$

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}-1 & 1\\ 1 & 1\end{pmatrix}\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \frac{1}{2}\begin{pmatrix}0 & 2\\ -2 & 0\end{pmatrix}\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \begin{pmatrix}\beta\\ -\alpha\end{pmatrix}$$



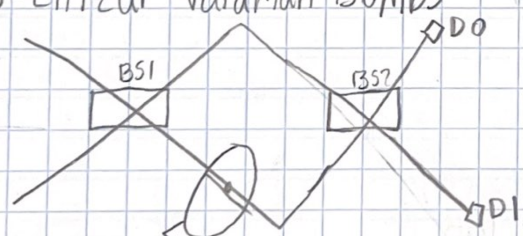
$$(BS1)\begin{pmatrix}0\\ 1\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}-1 & 1\\ 1 & 1\end{pmatrix}\begin{pmatrix}0\\ 1\end{pmatrix} = \begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{pmatrix}$$

$$(BS2)\begin{pmatrix}\frac{1}{\sqrt{2}}\\ 0\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}\frac{1}{\sqrt{2}}\\ 0\end{pmatrix} = \begin{pmatrix}\frac{1}{2}\\ \frac{1}{2}\end{pmatrix}$$

Outcome Blocked	P
photon at block	$\frac{1}{2}$
photon at D0	$\frac{1}{4}$
photon at D1	$\frac{1}{4}$

Outcome All Open	P
photon at D0	1
photon at D1	0

2.5: Elitzur-Vaidman Bombs



Outcome Bomb Defective	P
photon to D0	1
photon to D1	0
bomb explodes	0

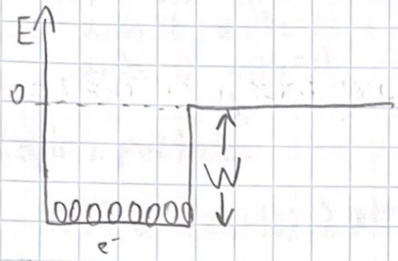
Outcome Bomb Good	P
Bomb explodes	$\frac{1}{2}$
Photon at D0	$\frac{1}{4}$
Photon at D1	$\frac{1}{4}$

3.1: The Photoelectric Effect

Polished metal plates irratiated may emit electrons "photo-electrons"
hence photoelectric current
There is threshold frequency ν_0; only for $\nu > \nu_0$ is there a current
Magnitude of current is proportional to light intensity
Energy of photoelectrons is independent of light intensity

[E_{e^-} increases linearly with ν of light]

$E = h\nu$



$W \equiv$ Work function

$E_{e^-} \cong \frac{1}{2}mv^2 = E_\gamma - W = h\nu - w$

Shine UV light with $\lambda = 290\,nm$ on metal with $W = 4.05\,eV$, what is the energy of E_{e^-} and what is their speed.

$E = h\nu = \frac{hc}{\lambda} = \frac{2\pi\hbar c}{\lambda}$

$\hbar = \frac{h}{2\pi} = 200\,MeV \cdot fm\ (197.33)$

$\frac{2\pi\,(197)\,MeV \cdot 10^{-15}m}{290 \cdot 10^{-9}nm} = \frac{(2\pi)(197)}{290}eV = 4.28\,eV$

$E_e = E_\gamma - W = (4.28 - 4.05)\,eV = 0.23\,eV$

$\frac{1}{2}mv^2 = \frac{1}{2}m_e c^2 \frac{v^2}{c^2} = \frac{1}{2}(511{,}000)\left(\frac{v}{c}\right)^2$

$v = 284\,km/s$

3.2: Units of h and Compton Wavelength

$$[h] = \frac{[E]}{[\nu]} = \frac{M\frac{L^2}{T^2}}{\frac{1}{T}} = \frac{ML^2}{T}$$

$$L \cdot M\frac{L}{T} = [r][p] = [L]$$

spin $\frac{1}{2}$ $|\vec{S}| = \frac{1}{2}\hbar$

$$[h] = [r][p]$$

m

$\frac{h}{p} \rightarrow \frac{h}{mc} \equiv$ Compton wavelength

What is the wavelength of a photon whose energy is the rest mass of a particle.

$$mc^2 = E_\gamma = h\nu = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{mc^2} = \frac{h}{mc}$$

$$\lambda_c(m)$$

$$\lambda_c(e) = \frac{h}{m_e c} = \frac{2\pi \hbar c}{m_e c^2} = \frac{2\pi(197.33\,\text{meV fm})}{0.511\,\text{meV}}$$

$$= 2426\,\text{fm} = 2.426\,\text{pm}$$

3.3: Compton Scattering

Photons quanta for energy is also for momentum

$E^2 - p^2c^2 = m^2c^4$

$E = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}$

$\vec{p} = \frac{m\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}}$

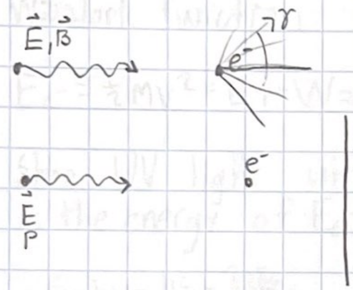
$\left(E = \frac{1}{2}mv^2, \vec{p} = m\vec{v} \Rightarrow E = \frac{p^2}{2m}\right)$

photon $m_\gamma = 0$, $E_\gamma = p_\gamma c$

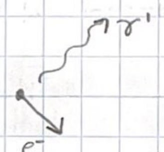
$p_\gamma = \frac{E_\gamma}{c} = \frac{h\nu_\gamma}{c} = \frac{h}{\lambda_\gamma}$

Photons scattering on electrons that are virtually free

Violation of Classical Thompson scattering

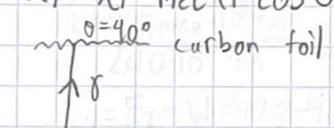


$\frac{d\sigma}{d\Omega} = \left(\frac{e^2}{mc^2}\right)^2 \frac{1}{2}(1+\cos^2\theta)$



photon looses energy $\lambda_f > \lambda_i$

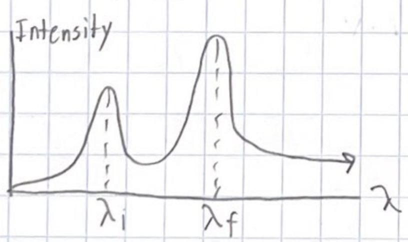
$\lambda_f = \lambda_i + \frac{h}{m_e c}(1-\cos\theta)$



Source Molybdenum

xrays $\lambda = 0.0709\,nm$

$E_i = 17.49\,keV$



$\lambda_i = 0.0709\,nm$

$\lambda_f = 0.0731\,nm$

$0.0022\,nm$

3.4: Louis de Broglie

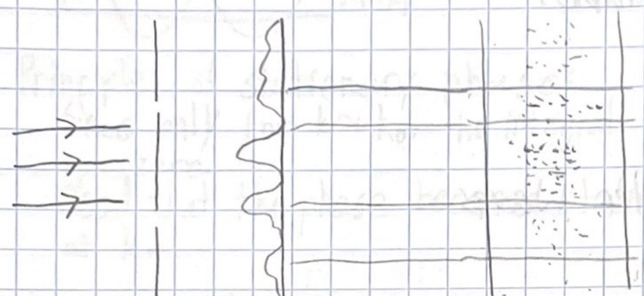
photon

Particle
$E, V, \vec{V}$

Wave
ν

Universal matter waves
Waves of probability amplitudes

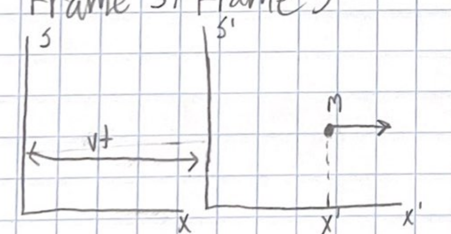
Particle of momentum p $\iff$ plane wave $\lambda = \frac{h}{p}$ (de Broglie wavelength)



4.1: De Broglie Wavelength in Different Frames

free particle with momentum p is associated to a wave where $\lambda = \frac{h}{p}$
$\Psi(x,t) \in \mathbb{C}$

$p = \frac{h}{\lambda} = \frac{h}{2\pi}\frac{2\pi}{\lambda} = \hbar k$
$\nu,\ \omega = 2\pi\nu$
Frame S, Frame S'



$t' = t$
$x' = x - vt$
$\frac{dx'}{dt} = \frac{dx}{dt} - v \Rightarrow \underline{V}' = \underline{V} - V$
$p' = p - mv$
$\lambda' = \frac{h}{p'} = \frac{h}{p - mv} \neq \frac{h}{p} = \lambda$

4.2: Galilean Transformation

$\phi = \text{phase} = kx - \omega t$

$\phi = k\left(x - \frac{\omega}{k}t\right) = \frac{2\pi}{\lambda}(x - Vt)$

$\phi = \frac{2\pi x}{\lambda} - \frac{2\pi V}{\lambda} t$

$\phi' = \phi = \frac{2\pi}{\lambda}(x - vt)$

$= \frac{2\pi}{\lambda}(x' + vt' - Vt')$

$= \frac{2\pi}{\lambda}x' - \frac{2\pi}{\lambda}V\left(1 - \frac{v}{V}\right)t'$

$\omega' = \omega\left(1 - \frac{v}{V}\right)$

$k' = k \Rightarrow \lambda' = \lambda$ for ordinary waves

Conclusion:
Ψ are not directly measurable:
Not Gallieleun invarient

$\Psi(x,t) \neq \Psi'(x',t')$

4.3: Frequency of the Matter Waves

$p = \hbar k$

$E = \hbar\omega \rightarrow \omega = \frac{E}{\hbar}$

$V_{phase} = \frac{\omega}{k} = \frac{E}{p} = \frac{\frac{1}{2}mv^2}{mv} = \frac{1}{2}v$

$V_{group} = \left.\frac{d\omega}{dk}\right|_k = \frac{dE}{dP} = \frac{d}{dp}\left(\frac{p^2}{2m}\right) = \frac{p}{m}$

Special relativity

$\left(\frac{E}{c}, \vec{p}\right)$ 4 vector

$\left(\frac{\omega}{c}, \vec{k}\right)$ 4 vector

$\left(\frac{E}{c}, \vec{p}\right) = \hbar\left(\frac{\omega}{c}, \vec{k}\right)$

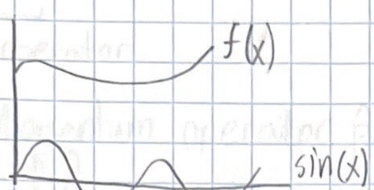
Photons: Einstein: $E = \hbar\omega$

4.4: Group Velocity

ωk

Velocity of a packet constructed by superposition of waves

$$\Psi(x,t)=\int dk\, \Phi(k) e^{i(kx-\omega(k)t)}, \quad kx-\omega(k)t=\varphi(k)$$



Principle of stationary phase:

Since only for $k \approx k_0$, the integral has a chance to be non zero.

Need that the phase becomes stationary with respect to k at k_0

$$\varphi(k)=kx-\omega(k)t$$

$$\left.\frac{d\varphi(k)}{dk}\right|_{k_0}=x-\left.\frac{d\omega(k)}{dk}\right|_{k_0}t=0$$

$$x=\left.\frac{d\omega}{dk}\right|_{k_0}t$$

4.5: Motion of a Wave Packet

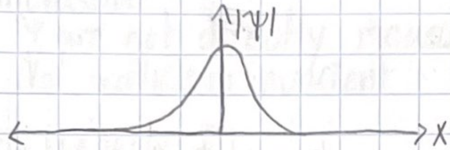
$$\Psi(x,t) = \int \Phi(k) e^{i(kx-\omega(k)t)} dk$$

$$\omega(k) = \omega(k_0) + (k-k_0)\frac{d\omega}{dk}\Big|_{k_0} + O((k-k_0)^2)$$

$$\Psi(x,t) = \int \Phi(k) e^{ikx} dk,\ e^{-i\omega(k_0)t},\ e^{-ik\frac{d\omega}{dk}|_{k_0}t},\ e^{ik_0\frac{d\omega}{dk}|_{k_0}t},\ e^{\text{neglagable}}$$

$$\Psi(x,t) = e^{-i\omega(k_0)t} e^{ik_0\frac{d\omega}{dk}|_{k_0}t} \int \Phi(k) e^{ik(x-\frac{d\omega}{dk}|_{k_0}t)} dk$$

$$|\Psi(x,t)| = |\Psi(x - \frac{d\omega}{dk}\Big|_{k=0} t, 0)|$$



4.6: Wave of a Free Particle

Particle E, P. $E = \hbar\omega$, $P = \hbar k$

$\sin(kx-\omega t)$
$\cos(kx-\omega t)$
$e^{ikx-i\omega t}$
$e^{-ikx+i\omega t}$

$$\left.\begin{array}{l}\sin(kx-\omega t)+\sin(kx+\omega t)\\ \cos(kx-\omega t)+\cos(kx+\omega t)\end{array}\right\}\text{problems}$$

$$e^{ikx-i\omega t} + e^{-ikx+i\omega t}$$

$$e^{-ikx}e^{i\omega t} + e^{ikx}e^{i\omega t} = 2\cos kx\, e^{i\omega t}$$

$$\Psi(x,t) = e^{-ikx-i\omega t}$$