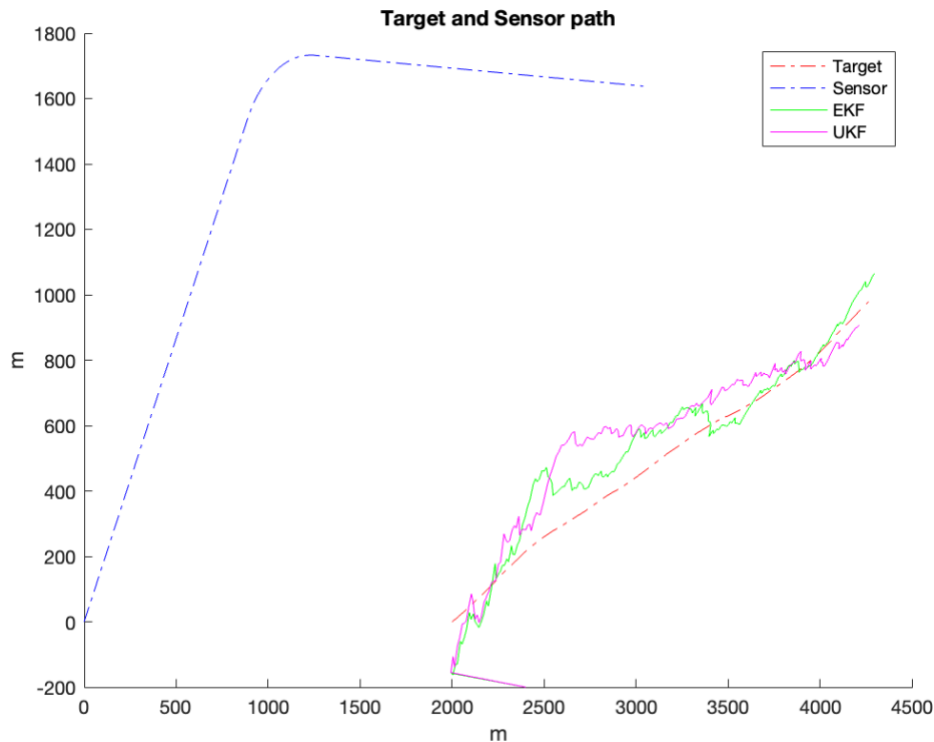


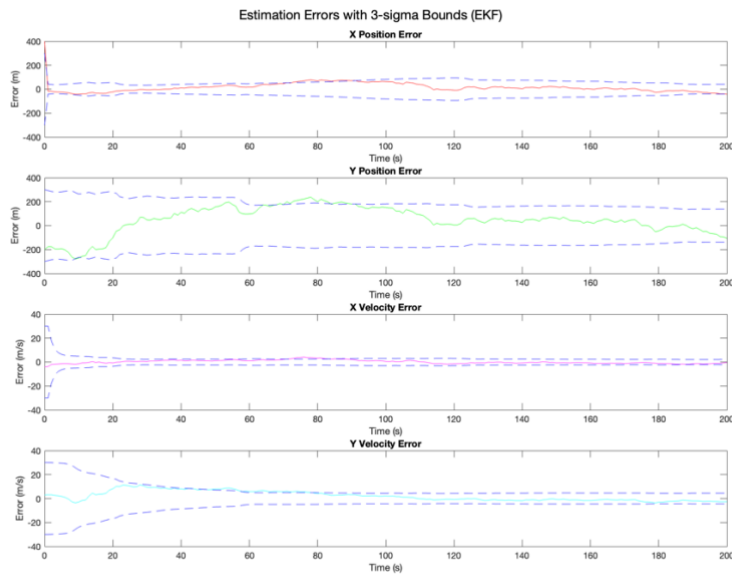
The first figure is about the result of exact target's path(red line) and exact sensor's path(blue line).



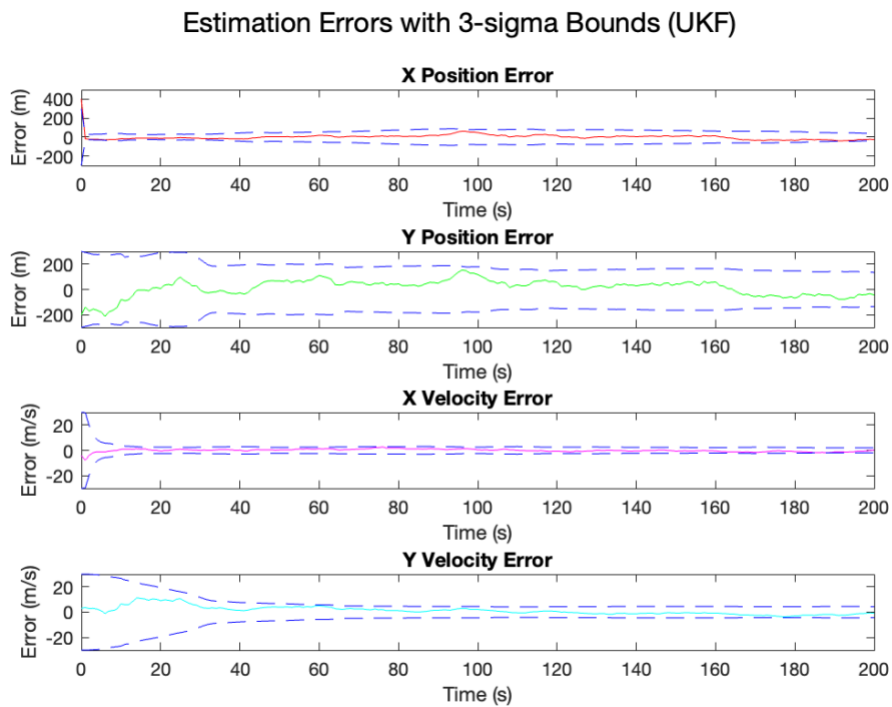
I added the estimated target's path driven by the Extended Kalman Filter (green, EKF) and the Unscented Kalman Filter (magenta, UKF) to the first figure. If we consider the distance between the target and sensor is far away from each other compared to the covariance matrix, EKF and UKF works well even there are some errors. According to the elapsed time of each filter, UKF needs more time to compute. It is expected because the sigma point filter is a spreading points and collect information at each point.

EKF of elapsed time : 0.042819 seconds
 UKF of elapsed time : 0.087806 seconds

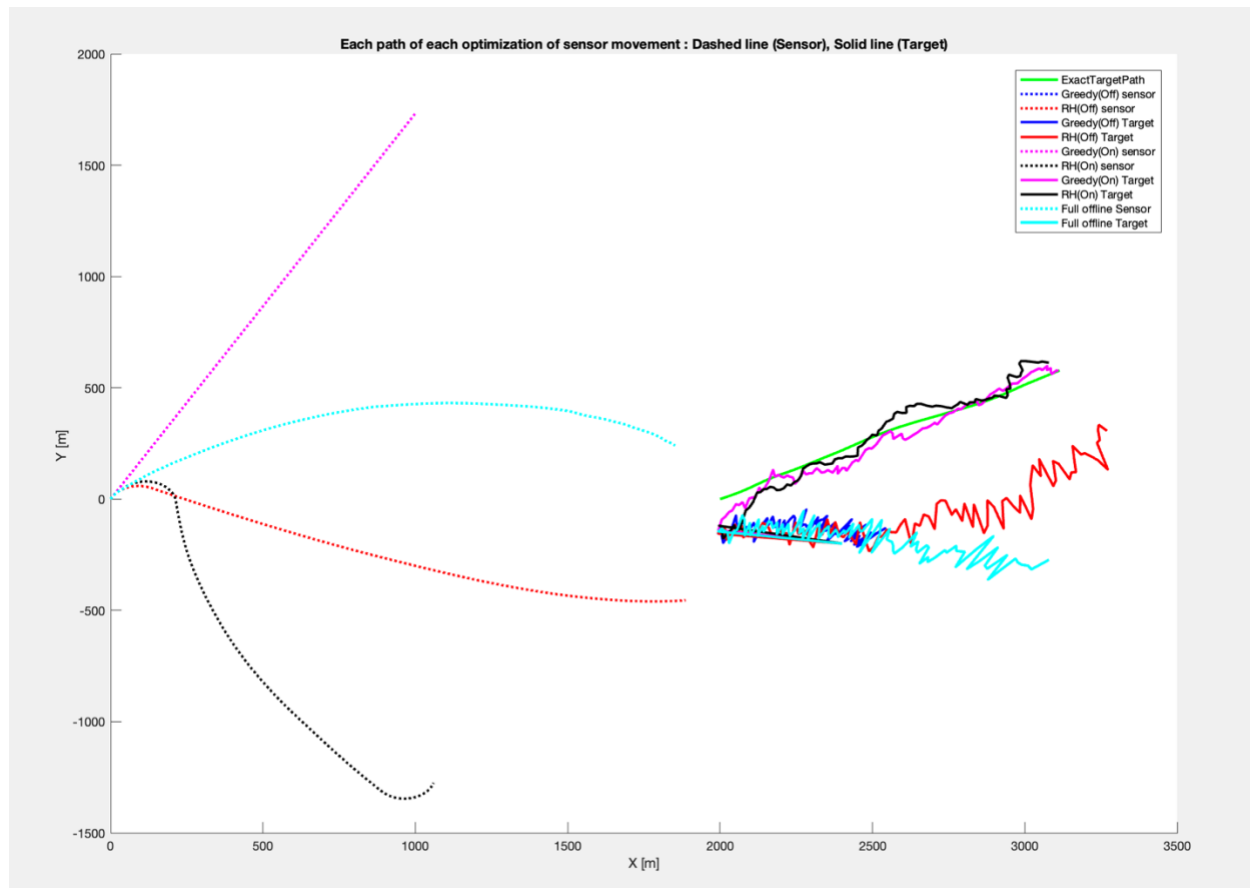
The second figure is the error analysis of EKF. The estimation by EKF has reasonable estimation, but a few points are outside or very close to 3 sigma bounds.



The third figure is the error analysis of using UKF. Unlike the EKF, the estimated state in almost all time are inside 3 sigma bounds. Both filters have similar performance, but UKF is a better than EKF according to the plot of 3 sigma bounds.



The last figure is about each path of each optimization method. Each optimization method has own color: Greedy offline(blue), Greedy online(magenta), Receding Horizon offline (red), Receding Horizon online (black), Full offline (cyan). Also, the sensor's trajectory of all methods are expressed by a dashed line and the target's trajectories are expressed by solid lines.



First, the greedy online(magenta) and offline(blue) have almost similar sensor's trajectory but both have different target performance. The greedy online has better performance of target estimation and it is supposed to be. Because in online method, the UKF runs at each time step. The RH online(black) and RH offline(red) have different sensor's trajectory by optimization. Similar with the greedy ones, the method in online has a better target estimation performance. The full offline optimization method is an optimization in every time step of control sequence. Even though, the target tracking performance is bad. In general, the greedy ones go upward relatively, and the RH ones go downward and the full offline goes to make a circle.

Secondly, the philosophy of offline (open-loop) and online (closed-loop) optimization methods is fundamentally different. Offline optimization (open-loop) computes an optimal control sequence for the entire trajectory in advance, assuming a predictable system and

target dynamics. Once the optimized path is determined, it is executed without modification, meaning the system does not incorporate new information during execution. Online optimization (closed-loop), on the other hand, continuously updates the control strategy based on real-time measurements, allowing the system to adjust dynamically as new data becomes available. This approach ensures adaptability to unexpected changes in the target's movement or external disturbances. While offline optimization can provide a theoretically optimal path with lower computational demands at runtime, online optimization offers greater robustness and accuracy by incorporating real-time feedback into the decision-making process.

Lastly, according to the result of each method, the most time consumed method is full offline method and it is about six minutes which is 5000 times bigger than the case of the smallest time (Greedy online). Also, greedy online and receding horizon online have the best estimated performance regarding to the smallest root mean square error (RMSE) to the exact target path.

Optimization Method	Elapsed Time (s)	RMSE
"Greedy Offline"	0.80579	567
"Receding Horizon Offline"	1.5908	373.79
"Greedy Online"	0.071894	76.215
"Receding Horizon Online"	1.5079	80.645
"Full Optimal"	358.27	537.77

So, the best optimization method of the estimation of target is the greedy online method. Because it is fast and has good performance. The worst choice is choosing full offline because it is slow and has bad performance.

Additional experiment : For RH, $N_c \rightarrow 5$.

Both sensors become more aggressive sensors to track the target. (Black line and red line)

