COMBINATORIAL DOUBLE AUCTIONS BASED ON SUBGRADIENT ALGORITHM

Fu-Shiung Hsieh

Dept. of Computer Science and Information Engineering
Chaoyang University of Technology,
Taiwan
fshsieh@cyut.edu.tw

Chung-Wei Huang

Dept. of Computer Science and Information Engineering
Chaoyang University of Technology
Taiwan

ABSTRACT

Due to the advantages of higher efficiency over fixed-price trading and the ability to discover equilibrium prices quickly, auction is a popular way of trading goods. In business-to-business e-commerce (B2B), many goods with complementarities or substitutabilities are being traded using auctions. Combinatorial auctions can be applied to improve the efficiency of trading in B2B marketplaces. In combinatorial auctions, a bidder can bid on a combination of goods with one limit price for the total combination. This improves the efficiency when the procurement of one good is dependent on the acquisition of another. Most of the combinatorial auctions studied in the literature are one-sided: either multiple buyers compete for commodities sold by one seller, or, multiple sellers compete for the right to sell to one buyer. Combinatorial double auctions in which both sides submit demand or supply bids are much more efficient than several one-sided auctions combined. However, combinatorial double auctions are notoriously difficult to solve from computation point of view. In this paper, we formulate the combinatorial double auction problem and propose an algorithm for finding near optimal solutions. The algorithm is developed by decomposing the combinatorial double auction problem into several subproblems and applying the subgradient algorithm to iteratively adjust the shadow prices for the subproblems.

1. INTRODUCTION

Combinatorial double auctions are different from combinatorial auctions in that the auctioneer of a combinatorial auction is a seller whereas the auctioneers of a combinatorial double auction are typically multiple sellers and buyers. In spite of the difference, combinatorial double auctions are closely related to combinatorial auctions. An excellent survey on combinatorial auctions can be found in [8] and [10]. In a combinatorial auction [8], bidders may place bids on combinations of items, which allows the bidders to express complementarities between items instead of having to speculate into an item's valuation about the impact of possibly getting other, complementary items. Allowing bids for bundles of items is the foundation of combinatorial auctions. There are, however, several problems with combinatorial auctions. Combinatorial auctions have been notoriously difficult to solve from a computational point of view [19]. Combinatorial auction is closely related to the set packing/knapsack problem. It deals with computational aspects and heuristics for solving what is known as the Winner Determination Problem of an auction [16].

The combinatorial auction problem can be modeled as a set packing problem (SPP), a well-known NP-complete problem [11]-[15]. Many algorithms have been developed for combinatorial auction problems. For example, in [9], the authors proposed a Lagrangian Heuristic for a combinatorial auction problem. Exact algorithms have been developed for the SPP problem, including a branch and bound search [15], iterative deepening A* search [14] and the direct application of available CPLEX IP solver [11]. Hsieh *et al.* also proposed computationally efficient algorithms based on Lagrangian relaxation [20]-[23].

We consider the combinatorial double auction problem in which there are multiple buyers and multiple sellers. Each buyer and seller places bids based on the required items to be purchased and the available items to be sold, respectively. Each seller places bids for each bundle of goods he can provide. We assume there is a mediator for the combinatorial double auction. The problem is to determine the winners.

The remainder of this paper is organized as follows. In Section 2, we present the winner determination problem formulation of combinatorial double auctions. In Section 3, we propose the subgradient based algorithms. In Section 4, we present the numerical examples and analyze the results of our solution approach. We conclude this paper in Section 5.

2. PROBLEM FORMULATION

In this paper, we first formulate the above combinatorial optimization problem as an integer programming problem. We then develop solution algorithms based on Lagrangian relaxation.

We assume that there are a set of buyers, a set of potential sellers and a mediator called the proxy buyer who consolidates the demands from the buyers. To fulfill the buyers' requirements and minimize cost, the proxy buyer must purchase the required items from the potential sellers at the minimal cost.

Figure 1 illustrates an application scenario in which Buyer 1 requests to purchase at least a bundle of items 1A, 2B and 3C from the market, and Buyer 2 requests to purchase at least a bundle of items 2A, 3B and 1C from the market .There are three bidders, Seller 1, Seller 2 and Seller 3 who place bids in the system. Suppose Seller 1 places two bids. The first bid is (2A, 2B, P_{11}) and the second bid is (1B, 1C, P_{12}), where P_{11} and P_{12} denote the prices of the first bid and the second bid. Seller 2 places two bids. The first bid is (1A, 1B, P_{21}) and the second bid is (1A, 2B, 2C, P_{22}). Seller 3 places two bids. The first bid is (1A, 1C, P_{31}) and the second bid is (1B, 2C, P_{32}). We assume that all the bids entered the auction are recorded. A bid is said to be active if it is in the solution. We assume that there is only one bid active for all the bids placed by the same bidder. For this example, the solution for this reverse auction problem is Seller1: (2A, 2B, P_{11}), Seller 2: (1A, 2B, 2C, P_{22}) and Seller 3: (1B, 2C, P_{32}). To formulate the problem, let's define the notation and key terms in this paper.

Buyer: A buyer is a participant who requests a set of items to be purchased.

Seller: A potential seller is a participant who provides a set of items to the mediator.

Buyer's requirements: The set of items requested by a buyer.

Seller's offer: The set of items that can be offered by a seller.

Items: Items are the goods or products requested by the buyers.

In a combinatorial double auction, there are many bidders to submit a tender. To model the combinatorial double auction problem, the bid must be represented mathematically. To formulate the problem, we following notations are defined.

Notations:

K : the number of items requested.

I : the number of potential sellers in a combinatorial double auction. Each $i \in \{1,2,3,....,I\}$ represents a seller.

N : the number of potential buyers in a combinatorial double auction. Each $n \in \{1,2,3,....,N\}$ represents a buyer.

d_{nhk} : the buyer- n 's desired units of the $k-th$ items in the $h-th$ request for tender, where $k \in \{1,2,3,....,K\}$.

j : $j-th$ bid submitted by the seller in a combinatorial double auction.

h : $h-th$ request for tender created by the buyer in a combinatorial double auction.

p_{ij} is a real positive number that denotes the price of the bundle corresponding to the $j-th$ bid submitted by the seller.

q_{ijk} is a nonnegative integer that denotes the quantity of the $k-th$ items in the $j-th$ bid submitted by the seller.

$b_{ij} = (q_{ij1}, q_{ij2}, q_{ij3},..., q_{ijK}, p_{ij})$: a vector to represent the $j-th$ bid submitted by seller i . The $j-th$ bid b_{ij} is actually an offer to deliver q_{ijk} units of items for each $k \in \{1,2,3,....,K\}$ a total price of p_{ij} .

p_{ij}^{WILL} : The reserved price of the $j-th$ bid submitted by the seller i .

x_{ij} : the variable to indicate the $j-th$ bid placed by seller i is active (x_{ij} =1) or inactive (x_{ij} =0).

p_{nh} is a real positive number that denotes the price of the bundle corresponding to the $h-th$ bid submitted by buyer n .

p_{nh}^{WILL} : The reserved price of the $h-th$ bid submitted by buyer n .

y_{nh} : the variable to indicate the $h-th$ bid placed by seller n is active (y_{nh} =1) or inactive (y_{nh} =0).

n_i : the number of bids placed by seller $i \in \{1,2,3,....,I\}$.

n_n : the number of bids placed by buyer $n \in \{1,2,3,....,N\}$.

The winner determination problem is formulated as an Integer Programming problem as follows.

Winner Determination Problem (WDP):

$$\min \sum_{i=1}^{I}\sum_{j=1}^{n_i} x_{ij}(p_{ij} - p_{ij}^{WILL}) + \sum_{n=1}^{N}\sum_{h=1}^{n_{ni}} y_{nh}(p_{nh}^{WILL} - p_{nh})$$

$$s.t. \sum_{i=1}^{I}\sum_{j=1}^{n_i} x_{ij} q_{ijk} \geq \sum_{n=1}^{N}\sum_{h=1}^{n_{ni}} y_{nh} d_{nhk} \;\forall k = 1,2,..., K \quad (2-1)$$

$$x_{ij} \in \{0,1\} \;\forall i \in \{1,2,3,....,I\} \quad (2-2)$$

$$y_{nh} \in \{0,1\} \;\forall n \in \{1,2,3,....,N\} \quad (2-3)$$

In WDP problem, we observe that the coupling among different operations is caused by the contention for resources through the minimal resource requirement constraints (2-1).

3. SOLUTION ALGORITHM

For a given Lagrange multiplier λ, the relaxation of constraints (2-1) decomposes the original problem into a number of bidder's subproblems (BS). These subproblems can be solved independently. That is, the Lagrangian relaxation results in subproblems with a highly decentralized decision making structure. Interactions among subproblems are reflected through Lanrange multipliers, which are determined by solving the following dual problem.

$\max_{\lambda \geq 0} L(\lambda)$, where

$$\mathrm{L}(\lambda) = Max[\sum_{i=1}^{I}\sum_{j=1}^{n_i} x_{ij}(p_{ij} - p_{ij}^{WILL}) + \sum_{n=1}^{N}\sum_{h=1}^{n_n} y_{nh}(p_{nh}^{WILL} - p_{nh})] +$$
$$\sum_{k=1}^{K}\lambda_k(\sum_{n=1}^{N}\sum_{h=1}^{n_n} y_{nh}d_{nhk} - \sum_{i=1}^{I}\sum_{j=1}^{n_i} x_{ij}q_{ijk})$$
$$s.t.\ x_{ij} \in \{0,1\}\ \forall i \in \{1,2,3,....,I\}$$
$$y_{nh} \in \{0,1\}\ \forall n \in \{1,2,3,....,N\}$$
$$= Max[\sum_{i=1}^{I}\sum_{j=1}^{n_i} x_{ij}(p_{ij} - p_{ij}^{WILL} - \sum_{k=1}^{K}\lambda_k q_{ijk}) +$$
$$Max[\sum_{n=1}^{N}\sum_{h=1}^{n_n} y_{nh}(p_{nh}^{WILL} - p_{nh} + \sum_{k=1}^{K}\lambda_k d_{nhk})$$
$$= \sum_{i=1}^{I} L_{si}(\lambda) + \sum_{n=1}^{N} L_{bn}(\lambda), \text{ with}$$
$$\mathrm{L_{si}}(\lambda)$$
$$= Max\ x_{ij}(p_{ij} - p_{ij}^{WILL} - \sum_{k=1}^{K}\lambda_k q_{ijk})$$
$$s.t.\ x_{ij} \in \{0,1\}$$
$$\mathrm{L_{bn}}(\lambda)$$
$$= Max[\sum_{n=1}^{N}\sum_{h=1}^{n_n} y_{nh}(p_{nh}^{WILL} - p_{nh} + \sum_{k=1}^{K}\lambda_k d_{nhk})$$
$$s.t.\ x_{nh} \in \{0,1\}$$

$L_{\mathrm{nj}}(\lambda)$ defines a bidder's subproblems (BS). Our methodology for finding a near optimal solution of WDP is developed based on the result of Lagrangian relaxation and decomposition. It consists of three parts as follows.

(1) An algorithm for solving subproblems

Given λ, the optimal solution to BS subproblem $L_{\mathrm{nj}}(\lambda)$ can be solved as follows.

$$x_{ij} = \begin{cases} 1\ if\ (p_{ij} - p_{ij}^{WILL} - \sum_{k=1}^{K}\lambda_k q_{ijk}) > 0 \\ 0\ if\ (p_{ij} - p_{ij}^{WILL} - \sum_{k=1}^{K}\lambda_k q_{ijk}) \leq 0 \end{cases}$$

$$y_{nh} = \begin{cases} 1\ if\ (p_{nh}^{WILL} - p_{nh} + +\sum_{k=1}^{K}\lambda_k d_{nhk}) > 0 \\ 0\ if\ (p_{nh}^{WILL} - p_{nh} + +\sum_{k=1}^{K}\lambda_k d_{nhk}) \leq 0 \end{cases}$$

(2) A subgradient method for solving the dual problem $\max_{\lambda \geq 0} L(\lambda)$

Let x^l be the optimal solution to the subproblems for given Lagrange multipliers λ^l of iteration l. We define the subgradient of $L(\lambda)$ as

$$g_k^l = \left.\frac{\partial L(\lambda)}{\partial \lambda_k}\right|_{\lambda_k^l} = \sum_{n=1}^{N}\sum_{h=1}^{n_n} y_{nh}d_{nhk} - \sum_{i=1}^{I=4}\sum_{j=1}^{n_i} x_{ij}q_{ijk},$$

where $k \in \{1,2,..., K\}$.

The subgradient method proposed by Polyak [9] is adopted to update λ as follows

$$\lambda_k^{l+1} = \begin{cases} \lambda_k^l + \alpha^l g_k^l\ if\ \lambda_k^l + \alpha^l \lambda_k^l \geq 0; \\ 0\ otherwise. \end{cases}$$

where $\alpha^l = c\dfrac{\overline{L} - L(\lambda)}{\Sigma_k (g_k^l)^2}$, $0 \leq c \leq 2$ and $\overline{L}$ is an estimate of the optimal dual cost. The iteration step terminates if α^l is smaller than a threshold. Polyak proved that this method has a linear convergence rate.

Iterative application of the algorithms in (1) and (2) may converge to an optimal dual solution (x^*, y^*, λ^*).

(3) A heuristic algorithm for finding a near-optimal $(\bar{x}, \bar{y})$, feasible solution based on the solution (x^*, y^*, λ^*) of the relaxed problem

The solution (x^*, y^*, λ^*) may result in one type of constraint violation due to relaxation: assignment of the quantity of items less than the demand of the items. Our heuristic scheme first checks all the demand constraints

$\sum_{i=1}^{I}\sum_{j=1}^{n_i} x_{ij} q_{ijk} \geq \sum_{n=1}^{N}\sum_{h=1}^{n_{ni}} y_{nh} d_{nhk} \forall k = 1,2,...,K$ that have not been satisfied.

Let $K^0 =$

$\{k \mid k \in \{1,2,3,....,K\}, \sum_{i=1}^{I}\sum_{j=1}^{n_i} x_{ij} q_{ijk} < \sum_{n=1}^{N}\sum_{h=1}^{n_{ni}} y_{nh} d_{nhk}\}$. K^0 denotes the set of demand constraints violated. Let $I^0 = \{i \mid i \in \{1,2,3,....,I\}, x_{ij}^* = 0\}$. I^0 denotes the set of sellers that is not a winner in solution x^*. To make the set of constraints K^0 satisfied, we first pick $k \in K^0$ with

$$k = \arg\min_{k \in K^0} \left(\sum_{n=1}^{N}\sum_{h=1}^{n_{ni}} y_{nh}^* d_{nhk} - \sum_{i=1}^{I}\sum_{j=1}^{n_i} x_{ij}^* q_{ijk}\right).$$

The heuristic algorithm proceeds as follows to make constraint k satisfied.

Select $i \in I^0$ with $i = \arg\min_{i \in \{1,2,...,I\}, q_{ijk} > 0} p_{ij}$ and set x_{ij}^* =1. After performing the above operation, we set $N^0 \leftarrow N^0 \setminus \{i\}$. If the violation of the k-th constraint cannot be completely resolved, the same procedure repeats. Eventually, all the constraints will be satisfied. We use $\bar{x}$ to denote the resulting feasible solution obtained from the above heuristics.

4. NUMERICAL RESULTS

Based on the proposed algorithms for combinatorial double auction, we conduct several examples to illustrate the validity of our method.

Example 1: Consider two buyers who will purchase a set of items specified as follows. The first bids and the second bids placed by the seven potential sellers' bids are shown in Table 2 and Table 3, respectively.

Table 1:

Seller \ Item	K#1	K#2	K#3	K#4
I #1	1	3	2	1
I #2	3	0	2	2

Table 2:

Buyer \ Item	K=1	K=2	K=3	K=4
N=1	1	0	3	0
N=2	0	2	0	1
N=3	3	1	1	2
N=4	2	1	0	3
N=5	4	3	2	1

$$q_{111} = 1,\ q_{112} = 3,\ q_{113} = 2, q_{114} = 1$$
$$q_{211} = 3,\ q_{212} = 0,\ q_{213} = 2, q_{214} = 2$$
$$p_{11} = 35,\ p_{21} = 25,$$
$$p_{ij}^{will} = 5$$

$$d_{111} = 1,\ d_{112} = 0,\ d_{113} = 3, d_{114} = 0$$
$$d_{211} = 0,\ d_{212} = 2,\ d_{213} = 0, d_{214} = 1$$
$$d_{311} = 3,\ d_{312} = 1,\ d_{313} = 1, d_{314} = 2$$
$$d_{411} = 2,\ d_{412} = 1,\ d_{413} = 0, d_{414} = 3$$
$$d_{511} = 4,\ d_{512} = 3,\ d_{513} = 2, d_{514} = 1$$
$$p_{11} = 45,\ p_{21} = 50, p_{31} = 90,\ p_{41} = 80, p_{51} = 100$$
$$p_{nh}^{will} = 6$$

$\bar{L} = 70$

Suppose we initialize the Lagrange multipliers as follows.

$\lambda_1 = 1, \lambda_2 = 1, \lambda_3 = 1, \lambda_4 = 1$

Our algorithm the subgradient algorithm converges to the following solution:

x_{11}^* =1, x_{21}^* =0, y_{11}^* =1, y_{21}^* =1 and y_{31}^*=0, y_{41}^*=0, y_{51}^*=0. By applying the heuristic algorithm, the solution is $\bar{x}_{11}$ =1, $\bar{x}_{21}$ =1, $\bar{y}_{11}$ =1, $\bar{y}_{21}$=1, $\bar{y}_{31}$=1, $\bar{y}_{41}$=0, $\bar{y}_{51}$=0. The duality gap of the solution is 4.15%. The duality gap is within 5%.

This means the solution methodology generates near optimal solution.Despite the duality gap is not zero, the solution $\bar{x}$ is also an optimal solution for this example.

5. CONCLUSIONS

Combinatorial double auctions enable buyers and sellers to bid on different combination of goods simultaneously and offer those items a price. Each seller places bids for each bundle of goods he can provide. Each buyer places the bids for the required products. We study the decision problem of the mediator between buyers and sellers in combinatorial double auctions. The mediator has to determine the winners for the buyers and sellers in combinatorial double auctions. We formulate a winner determination optimization problem for combinatorial double auction. The demands of the buyers and the goods supplied by the sellers impose constraints on determination of the winners. The problem is to determine the winners to maximize the total profit. The main results include: (1) a problem formulation for the combinatorial double auction problem; (2) a solution methodology based on Lagrangian relaxation and (3) analysis of numerical results based on our solution algorithms. Analysis of the numerical results shows that our algorithm can generate near-optimal solution within acceptable CPU time.

6. ACKNOWLEDGMENTS

This paper is supported in part by National Science Council of Taiwan under Grant NSC97-2410-H-324-017-MY3.

7. REFERENCES

[1] de Vries, Sven; Vohra, Rakesh V. “Combinatorial Auctions:A Survey, ” INFORMS Journal on Computing,(3):284–309, 2003.

[2] Guo, Y. Lim, A. Rodrigues, B. Tang, J. “Using a Lagrangian heuristic for a combinatorial auction problem,” In Proceedings of the 17th IEEE International Conference on Tools with Artificial Intelligence, 2005.

[3] A. Pekeč and M.H. Rothkopf. “Combinatorial auction design,” Management Science 49, pp. 1485–1503, 2003.

[4] A. Andersson, M. Tenhunen, and F. Ygge. “Integer programming for combinatorial auction winner determination,” In Proceedings of the Seventeenth National Conference on Artificial Intelligence, pages 39–46, 2000.

[5] Y. Fujishima, K. Leyton-Brown, and Y. Shoham. “Taming the computational complexity of combinatorial auctions:Optimal and approximate approaches,” In Sixteenth International Joint Conference on Artificial Intelligence, pages 548–553, 1999.

[6] Holger H. Hoos and Craig Boutilier. “Solving combinatorial auctions using stochastic local search,” In Proceedings of the Seventeenth National Conference on Artificial Intelligence, pages 22–29, 2000.

[7] Tuomas Sandholm. “Algorithm for optimal winner determination in combinatorial auctions, ” Artificial Intelligence, 135(1-2):1–54, 2002.

[8] Tuomas Sandholm, Subhash Suri, Andrew Gilpin, and David Levine. “CABOB: A fast optimal algorithm for combinatorial auctions.” In IJCAI, pages 1102–1108, 2001.

[9] Polyak, B. T. “Minimization of Unsmooth Functionals,” USSR Computational Math. and Math. Physics, vol. 9, pp. 14-29, 1969.

[10] T. Metty, R. Harlan, Q. Samelson, T. Moore, T. Morris, R. Sorensen, A. Schneur, O. Raskina, R. Schneur, J. Kanner, K. Potts and J. Robbins, “Reinventing the supplier negotiation process at Motorola,” Interfaces 35, pp. 7–23, 2005.

[11] J.D. Murray and R.W. White, “Economies of scale and economies of scope in multiproduct financial institutions: a study of British Columbia Credit Unions,” The Journal of Finance 38, pp. 887–902, 1983.

[12] M. Rothkopf, A. Pekeč and R. Harstad, “Computationally manageable combinational auctions,” Management Science 44, pp. 1131–1147, 1998.

[13] R.R. Vemuganti, “Applications of set covering, set packing and set partitioning models: a survey.” In: D.-Z. Du, Editor, Handbook of Combinatorial Optimization Vol. 1, Kluwer Academic Publishers, Netherlands, pp. 573–746, 1998.

[14] R. Gonen and D. Lehmann, “Optimal solutions for multi-unit combinatorial auctions: branch and bound heuristics,” The Proceedings of the Second ACM Conference on Electronic Commerce (EC'00) , pp. 13–20, 2000.

[15] J.L. Jones and G.J. Koehler, “Combinatorial auctions using rule-based bids,” Decision Support Systems 34, pp. 59–74, 2002.

[16] T. Sandholm, “Approaches to winner determination in combinatorial auctions,” Decision Support Systems 28, pp. 165–176, 2000.

[17] Fan, M., Stallaert, J. and Whinston, A.B., 1999. A web-based financial trading system. IEEE Computer 32 4, pp. 64–70.
[18] Ba, S., Stallaert, J. and Whinston, A.B., 2001. Optimal investment in knowledge within a firm using a market-mechanism. Management Science 47, pp. 1203–1219.
[19] Mu Xia, Jan Stallaert, Andrew B. Whinston, Solving the combinatorial double auction problem, European Journal of Operational Research, Volume 164, Issue 1, 1 July 2005, Pages 239-251.
[20]Fu-Shiung Hsieh "Combinatorial Auction with Minimal Resource Requirements," Lecture Notes in Artificial Intelligence, vol. 4570, pp. 1072-1077, 2007.
[21]Fu-Shiung Hsieh and Shih-Min Tsai, "Combinatorial Reverse Auction based on Lagrangian Relaxation," Proceedings of 2008 IEEE Asia-Pacific Services Computing Conference, pp. 329-334, 2008.
[22]Fu-Shiung Hsieh, "Combinatorial Reverse Auction Algorithm for Group Buying," The 2009 International Workshop on Mobile Systems, E-commerce, and Agent Technology (MSEAT 2009), 2009.
[23]Fu-Shiung Hsieh, "Combinatorial reverse auction based on revelation of Lagrangian multipliers," Decision Support Systems, Volume 48, Issue 2, pp. 323-330.