21.2 DISTRIBUTION OF CREDIT EXPOSURE

Credit exposure consists of the **current exposure**, which is readily observable, and the **potential exposure**, or future exposure, which is random. Define x as the potential value of the asset on the target date. We describe this variable by its probability density function $f(x)$. This is where market risk mingles with credit risk.

21.2.1 Expected and Worst Exposure

The **expected credit exposure** (ECE) is the expected value of the asset replacement value x, if positive, on a target date:

$$\text{Expected credit exposure} = \text{ECE} = \int_{-\infty}^{+\infty} \text{Max}(x, 0) f(x) dx \tag{21.2}$$

The **worst credit exposure** (WCE) is the largest (worst) credit exposure at some level of confidence. It is implicitly defined as the value that is not exceeded at the given confidence level p:

$$1 - p = \int_{\text{WCE}}^{\infty} f(x) dx \tag{21.3}$$

To model the potential credit exposure, we need to (1) model the distribution of risk factors and (2) evaluate the instrument given these risk factors. This process is identical to a market value-at-risk (VAR) computation except that the aggregation takes place at the counterparty level if contracts can be netted.

To simplify to the extreme, suppose that the payoff x, or net claim against a particular counterparty, is normally distributed with mean zero and volatility σ. The expected credit exposure is then

$$\text{ECE} = \frac{1}{2} E\left(x \mid x > 0 \right) = \frac{1}{2}\sigma\sqrt{\frac{2}{\pi}} = \frac{\sigma}{\sqrt{2\pi}} \tag{21.4}$$

Note that we divided by 2 because there is a 50% probability that the value will be positive. The worst credit exposure at the 95% level is given by

$$\text{WCE} = 1.645\sigma \tag{21.5}$$

Figure 21.1 illustrates the measurement of ECE and WCE for a normal distribution. Note that negative values of x are not considered.

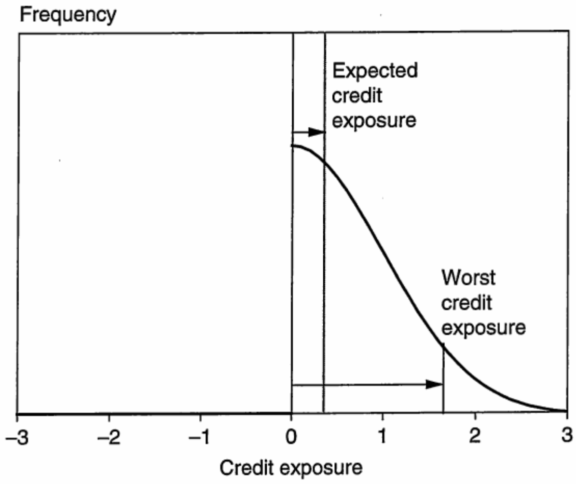

FIGURE 21.1 Expected and Worst Credit Exposures—Normal Distribution

21.2.2 Time Profile

The distribution can be summarized by the expected and worst credit exposures at each point in time. To summarize even further, we can express the average credit exposure by taking a simple arithmetic average over the life of the instrument.

The **average expected credit exposure** (AECE) is the average of the expected credit exposure over time, from now to maturity T:

$$\text{AECE} = (1/T)\int_{t=0}^{T} \text{ECE}_t\, dt \tag{21.6}$$

The **average worst credit exposure** (AWCE) is defined similarly:

$$\text{AWCE} = (1/T)\int_{t=0}^{T} \text{WCE}_t\, dt \tag{21.7}$$

21.2.3 Exposure Profile for Interest Rate Swaps

We now consider the computation of the exposure profile for an interest rate swap. In general, we need to define (1) the market risk factors, (2) the function and parameters for the joint stochastic processes, and (3) the pricing model for the swap. This is a good illustrative example for an instrument that is widely employed.

We start with a one-factor stochastic process for the interest rate, defining the movement in the rate r_t at time t as

$$dr_t = \kappa(\theta - r_t)dt + \sigma {r_t}^{\gamma} dz_t \tag{21.8}$$

as given in Chapter 4. The first term imposes **mean reversion**. When the current value of r_t is higher than the long-run value, the term in parentheses is negative, which creates a downward trend. More generally, the mean term could reflect the path implied in forward interest rates.

The second term defines the innovation, which can be given a normal distribution. An important issue is whether the volatility of the innovation should be constant or proportional to some power γ of the current value of the interest rate r_t. If the horizon is short, this issue is not so important because the current rate will be close to the initial rate.

When $\gamma = 0$, changes in yields are normally distributed, which is the Vasicek model (1977). As seen in a previous chapter, a typical volatility for *absolute* changes in yields is 1% per annum. A potential problem with this is that the volatility is the same whether the yield starts at 20% or 1%. As a result, the yield could turn negative, depending on the initial starting point and the strength of the mean reversion.

Another class of models is the lognormal model, which takes $\gamma = 1$. The model can then be rewritten in terms of $dr_t/r_t = d\ln(r_t)$. This specification ensures that the volatility shrinks as r gets close to zero, avoiding negative values. A typical volatility of *relative* changes in yields is 15% per annum, which is also the 1% value for changes in the level of rates divided by an initial rate of 6.7%.

For illustration purposes, we choose the normal process $\gamma = 0$ with mean reversion $\kappa = 0.02$ and volatility $\sigma = 0.25\%$ per month, which are realistic parameters based on recent U.S. data. The initial and long-run values of r are 6%. Typical simulation values are shown in Figure 21.2. Note how rates can deviate from their initial value but are pulled back to the long-term value of 6%.

This model is convenient because it leads to closed-form solutions. The distribution of future values for r is summarized in Figure 21.3 by its mean and two-tailed 90% confidence bands (called maximum and minimum values). The graph shows that the mean is 6%, which is also the long-run value. The confidence

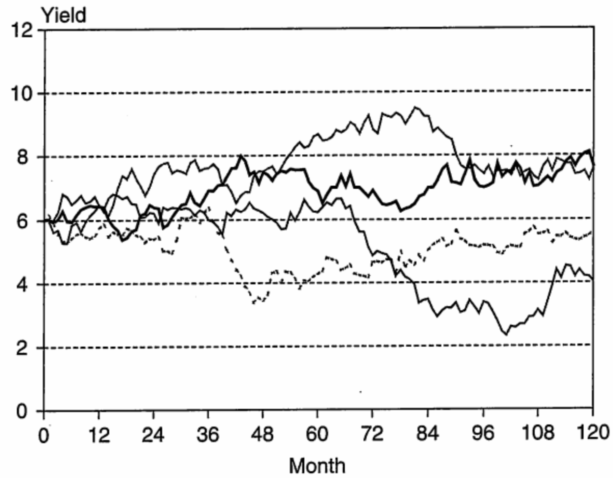

FIGURE 21.2 Simulation Paths for the Interest Rate

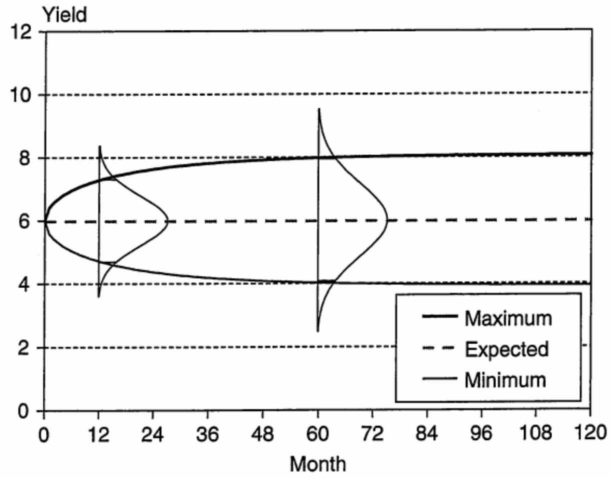

FIGURE 21.3 Distribution Profile for the Interest Rate

bands initially widen due to the increasing horizon, and then converge to a fixed value due to the mean reversion effect.

The next step is to value the swap. At each point in time, the current market value of the receive-fixed swap is the difference between the value of a fixed-coupon bond and a floating-rate note, as seen in Chapter 8:

$$V_t = B(F, t, T, c, r_t) - B(F, \text{FRN}) \tag{21.9}$$

Here, F is the notional amount or face value, c is the annualized *fixed* coupon rate, and T is the maturity date. The risk to the swap comes from the fact that the fixed leg has a coupon c that could differ from prevailing market rates. The principals are not exchanged.

Figure 21.4 illustrates the changes in cash flows that could arise from a drop in rates from 6% to 4% after five years. Assume the swap has notional of $N =$ \$100 million and pays semiannually. Every six months, the receive-fixed party is owed $\$100 \times (6 - 4)\% \times 0.5 = \$1$ million until the maturity of the swap. With 10 payments remaining, this adds up to a positive credit exposure of approximately \$10 million. More precisely, discounting over the life of the remaining payments gives \$8.1 million as of the valuation date.

In what follows, we assume that the swap receives fixed payments that are paid at a continuous rate instead of semiannually, which simplifies the example. Otherwise, there would be discontinuities in cash-flow patterns, and we would have to consider the risk of the floating leg as well. We also use continuous compounding. Defining N as the number of remaining years, the coupon bond value is

$$B(\$100, N, c, r) = \$100\frac{c}{r}\left[1 - e^{-rN}\right] + \$100e^{-rN} \tag{21.10}$$

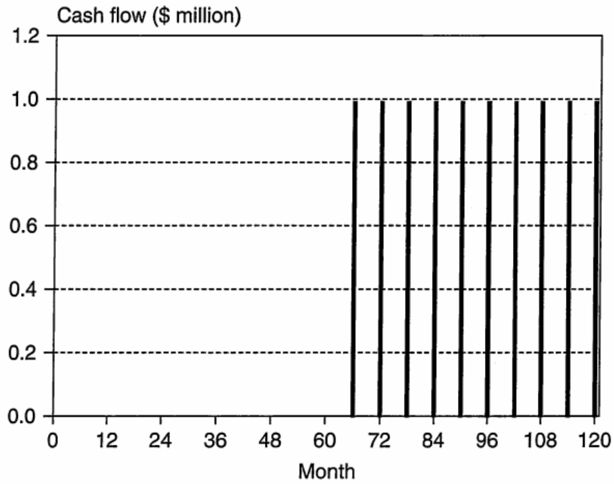

FIGURE 21.4 Net Cash Flows When Rates Fall to 4% After Five Years

as seen in the appendix to Chapter 1. The first term is the present value of the fixed-coupon cash flows discounted at the current rate r. The second term is the repayment of principal. For the special case where the coupon rate is equal to the current market rate ($c = r$), the market value is indeed \$100 for this par bond. The floating-rate note can be priced in the same way, but with a coupon rate that is always equal to the current rate. Hence, its value is always at par.

To understand the exposure profile of the coupon bond, we need to consider two opposing effects as time goes by:

- The **diffusion effect**, which increases the uncertainty in the interest rate
- The **amortization effect**, which decreases the bond's duration toward zero

The latter effect is described in Figure 21.5, which shows the bond's duration converging to zero. This explains why the bond's market value converges to the face value upon maturity, whatever happens to the current interest rate.

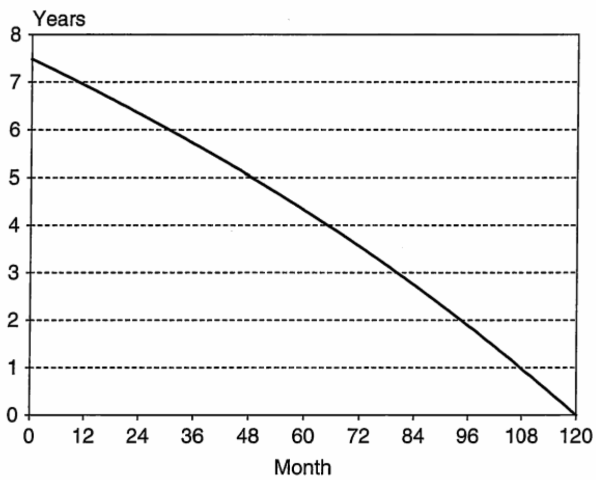

FIGURE 21.5 Duration Profile for a 10-Year Bond

Because the bond is a strictly monotonic function of the current yield, we can compute the 90% confidence bands by valuing the bond using the extreme interest rates range at each point in time. We use Equation (21.10) at each point in time in Figure 21.3. This exposure profile is shown in Figure 21.6.

Initially, the market value of the bond is \$100. After two or three years, the range of values is the greatest, from \$87 to \$115. Thereafter, the range converges to the face value of \$100. But overall, the fluctuations as a *proportion* of the face value are relatively small. Considering other approximations in the measurement of credit risk, such as the imprecision in default probability and recovery rate, assuming a constant exposure for the bond is not a bad approximation.

This is not the case, however, for the interest rate swap. Its value can be found by subtracting \$100 (the value of the floating-rate note) from the value of the coupon bond. Initially, its value is zero. Thereafter, it can take on positive or negative values. Credit exposure is the positive value only. Figure 21.7 presents the profile of the expected exposure and of the maximum (worst) exposure at the one-sided 95% level. It also shows the average maximum exposure over the whole life of the swap.

Intuitively, the value of the swap is derived from the difference between the fixed and floating cash flows. Consider a swap with two remaining payments and a notional amount of \$100. Its value is

$$
\begin{aligned}
V_t &= \$100\left[\frac{c}{(1+r)}+\frac{c}{(1+r)^2}+\frac{1}{(1+r)^2}\right]-\$100\left[\frac{r}{(1+r)}+\frac{r}{(1+r)^2}+\frac{1}{(1+r)^2}\right]\\
&= \$100\left[\frac{(c-r)}{(1+r)}+\frac{(c-r)}{(1+r)^2}\right] \qquad (21.11)
\end{aligned}
$$

Note how the principal payments cancel out and we are left with the discounted *net* difference between the fixed coupon and the prevailing rate $(c-r)$.

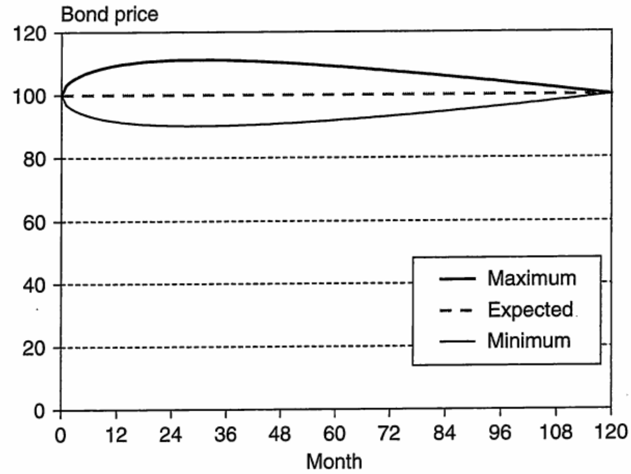

FIGURE 21.6 Exposure Profile for a 10-Year Bond

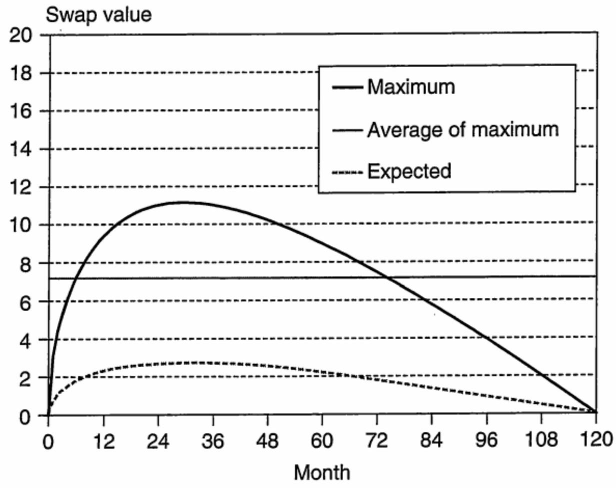

FIGURE 21.7 Exposure Profile for a 10-Year Interest Rate Swap

This information can be used to assess the expected exposure and worst exposure on a target date. The peak exposure occurs around the second year into the swap, or at about one-fourth of the swap's life. At that point, the expected exposure is about 3% to 4% of the notional, which is much less than that of the bond. The worst exposure peaks at about 10% to 15% of the notional. In practice, these values depend on the particular stochastic process used, but the exposure profiles will be qualitatively similar.

To assess the potential variation in swap values, we can make some approximations based on duration. Consider first the very short-term exposure, for which mean reversion and changes in durations are not important. The volatility of changes in rates then simply increases with the square root of time. Given a 0.25% per month volatility and 7.5-year initial duration, we can approximate the volatility of the swap value over the next year as

$$\sigma(V) = \$100 \times 7.5 \times 0.25\% \sqrt{12} = \$6.5 \text{ million}$$

Multiplying by 1.645, we get $10.7 million, which is close to the actual $9.4 million 95% worst exposure in a year reported in Figure 21.7.

The trade-off between declining duration and increasing risk can be formalized with a simple example. Assume that the bond's (modified) duration is proportional to the remaining life, $D = k(T - t)$ at any date t. The volatility from 0 to time t can be written as $\sigma(r_t - r_0) = \sigma\sqrt{t}$. Hence, the swap volatility is

$$\sigma(V) = [k(T - t)] \times \sigma\sqrt{t} \qquad (21.12)$$

To see where it reaches a maximum, we differentiate with respect to t:

$$\frac{d\sigma(V)}{dt} = [k(-1)]\sigma\sqrt{t} + [k(T - t)]\sigma\frac{1}{2\sqrt{t}}$$

Setting this to zero, we have

$$\sqrt{t} = (T - t)\frac{1}{2\sqrt{t}}, \quad 2t = (T - t)$$

which gives

$$t_{\text{MAX}} = (1/3)T \tag{21.13}$$

The maximum exposure occurs at one-third of the life of the swap. This occurs later than the one-fourth point reported previously because we assumed no mean reversion.

Further, we can check how this evolves with the maturity of the contract. At that point, the worst credit exposure will be

$$1.645\,\sigma(V_{\text{MAX}}) = 1.645\left[k(2/3)T\sigma\sqrt{T/3}\right] = \left[1.645k(2/3)\sigma\sqrt{1/3}\right]T^{3/2} \tag{21.14}$$

which shows that the WCE increases as $T^{3/2}$, which is faster than the maturity.

Figure 21.8 shows the exposure profile of a five-year swap. Here again, the peak exposure occurs at one-third of the swap's life. As expected, the magnitude is lower, with the peak expected exposure only about 1% of the notional.

Finally, Figure 21.9 displays the exposure profile when the initial interest rate is at 5% with a coupon of 6%. The swap starts in-the-money, with a current value of $7.9 million. With a long-run rate of 6%, the total exposure profile starts from a positive value, reaches a maximum after about two years, then converges to zero.

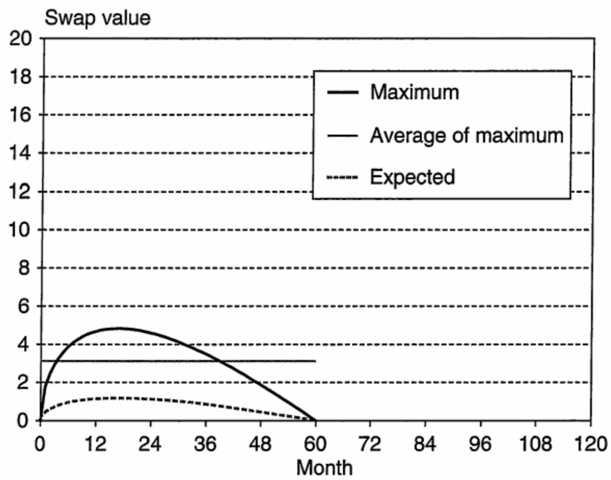

FIGURE 21.8 Exposure Profile for a Five-Year Interest Rate Swap

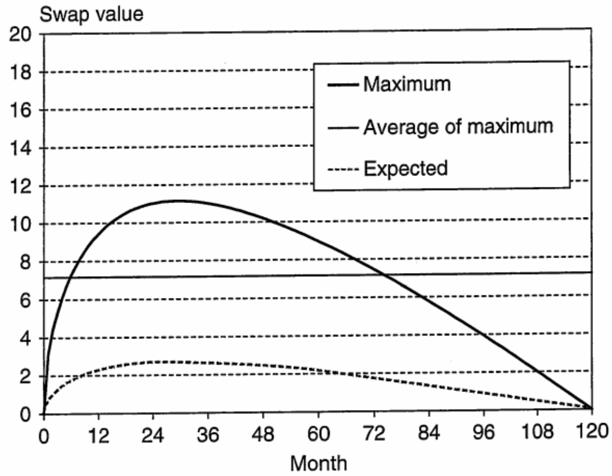

FIGURE 21.9 Exposure Profile for a 10-Year In-the-Money Swap

EXAMPLE 21.6: FRM EXAM 2004—QUESTION 43

In determining the amount of credit risk in a derivatives transaction, which of the following factors are used?

I. Notional principal amount of the underlying transaction
II. Current exposure
III. Potential exposure
IV. Peak exposure—the replacement cost in a worst case scenario

a. I and II
b. I, III, and IV
c. III and IV
d. II, III, and IV

EXAMPLE 21.7: FRM EXAM 2005—QUESTION 61

Assume that a bank enters into a USD 100 million, four-year annual pay interest rate swap, where the bank receives 6% fixed against 12-month LIBOR. Which of the following numbers best approximates the current exposure at the end of year 1 if the swap rate declines 125 basis points over the year?

a. USD 3,420,069
b. USD 4,458,300
c. USD 3,341,265
d. USD 4,331,382

EXAMPLE 21.8: PEAK EXPOSURE

Assume that the DV01 of an interest rate swap is proportional to its time to maturity (which at the initiation is equal to T). Assume that interest rate curve moves are parallel, stochastic with constant volatility, normally distributed, and independent. At what time will the maximum potential exposure be reached?

a. $T/4$
b. $T/3$
c. $T/2$
d. $3T/4$

EXAMPLE 21.9: FRM EXAM 2000—QUESTION 29

Determine at what point in the future a derivatives portfolio will reach its maximum potential exposure. All the derivatives are on one underlying, which is assumed to move in a stochastic fashion (variance in the underlying's value increases linearly with time passage). The derivatives portfolio's sensitivity to the underlying is expected to drop off as $(T-t)^2$, where T is the time from today until the last contract in the portfolio rolls off, and t is the time from today.

a. $T/5$
b. $T/3$
c. $T/2$
d. None of the above

EXAMPLE 21.10: FRM EXAM 2002—QUESTION 83

Assume that you have entered into a fixed-for-floating interest rate swap that starts today and ends in six years. Assume that the duration of your position is proportional to the time to maturity. Also assume that all changes in the yield curve are parallel shifts, and that the volatility of interest rates is proportional to the square root of time. When would the maximum potential exposure be reached?

a. In two months
b. In two years
c. In six years
d. In four years and five months

21.2.4 Exposure Profile for Currency Swaps

Exposure profiles are substantially different for other swaps. Consider, for instance, a currency swap where the notionals are 100 million U.S. dollars against 50 million British pounds (BP), set at an initial exchange rate of $S(\$/\text{BP}) = 2$.

The market value of a currency swap that receives foreign currency is

$$V_t = S_t(\$/\text{BP})B^*(\text{BP}50, t, T, c^*, r^*) - B(\$100, t, T, c, r) \tag{21.15}$$

Following the usual conventions, asterisks refer to foreign-currency values.

In general, this swap is exposed to domestic as well as foreign interest rate risk. When we just have two remaining coupons, the value of the swap evolves according to

$$\begin{aligned} V = S \times 50 &\left[\frac{c^*}{(1+r^*)} + \frac{c^*}{(1+r^*)^2} + \frac{1}{(1+r^*)^2}\right] \\ -\$100 &\left[\frac{c}{(1+r)} + \frac{c}{(1+r)^2} + \frac{1}{(1+r)^2}\right] \end{aligned} \tag{21.16}$$

Note that the principals do not cancel each other, unlike an interest rate swap. Instead, they are paid at maturity in different currencies, which is a major source of credit exposure.

In what follows, we will assume for simplicity that there is no interest rate risk, or that the value of the swap is dominated by currency risk. Further, we assume that the coupons are the same in the two currencies; otherwise there would be an asymmetrical accumulation of payments. As before, we have to choose a stochastic process for the spot rate. Say this is a lognormal process with constant variance and no trend:

$$dS_t = \sigma S_t \, dz_t \tag{21.17}$$

We choose $\sigma = 12\%$ annually, which is realistic, as seen in the chapter on market risk factors. This process ensures that the rate never becomes negative.

Figure 21.10 presents the exposure profile of a 10-year currency swap. Here there is no amortization effect, and exposure increases continuously over time. The peak exposure occurs at the end of the life of the swap. At that point, the expected exposure is about 10% of the notional, which is much higher than for the interest rate swap. The worst exposure is commensurately high, at about 45% of notional.

Although these values depend on the particular stochastic process and parameters used, this example demonstrates that credit exposures for currency swaps are far greater than for interest rate swaps, even with identical maturities.

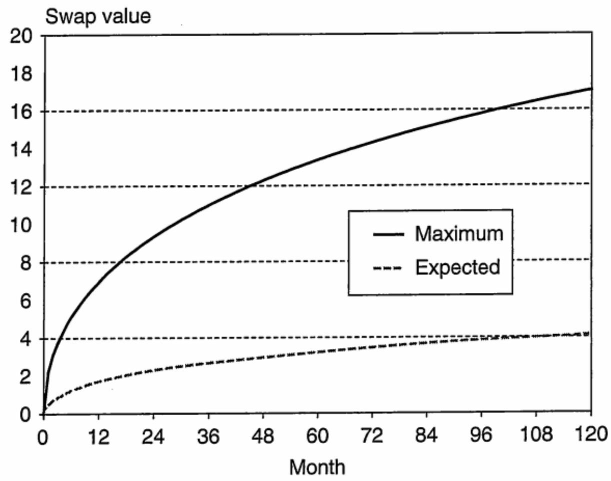

FIGURE 21.10 Exposure Profile for a 10-Year Currency Swap

21.2.5 Exposure Profile for Different Coupons

So far, we have assumed a flat term structure and equal coupon payments in different currencies, which creates a symmetric situation for the exposure for the long and short parties. In reality, these conditions will not hold, and the exposure patterns will be asymmetric.

Consider, for instance, the interest rate swap in Equation (21.11). If the term structure slopes upward, the coupon rate is greater than the floating rate, $c > r$, in which case the net payment to the party receiving fixed is initially positive. The value of the two-period swap can be analyzed by projecting floating payments at the forward rate:

$$V_t = \frac{(c - s_1)}{(1 + s_1)} + \frac{(c - f_{12})}{(1 + s_2)^2}$$

where s_1, s_2 are the one- and two-year spot rates, and f_{12} is the one- to two-year forward rate.

Example

Consider a \$100 million interest rate swap with two remaining payments. We have $s_1 = 5\%$, $s_2 = 6.03\%$ and hence, using $(1 + s_2)^2 = (1 + s_1)(1 + f_{12})$, we have $f_{12} = 7.07\%$. The coupon yield of $c = 6\%$ is such that the swap has zero initial value. The following table shows that the present value of the first payment (to the party receiving fixed) is positive and equal to \$0.9524. The second payment then must be negative, and is equal to $-$\$0.9524. The two payments exactly offset each other because the swap has zero value.

Time	Expected Spot	Expected Payment	Discounted
1	5%	$6.00 - 5.00 = +1.00$	$+0.9524$
2	7.07%	$6.00 - 7.07 = -1.07$	-0.9524
Total			-0.0000

This pattern of payments, however, creates more credit exposure to the fixed payer because it involves a payment in the first period offset by a receipt in the second. If the counterparty defaults shortly after the first payment is made, there could be a credit loss even if interest rates have not changed.

KEY CONCEPT

With a positively sloped term structure, the receiver of the floating rate (payer of the fixed rate) has a greater credit exposure than the counterparty.

A similar issue arises with currency swaps when the two coupon rates differ. Low nominal interest rates imply a higher forward exchange rate. The party that receives payments in a low-coupon currency is expected to receive greater payments later during the exchange of principal. If the counterparty defaults, there could be a credit loss even if rates have not changed.

KEY CONCEPT

The receiver of a low-coupon currency has greater credit exposure than the counterparty.

EXAMPLE 21.11: FRM EXAM 2000—QUESTION 47

Which one of the following deals would have the greatest credit exposure for a $1,000,000 deal size (assume the counterparty in each deal is an AAA-rated bank and has no settlement risk)?

a. Pay fixed in an Australian dollar (AUD) interest rate swap for one year.
b. Sell USD against AUD in a one-year forward foreign exchange contract.
c. Sell a one-year AUD cap.
d. Purchase a one-year certificate of deposit.

EXAMPLE 21.12: FRM EXAM 2001—QUESTION 8

Which of the following 10-year swaps has the highest potential credit exposure?

a. A cross-currency swap *after* two years
b. A cross-currency swap *after* nine years
c. An interest rate swap *after* two years
d. An interest rate swap *after* nine years

EXAMPLE 21.13: FRM EXAM 2004—QUESTION 14

BNP Paribas has just entered into a plain-vanilla interest-rate swap as a pay-fixed counterparty. Credit Agricole is the receive-fixed counterparty in the same swap. The forward spot curve is upward-sloping. If LIBOR starts trending down and the forward spot curve flattens, the credit risk from the swap will:

a. Increase only for BNP Paribas
b. Increase only for Credit Agricole
c. Decrease for both BNP Paribas and Credit Agricole
d. Increase for both BNP Paribas and Credit Agricole

21.3 EXPOSURE MODIFIERS

In a continuing attempt to decrease credit exposures, the industry has developed a number of methods to limit exposures. This section analyzes marking to market, margins and collateral, exposure limits, recouponing, and netting arrangements.

Other modifiers include **third-party guarantees** and purchasing **credit derivatives**. The former involve receiving, typically from a bank, a guarantee of payment should the counterparty fail. Credit derivatives will be covered in the next chapter.

21.3.1 Marking to Market

The ultimate form of reducing credit exposure is marking to market (MTM). **Marking to market** involves settling the variation in the contract value on a regular basis (e.g., daily). For OTC contracts, counterparties can agree to longer periods (e.g., monthly or quarterly). If the MTM treatment is symmetrical across the two counterparties, it is called **two-way marking to market**. Otherwise, if one party settles losses only, it is called **one-way marking to market**.

Marking to market has long been used by organized exchanges to deal with credit risk. The reason is that exchanges are accessible to a wide variety of investors, including retail speculators, who are more likely to default than others. On OTC markets, in contrast, institutions interacting with each other typically have an ongoing relationship. As one observer put it,

> *Futures markets are designed to permit trading among strangers, as against other markets which permit only trading among friends.*

With daily marking to market, the *current* exposure is reduced to zero. There is still, however, *potential* exposure because the value of the contract could change before the next settlement. Potential exposure arises from (1) the time interval between MTM periods and (2) the time required for liquidating the contract when the counterparty defaults.

In the case of a retail client, the broker can generally liquidate the position fairly quickly, within a day. When positions are very large, as in the case of brokers dealing with Long-Term Capital Management (LTCM), however, the liquidation period could be much longer. Indeed, LTCM's bailout was motivated by the potential disruption to financial markets had brokers attempted to liquidate their contracts with LTCM at the same time.

Marking to market introduces other types of risks, however:

- **Operational risk**, which is due to the need to keep track of contract values and to make or receive payments daily
- **Liquidity risk**, because the institution now needs to keep enough cash to absorb variations in contract values

Margins Potential exposure is covered by margin requirements. **Margins** represent cash or securities that must be advanced in order to open a position. The purpose of these funds is to provide a buffer against potential exposure.

Exchanges, for instance, require a customer to post an **initial margin** when establishing a new position. This margin serves as a performance bond to offset possible future losses should the customer default. Contract gains and losses are then added to the posted margin in the **equity account**. Whenever the value of this equity account falls below a threshold, set at a **maintenance margin**, new funds must be provided.

Margins are set in relation to price volatility and to the type of position, speculative or hedging. Margins increase for more volatile contracts. Margins are typically lower for hedgers because a loss on the futures position can be offset by a gain on the physical, assuming no basis risk. Some exchanges set margins at a level that covers the 99th percentile of worst daily price changes, which is a daily VAR system for credit risk.

Collateral Over-the-counter markets may allow posting securities as **collateral** instead of cash. This collateral protects against current and potential exposure. Typically, the amount of the collateral will exceed the funds owed by an amount known as the **haircut**. Collateral is typically managed within the ISDA **credit support annex** (CSA).

The haircut reflects both default risk and market risk. Safe counterparties will in general have lower haircuts. This also depends, however, on the downside risk of the asset. For instance, cash can have a haircut of zero, which means that there is full protection against current exposure. Government securities can require a haircut of 1%, 3%, and 8% for short-term, medium-term, and longer-term maturities, respectively. With greater price volatility, there is an increasing chance of losses if the counterparty defaults and the collateral loses value, which explains the increasing haircuts.

As an example, assume that hedge fund A enters swap with bank B. To mitigate A's credit risk, the two parties enter a collateral agreement that specifies the conditions under which B can ask for collateral. Now suppose the contract moves in-the-money for B, which requests $1 million in collateral from A. The funds are legally still the property of A but under the administration of B. If A defaults, B is entitled to sell the collateral and terminate the contract. Any positive excess value is returned to A. Conversely, if the collateral was not sufficient, B will have a claim against A.

21.3.2 Exposure Limits

Credit exposure can also be managed by setting **position limits** on the exposure to a counterparty. Ideally, these should be evaluated in a portfolio context, taking into account all the contracts between an institution and a counterparty.

To enforce limits, information on transactions must be centralized in middle-office systems. This generates an *exposure profile* for each counterparty, which can be used to manage credit line usage for several maturity buckets. Proposed new trades with the same counterparty should then be examined for their incremental effect.

These limits can be also set at the instrument level. In the case of a swap, for instance, an **exposure cap** requires a payment to be made whenever the value of the contract exceeds some amount. Figure 21.11 shows the effect of a $5 million cap on our 10-year swap. If, after two years, say, the contract suddenly moves into a positive value of $11 million, the counterparty would be required to make a payment of $6 million to bring the swap's outstanding value back to $5 million. This limits the worst exposure to $5 million and also lowers the average exposure.

21.3.3 Recouponing

Another method for controlling exposure at the instrument level is recouponing. **Recouponing** refers to a clause in the contract requiring the contract to be marked-to-market at some fixed date. This involves (1) exchanging cash to bring the MTM value to zero and (2) resetting the coupon or the exchange rate to prevailing market values.

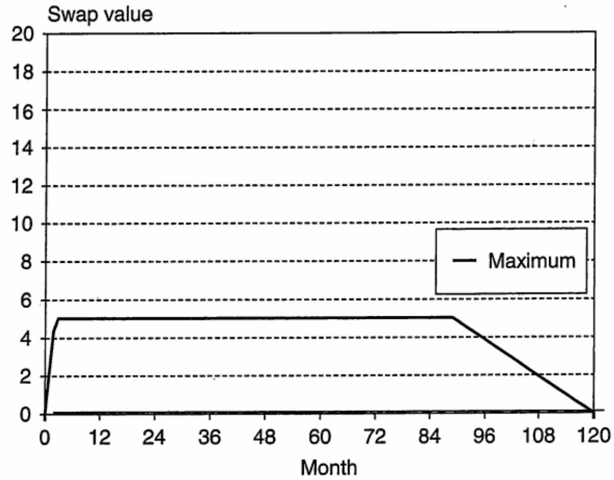

FIGURE 21.11 Effect of Exposure Cap

Figure 21.12 shows the effect of five-year recouponing on our 10-year swap. The exposure is truncated to zero after five years. Thereafter, the exposure profile is that of a swap with a remaining five-year maturity.

21.3.4 Netting Arrangements

Perhaps the most powerful mechanism for controlling exposures are **netting agreements**. These are now a common feature of standardized **master swap agreements** such as the one established in 1992 by the **International Swaps and Derivatives Association** (ISDA). This will be explained in more detail in Chapter 27.

The purpose of these agreements is to provide for the **netting** of payments across a set of contracts. In case of default, a counterparty cannot stop payments on contracts that have negative value while demanding payment on positive-value

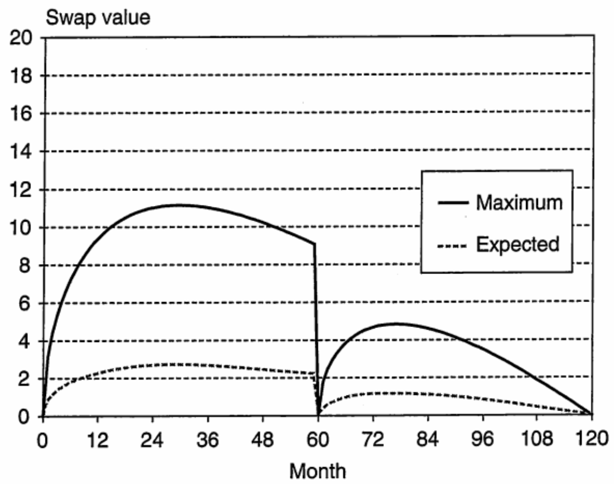

FIGURE 21.12 Effect of Recouponing After Five Years

contracts. As a result, this system reduces the exposure to the net value for all the contracts covered by the netting agreement. This prevents *cherry-picking* by the administrator of the defaulted counterparty.

Netting can be classified into three types:

1. **Payment netting** involves the daily off-setting of several claims in the same currency. An example is an interest rate swap, where only the net payment, floating against fixed, are exchanged.
2. **Novation netting** involves the cancellation of several contracts between the two parties, resulting in a replacement contract with new, net payments. As an example consider, a forward trade where A must pay 10 million euros in exchange for receiving 15 million dollars from B. In another trade, A must receive 5 million euros from B and pay 7 million dollars in exchange. Under novation, the two contracts are reduced to a payment of 5 million euros from A in exchange for a receipt of 8 million dollars from B.
3. **Close-out netting** involves the cancellation of all transactions under the master agreement in the event of bankruptcy or any other specified default event. The trades are then netted at market value.

Table 21.1 gives an example with four contracts. Without a netting agreement, the exposure of the first two contracts is the sum of the positive part of each, or \$100 million. In contrast, if the first two fall under a netting agreement, their value would offset each other, resulting in a net exposure of $\$100 - \$60 = \$40$ million. If contracts 3 and 4 do not fall under the netting agreement, the exposure is then increased to $\$40 + \$25 = \$65$ million.

To summarize, the **net exposure** with netting is

$$\text{Net exposure} = \text{Max}(V, 0) = \text{Max}\left(\sum_{i=1}^{N} V_i, 0\right) \tag{21.18}$$

TABLE 21.1 Comparison of Exposure with and without Netting

Contract	Contract Value	Exposure: No Netting	Exposure: With Netting for 1 and 2
	Under netting agreement		
1	+\$100	+\$100	
2	−\$60	+\$0	
Total, 1 and 2	+\$40	+\$100	+\$40
	No netting agreement		
3	+\$25	+\$25	
4	−\$15	+\$0	
Grand total, 1 to 4	+\$50	+\$125	+\$65

Without netting agreement, the **gross exposure** is the sum of all positive-value contracts:

$$\text{Gross exposure} = \sum_{i=1}^{N} \text{Max}(V_i, 0) \tag{21.19}$$

This is always greater than, or equal to, the exposure under the netting agreement.

The benefit from netting depends on the number of contracts N and the extent to which contract values covary. The larger the value of N and the lower the correlation, the greater the benefit from netting. It is easy to verify from Table 21.1 that if all contracts move into positive value at the same time, or have high correlation, there will be no benefit from netting.

Figures 21.13 and 21.14 illustrate the effect of netting on a portfolio of two swaps with the same counterparty. In each case, interest rates could increase or decrease with the same probability.

In Figure 21.13, the bank is long both a receive-fixed 10-year and five-year swap. The top panel describes the worst exposure when rates fall. In this case there is positive exposure for both contracts, which we add to get the total portfolio exposure. Whether there is netting or not does not matter, because the two positions are positive at the same time. The bottom panel describes the worst exposure when rates increase. Both positions, as well as the portfolio, have zero exposure.

In Figure 21.14, the bank is long the 10-year and short the five-year swap. When rates fall, the first swap has positive value and the second has negative

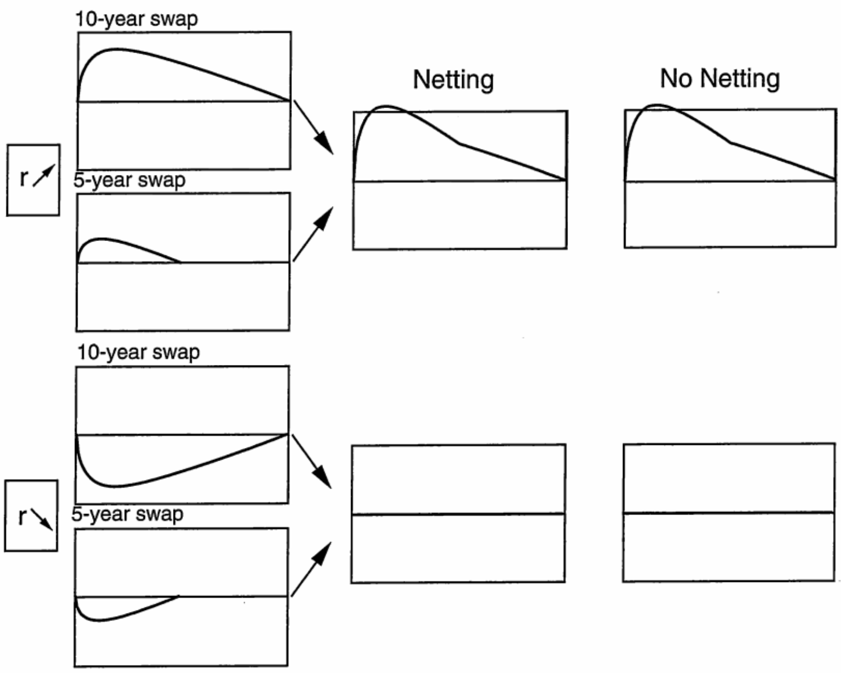

FIGURE 21.13 Netting with Two Long Positions

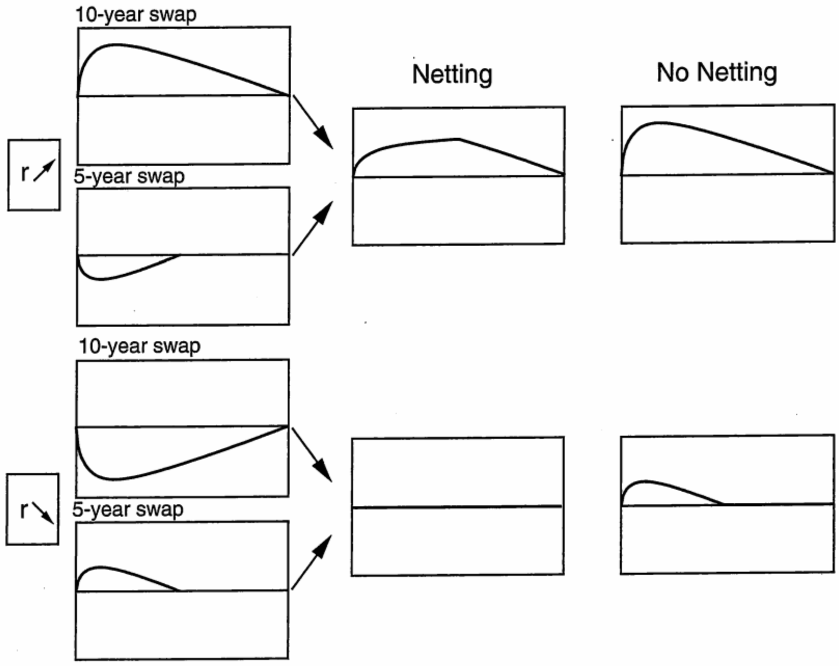

FIGURE 21.14 Netting with a Long and a Short Position

value. With netting, the worst exposure profile is reduced. In contrast, with no netting the exposure is that of the 10-year swap. Conversely, when rates increase, the swap value is negative for the first and positive for the second. With netting, the exposure profile is zero, whereas without netting it is the same as that of the five-year swap. This shows that netting is more effective with diversified positions.

Banks provide some information in their annual report about the benefit of netting for their current exposure. Without netting agreements or collateral, the **gross replacement value** (GRV) is reported as the sum of the worst-case exposures if all counterparties K default at the same time:

$$\text{GRV} = \sum_{k=1}^{K} (\text{Gross exposure})_k = \sum_{k=1}^{K} \left[\sum_{i=1}^{N_k} \text{Max}(V_i, 0) \right] \tag{21.20}$$

With netting agreements and collateral, the resulting exposure is defined as the **net replacement value** (NRV). This is the sum, over all counterparties, of the net positive exposure:

$$\text{NRV} = \sum_{k=1}^{K} (\text{Net exposure})_k = \sum_{k=1}^{K} \left[\text{Max}\left(\sum_{i=1}^{N_k} V_i, 0 \right) \right] \tag{21.21}$$

If collateral is held, this should be subtracted from the net exposure.

The effectiveness of netting can be assessed from BIS statistics for the OTC derivatives markets. As described in Chapter 5, the total notional amounts added up to \$596 trillion as of December 2007. The gross market value, defined as the summation of the positive part of all contracts, was estimated at \$15.8 trillion. The net credit exposure was reduced to \$3.3 trillion. Thus, netting reduces the exposure by 80%.

EXAMPLE 21.14: FRM EXAM 2002—QUESTION 89

If we assume that the VAR for the portfolio of trades with a given counterparty can be viewed as a measure of potential credit exposure, which of the following could *not* be used to decrease this credit exposure?

a. A netting agreement
b. Collateral
c. A credit derivative that pays out if the counterparty defaults
d. An offsetting trade with a different counterparty

EXAMPLE 21.15: FRM EXAM 2005—QUESTION 96

Which of the following statements correctly describes the impact of signing a netting agreement with a counterparty?

a. It will increase or have no effect on the total credit exposure.
b. It will decrease or have no effect on the total credit exposure.
c. It will increase exposure if exposure is net long and decrease exposure if it is net short.
d. Its impact is impossible to determine based on the available information.

EXAMPLE 21.16: FRM EXAM 2006—QUESTION 39

What are the benefits of novation?

a. Both parties are allowed to walk away from the contract in the event of default.
b. In a bilateral contract, it is specified that on default, the non-defaulting party nets gains and losses with the defaulting counterparty to a single payment for all covered transactions.
c. Financial market contracts can be terminated upon an event of default prior to the bankruptcy process.
d. Obligations are amalgamated with others.

EXAMPLE 21.17: FRM EXAM 2006—QUESTION 140

Company EFG is a large derivative market-maker that has many contracts with counterparty JKL, some transacted in the same legal jurisdiction and others across different legal jurisdictions. As a result, EFG has some contracts with JKL covered under legally enforceable netting agreement A, some contracts with JKL covered under legally enforceable netting agreement B, and some contracts with JKL with no netting agreement. Ignoring the effect of margin, if the current value (i.e., market value of the contract minus collateral and recovery value) and the netting agreement status of each contract with JKL are as shown below, what is EFG's current counterparty credit exposure to JKL?

- Contracts 1, 2, and 3 are covered under Netting Agreement A, with respective current values of USD 2,105; (-USD 3,319); USD 1,977.
- Contracts 4 to 7 are covered under Netting Agreement B, with respective current values of USD 5,876; (-USD 633); (-USD 2,335); USD 4,006.
- Contracts 8 and 9 are not covered by any Netting Agreement, with respective current values of USD 2,439; (-USD 1,504).

a. USD 8,612
b. USD 6,914
c. USD 14,899
d. USD 10,116

EXAMPLE 21.18: FRM EXAM 2003—QUESTION 24

Bank A, which is AAA-rated, trades a 10-year interest rate swap (semiannual payments) with bank B, rated A−. Because of bank B's poor credit rating, Bank A is concerned about its 10-year exposure. Which of the following measures help mitigate bank A's credit exposure to bank B?

I. Negotiate a CSA with bank B and efficiently manage the collateral management system
II. Execute the swap deal as a reset swap wherein the swap will be marked-to-market every six months
III. Execute the swap deal with a break clause in the fifth year
IV. Decrease the frequency of coupon payments from semiannual to annual

a. I only
b. IV only
c. I, II, III, and IV
d. I, II, and III

EXAMPLE 21.19: FRM EXAM 2002—QUESTION 73

Consider the following information. You have purchased 10,000 barrels of oil for delivery in one year at a price of $25/barrel. The rate of change of the price of oil is assumed to be normally distributed with zero mean and annual volatility of 30%. Margin is to be paid within two days if the credit exposure becomes greater than $50,000. There are 252 business days in the year. Assuming enforceability of the margin agreement, which of the following is the closest number to the 95% one-year credit risk of this deal governed under the margining agreement?

a. $50,000
b. $58,000
c. $61,000
d. $123,000

21.4 CREDIT RISK MODIFIERS

Credit risk modifiers operate on credit exposure, default risk, or a combination of the two. For completeness, this section discusses modifiers that affect default risk.

21.4.1 Credit Triggers

Credit triggers specify that if either counterparty's credit rating falls below a specified level, the other party has the right to have the swap cash settled. These are not exposure modifiers, but rather attempt to reduce the probability of default on that contract. For instance, if all outstanding swaps can be terminated when the counterparty rating falls below A, the probability of default is lowered to the probability that a counterparty will default when rated A or higher.

These triggers are useful when the credit rating of a firm deteriorates slowly, because few firms directly jump from investment-grade into bankruptcy. The increased protection can be estimated by analyzing transition probabilities, as discussed in a previous chapter. For example, say a transaction with an AA-rated borrower has a cumulative probability of default of 0.81% over 10 years. If the contract can be terminated whenever the rating falls to BB or below, this probability falls to 0.23%.

21.4.2 Time Puts

Time puts, or **mutual termination options**, permit either counterparty to terminate unconditionally the transaction on one or more dates in the contract. This feature decreases both the default risk and the exposure. It allows one counterparty to terminate the contract if the exposure is large and the other party's rating starts to slip.

Triggers and puts, which are types of **contingent requirements**, can cause serious trouble, however. They create calls on liquidity precisely in states of the world where the company is faring badly, putting additional pressure on the company's liquidity. Indeed, triggers in some of Enron's securities forced the company to make large cash payments and propelled it into bankruptcy. Rather than offering protection, these clauses can trigger bankruptcy, affecting all creditors adversely.

21.5 IMPORTANT FORMULAS

Credit exposure: $\mathrm{CE}_t = \mathrm{Max}(V_t, 0)$

Long options: $\mathrm{CE}_t = V_t$; short options, $\mathrm{CE}_t = 0$

Expected credit exposure (ECE): $\mathrm{ECE} = \int_{-\infty}^{+\infty} \mathrm{Max}(x, 0) f(x)dx$

Worst credit exposure (WCE): $1 - p = \int_{\mathrm{WCE}}^{\infty} f(x)dx$

Credit exposure for an interest rate swap, from:

$$V_t = B(F, t, T, c, r_t) - B(F, \mathrm{FRN})$$

Volatility of credit exposure for an interest rate swap: $\sigma(V_t) = [k(T-t)] \times \sigma\sqrt{t}$

Credit exposure for a currency swap, from:

$$V_t = S_t(\$/\mathrm{FC})B^*(F^*, t, T, c^*, r^*) - B(F, t, T, c, r)$$

Gross credit exposure: $\sum_{i=1}^{N} \mathrm{Max}(V_i, 0)$

Net credit exposure with netting: $\mathrm{Max}(V, 0) = \mathrm{Max}\left(\sum_{i=1}^{N} V_i, 0\right)$

Gross replacement value (GRV):

$$\mathrm{GRV} = \sum_{k=1}^{K}(\text{Gross exposure})_k = \sum_{k=1}^{K}\left[\sum_{i=1}^{N_k} \mathrm{Max}(V_i, 0)\right]$$

Net replacement value (NRV):

$$\mathrm{NRV} = \sum_{k=1}^{K}(\text{Net exposure})_k = \sum_{k=1}^{K}\left[\mathrm{Max}\left(\sum_{i=1}^{N_k} V_i, 0\right)\right]$$

minus collateral held

21.6 ANSWERS TO CHAPTER EXAMPLES

Example 21.1: FRM Exam 2006—Question 95

d. For a loss to occur, the exposure must be positive, (meaning rates move in your favor, and the counterparty must default.

Example 21.2: FRM Exam 2002—Question 93

d. Selling an option does not create credit exposures, because the premium has been received up front and the option can only create a future liability.

Example 21.3: FRM Exam 2006—Question 117

b. Selling an option does not create exposure, so answer d. is wrong. Longer horizons create a potential for larger price movements, so answer a. is wrong. The potential gain from being long is greater than being short. Prices can go up several times from the initial price for a long position. For a short position, the maximum gain is if the price goes to zero.

Example 21.4: FRM Exam 2004—Question 8

c. This is the only answer that involves truly selling an option, which has no credit exposure. A collar involves the sale and purchase of an option.

Example 21.5: FRM Exam 2001—Question 84

b. Being short an option creates no credit exposure, so answers c. and d. are false. With the short forward contract, a gain will be realized if the euro has depreciated.

Example 21.6: FRM Exam 2004—Question 43

d. All measures of exposure are important, current, potential, and peak. The notional amount, however, is not at risk.

Example 21.7: FRM Exam 2005—Question 61

a. The value of the fixed-rate bond is $6/(1+4.75\%)^1 + 6/(1+4.75\%)^2 + 106/(1+4.75\%)^3 = 103.420$, Subtracting \$100 for the floating leg gives an exposure of \$3.4 million. More intuitively, the sum of the coupon difference is 3 times $(6\% - 4.75\%)\$100 = \$1.25$, or around \$3.75 million without discounting.

Example 21.8: Peak Exposure

b. See Equation (21.14).

Example 21.9: FRM Exam 2000—Question 29

a. This question alters the variance profile in Equation (21.12). Taking now the variance instead of the volatility, we have $\sigma^2 = k(T-t)^4 \times t$, where k is a constant. Differentiating with respect to t,

$$\frac{d\sigma^2}{dt} = k[(-1)4(T-t)^3]t + k[(T-t)^4] = k(T-t)^3[-4t + T - t]$$

Setting this to zero, we have $t = T/5$. Intuitively, because the exposure profile drops off faster than in Equation (21.12), we must have earlier peak exposure than $T/3$.

Example 21.10: FRM Exam 2002—Question 83

b. Exposure is a function of duration, which decreases with time, and interest rate volatility, which increases with the square root of time. Define T as the original maturity and k as a constant. This give $\sigma(V_t) = k(T - t)\sqrt{t}$. Taking the derivative with respect to t gives a maximum at $t = (T/3)$. This gives $t = (6/3) =$ two years.

Example 21.11: FRM Exam 2000—Question 47

d. The CD has the whole notional at risk. Otherwise, the next greater exposure is for the forward currency contract and the interest rate swap. The short cap position has no exposure if the premium has been collected. Note that the question eliminates settlement risk for the forward contract.

Example 21.12: FRM Exam 2001—Question 8

a. The question asks about potential exposure for various swaps during their life. Interest rate swaps generally have lower exposure than currency swaps because there is no market risk on the principals. Currency swaps with longer remaining maturities have greater potential exposure. This is the case for the 10-year currency swap, which after two years has eight years remaining to maturity.

Example 21.13: FRM Exam 2004—Question 14

b. With an upward-sloping term structure, the fixed payer has greater credit exposure. He receives less initially, but receives more later. This backloading of payments increases credit exposure. Conversely, if the forward curve flattens, the fixed payer, i.e., BNP Paribas has less credit exposure. Credit Agricole must have greater credit exposure. Alternatively, if LIBOR drifts down, BNP will have to pay more, and its counterparty will have greater credit exposure.

Example 21.14: FRM Exam 2002—Question 89

d. An offsetting trade with a different party will provide no credit protection. If the first party defaults while the contract is in-the-money, there will be a credit loss.

Example 21.15: FRM Exam 2005—Question 96

b. Netting should decrease the credit exposure if contracts with the same counterparty have positive and negative values. In the worst case, that of one contract with positive value, there is no effect.

Example 21.16: FRM Exam 2006—Question 39

d. Answer a. is incorrect because this is a walk-away clause. Answer b. is incorrect because this is close-out netting. Answer c. is incorrect because this is a termination clause.

Example 21.17: FRM Exam 2006—Question 140

d. The sum of contracts 1, 2, and 3 is 763. The sum of contracts 4 to 7 is 6,914. We keep these two values because they are positive, and add the positive remaining contract value of 2,439, which gives a total of USD 10,116.

Example 21.18: FRM Exam 2003—Question 24

d. Collateral management will lower credit exposure, so Answer I. is correct. Resetting, or recouponing the swap, will also lower exposure. A break clause in five years will allow the marked-to-market, which also lowers exposure. On the other hand, decreasing the frequency of coupons will not change much the exposure. In fact, extending the period will increase exposure because there is a longer time to wait for the next payment, increasing the market will move in the favor or one counterparty.

Example 21.19: FRM Exam 2002—Question 73

c. The worst credit exposure is the \$50,000 plus the worst move over two days at the 95% level. The worst potential move is $\alpha\sigma\sqrt{T} = 1.645 \times 30\% \times \sqrt{(2/252)} =$ 4.40%. Applied to the position worth \$250,000, this gives a worst move of \$10,991. Adding this to \$50,000 gives \$60,991.

CHAPTER 22

Credit Derivatives and Structured Products

Credit derivatives are the latest tool in the management of portfolio credit risk. **Credit derivatives** are contracts whose value derives from the credit risk of an underlying obligor, corporate, sovereign, or multiname. They allow the exchange of credit risk from one counterparty to another. Credit derivatives initially grew from the need of banks to modify their credit exposure but since then have become essential portfolio management tools.

Like other derivatives, they can be traded on a standalone basis or embedded in some other instrument, such as a credit-linked note. This market has led to the expansion of **structured credit products**, through which portfolios of credit exposures are repackaged to better suit the needs of investors. This transformation relies on the securitization process, which was first applied to mortgage pools and was described in Chapter 7.

Section 22.1 presents an introduction to the size and rationale of these markets. Section 22.2 describes credit default swaps and their pricing. Other contracts such as total return swaps, credit spread forward and option contracts are covered in Section 22.3. Section 22.4 then presents credit structured products, including credit-linked notes and collateralized debt obligations (CDOs). Because of its importance, Section 22.5 describes the CDO market in more detail. Finally, Section 22.6 discusses the pros and cons of credit derivatives and structured products.

22.1 INTRODUCTION

22.1.1 Market Size

From 1996 to 2007, the market for credit derivatives is estimated to have grown from about \$40 billion in gross notional to more than \$62,000 billion, all of which is currently traded in **over-the-counter** (OTC) markets. As a reference, Chapter 7 has shown that the size of the global domestic corporate bond markets is \$6,300 billion. Including governments, financials and international bonds add up to a total of \$80,000 billion.

As is usual with OTC markets, however, only a fraction of this growth represents a net economic exchange of credit risk. A recent Fitch Ratings survey estimates that the ratio of gross-to-net exposures is 50, implying net exposures around $1,200 billion.

Gross exposures are very high because dealers had a practice of not cancelling existing trades but instead simply added new ones with offsetting characteristics. This practice, however, creates a backlog of paperwork, increases operational risk as well as counterparty risk.

As a result, the industry is starting to implement **portfolio compression**, which is a process that reduces the overall size and number of items in credit portfolios, without changing the risk parameters of the portfolio. In 2008 alone, $30,000 billion worth of contracts were cancelled, leading for the first time to a decline in notional amount during the year.

22.1.2 Markets for Exchanges of Risks

Credit derivatives have grown so quickly because they provide an efficient mechanism to exchange credit risk. While modern banking is built on the sensible notion that a portfolio of loans is less risky than single ones, banks still tend to be too concentrated in geographic or industrial sectors. This is because their comparative advantage is "relationship banking," which is usually limited to a clientele that banks know best. So far, it has been difficult to lay off this credit exposure, as there is only a limited market for secondary loans. In addition, borrowers may not like to see their bank selling their loans to another party, even for diversification reasons. Credit derivatives solve this dilemma by allowing banks to keep the loans on their books and to buy protection with credit derivatives.

In fact, credit derivatives are not totally new. **Bond insurance** is a contract between a bond issuer and a guarantor (a bank or insurer) to provide additional payment should the issuer fail to make full and timely payment. A **letter of credit** is a guarantee by a bank to provide a payment to a third party should the primary credit fail on its obligations. The **call feature** in corporate bonds involves an option on the risk-free interest rate as well as the credit spread. Indeed the borrower can also call back the bond should its credit rating improve. At an even more basic level, a long position in a **corporate bond**, is equivalent to a long position in a risk-free (meaning default-free) bond plus a short position in a credit default swap (CDS).

Thus, many existing instruments embed some form of credit derivative. What is new is the transparency and trading made possible by credit derivatives. Corporate bonds, notably, are difficult to short. This position can be replicated easily, however, by the purchase of a CDS contract. Thus, credit derivatives open new possibilities for investors, hedgers, and speculators.

22.1.3 Types of Credit Derivatives

Credit derivatives are over-the-counter contracts that allow credit risk to be exchanged across counterparties. They can be classified in terms of the following:

- *The underlying credit*, which can be either a single entity (single name) or a group of entities (multiname)

- *The exercise conditions*, which can be a credit event (such as default or a rating downgrade, or an increase in credit spreads)
- *The payoff function*, which can be a fixed amount or a variable amount with a linear or nonlinear payoff

The credit derivatives market includes plain-vanilla credit default swaps, total return swaps, credit spread forwards, and options. These instruments are bilateral OTC contracts. They also appear in credit structured products, which will be discussed later in this chapter. A recent survey breaks down the market into 33% for single-name CDSs, 30% for index CDSs, 16% for synthetic CDOs, 8% for tranche index trades, 3% for credit-linked notes, and 1% for credit spread options.[1] Thus the most common instruments are CDS contracts.

22.2 CREDIT DEFAULT SWAPS

22.2.1 Definition

In a **credit default swap** contract, a protection buyer (say A) pays a premium to the protection seller (say B), in exchange for payment if a credit event occurs. The **premium payment** can be a lump sum or periodic. The **contingent payment** is triggered by a credit event (CE) on the underlying credit, say a bond issued by company Y. The structure of this swap is described in Figure 22.1. Thus, these contracts represent the purest form of credit derivatives, as settlement payment only occurs in "default mode" (DM).

Note that these contracts are really options, not swaps. The main difference from a regular option is that the cost of the option is paid in installments instead of up front. When the premium is paid up front, these contracts are called *default put options*.[2] The annual payment is referred to as the **CDS spread**.[3]

Example

The protection buyer, call it A, enters a one-year credit default swap on a notional of $100 million worth of 10-year bonds issued by XYZ. The swap entails an annual payment of 50bp. The bond is called the *reference credit asset*.

At the beginning of the year, A pays $500,000 to the protection seller. Say that at the end of the year, company XYZ defaults on this bond, which now trades at 40 cents on the dollar. The counterparty then has to pay $60 million to A. If A holds this bond in its portfolio, the credit default swap provides protection against credit loss due to default.

[1] British Bankerss Association (2006), BBA Credit Derivatives Report 2006, London: BBA.

[2] Default swaps and default options are not totally identical instruments, however, because a default swap requires premium payments only until a triggering credit event occurs.

[3] This should not be confused with the bid–ask spread, which is the difference between the buying and selling rate. For instance, the bid rate may be 45bp, and the ask rate 55bp. So, the buyer would pay 0.55% annually to acquire protection. A protection seller would receive 0.45% only.

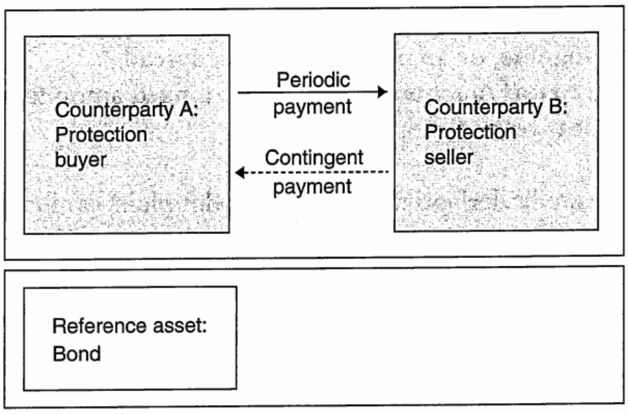

FIGURE 22.1 Credit Default Swap

Most CDS contracts are quoted in terms of an annual spread, with the payment made on a quarterly basis. Distressed names, however, can trade *up front*. For instance, on September 17, 2008, Washington Mutual was quoted at 44 points up front. This means that the buyer of protection on $100 million would have to pay $44 million up front plus the usual spread of 500bp per year. Wamu incurred a credit event on September 27, triggering payments on CDS contracts.

Default swaps are embedded in many financial products: Investing in a risky (credit-sensitive) bond is equivalent to investing in a risk-free bond plus selling a credit default swap. Say, for instance, that the risky bond sells at $90 and promises to pay $100 in one year. The risk-free bond sells at $95. Buying the risky bond is equivalent to buying the risk-free bond at $95 and selling a credit default swap worth $5 now. The up-front cost is the same, $90. If the company defaults, the final payoff will be the same.

KEY CONCEPT

A long position in a defaultable bond is economically equivalent to a long position in a default-free bond plus a short position in a CDS on the same underlying credit.

22.2.2 Settlement

Credit events must be subject to precise definitions. Chapter 19 provided such a list, drawn from the ISDA's master netting agreement. Ideally, there should be no uncertainty about the interpretation of a credit event. Otherwise, credit derivative transactions can create legal risks.

The payment on default reflects the loss to the holders of the reference asset when the credit event occurs. Define Q as the value of this payment per unit of notional. This can take a number of forms.

- **Cash settlement**, or a payment equal to the strike minus the prevailing market value of the underlying bond.
- **Physical delivery** of the defaulted obligation in exchange for a fixed payment.
- A **lump sum**, or a fixed amount based on some pre-agreed recovery rate. For instance, if the CE occurs, the recovery rate is set at 40%, leading to a payment of 60% of the notional.

Thus, the payoff on a credit default swap is

$$\text{Payment} = \text{Notional} \times Q \times I(\text{CE}) \tag{22.1}$$

where the indicator function $I(\text{CE})$ is 1 if the credit event has occurred and 0 otherwise.

The swap spread reflects both the probability of default and the loss given default, both of which are unknown. A slight variant on the usual CDS contract is the **binary credit default swap**, which pays a fixed amount $Q = 1$ if the credit event occurs. The two contracts can be combined to extract a market-implied estimate of the recovery rate.

With physical settlement, the contract usually defines a list of bonds that can be delivered. The bonds can trade at different prices but must all be exchanged for their face value. Naturally, the buyer of protection will select the cheapest bond, which creates a **delivery option**.

The growth of the CDS markets, however, has led to cases where the outstanding CDS notional far exceeds the available supply of bonds issued by a particular obligor. Cash settlement can be conducted through an **auction**, which defines the recovery rate. For instance, the final price for CDS contracts tied to Lehman Brothers Holdings was fixed on October 10, 2008 at a recovery rate of 8.625 cents on the dollar.[4] This meant that sellers of credit protection had to pay 91.375% of face value, an unexpectedly high fraction. Given an estimated $400 billion in outstanding CDS contracts, some feared that the settlement process would cause major disruptions to financial markets. In fact, the **Depository Trust & Clearing Corporation** (DTCC), which processes a large fraction of the trades, reported that only $5.2 billion had to change hands after the auction. This was because netting sharply reduced the gross exposures of $400 billion. Indeed, dividing by the estimated gross-to-net ratio of 50 gives a net exposure of only $8 billion. In addition, sellers of credit protection already had to post collateral. Thus, the CDS market has successfully handled a major default.

22.2.3 Pricing

CDS contracts can be priced by considering the present value of the cash flows on each side of the contract.

[4] Other notable auctions were Fannie Mae (91.51 cents for senior debt) and Washington Mutual (57 cents).

Define PV_t as the present value of a dollar paid at time t. For simplicity, assume that default occurs at the end of the year. As seen in Chapter 19, the marginal default rate from now to year t is $k_t = S_{t-1}d_t$, where S_t is the survival probability until the end of year t and d_t is the marginal probability of defaulting in year t. The survival probability is linked to the cumulative default probability $C_t = k_1 + k_2 + \cdots + k_t = 1 - S_t$.

Table 22.1 describes the annual default probabilities in the left panel. These represent typical market quotes for a credit initially rated BBB. The market-implied five-year cumulative default rate is 15.43%. Using $C_T = 1 - (1 - d)^T$, this gives an annual average default rate d of 3.30%. The second panel gives the discount factor assuming a risk-free interest rate of 6%.

Let us examine first the payoff payments. Upon default, the protection buyer receives the face value minus the recovery rate f, here assumed to be 40%. This occurs with probability k_t every year. The third panel in Table 22.1 illustrates the computations. On a notional of \$100, the PV of the expected payment in the first year is $k_1\,(1 - f) \times \$100 \times \mathrm{PV}_1 = 2.640\%(1 - 0.40) \times \$100 \times 0.9434 = 1.494$. Adding across the five years of the contract gives \$7.733.

In exchange, the protection buyer must make annual payments tied to a spread of s, defined in percent. In case of default, payments have to be made in arrears until the time of default then stops. For the first year, the PV of the expected payment is $s\$100 \times S_0 \times \mathrm{PV}_1 = (s/100)\$100 \times 1.000 \times 0.9434 = s0.943$. Here, $S_0 = 1.000$ because we are sure to make this first payment given that default happens at the end of the year. For the second year, this is $s\$100 \times S_1 \times \mathrm{PV}_2 = (s/100)\$100 \times 0.9736 \times 0.8900 = s0.867$. Summing across the five years gives $s3.986$.

The fair value of the spread is the number that sets the initial value of the CDS contract to zero. This solves

$$V = (\text{PV Payoff}) - s(\text{PV Spread}) = \left(\sum_{t=1}^{T} k_t(1 - f)\mathrm{PV}_t\right) - s\left(\sum_{t=1}^{T} S_{t-1}\mathrm{PV}_t\right) \tag{22.2}$$

In this case, the fair CDS spread solves $0 = 7.733 - s3.986$, which gives $s = 1.94\%$ because the spread was defined in percent. Note that this is very close to

TABLE 22.1 Payoffs on a Credit Default Swap

	Probability (%)				Discount	Payoff Payments		Spread Payments	
Year t	Cumul. C_t	Annual d_t	Marg. k_t	Survival S_t	Factor PV_t	Expected $k_t(1-f)$	PV	Expected sS_{t-1}	PV
1	2.64	2.640	2.640	0.9736	0.9434	1.584	1.494	$s1.000$	$s0.943$
2	5.48	2.917	2.840	0.9452	0.8900	1.704	1.517	$s0.974$	$s0.867$
3	8.57	3.269	3.090	0.9143	0.8396	1.854	1.557	$s0.945$	$s0.794$
4	11.89	3.631	3.320	0.8811	0.7921	1.992	1.578	$s0.914$	$s0.724$
5	15.43	4.018	3.540	0.8457	0.7473	2.124	1.587	$s0.881$	$s0.658$
Total			15.430		4.2124		7.733		$s3.986$

an approximation based on the annual average default rate times the loss given default, which is $3.30\% \times (1 - 0.40) = 1.98\%$.

Equation (22.2) can also be used to price an outstanding CDS contract. Assume for instance that the contract was entered with a spread of 1.50% and that the probabilities and interest rates in Table 22.1 represent current market conditions. The value of the CDS is then $V = 7.733 - 1.50 \times 3.986 = \$1.753$. This is a profit to the CDS buyer because the current spread is now greater than the locked-in value.

Also, Equation (22.2) can also be used to compute the spread duration of the contract. As an approximation, we can shock the market spreads $k_t(1 - f)$ upward by 1bp, which gives a gain of $\sum PV_t = 4.12$bp. This represents a (negative) spread duration of 4.12 years, slightly less than the maturity of the contract.

Note that the default probabilities used to price the CDS contract must be *risk-neutral* (RN) probabilities, not real-world probabilities. These RN probabilities π can be inferred from bond prices and CDS prices. For instance, assume that we observe a five-year CDS spread quote of 1.50%. Using the simplified approach, this gives $\pi = 1.50\%/(1 - 0.40) = 2.50\%$. For more precision, we could reverse the process in Table 22.1, using market quotes for one-, two-, three-, four-, and five-year CDS contracts to derive RN default probabilities for all the maturities.

Abstracting from counterparty risk, the CDS spread should be approximately equal to the difference between the yield on a corporate bond issued by the same obligor and the risk-free yield for the same maturity. If the CDS spread were markedly lower than this difference, an investor could make an arbitrage profit by buying the corporate bond, hedging pure interest rate risk by shorting a Treasury bond, and buying the CDS contract.

KEY CONCEPT

The CDS swap spread should approximately equal the yield on a corporate bond issued by the same obligor minus the risk-free yield.

In general, however, the **basis** between the CDS spread and the cash yield spread is slightly positive. To some extent, this is influenced by demand and supply considerations, including the availability of arbitrage capital. Because investors can only short credit by buying CDS contracts, this pushes up the spread. Otherwise, all else equal, the basis should be wider because the delivery option, which makes buying credit protection more attractive because the buyer is long the delivery option. In contrast, counterparty credit risk should decrease the basis because there is a risk the payoff may not be made if the credit event is triggered.

22.2.4 Counterparty Risk

It is important to realize that entering a credit swap does not eliminate credit risk entirely. Instead, the protection buyer decreases exposure to the reference credit

TABLE 22.2 CDS Spreads for Different Counterparties (Reference Obligation is Five-Year Bond Rated BBB)

	Counterparty Credit Rating			
Correlation	AAA	AA	A	BBB
0.0	194	194	194	194
0.2	191	190	189	186
0.4	187	185	181	175
0.6	182	178	171	159
0.8	177	171	157	134

Source: Adapted from Hull, J. and A. White (2001), Valuing Credit Default Swaps II: Modeling Default Correlations. *Journal of Derivatives* 8, 12–21.

Y but assumes new credit exposure to the CDS seller. Protection will be effective with a low correlation between the default risk of the underlying credit and of the counterparty. Just to be sure, the contracts may involve the posting of collateral from the protection seller.

Like options, these instruments are **unfunded**, meaning that each party is responsible for making payments (i.e., premiums and settlement amount) without recourse to other assets. In contrast, in a **funded** instrument, the protection seller makes a payment that could be used to settle any potential credit event. In the latter case, the protection buyer is not exposed to counterparty risk.

Table 22.2 illustrates the effect of the counterparty for the pricing of the CDS. If the counterparty is default free, the CDS spread on this BBB credit should be 194bp. The spread depends on the default risk for the counterparty as well as the correlation with the reference credit. In the worst case in the table, with a BBB rating for the counterparty and correlation of 0.8, protection is less effective, and the CDS spread is only 134bp.

EXAMPLE 22.1: FRM EXAM 2004—QUESTION 9

If an investor holds a five-year IBM bond, it will give him a return very close to the return of the following position:

a. A five-year IBM credit default swap on which he pays fixed and receives a payment in the event of default
b. A five-year IBM credit default swap on which he receives fixed and makes a payment in the event of default
c. A five-year U.S. Treasury bond plus a five-year IBM credit default swap on which he pays fixed and receives a payment in the event of default
d. A five-year U.S. Treasury bond plus a five-year IBM credit default swap on which he receives fixed and makes a payment in the event of default

EXAMPLE 22.2: FRM EXAM 2007—QUESTION 18

Suppose the return on U.S. Treasuries is 3% and a risky bond is currently yielding 15%. A trader you supervise claims to be able to make an arbitrage trade earning 5% using U.S. Treasuries, the risky bond, and the credit default swap. Which of the following could be the trader's strategy, and what is the CDS premium?

a. Go long the Treasury, short the risky bond, and sell the credit default swap with a premium of 7%.
b. Go long the Treasury, short the risky bond, and buy the credit default swap with a premium of 6%.
c. Short the Treasury, invest in the risky bond, and buy the credit default swap with a premium of 7%.
d. Short the Treasury, invest in the risky bond, and sell the credit default swap with a premium of 6%.

EXAMPLE 22.3: FRM EXAM 2007—QUESTION 120

Bank A makes a USD 10 million five-year loan and wants to offset the credit exposure to the obligor. A five-year credit default swap (CDS) with the loan as the reference asset trades on the market at a swap premium of 50 basis points paid quarterly. In order to hedge its credit exposure, bank A

a. Sells the five-year CDS and receives a quarterly payment of USD 50,000.
b. Buys the five-year CDS and makes a quarterly payment of USD 12,500.
c. Buys the five-year CDS and receives a quarterly payment of USD 12,500.
d. Sells the five-year CDS and makes a quarterly payment of USD 50,000.

EXAMPLE 22.4: FRM EXAM 2004—QUESTION 50

The table below shows the bid-ask quotes by UBS for CDS spreads for companies A, B, and C. CSFB has excessive credit exposure to company C and wants to reduce it through the CDS market.

	1 Year	3 Year	5 Year
A	15/25	21/32	27/36
B	43/60	72/101	112/152
C	71/84	93/113	141/170

Since the furthest maturity of its exposure to C is three years, CSFB buys a USD 200 million three-year protection on C from UBS. In order to make its purchase of this protection cheaper, based on its views on companies A and B, CSFB decides to sell USD 300 million five-year protection on company A and to sell USD 100 million one-year protection on company B to UBS. What is the net annual premium payment made by CSFB to UBS in the first year?

a. USD 1.02 million
b. USD 0.18 million
c. USD 0.58 million
d. USD 0.62 million

EXAMPLE 22.5: FRM EXAM 1999—QUESTION 135

The Widget Company has outstanding debt of three different maturities as outlined in the table.

	Widget Co. Bonds		U.S. T-Bonds	
Maturity	Price	Coupon	Price	Coupon
one year	100	7.00%	100	6.00%
five years	100	8.50%	100	6.50%
10 years	100	9.50%	100	7.00%

All Widget Company debt ranks pari passu and contains cross default provisions. The recovery value for each bond is 20%. The correct price for a one-year CDS with the Widget Company, 9.5% 10-year bond as a reference asset is

a. 1.0% per annum
b. 2.0% per annum
c. 2.5% per annum
d. 3.5% per annum

EXAMPLE 22.6: FRM EXAM 2004—QUESTION 65

When an institution has sold exposure to another institution (i.e., purchased protection) in a CDS, it has exchanged the risk of default on the underlying asset for which of the following?

a. Default risk of the counterparty
b. Default risk of a credit exposure identified by the counterparty
c. Joint risk of default by the counterparty and of the credit exposure identified by the counterparty
d. Joint risk of default by the counterparty and the underlying asset

EXAMPLE 22.7: FRM EXAM 2007—QUESTION 85

Bank A has exposure to USD 100 million of debt issued by company R. Bank A enters into a credit default swap transaction with bank B to hedge its debt exposure to company R. Bank B would fully compensate bank A if company R defaults in exchange for a premium. Assume that the defaults of bank A, bank B, and company R are independent and that their default probabilities are 0.3%, 0.5%, and 3.6%, respectively. What is the probability that bank A will suffer a credit loss in its exposure to company R?

a. 4.1%
b. 3.6%
c. 0.0108%
d. 0.0180%

EXAMPLE 22.8: FRM EXAM 2005—QUESTION 111

You enter into a credit default swap with bank B that settles based upon the performance of company C. Assuming that bank B and company C have the same initial credit rating and everything else remains the same, what is the impact on the value of your credit default swap if bank B buys company C?

a. The credit default swap value increases.
b. The credit default swap value remains the same.
c. The credit default swap value decreases.
d. Impossible to determine based on the information provided.

22.3 OTHER CONTRACTS

22.3.1 CDS Variants

Credit default swaps can also be written on multiple names. For instance, the **first-of-basket-to-default swap** gives the protection buyer the right to deliver *one and only one* defaulted security out of a basket of selected securities. Because the protection buyer has more choices—that is, default can occur across a basket instead of just one reference credit—this type of protection will be more expensive than a single credit swap, all else kept equal. The price of protection also depends on the correlation between credit events. The lower the correlation, the more expensive the swap. Conversely, the higher the correlation, the lower the swap rate. To illustrate this point, consider the extreme case of perfect correlation. In such case, all underlying credits default at the same time, and this basket swap is equivalent to a regular single-name CDS.

With an **Nth-to-default swap**, payment is triggered after N defaults in the underlying portfolio, but not before. When N is large, the cost of protection will be high when the default correlation is high, making it more likely that N names will default simultaneously.

CDS indices are widely used to track the performance of this market. The iTraxx indices cover the most liquid names in European and Asian credit markets. The North American and emerging markets are covered by the CDX indices. For example, the CDX.NA.IG index is composed of 125 investment-grade entities domiciled in North America. The CDX.NA.HY index covers 100 non-investment-grade (high yield) borrowers. The CDX.EM index covers borrowers from emerging markets. The indices are rebalanced every six months. Because these contracts are very liquid, and trade at tight bid–ask spreads, they provide an easy way to buy and sell marketwide or sectoral credit risk. These indices also have tradable tranches, using the CDO methodology described later in this chapter.

CDS indices trade at tight spreads, even more narrow than for single-name contracts. Assume for instance that a dealer quotes 201/203 for a five-year CDX.NA.IG. A trader wants \$80,000 of protection on each of the 125 companies in the index, which add up to a notional of \$10 million. The total cost is $0.0203 \times \$10,000,000 = \$203,000$.[5] If one company defaults, the buyer receives the usual CDS payment, and the notional is then reduced by \$80,000.

22.3.2 Total Return Swaps

A **total return swap** (TRS) is a contract where one party, called the protection buyer, makes a series of payments linked to the total return on a reference asset. In exchange, the protection seller makes a series of payments tied to a reference rate, such as the yield on an equivalent Treasury issue (or LIBOR) plus a spread. If the price of the asset goes down, the protection buyer receives a payment from the counterparty; if the price goes up, a payment is due in the other direction. The structure of this swap is described in Figure 22.2.

[5] In practice, the pattern of payment is more complicated, with a fixed coupon and an initial price that depends on the quoted spread.

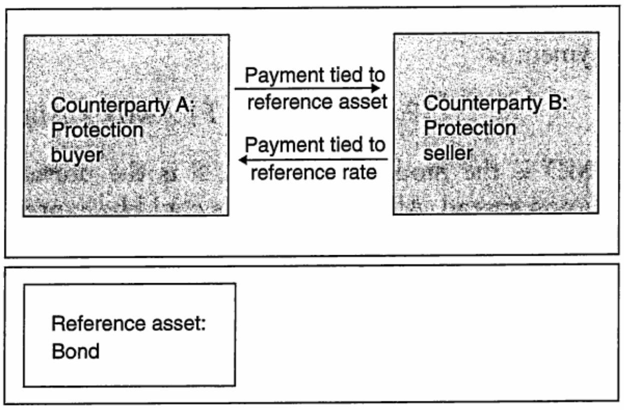

FIGURE 22.2 Total Return Swap

This type of swap is tied to changes in the market value of the underlying asset and provides protection against credit risk in a marked-to-market (MTM) framework. For the protection buyer, the TRS removes all the economic risk of the underlying asset without selling it. Unlike a CDS, the TRS involves both credit risk and market risk, the latter reflecting pure interest rate risk.

Example

Suppose that a bank, call it bank A, has made a \$100 million loan to company XYZ at a fixed rate of 10%. The bank can hedge its exposure by entering a TRS with counterparty B, whereby it promises to pay the interest on the loan plus the change in the market value of the loan in exchange for LIBOR plus 50bp. If the market value of the loan decreases, the payment tied to the reference asset will become negative, providing a hedge for the bank.

Say that LIBOR is currently at 9% and that after one year, the value of the loan drops from \$100 to \$95 million. The *net* obligation from bank A is the sum of

- Outflow of $10\% \times \$100 = \$10$ million, for the loan's interest payment
- Inflow of $9.5\% \times \$100 = \$9.5$ million, for the reference payment
- Outflow of $\frac{(95-100)}{100}\% \times \$100 = -\$5$ million, for the movement in the loan's value

This sums to a net receipt of $-10 + 9.5 - (-5) = \$4.5$ million. Bank A has been able to offset the change in the economic value of this loan by a gain on the TRS.

22.3.3 Credit Spread Forward and Options

These instruments are derivatives whose value is tied to an underlying credit spread between a risky and risk-free bond.

In a **credit spread forward contract**, the buyer receives the difference between the credit spread at maturity and an agreed-upon spread, if positive. Conversely, a payment is made if the difference is negative. An example of the formula for the cash payment is

$$\text{Payment} = (S - F) \times \text{MD} \times \text{Notional} \tag{22.3}$$

where MD is the modified duration, S is the prevailing spread, and F is the agreed-upon spread. Alternatively, this could be expressed in terms of prices:

$$\text{Payment} = [P(y + F, \tau) - P(y + S, \tau)] \times \text{Notional} \tag{22.4}$$

where y is the yield to maturity of an equivalent Treasury, and $P(y + S, \tau)$ is the present value of the security with τ years to expiration, discounted at y plus a spread. Note that if $S > F$, the payment will be positive as in the previous expression.

In a **credit spread option contract**, the buyer pays a premium in exchange for the right to "put" any increase in the spread to the option seller at a predefined maturity:

$$\text{Payment} = \text{Max}(S - K, 0) \times \text{MD} \times \text{Notional} \tag{22.5}$$

where K is the predefined spread. The purchaser of the option buys credit protection, or the right to put the bond to the seller if it falls in value. The payout formula could also be expressed directly in terms of prices, as in Equation (22.4).

Example

A credit spread option has a notional of \$100 million with a maturity of one year. The underlying security is an 8% 10-year bond issued by corporation XYZ. The current spread is 150bp against 10-year Treasuries. The option is European type with a strike of 160bp.

Assume that, at expiration, Treasury yields have moved from 6.5% to 6% and the credit spread has widened to 180bp. The price of an 8% coupon, nine-year semiannual bond discounted at $y + S = 6 + 1.8 = 7.8\%$ is \$101.276. The price of the same bond discounted at $y + K = 6 + 1.6 = 7.6\%$ is \$102.574. Using the notional amount, the payout is $(102.574 - 101.276)/100 \times \$100{,}000{,}000 = \$1{,}297{,}237$.

EXAMPLE 22.9: FRM EXAM 2005—QUESTION 14

Sylvia, a portfolio manager, established a Yankee bond portfolio. However, she wants to hedge the credit and interest rate risk of her portfolio. Which of the following derivatives will best fit Sylvia's need?

a. A total return swap
b. A credit default swap
c. A credit-spread option
d. A currency swap

EXAMPLE 22.10: FRM EXAM 2007—QUESTION 69

A bank holds USD 60 million of 10-year 6.5% coupon bonds that are trading at a clean price of USD 101.82. The bank is worried by the exposure due to these bonds but cannot unwind the position for fear of upsetting the client. Therefore, it purchases a total return swap (TRS) in which it receives annual LIBOR+100 bp in return for the marked-to-market return on the bond. For the first year, LIBOR sets at 6.25%, and by the end of the year the clean price of the bonds is at USD 99.35. The net receipt/payment for the bank in the total return swap will be to

a. Receive USD 1.93 million
b. Receive USD 2.23 million
c. Pay USD 2.23 million
d. Pay USD 1.93 million

EXAMPLE 22.11: FRM EXAM 2000—QUESTION 61

(Complex—use the valuation formula with prices) A credit-spread option has a notional amount of $50 million with a maturity of one year. The underlying security is a 10-year, semiannual bond with a 7% coupon and a $1,000 face value. The current spread is 120bp against 10-year Treasuries. The option is a European option with a strike of 130bp. If at expiration, Treasury yields have moved from 6% to 6.3% and the credit-spread has widened to 150bp, what will be the payout to the buyer of this credit-spread option?

a. $587,352
b. $611,893
c. $622,426
d. $639,023

22.4 STRUCTURED PRODUCTS

22.4.1 Creating Structured Products

Structured products generally can be defined as instruments created to meet specific needs of investors or borrowers that cannot be met with conventional financial instruments.

A typical example is retail demand for investments that participate in the appreciation of stock markets but also preserve capital. The payoff profile of the product can be replicated from a combination of existing, or sometimes new instruments. In this case, for example, the payoff can be replicated by an investment in a risk-free bond with notional equal to the guaranteed capital, plus long positions in a call option, either through direct investment in a portfolio of options or indirectly replicated through dynamic trading. This instrument is a **principal-protected note**, and can be indexed to a variety of markets, including equities, currencies, and commodities.

In recent years, the market for credit structured products has expanded enormously. The advent of credit derivatives has made possible a flurry of innovative products where payoffs are linked to credit events.

22.4.2 Credit-Linked Notes

Credit-linked notes (CLN) are structured securities that combine a credit derivative with a regular bond. In a CLN, the buyer of protection transfers credit risk to an investor via an intermediary bond-issuing entity. This entity can be the buyer itself or a special purpose vehicle.

An example of the first case is a bank with exposure to an emerging country, say Mexico. The bank issues a note with an embedded short position in a credit default swap on Mexico. The note is a liability on the bank's balance sheet. Investors receive a high coupon but will lose some of the principal if Mexico defaults on its debt. This structure achieves its goal of reducing the bank's exposure if Mexico defaults. In this case, because the note is a liability of the bank, the investor is exposed to a default of either Mexico or of the bank.

An example of the SPV structure is provided in Figure 22.3. In this case, the investor's initial funds are placed in a top-rated investment that pays LIBOR plus a spread of Y bp. The SPV takes a short position in a credit default swap, for an additional annual receipt of X bp. The annual payment to the investor is then LIBOR $+ Y + X$. In return for this higher yield, the investor must be willing to lose some of the principal should a default event occur.

Relative to a regular investment in, say, a note issued by the government of Mexico, this structure may carry a higher yield if the CDS spread is greater than the bond yield spread. This structure may also be attractive to investors who are precluded from investing directly in derivatives.

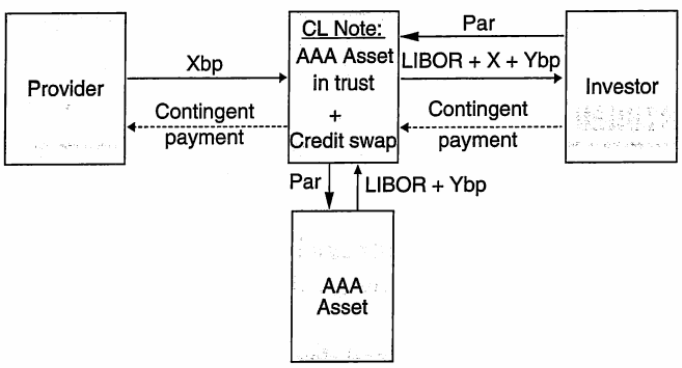

FIGURE 22.3 Credit-Linked Note

22.4.3 Collateralized Debt Obligations

Much of financial engineering is about repackaging financial instruments to make them more palatable to investors, creating value in the process. In the 1980s, **collateralized mortgage obligations** (CMOs) brought mortgage-backed securities to the masses by repackaging their cash flows into **tranches** with different characteristics.

The same magic is performed with **collateralized debt obligations** (CDOs), which are securities backed by a pool of debt. **Collateralized bond obligations** (CBOs) and **collateralized loan obligations** (CLOs) are backed by bonds and loans, respectively. Figure 22.4 illustrates a typical CDO structure.[6]

The first step is to place a package of corporate bonds in a **special-purpose vehicle** (SPV). Assume that we have a total of $1,000 million, representing exposures of $10 million to 100 entities. Multiple tranches are then issued by the SPV, with a specified **waterfall** structure, or priority of payments to the various tranches. Tranches are categorized as senior, mezzanine, and subordinated or equity. In the simplest structure, the SPV is ideally a passive entity. It redistributes cash flows according to well-defined rules. There is no need for other management action.

In this example, 80% of the capital structure is apportioned to tranche A, which has the highest credit rating of Aaa, using Moody's rating, or AAA. It pays LIBOR + 45bp, for example. Other tranches have lower priority and rating. These intermediate, *mezzanine*, tranches are typically rated A, Baa, Ba, or B (A, BBB, BB, B, using S&P's ratings). For instance, tranche C would absorb losses from 3% to 10%. These numbers are called, respectively, the **attachment point** and the **detachment point**.

At the bottom comes the equity tranche, which is not rated. Due to leverage, the return on the equity tranche can be very high if there is no default. In

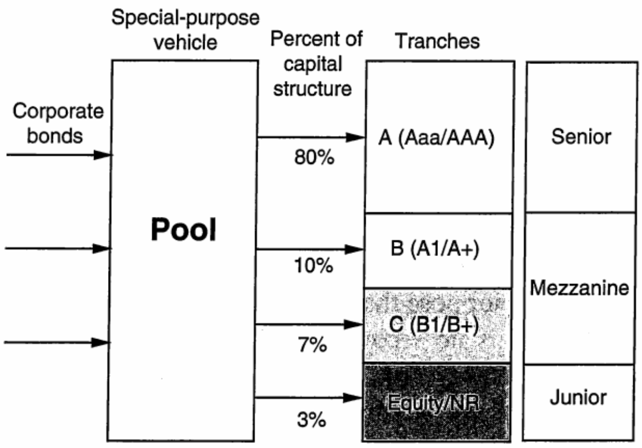

FIGURE 22.4 Collateralized Debt Obligation Structure

[6] This structure has similarities with the CLN structure. The differences are that CDOs are always issued by a SPV, involve a pool with a large number of underlying assets, and are usually tranched.

exchange, this is exposed to the first dollar loss in the portfolio. Special conventions apply to trading in equity tranches. The investor first pays the notional amount, which is \$30 million in this case. In exchange, this protection seller receives a spread, called running spread, and an up-front fee. This fee is quoted in percent and is typically around 40% for an investment-grade CDO. In this case, the investor would get $40\% \times \$30 = \$12$ million up front, and the running spread, say 500bp.

Cumulative losses of \$20 million would reduce the notional of the equity tranche to $\$30 - \$20 = \$10$ million. The running spread then applies to the new notional of \$10 million.

For losses amounting to \$45 million, the first tranche is wiped out, and investors in tranche C receive only $\$70 - \$15 = \$55$ million back. Thus, the rating enhancement for the senior classes is achieved through prioritizing the cash flows. Rating agencies have developed internal models to rate the senior tranches based on the probability of shortfalls due to defaults.

Whatever transformation is brought about, the resulting package must obey some basic laws of conservation. For the underlying and resulting securities, we must have the same cash flows at each point in time, apart from transaction costs. As a result, this implies (1) the same total market value, and (2) the same risk profile, both for interest rate and default risk. The weighted duration of the final package must equal that of the underlying securities. The expected default rate, averaged by market values, must be the same. So, if some tranches are less risky, others must bear more risk. Like CMOs, CDOs are structured so that most of the tranches have less risk than the collateral. Inevitably, the remaining **residual** tranche is more risky. This is sometimes called "toxic waste." If this residual is cheap enough, however, some investors should be willing to buy it. Oftentimes, the institution sponsoring the CDO will retain the most subordinate equity tranche to convince investors of the quality of the pool. Credit investors have developed sophisticated trading strategies that involve going long and short different tranches of these capital structures.

Example: Correlation Trading

Take a synthetic \$1 billion CDO with 100 names worth \$10 million each. Say that the equity tranche is \$30 million, which represents the first 3% of losses. For simplicity, suppose that all premiums are measured in net present value terms and that there is no recovery. The investor gets paid \$15 million up front for assuming the equity risk, so his worst net loss on the tranche is \$15 million. He then hedges by buying CDS on the same 100 names, with notional of \$3 million each. The present value of the spread is 2%, which gives a payment of $\$300 \times 2\% = \$6$ million. If there is no default, the principal is returned, and the net gain is $\$15 - \$6 = \$9$ million. If all 100 names default, the position loses the principal of the equity and gains the CDS payments, which gives $(\$15 - \$30) - \$6 + \$300 = \$279$ million. Of course, this is very unlikely.

We need to explore other scenarios that could generate losses. If only three names default, the equity tranche is wiped out. The investor then exercises three CDS contracts and unwinds the 97 remaining CDS hedges, which are no longer necessary. As a worst-case situation, suppose the CDS spreads have tightened and that the contracts are sold for \$4.8 million. This translates into a net loss of $(\$15 - \$30) - \$6 + (3 \times \$3) + \$4.8 = -\$7.2$ million.

EXAMPLE 22.12: FRM EXAM 2004—QUESTION 63

A CDO, consisting of three tranches, has an underlying portfolio of n corporate bonds with a total principal of USD N million. Tranche 1 has 10% of N and absorbs the first 10% of the default losses. Tranche 2 has 20% of N and absorbs the next 20% of default losses. The final tranche 3 has 70% of N and absorbs the residual default loss. Which of the following statements are true?

I. Tranche 2 has the highest yield.
II. Tranche 1 is usually called "toxic waste."
III. Tranche 3 would typically be rated as AAA by S&P.
IV. Tranche 3 has the lowest yield.

a. I only
b. IV only
c. II, III, and IV only
d. II and IV only

EXAMPLE 22.13: FRM EXAM 2001—QUESTION 12

A pool of high yield bonds is placed in a SPV and three tranches (including the equity tranche) of bonds are issued collateralized by the bonds to create a Collateralized Bond Obligation (CBO). Which of the following is true?

a. At fair value the value of the issued bonds should be less than the collateral.
b. At fair value the total default probability, weighted by size of issue, of the issued bonds should equal the default probability of the collateral pool.
c. The equity tranche of the CBO has the least risk of default.
d. The yield on the low risk tranche must be greater than the yield on the collateral pool.

EXAMPLE 22.14: FRM EXAM 2002—QUESTION 32

A CBO (Collateralized Bond Obligation) consists of several tranches of notes from a repackaging of corporate bonds, ranging from equity to super senior. Which of the following is generally true of these structures?

a. The total yield of all the CBO tranches is slightly less than the underlying repackaged bonds to allow the issuer to recover their fees/costs/profits.
b. The super senior tranche has expected loss rate higher than the junior tranche.
c. The super senior tranche is typically rated below AAA and sold to bond investors.
d. The equity tranche does not absorb the first losses of the structure.

EXAMPLE 22.15: FRM EXAM 2007—QUESTION 130

A three-year, credit-linked note (CLN) with underlying company Z has a LIBOR + 60bps semi-annual coupon. The face value of the CLN is USD 100. LIBOR is 5% for all maturities. The current three-year CDS spread for company Z is 90bps. The fair value of the CLN is closest to

a. USD 100.00
b. USD 111.05
c. USD 101.65
d. USD 99.19

22.5 CDO MARKET

22.5.1 Balance Sheet and Arbitrage CDOs

Table 22.3 describes the evolution of the CDO market from 2004 to 2008. From 2004 to 2006, the market doubled every year, reaching $521 billion in new issues during 2006. As a result of the credit crisis, however, issuances have spiraled down.

CDO transactions are typically classified by purpose, as balance sheet or arbitrage. The primary goal of **balance sheet CDOs** is to move loans off the balance sheet of commercial banks to lower regulatory capital requirements.

In contrast, **arbitrage CDOs** are designed to capture the spread between the portfolio of underlying securities and that of highly rated, overlying, tranches. Because CDO senior tranches should be relatively safe due to diversification effects,

TABLE 22.3 Evolution of the CDO Market Annual Issues ($ Millions)

Explanation	2004	2006	2008
By Type:			
Cash flow	119,531	410,504	40,321
Synthetic	37,237	66,503	1,191
Market value	650	43,638	14,581
By Purpose:			
Arbitrage	146,998	454,971	45,124
Balance sheet	10,420	65,674	10,967
Total	157,418	520,645	56,093

Source: Bond Market Association

they pay a tight spread over LIBOR. The arbitrage profit then goes into the equity tranche (but also into management and investment banking fees).

Senior tranches also seem attractive for investors. Generally, AA-rated corporate borrowers pay LIBOR. Higher credit pay rates below LIBOR. A typical AAA-rated senior CDO tranche, however, pays a higher rate than LIBOR. This explains why investors were attracted to this market, putting blind faith into the credit ratings. Some commentators wondered "whether this is a true arbitrage or involves some type of model risk." By now, it is clear that the models used by the credit rating agencies were flawed (see also Chapter 19).

22.5.2 Cash Flow and Synthetic CDOs

Credit risk transfer can be achieved by cash flow or synthetic structures. The example in Figure 22.4 is typical of traditional, or *funded*, **cash flow CDOs**. The physical assets are sold to a SPV and the underlying cash flows used to back payments to the issued notes.

In contrast, the credit risk exposure of **synthetic CDOs** is achieved with credit default swaps. We know that a long position in a defaultable bond is equivalent to a long position in a default-free bond plus a short position in a CDS. Synthetic CDOs create higher yields by first, funding or placing the initial investment in default-free, or Treasury, securities, and second, selling a group of CDSs to replicate a cash flow CDO.

Synthetic CDOs offer several advantages. First, they are easier to manage than cash flow CDOs. In case of bankruptcy of one of the underlying credits, the management of a cash flow SPV has to take part in the bankruptcy process. With a short CDS position that is cash settled, there is no need for the SPV to get involved with the bankruptcy process. Second, the issue does not need to be fully funded. In a **full capital structure CDO**, the total notional amount of notes issued is equal to the total notional amount of the underlying portfolio. So, it is fully funded. In contrast, a **single tranche CDO** is a **bespoke** transaction where the bank and the investor agree on the terms of a deal including size, credit rating, and underlying credits.[7] Effectively, the bank holds the rest of the capital structure and does not place it.

Take as an example a \$10 million tranche rated A+ paying a coupon of six-month LIBOR plus 111bp, on a reference portfolio of \$1,000 million of 100 North American investment-grade entities. The attachment point is 5%. The detachment point is 6%. The investor will receive the promised payments as long as cumulative losses in the reference portfolio remain below 5%. Above that, the investor will have to take a loss. For example, if cumulative dollar losses on the portfolio amount to \$52 million, or 5.2% of total, the investor will lose $(5.2 - 5.0)/(6.0 - 5.0) = 20\%$ of his capital, which is \$2 million in this case. The coupon then applies to the reduced notional amount. A bank can sell such structure to an investor without fully funding it.

22.5.3 Cash Flow and Market Value CDOs

In the case of cash flow CDOs, payments to investors solely come from collateral cash flows. In contrast, with **market value CDOs**, payments are made from collateral cash flows as well as sales of collateral. If the market value of the collateral falls below some level, payments to the equity tranche are suspended. This creates more flexibility for the portfolio manager.

Credit rating agencies analyze the quality of credit structures using **overcollateralization ratios** (OC). This ratio measures how many times the collateral can cover the SPV liabilities. For a market value CDO, define V as the market value of assets and D as the par value of liabilities. The OC ratio is then defined as:

$$\mathrm{OC} = \frac{V}{D} \tag{22.6}$$

This must be high enough to ensure sufficient coverage of liabilities.

Alternatively, the par value of cumulative tranches, starting from the top, must be kept below the market value of assets times an **advance rate**. In our CDO example, the notional for the first tranche is \$800 million. This must be kept below the value of assets, say \$1,000 million, times an advance rate of 85%. Because $\$800 < \$850$, the first tranche in the structure passes the test. For the first two tranches, the advance rate is 95%, and the test gives $\$800 + \$100 < \$950$, so this passes the test as well. A structure that fails the overcolleralization test risks downgrading. Such failure can be cured by selling some of the assets and repaying some of the tranches or issuing more equity.

For a cash flow CDO, the ratio uses the par value of total assets in the numerator. Another ratio, the **interest rate coverage ratio** (IC), is also used to assess the quality of a credit structure. This is computed as the total interest payment to be received by the collateral divided by the interest liability of each tranche and above.

[7] The term "bespoke" was originally used to describe clothing made to customer's specification. The term comes from the word *bespeak*, meaning to ask for or order something.