SAGEMATH

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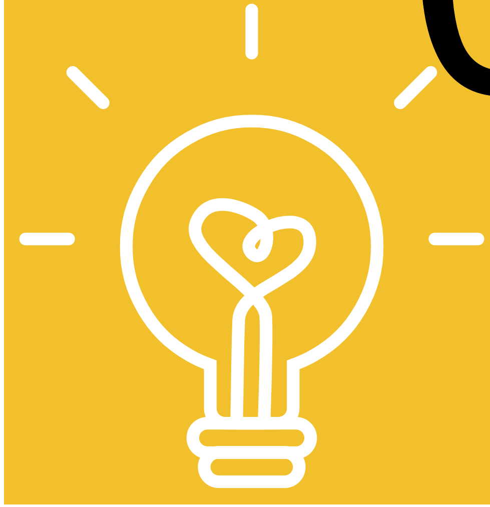
Some contents from June 1st 2017, Mini-course HSE Moscow
by Vincent Delecroix, CNRS researcher at LaBRI Bordeaux (France)
And “Open source in OR in ORNET, 9 September 2011

Outline

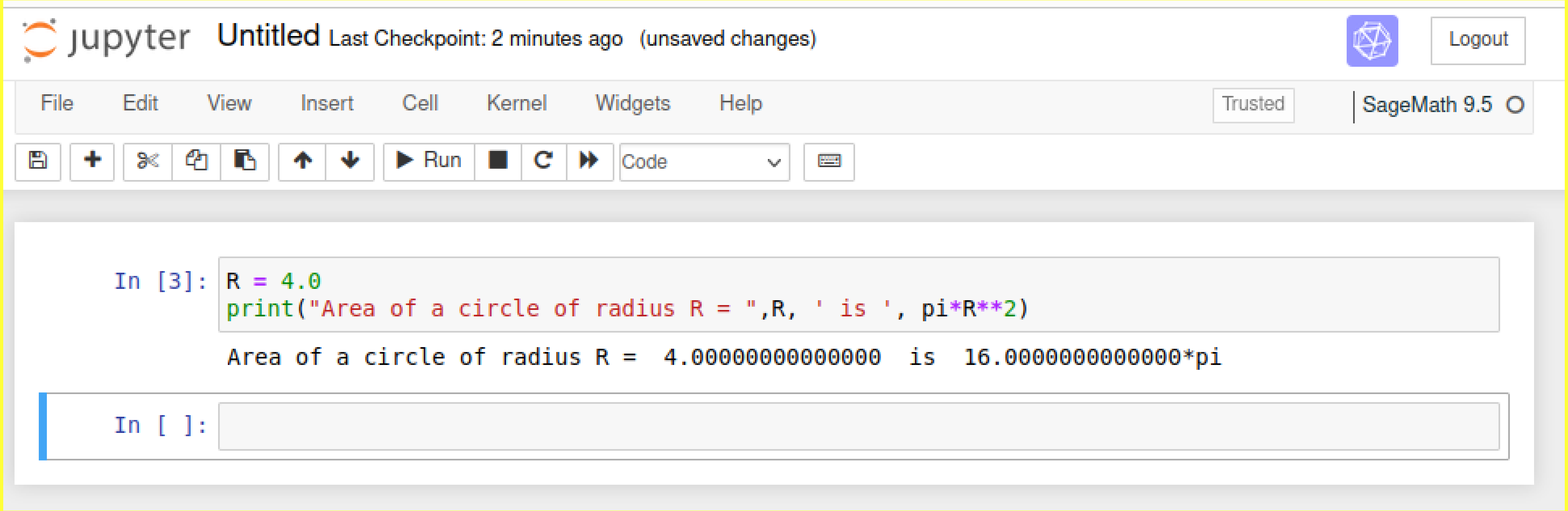
- Why do you want (good) math software?
- SageMath
- Sagemath Architecture
- Sagemath examples



01 Reason for Math Software

Why do you want (good) math software?

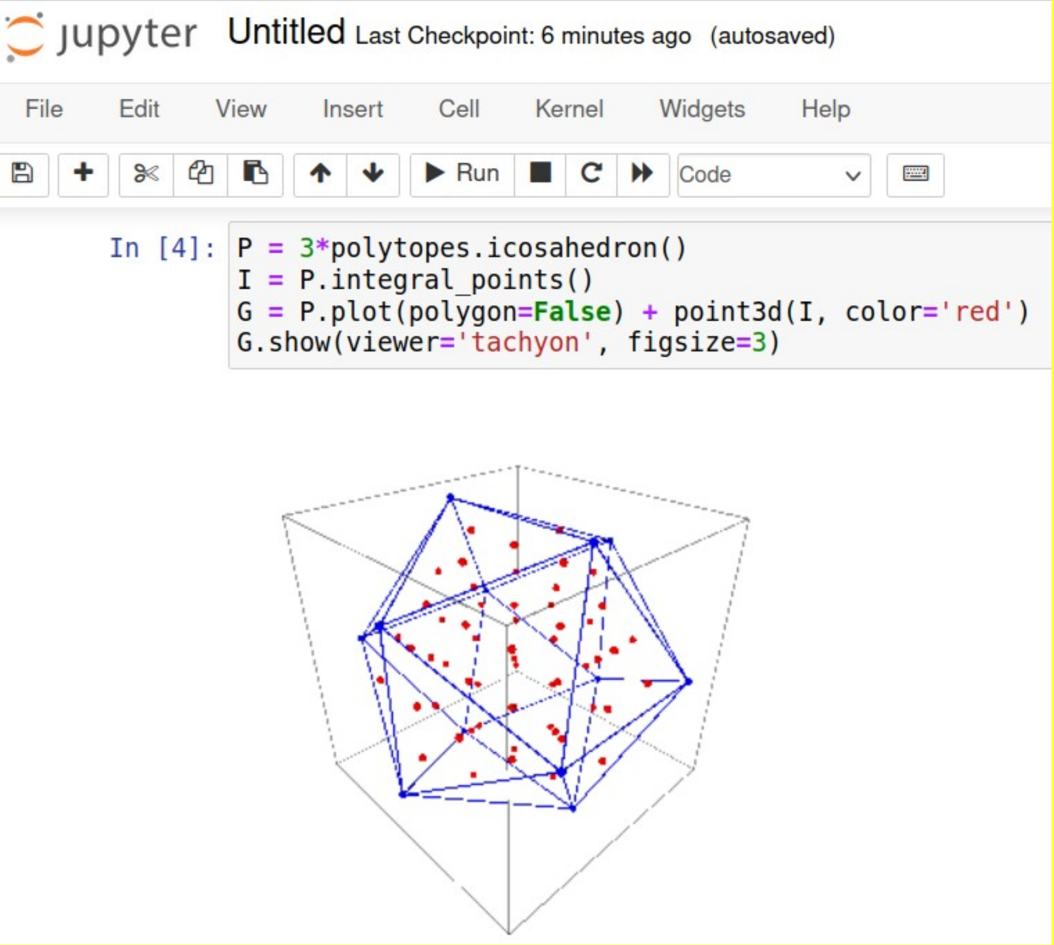
- Verify small computations



```
In [3]: R = 4.0
        print("Area of a circle of radius R = ",R, ' is ', pi*R**2)

        Area of a circle of radius R =  4.00000000000000  is  16.0000000000000*pi

In [ ]:
```

Why do you want (good) math software?

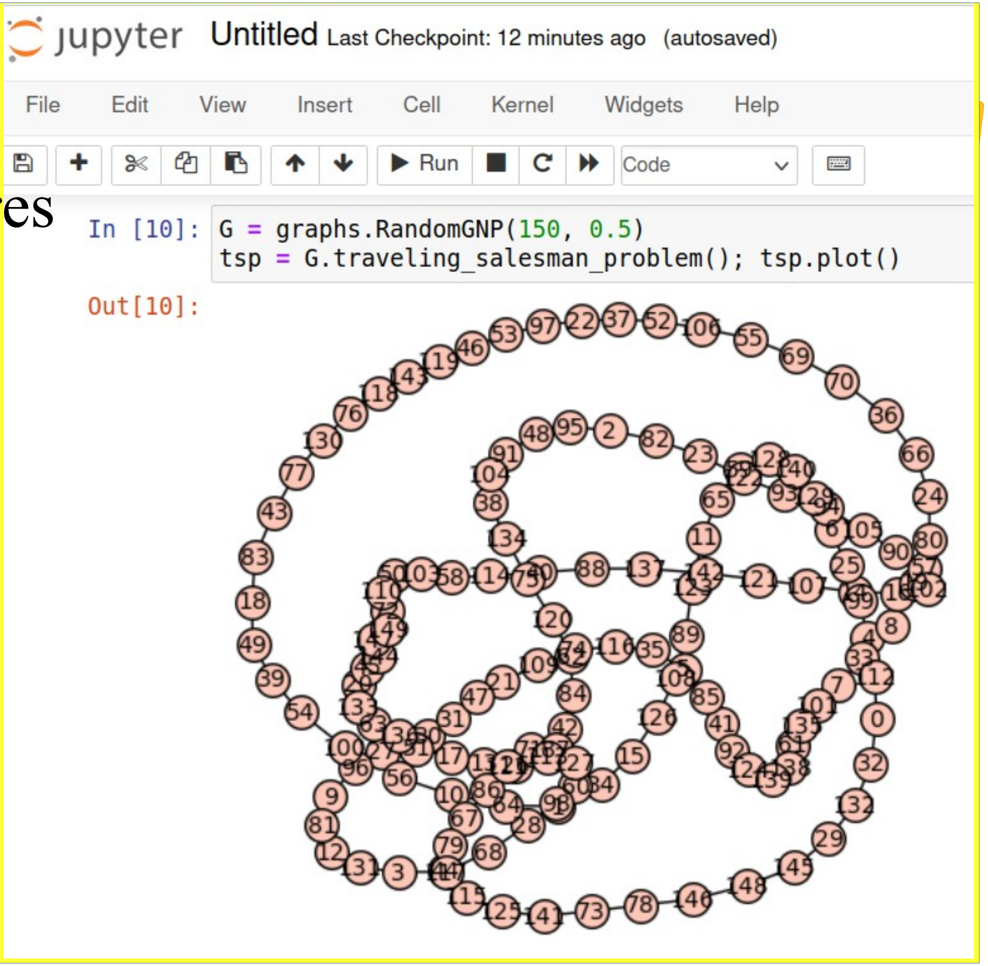
- Verify small computations
- Make nice illustration



```
P = 3*polytopes.icosahedron()
I = P.integral_points()
G = P.plot(polygon=False) + point3d(I, color='red')
G.show(viewer='tachyon', figsize=3)
```

Why do you want (good) math software?

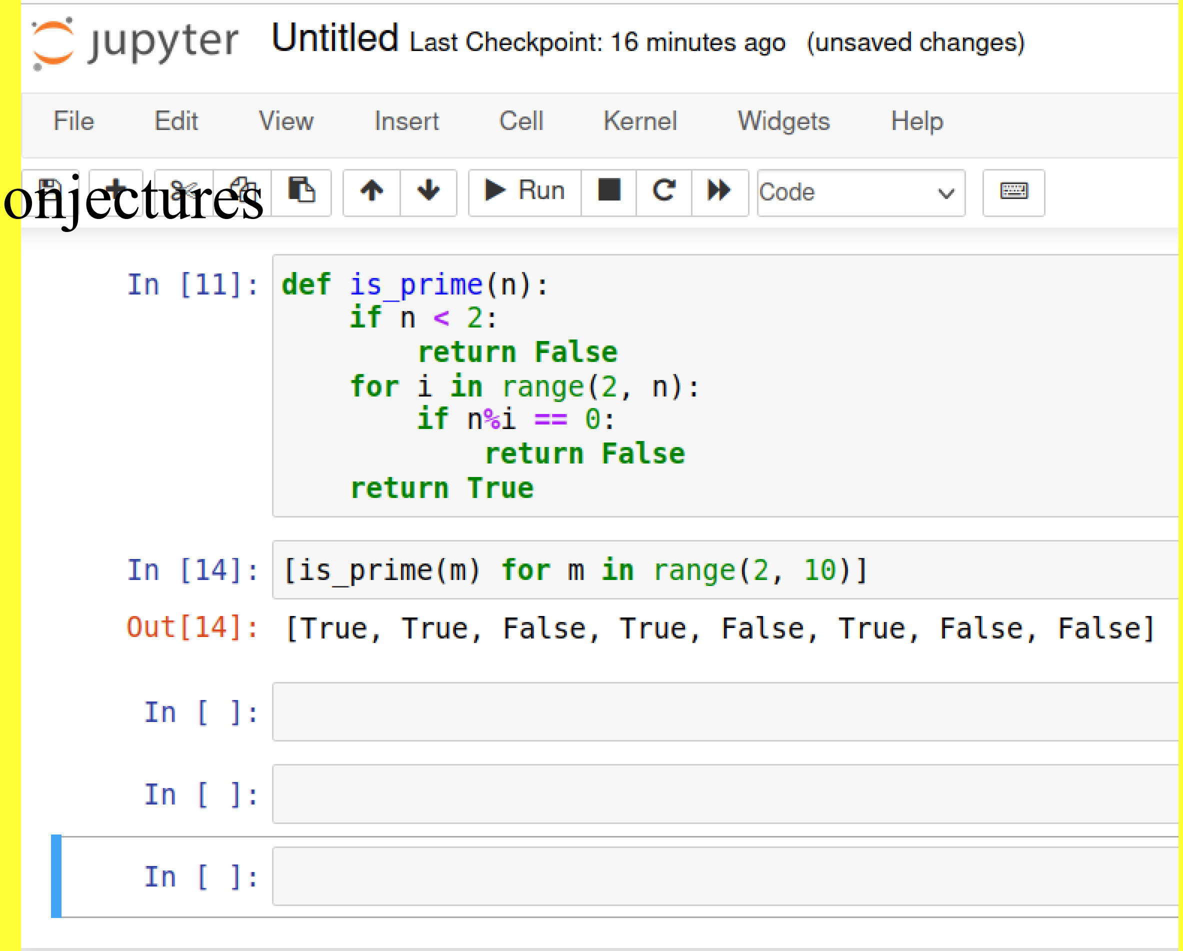
- Verify small computations
- Make nice illustration
- Make huge computations, test conjectures



```
In [10]: G = graphs.RandomGNP(150, 0.5)
         tsp = G.traveling_salesman_problem(); tsp.plot()
Out[10]:
```

Why do you want (good) math software?

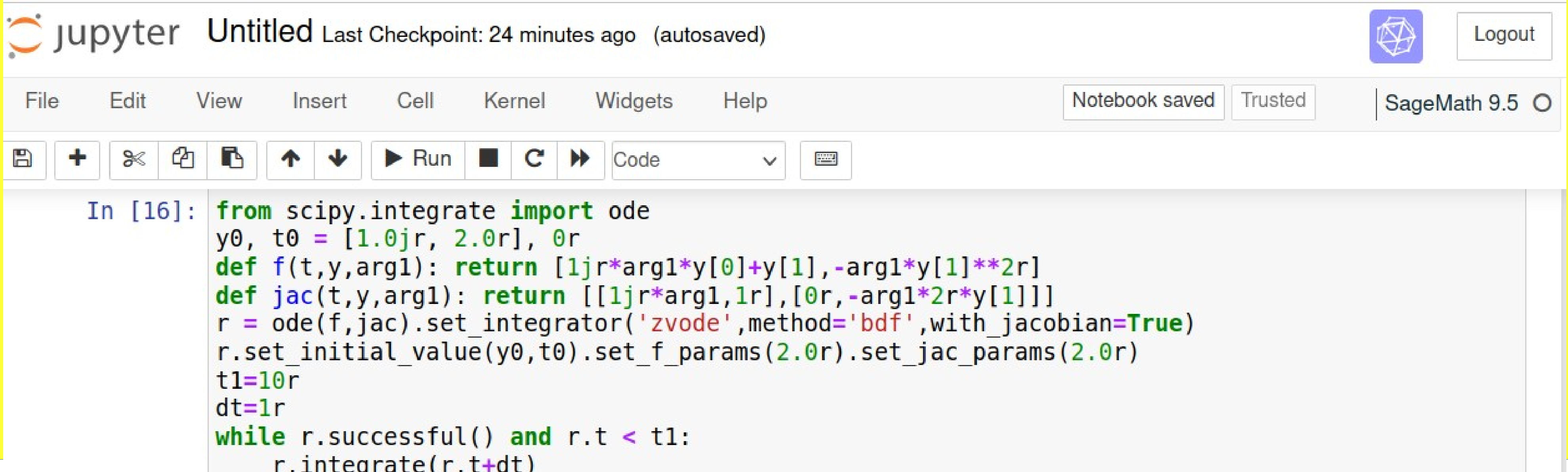
- Verify small computations
- Make nice illustration
- Make huge computations, test conjectures
- Develop new algorithms



```
In [11]: def is_prime(n):
             if n < 2:
                 return False
             for i in range(2, n):
                 if n%i == 0:
                     return False
             return True

In [14]: [is_prime(m) for m in range(2, 10)]

Out[14]: [True, True, False, True, False, True, False, False]

In [ ]:

In [ ]:

In [ ]:
```

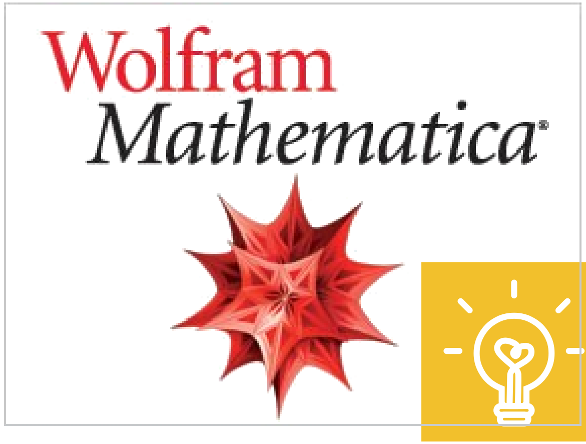
Why do you want (good) math software?

- Verify small computations
- Make nice illustration
- Make huge computations, test conjectures
- Develop new algorithms
- Experimental physics or mathematics, make conjectures



```
from scipy.integrate import ode
y0, t0 = [1.0jr, 2.0r], 0r
def f(t,y,arg1): return [1jr*arg1*y[0]+y[1],-arg1*y[1]**2r]
def jac(t,y,arg1): return [[1jr*arg1,1r],[0r,-arg1*2r*y[1]]]
r = ode(f,jac).set_integrator('zvode',method='bdf',with_jacobian=True)
r.set_initial_value(y0,t0).set_f_params(2.0r).set_jac_params(2.0r)
t1=10r
dt=1r
while r.successful() and r.t < t1:
    r.integrate(r.t+dt)
```

Four commercial math software



One department license

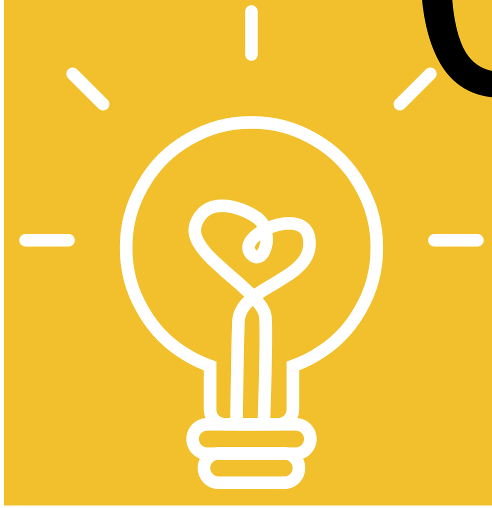
University license

Four commercial math software

- These software are expensive.
- No way to verify how they work.
- Sometimes impossible to get bugs corrected.
- They can disappear.

02

Sagemath Software

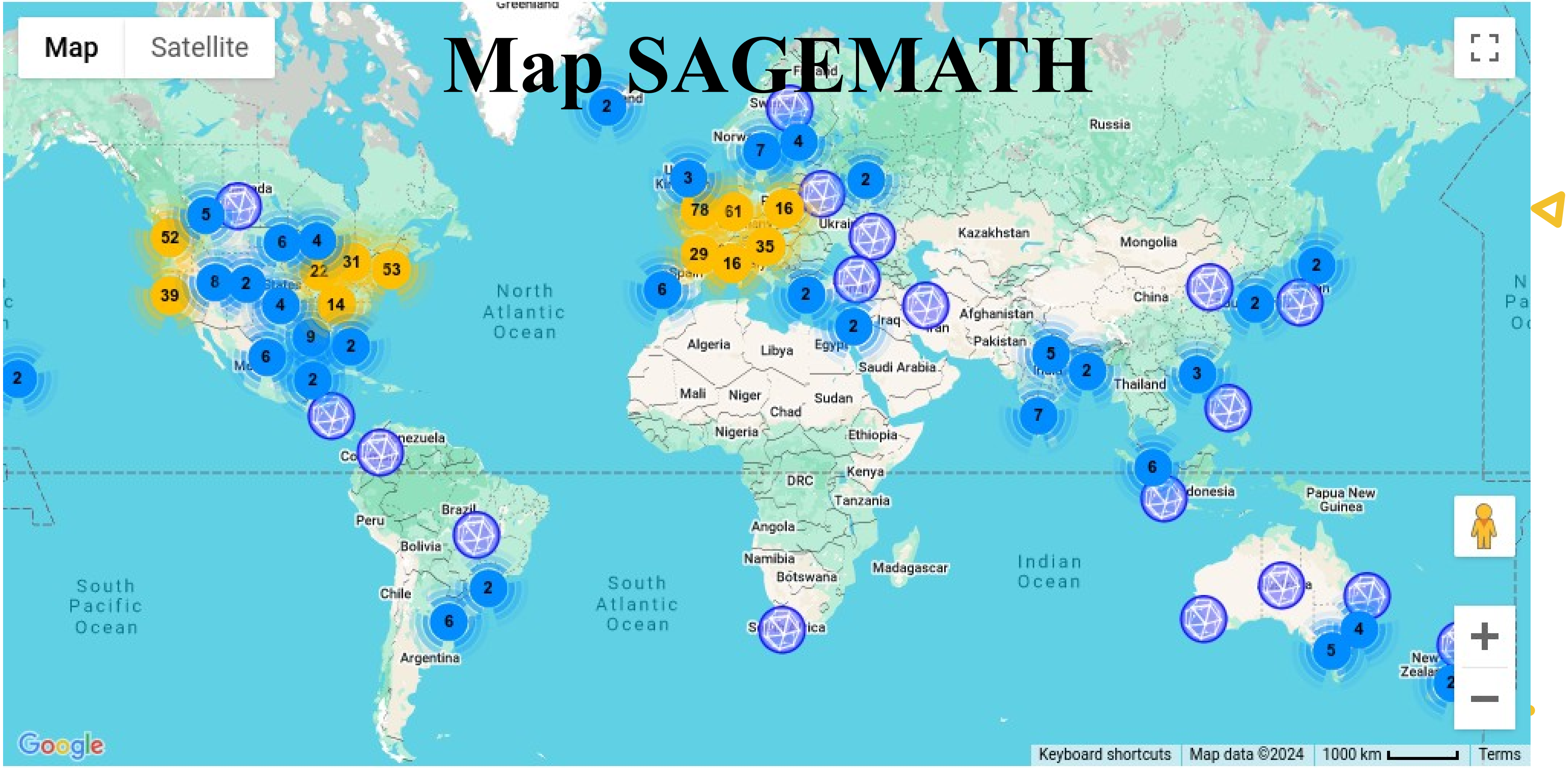


Free mathematical software

- Since the 80’s, there were GAP, R, Maxima, PARI/GP. Each of them is specialized. SageMath is an international project started in 2005 and aims to cover a large range of mathematics.

https://www.sagemath.org/development-map.html

Map SAGEMATH



https://www.sagemath.org/development-map.html

Four important ingredients

01 Open source license

GPL v3 Free Software

Free as in Freedom

02 Popular programming language

python™

03 Notebook interface

jupyter

iPython
python
GAP
NTL
PARI/GP
SINGULAR
GMP
MPFR
matplotlib
mwrank
Gitoro

04 Hundreds of free mathematical libraries

Open source license

- SageMath is free.
- Can be downloaded from internet https://www.sagemath.org/
- Source code at https://git.sagemath.org/sage.git/
- Contributions open to anyone https://trace.sagemath.org

Freeness will remain forever and is guaranteed by the GNU GPL license.

python™

- Python is a very popular programming language that is easy to learn and close to mathematical notation. The set

$$\{x \in \{1,\ldots,20\} : 2x^2-1 \text{ is prime}\}$$

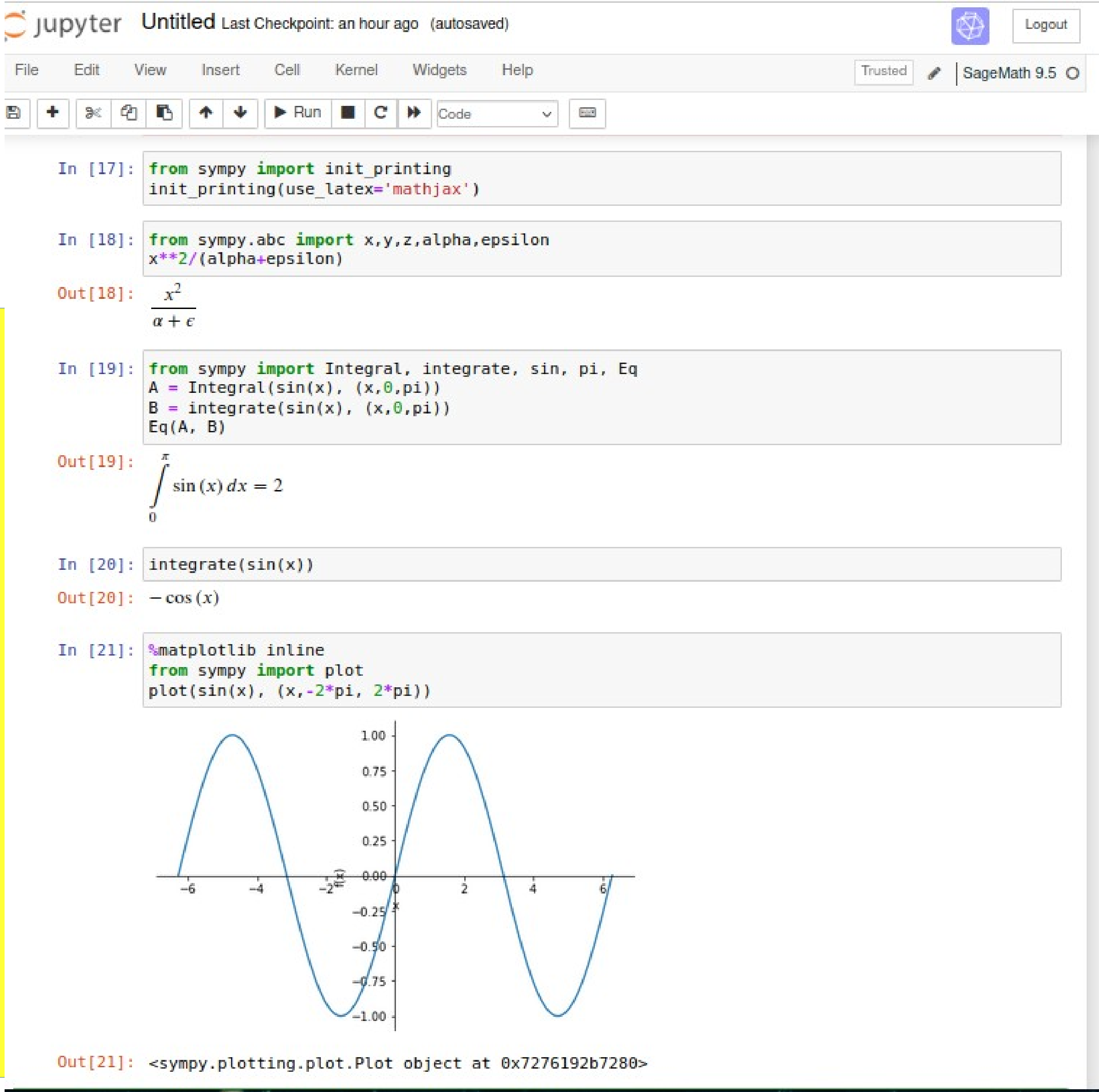
can be constructed in Sage as

```
[x for x in [1..20] if is_prime(2*x^2-1)]
```

Python is used for many other purposes: web programming, script language, biology, data analysis, etc.

- Jupyter is generic web interface for programming language. It can be used with many different languages and softwares: Sage, PARI/GP, C, C++, etc. The list of kernels can be found at

  https://github.com/jupyter/jupyter/wiki/Jupyter-kernels



```
In [17]: from sympy import init_printing
         init_printing(use_latex='mathjax')

In [18]: from sympy.abc import x,y,z,alpha,epsilon
         x**2/(alpha+epsilon)
```

Out[18]: $\frac{x^2}{\alpha + \epsilon}$

```
In [19]: from sympy import Integral, integrate, sin, pi, Eq
         A = Integral(sin(x), (x,0,pi))
         B = integrate(sin(x), (x,0,pi))
         Eq(A, B)
```

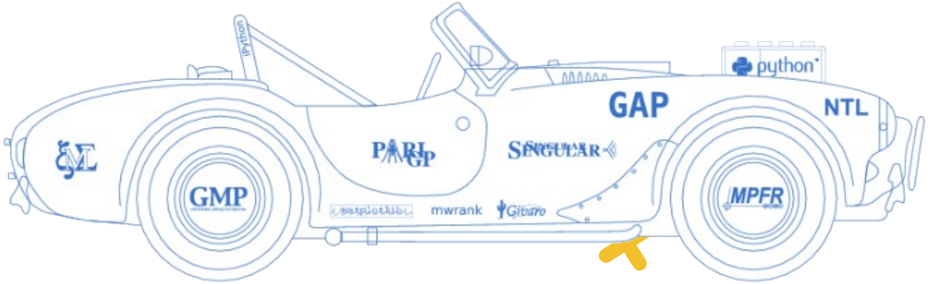
Out[19]: $\int_0^\pi \sin(x)\,dx = 2$

```
In [20]: integrate(sin(x))
```

Out[20]: $-\cos(x)$

```
In [21]: %matplotlib inline
         from sympy import plot
         plot(sin(x), (x,-2*pi, 2*pi))
```

```
Out[21]: <sympy.plotting.plot.Plot object at 0x7276192b7280>
```



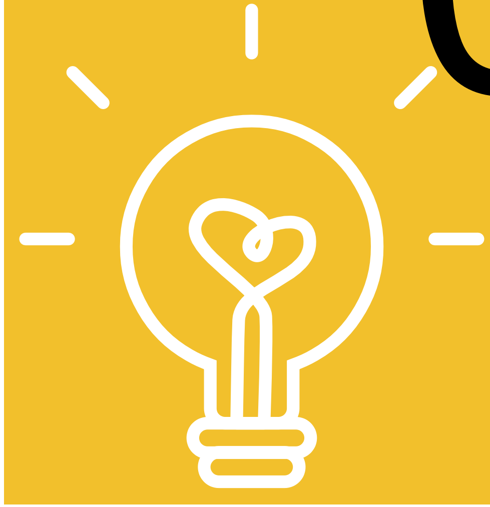
Sage is built on top of hundreds of scientific libraries and software.

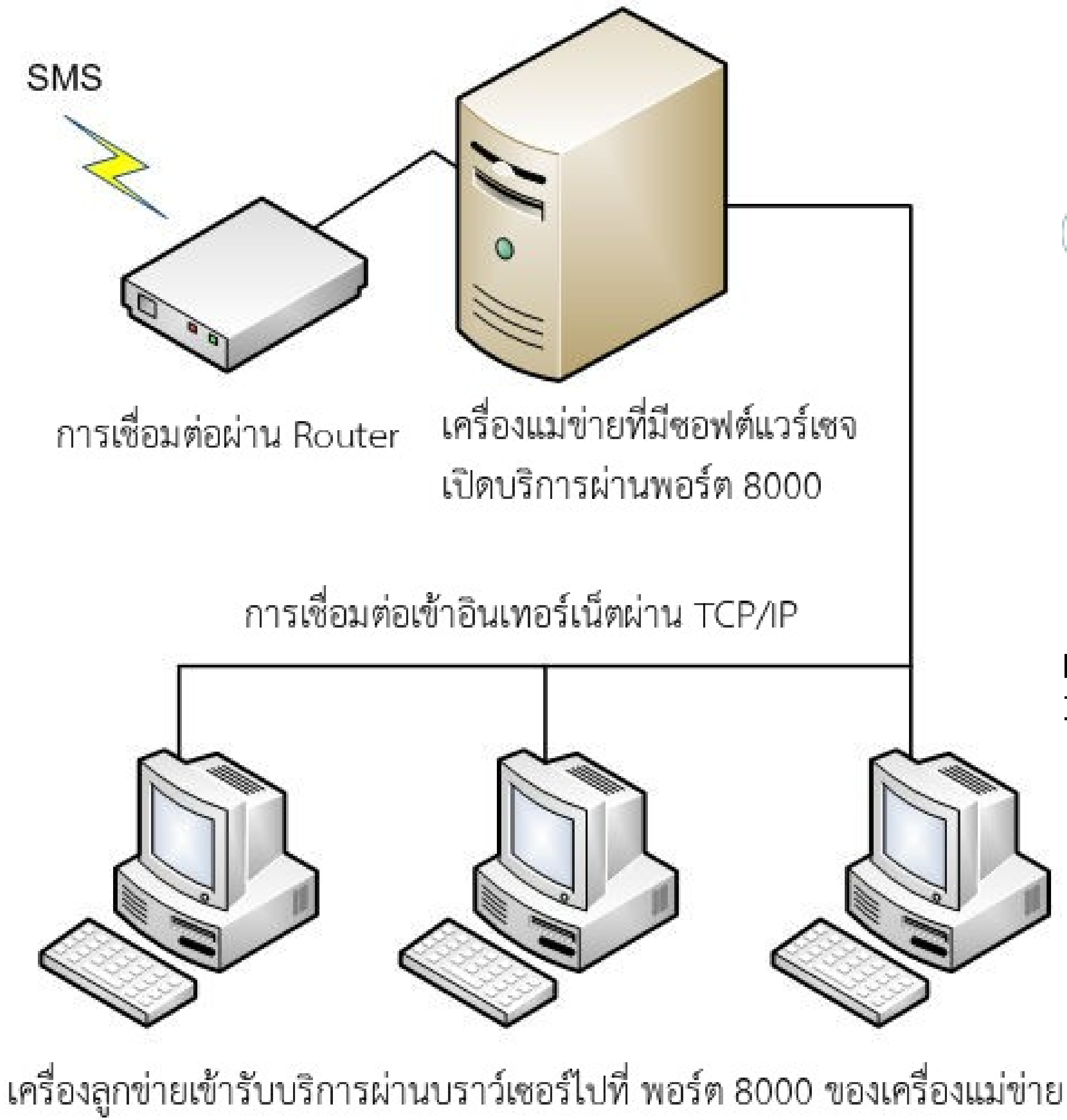
- GMP or MPIR:C library for arbitrary precision integers
- Flint:a C library for algebra
- Linbox:C++ library for exact linear algebra
- PARI/GP:a CAS for number theory
- GAP:a CAS for group computations
- Complete list at

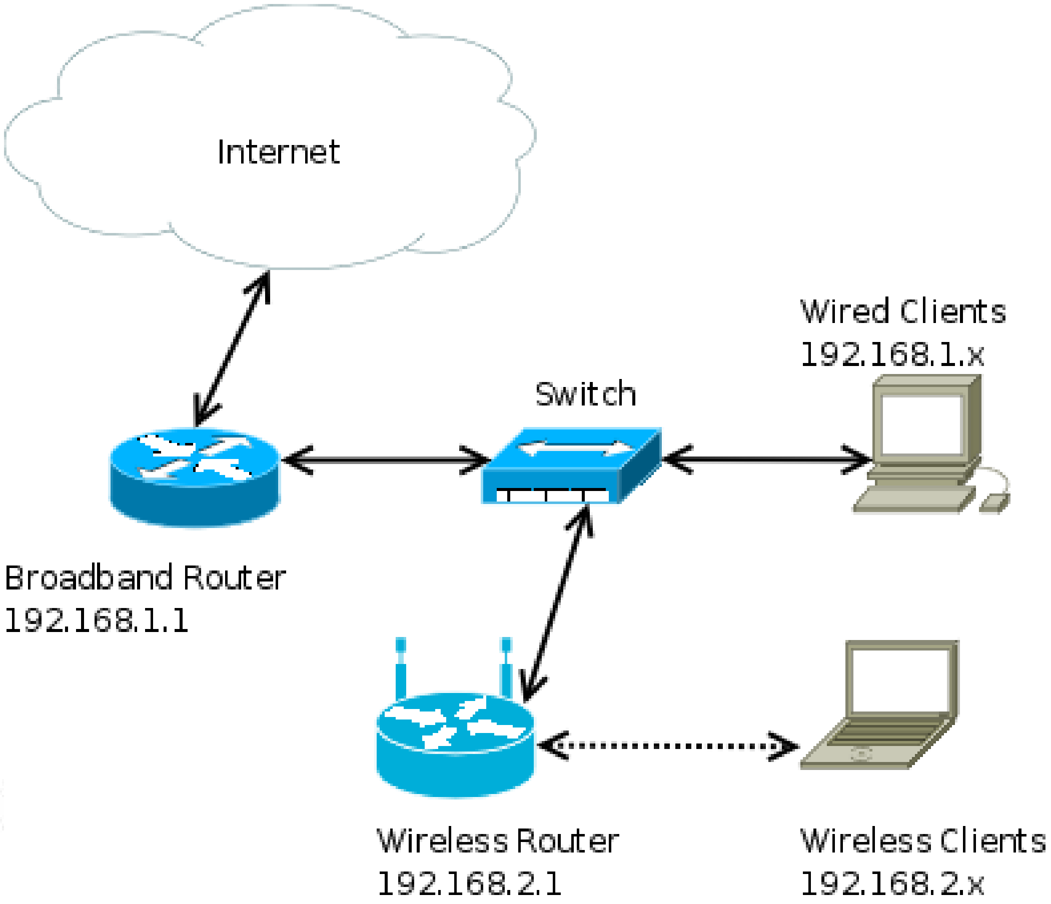
http://www-ftp.lip6.fr/pub/math/sagemath/spkg/upstream/

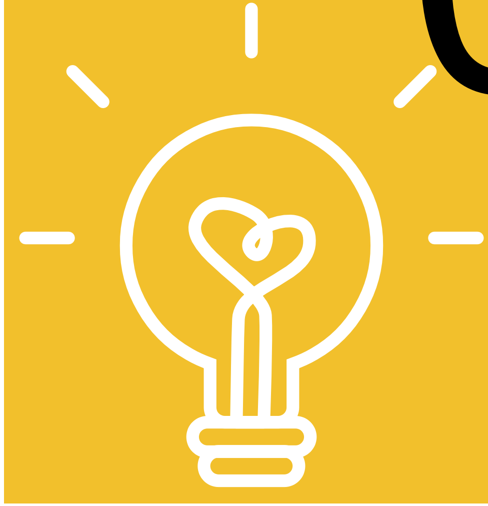
03

Sagemath architecture

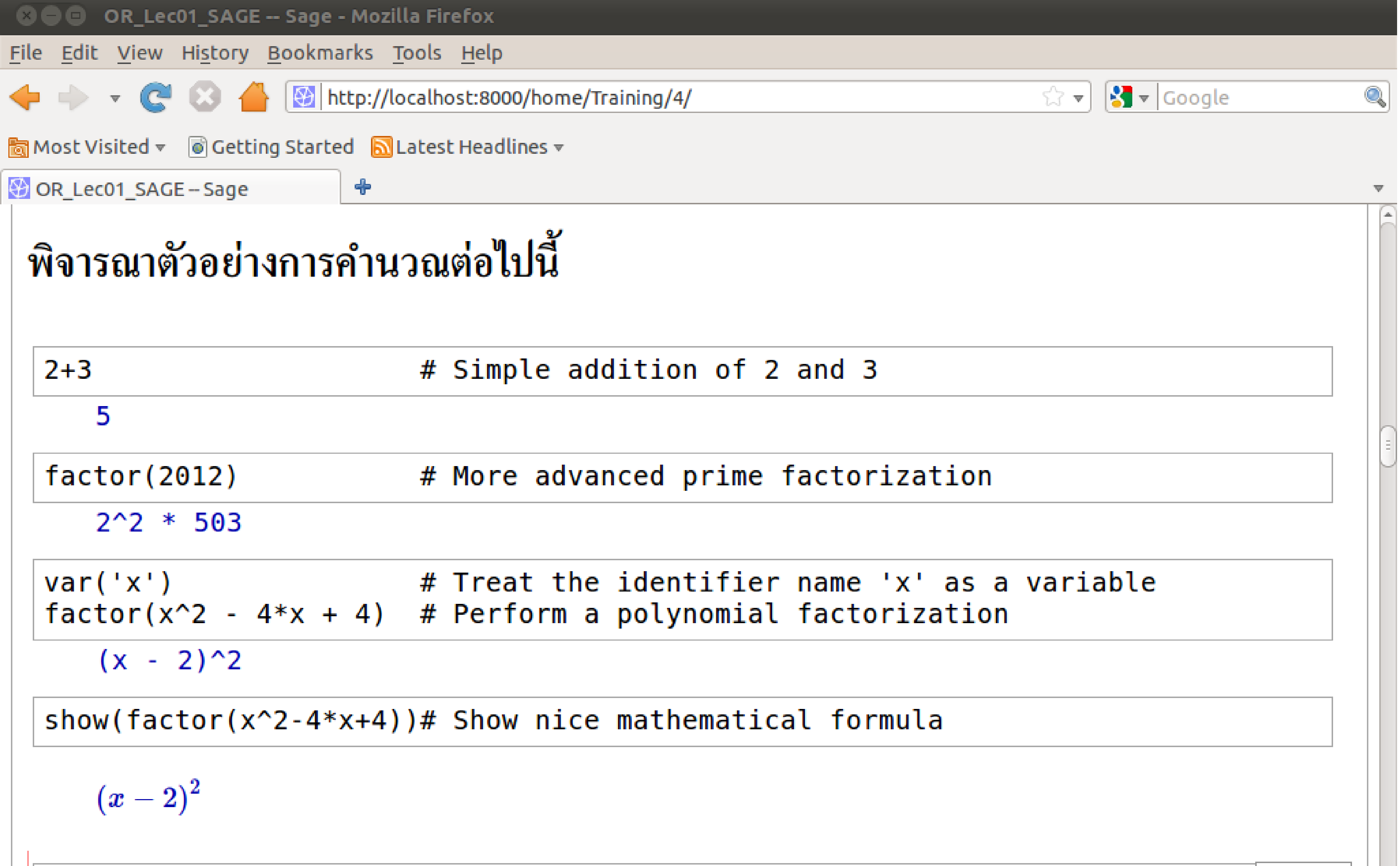



SMS
การเชื่อมต่อผ่าน Router
เครื่องแม่ข่ายที่มีซอฟต์แวร์เซจ
เปิดบริการผ่านพอร์ต 8000
การเชื่อมต่อเข้าอินเทอร์เน็ตผ่าน TCP/IP
เครื่องลูกข่ายเข้ารับบริการผ่านบราว์เซอร์ไปที่ พอร์ต 8000 ของเครื่องแม่ข่าย


Internet
Wired Clients
192.168.1.x
Switch
Broadband Router
192.168.1.1
Wireless Router
192.168.2.1
Wireless Clients
192.168.2.x



04 Sagemath Examples



พิจารณาตัวอย่างการคำนวณต่อไปนี้

```
2+3                      # Simple addition of 2 and 3
```

```
    5
```

```
factor(2012)             # More advanced prime factorization
```

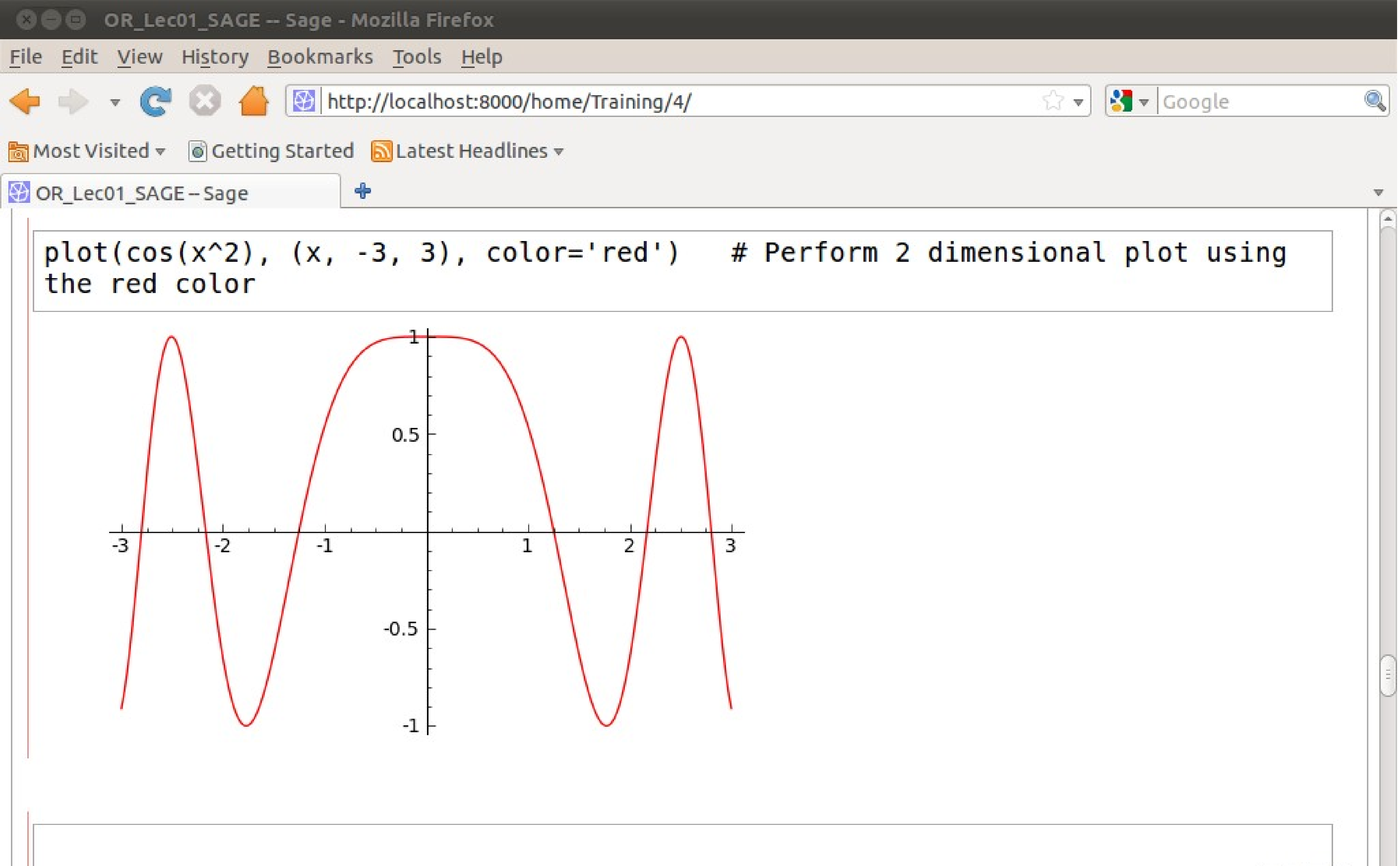
```
    2^2 * 503
```

```
var('x')                 # Treat the identifier name 'x' as a variable
factor(x^2 - 4*x + 4)    # Perform a polynomial factorization
```

```
    (x - 2)^2
```

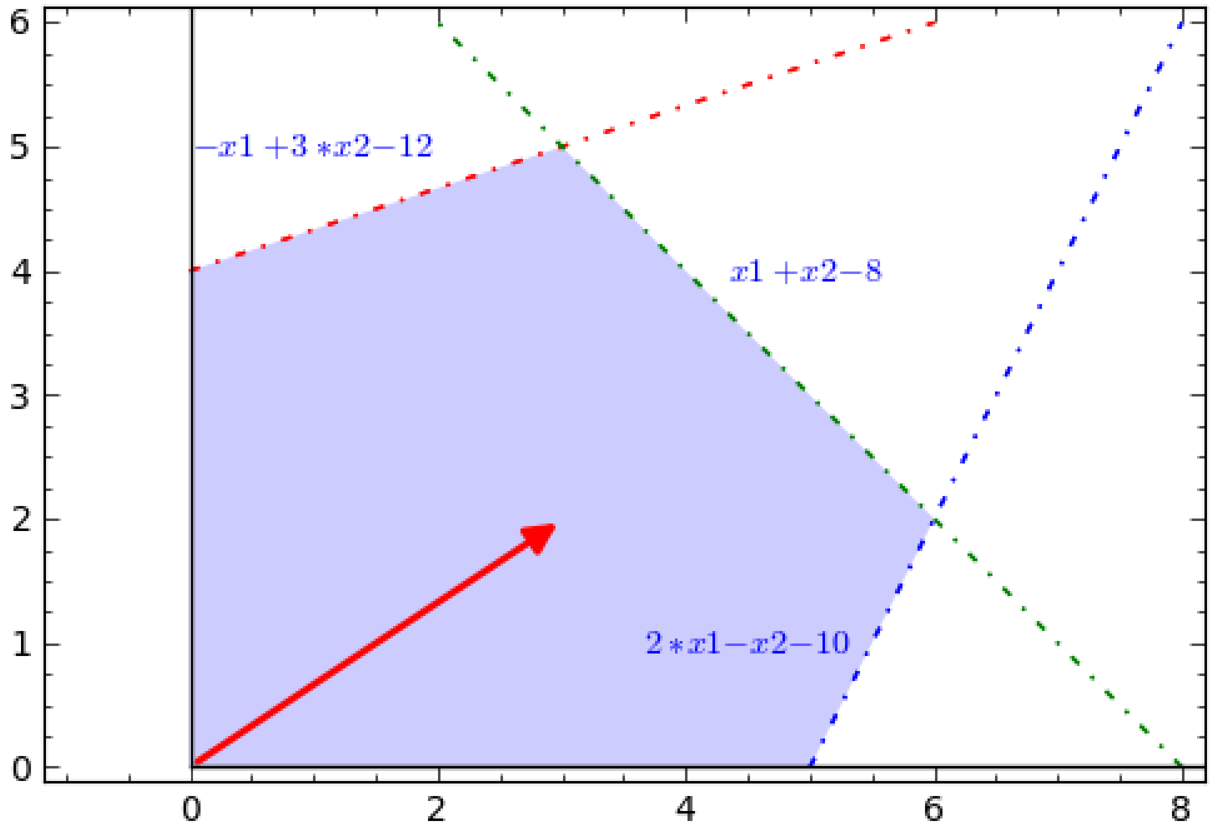
```
show(factor(x^2-4*x+4))# Show nice mathematical formula
```

$$(x-2)^2$$

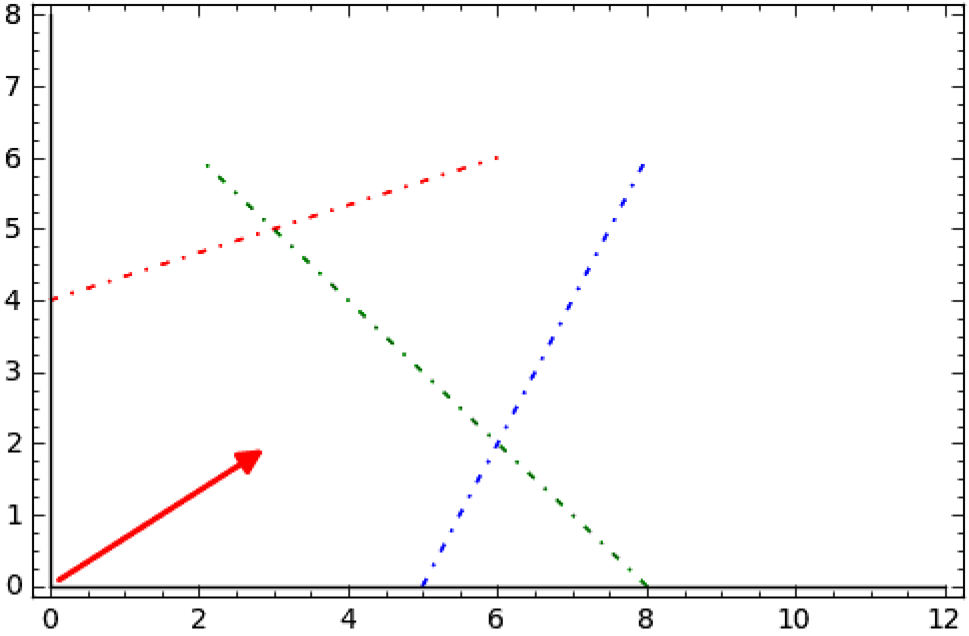


```
plot(cos(x^2), (x, -3, 3), color='red')   # Perform 2 dimensional plot using
the red color
```

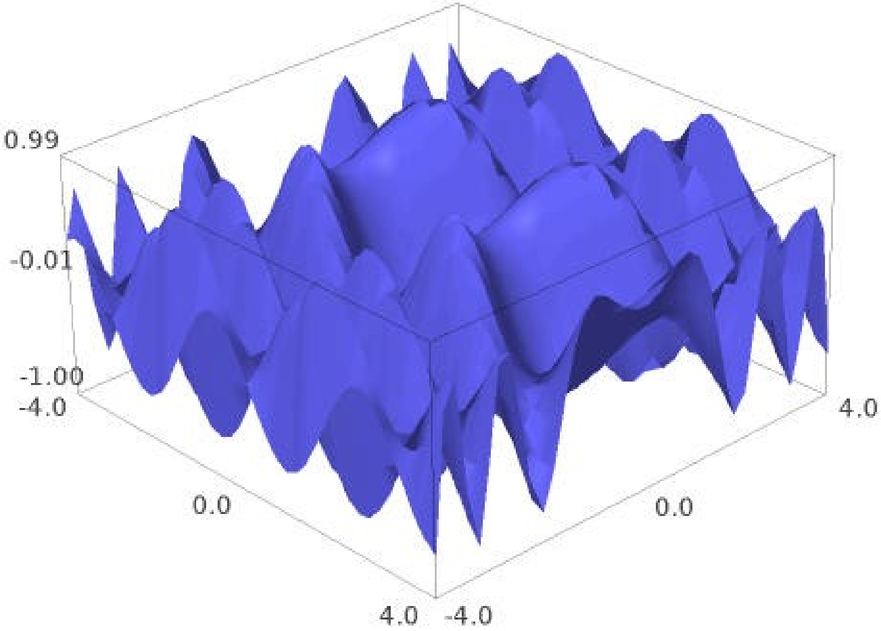
Draw a feasible region

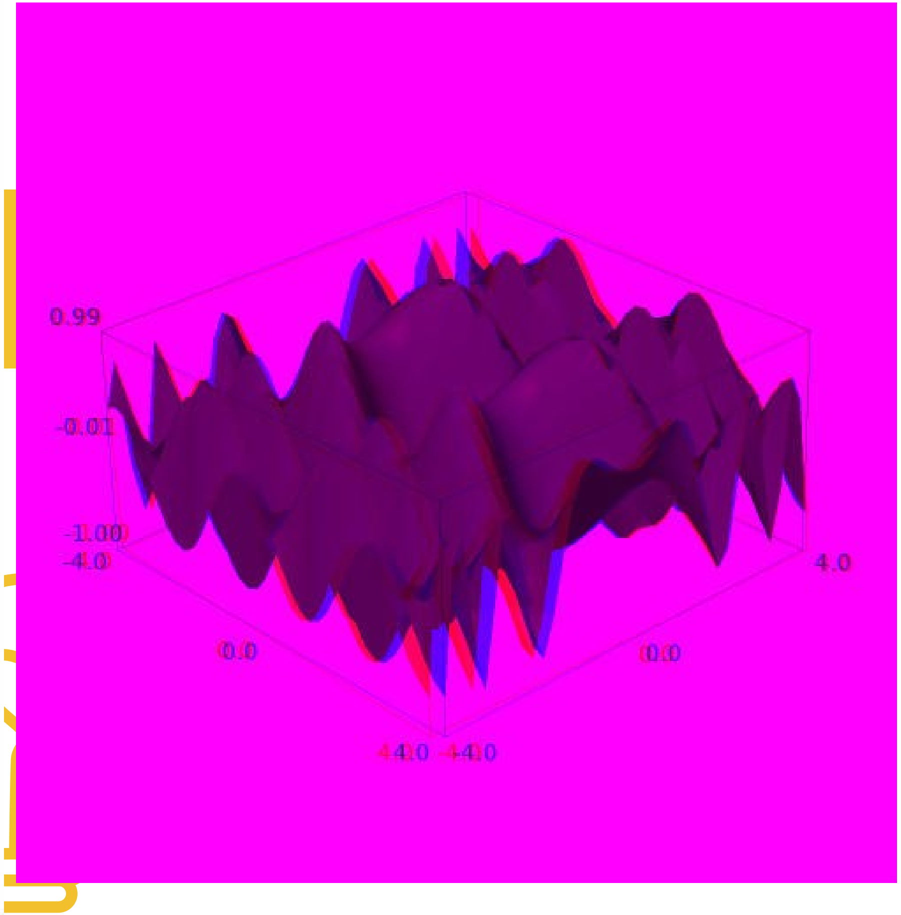


Create animation

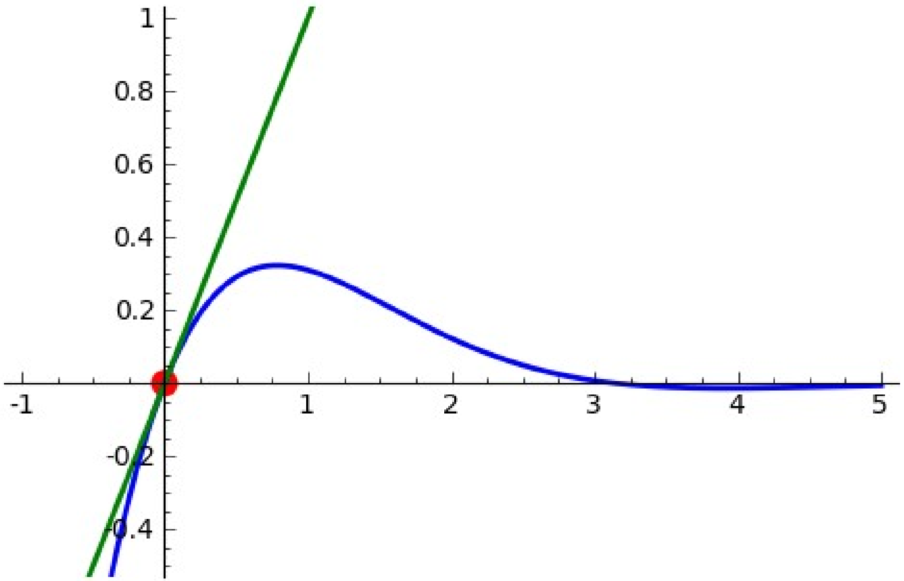


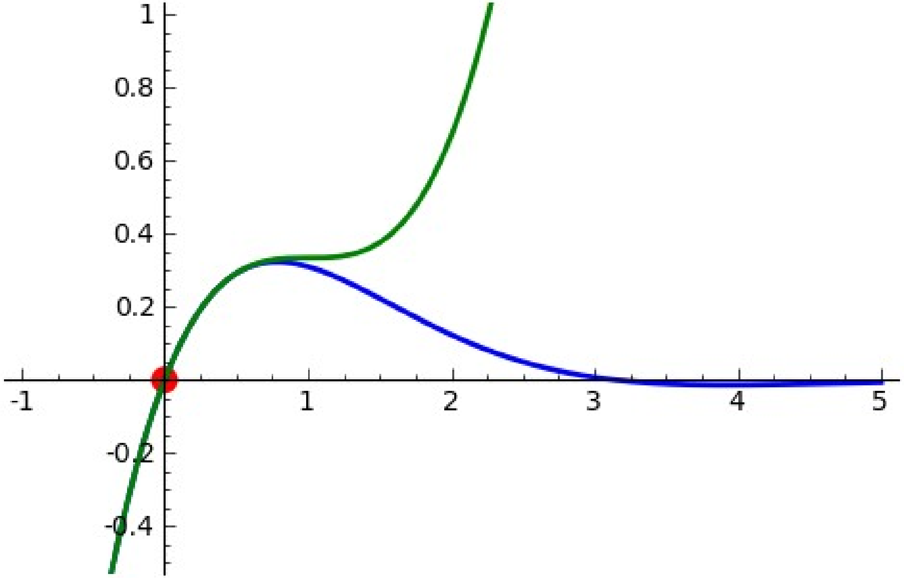
Create 3D



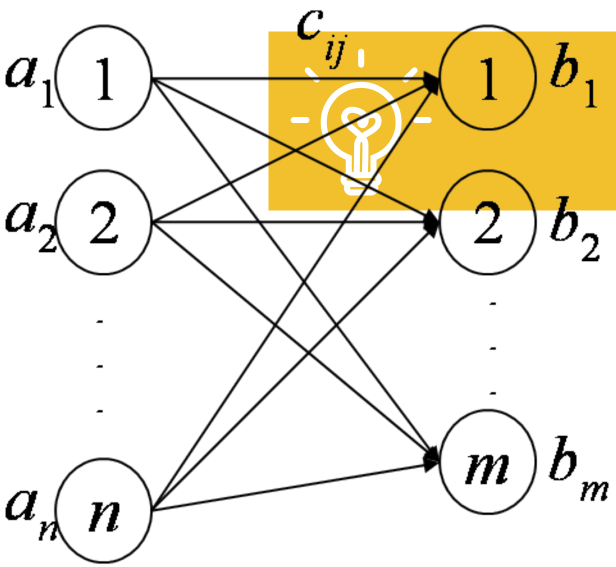


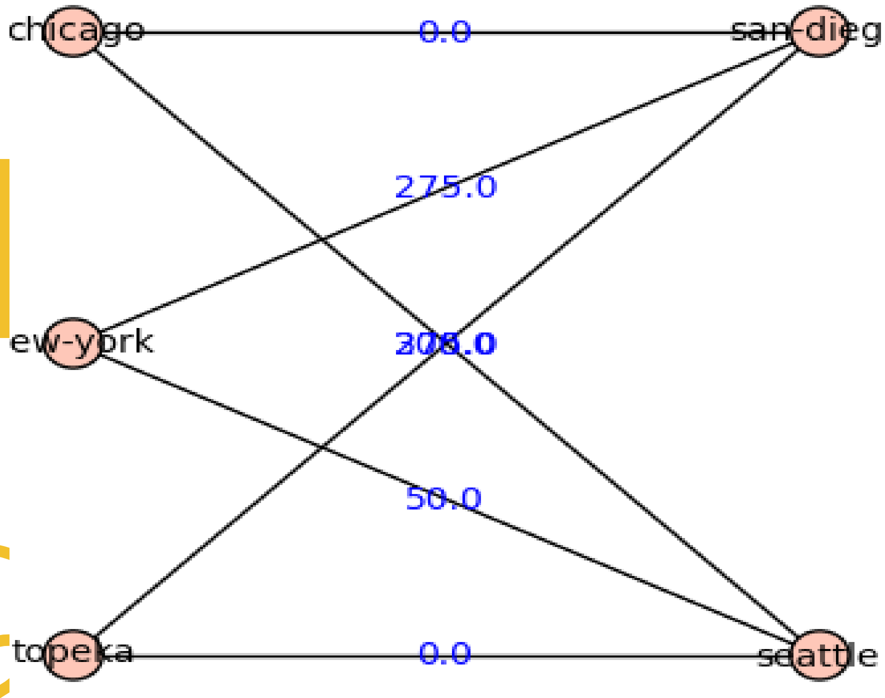
Interactive plotting



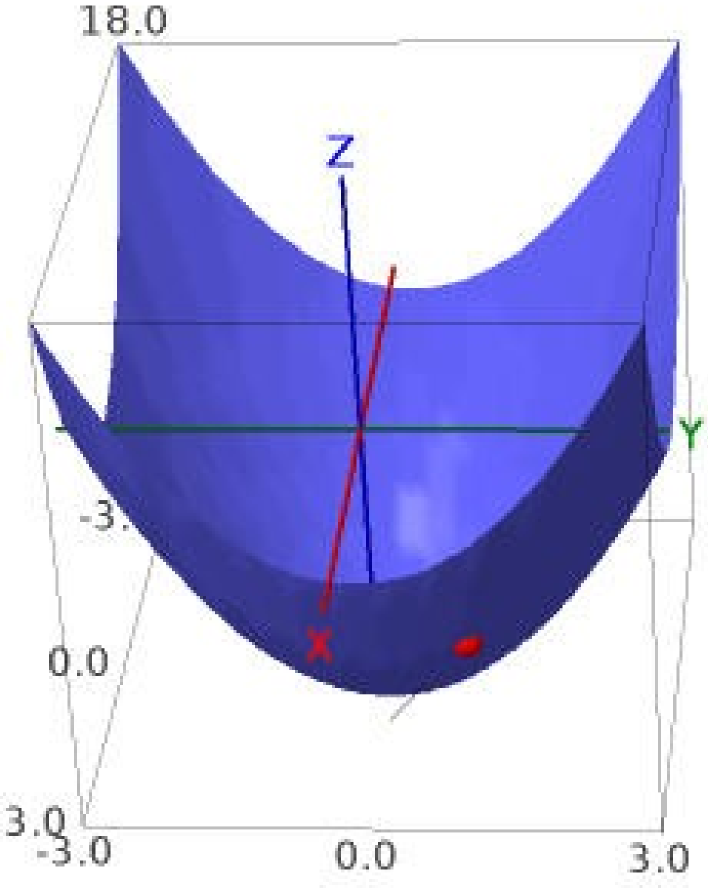


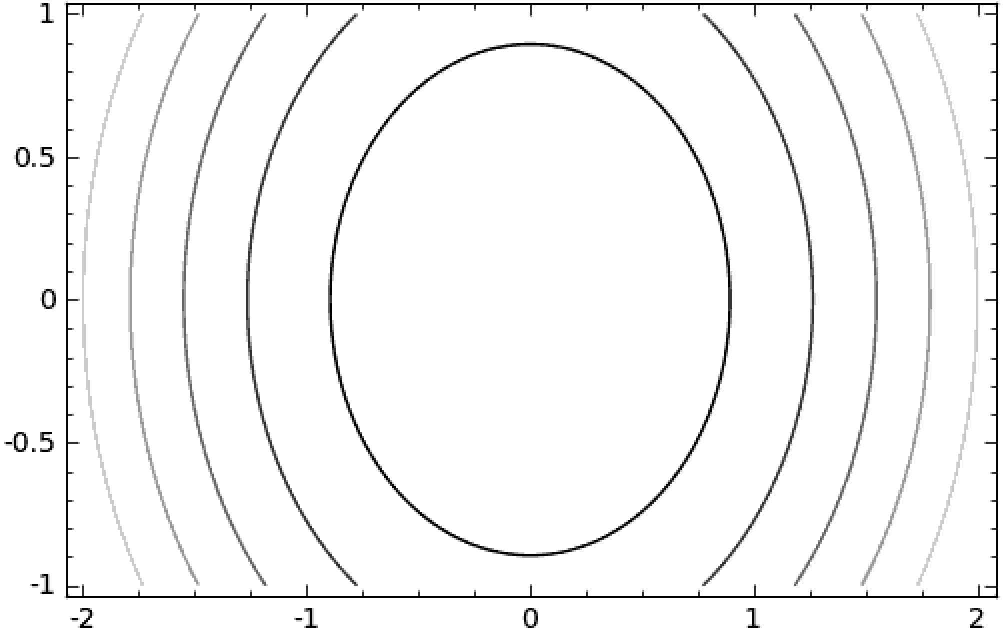
Solving OR problem





Solving nonlinear program





References

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THANK YOU

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