Example: Computing Fees

The most common fee structure consists of (1) a management fee that is a fixed proportions of assets under management, typically 2%, and (2) an incentive fee, which is typically 20% of the profits over a year when positive. Sometimes, the incentive fee is paid after the performance exceeds a **hurdle rate**, such as LIBOR.

As an example, assume that the Net Asset Value (NAV) goes from 100 to 120 over the year before fees (gross) and that LIBOR is at 5%. Fees are at the usual 2 and 20. The total fee is $100 \times 2\% + [(120 - 102) - 0.05 \times 100] \times 20\%$. In this case, the gross return is 20%, the total fees to the manager are 4.6%, and the net return is 15.4%. Without a hurdle, the net return would be 14.4%.

17.2 LEVERAGE, LONG, AND SHORT POSITIONS

Hedge funds can achieve leverage and implement short-sales through their **prime broker** (PB). PBs provide various back-office services to hedge funds, including trade reconciliation (clearing and settlement), custody services, and risk management, as well as record keeping. In addition, they provide credit lines for financing leverage and short selling capabilities.

To understand the mechanics of hedge funds, we need to describe how stock borrowing and margins work. In typical corporate balance sheet analysis, **balance sheet leverage** is defined as the ratio of balance sheet assets over equity. This simplistic measure, however, assumes that all the risk is coming from the assets, or that the future value of liabilities is known. Such definition is not adequate for hedge funds, or most financial institutions, for that matter. In these cases, both assets and liabilities, long and short positions, are risky.

In what follows, we illustrate the use of long and short positions in stocks. This analysis, however, can be extended to any asset that can be shorted, subject to its own specific margin requirements.

17.2.1 Long Position

Let us start with the simplest case, which is a long position in a risky asset. Consider an investor with \$100 (say millions) invested in one stock. This can be achieved with \$100 of investor equity. Or the investor can borrow. Suppose the broker requires a 50% margin deposit, which is the minimum requirement under **Regulation T** in the United States. The investor needs to invest only \$50 and the remainder is provided by the broker, who gives a \$50 loan. The balance sheet of the position is described below, with the risky entry in bold. Defining leverage L as the ratio of assets over equity, the leverage of this position is 2 to 1.

Assets	Liabilities
\$100 **Long stock**	\$50 Broker loan
	\$50 Equity

The risk is that of a *decrease* in the value of the stock. For instance, a loss of \$1, which is 1% of the value of the stock, translates into a \$1 loss in the value of equity, which is a 2% loss in relative terms. Thus, movements in the asset value are magnified by the leverage factor L. If there is no leverage ($L = 1$), the worst loss occurs when the stock price goes to zero.

The rate of return on the equity is the summation of L times the rate of return on the long stock position R_S minus $(L - 1)$ times the cost of the loan, R_F:

$$R_E = LR_S - (L - 1)R_F = R_F + L(R_S - R_F) \tag{17.1}$$

Hence, the volatility of equity will be L times that of the stock position. Similarly, the beta and idiosyncratic volatility are multiplied by the leverage:

$$\beta_E = L\beta_S \tag{17.2}$$

Leverage amplifies returns but also creates more risk.

Note that leverage can also be obtained by using derivatives, instead of cash instruments. This includes single stock futures, contracts for differences, or equity swaps. If a stock futures position can be entered with a margin of only 10%, the economically equivalent liabilities would consist of a loan of \$90 plus equity of \$10. The dollar exposure is still the same, at \$100, but the leverage is now much higher than before, at 10 to 1.

17.2.2 Short Position

Consider next a situation where the investor wants to short the stock instead. Under a stock loan agreement, the owner of a stock lends the stock to our investor in exchange for cash and a future demand to get the stock back. In the meantime, the investor must pass along any cash flow on the stock, such as dividends, to the original owner.[1] When the operation is reversed, the stock lender returns the cash plus the short-term interest rate minus a **stock loan fee**. This is typically 20 basis points (bp) for most stocks but can reach 400bp for stocks that are hard to borrow (said to be "on special"). In the meantime, the stock lender will have invested the cash, thus earning a net fee of 20bp. From the viewpoint of the stock lender, this is an easy way to increase the return on the stock by a modest amount.

The stock borrower will now sell the stock in anticipation of a fall in the price. The sale, however, will go through a broker, who will not allow the seller to have full access to the sales proceeds. In the United States, under Regulation T, the broker keeps 50% of the sales proceeds. This margin, which can be posted as any security owned clear by the investor, imposes a limit on the investor's leverage.

So, the investor receives \$100 worth of stocks, sells it, and keeps at least \$50 as margin with the broker. The hope is for a fall in the stock price, so that the stock can be repurchased later at a lower price.

[1] Traditional stock loans are made on a day-to-day basis. The lender can demand the return of the stock at any time, with a three-day period for delivery.

All of the cash flows are arranged at the same time. The investor needs to send \$100 to the stock lender, half of which will come from the remaining proceeds and the other half from the equity invested, or own funds. The balance sheet for the short position is described below, with the risky entry in bold. Here, leverage can be defined as the ratio of the absolute value of the short position to the equity, which is 2 to 1. As in the previous long-only case, we have a position of \$50 in equity leveraged into a position of \$100 in stocks. Regulation T imposes a maximum leverage ratio of 2, which is the inverse of the 50% of the short-sales proceeds kept by the broker.

Assets	Liabilities
\$100 Cash lent to stock owner	\$100 **Short stock**
\$50 Margin at the broker	\$50 Equity

Here, the risk is that of an *increase* in the value of the stock. If the stock price goes up by \$1, or 1%, the equity loses \$1, which is 2% in relative terms. This ratio equal the leverage of 2. The beta of the equity is now negatively related to that of the stock:

$$\beta_E = -L\beta_S \tag{17.3}$$

Short positions are intrinsically more risky than long positions, however. This is because the distribution of prices is asymmetrical. The price has a lower bound of zero but has unlimited upper values, albeit with decreasing probabilities. With a long position, the most that could be lost is \$100 million. With a short position, the price could go from \$100 to \$200 or even higher, in which case the dollar loss would exceed \$100 million.

17.2.3 Long and Short Positions

Consider now a typical hedge fund, which has both long and short positions. Say the initial capital is \$100. This is the equity, or **net asset value** (NAV). The fund could buy \$100 worth of stocks and short \$100 worth of stocks as above. Part of the long stock position can be used to satisfy the broker's minimum margin requirement of \$50 for shorting the stock. The balance sheet for the long and short positions is described below, with the risky entries in bold.

Assets	Liabilities
\$100 **Long stock**	
\$100 Cash lent to stock owner	\$100 **Short stock**
	\$100 Equity

Let us now turn to traditional risk measures. Define V_L, V_S, and V_E as the (absolute) dollar values of the long stock positions, short stock positions, and equity, respectively. V_A is the value of total assets. If β_L and β_S are the betas of the long and short stock positions, the total dollar beta is

$$(\beta_L V_L - \beta_S V_S) = \beta_E V_E \tag{17.4}$$

which defines the net beta of equity, or β_E. This net measure of systematic risk, however, ignores idiosyncratic risk.

Traditional **leverage** is commonly used as a risk measure

$$\text{Leverage} = \frac{V_A}{V_E} = \frac{\text{Long stock positions plus cash}}{\text{Equity}} \tag{17.5}$$

In our example, this is $(\$100 + \$100)/(\$100) = 2$. If cash is ignored, **long leverage** is 1. This, however, ignores the hedging effect of short stock positions, so it is inadequate.

Using gross amounts, **gross leverage** is

$$\text{Gross Leverage} = \frac{V_L + V_S}{V_E} = \frac{\text{Long positions plus absolute value of short positions}}{\text{Equity}} \tag{17.6}$$

In our example, this is $(\$100 + \$100)/(\$100) = 2$.

Gross leverage is often used as a rough measure of hedge fund risk. This measure, however, fails to capture the systematic risk of the equity position adequately. If the long and short positions have the same value and market beta, the net beta is zero, so there is no directional market risk. In the limit (even though there would be no reason to do so), if the long and short positions are invested in the same stock, there is no risk. Yet, gross leverage is high.

Another definition sometimes used is **net leverage**, which is

$$\text{Net Leverage} = \frac{V_L - V_S}{V_E} = \frac{\text{Long positions minus absolute value of short positions}}{\text{Equity}} \tag{17.7}$$

In our example, this is $(\$100 - \$100)/(\$100) = 0$.

Net leverage is also inadequate as a risk measure. Although it roughly accounts for systematic risk, it fails to take into account potential divergences in the value of the long and short positions. It is only appropriate under restrictive assumptions. For example, if the betas of the long and short positions are the same, then the equity beta is

$$\beta_E = \frac{\beta_L(V_L - V_S)}{V_E} = \beta_L \times \text{Net Leverage} \tag{17.8}$$

so this net leverage term measures the multiplier applied to the beta of the long position. This totally ignores idiosyncratic risk, however, which is precisely the type of risk that the hedge fund manager should take views on. In conclusion, these leverage measures should only be viewed as rough indicators of risk. They are robust and easy to compute, however.

This is why the industry has moved to more comprehensive position-based risk measures. VAR, for example, accounts for the size of positions, volatilities, as well as correlations between assets and liabilities. As such, it is a superior measure of the risk of loss.

EXAMPLE 17.1: FRM EXAM 2006—QUESTION 41

A hedge fund is long \$315 million in certain stocks and short \$225 million in other stocks. The hedge fund's equity is \$185 million. The fund's overall beta is 0.75. Calculate the gross and net leverage.

a. 2.91 and 0.48
b. 2.18 and 0.36
c. 2.91 and 0.36
d. 2.18 and 0.48

EXAMPLE 17.2: HEDGING AND RETURNS

Continuing with the previous question, assume the stock market went up by 20% last year. Ignore the risk-free rate and idiosyncratic risk, and assume the average beta of both long and short positions is one. Over the same period, the return on the fund should be about

a. 20%
b. 15%
c. 10%
d. 5%

EXAMPLE 17.3: FRM EXAM 2004—QUESTION 2

A relative value hedge fund manager holds a long position in Asset A and a short position in Asset B of roughly equal principal amounts. Asset A currently has a correlation with Asset B of 0.97. The risk manager decides to overwrite this correlation assumption in the variance-covariance based VAR model to a level of 0.30. What effect will this change have on the resulting VAR measure?

a. It increases VAR.
b. It decreases VAR.
c. It has no effect on VAR, but changes profit or loss of strategy.
d. Do not have enough information to answer.

17.3 HEDGE FUNDS: MARKET RISKS

17.3.1 Types of Market Risks

Hedge funds are a much more heterogeneous group of investment managers than others. They follow a great variety of strategies, which can be classified into different styles. More generally, they can be categorized into taking directional or nondirectional risks.

- **Directional risks** involve exposures to the direction of movements in major financial market variables. These directional exposures are measured by first-order or linear approximations such as
  - **Beta** for exposure to general stock market movements
  - **Duration** for exposure to the level of interest rates
  - **Spread duration** for exposure to movements in credit spreads
  - **Delta** for exposure of options to the price of the underlying asset
- **Nondirectional risks** involve other remaining exposures, such as nonlinear exposures, exposures to hedged positions, and exposures to volatilities. These nondirectional exposures are measured by exposures to differences in price movements, or quadratic exposures such as
  - **Basis risk**, which involves differences in prices of related assets
  - **Convexity risk**, which involves quadratic effects for interest rates
  - **Gamma risk**, which involves quadratic effects with options
  - **Volatility risk**, which involves movements in the volatility

Directional trades can take long or short positions on the major risk factors, such as equities, currencies, fixed-income instruments, and commodities. As a result, directional positions have greater volatilities than nondirectional ones. For funds that take directional risks, total portfolio risk is controlled through diversification across sources of risks, across trading strategies, and with risk limits.

Many categories of hedge funds are hedged against directional risks. As a result, they are exposed to nondirectional risks. Such strategies need to take long *and* short positions in directional trades. The example we gave in the previous section was long \$100 in a stock offset by a short position worth \$100 in another stock. Such strategy has little directional risk to the stock market, but is exposed to changes in the relative value of the two stocks. Limiting risk also limits rewards, however. As a result, nondirectional strategies are often highly leveraged in order to multiply gains from taking nondirectional bets.

17.3.2 Hedge Fund Styles

Hedge funds can be classified into various styles, reflecting the type of trading and markets they are exposed to. Table 17.1 lists various hedge fund styles. To some extent, this classification is arbitrary. Definitions of categories vary within the industry. Different hedge fund index providers, for example, use different

TABLE 17.1 Hedge Fund Styles

Style	AUM ($b)	Nb. of Funds	Risk (%pa)	Description
Directional Strategies				
Long/short equity	424	975	11%	Combination of long and short equity positions with net long bias
Emerging markets	115	205	12%	Equity and bond positions in emerging countries, with net long bias
Global macro	122	148	11%	Long and/or short positions across all asset classes
Nondirectional Strategies				
Relative value:				
Equity market neutral	71	183	6%	Combination of long and short equity positions with net beta close to zero
Fixed-income arbitrage	59	138	6%	Offsetting long and short positions in fixed-income securities
Convertible arbitrage	39	79	5%	Long positions in convertible bonds hedged for stock risk and interest risk
Event driven:	318	289	6%	
Merger arbitrage, Distressed securities, Credit hedging				Positions driven by corporate events such as mergers, reorganizations, and bankruptcy proceedings
Fund Structure				
Managed futures	57	235	17%	Positions in futures and option contracts (includes CTAs)
Multistrategy	181	262	9%	Combinations of hedge fund strategies in the same fund
Funds of funds		872	6%	Diversified portfolios of hedge funds

Source: TASS database, sample of live funds reporting in U.S. dollars as of December 2007. Risk is cross-sectional average of annualized volatility over the last four years.

classifications, even though the underlying pool of hedge funds is similar. Classifications can also lose meaning if hedge funds managers change strategies over time.

The table also reports the number of existing funds in each group, as well as their typical risk, measured as the annual standard deviation averaged across all funds.[2] Styles are listed in order of decreasing risk.

Table 17.2 presents the performance of typical hedge fund indices, measured over the period 1994 to 2007. CSFB builds each sector index as a value-weighted average of eligible funds. The table shows the compound growth, volatility, beta to the S&P stock index, return in excess of cash, and the alpha from a regression on equities (as discussed in the previous chapter).

The table shows that the overall hedge fund index returned 10.3% over this period, which is above the 9.1% return of the S&P index, with much lower

[2] Note that the risk measures are for live funds only. Hence, the data are subject to survivorship bias. The risk of existing funds is less than that of dead funds.

TABLE 17.2 Hedge Fund Performance: CSFB Indices, 1994 to 2007

	Growth	Volatility	Beta	Exc. Ret.	Alpha
Overall Index	10.3%	7.5%	0.26	6.0%	4.7%
Sectors:					
Long/short equity	11.2%	9.8%	0.42	6.9%	4.9%
Short biased	−1.1%	16.9%	−0.91	−5.4%	−0.9%
Emerging markets	8.7%	15.4%	0.52	4.5%	2.0%
Global macro	13.5%	10.4%	0.15	9.2%	8.5%
Equity market neutral	9.7%	2.8%	0.07	5.4%	5.1%
Fixed-income arbitrage	5.6%	4.1%	0.02	1.3%	1.2%
Convertible arbitrage	7.7%	4.9%	0.06	3.4%	3.1%
ED-Merger arbitrage	7.6%	4.1%	0.13	3.3%	2.7%
ED-Distressed securities	12.3%	6.2%	0.24	8.0%	6.8%
ED-Credit hedging	10.4%	6.1%	0.21	6.1%	5.1%
Managed futures	6.7%	12.0%	−0.11	2.4%	2.9%
Multistrategy	9.1%	4.4%	0.04	4.8%	4.6%
Benchmarks:					
Cash	4.3%	0.0%	0.00	0.0%	0.0%
S&P Index	9.1%	14.1%	1.00	4.8%	0.0%
Treasury Index	6.1%	4.5%	−0.04	1.8%	
High Yield Index	6.4%	6.7%	0.26	2.1%	

Notes: Excess returns are measured relative to one-month London Interbank Deposit (LIBID). Alpha is measured from a market model regression on the S&P index.

volatility. The index, however, has slightly positive beta of 0.26, so part if its performance is due to the equity premium. The last column shows an alpha of 4.7%, which is significant.

Also note that the volatility in this table is not directly comparable to that in Table 17.1, because this is the volatility of a portfolio, instead of the average fund volatility. In contrast, the beta of a portfolio is a weighted average of the fund betas, so the portfolio beta gives a good indication of the typical beta of individual funds.[3]

Long/Short Equity The first category consists of directional strategies. These include **long/short equity funds**, which as Table 17.1 shows, is the most prevalent strategy. These funds are not market neutral. Most have a long bias, e.g., 100% of NAV in long positions, and 50% in short positions. Table 17.2 indeed shows a beta of 0.42.

A related category consists of **short biased** funds, which are net short. Table 17.2 shows a negative beta, close to −1. Another category is **emerging markets**, which consists of equity and bond positions in emerging countries, such as Brazil, Russia, India, and China.

[3] In addition, the numbers are not directly comparable across the two tables because they are measured over different periods.

These funds are exposed to the general market risk factor, in addition to sector and idiosyncratic risks. Because of leverage, volatility is high, at 11% on average across all such funds. This is on the order of the volatility of an unleveraged position in the S&P 500.

Global Macros Next are **global macros funds**, which take directional, leveraged bets on global asset classes, equities, fixed-income, currencies, and commodities. Because they span so many markets, these funds do not have a homogeneous risk profile. An example is the Soros fund that shorted the British pound against the German mark just before its devaluation, leading to a reported gain of $1 billion for the hedge fund. This group is close to **global tactical asset allocation** (GTAA), which is a traditional investment manager category. GTAA managers take positions across national stock markets, fixed-income markets, and currencies to take advantage of short-term views, often through derivatives.

These funds are exposed to a number of general market risk factors, in addition to sector and idiosyncratic risks. The average volatility is 11%. This is less than the previous category because these funds also invest in other markets, which are less volatile than equities.

We now turn to nondirectional strategies. The first three categories are sometimes called **relative value funds**, because they rely on comparisons of securities with similar characteristics, buying the cheap ones while selling the expensive ones in the hope of future convergence.

Equity Market Neutral The first group is **equity market neutral funds**, which attempt to maintain zero beta through balanced long and short positions in equity markets. These funds may or may not be neutral across other risk factors, including industries, styles, and countries.

So, these funds are exposed to these other risk factors (industries, styles, countries) in addition to idiosyncratic, stock-specific risks. Balance sheet leverage is typically three times on each side, i.e., both longs and shorts add up to 300% of equity. The average volatility is 6%, which is much less than that of equity indices, due to the hedging effect of the short positions.

Fixed-Income Arbitrage The next group is **fixed-income arbitrage funds**. This is a generic term for a number of strategies that involve fixed-income securities and derivatives. The hedge fund manager assesses the relative value of various fixed-income instruments. For instance, if the on-the-run bond is expensive relative to the off-the-run bond, the fund would buy the undervalued security and sell the expensive one. This position has a net duration close to zero but is exposed to the spread between the two securities. Other examples include taking positions in swap spreads, or in asset-backed securities when their option-adjusted spread is high. This group includes mortgage arbitrage.

These funds avoid directional exposures to interest rates but are exposed to other nondirectional risks, such as spread risk. Due to the small expected profit of each trade, fixed-income arbitrage funds are highly leveraged, with leverage ratios ranging from 10 to 25.

Example: LTCM's Bet

Long-Term Capital Management (LTCM) started as a fixed-income arbitrage fund, taking positions in relative value trades, such as duration-matched positions in long swap, short Treasuries. It started the year 1998 with \$4.7 billion in equity capital.

On August 21, 1998, the 10-year Treasury yield dropped from 5.38% to 5.32%. The swap rate, in contrast, increased from 6.01% to 6.05%. This divergence was highly unusual. Assuming a notional position of \$50 billion and modified duration of eight years, this leads to a value change of $-8\times (5.32-5.38)/100 \times \$50{,}000 = +\$240$ million on a long Treasury position and $-8 \times (6.05-6.01)/100 \times \$50{,}000 = -\$160$ million on a long swap position. As the spread position is long the swap and short Treasury, this leads to a total loss of \$400 million, close to 10% of capital.

LTCM also took positions in option markets, selling options when they were considered expensive and dynamically hedging to maintain a net delta of zero. Implied volatilities went up sharply on August 21, leading to further losses on the option positions. On that day, LTCM's reported loss was \$550 million.

The average volatility of this group is 6%. The distribution of payoffs is typically asymmetric, however. Swap spreads, for example, cannot narrow below zero but can increase to very large values, and have done so. This asymmetry in the distribution of spreads is reflected in that of profits. Such hedge funds have negatively skewed distribution. When they lose money, they can lose large amounts.

Convertible Arbitrage The last group in the relative value category is **convertible arbitrage funds**. The hedge fund manager assesses the relative value of convertible bonds using proprietary option pricing models. If the convertible bond is cheap, the hedge fund buys the bond while hedging the major risks.

Because a convertible bond involves a long call option position, it has positive delta with respect to the underlying stock. Therefore, the manager should short the stock to bring the net delta of the position close to zero. Typically, interest rate risk is hedged by shorting Treasury bonds, or T-bond futures. Sometimes, credit spread risk is hedged by buying credit default swaps.

These funds avoid directional exposures to interest rates but are exposed to other nondirectional risks, such as spread risk. Being typically long convertible bonds, the long option position creates positive gamma and vega (long implied volatility). The bond position creates positive convexity, unless the bond is callable. This strategy is also exposed to corporate event risk, such as default (if not hedged) and takeover. Leverage is moderate. Typically, the long convertible bond position is no more than three times equity. The average volatility of this group is 5%, which is fairly low, in part because of illiquidity.

Event Driven The next group includes **event-driven funds**, which attempt to capitalize on the occurrence of specific corporate events. This group includes **merger arbitrage funds** and **distressed securities funds.**

Let us focus first on merger arbitrage funds, also known as **risk arbitrage funds. Mergers and acquisitions** are transactions that combine two firms into one new firm.[4] The parties can be classified as the **acquiring firm**, or bidder, which initiates the offer and the **target firm**, or acquired firm, which receives the offer. The bidder offers to buy the target at a **takeover premium**, which is the difference between the offer price and the target's stock price before the bid. This premium is typically high, averaging 50% of the initial share price.

Upon the announcement of the merger, the price of the target firm reacts strongly, increasing by, say, 40%. This still falls short of the takeover price, due to the uncertainty as to whether the transaction will occur. The completion rate is 83% on average, so there is always a possibility the transaction could fail. When this happens, the target firm typically suffers a large price drop. As a result, it is important to diversify by spreading the portfolio over many deals.

Offers can take the form of cash or stock of the bidding company. For a cash deal, the risk arbitrage position simply consists of buying the target's stock, and hoping the price will eventually move to the takeover price. For a stock deal, the bidder offers to exchange each target share for Δ shares of the bidder. The risk arbitrage position then consists of a long position in the target offset by a short position of Δ in the bidder's stock. These positions generate an average annualized excess return of 10%.[5]

The volatility of this group is relatively low. Because the stochastic process for the target's stock price changes after the announcement, traditional position-based risk measures are not appropriate measures of risk.[6]

Example: Exxon–Mobil Merger

On December 1, 1998, Exxon confirmed that it had agreed to buy Mobil, another major oil company, in a transaction valued at $85 billion, which was the biggest acquisition ever. The deal created the world's largest traded oil company, with a market capitalization of $250 billion. Under the terms of the agreement, each shareholder of Mobil would receive $\Delta = 1.32015$ shares of Exxon in exchange.

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[4] These are sometimes called takeovers. Takeovers can take the form of mergers or tender offers. Mergers are negotiated directly with the target manager's, approved by the board of directors, and then by shareholder vote. Tender offers are offers to buy shares made directly to target shareholders.

[5] This is a risk-adjusted excess return. These profits, however, seem to be related to limits to arbitrage, as there are lower for firms that are large and have low idiosyncratic risk. See Baker, Malcolm and Serkan Savasoglu, 2002, Limited Arbitrage in Mergers and Acquisitions, *Journal of Financial Economics* 64, 91–115.

[6] See Jorion, P., 2008, Risk Management for Event-Driven Funds, *Financial Analysts Journal* 64, 61–73.

Before the announcement, the initial prices of Mobil and Exxon were \$78.4 and \$72.7, respectively, which implies a modest premium of $(1.32016 \times \$72.7)/\$78.4 - 1 = 22\%$. Over the three days around the announcement, Mobil's stock price went up by +6.9% to \$84.2 and Exxon's price went down by −1.5% to \$71.6. This stock price reaction is typical of acquisition announcements.

The exchange was consummated on November 30, 1999, after regulatory and shareholder approval. On that day, the respective stock prices for Mobil and Exxon were \$104.4 and \$79.3. Multiplying the latter by 1.32015, we get \$104.7, which is close to the final stock price for Mobil. So, the two prices converged to the same converted value. The profit from the risk arbitrage trade was $(\$104.4 - \$84.2) - 1.32016(\$79.3 - \$71.6) = \$10.0$ per share.

Event-driven funds also include **distressed securities funds**, which take positions in securities, debt or equity, of firms in financial difficulty. In such situations, the hedge fund manager needs to assess the effect of restructuring or the bankruptcy process on the market price of the securities. This requires an evaluation of the financial situation of the firm, as well as a good understanding of legal issues involved. If, for instance, the debt of a bankrupt company trades at 40 cents on the dollar, the hedge fund would benefit if the total payment after reorganization is 50 cents. Such funds are also actively involved in the bankruptcy processes and the reorganization plans.

These funds are exposed to event risks, that is, that the takeover or reorganization fails. They may also be exposed to equity market risk and interest rate risk if these exposures are not hedged. Because distressed securities do not trade actively, there is also liquidity risk. Leverage for event-driven funds is low to moderate, no more than two times.

The average volatility for event-driven funds is 6%, which is fairly low. This, however, hides the fact that the distribution of payoffs is asymmetric. Typically, the upside is more limited than the downside, should the takeover or reorganization fail. So, these funds are short volatility, or exposed to rare events. Because of the unusual nature of the event, measures of risk based on historical returns can be inaccurate for forecasting risk.

Managed Futures Funds The next category of hedge funds differs from others on the basis of the fund structure. **Managed futures funds** consist of managers who use commodity and financial futures and options traded on organized exchanges. Trading strategies often involve **technical trading**, where positions depend on patterns in price histories. Leverage is high, leading to high volatility.

These funds have directional exposures to all the markets that have listed futures contracts. Their risk factors overlap with global macro funds. GTAA strategies, for instance, often involve stock index and currency futures. The average volatility of this group is 17%, which is fairly high.

Multistrategy Funds Next, **multistrategy funds** are hedge funds that cover combinations of previously described fund strategies. One advantage of such funds is that they can reallocate capital quickly from strategy to another. They also provide automatic diversification across strategies. The average volatility of this group is 9%.

On the other hand, multistrategy funds trend to be more concentrated in one type of strategy, and do not provide as much diversification as funds of funds, described below. Amaranth, for example, initially started as a multistrategy fund focused on convertible bond arbitrage, then morphed into a natural gas trading operation, and eventually blew up. Because all strategies are run within the same fund, a large loss in one strategy may affect the capital of other strategies. In other words, strategies are not firewalled, unlike in the fund of funds structure.

Funds of Funds Finally, **funds of funds**, also called **multimanager funds**, are portfolios of hedge funds. These add value by careful selection of styles and investment managers. They also perform essential functions, such as the due diligence process when evaluating new managers and their continuous monitoring. Funds of funds can take views on strategies, increasing allocation to strategies that are expected to perform better.

Funds of funds charge additional management fees on top of those levied by the underlying funds, typically around 1%. On the other hand, because of their size, funds of funds can negotiate lower fees from the hedge fund managers.

Relative to multistrategy funds, funds of funds have higher fees. This cost difference, however, is offset by the fact that a fund of funds has access to the best managers, who generally want to run their own fund, thus creating better performance.

Also, funds of funds have lower risk of losses due to blowups than multistrategy funds, where the entire investment can be lost, as in the case of Amaranth. This is for two reasons. First, funds of funds are generally better diversified across strategies than multistrategy funds. Second, the hedge funds in a fund of fund pool are legally separate from each other. As a result, a blowup in one fund will not contaminate the rest of the portfolio, as in the case of a multistrategy fund.

Funds of funds provide convenient access to a diversified portfolio of hedge funds. Because hedge funds have minimum investments amounts, such diversification is difficult to achieve for small mandates allocated to hedge funds. For example, a $100 million allocation to hedge funds can be realistically invested to at most 10 hedge funds. A typical fund of funds, in contrast, will invest in 50 funds. Funds of funds provide economies of scale in the due diligence and risk monitoring processes. Funds of funds can also negotiate greater capacity and better liquidity than other investors.

Table 17.1 shows that the average volatility of this group is 6%. This low number reflects effective diversification across managers and styles.

A related category is **hedge fund indices**. These are unmanaged, passive baskets of hedge funds. In practice, however, their fees are similar to those charged by funds of funds, unlike fees for indexed mutual funds, which are much lower, around 0.10%. Also relatively new are **collateralized fund obligations** (CFOs), which are pools of hedge funds whose total payoff is sliced into various tranches, much like CMOs.

This list makes it clear that hedge funds are a very heterogeneous group. They are exposed to a wide variety of risk factors, follow different trading rules, and have varying levels of leverage and risk. The common element, however, is the need to manage risk.

EXAMPLE 17.4: RISKS IN FIXED-INCOME ARBITRAGE

Identify the risks in a fixed-income arbitrage strategy that takes long positions in interest rate swaps hedged with short positions in Treasuries.

a. The strategy could lose from decreases in the swap-Treasury spread.
b. The strategy could lose from increases in the Treasury rate, all else fixed.
c. The payoff in the strategy has negative skewness.
d. The payoff in the strategy has positive skewness.

EXAMPLE 17.5: RISKS IN CONVERTIBLE ARBITRAGE

Identify the risks in a convertible arbitrage strategy that takes long positions in convertible bonds hedged with short positions in Treasuries and the underlying stock.

a. Short implied volatility
b. Long duration
c. Long stock delta
d. Positive gamma

EXAMPLE 17.6: RISKS IN MERGER ARBITRAGE—I

A major acquisition has just been announced, targeting Company B. The bid from Company A is an exchange offer with a ratio of 2. Just after the announcement, the prices of A and B are \$50 and \$90, respectively. A hedge fund takes a long position in Company B hedged with A's stock. After the acquisition goes through, the prices move to \$60 and \$120. For each share of B, the gain is

a. \$30
b. \$20
c. \$10
d. \$0 since the acquisition is successful

EXAMPLE 17.7: RISKS IN MERGER ARBITRAGE—II

Suppose the payoff from a merger arbitrage operation is \$5 million if successful, −\$20 million if not. The probability of success is 83%. The expected payoff on on the operation is

a. \$5 million
b. \$0.75 million
c. \$0 since markets are efficient
d. Symmetrically distributed

EXAMPLE 17.8: FRM EXAM 2005—QUESTION 47

The Big Bucks Hedge fund has the following description of its activities. It uses simultaneous long and short positions in equity with a net beta close to zero. Which of the following statements about Big Bucks are correct?

I. It uses a directional strategy.
II. It is a relative value hedge fund.
III. This fund is exposed to idiosyncratic risks.

a. I and II
b. II and III
c. I and III
d. II only

17.4 HEDGE FUNDS: SPECIFIC RISKS

17.4.1 Agency Risk

Hedge fund managers act as agents for investors. This can cause misalignment of incentives, however. Incentive fees make a payment that is a fraction of the profits, if positive. As a result, the hedge fund manager is long an option. Because the value of an option increases with the volatility, the fund manager may have an incentive to increase the risk of the fund.

Another potential problem is that of **style drift**, which occurs when managers change their investment patterns or stray into new markets.

This type of behavior can be minimized in a number of ways. Most importantly, hedge fund managers should invest a large fraction of their personal wealth in the fund they manage. This lessens the incentive to take on too much risk.

Some risk monitoring occurs at the level of the prime broker (PB). Because the PB is primarily concerned about the risk of loss from lending to the hedge fund, however, its interests do not align with those of fund investors. For example, a lender may use margin calls to force liquidation of the fund asset at distressed prices. As long as there is excess collateral, the lender would be protected, but at the expense of investors.

The incentive to take risk is also lessened with **high water marks**, also known as loss carryforward provisions.[7]

The manager only receives performance fees to the extent that the current NAV exceeds the highest NAV previously achieved. Suppose, for instance, that the NAV changes from $100 to $130 to $120 to $140 in four consecutive years. The first year, the performance fee would apply to $30. The second year, there is no performance fee because the fund lost money. The third year, the performance fee only applies to the portion of $140 in excess of $130, which is the highest previous NAV. This mechanism, however, may not provide complete protection if the watermark is too high. In this case, the fund manager may choose to close the fund and to start a new one (if investors can be found).

17.4.2 Liquidity and Leverage Risk

Hedge funds take leveraged positions to increase returns, especially with nondirectional trades such as fixed-income arbitrage, where the expected return on individual trades is generally low.

Perversely, this creates other types of risks, including **liquidity risk**. This strategy indeed failed for Long-Term Capital Management (LTCM), a highly leveraged hedge fund that purported to avoid directional risks. LTCM had a leverage ratio of 25 to 1. It had grown to $125 billion in assets, four times the asset size of the next larger hedge fund. Once the fund started to accumulate losses, it became difficult to cut positions given its size. LTCM also had to meet margin calls from brokers. The fund ended up losing $4.4 billion, or 92% of its equity.

Table 17.3 links sources of liquidity risk to a hedge fund balance sheet. Liquidity risk arises on the asset side and is a function of the size of the positions as well as of the price impact of a trade. On the liabilities side, funding risk arises when the hedge fund cannot rollover funding from its broker, or when losses in marked-to-market positions or increases in haircuts lead to cash outflows. This is often a major source of risk for hedge funds, as the failure to meet margin calls can lead the lender to seize the collateral, forcing liquidation of the fund. Finally, funding risk also arises when the fund faces investor redemptions.

TABLE 17.3 Sources of Liquidity Risk

Assets	Liabilities
Size of position	Funding
Price impact	Mark-to-market, haircuts
	Equity
	Investor redemptions

[7] Sometimes, a **claw-back** provision is included, which requires the fund manager to pay back performance fees when the value of the fund drops.

The price impact function is instrument-specific. For example, major currencies, large stocks, Treasury bills, and Treasury bonds are very liquid, meaning that large amounts can be transacted without too much effect on the price. Other markets are by nature less liquid. For instance, minor currencies, small stocks, and most corporate debt instruments are generally illiquid.

LTCM dealt with mostly liquid instruments but was exposed to liquidity risk due to the sheer size of its positions. This is why hedge funds often say they have a **maximum capacity**. Beyond that optimal size, trading becomes difficult due to market impact.

Another type of risk that is exacerbated by leverage is **model risk**. This occurs when the investment strategy relies on valuation or risk models that are flawed. Due to leverage, small errors in the model can create big errors in the risk measure. Indeed, LTCM's risk measurement system was deficient, leading to an fatal underestimation of the amount of capital required to support its positions.

Some categories of hedge funds have intrinsic liquidity risk because the instruments are thinly traded, implying a large price impact for most trades. This is the case with convertible bonds and especially so with distressed securities. Because these funds invest in thinly traded securities, liquidity risk arises even for small funds.

Typically, funds with greater liquidity risk impose a longer **lockup period** and **redemption notice period**. The former refers to the minimum time period during which investor money is to be held in the fund. The latter refers to the period required to notify the fund of an intended redemption. Lockup periods average three months, and can extend to five years. Advance notice periods average 30 days. Funds also often have **gates**, which limit the amount of withdrawals to a fraction of the net assets. In the extreme, funds might be able to impose an outright **suspension** of redemptions.

Instrument liquidity risk creates a major problem for the measurement of risk. Typically, funds report their **net asset value** at the end of each month. If transaction prices are not observed at the end of the month, the valuation may be using a price from a trade that occurred in the middle of the month. This price is called a **stale prices** because it is "old" and does not reflect a market-clearing trade on the day of reporting. Unfortunately, this will distort the reported NAVs as well as the risk measures.

The first effect is that the reported monthly volatility will be less than the true volatility. This is because prices are based on trades during the month, which is similar to an averaging process. Averages are less volatile than end-of-period values.

The second effect is that monthly changes will display positive autocorrelation. A movement in one direction will be only partially captured using prices measured during the month. The following month, part of the same movement will show up in the return. This positive autocorrelation substantially increases the risk over longer horizons. Consider for instance the extrapolation of a one-month volatility to two months. The usual adjustment factor is $\sqrt{T} = \sqrt{2} = 1.41$. With autocorrelation of $\rho = 0.5$, this adjustment factor is instead $\sqrt{(1 + 1 + 2\rho)} = \sqrt{2(1 + 0.5)} = 1.73$. The true risk is understated by $(1.73 - 1.41)/1.73 = 18\%$.

This effect increases with the length of the horizon. As a result, the annualized volatility presented in Table 17.1, which extrapolates monthly volatility using the square root of time, may understate the true annual risk. Long-term measures of risk must specifically account for the observed autocorrelation.

A third, related effect is that measures of systematic risk will be systematically biased downward. If the market goes up during a month, only a fraction of this increase will be reflected in the NAV, leading to beta measures that are too low. Corrections to the beta involve measuring the portfolio's beta with the contemporaneous market return plus the beta with respect to the one-month lagged return plus the beta with respect to the one-month future return. With thin trading, the sum of these three betas should be higher than the contemporaneous beta, and also closer to the true systematic risk.[8]

Leverage can create other problems, which can be classified as **crowded trade risk.** This arises when many leveraged investors are on the same side of a trade.[9] A loss in their portfolio may require them to post additional margin, which may be satisfied by several funds selling similar assets at the same time, which can create disruptions in markets. Apparently, this explains why many **quant funds**, which are generally equity market neutral (EMN) strategies driven by quantitative models, suffered heavy losses in August 2007. The story is that a large multistrategy fund lost money on credit trades and then liquidated its equity positions, because these are more liquid than others. This caused large losses in EMN portfolios, both on the long and short sides, which was highly unusual.

This is not just a problem with leverage, however. Any mechanistic trading rule that involves cutting positions after a loss is incurred may have similar effects if there is an imbalance between demand and supply. This includes, for instance, stop losses, which are equivalent to synthetically replicating long positions in options.

17.4.3 Leverage and Counterparty Risk

Leverage also create another type of risk, which is **counterparty risk.** Hedge funds that use leverage give collateral to prime brokers. **Hypothecation** is the pledge of client-owned securities in a margin account to secure a loan. The broker then has the right to **rehypothecate** the securities to another party. If the broker goes bankrupt, however, the rehypothecated assets become part of the claims against the broker.

In the case of the Lehman failure, $22 billion of out the $40 billion held by Lehman's European prime brokerage had been rehypothecated. As a results,

[8] This correction is called the *Dimson beta*. See Dimson, E. (1979), Risk Measurement When Shares are Subject to Infrequent Trading, *Journal of Financial Economics* 7, 197–226.

[9] Of course, other investors must be on the other side of the trade. This classification supposes that the other side is not so leveraged, otherwise trades could be crossed without much effect on prices. In practice, positions are confidential, and it is impossible to know who is on which side of a trade, except anecdotally.

hedge funds trying to reclaim these assets found themselves in the line of general creditors with claims against Lehman.

17.4.4 Fraud Risk

A last issue, especially with complex or illiquid assets, is **improper valuation of assets**. This problem arises when assets do not have market-clearing prices at the end of the reporting period and when fund managers calculate the NAV themselves. As a result, some unscrupulous hedge fund managers have succumbed to the temptation to misreport the value of the fund's assets in order to hide their trading losses.[10] Others have even stolen investors' assets.

Indeed, a recent study has shown that valuation problems played a role in 35% of hedge fund failures, and that 57% of those valuation problems were caused by fraud or misrepresentation.[11] The growth of the hedge fund industry, along with the increasing occurrence of fraud, explains why the Securities and Exchange Commission (SEC) has issued a new rule in December 2004 that requires hedge funds to *register* as investment advisors.[12] Registration gives the SEC the authority to conduct examinations of hedge fund activities. The goal is to help to identify compliance problems at an early stage and to provide a deterrent to fraud. Registration also requires the hedge fund to designate a **chief compliance officer**. In June 2006, this registration requirement was annulled by the U.S. Court of Appeals. In practice, however, the majority of U.S.-based hedge funds have voluntarily registered as investment advisors. Registration is often required by investors as a precondition for investing.

The possibility of fraud can be lessened when a fund has an independent **administrator**. Administrators perform day-to-day administrative duties associated with running a fund, in particular financial and tax reporting. They calculate net asset values, maintain the statutory books and records, and provide shareholder services. In addition, an outside **auditor** provides important additional information. Auditors issue a written opinion upon the fair presentation of the fund's financial statements, typically on an annual basis. To protect against theft, investors should insist on an external **custodian**, which is a financial institution such as a bank or trust company that holds the fund's assets. Usually a fund's prime broker will perform the role of custodian.

Example: Ponzi Scheme

The term **Ponzi scheme** is attributed to Carlo Ponzi, who in 1919 established an inventive pyramid scheme using new investor funds to repay earlier investors. The

[10] A 2003 report by the SEC, however, notes that there is no evidence that hedge fund advisors engage *disproportionately* in fraudulent activities.
[11] See Kundro, C. and S. Feffer (2003), Valuation Issues and Operational Risk in Hedge Funds, *Capco White Paper*.
[12] This rule applies to U.S.-based hedge funds, and to non-U.S. funds that have at least fourteen U.S. investors. Funds with less than $25 million under management do not have to comply. The rule took effect in February 2006.

investment was based on a relative-value trade, in which postal coupons were bought overseas for the equivalent of one U.S. cent and resold for six American one-cent stamps. After transaction costs were factored in, however, the trade was unprofitable. Nevertheless, thousands of people invested with him, lured by a promise of 50% return in 90 days. Ultimately, he lost $140 million of investor funds, in today's dollars, and was jailed for fraud.

The most famous case of a Ponzi scheme was perpetrated by **Bernard Madoff**, who was arrested in December 2008 after admitting to defrauding investors of perhaps $50 billion through his brokerage firm, Bernard Madoff Investment Securities (BMIS). BMIS was established in 1960 and by the end of 2007 was managing about $17 billion in hedge funds investments. Returns to initial investors were paid using new investments. The scheme collapsed when investors sought about $7 billion in redemptions during 2008, which could not be met. Ponzi schemes only work as long as new money flows in.

Investors had been attracted by the high and steady returns from Madoff's funds, which turned out to have been fabricated. Many leading funds of funds, however, had performed due diligence on Madoff and had spotted enough warning signs to stay clear of him. BMIS acted as custodian of investor assets, had no external administrator, and relied on an unknown three-person audit firm. In addition, some people who had analyzed the options-trading strategy had concluded that it was infeasible because it would have implied trading volumes far in excess of exchange-traded volumes. This illustrates the value of a thorough due diligence process.

17.4.5 Regulatory Risks

Finally, hedge funds are also subject to **regulatory risks**. This is the risk of loss due to regulatory changes. In September 2008, for example, many countries put in place outright bans on short-selling equities.

This ban created havoc with strategies that rely on short sales to hedge. For example, the convertible bond arbitrage sector suffered extreme losses during these months. Statistical arbitrage funds, which take long and short positions in stocks, had to withdraw from many markets.

There is broad consensus, however, that such bans are ineffective at stopping prices from falling.[13] Such bans also have far-reaching consequences. In the case of convertible bonds, for example, the market freeze prevented many institutions, including banks, that traditionally issue convertible bond debt to raise new funds. In addition, such bans cause investors to withdraw from markets, sapping liquidity, which actually increases volatility. Bans on short-selling removed hedge funds' main mechanism for risk management, which led to an acceleration of withdrawal of capital when the market needed it most.

[13] Marsh, I. and N. Niemer (2008), The Impact of Short Sales Restrictions, *Working Paper*, Cass Business School, London.

EXAMPLE 17.9: LIQUIDITY RISKS

Asset liquidity risk is most pronounced for

a. A $10 million position in distressed securities
b. A $10 million position in Treasury bonds
c. A $100 million position in distressed securities
d. A $100 million position in Treasury bonds

EXAMPLE 17.10: FRM EXAM 2007—QUESTION 62

You are asked to estimate the exposure of a hedge fund to the S&P 500. Though the fund claims to mark to market weekly, it does not do so and marks to market once a month. The fund also does not tell investors that it simply holds an Exchange Traded Fund (ETF) indexed to the S&P 500. Because of the claims of the hedge fund, you decide to estimate the market exposure by regressing weekly returns of the fund on the weekly return of the S&P 500. Which of the following correctly describes a property of your regression estimates?

a. The intercept of your regression will be positive, showing that the fund has a positive alpha when estimated using an OLS regression.
b. The beta will be misestimated because hedge fund exposures are nonlinear.
c. The beta of your regression will be one because the fund holds the S&P 500.
d. The beta of your regression will be zero because the fund returns are not synchronous with the S&P 500 returns.

EXAMPLE 17.11: FRM EXAM 2006—QUESTION 112

For a portfolio of illiquid assets, hedge fund managers often have considerable discretion in portfolio valuation at the end of each month and may have incentives to smooth returns by marking values below actual in high-return months and above actual in low-return months. Which of the following is *not* a consequence of return smoothing over time?

a. Higher Sharpe ratio
b. Lower volatility
c. Higher serial correlation
d. Higher market beta

17.5 DEALING WITH HEDGE FUND RISKS

Because of these risks, hedge funds need to be monitored closely. This starts with **due diligence**, which is the process of systematically investigating the fund before investing. On the operational side, this involves an analysis of the fund documents, of the key personnel (including background checks), of the fund service providers (administrator, prime broker, legal counsel, auditor), of the regulatory registration, and of the operations and valuation procedures. On the investment side, this involves an analysis of the investment strategy, of the risk factors, and of the risk control systems. Once a hedge fund manager is hired, some components of this due diligence process need to be verified periodically.

Without information about the positions, however, this process is rather incomplete. It is very difficult to detect style drift, for example, from historical returns. Because returns are typically provided at monthly intervals, structural breaks can generally be identified only after a few years.

17.5.1 Hedge Fund Transparency Issues

Hedge funds, however, are generally reluctant to reveal information about their positions. This lack of transparency has serious disadvantages for investors, however.

Disclosure allows *risk monitoring* of the hedge fund, which is especially useful with active trading. This can help to avoid situations where the hedge fund manager unexpectedly increases leverage or changes style. Closer monitoring of the fund can also decrease the probability of fraud or misvaluation of assets.

Disclosure is also important for *risk aggregation*. The investor should know how the hedge fund interacts with other assets in the portfolio. Whether the hedge fund has a positive or negative correlation with the rest of the portfolio affects the total portfolio risk.

Example: Why Risk Aggregation?

Aggregation of positions is important to identify potential concentrations to individual names, or companies. The story is that of a large pension fund, which had allocated assets to outside managers investing in corporate bonds, growth equities, and value equities. In 2000, Enron was rated investment-grade and viewed as a growth stock, reflecting its high stock price of $90. The fund had positions in Enron corporate bonds and in Enron stock, through the growth manager.

As negative news unfolded the following year, the stock price fell to $15 by October 2001. Many saw this decline as a great opportunity to buy the stock. The pension fund's value managers started to buy the stock. As the same time, its other managers did not have the discipline to sell. By December 2001, the stock price had fallen to $0.03. The pension fund ended up with large holdings of Enron stock and bonds, and a huge loss due to its failure to identify this concentration risk.

Greater disclosure is often resisted on the grounds that it would disclose *proprietary information*, leading to the possibility of a third party trading against the hedge fund. This threat, however, comes from the broker-dealer community, generally not from investors. If this is an issue, confidentiality agreements should prevent leakages of sensitive information. Hedge funds generally prefer to release such information to investors with no trading operations, whether directly or through affiliates, who would not be able to profit from this information.

Another argument sometimes advanced is the *lack of investor sophistication.* In other words, disclosing positions would give too much information to investors who might not able to use it.

17.5.2 Solutions for Transparency

These arguments can be addressed with a number of solutions. The first consists of external risk measurement services. These firms have access to the individual positions of hedge funds, with the proper confidentiality agreements, and provide aggregate risk measures to investors. They release only exposures to major risk factors, such as net duration, net systematic risk, and so on. This solution neatly solves the problems of risk aggregation and managers' widespread reluctance to disclose detailed information about their positions. Another solution is to go through a fund of funds that has position-level information. Although this is still fairly rare, such fund of funds can perform the risk measurement and monitoring function for the investor, thereby justifying some of its added fee.

As will be seen in the chapter on operational risk, sound risk management relies heavily on the principle of independence. Some hedge funds are now using external companies that provide independent valuation services. Figure 17.2 describes a best-practice architecture for valuation and risk measurement in hedge funds.

Portfolio positions are recorded by the prime broker, who can provide portfolio valuation. The hedge fund, however, may also use external service providers for valuation and risk measurement, both of which are based on the portfolio positions. All of this information is then fed into the portfolio and risk measurement system for the hedge fund manager.

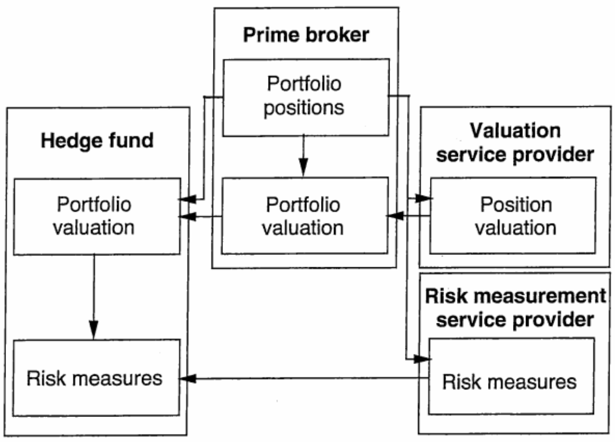

FIGURE 17.2 Architecture of Hedge Fund Risk Systems

EXAMPLE 17.12: TRANSPARENCY

Investors should insist on learning about the positions of hedge funds because

a. They want to trade ahead of the hedge fund.
b. They do not understand the trading strategies behind the positions.
c. They want to aggregate the risk of hedge funds with the rest of their portfolio.
d. They receive the information from the prime broker, anyway.

EXAMPLE 17.13: FRM EXAM 2003—QUESTION 51

As one of your duties as the chief risk officer for a fund of funds, you evaluate the risk management of candidate hedge funds. In your evaluation of a newly organized two-person hedge fund, which is your primary consideration?

a. Risk reporting structure
b. Investment style
c. Assets under management
d. Last month's return

17.6 IMPORTANT FORMULAS

Net Beta: $(\beta_L V_L - \beta_S V_S) = \beta_E V_E$

Leverage: $\frac{V_A}{V_E} = \frac{\text{Long stock positions plus cash}}{\text{Equity}}$

Gross Leverage: $\frac{V_L+V_S}{V_E} = \frac{\text{Long positions plus absolute value of short positions}}{\text{Equity}}$

Net Leverage: $\frac{V_L-V_S}{V_E} = \frac{\text{Long positions minus absolute value of short positions}}{\text{Equity}}$

17.7 ANSWERS TO CHAPTER EXAMPLES

Example 17.1: FRM Exam 2006—Question 41

c. The gross leverage is $(315 + 225)/185 = 2.9$. The net leverage is $(315 - 225)/185 = 0.5$. Note that beta is not needed for this calculation.

Example 17.2: Leverage and Returns

c. The net return on the stock portfolio is $(\beta_L \$315 - \beta_S \$225) \times 20\%$. With betas of 1, this is \$18 million. Given that the equity is \$185 million, the rate of return is about 10%. The rate of return is less than that on the market because most of the exposure to the market is hedged.

Example 17.3: FRM Exam 2004—Question 2

a. Because the position is both long and short, high correlation implies low risk. Conversely, lowering correlation increases risk.

Example 17.4: Risks in Fixed-Income Arbitrage

c. The strategy has no exposure to the level of rates but is exposed to a widening of the swap-Treasury spread. Assume for instance that the swap and Treasury rates are initially 5.5% and 5%. If these rates change to 5.3% and 4.5%, for example, both values for the swap and the Treasury bond would increase. Because the drop in the Treasury rate is larger, however, the price of the Treasury bond would fall more than the swap, leading to a net loss on the position. The strategy should *gain* from decreases in the swap-Treasury spread, so a. is wrong. The strategy should *gain* from increases in the Treasury rate, all else equal, so b. is wrong. Finally, the distribution of the payoff depends on the distribution of the swap-Treasury spread. Because this cannot go below zero, there is a limit on the upside. The position has negative skewness, so c. is correct.

Example 17.5: Risks in Convertible Arbitrage

d. This position is hedged against interest rate risk, so b. is wrong. It is also hedged against directional movements in the stock, so c. is wrong. The position is long an option (the option to convert the bond into the stock) so is long implied volatility, so a. is wrong. Long options positions have positive gamma.

Example 17.6: Risks in Merger Arbitrage—I

c. The position is long one share of Company B offset by a short position in two shares of Company A. The payoff is $(\$120 - \$90) - 2(\$60 - \$50) = \$30 - \$20 = \$10$.

Example 17.7: Risks in Merger Arbitrage—II

b. The expected payoff is the sum of probabilities times the payoff in each state of the world, or $83\% \times \$5 + 17\% \times (-\$20) = \$4.15 - \$3.40 = \$0.75$. Note that the distribution is highly asymmetric, with a small probability of a large loss.

Example 17.8: FRM Exam 2005—Question 47

b. This fund has zero beta, so is a relative value fund. It is, however, exposed to idiosyncratic, stock-specific risk.

Example 17.9: Liquidity Risks

c. Asset liquidity risk is a function of the size of the position and the intrinsic liquidity of the instrument. Distressed securities trade much less than Treasury bonds, so have more liquidity risk. A $100 million is more illiquid than a $10 million in the same instrument.

Example 17.10: FRM Exam 2007—Question 62

d. The weekly returns are not synchronized with those of the S&P. As a result, the estimate of beta from weekly data will be too low.

Example 17.11: FRM Exam 2006—Question 112

d. Illiquidity creates an *understatement* of the total risk measure. As a result, the Sharpe ratio will be artificially higher. Illiquidity creates trends in returns (higher serial correlation), as market shocks during a month will be partially recorded in two consecutive months. Illiquidity, however, biases down the market beta.

Example 17.12: Transparency

c. Risk aggregation is an important reason for investor to learn about the positions of their investment in hedge funds. Answer a. is incorrect because front-running the hedge fund would be a reason *not* to disclose position information. Answer b. is incorrect because misunderstanding the trading strategies would be a reason *not* to require position information. Answer d. is incorrect because they do not receive position information from the prime broker.

Example 17.13: FRM Exam 2003—Question 51

b. The market risk of the fund is primarily driven by the investment style. Answer a. about the risk reporting structure, is not too important for this fund because there will not be much separation of duties, anyway, for such a small team.

PART Five

Credit Risk Management

CHAPTER 18

Introduction to Credit Risk

Credit **risk** is the risk of an economic loss from the failure of a counterparty to fulfill its contractual obligations. Its effect is measured by the cost of replacing cash flows if the other party defaults.

This chapter provides an introduction to the measurement of credit risk. The study of credit risk has undergone vast developments in the last few years. Fuelled by advances in the measurement of market risk, institutions are now, for the first time, attempting to quantify credit risk on a portfolio basis.

Credit risk, however, offers unique challenges. It requires constructing the distribution of default probabilities, of loss given default, and of credit exposures, all of which contribute to credit losses and should be measured in a portfolio context. In comparison, the measurement of market risk using value at risk is a simple affair. These challenges explain why many of these models performed poorly during the credit crisis that started in 2007.

For most institutions, however, market risk pales in significance compared with credit risk. Indeed, the amount of risk-based capital for the banking system reserved for credit risk is vastly greater than that for market risk. The history of financial institutions has also shown that the biggest banking failures were due to credit risk.

Credit risk involves the possibility of nonpayment, either on a future obligation or during a transaction. Section 18.1 introduces **settlement risk**, which arises from the exchange of principals in different currencies during a short window, typically a day. We discuss exposure to settlement risk and methods to deal with it.

Traditionally, however, credit risk is viewed as **presettlement risk**, which arises during the life of the obligation. Section 18.2 analyzes the components of a credit risk system and the evolution of credit risk measurement systems. Section 18.3 then shows how to construct the distribution of credit losses for a portfolio given default probabilities for the various credits in the portfolio.

The key drivers of portfolio credit risk are the correlations between defaults. Section 18.4 takes a fixed $100 million portfolio with an increasing number of obligors and shows how the distribution of losses is dramatically affected by correlations.

18.1 SETTLEMENT RISK

18.1.1 Presettlement versus Settlement Risk

Counterparty credit risk consists of both presettlement and settlement risk. **Presettlement risk** is the risk of loss due to the counterparty's failure to perform on an obligation during the life of the transaction. This includes default on a loan or bond or failure to make the required payment on a derivative transaction. Presettlement risk exists over long periods—years—starting from the time it is contracted until settlement.

In contrast, **settlement risk** is due to the *exchange* of cash flows and is of a much shorter-term nature. This risk arises as soon as an institution makes the required payment and exists until the offsetting payment is received. This risk is greatest when payments occur in different time zones, especially for foreign exchange transactions where notionals are exchanged in different currencies. Failure to perform on settlement can be caused by counterparty default, liquidity constraints, or operational problems.

Most of the time, settlement failure due to operational problems leads to minor economic losses, such as additional interest payments. In some cases, however, the loss can be quite large, extending to the full amount of the transferred payment. An example of major settlement risk is the 1974 failure of Herstatt Bank. The day the bank went bankrupt, it had received payments from a number of counterparties but defaulted before payments were made on the other legs of the transactions.

18.1.2 Managing Settlement Risk

In March 1996, the Bank for International Settlements (BIS) issued a report warning that the private sector should find ways to reduce settlement risk in the $1.2 trillion-a-day global foreign exchange market.[1] The report noted that central banks had "significant concerns regarding the risk stemming from the current arrangements for settling FX trades." It explained that "the amount at risk to even a single counterparty could exceed a bank's capital," which creates **systemic risk**. The threat of regulatory action led to a reexamination of settlement risk.

Examining the various stages of a trade, its status can be classified into five categories:

1. *Revocable:* When the institution can still cancel the transfer without the consent of the counterparty
2. *Irrevocable:* After the payment has been sent and before payment from the other party is due
3. *Uncertain:* After the payment from the other party is due but before it is actually received

[1] Committee on Payment and Settlement Systems (1996), *Settlement Risk in Foreign Exchange Transactions*, BIS (online), available at http://www.bis.org/publ/cpss17.pdf.

4. *Settled:* After the counterparty payment has been received
5. *Failed:* After it has been established that the counterparty has not made the payment

Settlement risk occurs during the periods of irrevocable and uncertain status, which can take from one to three days.

While this type of credit risk can lead to substantial economic losses, the short-term nature of settlement risk makes it fundamentally different from presettlement risk. Managing settlement risk requires unique tools, such as **real-time gross settlement** (RTGS) systems. These systems aim at reducing the time interval between the time an institution can no longer stop a payment and the receipt of the funds from the counterparty.

Settlement risk can be further managed with netting agreements. One such form is **bilateral netting**, which involves two banks. Instead of making payments of gross amounts to each other, the banks tot up the balance and settle only the net balance outstanding in each currency. At the level of instruments, netting also occurs with **contracts for differences** (CFD). Instead of exchanging principals in different currencies, the contracts are settled in dollars at the end of the contract term.[2]

The next step up is a **multilateral netting system**, also called **continuous-linked settlements**, where payments are netted for a group of banks that belong to the system. This idea became reality when the **CLS Bank**, established in 1998 with 60 bank participants, became operational on September 9, 2002. Every evening, CLS Bank provides a schedule of payments for the member banks to follow during the next day. Payments are not released until funds are received and all transaction confirmed. The risk now has been reduced to that of the netting institution. In addition to reducing settlement risk, the netting system has the advantage of reducing the number of trades between participants, by up to 90%, which lowers transaction costs.

EXAMPLE 18.1: FRM EXAM 2000—QUESTION 36

Settlement risk in foreign exchange is generally due to

a. Notionals being exchanged
b. Net value being exchanged
c. Multiple currencies and countries involved
d. High volatility of exchange rates

[2] These are similar to **nondeliverable forwards**, which are used to trade emerging-market currencies outside the jurisdiction of the emerging-market regime and are also settled in dollars.

EXAMPLE 18.2: FRM EXAM 2000—QUESTION 85

Which one of the following statements about multilateral netting systems is *not* accurate?

a. Systemic risks can actually increase because they concentrate risks on the central counterparty, the failure of which exposes all participants to risk.
b. The concentration of risks on the central counterparty eliminates risk because of the high quality of the central counterparty.
c. By altering settlement costs and credit exposures, multilateral netting systems for foreign exchange contracts could alter the structure of credit relations and affect competition in the foreign exchange markets.
d. In payment netting systems, participants with net-debit positions will be obligated to make a net settlement payment to the central counterparty that, in turn, is obligated to pay those participants with net-credit positions.

18.2 OVERVIEW OF CREDIT RISK

18.2.1 Drivers of Credit Risk

We now examine the drivers of credit risk, traditionally defined as presettlement risk. Credit risk measurement systems attempt to quantify the risk of losses due to counterparty default. The distribution of credit risk can be viewed as a compound process driven by these variables:

- **Default**, which is a discrete state for the counterparty—either the counterparty is in default or not. This occurs with some **probability of default** (PD).
- **Credit exposure** (CE), which is the economic or market value of the claim on the counterparty. It is also called **exposure at default** (EAD) at the time of default.
- **Loss given default** (LGD), which represents the fractional loss due to default. As an example, take a situation where default results in a **fractional recovery rate** of 30% only. LGD is then 70% of the exposure.

Traditionally, credit risk has been measured in the context of loans or bonds for which the exposure, or economic value, of the asset is close to its notional, or face value. This is an acceptable approximation for bonds but certainly not for derivatives, which can have positive or negative value. Credit exposure is defined as the positive value of the asset:

$$CE_t = \mathrm{Max}(V_t, 0) \tag{18.1}$$

This is so because if the counterparty defaults with money owed to it, the full amount has to be paid.[3] In contrast, if the counterparty owes money, only a fraction may be recovered. Thus, presettlement risk only arises when the contract's replacement cost has a positive value to the institution (i.e., is in-the-money).

18.2.2 Measurement of Credit Risk

The evolution of credit risk measurement tools has gone through these steps:

Notional amounts, adding up simple exposures

Risk-weighted amounts, adding up exposures with a rough adjustment for risk

Notional amounts combined with credit ratings, adding up exposures adjusted for default probabilities

Internal portfolio credit models, integrating all dimensions of credit risk

Initially, risk measures were based on the total notional amount. A multiplier, say 8%, was applied to this amount to establish the amount of required capital to hold as a reserve against credit risk.

The problem with this approach is that it ignores variations in the probability of default. In 1988, the Basel Committee instituted a rough categorization of credit risk by *risk class*, providing risk weights to multiply notional amounts. This was the first attempt to force banks to carry capital in relation to the risks they were taking. The Basel rules are explained in more detail in Chapter 29.

These risk weights proved to be too simplistic, however, creating incentives for banks to alter their portfolios in order to maximize their shareholder returns subject to the Basel capital requirements. This had the perverse effect of introducing more risk into the balance sheets of commercial banks, which was certainly not the intended purpose of the 1988 rules. As an example, there was no differentiation between AAA-rated and C-rated corporate credits. Because loans to lower quality credits are generally more profitable than those to high quality credits but require the same amount of regulatory capital, some banks have responded by shifting its loan mix toward lower-rated credits. This led to the Basel II rules, which allow banks to use their own internal or external credit ratings. These credit ratings provide more refined representation of credit risk.

Even with these improvements, the credit risk charges are computed separately for all exposures and then added up. This approach may not properly account for diversification effects, unlike internal portfolio credit models. Such models, however, create special challenges and as a result, are still not accepted by the Basel Committee for capital adequacy requirements.

[3] This is due to *no walk-away clauses*, explained in Chapter 27.

TABLE 18.1 Comparison of Market Risk and Credit Risk

Item	Market Risk	Credit Risk
Sources of risk	Market risk only	Default risk, recovery risk, market risk
Distributions	Mainly symmetric, perhaps fat tails	Skewed to the left
Time horizon	Short term (days)	Long term (years)
Aggregation	Business/trading unit	Whole firm vs. counterparty
Legal issues	Not applicable	Very important

18.2.3 Credit Risk versus Market Risk

The tools recently developed to measure market risk have proved invaluable in assessing credit risk. Even so, there are a number of major differences between market and credit risks, which are listed in Table 18.1.

As mentioned previously, credit risk results from a compound process with three types of risk. The nature of this risk creates a distribution that is strongly skewed to the left, unlike most market risk factors. This is because credit risk is akin to short positions in options. At best, the counterparty makes the required payment and there is no loss. At worst, the entire amount due is lost.

The time horizon is also different. Whereas the time required for corrective action is relatively short in the case of market risk, it is much longer for credit risk. Positions also turn over much more slowly for credit risk than for market risk, although the advent of credit derivatives has made it easier to hedge credit risk.

Finally, the level of aggregation is different. Limits on market risk may apply at the level of a trading desk, business units, and eventually the whole firm. In contrast, limits on credit risk must be defined at the counterparty level, for all positions taken by the institution.

Legal issues are also very important for credit risk. Exposures can be controlled by netting contracts, for example. Finally, an event of default invariably puts in motion a level process.

18.3 MEASURING CREDIT RISK

18.3.1 Credit Losses

To develop the intuition of credit models, let us start with a simple case where losses are due to the effect of defaults only. This is what is called **default mode.** In other words, there is no intermediate marking-to-market. This example would be typical of loans held in a banking book as opposed to bonds held in a trading account.

The distribution of credit losses from a portfolio of N instruments issued by different obligors can be described as

$$\text{Credit loss} = \sum_{i=1}^{N} b_i \times \mathrm{CE_i} \times (1 - f_i) \tag{18.2}$$

where

- b_i is a (Bernoulli) random variable that takes the value of 1 if default occurs and 0 otherwise, with probability p_i, such that $E[b_i] = p_i$
- $\mathrm{CE_i}$ is the credit exposure at the time of default
- f_i is the recovery rate, or $(1 - f_i)$ the loss given default

18.3.2 Joint Events

In theory, all of these could be random variables. For what follows, we will assume that the only random variable is the default indicator b.

Equation (18.2) then shows that the expected credit loss is

$$E[\mathrm{CL}] = \sum_{i=1}^{N} E[b_i] \times \mathrm{CE_i} \times (1 - f_i) = \sum_{i=1}^{N} p_i \times \mathrm{CE_i} \times (1 - f_i) \tag{18.3}$$

The dispersion in credit losses, however, critically depends on the correlations between the default events.

Closed-form solutions can be easily derived when the default events are statistically independent. This simplifies the analysis considerably, as the probability of any joint event is simply the product of the individual event probabilities:

$$p(A \text{ and } B) = p(A) \times p(B) \tag{18.4}$$

At the other extreme, if the two events are perfectly correlated, that is, if B always defaults when A defaults, we have

$$p(A \text{ and } B) = p(B \mid A) \times p(A) = 1 \times p(A) = p(A) \tag{18.5}$$

when the marginal probabilities are equal, $p(A) = p(B)$.

Suppose, for instance, that the marginal probabilities are $p(A) = p(B) = 1\%$. Then the probability of the joint event is 0.01% in the independence case and 1% in the perfect-correlation case.

More generally, one can show that the probability of a joint default depends on the marginal probabilities and the correlations. As we have seen in Chapter 2,

TABLE 18.2 Joint Probabilities (Default Correlation = 0.5)

	B		
A	Default	No Default	Marginal
Default	0.00505	0.00495	0.01
No default	0.00495	0.98505	0.99
Marginal	0.01	0.99	

the expectation of the product can be related to the covariance

$$E[b_A \times b_B] = \mathrm{Cov}[b_A, b_B] + E[b_A]E[b_B] = \rho\sigma_A\sigma_B + p(A)p(B) \tag{18.6}$$

Given that b_A is a Bernoulli variable, Chapter 2 has shown that its standard deviation is $\sigma_A = \sqrt{p(A)[1-p(A)]}$ and similarly for b_B. We then have

$$p(A \text{ and } B) = \mathrm{Corr}(A, B)\sqrt{p(A)[1-p(A)]}\sqrt{p(B)[1-p(B)]} + p(A)p(B) \tag{18.7}$$

For example, if the correlation is unity and $p(A) = p(B) = p$, we have

$$p(A \text{ and } B) = 1 \times [p(1-p)]^{1/2} \times [p(1-p)]^{1/2} + p^2 = [p(1-p)] + p^2 = p$$

as shown in Equation (18.5).

If the correlation is 0.5 and $p(A) = p(B) = 0.01$, however, then we have $p(A \text{ and } B) = 0.00505$, which is only half of the marginal probabilities. This example is illustrated in Table 18.2, which lays out the full joint distribution. Note how the probabilities in each row and column sum to the marginal probability. From this information, we can infer all remaining probabilities. For example, the probability of B not defaulting when A is in default is $0.01 - 0.00505 = 0.00495$. This allows us to fill the entire table with joint probabilities.

18.3.3 An Example

As an example of credit loss distribution, consider a portfolio of $100 million with three bonds, A, B, and C, with various probabilities of default. To simplify, we assume (1) that the exposures are constant, (2) that the recovery in case of default is zero, and (3) that default events are independent across the three issuers.

Table 18.3 displays the exposures and default probabilities. We can easily compute the expected loss as $E[\mathrm{CL}] = \sum p_i \times \mathrm{CE}_i = 0.05 \times 25 + 0.10 \times 30 + 0.20 \times 45$, or

$$E[\mathrm{CL}] = \sum p_i \times \mathrm{CE}_i = \$13.5 \text{ million}$$

This is the average credit loss over many repeated, hypothetical "samples." This computation is very easy and does not require any information about the

TABLE 18.3 Portfolio Exposures, Default Risk, and Credit Losses

		Issuer	Exposure	Probability	
		A	\$25	0.05	
		B	\$30	0.10	
		C	\$45	0.20	

Default i	Loss L_i	Probability $p(L_i)$	Cumulative Probability	Expected $L_i p(L_i)$	Variance $(L_i - EL_i)^2 p(L_i)$
None	\$0	0.6840	0.6840	0.000	120.08
A	\$25	0.0360	0.7200	0.900	4.97
B	\$30	0.0760	0.7960	2.280	21.32
C	\$45	0.1710	0.9670	7.695	172.38
A, B	\$55	0.0040	0.9710	0.220	6.97
A, C	\$70	0.0090	0.9800	0.630	28.99
B, C	\$75	0.0190	0.9990	1.425	72.45
A, B, C	\$100	0.0010	1.0000	0.100	7.53
Sum				\$13.25	434.7

distribution other than default probabilities. Also note that we define the loss as a positive number, which is the usual convention.

On the other hand, we do need to described the full distribution to derive a worst-loss measure. This is done in the second panel, which lists all possible states. In state 1, there is no default, which has a probability of $(1-p_1)(1-p_2)(1-p_3) = (1-0.05)(1-0.10)(1-0.20) = 0.684$, given independence. In state 2, bond A defaults and the others do not, with probability $p_1(1-p_2)(1-p_3) = 0.05(1-0.10)\,(1-0.20) = 0.036$ (and so on for the other states).

Figure 18.1 plots the frequency distribution of credit losses. The table also shows how to compute the variance as

$$V[\mathrm{CL}] = \sum_{i=1}^{N} (L_i - E[\mathrm{CL}_i])^2 p(L_i) = 434.7$$

which gives a standard deviation of $\sigma(\mathrm{CL}) = \sqrt{434.7} = \$20.9$ million.

Alternatively, we can express the range of losses with a 95% quantile, which is the lowest number CL_i such that

$$P(\mathrm{CL} \leq \mathrm{CL}_i) \geq 95\% \qquad (18.8)$$

Because losses are recorded as positive numbers, the quantile is the lowest loss such that the cumulative probability of a lower loss is at or just above 95%.

From Table 18.3, we see that the fourth row has a cumulative probability of 0.9670, just above 0.95 and corresponds to a loss of \$45 million. Figure 18.2 plots the cumulative distribution function and shows that the 95% quantile is \$45 million.

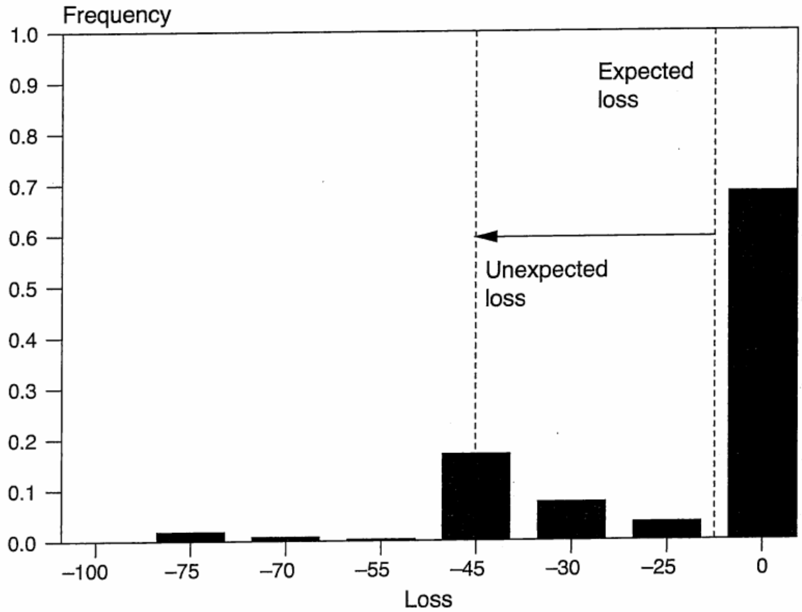

FIGURE 18.1 Distribution of Credit Losses

In terms of deviations from the mean, this gives an unexpected loss of 45 – 13.2 = \$32 million. This is a measure of **credit VAR.** Sometimes, however, credit VAR is measured as the total loss, or \$45 million in this case.

This very simple three-bond portfolio provides a useful example of the measurement of the distribution of credit risk. It shows that the distribution is skewed to the left. In addition, the distribution has irregular "bumps" that correspond to

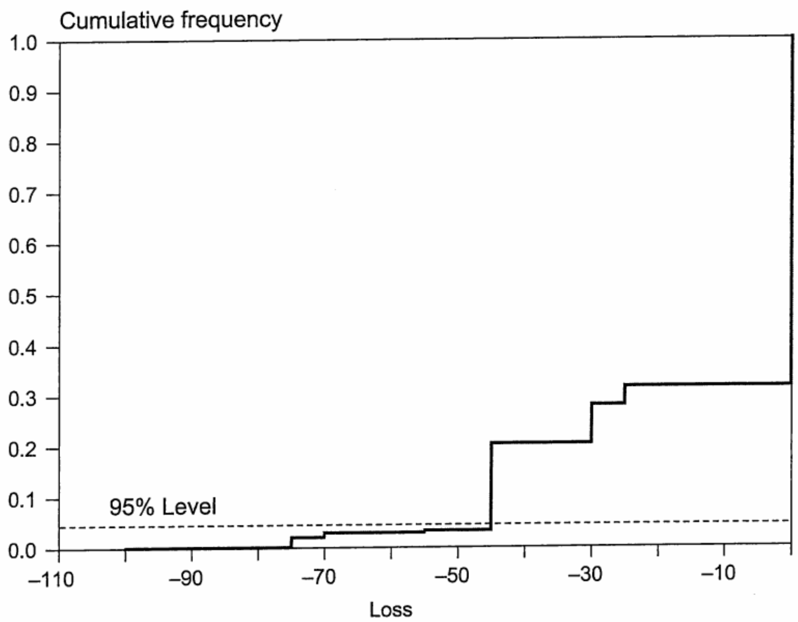

FIGURE 18.2 Cumulative Distribution of Credit Losses

the default events. The chapter on managing credit risk will further elaborate on this point.

KEY CONCEPT

The expected credit loss depends on default probabilities but not on default correlation. In contrast, the higher the default correlation, the higher the unexpected credit loss.

EXAMPLE 18.3: FRM EXAM 2002—QUESTION 130

You have granted an unsecured loan to a company. This loan will be paid off by a single payment of $50 million. The company has a 3% chance of defaulting over the life of the transaction and your calculations indicate that if they default you would recover 70% of your loan from the bankruptcy courts. If you are required to hold a credit reserve equal to your expected credit loss, how great a reserve should you hold?

a. $450,000
b. $750,000
c. $1,050,000
d. $1,500,000

EXAMPLE 18.4: FRM EXAM 2003—QUESTION 17

An investor holds a portfolio of $100 million. This portfolio consists of A-rated bonds ($40 million) and BBB-rated bonds ($60 million). Assume that the one-year probabilities of default for A-rated and BBB-rated bonds are 3% and 5%, respectively, and that they are independent. If the recovery value for A-rated bonds in the event of default is 70% and the recovery value for BBB-rated bonds is 45%, what is the one-year expected credit loss from this portfolio?

a. $1,672,000
b. $1,842,000
c. $2,010,000
d. $2,218,000

EXAMPLE 18.5: FRM EXAM 2007—QUESTION 73

A portfolio consists of two bonds. The credit-VAR is defined as the maximum loss due to defaults at a confidence level of 98% over a one-year horizon. The probability of joint default of the two bonds is 1.27%, and the default correlation is 30%. The bond value, default probability, and recovery rate are USD 1,000,000, 3%, and 60% for one bond, and USD 600,000, 5%, and 40% for the other. What is the expected credit loss of the portfolio?

a. USD 0
b. USD 9,652
c. USD 20,348
d. USD 30,000

EXAMPLE 18.6: FRM EXAM 2007—QUESTION 74

Continuing with the previous question, what is the best estimate of the unexpected credit loss (away from the ECL), or credit VAR, for this portfolio?

a. USD 570,000
b. USD 400,000
c. USD 360,000
d. USD 370,000

EXAMPLE 18.7: FRM EXAM 2007—QUESTION 102

Suppose Bank Z lends EUR 1 million to X and EUR 5 million to Y. Over the next year, the PD for X is 0.2 and for Y is 0.3. The PD of joint default is 0.1. The loss given default is 40% for X and 60% for Y. What is the expected loss of default in one year for the bank?

a. EUR 0.72 million
b. EUR 0.98 million
c. EUR 0.46 million
d. EUR 0.64 million

EXAMPLE 18.8: FRM EXAM 2004—QUESTION 46

Consider an A-rated bond and a BBB-rated bond. Assume that the one-year probabilities of default for the A- and BBB-rated bonds are 2% and 4%, respectively, and that the joint probability of default of the two bonds is 0.15%. What is the default correlation between the two bonds?

a. 0.07%
b. 2.6%
c. 93.0%
d. The default correlation cannot be calculated with the information provided.

18.4 CREDIT RISK DIVERSIFICATION

Modern banking was built on the sensible notion that a portfolio of loans is less risky than are single loans. As with market risk, the most important feature of credit risk management is the ability to diversify across defaults.

To illustrate this point, Figures 18.3a through 3c presents the distribution of losses for a $100 million loan portfolio. The probability of default is fixed at 1%. If default occurs, recovery is zero.

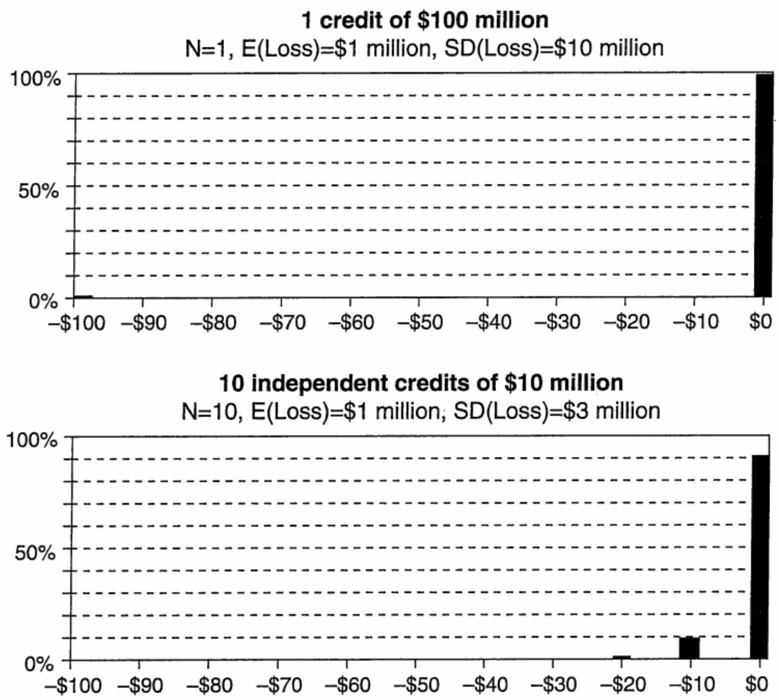

FIGURE 18.3a Distribution of Credit Losses

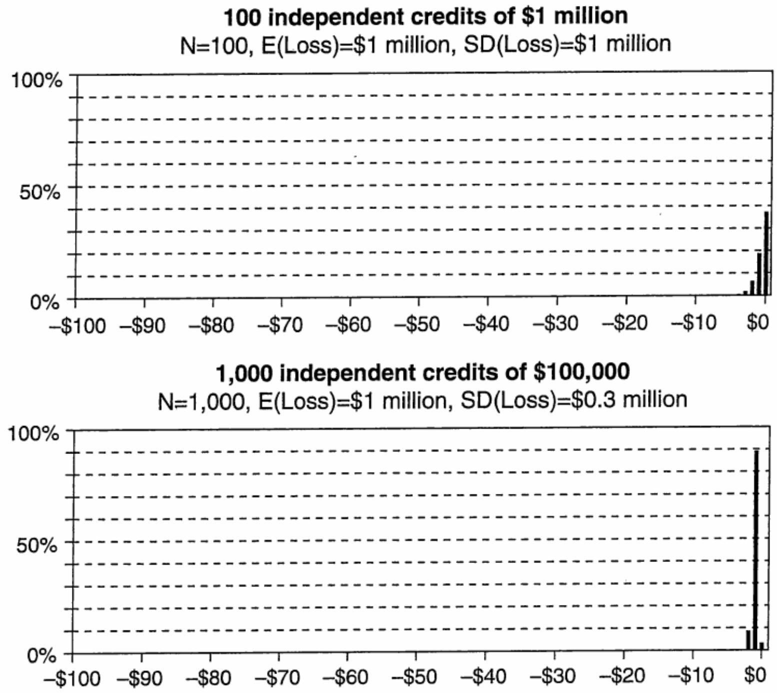

FIGURE 18.3b Distribution of Credit Losses

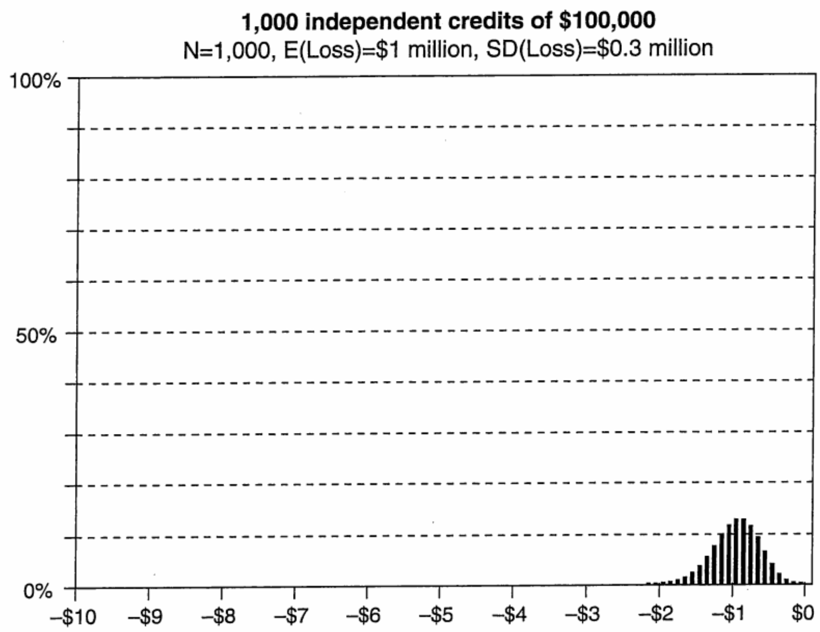

FIGURE 18.3c Distribution of Credit Losses

In the first panel, we have one loan only. We can either have no default, with probability 99%, or a loss of \$100 million with probability 1%. The expected loss is

$$\text{EL} = 0.01 \times \$100 + 0.99 \times 0 = \$1 \text{ million}$$

The problem, of course, is that, if default occurs, it will be a big hit to the bottom line, possibly bankrupting the lending bank.

Basically, this is what happened to Peregrine Investments Holdings, one of Hong Kong's leading investment banks which shut down due to the Asian crisis of 1997. The bank failed in large part from a single loan to PT Steady Safe, an Indonesian taxi-cab operator, that amounted to \$235 million, a quarter of the bank's equity capital.

In the case of our single loan, the spread of the distribution is quite large, with a variance of 99, which implies a standard deviation (SD) of about \$10 million. Simply focusing on the standard deviation, however, ignores the severe skewness in the distribution.

In the second panel, we consider 10 loans, each for \$10 million. The total notional is the same as before. We assume that defaults are independent. The expected loss is still \$1 million, or $10 \times 0.01 \times \$10$ million. The SD, however, is now \$3 million, much less than before.

Next, the third panel considers 100 loans of \$1 million each. The expected loss is still \$1 million, but the SD is now \$1 million, even lower. Finally, the fourth panel considers 1,000 loans of \$100,000, which create a SD of \$0.3 million.

For comparability, all these graphs use the same vertical and horizontal scale. This, however, does not reveal the distributions fully. This is why the fifth panel expands the distribution with 1,000 counterparties, which looks similar to a normal distribution. This reflects the **central limit theorem**, which states that the distribution of the sum of *independent* variables tends to a normal distribution. Remarkably, even starting from a highly skewed distribution, we end up with a normal distribution due to diversification effects.

In addition, the spread of the distribution becomes very small. This explains why portfolios of consumer loans, which are spread over a large number of credits, are less risky than typical portfolios of corporate loans.

With N events that occur with the same probability p, define the variable $X = \sum_{i=1}^{N} b_i$ as the number of defaults (where $b_i = 1$ when default occurs). The expected credit loss on our portfolio is then

$$E[\text{CL}] = E[X] \times \$100/N = pN \times \$100/N = p \times \$100 \tag{18.9}$$

which depends not on N but rather on the average probability of default and total exposure, \$100 million. When the events are independent, the variance of this variable is, using the results from a binomial distribution,

$$V[\text{CL}] = V[X] \times (\$100/N)^2 = p(1-p)N \times (\$100/N)^2 \tag{18.10}$$

which gives a standard deviation of

$$\mathrm{SD}[\mathrm{CL}] = \sqrt{p(1-p)} \times \$100/\sqrt{N} \tag{18.11}$$

For a constant total notional, this shrinks to zero as N increases.

We should note the crucial assumption that the credits are independent. When this is not the case, the distribution will lose its asymmetry more slowly. Even with a very large number of consumer loans, the dispersion will not tend to zero because the general state of the economy is a common factor behind consumer credits. Indeed, many more defaults occur in a recession than in an expansion. This is one of the reasons for being suspicious of credit risk models that have been calibrated over periods of expansion.

Institutions loosely attempt to achieve diversification by **concentration limits.** In other words, they limit the extent of exposure, say loans, to a particular industrial or geographical sector. The rationale behind this is that defaults are more highly correlated within sectors than across sectors. Conversely, **concentration risk** is the risk that too many defaults could occur at the same time.

The distributions in this section were derived using closed-form solutions, which assume *homogeneity*, or the same probability of default p, and independence. In this situation, all the possible states of the world can be described by using the **binomial expansion.** This general theorem states that

$$(x+y)^N = a_0x^N + a_1x^{N-1}y^1 + a_2x^{N-2}y^2 + \ldots + a_{N-1}x^1y^{N-1} + a_Ny^N \tag{18.12}$$

where the coefficients a_i in this expansion are the number of combinations of N things taken i at a time, or

$$a_i = \binom{N}{i} = \frac{N!}{i!(N-i)!} \tag{18.13}$$

If we define $x = p$ and $y = 1 - p$, the expansion must sum to unity, and each term gives the probability of this particular combination of events.

$$1 = p^N + Np^{N-1}(1-p)^1 + \tfrac{N(N-1)}{2}p^{N-2}(1-p)^2 + \ldots + Np^1(1-p)^{N-1} + (1-p)^N \tag{18.14}$$

As an example, with $N = 3$, we have

$$1 = p^3 + 3p^2(1-p) + 3p(1-p)^2 + (1-p)^3 \tag{18.15}$$

The first term gives the probability of three defaults. The second is the probability of exactly two defaults, which involves three events.[4] The last term gives the

[4] The events are (1) bond 1 in no default with bonds 2 and 3 in default, (2) bond 2 in no default with others in default, and (3) bond 3 in no default with others in default.

probability of no defaults. This is a convenient decomposition but is only valid when defaults are independent.

EXAMPLE 18.9: FRM EXAM 2002—QUESTION 92

A portfolio of bonds consists of five bonds whose default correlation is zero. The one-year probabilities of default of the bonds are: 1%, 2%, 5%, 10%, and 15%. What is the one-year probability of no default within the portfolio?

a. 71%
b. 67%
c. 85%
d. 99%

EXAMPLE 18.10: FRM EXAM 2004—QUESTION 15

There are 10 bonds in a credit default swap basket. The probability of default for each of the bonds is 5%. The probability of any one bond defaulting is completely independent of what happens to the other bonds in the basket. What is the probability that exactly one bond defaults?

a. 5%
b. 50%
c. 32%
d. 3%

18.5 IMPORTANT FORMULAS

Credit loss $= \sum_{i=1}^{N} b_i \times \mathrm{CE_i} \times (1 - f_i)$

Joint probability with independence: $p(A \text{ and } B) = p(A) \times p(B)$

Joint probability:
$p(A \text{ and } B) = \mathrm{Corr}(A, B)\sqrt{p(A)[1 - p(A)]}\sqrt{p(B)[1 - p(B)]} + p(A)p(B)$,
using $E[b_A \times b_B] = \mathrm{Cov}[b_A, b_B] + E[b_A]E[b_B]$

Binomial expansion:
$1 = p^N + Np^{N-1}(1 - p)^1 + \frac{N(N-1)}{2} p^{N-2}(1 - p)^2 + \ldots + Np^1(1 - p)^{N-1} + (1 - p)^N$

18.6 ANSWERS TO CHAPTER EXAMPLES

Example 18.1: FRM Exam 2000—Question 36

a. Settlement risk is due to the exchange of notional principal in different currencies at different points in time, which exposes one counterparty to default after it has made payment. There would be less risk with netted payments.

Example 18.2: FRM Exam 2000—Question 85

b. Answers c. and d. are both correct. Answers a. and b. are contradictory. A multilateral netting system concentrates the credit risk into one institution. This could potentially create much damage if this institution fails.

Example 18.3: FRM Exam 2002—Question 130

a. The Expected Credit Loss (ECL) is the notional amount times the probability of default times the loss given default. This is $\$50,000,000 \times 0.03 \times (1 - 70\%) = \$450,000$.

Example 18.4: FRM Exam 2003—Question 17

c. The expected loss is $\sum_i p_i \times \text{CE}_i \times (1 - f_i) = \$40,000,000 \times 0.03(1 - 0.70) + \$60,000,000 \times 0.05(1 - 0.45) = \$2,010,000$.

Example 18.5: FRM Exam 2007—Question 73

d. The ECL is for the first bond $1,000,000 \times 3\% \times (1 - 60\%) = 12,000$, and for the second $600,000 \times 5\% \times (1 - 40\%) = 18,000$. This adds up to \$30,000. Note that this number does not depend on the default correlation.

Example 18.6: FRM Exam 2007—Question 74

d. Here, the joint default probability matters. If the two bonds default, the loss is $\$1,000,000 \times (1 - 40\%) + \$600,000 \times (1 - 60\%) = \$400,000 + \$360,000 = \$760,000$. This will happen with probability 1.27%. The next bigger loss is \$400,000, which has probability of $3.00 - 1.27 = 1.73\%$. Its cumulative probability must be $100.00 - 1.17 = 98.73\%$. This is slightly above 98%, so \$400,000 is the quantile at the 98% level of confidence or higher. Subtracting the mean gives \$370,000.

Example 18.7: FRM Exam 2007—Question 102

b. The joint PD does not matter for the ECL. This is $ECL = 1 \times 20\% \times 40\% + 5 \times 30\% \times 60\%, = 0.08 + 0.90$, or EUR 0.98 million.

Example 18.8: FRM Exam 2004—Question 46

b. From Equation (18.7), the default correlation is $\text{Corr}(A, B) = [p(A \text{ and } B) - p(A)p(B)]/\{\sqrt{p(A)[1 - p(A)]}\sqrt{p(B)[1 - p(B)]}\} = [0.0015 - 0.02 \times 0.04]/\{\sqrt{0.02[1 - 0.02]}\sqrt{0.04[1 - 0.04]}\} = 0.025516$.

Example 18.9: FRM Exam 2002—Question 92

a. Because the events are independent, the joint probability is given by the product $(1 - p_1)(1 - p_2)(1 - p_3)(1 - p_4)(1 - p_5) = (1 - 1\%)(1 - 2\%)(1 - 5\%)(1 - 10\%)(1 - 20\%) = 70.51\%$.

Example 18.10: FRM Exam 2004—Question 15

c. Using the second term in Equation (18.14), we have $a_1 = \binom{10}{1} = 10$, and the probability is $10p^1(1 - p)^9 = 10 \times 0.05 \times (1 - 0.05)^9 = 0.315$.

CHAPTER 19

Measuring Actuarial Default Risk

Default risk is the primary driver of credit risk. It is represented by the **probability of default** (PD). When default occurs, the actual loss is the combination of **exposure at default** (EAD) and **loss given default** (LGD).

Default risk can be measured using two approaches: (1) **actuarial methods**, which provide "objective" measures of default rates, usually based on historical default data; and (2) **market-price methods**, which infer from traded prices of debt, equity, or credit derivatives "risk-neutral" measures of default risk.

Risk-neutral measures were introduced earlier in relation to option prices. They provide a useful shortcut for asset *pricing*. A major benefit of risk-neutral measures is that they are forward looking and responsive to the latest news because they are based on current market prices.

For *risk management* purposes, however, they are contaminated by the effect of risk premiums and contain measures of loss given default. As a result, they do not directly measure default probabilities. In contrast, objective measures describe the "actual" or "natural" probability of default.

Actuarial measures of default probabilities are provided by **credit rating agencies**, which classify borrowers by credit ratings that are supposed to quantify default risk. Such ratings are **external** to the firm. Similar techniques can be used to develop **internal** ratings.

Ratings usually start with **accounting variables models**, which relate the occurrence of default to a list of accounting variables, or more generally firm characteristics. Statistical techniques such as discriminant analysis can be used to examine how these variables are related to the occurrence or nonoccurrence of default. Accounting variables are augmented by information from financial markets and about the economic environment. Rating agencies also have access to management and private information about the firm.

This chapter focuses on actuarial measures of default risk. Market-based measures of default risk will be examined in the next chapter. Section 19.1 examines first the definition of a credit event. Section 19.2 then examines credit ratings, describing how historical default rates can be used to infer default probabilities. Recovery rates are analyzed in Section 19.3. Finally, Section 19.4 broadly discusses corporate and sovereign credit risk, as well as the role of credit rating agencies.

19.1 CREDIT EVENT

A credit event is a discrete state of the world. Either it happens or does not, depending on the definition of the event, which must be framed as precisely as possible.

The definition of default for a bond obligation is very narrow. Default on a bond occurs when payment on that bond is missed. The state of **default** is defined by **Standard & Poor's** (S&P), a credit rating agency, as

> *The first occurrence of a payment default on any financial obligation, rated or unrated, other than a financial obligation subject to a bona fide commercial dispute; an exception occurs when an interest payment missed on the due date is made within the grace period.*

Default on a particular bond, however, generally reflects the creditor's financial distress and is typically accompanied by default on other obligations. This is why rating agencies give a credit rating for the *issuer* in addition to a rating for specific bonds. The rating for specific bonds can be higher or lower than this issuer rating, depending on their relative priority.

This definition, however, needs to be defined more precisely for credit derivatives, whose payoffs are directly related to credit events. We will cover credit derivatives in Chapter 22. The definition of a **credit event** has been formalized by the **International Swaps and Derivatives Association** (ISDA), an industry group, which lists these events:

- **Bankruptcy**, which is defined as a situation involving either of
  - The *dissolution* of the obligor (other than merger)
  - The *insolvency*, or inability to pay its debt
  - The *assignment* of claims
  - The *institution of bankruptcy proceeding*
  - The *appointment of receivership*
  - The *attachment of substantially all assets by a third party*
- **Failure to pay**, which means failure of the creditor to make due payment; this is usually triggered after an agreed-upon grace period and when the payment due is above a certain amount
- **Obligation/cross default**, which means the occurrence of a default (other than failure to make a payment) on any other similar obligation
- **Obligation/cross acceleration**, which means the occurrence of a default (other than failure to make a payment) on any other similar obligation, resulting in that obligation becoming due immediately
- **Repudiation/moratorium**, which means that the counterparty is rejecting, or challenging, the validity of the obligation
- **Restructuring**, which means a waiver, deferral, or rescheduling of the obligation with the effect that the terms are less favorable than before

Notably, credit events occurred in 2008 for Fannie Mae and Freddie Mac (receivership), Lehman Brothers Holdings and Washington Mutual (bankruptcy),