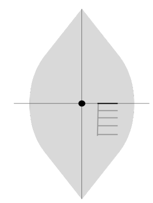
MULTIVARIABLE DERIVATIVES ACTIVITY
MATH 32

9/28/22

(1) (a) What are the names of your classmates working on this with you?

(b) Are apples or oranges better, and why?



(2) Look at the football-shaped surface. It should be oriented like this:

(a) If the surface is given by $f(x, y)$, what is f_x at the black dot?

(b) What is f_y at the black dot?

(c) Is f_y positive or negative at the black toothpick? How do you know?

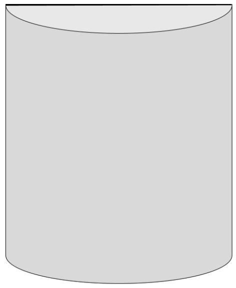
(d) Is f_x positive or negative at the black toothpick? How do you know?

(e) Is f_{yy} positive or negative at the black toothpick? How do you know?

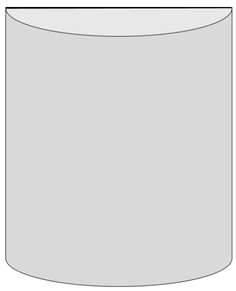
(f) Is f_{yx} positive or negative at the toothpick *next to* the black toothpick? How do you know?

(3) Now, look at the half-cylinder. Suppose the surface is given by $g(x, y)$.

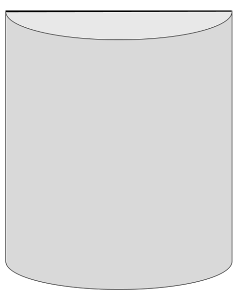
(a) Identify all the places where $\nabla g = \vec{0}$.



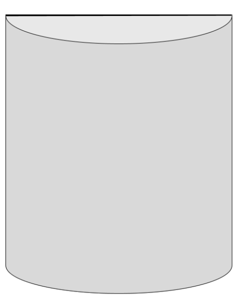
(b) Identify all the places where $\nabla g = \langle 0, a\rangle$ with $a > 0$.



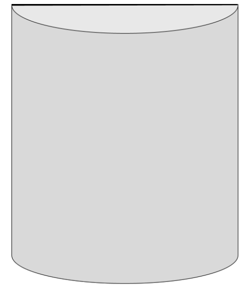
(c) Identify all the places where $\nabla g = \langle 0, a\rangle$ with $a < 0$



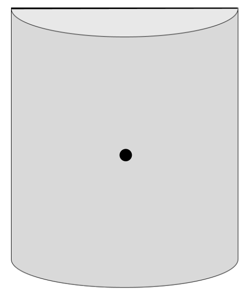
(d) Identify all the places where $\nabla g = \langle a, 0\rangle$ with $a > 0$.



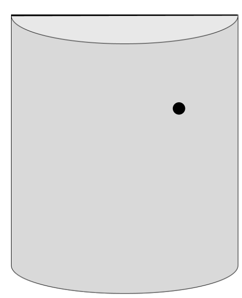
(e) If $\vec{v} = \langle 1, 1\rangle$, identify where $D_{\vec{v}} g$ is positive.



(f) Find all $\vec{v}$ such that $D_{\vec{v}} g = 0$ at the black dot.



(g) Find all $\vec{v}$ such that $D_{\vec{v}} g = 0$ at the black dot.



(4) Look at exercises (2) and (3) above. If you wanted to find the maximum of a surface, how would you be able to do it using only the fomula without graphing it?

(5) If you finish early, confirm your answers to (2) and (3) by calculation. The football is approximately given by

$$f(x,y) = \sqrt{1 - \frac{x^2}{16} - \frac{y^2}{4}}$$

where the black dot is located near $(0,0)$ and the black toothpick is located near $(0,1)$.

The half-cylinder is approximately given by

$$g(x,y) = \sqrt{1-y^2}, \quad -2 \leq x \leq 2$$

where $(0,0)$ is located near the center of the surface.

Neither figure is to scale, but they do capture the general shape.