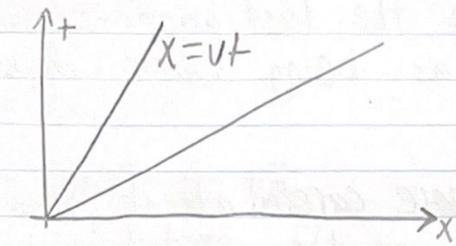
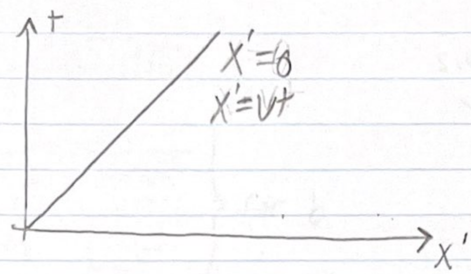
§1: Reference Frames



$x=ct$



$$\begin{cases} x=ct \\ x'=x-vt \\ t'=t \text{ (Newton)} \end{cases}$$

$x'=ct-vt=(c-v)t'$

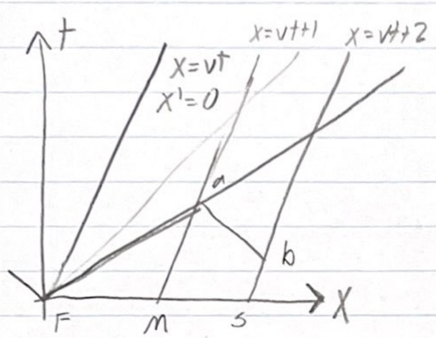
$x=-ct$

$x'=-(c+v)t$

2: Reference Frames

Something is wrong with the last page because simultinaety is defined as being frame dependent

Einstein wants to be more careful about how time is defined based on the postulate that the speed of light is constant.



$X=ct$

Let $c=1 \therefore x=t$ to find

$X=vt+1 \Rightarrow t=vt+1$

$\Rightarrow t(1-v)=1$

$\left.\begin{aligned} &\Rightarrow t_a=\frac{1}{1-v} \\ &\Rightarrow X_a=\frac{1}{1-v} \end{aligned}\right\}$ for a

for a

§1: Reference Frames

$$x + t = \frac{2}{1-v}$$

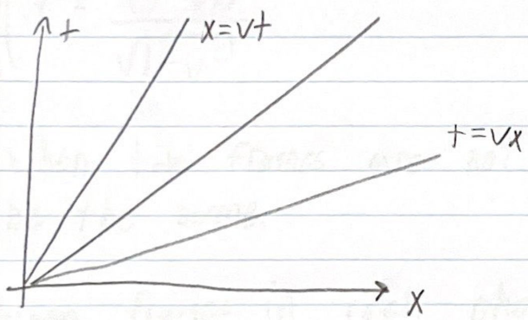
$$x = vt = 2$$

$$\Rightarrow t(1+v) = \frac{2}{1-v} - 2$$

$$\Rightarrow t(1+v) = \frac{2v}{1-v}$$

$$\Rightarrow t = \frac{2v}{(1-v)(1+v)}$$

$$\left.\begin{aligned} &\Rightarrow t_b = \frac{2v}{(1-v)^2} \\ &\Rightarrow x_b = \frac{2}{1-v^2} \end{aligned}\right\} \text{for } b$$



§1: Reference Frames

$x'=0$ when $x=vt$

$x'=x-vt$

but really define it as a function:

$x'=(x-vt)f(v)$

$t'=0$ when $t=xv$

$t'=(t-vx)f(v)$

$x=(x'+vt')f(v)$

$t=(t'+vx')f(v)$

§1: Reference Frames

$$x = (x-vt)f^2(v) + v(t-vx)f^2(v)$$

$$= xf^2 + v^2 x f^2$$

$$= x(1-v^2)f^2$$

$$f = \frac{1}{\sqrt{1-v^2}}$$

Thus:

$$\begin{cases} x' = \dfrac{(x-vt)}{\sqrt{1-v^2}} \\ t' = \dfrac{(t-vx)}{\sqrt{1-v^2}} \end{cases}$$

When two frames are not moving, they should be the same.

When frames in rest, choose positive case thus $x = x'$ & $t = t'$

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$$\left.\begin{aligned} x' &= \frac{x-vt}{\sqrt{1-v^2}} \\ t' &= \frac{t-vx}{\sqrt{1-v^2}} \end{aligned}\right\} \text{Lorentz Transforms}$$

Equations must be dimensionally consistant

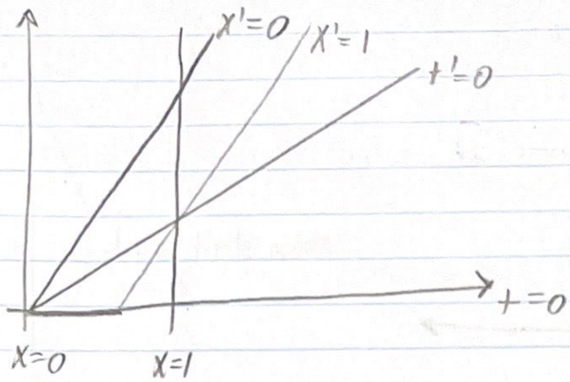
so set the following revision:

$$x' = \frac{x-vt}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$t' = \frac{x-\frac{v}{c^2}t}{\sqrt{1-\frac{v^2}{c^2}}}$$

Make corrections near the speed of light

9: Reference Frames



$$x' = \frac{x - v^2 x}{\sqrt{1-v^2}}$$

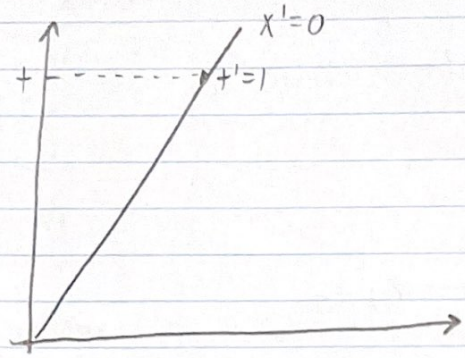
$$= \frac{1-v^2}{\sqrt{1-v^2}}$$

$$= \sqrt{1-v^2}$$

$y' = y$

$z' = z$

§1: Reference Frames



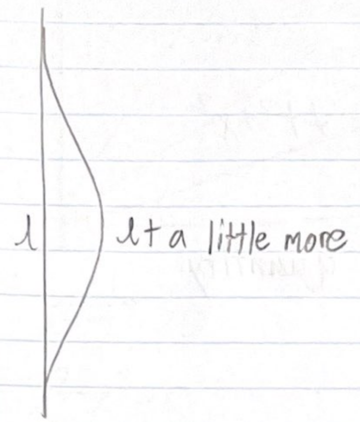
$$\left.\begin{aligned} x &= \frac{x' + vt'}{\sqrt{1-v^2}} \\ t &= \frac{t' + vx'}{\sqrt{1-v^2}} \end{aligned}\right\}$$

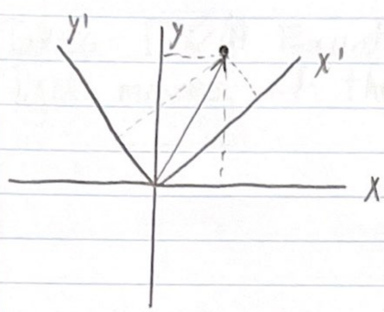
$t'=1, x'=0$

$$t = \frac{1}{\sqrt{1-v^2}}$$

twin paradox: reverse motion of the $t'=1$ and send it back to normal worldline. Did one time travel?

§1: Reference Frames





Everyone agrees on distance from origin

$x^2+y^2=x'^2+y'^2$

$t'^2+x'^2=t^2+x^2$

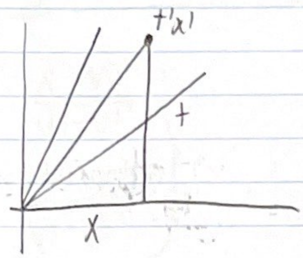
$t^2+v^2x^2$

§1: Reference Frames

$$t'^2+x'^2=t^2+x^2$$

$$\frac{t^2+v^2x^2-2vtx}{1-v^2}+\frac{x^2+v^2t^2-2vtx}{1-v^2}\neq t^2+x^2$$

$$\frac{-x^2(-v^2+1)+t^2(1-v^2)}{1-v^2}=\text{invariant quantity}$$



t^2-x^2 is invariant

$$t'^2=t^2-x^2$$

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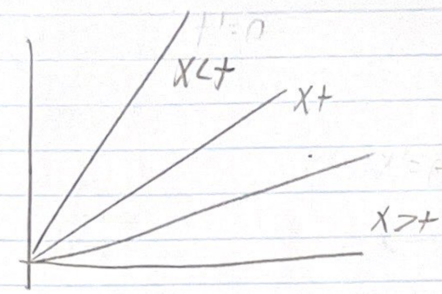
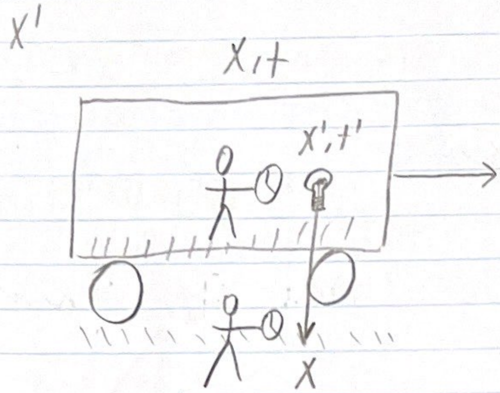


$x^2 - t^2 > 0$

$t^2 - x^2 > 0$

Takes E&M Equations as laws of physics so light moves at the speed of light

§2: Particle Motion



$$x' = \frac{x - vt}{\sqrt{1-v^2}}$$

$$t' = \frac{t - vx}{\sqrt{1-v^2}}$$

§2: Particle Motion

$$x = \frac{x' + vt'}{\sqrt{1-v^2}}$$

$$t = \frac{t' + vx'}{\sqrt{1-v^2}}$$

Imagine a car in the box car t'' & x''

$$x'' = \frac{x' - ut'}{\sqrt{1-u^2}}$$

$$t' = \frac{t' - ux'}{\sqrt{1-u^2}}$$

§2: Particle Motion

t'' & x'' redefined

$$t'' = \frac{x - vt}{\sqrt{1-v^2}\sqrt{1-u^2}} - \frac{u(t-vx)}{\sqrt{1-v^2}\sqrt{1-u^2}}$$

$$= \frac{(1+uv)x - (v+u)t}{\sqrt{1-v^2}\sqrt{1-u^2}}$$

$$x = \frac{u+v}{1+uv} t$$

$$\therefore x'' = \frac{x - wt}{\sqrt{1-w^2}}$$

$$\therefore t'' = \frac{t - wx}{\sqrt{1-w^2}}$$

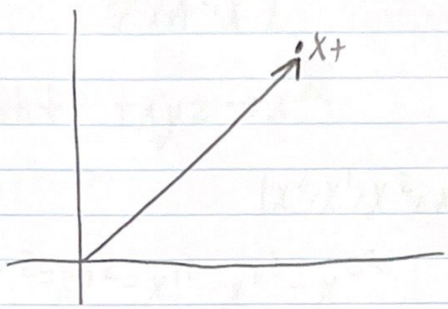
$$w = \frac{u+v}{1+\frac{uv}{c^2}}$$

S2: Particle Motion

$u = 0.01$

$v = 0.01$

$$\Rightarrow w = \frac{u+v}{1+uv} = \frac{0.02}{1+0.0001}$$

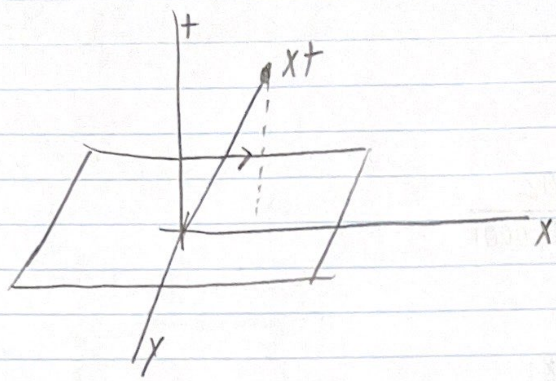


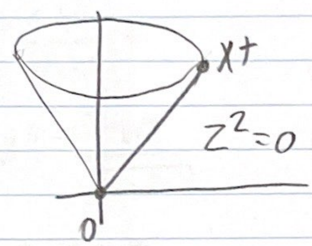
$$\tau^2 = t^2 - x^2$$

$$= t'^2 - x'^2$$

$$= t''^2 - x''^2$$

§2: Particle Motion





$t^2 - x^2 = 0$

$\Downarrow$

$t^2 - (x^2 + y^2 + z^2) = 0$

§2: Particle Motion

$\to xyz(t)$

$x^1 \to (x, y, z)$

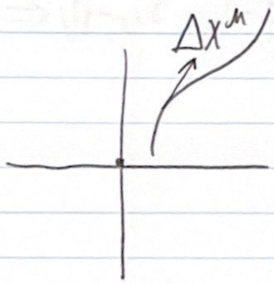
(x^1, x^2, x^3)

With t: $txyz \to x^\mu$

(x^0, x^1, x^2, x^3)

$\tau^2 = x^{0\,2} - x^{1\,2} - x^{2\,2} - x^{3\,2}$

$$\left(x'\right) = \begin{pmatrix} 1 & -v \\ -v & 1 \end{pmatrix} \left(x\right)$$



$\Delta x^\mu = (\Delta t, \Delta x, \Delta y, \Delta z)$

§2: Particle Motion

$$\frac{\Delta x^{\mu}}{\Delta \tau} = u^{\mu}$$

$$(\Delta \tau)^2 = (\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2$$

↑

Invariant spacetime difference

We want a theory for a motion of particles so we get dynamics later.

Generalization of Newton