# Cal - Cul - aTe

#### **FUNDAMENTALS TO FRONTIERS**

A bird never soars too high If it soars with its own wings - Blake

# Introduction

Mathematics is the supreme path of intellect.

All other pursuits are derivative of generalized & simplified mathematical structures. Once the mathematical-framework is conquered then all else kneels in submission.

Mathematician seeks to assess a complex situation into a process of reduction until only independent irreducible-elements remain.

Next the relationships of irreducible-elements lead to theorems and basic-structures.

The power of the art is to rebuild wild-systems into predictable-containment.

Simplification is the greatest pursuit of the mathematician.

The highest intellectual-pursuit demands the greatest investment.

Youth-long academic training is only the pre-requisite, any is sufficient.

30 years is the minimum age to begin the serious mathematical journey.

Ten years are to be devoted to the fundamentals at an average struggle of 3 hour daily struggles.

The journey must be accounted to purge self-delusion prone to solitary expeditions.

The only resources required are books & art-supplies.

No mentor. No calculator.

Mathematical progression is a process of hardening layers.

Linearity, imposed by the structure of books, is unnatural to mathematics.

#### Math is a bush

Tend growth at each layer of branch.

Foundational layers are most essential.

There are various heights, but all must be checked else exist droop or breakage. Foundations are not tended once, and then discarded for more fruitful branches. Each layer is revisited from trunk to extension in full, as season permits.

There is no singular pinnacle of mathematics.

It is the resources of a solid-diverse-system that breaks down complexity. Unlike a tree, the deficiency in any layer of a well-groomed bush is obvious.

## Proof

Truths are built by proofs.

Each truth must be naturalized into the mind until the process becomes intuitive.

Intuition is able to construct the solution from consequents of known truths.

Deductive-reasoning is basing the foundation of theory upon generally-accepted memorized formula.

Formula memorization is the weak-link in a strong chain of reason that will arrive an intelligent mind into a false conclusion.

Intuition is immune to false affirmation.

The destination, as is each step, is known as proven or remains unresolved.

Proofs are not created equal.

Necessary & sufficient attitude of minimum effort has no place here.

The exposition of proof is the expression of the powerful mind.

Genius is not easily convinced but is assured by perspectives & applications.

Generalized-abstraction has a role in preliminary-design and in the establishment of key-waypoints.

Calculate-exhaustion is the distinctive trait separating the mathematician from the intellectual-poser.

'The question of numerical calculation of the limit ...

'One of great practical significance,

'Is usually in theoretical-considerations of very second-rate importance

'From a theoretical point-of-view, all modes of representation for a real number are precisely equivalent

'Theoretic representation of a number is the Dedekind section

[Insufficient in any practical application]

'The representation of a real number by a sequence may be considered as the most general mode of representation

[Critical in both application & theory]

- Konrad Knopp p. 79

Exhaustive application, a sequence, fulfills theoretic requirements.

Dedekind sections are neither necessary nor sufficient; yet their simplicity make them ideal for generalization.

Sterile & pathetic definitions of abstraction are poor foundations in both application & theory.

The universe has a tendency to pretend submission as it works chaos in nuanced subtle folds of nature.

Exhaustive application breeds humility, ever searching for the outlier. Abstraction breeds arrogance of the mind, proclaiming containment.

#### **Historic state**

Modern Mathematics has been perverted by Jewish influence.

The ancient geometer nor the renaissance scientist would recognize affiliation with the modern academic.

Germany was the historical champion of truth-by-exhaustion.

A direct intellectual descendent of the Archimedean style.

The forefathers of Knopp's methods all predate the Jewish infiltration.

In this era, the 1800s, the legendary status of Germans attained its height at the global-head of mathematics, but by the early 1900s there was a sharp decline.

After which there were no longer masters produced from Germany.

By 1933, a third of all mathematic professors in Germany were Jewish, less than 1% of the population.

America became damned to the same fate; unfortunately aborted before any greatness attained.

In the 1950s American institutions were fully infiltrated by the invasive species of jews, both in university positions and academic publishing.

A race less than 2% of the population, secured 75% of academic publishing contracts.

The most fertile application of the mind, math, now reduced to a trivial status in the academic world.

Memorization & mimicry in circus-performances of soul-less puppets who inherited the fate of a god-forsaken-people.

Universities justify exorbitant fees with the product of 'advanced-methods' no other university can produce.

Students, in each institution, are corralled like sheep into a fast-paced maze that indoctrinate a unique dance that appears esoteric & advanced.

Superficial treatment encodes 'answers', which no unaffiliate can answer. The purpose is to gate-keep via the pretended superiority of a trick-question.

This magic-trick justifies the enormous cost of the the education.

There is no incentive for the institution to build foundations.

There is only the race to jazz-hands to dazzle an audience and ultimately justify privileged placement.

Abstraction is the tool of the mediocre to pretend mastery.

Outlines are a means to an end arrived at by refined techniques which open source expertise.

The purpose of this book is to rebuild the path to timeless mathematics.

Upon this journey no jew can conspire place. Here position is earned.

## Scope

Upon this journey, you will return humbly to topics once believed were mastered in childhood.

All masters have humbled themselves to ashes & dirt.

To look upon a concept honestly takes time.

Perception of mastery is the constant deceiver.

Lifelong scope transmogrifies the pursuit, and its elements, into a new species.

Progression is no longer defined as a process with a beginning nor end.

Progression becomes a garden that blooms in the calculation.

Each bud or cluster is the endgame.

The technique required becomes the product.

The foundation becomes the source and the fruit of complex structures become trivial. Foundational structures are used in every endeavor, but the frontier superstructures are objects of last-resort where mastery is relinquished.

### Measure of the man

Noob: first year. Surveyor of fields of labor.

Journeyman: second & third year. Attain self-image by trial & error.

Craftsman: fourth to eighth year. Build unique creations.

Expert: eight to twenty years. The working man. Master: no peer. 10 years paid into specialized focus.

Time of devotion is the unit of measure.

Reduction of complexity to simplicity is a process unique to the individual strength.

In this journey the roles of foundation & frontier are truly inverse.

Foundational concept are the workhorse and frontier-structures are unavoidable garnish.

The measure of ability is natural to the world.

Peer competition can only designate a superior.

Third-party choice of a complex system.

At the end of the period the superior mind will be plain or else peer-ship confirmed.

## Mastery by calculation

Books are force a non-linear-state into linearity. This book adds a novel technique to add layers.

The entire tract of the book is reworked according to the 5 layers of measure. Perspective, techniques, structures and definitions will evolve with each treatment.

[ N = 10 ]

Training must be long enough to foster and allow contemplation at each step. If a step is worth understanding, then it is worth a long & careful consideration.

Open a math book and you will not find math.

This approach sets mathematical computations as the lecturer.

Problems are set into long relationships that are carried until the end. The set of problems are curated & designed to serve as a bridge to deeper foundations. Layer calculation will refine the graph to truly show distinct benefit of techniques. Each layer will construct a problem into a intricate study. Compilation of the constructions into a unified grand whole.

Each step in this journey is towards the ultimate grand artwork. Each is exposed as a necessary part of a cyclopean whole.

## **Determinate Space**

There exist no irrational, imaginary, nor infinities.

Mathematics is the science of measure.

To measure there first must exist a unit.

A unit then imposes a logical scale of a specific depth.

Square-root-of-two, pi, e and all other numbers will exist as unique discrete points in space.

There do not exist an infinite amount of possible numbers between any two points. In a system grounded by scale, there is a finite distance between any two points.

The square root of negative one is proof that there is no solution.

## **Diety**

Gods of this religion: Euclid, Euler, Archimedes, Boole.

Euclid it the god of reason.

Axiomatic construction of geometry from given to consequent.

The Elements must be studied life-long.

Euler is the god of arithmetic-exhaustion.

Elements of Algebra, Analysis of the Infinite, Differential Calculus, Integral Calculus.

These works were built after wisdom to highlight what still retained his expert curiosity.

Archimedes is the god of exhaustion.

His works are the origin of herculean ventures.

Know what you have mastered.

By sheer volume of those simple techniques, arrive to any conclusion.

Approximate error of the result.

Boole is the god of individuality.

Self-taught in all his avenues of thought.

Restructure mathematic thought by power of self mastery.

Humble great institutions into a open-eyed student.

## **Ritual**

Daily worship to wrangle the 3 hour average can only be attained by obsession. This religion is the most ancient. Its gods are real men. Its powers are unfathomable.

Linen, wool & silk offer comforts for the long delves into study.

Your soul in commit to a journey. Outfit is the outward expression of dedication to diligence. Respect of appearance has internal & external ramifications.

Solitary plunges into the works of the dead will summon spirits of the departed to impart truths to the modern acolyte.

In order for the spirits to engage there must be no doubt as to the resolve.

Presence of vibe must be cultivated to command devotion of the dead.

Bronze basin of ashes leveled. Resin, sage, sandalwood, palo-santo, & other ground into powder. Incense in the basin begins the study.

A sacred cup and thermos of coffee  $\!\!/$  tea.

A plate of treats to invigorate the mind.

Light & sound are also set to a cohesive ambience to foster spiritual resolve. Together the mind & the body & the spirit all united for the singular purpose of intellectual perfection.

## Wizard's Abacus

Set as a relic necklace. The device embodies time devoted upon path.

What is gained is not lost.

Only resilience will produce a true sorcerer of mathematical mysteries.

This artifact will become the most precious treasure.

It represents what has been accomplished and it contains a discrete growth for hope.

The abacus is not meant to be undone.

Four types of beads.

1 hour beads: 10

Clip spacer to distinguish done from undone.

10 hour beads: 10

Twine tie to separate done from undone.

100 hour beads: 10 Copper wire delimiter. 1000 hour beads:10

Permanent silver delimiter installed after achievement.

40 beads complete 11,110 hours.

State is public. Wear it proud.

## Altar

Massive standing desk. Gradated foam mat to keep stance dynamic. Long hours are best engaged standing.

Stack of books. The older the books the better. Do not acquire library too quickly. Age & experience are the only untainted sources to base investments. Fewer books keep focus sharp. Libraries exist to enrich diversity.

Crystal ball is your constant companion. Show it the measure of your soul. Years will create a magical bond until what looks back is saturated in reverence for your dedication.

Large paper allow for a grand calculations. Ink, nibs, pens, markers, straight-edge, compass.

Sand/dirt consecrated into a vase. Pour in a circle to begin daily commitment.

Cultivate an environment which is able to uplift the soul from its daily cares into a transcendental state above all other worry.

Keep the space sacred for solitude.

## **Foundations**

' gladly avail myself of the opportunity of inscribing to you, 'For a second time, a work of mine on Algebra, 'As a sincere tribute of my respect, affection and gratitude

. . .

'I continue to devote some portion of the leisure at my command,
'To the completion of an extensive Treatise,
'Embracing the more important departments of Analysis
'The execution of which I have long contemplated

'I have separated arithmetical from algebra
'I have devoted the present volume entirely
'To the exposition f the principles of arithmetic
'Application to the theory of numbers and of arithmetical processes

'To obviate the confusion, obscurity, and false reasoning which thence arises 'A short statement of the distinct and proper provinces of these two sciences 'Arithmetic & Algebra, will make this difficulty sufficiently manifest.

'Numbers and the operations... in their ordinary meaning only
We must suppose the numbers to be quantities of the same kind
When a first number is reduced by a second number
We must suppose the first number be greater than the second number
'Therefore homogeneous with it

'Numerical fractions which have NOT a common denominator, are NOT homogenous 'And are incapable of addition and subtraction

> Homogenous subtraction Homogenous division

'Abstract numbers: all results whatsoever, including negative quantities 'Which are not strictly deducible as legitimate conclusions 'From the definitions of the several operations, 'Must be rejected as impossible, or as foreign to Arithmetic

'The permanence of forms, constitutes the great and fundamental principle of Algebra
'An extension to our notion of number
'The generalizations of Algebra are the generalizations of reasoning
'And not of form

'Algebra adopts the rules of Arithmetic 'But removes altogether the restrictions Of homogeneity

'Upon this principle we shall be enabled to give a consistent interpretation

'A student who is not only familiar with the results of Arithmetic
'But likewise with the limitations it imposes
'Will be in a condition to comprehend and appreciate
'The whole extent of the legitimate conclusions which it furnishes

'Acquire the habit of observing not merely what is within 'But what is without the just and proper boundaries of the science

'He will be thus enabled to appreciate at once the origin and the full extent 'Of the principle of the permanence of equivalent forms.

- George Peacock 1842

## **Denomination**

Habitualization of fractions has trivialized the nuanced theory for the sake of concise treatment necessary for the memorized mimicry of Modern Math.

A fraction represents a ratio of the numerator by the denominator. The denominator is theoretically distinct from the numerator. Numerator acts as a general number whose unit is the denominator. Denominator is a number encapsulated by the concept of a base scale.

Denomination of scale.

Space in consideration is unique to the denomination.

Denominators of different scale PRODUCE numerators entirely unrelated. Numerator of scale 1 NONCOMESURATE to a numerator os scale 10. Different scale of space produces different numerators.

Denominate by zero.

Space can NOT be scaled by zero.

Unit of scale changed to zero means nothing.

How many parts of magnitude zero can a quantity be? Nonsense. Measure the quantity of an item using a ruler that by definition does not exist. Nonsense.

Zero is not a number. It is a statement that nothing exists. Hence any number can serve to denominate space. O = NULL

A fraction is the operation of division, contained by sanity, only valid within the principle of homogenous space.

'The whole of higher analysis may be regarded as a field for the application of Infinite Series

'For all limiting processes — including differentiation & integration —

'Are based on the investigation of Infinite Sequences or of Infinite Series

'My aim is to give a comprehensive account of all the investigations of higher analysis

'In which Infinite Series are chief objects of interest
'To start at the very beginning and lead on to the extensive frontiers of present day research

'Without in the least abandoning exactness

'With the object of providing the student with a convenient introduction

'And of giving him an idea of its rich & fascinating variety

'I have taken pains to put practical applications in the forefront 'And to leave mere playing with theoretical niceties alone

'The foundation on which the structure of higher analysis rests is the Theory of Real Numbers 'Calculus, the men who developed it, of who Euler is chief 'Too intoxicated by the mighty stream of learning springing from the newly-discovered sources

'To feel obliged to criticize fundamentals.

'Critical analysis ventured to examine the fundamental conceptions 'Chiefly owing to the powerful influence of Gauss.

'Nearly a century had to pass, however, before the most essential matters could be considered thoroughly cleared up

'Nowadays rigor in connection with the underlying number concept is the most important requirement 'In the treatment of any mathematical subject 'The last word on the matter has been uttered '—by Weierstrass in 1860s, and by Cantor & Dedekind in 1872

'No lecture or treatise dealing with fundamental parts of higher analysis 'Can claim validity unless it takes the refined concept of the real number as its starting point

'Theory of Infinite Series would be up in the clouds throughout 'If it were not firmly based upon the system of real numbers

- Konrad Knopp 1921

## Set, Sequence & Series

Set: is a group of unordered-numbers.

$$S = \{ 3, 34, 22/7, .66666, 2.718, ... \}$$

Sequence: is a set of <u>ordered-numbers</u>. Elements of the set are mapped into a natural progression 1, 2, ..., N.

A[0] = 1.4

A[1] = 1.41

A[2] = 1.414

Series: is a set of <u>ordered-numbers</u> connected by <u>addition</u>. A series is a sequence with each term added into a singular whole magnitude.

$$B[0] + B[1] + B[2] + ... + B[N]$$

Cal Cul aTe requires no calculator nor table of values.

Series allows advanced mathematics to be not only understood, but the calculations give a foundation unaccessible to theoretical approaches.

Rigor & abstracted-generalizations are up in the clouds until firmly grounded by discrete calculations. Both their role is to orient & simplify the implementation.

Modern Mathematics is dependent upon truncated tables accessed by calculators.

For the Modern Mathematician has deluded himself to believe that symbolic representation of the permanence of forms is sufficient, hands-on-work.

Blackbox processing thru calculators & computers are the handicaps of the intellectual poser.

Series is the limiting process of the practical mathematician which opens the field of study traditionally only accessible to the intellectual-poser.

Series uniquely grounds a solution by discrete perfection, with all error explicit.

## **Least Common Denominator**

The heart of Sequences & Series is the homogenized denomination.

Two quantities MUST be homogenized before comparison, before arithmetic operation.

Compare 3.14 & 22/7

Neither the numerical-representation nor the denomination is homogenized.

When a Sequence is listed the numbers must be homogenized first.

Order is unknown. Nor is it known wether the sequence of numbers expresses tendency (This is the central question of Sequences).

Homogenization is by setting a Least Common Denominator for the entire set and then adjusting each numerator.

This paragraph is entirely inherited by Series.

In order to add a sequence of numbers, there must be a universal denominator with adjusted numerators.

Be aware, mathematics is the most difficult of intellectual paths.

Be aware, Cal Cal aTe is the most difficult & exact of mathematical paths.

Homogenization of a sequence is the most labor-intensive efforts in mathematics.

## General Method of Homogenization:

- 1) Progress from first to next, then result to next, until last
- 2) Prime factorize both numbers in the denominator
- 3) Join all factors, but only the highest powers of each to obtain LCD
- 4) for each denominator, remove all factors in common to the LCD, then multiply the numerator by this number, homogenization is obtained.
- 5) Now a sequence can be compared and a series can be summed.

'Whatever is capable of increase or diminution is called a magnitude 'Different kinds of magnitude ... is the origin of the different branches of Mathematics 'Each being employed on a particular kind of magnitude

'We cannot determine any quantity, except by 'Considering some other quantity of the same kind as known 'And pointing out their mutual relation

'The measure of magnitude of all kinds, is reduced to this:

'Fix at pleasure upon any one known magnitude of the same species

'With that which is to be determined

'Consider it as the unity
'Then determine the proportion of the proposed magnitude to this known measure

'From this it appears that all magnitudes may be expressed by numbers 'The foundation of all the Mathematical Sciences must be laid 'In a complete treatise on the science of Numbers 'And in an accurate examination of the different possible methods of calculation

- Euler 1765

## **Euler's Utmost Subjects**

Fractal Invariance, Infinite Fractions, Negative Space, Transpositions, Imaginary

'It is evident therefore how essential it is, in all problems 'To consider the circumstances of the question attentively 'In order to deduce from it an equation that shall express by letters the numbers sought 'The whole art consists in resolving those equations 'Or deriving from them the values of the unknown numbers

'We must remark, in the first place, the diversity which subsists amount the questions 'In some, we seek only for one unknown quantity 'In others, we have to find two or more 'It is to be observed, with regard to this last case 'In order to determine them all, 'We must deduce from the circumstances, or the conditions of the problem 'As many equations as there are unknown quantities

'An equation consists of two parts separated by the sign of equality '=' We are often obliged to perform a great number of transformations on those two parts 'In order to deduce from them the value of the unknown quantity 'These transformations must be all founded on the following principles 'Two qualities remain equal whether we add to them, or subtract from them, equal quantities 'Whether we multiply them or divide them, by the same number Whether we raise them both to the same power, or extract their roots of the same degree 'Lastly, whether we take the logarithms of those quantities

- Euler 1765

Arithmetic

Fractions: most important department of Arithmetic

Proportion

Equality, Systems of Equality

Inequality & Distance & delta-epsilon

Parametric Representation

Conditional Inequality

Double & Triple Inequality

Incommensurable

Permutation

Binomial Structure

Compounds, Quadratic, Cubes

GCD

**BeZout** 

Prime Factorization, must exist a prime divisor less than the square root of the number.

LCM

Base

Power

Root

Fractional Exponents

Log

Polynomial Division

Infinite Series

Continued Fraction

Difference, Arithmetic Proportion & Arithmetic Progression

Geometric Proportion & Progression

Calculation of Interest & e

First Degree Equations

Quadratic Equations

**Cubic Equations** 

Fourth Degree

Geometry & Numerics

Space {Natural, Integral, Fractal, Continuous, Polar, Inequality, Inverse}

Numeric {Integer, Fraction, Irrational, Imaginary, Logarithmic, Decimal, Segisesimal, Rad}

Geometry { points, lines, body, sets, angles, triangles, parallelogram, polygon, circle, irrational }

Lines { slope, perpendicular, projection, systems, vector, matrix, dot, determinant, reduction, bezout}

1/x irrational

Logarithm & Exponential

Curves { compounds, roots, quadratic, cubic, conics, hyperbola

Periodic { trig, polar, imaginar, modulus-congruence }

Convergence { traversal(cauchy product), null, nest, mesh, set-section, conditional, absolute }

Limit {graphical-approach, sequence-table, sets, series, interval, sets }

Finite Differences

Rates of Change { diff-quotient, product, reciprocal, chain} [t^2 falling body]

Continuity { interval, hood

Analysis { bounds, intermediate-value, mean-value, extrema }

Integration { Riemann, part, substitution, improper }

Tools: add, sub, mul, div, greater, less, distance, gcd, lcm, decimal, (n:r), factorial, arith-progression, geom-progression

Structures: compounds, polynomial, set, sequence, series, continued fraction Geometry Tools { perimeter, area, volume, surface, tangent, perpendicular, bisect, circumscribe, triangulation, sector, ratio, congruence, transformation}