Calculation of rainfall
on an arbitrary two-dimensional surface

Preface

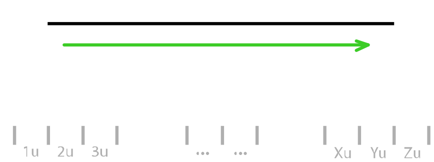
This article describes an algorithm that calculates a rainfall in a finite two-dimensional surface defined by its heights.

Problem: Calculate a volume of water which remained after the "rain" on a 2D "surface" (in **units** - *a dimensionless quantity*).
Input: An array of integer numbers (describes profile's heights of a "surface").
Output: A number, further called "volume"

Terms

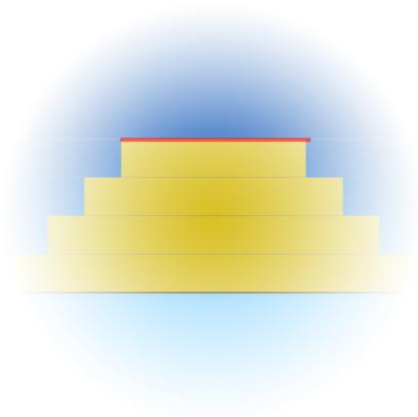
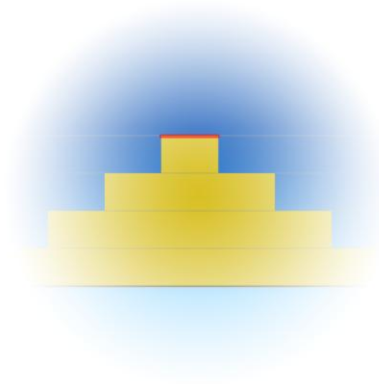
To solve this problem, we will use an object-oriented approach that gives us the opportunity to abstract from pure mathematics and describe the algorithm in terms and concepts that are accessible to understanding of a wide audience.

Plateau

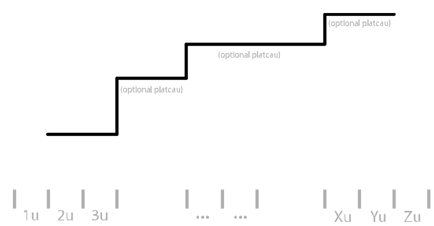


A continuous set of points (heights), that have same value. A plateau may consist of one single point as well.

Examples:

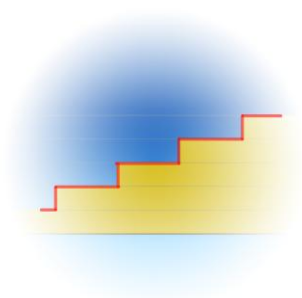
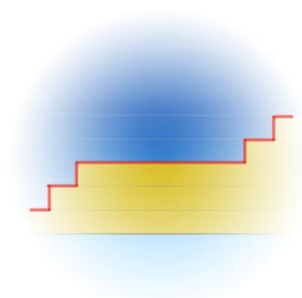
 

Ascent

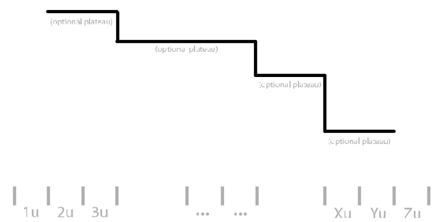


A set of points where each next point has value higher than the previous (may include one or more plateaus in the middle).

Examples:

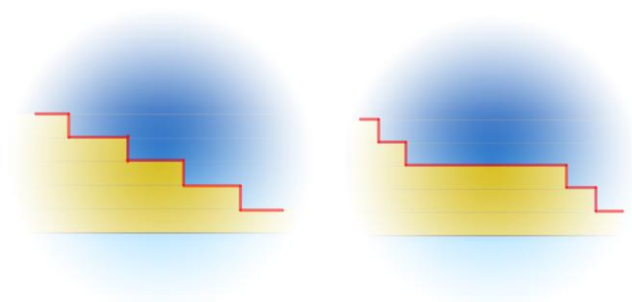
 

Descent

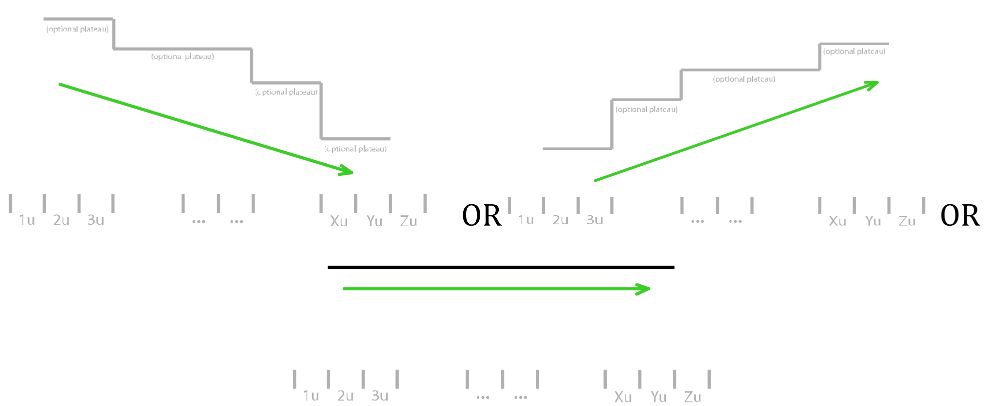


A set of points where each next point has value lower than the previous (may include one or more plateaus in the middle).

Examples:

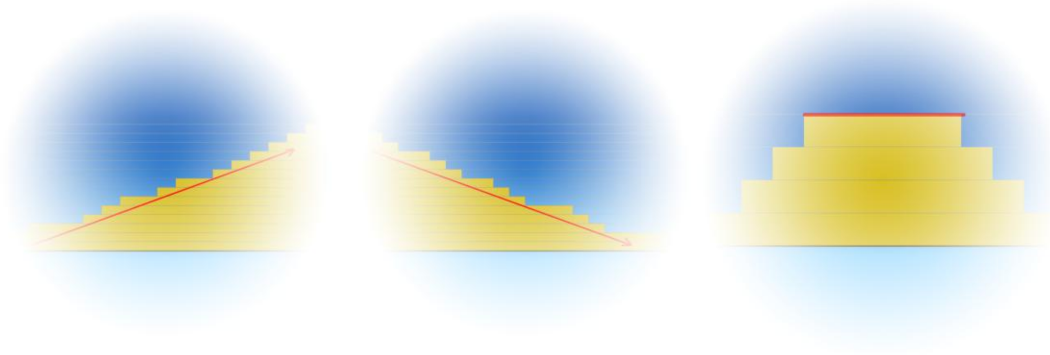


Trend

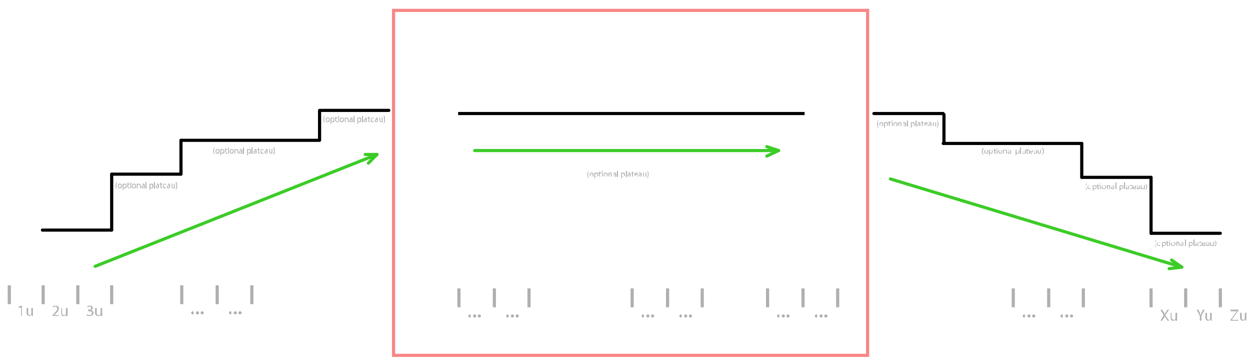


A set of constantly ascending values (and optionally plateaus in the middle) OR a set of constantly descending values (and optionally plateaus in the middle) OR a set of the same values that constitute a plateau.

Examples:

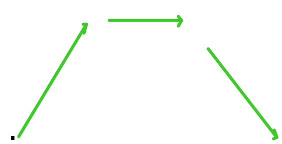


Peak

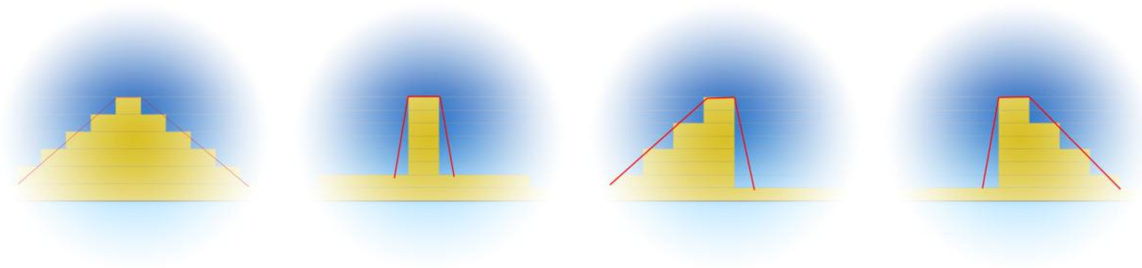


A highest point (a set of points, in the case of a plateau) in a sequence of the values which corresponds to a pattern: when **values are ascending, then form a plateau (one or more equal values), then descending**.

A peak pattern looks like this: 1) trend "*UP", 2) trend "plateau" OR a single point, trend "DOWN"*

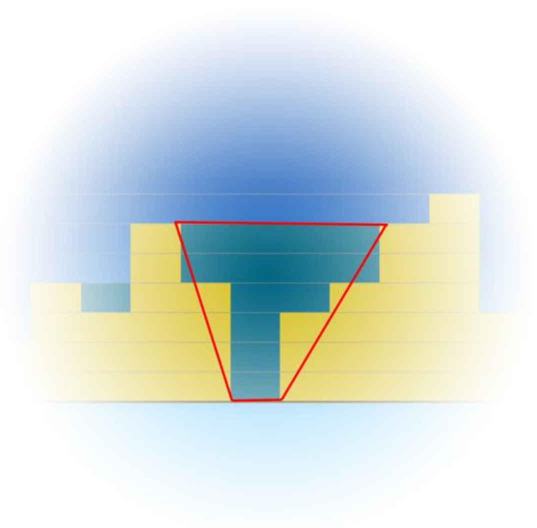


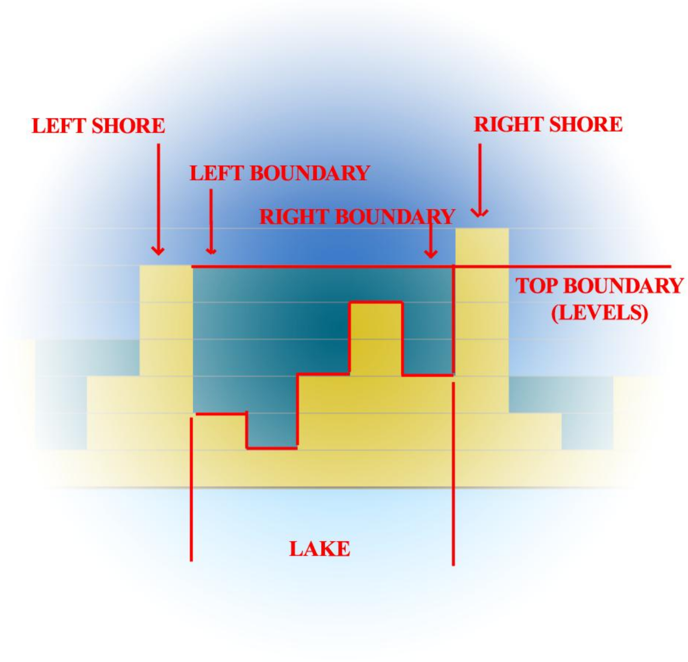
Examples:



Lake

An area between a descent and an ascent, defined by its **left and right boundaries** , which's **top boundary (the level)** is limited by the lower of two peaks of that trends**. Any lake may exist only between exactly two peaks - left and right**.

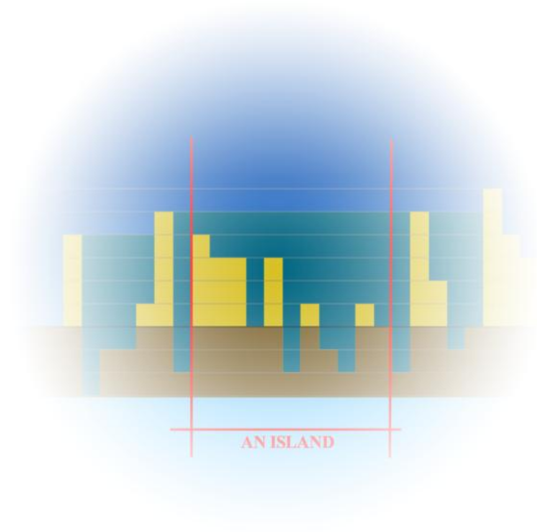




Island

Island is an arbitrary subset of the peaks (one or more) from the all available peaks.

Example:



Algorithm Implementation

In the mentioned terms, in bird's-eye view, the algorithm may be described in the next steps:

1. **Step 1**: Find all "peaks" defined by the means of their heights (with the "trend up-plateau-trend down" pattern);

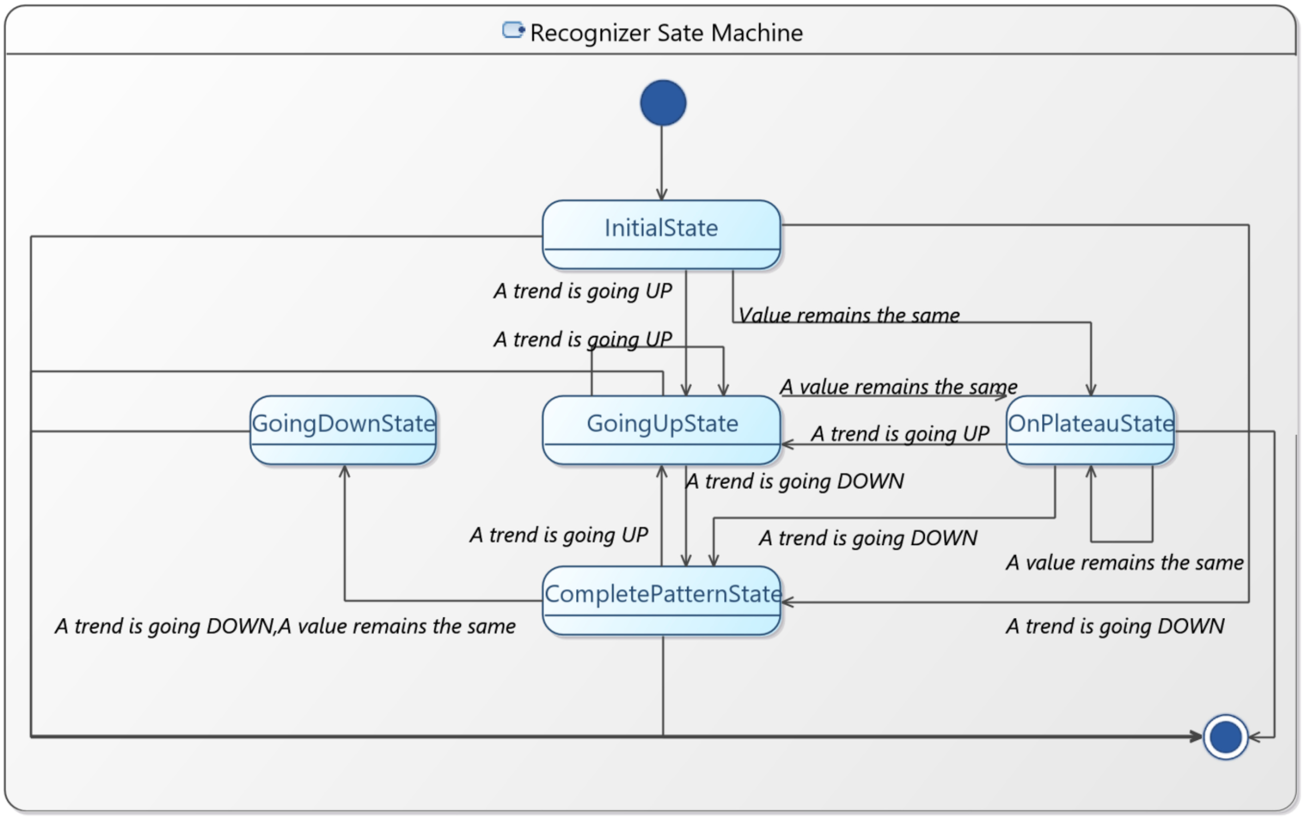
2. **Step 2**: Find all the "lakes" that exists among the revealed "peaks";

3. **Step 3**: Calculate the integral sum of volume of each such lake;

Step 1. Find all "peaks" defined by the means of their heights

To find the peaks we will reuse a state machine, that should recognize the up-plateau-down pattern.

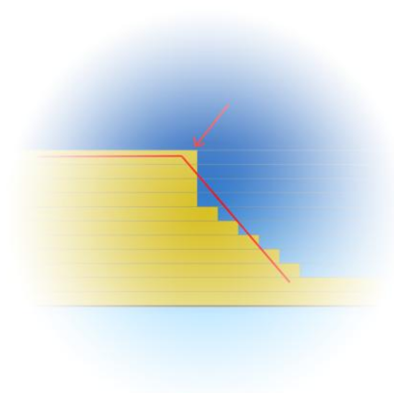
Peak recognizer state machine is following the values trends and collects the indexes of that values in the income heights array. Here is the map of state transitions:



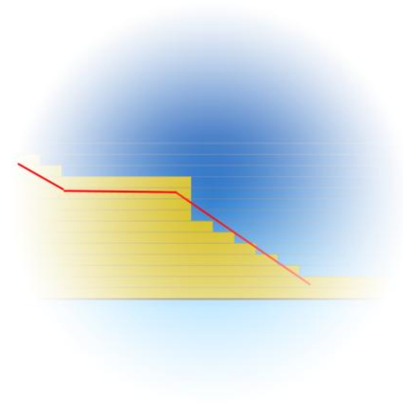
Each state takes its place as a respond to the values trend changing event. Values become "run" in a different (than the current) direction - a new state is activated.

To understand the transitions, let's take a look at the few simple rules underlying these operations:

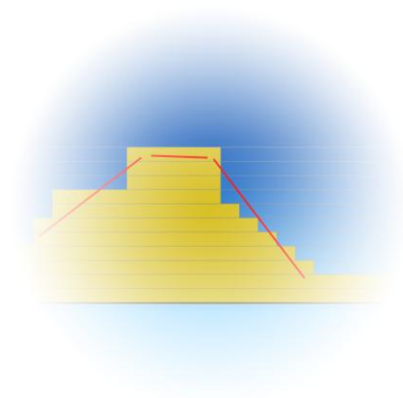
1. Actually, an **every new peak starts only when a new trend is going down after a plateau or a single highest value**;



2. At the same time - the mandatory prerequisite for 1) is that **there should be an ascent or an initial plateau before it**. What does it mean: a descent after a plateau (single point) which is also following after a descent - doesn't necessary marks a peak.
   See this exclusion example (a descent, after a plateau, after another descent - this is certainly not a peak):



3. This is where the **"Initial State"** comes to be useful - it guarantees that the 2) prerequisite is true at the very beginning. After that the pattern recognition is managed by the corresponding state - **up -(optional plateau) - down**.



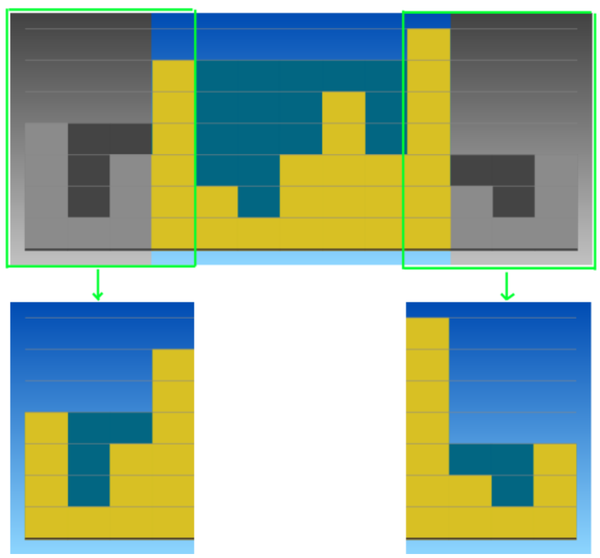
4. And so on - every state is collecting indexes of heights until their trends conforms to the "up-plateau-down" pattern. And after the **plateau-down** stage we may find the machine into the "**CompletePatternState**" which contains the plateau of the similar points (an array of indexes) or a single highest point (if there were no other points at the same level) - **an instance of the Peak object**.

5. Only the "**CompletePatternState**" provides a set of highest values indexes on a particular range of indexes. Being in " CompletePatternState" means that the "up - optional plateau - down" pattern was recognized.

Step 2. Find all the "lakes" that exists among the revealed "peaks"

This is the recursive iterative step, which's main operation are:

1. Find exactly two highest peaks - **L** (highest from the left) and **R** (highest from the right);
2. Flood the entire space between them (including other peaks that are less that the mentioned two) forming a lake;
3. Collect the lake to a **lakes' store**;
4. Break the current island on a two - leftmost peaks (including L), and rightmost peaks (including R);
5. Repeat from the **step 1)** for each such island (that contains more than one peak - a lake can exist only between at least two peaks)



Step 3: Calculate the integral volume of each lake

When all islands are processed, there is only step left - run trough the lakes collection and sum all lakes volumes. **A lake volume is a difference among lakes level and lakes bottom at each index of the input date, over which the lake does exist.**

Pseudo Code

```
Peak[]PEAKSINRANGE(Peak[] peaks, start, end)
BEGIN
  for peak in peaks
    if (peak.firstIndex between start and end) OR (peak.lastIndex between start and end)
      result += peak
  return result
END
```

```
highest1, highest2 FINDTWOHIGHESTPEAKSONISLAND(Peak[] island)
BEGIN
  highest1, highest2 = undefined

  for peak in peaks
    if highest1 is undefined
      highest1 = peak
      continue
    else if highest2 is undefined
      highest2 = peak
      continue

    if highest1.height < peak.height
      if highest1.height >= highest2.height
        highest2 = peak
      else
        highest1 = peak
      continue

    if highest2.height < peak.height
      highest2 = peak

  return highest1, highest2
END
```

```
leftBoundary, rightBoundary, level CALCULATELAKEBOUNDARIES(Peak left, Peak right, int[] heights)
BEGIN
  leftShore = left.lastIndex
  rightShore = right.firstIndex
  leftBoundary = leftShore + 1
  rightBoundary = rightShore - 1

  int level = min(left.height, right.height)
  if "Left peak is the base"  // left.height == level
    for index from:rightBoundary downto leftBoundary
      if heights[index] >= level
        rightBoundary = index - 1
       else
        break
   else                       // right.height == level
    for index from:leftBoundary to rightBoundary
      if heights[index] >= level
        leftBoundary = index + 1
       else
        break
  return leftBoundary, rightBoundary, level
END
```

```
VOID DISCOVERLAKES(QUEUE(Peak[]) islands, Lake[] accumulator, int[] heights)
BEGIN
  Peak[] island = islands.pollFirst()
  if peaks.length < 2
```

```
    discoverLakes(islands, accumulator, heights)
    return

  // Step 1.
  leftPeak, rightPeak = findTwoHighestPeaksOnIsland(island)

  // Step 2.
  leftBoundary, rightBoundary, level = calculateLakeBoundaries(leftPeak, rightPeak, heights)
  lake = new Lake(leftBoundary, rightBoundary, level)
  lakesAccumulator += lake

  // Step 3.
  Peak[] leftIsland = peaksInRange(peaks, 0, leftPeak.firstIndex)
  if leftIsland.length > 1
    islands += leftIsland

  Peak[] rightIsland = peaksInRange (peaks, rightPeak.lastIndex, <infinity>)
  if rightIslandlength > 1
    islands += rightIsland

  discoverLakes(islands, lakesAccumulator, heights)
END
```

```
Peak[] DISCOVERPEAKS(int[] heights)
BEGIN
  state = PeakMatcherStateMachine.start
  for height in heights
    nextState = state.nextStep(height)
    if nextState.matchPattern()
      peaks += nextState.peak
      state = nextState
  return peaks
END
```

Algorithm Complexity

Routine	Complexity
PEAKSINRANGE	O(n)
FINDTWOHIGHESTPEAKSONISLAND	O(n)
CALCULATELAKEBOUNDARIES	O(n)
DISCOVERPEAKS	O(n)
DISCOVERLAKES	O(n) On each its iteration the procedure processes a subset of islands. This is equivalent to a "for" cycle, where each iteration is processing a decreasing subset of N. Where N is a number of remaining Peaks that tends to 1 or 0 (thus such islands will not be processed).
Total	O(n)

Memory Consumption

As the result this algorithm returns a list of "levels" which size equals to the input data size. Thus the resulting memory consumption is equals 2 * SIZEOF(input data) + SIZEOF(total volume) which is the same as O(n).