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In [1]: # DSC530-T302
# Stephen Smitshoek
# Week06
# Exercise 8-1 and 8-2
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In [2]: from __future__ import print_function, division

import thinkstats2
import thinkplot

import math
import random
import numpy as np
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In [3]: def MeanError(estimates, actual):
        """Computes the mean error of a sequence of estimates.

        estimate: sequence of numbers
        actual: actual value

        returns: float mean error
        """
        errors = [estimate-actual for estimate in estimates]
        return np.mean(errors)
```

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In [4]: def RMSE(estimates, actual):
        """Computes the root mean squared error of a sequence of estimates.

        estimate: sequence of numbers
        actual: actual value

        returns: float RMSE
        """
        e2 = [(estimate-actual)**2 for estimate in estimates]
        mse = np.mean(e2)
        return math.sqrt(mse)
```

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In [5]: def Estimate1(n=7, m=1000):
        """Evaluates RMSE of sample mean and median as estimators.

        n: sample size
        m: number of iterations
        """
        mu = 0
        sigma = 1

        means = []
        medians = []
        for _ in range(m):
            xs = [random.gauss(mu, sigma) for _ in range(n)]
            xbar = np.mean(xs)
            median = np.median(xs)
            means.append(xbar)
            medians.append(median)

        print('Experiment 1')
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print('mean error xbar', MeanError(means, mu))
print('mean error median', MeanError(medians, mu))
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In [6]: def Estimate2(n=7, m=1000):
        """Evaluates S and Sn-1 as estimators of sample variance.

        n: sample size
        m: number of iterations
        """

        mu = 0
        sigma = 1

        estimates1 = []
        estimates2 = []
        for _ in range(m):
            xs = [random.gauss(mu, sigma) for _ in range(n)]
            biased = np.var(xs)
            unbiased = np.var(xs, ddof=1)
            estimates1.append(biased)
            estimates2.append(unbiased)

        print('Experiment 2')
        print('rmse biased', RMSE(estimates1, sigma**2))
        print('rmse unbiased', RMSE(estimates2, sigma**2))
```

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In [7]: def Estimate3(n=7, m=1000):
        """Evaluates L and Lm as estimators of the exponential parameter.

        n: sample size
        m: number of iterations
        """

        lam = 2

        means = []
        medians = []
        for _ in range(m):
            xs = np.random.exponential(1/lam, n)
            L = 1 / np.mean(xs)
            Lm = math.log(2) / np.median(xs)
            means.append(L)
            medians.append(Lm)

        print('Experiment 3')
        print('rmse L', RMSE(means, lam))
        print('rmse Lm', RMSE(medians, lam))
        print('mean error L', MeanError(means, lam))
        print('mean error Lm', MeanError(medians, lam))
```

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In [8]: def SimulateSample(mu=90, sigma=7.5, n=9, m=1000):
        """Plots the sampling distribution of the sample mean.

        mu: hypothetical population mean
        sigma: hypothetical population standard deviation
        n: sample size
        m: number of iterations
        """

        def VertLine(x, y=1):
            thinkplot.Plot([x, x], [0, y], color='0.8', linewidth=3)
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means = []
for _ in range(m):
    xs = np.random.normal(mu, sigma, n)
    xbar = np.mean(xs)
    means.append(xbar)

stderr = RMSE(means, mu)
print('standard error', stderr)

cdf = thinkstats2.Cdf(means)
ci = cdf.Percentile(5), cdf.Percentile(95)
print('confidence interval', ci)
VertLine(ci[0])
VertLine(ci[1])

# plot the CDF
thinkplot.Cdf(cdf)
thinkplot.Save(root='estimation1',
                xlabel='sample mean',
                ylabel='CDF',
                title='Sampling distribution')

```

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In [9]: def simulate_experiment(n=10, lam=2, m=1000):
def VertLine(x):
    thinkplot.Plot([x, x], [0, 1], color='0.8', linewidth=3)

L_estimates = []
for _ in range(m):
    xs = np.random.exponential(1.0/lam, n)
    L = 1 / np.mean(xs)
    L_estimates.append(L)

cdf = thinkstats2.Cdf(L_estimates)
ci = cdf.Percentile(5), cdf.Percentile(95)
stderr = RMSE(L_estimates, lam)

print('The 90% Confidence Interval is {} to {}'.format(round(ci[0], 2), round(ci[1], 2)))
print('The standard error is {}'.format(round(stderr, 2)))

VertLine(ci[0])
VertLine(ci[1])
thinkplot.Cdf(cdf)

```

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In [10]: print("m=10")
Estimate1(m=10)
print("\nm=10,000")
Estimate1(m=10000)
print("\nBoth the mean and median trend towards zero as the number of experiments increases. It is unlikely that neither of them are biased.")

```

```

m=10
Experiment 1
mean error xbar 0.13822209624935694
mean error median 0.25140235875528316

```

```

m=10,000
Experiment 1
mean error xbar 0.002117238023748525
mean error median 0.00301923665666006

```

Both the mean and median trend towards zero as the number of experiments increases, therefore it is likely that neither of them are biased.

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In [11]: Estimate2()
print("\nBoth S^2 and S(n-1)^2 yield similar results with S^2 being slightly smaller."

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Experiment 2
rmse biased 0.5008320018577254
rmse unbiased 0.5570055288254314

```

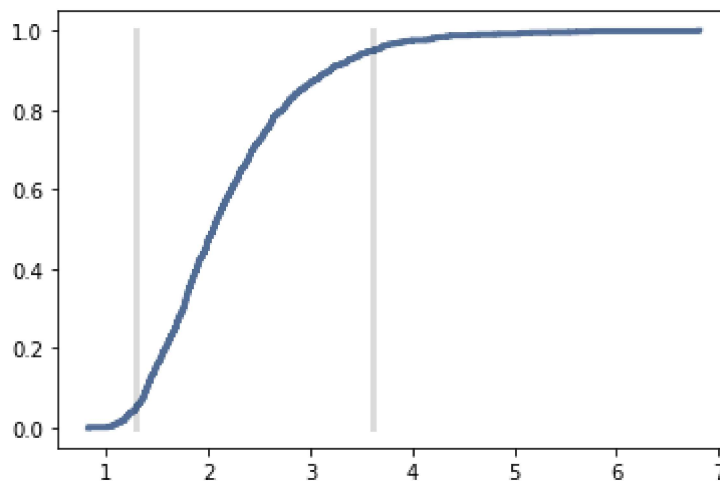
Both S^2 and $S(n-1)^2$ yield similar results with S^2 being slightly smaller.

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In [12]: simulate_experiment(n=10)

```

The 90% Confidence Interval is 1.3 to 3.62
The standard error is 0.78

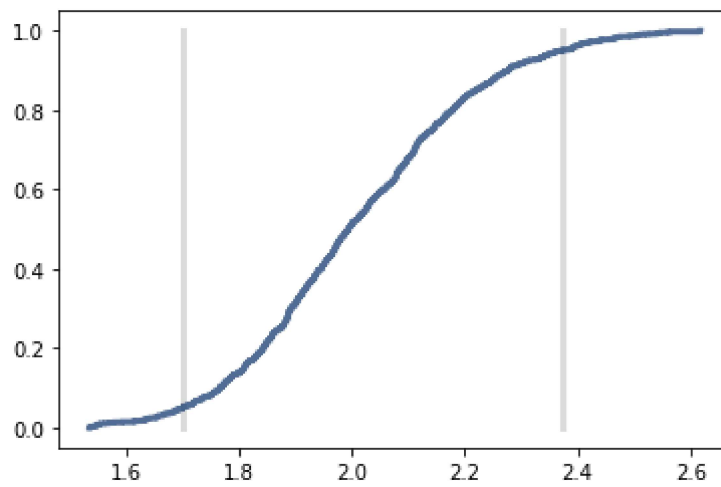


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In [13]: simulate_experiment(n=100)

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The 90% Confidence Interval is 1.7 to 2.37
The standard error is 0.2



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In [14]: simulate_experiment(n=1000)
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The 90% Confidence Interval is 1.9 to 2.1
The standard error is 0.06

