1   DIS 0B

1.1   Intro

- My OH is Monday 1-2 and Tuesday 3-4 in Cory 212.

- Email is first.last@

- 3rd year cs + math major

- hobbies?

1.1.1   Some CS70 advice

- Goal: enhance problem solving techniques/approach

- Don't fall behind on content, catching up will not be fun

- problems, problems, more problems

- Ask lots of questions (imperative for strong foundation)

- Don't stress, we're in this ride together

1.2   Propositional Logic

Relevant notation:

- $\wedge$ = and

- $\vee$ = or

- $\neg$ = not

- $\implies$ = implies

- $\exists$ = there exists

- $\forall$ = forall

- $\mathbb{N}$ = natural numbers $\{0, 1, \ldots\}$

- $a|b$ = a divides b

$P \implies Q$ is an example of an implication. We can read this as "If P, then Q." An implication is false only when P is true and Q is false. If P is false, the implication is vacuously true.

> **Definition 1.1 (Contrapositive)**
> If $P \implies Q$ is an implication, then the implication $\neg Q \implies \neg P$ is known as the **contrapositve**.

An important identity is that $P \implies Q \equiv \neg Q \implies \neg P$.

1.3   Proofs

Induction will be in its own section.

Different methods.

1.3.1 Direct proof

Want to show $P \implies Q$ by assuming P and logically concluding Q.

1.3.2 Contraposition

Want to show $P \implies Q$ by equivalently proving $\neg Q \implies \neg P$.

1.3.3 Contradiction

Want to show P. We do this by assuming $\neg P$ and concluding $R \wedge \neg R$.

Why? Idea is that if we can show the implication $\neg P \implies (R \wedge \neg R)$ is True, this is the same as showing $\neg P \implies F$ is True. The contraposition gives $T \implies P$.

1.3.4 Cases

Break up a problem into multiple cases i.e. odd vs even.

2 DIS 1A

2.1 Induction

Goal of induction is to show $\forall n P(n)$.

2.1.1 (Weak) Induction

- Prove $P(0)$ is true (or relevant base cases), then $\forall n \in \mathbb{N} \, (P(n) \implies P(n+1))$.

- Induction dominoes analogy!

- Sometimes you might have multiple base cases (Problem about $4x + 5y$ in Notes 3)

2.1.2 Strengthening the Hypothesis

Sometimes proving $P(n) \implies P(n+1)$ is not straightforward with induction. In such a scenario, we can try to introduce a (stronger) statement $Q(n)$. We want to construct Q such that $Q(n) \implies P(n)$. Inducting on Q proves P.

2.1.3 Strong Induction

- Prove $P(0)$ is true (or relevant base cases), then $\forall n \, ((P(0) \wedge P(1) \wedge \cdots \wedge P(n)) \implies P(n+1))$.