# Contents

# 1 DIS 0B

## 1.1 Intro

- My OH is Monday 1-2 and Tuesday 3-4 in Cory 212.
- Email is first.last@
- 3rd year cs + math major
- hobbies?

### 1.1.1 Some CS70 advice

- Goal: enhance problem solving techniques/approach
- Don't fall behind on content, catching up will not be fun
- problems, problems, more problems
- Ask lots of questions (imperative for strong foundation)
- Don't stress, we're in this ride together

## 1.2 Propositional Logic

Relevant notation:

- $\wedge$ = and
- $\vee$ = or
- $\neg$ = not
- $\implies$ = implies
- $\exists$ = there exists
- $\forall$ = forall
- $\mathbb{N}$ = natural numbers $\{0, 1, \ldots\}$
- $a|b$ = $a$ divides $b$

$P \implies Q$ is an example of an implication. We can read this as "If $P$, then $Q$." An implication is false only when $P$ is true and $Q$ is false. If $P$ is false, the implication is vacuously true.

**Definition 1.1 (Contrapositive)**
If $P \implies Q$ is an implication, then the implication $\neg Q \implies \neg P$ is known as the **contrapositve**.

An important identity is that $P \implies Q \equiv \neg Q \implies \neg P$.

## 1.3 Proofs

Induction will be in its own section.

Different methods.

#### 1.3.1 Direct proof

Want to show $P \implies Q$ by assuming $P$ and logically concluding $Q$.

#### 1.3.2 Contraposition

Want to show $P \implies Q$ by equivalently proving $\neg Q \implies \neg P$.

#### 1.3.3 Contradiction

Want to show $P$. We do this by assuming $\neg P$ and concluding $R \wedge \neg R$.

Why? Idea is that if we can show the implication $\neg P \implies (R \wedge \neg R)$ is True, this is the same as showing $\neg P \implies F$ is True. The contraposition gives $T \implies P$.

#### 1.3.4 Cases

Break up a problem into multiple cases i.e. odd vs even.

# 2 DIS 1A

## 2.1 Induction

Goal of induction is to show $\forall n P(n)$.

### 2.1.1 (Weak) Induction

- Prove $P(0)$ is true (or relevant base cases), then $\forall n \in \mathbb{N}\,(P(n) \implies P(n+1))$.
- Induction dominoes analogy!
- Sometimes you might have multiple base cases (Problem about $4x + 5y$ in Notes 3)

### 2.1.2 Strengthening the Hypothesis

Sometimes proving $P(n) \implies P(n+1)$ is not straightforward with induction. In such a scenario, we can try to introduce a (stronger) statement $Q(n)$. We want to construct $Q$ such that $Q(n) \implies P(n)$. Inducting on $Q$ proves $P$.

### 2.1.3 Strong Induction

- Prove $P(0)$ is true (or relevant base cases), then $\forall n\,((P(0) \wedge P(1) \wedge \cdots \wedge P(n)) \implies P(n+1))$.
- Dominoes analogy, but emphasis on the difference between weak and strong induction (assuming middle domino works vs everything from start to middle).

### 2.1.4 Weak vs Strong

A common point of confusion is when one should use strong induction in lieu of weak induction. Strong induction **always** works whenever weak induction works. However, there may be scenarios in which the induction hypothesis to prove $n = k + 1$ requires more information than just $n = k$. A scenario like this requires strong induction.