INTRODUCTION TO LINE INTEGRALS
MATH 32

11/4/22

All of the models today are scaled 1 inch $\approx$ 1 unit. You may want to round everything to a few decimal places to make the computations easier.

(1) (a) What are the names of your classmates working on this with you?

(b) When is it appropriate to start playing Christmas music on the radio?

(2) You want to find the area of the figure whose height is given by $z = 2 + x^2y$ and whose base is the line segment $y = 2x$ with $-1 \leq x \leq 1$. (See Figure I).

(a) We start by approximating the figure with 2 rectangles. (See Figure II.)

(i) The height of $A1$ is 4, since at $(1, 2)$, $z = 4$. If we got the height of $A2$ from the height of the function at $(0, 0)$, what is the height of $A2$?

(ii) What is the width of each rectangle? (Hint: use Pythagorean Theorem)

(iii) Add the areas of the two rectangles to approximate the desired area.

(b) Now let's approximate the figure with 4 rectangles. (See Figure III)

(i) To find the height of each rectangle, we used the points $(-\frac{1}{2}, -1)$, $(0, 0)$, $(\frac{1}{2}, 1)$, and $(1, 2)$. What are the heights of each rectangle?

(ii) What is the width of each rectangle?

(iii) What is the new approximation for the area?

(c) To find a better approximation for the area, we should use even more rectangles!

(i) A parametric equation for the line is $\langle t, 2t \rangle$, where $-1 \leq t \leq 1$. If we use the point (x, y) to get the height of our rectangles, what is the height of each rectangle with respect to t?

(ii) If we take points Δt increments apart, what is the width of each rectangle?

(iii) Take a guess: what integral (with respect to t) would give you the area if we let $\Delta t \to 0$?

(3) Now, let's find the area of the figure whose height is given by $z = 2 + x^2 y$ and whose base is the semicircle $y = \sqrt{1 - x^2}$. (Figure IV) Let's parametrize the circle with

$$r(t) = \langle \cos t, \sin t \rangle, \ 0 \leq t \leq \pi$$

(a) Let's approximate the figure with 4 rectangles. (See Figure V)

(i) What should the width of each rectangle be? We want to think of the base of each rectangle as straight and not curved.

(ii) Based on t (not x), I used a left-hand approximation and equally-sized spacings for t. What is the height of each rectangle?

(iii) What is the approximate area based on these 4 rectangles?

(b) Set up an integral (with respect to t) to calculate the total area if we used infinitely many rectangles, and evaluate it.

(4) Let's do the same as the above, but using

$$\langle t, \sqrt{1-t^2}\rangle, \ -1 \leq t \leq 1$$

for the circle.

(a) Using equally spaced values for t and a left-hand approximation, what is the height of each rectangle?

(b) What is the width of each rectangle? Note: they're not all the same size!

(c) What is the approximation for the area given by these 4 rectangles?

(d) Set up an integral (with respect to t) to calculate the total area if we used infinitely many rectangles, and evaluate it.

(e) Is your answer the same as (2)(b)? (If not, it means you made a mistake.)

(5) Can you think of a general formula: if you have a figure whose height is given by $z = f(x, y)$ and whose base is along a curve $r(t) = \langle x(t), y(t)\rangle$, $a \leq t \leq b$, what is its area?