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A COMPARATIVE STUDY OF TWO COMBINATORIAL REVERSE AUCTION MODELS

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Abstract

Online group-buying is one of the most innovative business models employed by many companies. From the perspective of buyers, quantity based discounts provide a huge incentive to form coalitions and take advantage of lower prices without ordering more than their actual demand. Traditional group-buying mechanisms are usually based on a single item and uniform cost sharing. One way to reduce the cost for acquiring the required items is to take into account the complementarities between items provided by the sellers. By holding a combinatorial reverse auction, the total cost to acquire the required items will be significantly reduced due to complementarities between items. However, combinatorial reverse auctions suffer from high computational complexity. If there are multiple buyers, there are two different business models for procurement based on combinatorial reverse auctions: (1) independent combinatorial reverse auctions: each buyer may hold a combinatorial reverse auction independently and (2) combinatorial reverse auctions based on group buying: multiple buyers delegate the auction to a group buyer and the group buyer holds only one combinatorial reverse auction for all the buyers. In developing an effective tool to support the decision of multiple buyers' procurement, a comparative study on the performance and efficiency of these two different business models is needed. In this paper, we compare the performance as well as the computational efficiency for these two combinatorial reverse auction models. Our analysis indicates that group buying combinatorial reverse auction outperforms multiple separate combinatorial reverse auctions not only in performance but also in efficiency.

Keywords: Combinatorial auction, Winner determination, Group buying, e-Commerce.

1   INTRODUCTION

Online group-buying is one of the most innovative business models employed by many companies. The rationale of group-buying is due to demand aggregation, which benefits sellers, offering lower marketing costs and coordinated distribution channels, as well as buyers, who enjoy lower costs for product purchases (Dolan 1987). The Internet provides a natural platform for demand aggregation and facilitates group-buying. From the perspective of buyers, quantity based discounts provide a huge incentive to form coalitions and take advantage of lower prices without ordering more than their actual demand. By forming a coalition, buyers can also improve their bargaining power and negotiate more advantageously with sellers to purchase at a lower price.

There are several studies on group-buying in existing literature. Kauffman and Wang (2001;2002) examine group-buying as a dynamic pricing mechanism. They offer valuable insights for distributed group-buying mechanisms under uniform cost sharing. Anand and Aron (2003) analyze the value of group-buying and the optimal price curve from a seller's perspective. They assume atomic buyers, whose individual participation in a buying group has no influence on the price. Chen et al. (2002) analyze buyers' bidding strategies in a group-buying auction considering limited supply of an item and private information of buyers. Chen et al. (2007) also compare group-buying with fixed pricing mechanisms. Cuihong Li, Katia Sycara and Alan Scheller-Wolf (2009) introduce the concept of combinatorial coalition formation, which allows buyers to announce reserve prices for combinations of items. These reserve prices, along with the sellers' price-quantity curves for each item, are used to determine the formation of buying groups for each item. The objective is to maximize buyers' total surplus. A two-stage process is proposed to realize the implementation. In the first stage, customer preferences are expressed, and in the second stage, buying groups are formed and cost shares are determined. In the first stage, the market coordinator has to capture the preference information of each buyer for all possible item combinations of interest. The second stage uses buyers' preferences over combinations of items to determine the configuration of buying groups and the payments of each buyer. This is analogous to the winner determination problem in combinatorial auctions, namely determining the item allocation and bidders' payments based on the package bids.

Traditional group-buying mechanisms are usually based on a single item and uniform cost sharing. All the existing literature mentioned above does not address how to take the advantage of the complementarities between items to reduce the cost for acquiring the required items. One way to reduce the cost for acquiring the required items is to take into account the complementarities between items provided by the sellers. Combinatorial reverse auctions allow one to take advantage of the complementarities between items provided by the sellers. Combinatorial reverse auctions are different from combinatorial auctions in that the auctioneer of a combinatorial reverse auction is a buyer whereas the auctioneer of a combinatorial auction is a seller. By holding a combinatorial reverse auction, the total cost to acquire the required items will be significantly reduced due to complementarities between items. Applying combinatorial reverse auctions in corporations' procurement processes can lead to significant savings (Metty, Harlan, Samelson, Moore, Morris, Sorensen, Schneur, Raskina, Schneur, Kanner, Potts, and Robbins 2005; Murray, and White 1983). Therefore, a single buyer may also benefit from combinatorial reverse auctions.

However, combinatorial reverse auctions suffer from high computational complexity. In spite of the difference, combinatorial reverse auctions are closely related to combinatorial auctions. An excellent survey on combinatorial auctions can be found in (de Vries & Vohra 2003; Pekeč & Rothkopf 2003). Allowing bids for bundles of items is the foundation of combinatorial auctions. In a combinatorial auction (de Vries & Vohra 2003), bidders may place bids on combinations of items, which allows the bidders to express complementarities between items instead of having to speculate into an item's valuation about the impact of possibly getting other, complementary items. There are, however, several problems with combinatorial auctions. Combinatorial auctions have been notoriously difficult to solve from a computational point of view (Rothkopf, Pekeč and Harstad 1998). The combinatorial auction problem can be modelled as a set packing problem (SPP) (Vemuganti, 1998), a well-known NP-complete problem (Andersson, Tenhunen, and Ygge 2000; Fujishima, Leyton-Brown, and

Shoham 1999; Holger, Hoos and Boutilier 2000; Sandholm 2002; Sandholm, Suri, Gilpin and Levine 2001). It deals with computational aspects and heuristics for solving what is known as the Winner Determination Problem of an auction (Gonen, and Lehmann 2000; Jones and Koehler 2002; Sandholm 2000). Many algorithms have been developed for combinatorial auction problems. For example, in (Guo, Lim, Rodrigues and Tang 2005), the authors proposed a Lagrangian Heuristic for a combinatorial auction problem. Hsieh et al. also proposed computationally efficient algorithms based on Lagrangian relaxation (Hsieh 2007; Hsieh & Tsai 2008; Hsieh 2009; Hsieh, 2010). Exact algorithms have been developed for the SPP problem, including a branch and bound search (Sandholm, Suri, Gilpin and Levine 2001), iterative deepening A* search (Sandholm 2002) and the direct application of available CPLEX IP solver (Andersson, Tenhunen and Ygge 2000).

If there are multiple buyers, there are two different business models for procurement based on combinatorial reverse auctions: (1) independent combinatorial reverse auctions: each buyer may hold a combinatorial reverse auction independently and (2) combinatorial reverse auctions based on group buying: multiple buyers delegate the auction to a group buyer and the group buyer holds only one combinatorial reverse auction for all the buyers. In developing an effective tool to support the decision of multiple buyers' procurement, a comparative study on the performance and efficiency of these two different business models is needed. Two interesting questions are: (1) which model outperforms the other one? (2) which model is more efficient?

To assess the advantages and effectiveness of applying combinatorial reverse auctions in group-buying quantitatively, further study is needed. This motivates us to study the performance as well as the efficiency of group-buying based on combinatorial reverse auctions. In this paper, we first propose two business models to describe the application of combinatorial reverse auction by a single buyer and a group of buyers, respectively. To compare the effectiveness of these two business models, we then formulate the optimization problems for these two business models. As the computational complexities to solve these two problems are NP-complete, we propose subgradient algorithms to find approximate solutions for these two problems and compare the performance as well as the efficiency of the subgradient algorithms for solving these two problems.

   Typically, a combinatorial reverse auction has an auctioneer, who is the buyer. However, in group-buying, there are many buyers. To apply combinatorial reverse auction to the problem setting of group-buying, we consider a combinatorial reverse auction problem in which there are multiple buyers and multiple sellers. We propose the concept of proxy buyer to deal with this problem. The proxy buyer consolidates the demands from the buyers and then holds a combinatorial reverse auction to try to obtain the goods from a set of sellers who can provide the goods. Each seller places bids for each bundle of goods he can provide. The problem is to determine the winners to minimize the total cost for the proxy buyer.

   The remainder of this paper is organized as follows. In Section 2, we present the winner determination problem formulation of the combinatorial reverse auction in group-buying. In Section 3, we propose the Lagrangian relaxation algorithms. In Section 4, we present the numerical examples and analyze the results of our solution approach. We conclude this paper in Section 5.

2 TWO COMBINATORIAL REVERSE AUCTION MODELS

In this section, we illustrate different ways to meet multiple buyers' requirements with combinatorial reverse auctions. We introduce two different combinatorial reverse auction models. In the first model, each buyer holds a combinatorial reverse auction independently. The solution that meets all the buyers' requirements can be obtained by solving N combinatorial reverse auction subproblems. Each subproblem is solved by applying any existing combinatorial reverse auction algorithm. In the second model, a virtual group buyer is created. The group buyer's requirements consolidate all the buyers' requirements. The bids placed by the potential bidders in the first model are regarded as the bids placed to the group buyer. The solution that meets the group buyer's requirements (and hence all the buyers' requirements) is found by solving the combinatorial reverse auction problem for the group buyer. In fact, the bids placed by the potential bidders in the first model are a subset of bids placed to the group buyer.

Model 1: Independent Combinatorial Reverse Auctions

Example 1: Figure 1 illustrates an application scenario in which Buyer 1 wants to purchase at least a bundle of items 1A, 1B and 1C from the market and Buyer 2 wants to purchase a bundle of items 1C and 1D. There are four bidders, Seller 1, Seller 2, Seller 3 and Seller 4 who place bids in the system. Suppose Seller 1 places the bid (1A,1C, p1) on Buyer 1, where p11 denotes the prices of the bid. Seller 2 places the bid (1B, 1C, p2) on Buyer 1. Seller 3 places the bid (1D, p3) on Buyer 2. Seller 4 places the bid (1C, 1D, p4) on Buyer 2. We assume that all the bids entered the auction are recorded. A bid is said to be active if it is in the solution. We assume that there is only one bid active for all the bids placed by the same bidder. For this example, the solution for this combinatorial reverse auction problem is Seller1: (1A, 1C, p1), Seller 2: (1B, 1C, p2) and Seller 4: (1C,1D, p4). The overall cost of this solution is p1+p2+p4.
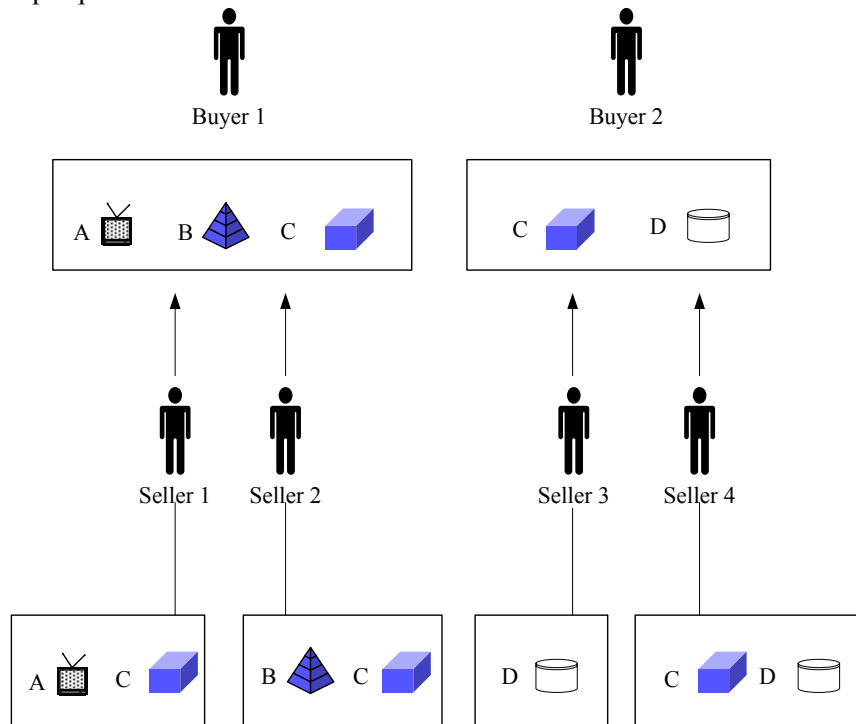

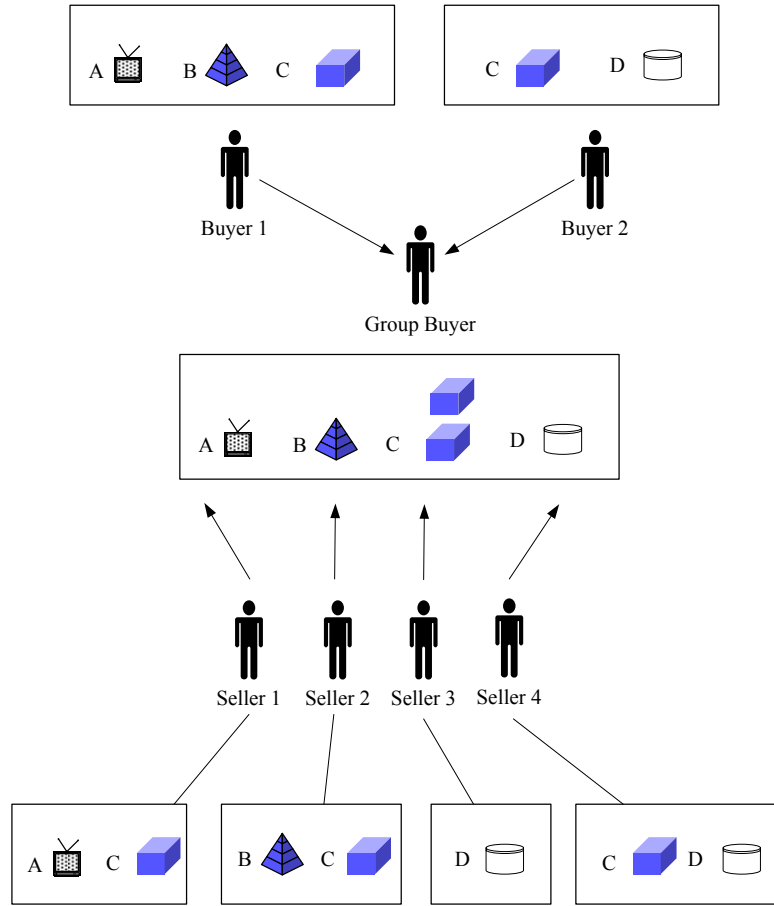*Figure 1. Two Independent Combinatorial Reverse Auctions*

Figure 2. Combinatorial Reverse Auction based on Group Buying

Model 2: Combinatorial Reverse Auction based on Group Buying

Example 2: Figure 2 illustrates an application scenario in which Buyer 1 wants to purchase at least a bundle of items 1A, 1B and 1C from the market and Buyer 2 wants to purchase a bundle of items 1C and 1D. Instead of holding two independent combinatorial reverse auctions, Buyer 1 and Buyer 2 delegate the combinatorial reverse auction to a Group Buyer. The Group Buyer consolidates all the requirements of Buyer 1 and Buyer 2 and holds only one combinatorial reverse auction. Suppose there are four bidders, Seller 1, Seller 2, Seller 3 and Seller 4 who place bids on the Group Buyer. Suppose Seller 1 places the bid: (1A, 1C, p1), where p11 denotes the prices of the bid. Seller 2 places the bid: (1B, 1C, p2). Seller 3 places the bid: (1D, p3). Seller 4 places the bid: (1C, 1D, p4). We assume that all the bids entered the auction are recorded. A bid is said to be active if it is in the solution. We assume that there is only one bid active for all the bids placed by the same bidder. For this example, the solution for this combinatorial reverse auction problem is Seller1: (1A, 1C, p1), Seller 2: (1B, 1C, p2) and Seller 3: (1D, p3). The overall cost of this solution is p1+p2+p3, which is lower than that of Model 1.

Suppose $p3 \le p4$. Then the overall cost of the solution of Model 2 is no greater than that of Model 1. This example illustrates that the overall cost of the combinatorial reverse auction based on group buying is no greater than that of two independent combinatorial reverse auctions even if only the bids submitted in the two independent combinatorial reverse auctions are considered.

3   PROBLEM FORMULATION OF TWO AUCTION MODELS

Motivated by the above examples, it is interesting to compare the two models of combinatorial reverse auction from the performance and computational efficiency aspects. In the remainder of this paper, we first formulate the problem and then propose solution methodology for these two problems. We then compare the performance and computational efficiency by numerical examples.

3.1   Combinatorial Reverse Auction for a Single Buyer

We first formulate the optimization problem for Model 1: Combinatorial Reverse Auction for a Single Buyer.

Consider a buyer i who requests a set of items to be purchased. Let K denote the number of items requested. Let d_{ik} denote the desired units of the $k-th$ item, where $k \in \{1,2,3,....,K\}$. In a combinatorial reverse auction, there are many bidders. Let N_i denote the number of bidders in the combinatorial reverse auction. Each $n \in \{1,2,3,....,N_i\}$ represents a bidder. To model the combinatorial reverse auction problem, the bid must be represented mathematically. We use a vector $b_{nj} = (q_{nj1}, q_{nj2}, q_{nj3}, ..., q_{njK}, p_{nj})$ to represent the $j-th$ bid submitted by bidder n, where q_{njk} is a nonnegative integer that denotes the quantity of the $k-th$ items and p_{nj} is a real positive number that denotes the price of the bundle. As the quantity of the $k-th$ items cannot exceed the quantity d_k, it follows that the constraint $0 \leq q_{njk} \leq d_k$ must be satisfied. The $j-th$ bid b_{nj} is actually an offer to deliver q_{njk} units of items for each $k \in \{1,2,3,....,K\}$ a total price of p_{nj}. Let J_n denote the number of bids placed by bidder $n \in \{1,2,3,....,N_i\}$. Let J denote the maximum number of bids that a bidder can place in each round of combinatorial reverse auction. That is, $J = \max_{n \in \{1,2,...,N\}} J_n$. To formulate the problem, we use the variable x_{nj} to indicate the $j-th$ bid placed by bidder n is active ($x_{nj}=1$) or inactive ($x_{nj}=0$). The winner determination problem can be formulated as an Integer Programming problem as follows.

Winner Determination Problem (WDP) for a Single Buyer i

$$\min \sum_{n=1}^{N_i} \sum_{j=1}^{J_n} x_{nj} p_{nj}$$

$$s.t. \sum_{n=1}^{N_i} \sum_{j=1}^{J_n} x_{nj} q_{njk} \geq d_{ik} \, \forall k = 1,2,...,K \quad (2-1)$$

$$x_{nj} \in \{0,1\} \quad\quad\quad\quad\quad\quad (2-2)$$

Condition (2-1) in WDP assumes "free disposal" as the total quantity offered by the winners must be greater than or equal to the desired quantity of the buyer. If there are more quantities provided than needed, we can dispose of the surplus with no additional cost. One way to reduce the computational burden in solving the WDP is to adopt Lagrangian relaxation approach to set up a fictitious market to determine an allocation and prices in a decentralized way to adapt to dynamic environments where bidders and items may change from time to time. The buyer announces which sets of items and sets prices for them. If two or more agents compete for the same item, the buyer adjusts the price vector. This saves bidders from specifying their bids for every possible combination and the buyer from having to process each bid function. The bundle associated with the bid is tentatively assigned to that bidder only if the price of the bid is the lowest.

3.2 Combinatorial Reverse Auction for Multiple Buyers based on Group Buying

The optimization problem for Model 2: Combinatorial Reverse Auction for Multiple Buyers based on Group Buying is formulated as follows.

Consider a buyer who requests a set of items to be purchased. Let K denote the number of items requested. Let I denote the number of buyers in a combinatorial auction. Each $i \in \{1,2,3,...., I\}$ represents a buyer. Let d_{ik} denote the desired units of the $k-th$ items, where $k \in \{1,2,3,...., K\}$. In a combinatorial auction, there are many bidders to submit a tender. Let N denote the number of bidders in a combinatorial auction. To model the combinatorial reverse auction problem, the bid must be represented mathematically. We use a vector $b_{nj} = (q_{nj1}, q_{nj2}, q_{nj3},..., q_{njK}, p_{nj})$ to represent the $j-th$ bid submitted by bidder n, where q_{njk} is a nonnegative integer that denotes the quantity of the $k-th$ items and p_{nj} is a real positive number that denotes the price of the bundle. As the quantity of the $k-th$ items cannot exceed the quantity d_{ik}, it follows that the constraint $0 \leq q_{njk} \leq \sum_{i=1}^{I} d_{ik}$ must be satisfied.

The $j-th$ bid b_{nj} is actually an offer to deliver q_{njk} units of items for each $k \in \{1,2,3,...., K\}$ a total price of p_{nj}. Let J_n denote the number of bids placed by bidder $n \in \{1,2,3,...., N\}$. To formulate the problem, we use the variable x_{nj} to indicate the $j-th$ bid placed by bidder n is active (x_{nj}=1) or inactive (x_{nj}=0). The winner determination problem can be formulated as an Integer Programming problem as follows.

Winner Determination Problem (WDP) for Group Buyer

$$\min \sum_{n=1}^{N} \sum_{j=1}^{J_n} x_{nj} p_{nj}$$

$$s.t. \sum_{n=1}^{N} \sum_{j=1}^{J_n} x_{nj} q_{njk} \geq \sum_{i=1}^{I} d_{ik} \; \forall k = 1,2,...,K \quad (2-3)$$

$$x_{nj} \in \{0,1\} \quad\quad\quad\quad\quad\quad (2-4)$$

In WDP problem, we observe that the coupling among different operations is caused by the contention for resources through the minimal resource requirement constraints (2-1).

4 SOLUTION ALGORITHMS

One way to reduce the computational complexity in solving the Winner Determination Problem (WDP) for combinatorial reverse auction is to set up a fictitious market to determine an allocation and prices in a decentralized way to adapt to dynamic environments where bidders and items may change from time to time. In this paper, we apply Lagrangian relaxation technique to develop solution algorithms for WDP.

Lagrangian relaxation provides a systematic approach to determine an allocation and prices based on the introduction of Lagrange multipliers, which set prices for each item to be purchased by the buyer. Lagrange multipliers can often be given the economic interpretation as marginal costs for using the items when they are used to relax demand constraints. In our relaxation procedure above, the Lagrange multipliers λ_k is used to relax the demand constraints of item k. Lagrange multipliers λ_k can be interpreted as the marginal benefit of using an additional unit of item k. If two or more sellers compete for the same item, the price will be adjusted. This saves bidders from specifying their bids for every possible combination and the buyer from having to process each bid function. Based on the price for the individual items, bidders submit bids. The bundle associated with a bid is tentatively assigned to that bidder only if the price of the bid is the lowest. Based on the iterative price adjustment mechanism, a solution will be obtained.

Our methodology for finding a near optimal solution of the WDP for a Group Buyer and the WDP for a Single Buyer consists of three parts: (1) an algorithm for solving subproblems, (2) a subgradient method for solving the dual problem and (3) a heuristic algorithm for finding a near-optimal feasible solution. Due to the limited space for presentation, only the solution of the Combinatorial Reverse Auction for Group Buyer will be presented in this paper. By applying a similar procedure, the solution algorithm of the WDP for a Single Buyer can also be derived. For the WDP for a Group Buyer, we define the following dual problem.

$$\max_{\lambda \geq 0} L(\lambda) \text{, where}$$

$$L(\lambda) = \sum_{k=1}^{K} \lambda_k (\sum_{i=1}^{I} d_{ik}) + \sum_{n=1}^{N} \sum_{j=1}^{J_n} L_{nj}(\lambda) \text{, with}$$

$$L_{nj}(\lambda) = \min(p_{nj} - \sum_{k=1}^{K} \lambda_k q_{njk})$$

$$s.t. \ x_{nj} \in \{0,1\}$$

$L_{nj}(\lambda)$ defines a bidder's subproblems (BS). Our methodology for finding a near optimal solution of WDP consists of three parts as follows.

(1) An algorithm for solving subproblems Given λ, solve the BS subproblem $L_{nj}(\lambda)$.

(2) A subgradient method for solving the dual problem $\max_{\lambda \geq 0} L(\lambda)$.

The subgradient method proposed by Polyak [9] is adopted to update λ as follows

$$\lambda_k^{l+1} = \begin{cases} \lambda_k^l + \alpha^l g_k^l & \text{if } \lambda_k^l + \alpha^l \lambda_k^l \geq 0; \\ 0 & \text{otherwise.} \end{cases}$$

where $\alpha^l = c \dfrac{\overline{L} - L(\lambda)}{\sum\limits_{k} (g_k^l)^2}$, $0 \leq c \leq 2$ and $\overline{L}$ is an estimate of the optimal dual cost.

(3) A heuristic algorithm for finding a near-optimal $\overline{x}$, feasible solution based on the solution (x^*, λ^*) of the relaxed problem.

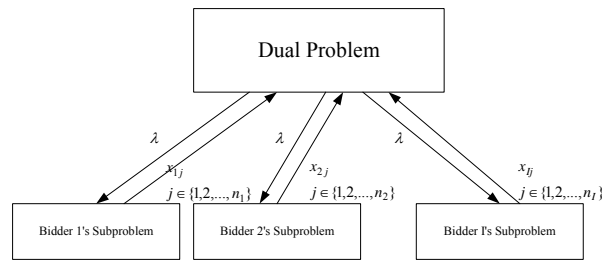Figure 3 shows the architecture of the subgradient algorithm for solving WDP.

Figure 3 Architecture of the subgradient algorithm for solving WDP

5   NUMERICAL RESULTS AND ANALYSIS

Based on the subgradient algorithms, we conduct several examples to compare the effectiveness of the two combinatorial reverse auction models.

Example 3: Buyer 1 and Buyer 2 need to acquire the required items from the market. There are four different types of items. The requirements of Buyer 1 are one type-1 item, one type-2 item and one type-3 item. The requirements of Buyer 2 are one type-3 item and one type-4 item.

To compare the effectiveness of Model 1 and Model 2, the following problems are formulated and solved by the subgradient algorithms.

Problems and Solution for Model 1:
Problem 1-1: Suppose Model 1 is applied to Buyer 1. Buyer 1 holds the following combinatorial reverse auction. Suppose there are two bidders that place bids on Buyer 1's combinatorial reverse auction. There are two bids submitted by the two bidders, one for each bidder.
The data for Buyer 1's combinatorial reverse auction are as follows:

$N_1 = 2, \quad J = 1, \quad K = 3$

$d_{11} = 1, d_{12} = 1, d_{13} = 1.$

$q_{111} = 1, q_{112} = 0, q_{113} = 1,$

$q_{211} = 0, q_{212} = 1, q_{213} = 1,$

$P_{11} = 100, \quad P_{21} = 60.$

Applying the subgradient algorithm to Problem 1-1, the solution is $\bar{x}_{11} = 1, \ \bar{x}_{21} = 1$.
The cost is 160.
CPU time is 15 (millisecond).

Problem 1-2: Suppose Model 1 is applied to Buyer 2. Buyer 2 holds the following combinatorial reverse auction. Suppose there are two bidders that place bids on Buyer 2's combinatorial reverse auction. There are two bids submitted by the two bidders, one for each bidder.
The data for Buyer 2's combinatorial reverse auction are as follows:
As there are two bidders that place bids on Buyer 2's combinatorial reverse auction, $N_2 = 2$.

$N_2 = 2, \quad J = 1, \quad K = 2$

$d_{21} = 1, d_{22} = 1.$

$q_{111} = 0, q_{112} = 1,$

$q_{211} = 1, q_{212} = 1.$

$P_{11} = 80, \quad P_{21} = 120.$

Applying the subgradient algorithm to Problem 1-2, the solution is $x_{11} = 0, \ x_{21} = 1$.
The cost is: 120.
CPU time is 12 (millisecond).

Based on the solutions for Problem 1-1 and Problem 1-2, the cost of the solution for Model 1 is 280 and the CPU time is 27.

Problem and Solution for Model 2:
Problem 2: Suppose Model 2 is applied to Buyer 1 and Buyer 2. The Group Buyer holds a combinatorial reverse auction for Buyer 1 and Buyer 2.
The data for Group Buyer's combinatorial reverse auction are as follows:

$N = 4, \quad J = 1, \quad K = 4, \quad I = 2,$

$d_{11} = 1, d_{12} = 1, d_{13} = 1, \quad d_{14} = 0,$

$d_{21} = 0, d_{22} = 0, d_{23} = 1, \quad d_{24} = 1,$

The two bids submitted by the two bidders are as follows:

$q_{111} = 1, q_{112} = 0, q_{113} = 1, q_{114} = 0,$

$q_{211} = 0, q_{212} = 1, q_{213} = 1, q_{214} = 0,$

$q_{311} = 0, q_{312} = 0, q_{313} = 0, q_{314} = 1,$

$q_{411} = 0, q_{412} = 0, q_{413} = 1, q_{414} = 1,$

$P_{11} = 100, \quad P_{21} = 60, \quad P_{31} = 80, \quad P_{41} = 120 .$

Applying the subgradient algorithm to Problem 2, the solution is $x_{11}=1, \ x_{21}=1, \ x_{31}=1, \ x_{41}=0$.

The cost is 240.

The CPU time is 19 (millisecond).

Based on the solutions for Problem 2, the cost of the solution for Model 2 is 240 and the CPU time is 19.

For Example 3, Model 2 outperforms Model 1 not only in cost but also in computational efficiency. In addition to the aforementioned example, we have also conducted simulations for several examples to compare the computational efficiency of Model 1 and Model 2. The results are summarized in Figure 4. Figure 4 illustrates that the CPU time for Model1 is significantly longer than that of Model 2 as the number of bidders grows. The solution of the cost for each example by applying Model 1 are greater than that obtained by applying Model 2. These results indicate that combinatorial reverse auction based on group buying yields better performance and computational efficiency than holding several independent combinatorial reverse auction.s
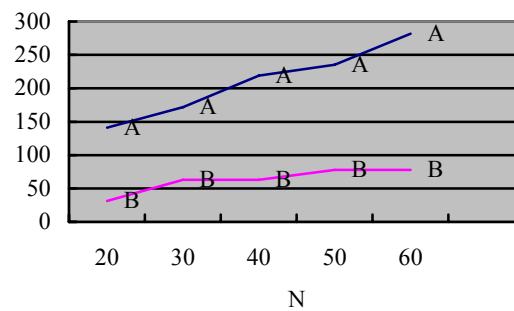

Figure 4 CPU time (in millisecond) respect to N

A: Model1

B: Model2

6   CONCLUSION

Combinatorial reverse auctions can be applied by a buyer to acquire the required products at the minimal cost. If there are multiple buyers, buyers may either hold multiple combinatorial reverse auctions independently. Alternatively, buyers may delegate the auction to a group buyer and the group buyer holds only one combinatorial reverse auction for all the buyers. To study the effectiveness of the two business models for procurement based on combinatorial reverse auctions, we formulate two winner determination optimization problems. The first formulation is for independent combinatorial reverse auctions, where each buyer may hold a combinatorial reverse auction independently. The second formulation is for combinatorial reverse auctions based on group buying, where multiple buyers delegate the auction to a group buyer and the group buyer holds only one combinatorial reverse auction for all the buyers. Due to computational complexity, these two problems cannot be solved exactly. We solve these two problems by applying the Lagrangian relaxation technique. The solution methodology consists of three parts: (1) an algorithm for solving bidders' subproblems by exploiting their individual structures; (2) a subgradient method for solving the non-differentiable dual problem and (3) a heuristic algorithm for finding a near-optimal, feasible solution. Based on the proposed algorithms for the two problems, we compare the efficiency and performance of the two combinatorial reverse auction models. Numerical results indicate that holding a Combinatorial Reverse Auction based on Group Buying yields better performance than holding several independent Combinatorial Reverse Auctions. Moreover, the computational efficiency of Combinatorial Reverse Auction based on Group Buying is also significantly better than several independent Combinatorial Reverse Auctions. If there is no common item in the requirements of different buyers, our analysis and numerical results suggest that Model 2 should be used due to its advantage in computational efficience although the costs of the solutions are the same. If there exist common items in the requirements of different buyers, the solution of Model 2 is always better than Model 1. In this case, suggest that Model 2 should be used due to its advantage in computational efficiency as well as the cost of the solution.

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