22.5.4 Static and Managed CDOs

Finally, CDOs differ in the management of the asset pool. In **static CDOs**, the asset pool is basically fixed. In contrast, with **managed CDOs**, a portfolio manager is allowed to trade actively the underlying assets.

This has all the usual benefits and disadvantages of active management. One benefit is the ability to unwind assets with decreasing credit quality, to buy undervalued securities, and to sell overvalued securities. With managed CDOs, investors face credit risk as well as poor management risk, however. In addition, they pay management fees.

22.5.5 Other Products

As the market for CDOs has expanded, new products have appeared. For instance, a CDO can invest in CDO tranches instead of individual credits. This is a **CDO-squared** structure. The main benefit of this structure is the greater degree of diversity. A typical single-layer CDO references 50 to 100 corporate credits. A CDO-squared has five to 10 one-layer CDOs, and is thus exposed to 250 to 1,000 names. There was even talk of a further structure, called CDO-cubed.

The market now also trades credit default swaps on asset-backed securities (ABS) tranches, called **ABCDS**. Most commonly, the assets are backed by home equity and commercial property loans. In the past, it was difficult to short such ABSs. Buying an ABCDS is equivalent to acquiring protection, or shorting the security. This opens up new possibilities to implement relative value trades or to hedge this type of risk. The market has developed rapidly thanks to standardized ISDA documentation as well as the establishment of a benchmark index, the **ABX index**, which contains 20 home equity securities.

ABCDS are complex instruments. A corporate CDS makes a payment if the underlying company suffers a credit event. In contrast, with an ABS, the issuing SPV cannot go bankrupt but defaults can occur for individual loans in the pool. Also, the notional amount is not fixed but amortizes over time as principal is paid back on the loans.

These ABCDSs are similar to **CDSs on CDOs**, which are credit default swaps on CDO tranches, usually the senior ones. These instruments provide an efficient way to short sell the market. Dealers who are arranging cash CDOs can buy the CDSs to hedge their exposure, for example.

Another recent innovation is the **constant proportional debt obligation** (CPDO). CPDOs offer protection on a portfolio of corporate credits such as the iTraxx European CDS index. The transaction is highly levered and dynamically adjusted, getting rid of the credits that deteriorate over time and changing the leverage as spreads vary. This creates a structure that is rated AAA yet pays LIBOR plus 200bp. These new instruments looked very attractive in a benign environment of stable or falling credit spreads. For risk managers, however, their risk profile is difficult to assess due to their dynamic nature. The credit crisis caused considerable losses to many CPDOs, some of which have already defaulted.

EXAMPLE 22.16: FRM EXAM 2003—QUESTION 7

A standard synthetic CDO references a portfolio of 10 corporate names. Assume the following. The total reference notional is X, and the term is Y years. The reference notional per individual reference credit name is $X/10$. The default correlations between the individual credit names are all equal to one. The single-name CDS spread for each individual name is 100 bp, for a term of Y years. The assumed recovery rate on default for all individual reference credits is zero in all cases. The synthetic CDO comprises two tranches, a 50% junior tranche priced at a spread J, and a 50% senior tranche priced at spread S. All else constant, if the default correlations between the individual reference credit names are reduced from 1.0 to 0.7, what is the effect on the relationship between the junior tranche spread J and the senior tranche spread S?

a. The relationship remains the same.
b. S increases relative to J.
c. J increases relative to S.
d. The effect cannot be determined given the data supplied.

EXAMPLE 22.17: FRM EXAM 2007—QUESTION 81

A bank is considering buying (i.e., selling protection on) an AAA-rated super senior tranche [10% – 11%] of a synthetic collateralized debt obligation (CDO) referencing an investment-grade portfolio. The pricing of the tranche assumes a fixed recovery of 40% for all names. All else being equal, which one of the following four changes will make the principal invested more risky?

a. An increase in subordination of 1%, i.e., investing in the [11% – 12%] tranche
b. An increase in the tranche thickness from 1% to 3%, i.e., investing in the [10% – 13%] tranche
c. Using a recovery rate assumption of 50%
d. An increase in default correlation between names in the portfolio

EXAMPLE 22.18: FRM EXAM 2007—QUESTION 10

Consider the following homogeneous reference portfolio in a synthetic CDO: Number of reference entities, 100; CDS spread, $s = 150$bps; recovery rate $f = 50\%$. Assume that defaults are independent. On a single name the annual default probability is constant over five years and obeys the relation: $s = (1 - f)\text{PD}$. What is the expected number of defaulting entities over the next five years, and which of the following tranches would be entirely wiped out (loses 100% of the principal invested) by the expected number of defaulting entities?

a. 14 defaults and a [3% – 14%] tranche would be wiped out.
b. Three defaults and a [0% – 1%] tranche would be wiped out.
c. Seven defaults and a [2% – 3%] tranche would be wiped out.
d. 14 defaults and a [6% – 7%] tranche would be wiped out.

22.6 CONCLUSIONS

Credit products are by far the fastest growing segment of financial derivatives. Credit default swaps have become mainstream products, and now are actively traded for a large variety of names.

The rapid growth of the credit derivatives market is the best testimony of their usefulness. These instruments are superior risk management tools, allowing the *transfer of risks*. Table 22.4 breaks down the market by participants, taken from the BBA survey. a survey by the British Bankerss Association (BBA).

Banks are net buyers of credit protection, which is a hedge against their lending business. This helps explain why banks weathered the 2001 recession rather well, in spite of large corporate (WorldCom and Enron) and sovereign (Argentina) defaults. Most of these bank exposures had been sold. On the other side are insurance companies, which are net sellers of credit protection. This is akin to selling insurance.

TABLE 22.4 Buyers and Sellers of Credit Protection

Type of Institution	Percentage		
	Buyer	Seller	Net
Banks	59	44	+15
Insurers	6	17	−11
Hedge funds	28	32	−4
Others	7	7	0
Total	100	100	0

Source: BBA Credit Derivatives Report 2006

On the other hand, structured products have spread risk all over the world, which has created contagion effects when subprime-backed assets started to go bad. In addition, insurance companies such as AIG have ended up selling too much protection. Because of its size, a failure by AIG would have probably caused systemic risk (the case of AIG is discussed in Chapter 25).

Standard credit derivatives such as credit default swaps, however, have many benefits. During 2007 and 2008, this market has remained fairly liquid unlike the cash bond markets. This CDS market creates transaction prices that provide useful information about the cost of credit to outside observers. In other words, they provide *price discovery*. CDS contracts also allow *transactional efficiency*, because they have lower transaction costs than the cash markets.

On the downside, the growth of this market has created *operational risk* because of backlogs in the processing of trades. Regulators have pushed the industry to improve the operational infrastructures, including more automated trade processing.

In addition, *counterparty risk* has become an issue in the wake of Lehman's failure. Lehman was a major player in the CDS market. This explains the push for a *centralized clearinghouse*. As in the case of the CLS Bank discussed in Chapter 18, this would allow multilateral netting of contracts and generally decrease counterparty risk. This would also make trading more transparent and, for some contracts, more liquid.

Credit derivatives also introduce a new element of risk, which is *legal risk*. Parties may not agree on the terms of the trade in case of default. Even with full confirmation of the trade, parties sometimes squabble over the definition of a credit event. Such disagreement occurred during the Russian default as well as notable debt restructurings and demergers. The widespread use of ISDA confirmation agreements helps resolve some of this uncertainty.

Overall, credit default swaps are likely to continue to thrive because they provide many benefits to financial market participants. In contrast, the future of complex credit securitizations is more clouded. This market has evolved from *regulatory arbitrage*—that is, attempts to defeat capital requirements by laying off loan credit risk through securitizations. As discussed in Chapter 7, the recent credit crisis has revealed serious flaws in the securitization process, which created complex instruments. Evaluating these complex structures requires sophisticated portfolio credit risk models, which are covered in the next chapter. Indeed the losses suffered on many of these structures were the trigger for the credit crisis that started in 2007.

22.7 IMPORTANT FORMULAS

Payoff on a credit default swap:

$$\text{Payment} = \text{Notional} \times Q \times I(\text{CE})$$

Payoff on a credit spread forward contract:

$$\text{Payment} = (S - F) \times \text{MD} \times \text{Notional}$$

$$\text{Payment} = [P(y + F, \tau) - P(y + S, \tau)] \times \text{Notional}$$

Valuation of a CDS contract:

$$V = (\text{PV Payoff}) - s(\text{PV Spread}) = \left(\sum_{t=1}^{T} k_t(1-f)\text{PV}_t\right) - s\left(\sum_{t=1}^{T} S_{t-1}\text{PV}_t\right)$$

22.8 ANSWERS TO CHAPTER EXAMPLES

Example 22.1: FRM Exam 2004—Question 9

d. A long corporate bond position is equivalent to a long Treasury bond position plus a short CDS.

Example 22.2: FRM Exam 2007—Question 18

c. The bond yield spread is $15 - 3 = 12\%$. So, the trader could buy the corporate bond and hedge the interest rate risk by shorting the Treasury. To protect against default risk, he should buy a CDS on the same obligor at a spread of y. The total profit must be $12\% - y = 5\%$. Hence, the CDS spread must be 7%.

Example 22.3: FRM Exam 2007—Question 120

b. The bank should buy the swap to protect against default. The quarterly payment will be $\$10\text{M} \times 0.50\%/4 = \$12{,}500$.

Example 22.4: FRM Exam 1999—Question 135

a. Because all bonds rank equally, all defaults occur at the same time and have the same loss given default. Therefore the cash flow on the one-year credit swap can be replicated (including any risk premium) by going long the one-year Widget bond and short the 1-year T-bond.

Example 22.5: FRM Exam 2004—Question 50

a. The payment is $200 \times 113 - 300 \times 27 - 100 \times 43$, which translates into \$1.02 million.

Example 22.6: FRM Exam 2004—Question 65

d. The protection buyer is exposed to the joint risk of default by the counterparty and underlying credit. If only one defaults, there is no credit risk.

Example 22.7: FRM Exam 2007—Question 85

d. For a loss to occur, both bank B and company R must default. The joint probability of default by B and R is 0.5% times 3.6%, which gives 0.018%.

Example 22.8: FRM Exam 2005—Question 111

c. If bank B buys company C, the two entities B and C will default at the same time. This increase in the default correlation makes the CDS contract less valuable. In Table 22.2, the fair CDS spread decreases when the correlation increases. Given that the existing CDS contract has a fixed spread, this event should decrease the value of the outstanding contract.

Example 22.9: FRM Exam 2005—Question 14

a. A TRS will provide protection against both interest rate and credit risk, as it is indexed to the bond portfolio value. A CDS or CS option only provide protection against credit risk. There is no currency risk in Yankee bonds, which are denominated in dollars, anyway.

Example 22.10: FRM Exam 2007—Question 69

a. On the LIBOR leg, the bank receives 7.25. In exchange, it pays the return on the bond, which is the coupon of 6.5% plus the relative return of $(99.35 - 101.82)/100 = -2.47\%$. This gives a receipt of $\$60 \times (7.25 - 6.5 + 2.47)/100 = 1.932$.

Example 22.11: FRM Exam 2000—Question 61

c. We need to value the bond with remaining semiannual payments for nine years using two yields, $y + S = 6.30 + 1.50 = 7.80\%$ and $y + K = 6.30 + 1.30 = 7.60\%$. This gives \$948.95 and \$961.40, respectively. The total payout is then $\$50{,}000{,}000 \times [\$961.40 - \$948.95]/\$1{,}000 = \$622{,}424$.

Example 22.12: FRM Exam 2004—Question 63

c. The equity tranche, tranche 1, must have the highest yield, and is sometimes called "toxic waste" because it has the highest risk. Conversely, tranche 3 would have the highest credit rating and the lowest yield.

Example 22.13: FRM Exam 2001—Question 12

b. The market values and weighted probability of default should be equal for the collateral and various tranches. So, a. is wrong. The equity tranche has the highest risk of default, so c. is wrong. The yield on the low-risk tranche must be the lowest, so d. is wrong.

Example 22.14: FRM Exam 2002—Question 32

a. In the absence of transaction costs or fees, the yield on the underlying portfolio should be equal to the weighted average of the yields on the different tranches.

With costs, however, the CBO yield will be slightly less. Otherwise, the senior tranche is typically rated AAA, has the lowest loss rate of all tranches, and absorbs the last loss on the structure.

Example 22.15: FRM Exam 2007—Question 130

d. Because the current CDS spread is greater than the coupon, the CLN must be selling at a discount. The only solution is d. More precisely, we can use the spread duration from Equation (22.2), which is the sum of the present value factor over three years. Assuming a flat term structure, this is $\sum \mathrm{PV}_t = 0.952 + 0.907 + 0.864 = 2.72$ years. Multiplying by $(90 - 60) = 30$bp gives a fall of 0.81%, which gives \$99.19.

Example 22.16: FRM Exam 2003—Question 7

c. If the correlation is one, all names will default at the same time, and the junior and senior tranche will be equally affected. Hence, their spread should be 100bp, which is the same as for the collateral. With lower correlations, the losses will be absorbed first by the junior tranche. Therefore, the spread on the junior tranche should be higher, which is offset by a lower spread for the senior tranches.

Example 22.17: FRM Exam 2007—Question 81

d. Increasing the subordination will make the senior tranche less risky because there is a thicker layer beneath to absorb losses. Increasing the thickness of the tranche will make it less likely to be wiped out, so is less risky. An increase in the default correlation will increase the risk. In the limit, if all assets default at the same time, all tranches will suffer a loss.

Example 22.18: FRM Exam 2007—Question 10

d. The annual marginal PD is $d = 1.5\%/(1 - 0.50) = 3.00\%$. Hence, the cumulative PD for the five years is $d + S_1 d + S_2 d + S_3 d + S_4 d = 3\%(1 + 0.970 + 0.941 + 0.913 + 0.885) = 14.1\%$, where the survival rates are $S_1 = (1 - 3\%) = 0.970$, $S_2 = S_1(1 - 3\% = 0.941$, and so on. The expected number of defaults is therefore $100 \times 14.1\%$, or 14. With a recovery rate of 50%, the expected loss is 7% of the notional. So, all the tranches up to the 7% point are wiped out.

CHAPTER 23

Managing Credit Risk

Previous chapters have explained how to estimate the various input to portfolio credit risk models, including default probabilities, credit exposures, and recovery rates for individual credits. We now turn to the measurement of credit risk for the overall portfolio.

In the past, credit risk was measured on a standalone basis, in terms of a "yes" or "no" decision by a credit officer. Some consideration was given to portfolio effects through very crude credit limits at the overall level. Portfolio theory, however, teaches us that risk should be viewed in the context of the contribution to the total risk of a portfolio, not in isolation. The new credit risk models measure risk on a portfolio basis.

While this fundamental focus on diversification also exists in market risk, credit risk is markedly more complex. In particular, it is difficult to estimate probabilities and correlations of default events. These correlations, however, are essential drivers of diversification benefits.

Section 23.1 introduces the distribution of credit losses. This has two major features. The first is the expected credit loss, which is essential information for pricing and reserving purposes, as explained in Section 23.2. The second component is the unexpected credit loss, or worst deviation from the expected loss at some confidence level. Section 23.3 shows how this credit value at risk (CVAR), like market VAR, can be used to determine the amount of capital necessary to support a position. Section 23.4 then provides an overview of recently developed credit risk models, including CreditMetrics, CreditRisk+, the KMV model, and Credit Portfolio View. Finally, Section 23.5 gives some concluding comments. Given the complexity of these models, it is essential for risk managers to understand their weak spots and limitations.

23.1 MEASURING THE DISTRIBUTION OF CREDIT LOSSES

23.1.1 Steps

The previous chapters provided a detailed analysis of the various components of credit models, which include default probabilities, credit exposures, and recovery rates. We can now pool this information to measure the distribution of losses due to credit risk. For simplicity, we initially consider only losses in **default mode** (DM), that is, losses due to defaults instead of changes in market values.

For one instrument, the potential credit loss is

$$\text{Credit loss} = b \times \text{CE} \times \text{LGD} \tag{23.1}$$

which involves the random variable b that takes on the value of 1 when the discrete state of default occurs, with probability of default (PD) p; the credit exposure, also called exposure at default (EAD); and the loss given default (LGD). With this definition, the credit loss is positive.

For a portfolio of N counterparties, the credit loss (CL) is

$$\text{CL} = \sum_{i=1}^{N} b_i \times \text{CE}_i \times \text{LGD}_i \tag{23.2}$$

where CE_i is now the total credit exposure to counterparty i, across all contracts and taking into account netting agreements.

The distribution of credit loss is quite complex. Typically, information about credit risk is described by the **net replacement value** (NRV), which is

$$\text{NRV} = \sum_{i=1}^{N} \text{CE}_i \tag{23.3}$$

evaluated at the current time. This is the most that could be lost if all parties defaulted at the same time ($b_i = 1$) and if there was no recovery ($\text{LGD}_i = 1$). This is not very informative, however. The NRV, which is often disclosed in annual reports, is equivalent to using notionals to describe the risks of derivatives portfolios. It does not take into account the probability of default or correlations across defaults and exposures.

Chapter 18 gave an example of a loss distribution for a simple portfolio with three counterparties. This example was tractable, as we could enumerate all possible states. In general, we need to consider many more credit events. We also need to account for movements and co-movements in risk factors, which drive exposures, uncertain recovery rates, and correlations among defaults. This can be done with the help of *Monte Carlo simulations*. Once this is performed for the entire portfolio, we obtain a distribution of credit losses on a target date. Figure 23.1 describes a typical distribution of credit profits and losses. A later section will illustrate the construction of this distribution as provided by commercial models.

We note that the distribution of credit P&L is *highly skewed to the left*, in contrast to that of market risk factors, which is in general roughly symmetrical. This credit distribution is similar to a short position in an option. This is one of the essential insights of the Merton model, which equates a risky bond to a risk-free bond plus a short position in an option.

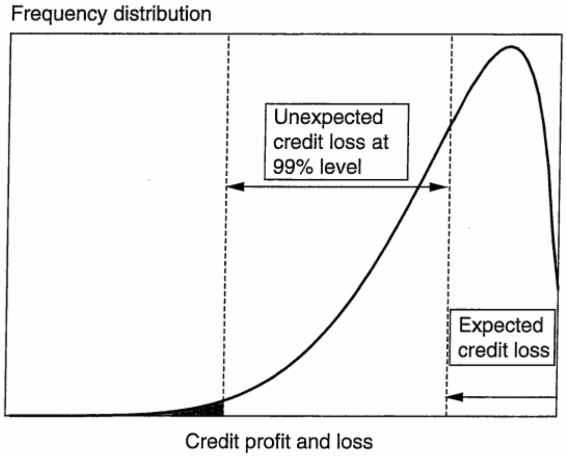

FIGURE 23.1 Distribution of Credit Losses

23.1.2 Major Features

This distribution can be described by:

- **Expected credit loss** (ECL). The **expected credit loss** represents the average credit loss. The *pricing* of the portfolio should be such that it covers the expected loss. In other words, the price should be advantageous enough to offset average credit losses. In the case of a bond, the price should be low enough, or the yield high enough, to compensate for expected losses. In the case of a derivative, the bank that takes on the credit risk should factor the expected loss into the pricing of its product. Loan loss reserves should be accumulated as a **credit provision** against expected losses. Focusing only on the default variables, the ECL depends solely on default probabilities.
- **Unexpected credit loss** (UCL). The **worst credit loss** represents the loss that will not be exceeded at some level of confidence, typically 99.9%. This is basically the quantile of the distribution. Taking the deviation from the expected loss gives the **unexpected credit loss**. The institution should have enough equity capital to cover the unexpected loss. Focusing only on the default variables, the UCL depends both on default probabilities and default correlations.

23.1.3 Effect of Correlations

The key to this approach is measuring risk at the top, portfolio level. This approach can also reveal the effect of correlations between and across risk types.

At the top of the list are the correlations across default events b_i. With low correlations and many obligors, the distribution will be narrow, as illustrated in Chapter 18. In this case, a bank could leverage up its equity several times. In its simplest version, the Basel Accord requires a minimum ratio of equity to assets of 8%, implying a maximum leverage ratio of 12.5. In contrast, high correlations will lead to simultaneous defaults, which extend the tail of the distribution and increase the UCL. In the limit, with perfect correlations, the worst loss at a fixed confidence level is the entire notional amount of the portfolio. In this case, the bank cannot have any leverage. Its assets must be covered by the same amount of equity.

Correlation can also occur across the default event and exposure, i.e., between b_i and CE. For example, **wrong-way trades** are positions where the exposure is positively correlated with the probability of default. Before the Asian crisis, for instance, many U.S. banks had lent to Asian companies in dollars, or entered equivalent swaps. Many of these Asian companies did not have dollar revenues but instead were speculating, reinvesting the funds in the local currency. When currencies devalued, the positions were in-the-money for the U.S. banks, but could not be collected because the counterparties had defaulted. Conversely, **right-way trades** occur when the transaction is a *hedge* for the counterparty—for instance, when a loss on its side of the trade offsets an operating gain.

KEY CONCEPT

Credit risk is lowered for right-way trades, where the counterparty is using the trade as a hedge. Conversely, wrong-way trades create a positive correlation between the credit exposure and the probability of default.

Another pernicious correlation is between the default event and the loss given default, i.e., between b_i and LGD. Figure 23.2 plots the recovery rate from Moody's over the period 1982 to 2007 against the speculative-grade default rate during that year. During recession years, such as 1990, 1991, and 2001, the recovery rate on unsecured senior bonds was markedly lower than during other years. The default rate was also very high during these years. This effect will extend the tail of the credit loss distribution. If ignored, the credit VAR measure will underestimate risk.

EXAMPLE 23.1: CREDIT PROVISIONS

Credit provisions should be taken to cover all of the following *except*

a. Nonperforming loans
b. The expected loss of a loan portfolio
c. An amount equal to the VAR of the credit portfolio
d. Excess credit profits earned during below-average-loss years

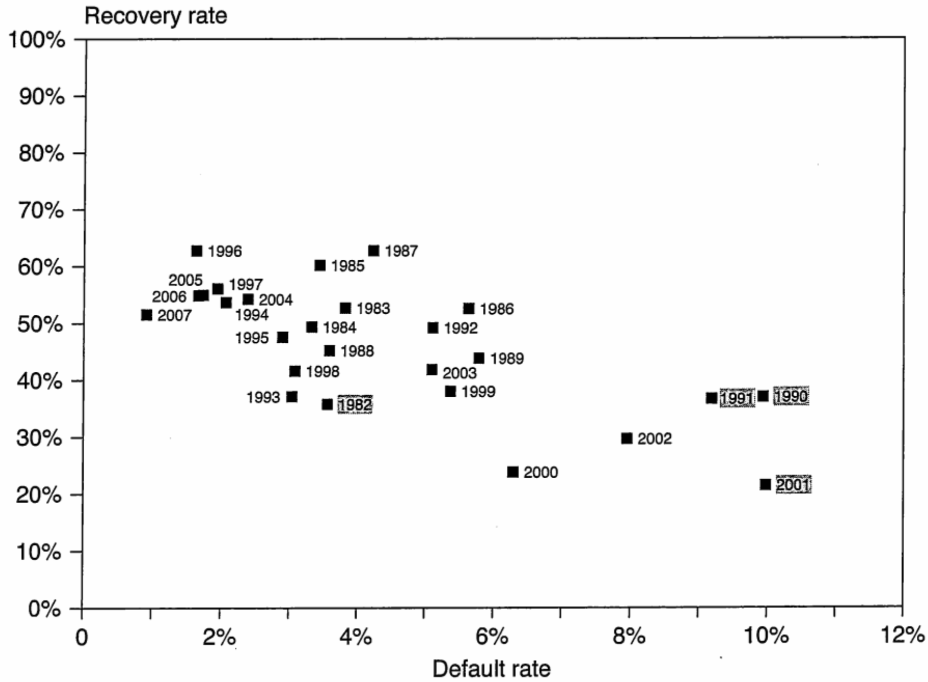

FIGURE 23.2 Recovery Rates and Default Rates

EXAMPLE 23.2: FRM EXAM 2002—QUESTION 74

Following is a set of identical transactions. Assuming all counterparties have the same credit rating, which transaction should preferably be executed?

a. Buying gas from a trading firm
b. Buying gas from a gas producer
c. Buying gas from a distributor
d. Indifferent between a., b., and c.

23.2 MEASURING EXPECTED CREDIT LOSS

23.2.1 Expected Loss over a Target Horizon

For pricing purposes, we need to measure the expected credit loss, which is

$$E[\mathrm{CL}] = \int f(b, \mathrm{CE}, \mathrm{LGD})(b \times \mathrm{CE} \times \mathrm{LGD})\, db\, d\mathrm{CE}\, d\mathrm{LGD} \qquad (23.4)$$

If the random variables are independent, the joint density reduces to the product of densities. We have

$$E[\mathrm{CL}] = \left[\int f(b)(b)\,db\right]\left[\int f(\mathrm{CE})(\mathrm{CE})\,d\mathrm{CE}\right]\left[\int f(\mathrm{LGD})(\mathrm{LGD})\,d\mathrm{LGD}\right] \tag{23.5}$$

which is the product of the expected values. In other words,

$$\text{Expected credit loss} = \text{Prob[default]} \times E[\text{Credit exposure}] \times E[\text{LGD}] \tag{23.6}$$

As an example, the actuarial expected credit loss on a BBB-rated \$100 million five-year bond with 47% recovery rate is $E[\mathrm{CL}] = 2.28\% \times \$100{,}000{,}000 \times (1 - 47\%) = \$1.2$ million. Note that this expected loss is the same whether the bank has one \$100 million exposure or 100 exposures worth \$1 million each. The distributions, however, will be very different with more credits.

23.2.2 The Time Profile of Expected Loss

So far, we have focused on a fixed horizon, say a year. For pricing purposes, however, we need to consider the total credit loss over the life of the asset. This should involve the time profile of the exposure, the probability of default, and the discounting factor. Define PV_t as the present value of a dollar paid at time t.

The **present value of expected credit losses** (PVECL) is obtained as the sum of the discounted expected credit losses at each time step:

$$\mathrm{PVECL} = \sum_t E\left[\mathrm{CL}_t\right] \times \mathrm{PV}_t = \sum_t \left[k_t \times \mathrm{ECE}_t \times (1-f)\right] \times \mathrm{PV}_t \tag{23.7}$$

where the probability of default is $k_t = S_{t-1}\, d_t$, or the probability of defaulting at time t, conditional on not having defaulted before.

Alternatively, we could simplify by using the average default probability and average exposure over the life of the asset:

$$\mathrm{PVECL}_A = \mathrm{Ave}[k_t] \times \mathrm{Ave}[\mathrm{ECE}_t] \times (1-f) \times \left[\sum_t \mathrm{PV}_t\right] \tag{23.8}$$

This approach, however, may not be as good an approximation when default risk and exposure profile are correlated over time. For example, currency swaps with highly rated counterparties have an exposure and a default probability that increase with time. Due to this correlation, taking the product of the averages understates credit risk. In other cases, it could overstate credit risk.

An even simpler approach, when ECE is constant, considers the final maturity T only, using the cumulative default rate c_T and the discount factor PV_T:

$$\mathrm{PVECL}_F = c_T \times \mathrm{ECE} \times (1-f) \times \mathrm{PV}_T \tag{23.9}$$

TABLE 23.1 Computation of Expected Credit Loss for a Swap

	P(default) (%)			Exposure	LGD	Discount	Total
Year t	c_t	d_t	k_t	ECE_t	$(1 - f)$	PV_t	$PVECL_t$
1	0.22	0.220	0.220	$1,660,000	0.55	0.9434	$1,895
2	0.54	0.321	0.320	$1,497,000	0.55	0.8900	$2,345
3	0.88	0.342	0.340	$1,069,000	0.55	0.8396	$1,678
4	1.55	0.676	0.670	$554,000	0.55	0.7921	$1,617
5	2.28	0.741	0.730	$0	0.55	0.7473	$0
Total			2.280			4.2124	$7,535
Average			0.456	$956,000			
$PVECL_A$			0.456	$956,000	0.55	4.2124	= $10,100

23.2.3 Examples

Table 23.1 shows how to compute the PVECL. We consider a five-year interest rate swap with a counterparty initially rated BBB and a notional of $100 million. The discount factor is 6% and the recovery rate 45%. We also assume that default can occur only at the end of each year. This analysis is similar to that for a credit default swap in Chapter 22. For simplicity, we use here real-world default probabilities.

In the first column, we have the cumulative default probability, c_t, for a BBB-rated credit from years 1 to 5, expressed as a percentage. The second column shows the marginal probability of defaulting during that year, d_t, and the third column shows the probability of defaulting in each year, conditional on not having defaulted before, $k_t = S_{t-1}d_t$. The fourth column reports the end-of-year expected credit exposure, ECE_t. The fifth column shows the constant LGD. The sixth column displays the present value factor, PV_t.

The final column gives the product $[k_t ECE_t(1 - f)PV_t]$. The first entry, for example is $0.220\% \times \$1{,}660{,}000 \times 0.55 \times 0.9434 = \$1{,}895$. Summing across years gives $7,535 on a swap with a notional of $100 million, or 0.007% of principal. This is very small, less than 1 basis point. So the expected credit loss on an interest rate swap is minuscule. Basically, the expected credit loss is very low due to the small exposure profile. For a regular bond or currency swap, the expected loss is much greater.

The last line shows a shortcut to the measurement of expected credit losses based on averages, from Equation (23.8). The average annual default probability is 0.456. Multiplying by the average exposure, $956,000, the LGD, and the sum of the discount rates gives $10,100. This is on the same order of magnitude as the exact calculation.

Table 23.2 details the computation for a bond assuming a constant exposure of $100 million. The expected credit loss is $1.020 million, about 100 times larger than for the swap. This is because the exposure is also about 100 times larger.

As in the previous table, the next line shows results based on averages. Here the expected credit loss is $1.056 million, very close to the exact number, as there is no variation in credit exposures over time.

TABLE 23.2 Computation of Expected Credit Loss for a Bond

	P(default) (%)			Exposure	LGD	Discount	Total
Year t	c_t	d_t	k_t	ECE_t	$(1 - f)$	PV_t	PVECL_t
1	0.22	0.220	0.220	\$100,000,000	0.55	0.9434	\$114,151
2	0.54	0.321	0.320	\$100,000,000	0.55	0.8900	\$156,639
3	0.88	0.342	0.340	\$100,000,000	0.55	0.8396	\$157,009
4	1.55	0.676	0.670	\$100,000,000	0.55	0.7921	\$291,887
5	2.28	0.741	0.730	\$100,000,000	0.55	0.7473	\$300,024
Total			2.280			4.2124	\$1,019,710
Average			0.456	\$100,000,000			
PVECL_A			0.456	×\$100,000,000	×0.55	×4.2124	= \$1,056,461
PVECL_F	2.280			×\$100,000,000	×0.55	×0.7473	= \$937,062

We could also take the usual shortcut and simply compute an expected credit loss given by the cumulative five-year default rate times \$100 million times the loss given default, which is \$1.254 million. Discounting to the present, we get \$0.937 million, close to the previous result.

EXAMPLE 23.3: FRM EXAM 2003—QUESTION 26

Which of the following loans has the lowest credit risk?

Loan	1 Year Probability of Default	Loss Given Default	Remaining Term (Months)
a.	1.99%	60%	3
b.	0.90%	70%	9
c.	1.00%	75%	6
d.	0.75%	50%	12

EXAMPLE 23.4: FRM EXAM 2007—QUESTION 38

Mr. Rosenquist, asset manager, holds a portfolio of SEK 200 million, which consists of BBB-rated bonds. Assume that the one-year probability of default is 4%, the recovery rate is 60%, and defaults are uncorrelated over the years. What is the two-year cumulative expected credit loss on Mr. Rosenquist's portfolio?

a. SEK 6.40 million
b. SEK 6.27 million
c. SEK 9.60 million
d. SEK 9.48 million

23.3 MEASURING CREDIT VAR

23.3.1 Credit VAR over a Target Horizon

Credit VAR is defined as the unexpected credit loss at some confidence level. Using the measure of credit loss in Equation (23.1), we construct a distribution of the credit loss $f(\mathrm{CL})$ over a target horizon. At a given confidence c, the worst credit loss (WCL) is defined such that

$$1 - c = \int_{\mathrm{WCL}}^{\infty} f(x)dx \tag{23.10}$$

The credit VAR is then measured as the deviation from ECL

$$\mathrm{CVAR} = \mathrm{UCL} = \mathrm{WCL} - \mathrm{ECL} \tag{23.11}$$

where all losses are defined as positive numbers.

This CVAR number should be viewed as the economic capital to be held as a buffer against *unexpected* losses. Its application is fundamentally different from the *expected* credit loss, which is additive across obligors and can be aggregated over time.

Instead, the CVAR is measured over a target horizon, say one year, which is deemed sufficient for the bank to take corrective actions should credit problems start to develop. Corrective action can take the form of exposure, reduction, or adjustment of economic capital, all of which take considerably longer than the typical horizon for market risk.

23.3.2 Using CVAR to Manage the Portfolio

Once credit VAR is measured, it can be managed. The portfolio manager can examine the trades that contribute most to CVAR. If these trades are not particularly profitable, they should be eliminated.

The **marginal contribution to risk** can also be used to analyze the incremental effect of a proposed trade on the total portfolio risk. As in the case of market risk, individual credits should be evaluated on the basis not only of their standalone risk, but also of their contribution to the portfolio risk. For the same expected return, a trade that lowers risk should be preferable over one that adds to the portfolio risk. Such trade-offs can be made only with a formal measurement of portfolio credit risk, however.

This marginal analysis can also help to establish the **remuneration of capital** required to support the position. Say the distribution has an ECL of \$1 billion and UCL of \$5 billion. The bank then needs to set aside \$5 billion just to cover random deviations from expected credit losses. This equity capital, however, will require remuneration. So, the pricing of loans should cover not only expected losses, but also the remuneration of this economic capital. This is what we call a *risk premium* and explains why observed credit spreads are larger than necessary simply to cover actuarial losses.

EXAMPLE 23.5: CREDIT VAR FOR ONE BOND

A risk analyst is trying to estimate the credit VAR (CVAR) for a risky bond. CVAR is defined as the maximum unexpected loss at a confidence level of 99.9% over a one-month horizon. Assume that the bond is valued at $1,000,000 one month forward, and the one-year cumulative default probability is 2% for this bond. What is the best estimate of the CVAR for the bond, assuming no recovery?

a. $20,000
b. $1,682
c. $998,318
d. $0

EXAMPLE 23.6: CREDIT VAR FOR TWO BONDS

A risk analyst is trying to estimate the credit VAR for a portfolio of two risky bonds. The credit VAR is defined as the maximum unexpected loss at a confidence level of 99.9% over a one-month horizon. Assume that each bond is valued at $500,000 one month forward, and the one-year cumulative default probability is 2% for each of these bonds. What is the best estimate of the credit VAR for this portfolio, assuming no default correlation and no recovery?

a. $841
b. $1,682
c. $998,318
d. $498,318

EXAMPLE 23.7: FRM EXAM 2005—QUESTION 122

You are the credit risk manager for Bank Happy. Bank Happy holds Treasuries for USD 500 million, one large loan that has a positive probability of default for USD 400 million and another loan that has a positive probability of default for USD 100 million. The defaults are uncorrelated. The bank computes a credit VAR at 1% using CreditRisk+. Which of the following statements made about the VAR by the analyst who works for you is necessarily *wrong*?

a. The VAR or WCL can be equal to zero.
b. The expected loss on the portfolio exceeds the VAR.
c. The expected loss on the portfolio is necessarily smaller than the VAR.
d. None of the above statements is wrong.

23.4 PORTFOLIO CREDIT RISK MODELS

Portfolio credit risk models can be classified according to their approaches. This section also describes the four main portfolio credit models.

23.4.1 Approaches to Portfolio Credit Risk Models

Table 23.3 summarizes the essential features of portfolio credit risk models in the industry.

Model Type **Top-down models** group credit risks using single statistics. They aggregate many sources of risk viewed as *homogeneous* into an overall portfolio risk, without going into the details of individual transactions. This approach is appropriate for retail portfolios with large numbers of credits, but less so for corporate or sovereign loans. Even within retail portfolios, top-down models may hide specific risks, by industry or geographic location.
Bottom-Up models account for features of each instrument. This approach is most similar to the structural decomposition of positions that characterizes market VAR systems. It is appropriate for corporate and capital market portfolios. Bottom-up models are also most useful for taking corrective action, because the risk structure can be reverse-engineered to modify the risk profile.

Risk Definitions **Default-mode models** consider only outright default as a credit event. Hence any movement in the market value of the bond or in the credit rating is irrelevant.
Mark-to-market models consider changes in market values and ratings changes, including defaults. These fair market value models provide a better assessment of

TABLE 23.3 Comparison of Credit Risk Models

	CreditMetrics	CreditRisk+	KMV	CreditPf.View
Originator	J.P. Morgan	Credit Suisse	KMV	McKinsey
Model type	Bottom-up	Bottom-up	Bottom-up	Top-down
Risk definition	Market value (MTM)	Default losses (DM)	Default losses (MTM/DM)	Market value (MTM)
Risk drivers	Asset values	Default rates	Asset values	Macro factors
Credit events	Rating change/ default	Default	Continuous default prob.	Rating change/ default
Probability	Unconditional	Unconditional	Conditional	Conditional
Volatility	Constant	Variable	Variable	Variable
Correlation	From equities (structural)	Default process (reduced-form)	From equities (structural)	From macro factors
Recovery rates	Random	Constant within band	Random	Random
Solution	Simulation/ analytic	Analytic	Analytic	Simulation

risk, which is consistent with the holding period defined in terms of the liquidation period.

Models of Default Probability **Conditional models** incorporate changing macroeconomic factors into the default probability through a functional relationship. Notably, we observe that the rate of default increases in a recession.
Unconditional models have fixed default probabilities and tend to focus on borrower- or factor-specific information. Some changes in the environment, however, can be allowed by manually changing the input parameters.

Models of Default Correlations Because default correlations are not directly observed for the obligors in the portfolio, they must be inferred from a model.
Structural models explain correlations by the joint movements of assets, for example, stock prices. For each obligor, this price is the random variable that represents movements in default probabilities.
Reduced-form models explain correlations by assuming a particular functional relationship between the default probability and "background factors." For example, the correlation between defaults across obligors can be modeled by the loadings on common risk factors, say, industrial and country.

23.4.2 CreditMetrics

CreditMetrics, published in April 1997 by J.P. Morgan, was an early portfolio credit risk model. The system is a "bottom-up" approach where credit risk is driven by movements in bond ratings taken from a transition matrix.

In this model, credit quality is measured by a *latent variable*, unobserved, which can be interpreted as the value of assets of the obligor. This value is related to the value of the equity, which is the source of correlations across obligors because equity prices are observable. When the value of the asset falls below some floor, the obligor is assumed to be in a state of default. Thus, this class of models includes three types of random variables, (1) equity value, (2) asset value, and (3) default indicator.

The components of the system are described in Figure 23.3.

Measurement of Exposure by Instrument This starts from the user's portfolio, decomposing all instruments by their exposure and assessing the effect of market volatility on expected exposures on the target date. The range of covered instruments includes bonds and loans, swaps, receivables, commitments, and letters of credit.

Distribution of Individual Default Risk This step starts with assigning each instrument to a particular credit rating. Credit events are then defined by rating migrations, which include default, through a matrix of migration probabilities.

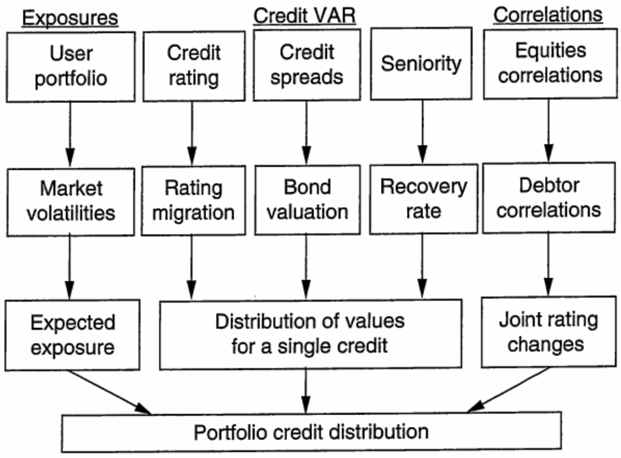

FIGURE 23.3 Structure of CreditMetrics

Thus movements in default probabilities are discrete. After the credit event, the instrument is valued using credit spreads for each rating class. In the case of default, the distributions of recovery rates are used from historical data for various seniority classes.

This is illustrated in Figure 23.4. We start from a bond or credit instrument with an initial rating of BBB. Over the horizon, the rating can jump to eight new values, including default. For each rating, the value of the instrument is recomputed, for example, to \$109.37 if the rating goes to AAA, or to the recovery value of \$51.13 in case of default. Given the state probabilities and associated values, we can compute an expected bond value, which is \$107.09, and a standard deviation of \$2.99.

Changes in the credit rating are driven by the latent factor, which is the asset value. Each asset value has a standard normal distribution with cutoff points

		Probability (p_i)	Value (V_i)	Exp. $\Sigma(p_i V_i)$	Var. $\Sigma p_i(V_i-m)^2$
	AAA	0.02%	\$109.37	0.02	0.00
	AA	0.33%	\$109.19	0.36	0.01
	A	5.95%	\$108.66	6.47	0.15
BBB	BBB	86.93%	\$107.55	93.49	0.19
	BB	5.30%	\$102.02	5.41	1.36
	B	1.17%	\$98.10	1.15	0.95
	CCC	0.12%	\$83.64	0.10	0.66
	Default	0.18%	\$51.13	0.09	5.64
		Sum = 100.00%		m = \$107.09	σ^2 = 8.95
					SD = \$2.99

FIGURE 23.4 Building the Distribution of Bond Values

TABLE 23.4 Cutoff Values for Simulations

Rating i	Prob. p_i	Cum. Prob. $N(z_i)$	Cutoff z_i
AAA	0.02%	100.00%	
AA	0.33%	99.98%	3.54
A	5.95%	99.65%	2.70
BBB	86.93%	93.70%	1.53
BB	5.30%	6.77%	−1.49
B	1.17%	1.47%	−2.18
CCC	0.12%	0.30%	−2.75
Default	0.18%	0.18%	−2.91

selected to represent the probabilities of changes in credit ratings. Table 23.4 illustrates the computations for our BBB credit. From Figure 23.4, there is a 0.18% probability of going from BBB into the state of default. We choose z_1 such that the area to its left is $N(z_1) = 0.18\%$. This gives $z_1 = -2.91$. Next, we need to choose z_2 so that the probability of falling between z_1 and z_2 is 0.12%, or that the total left-tail probability is $N(z_2) = 0.18\% + 0.12\% = 0.30\%$. This gives $z_2 = -2.75$, and so on.

Correlations among Defaults Correlations among defaults are inferred from correlations between asset values. These in turn are taken from correlations across *equity indices*. Each obligor is mapped to an industry and a geographical sector, using pre-assigned weights. Correlations are inferred from the co-movements of the common risk factors, using a database with some 152 country–industry indices, 28 country indices, and 19 worldwide–industry indices.

As an example, company 1 may be such that 90% of its volatility comes from the U.S. chemical industry. Using standardized returns, we can write

$$r_1 = 0.90\, r_{\mathrm{US,Ch}} + k_1\epsilon_1$$

where the residual ϵ is uncorrelated with other variables. Because the total volatility is normalized to 1, we must have $k_1 = \sqrt{1 - 0.9^2} = 0.44$.

Next, suppose that company 2 has a 74% weight on the German insurance index and 15% on the German banking index:

$$r_2 = 0.74\, r_{\mathrm{GE,In}} + 0.15\, r_{\mathrm{GE,Ba}} + k_2\epsilon_2$$

The correlation between asset values for the two companies is

$$\rho(r_1, r_2) = (0.90 \times 0.74)\rho(r_{\mathrm{US,Ch}}, r_{\mathrm{GE,In}}) + (0.90 \times 0.15)\rho(r_{\mathrm{US,Ch}}, r_{\mathrm{GE,Ba}})$$
$$\rho(r_1, r_2) = (0.90 \times 0.74)0.15 + (0.90 \times 0.15)0.08 = 0.11$$

CreditMetrics then uses simulations of the joint asset values, assuming a multivariate normal distribution with the prespecified correlations. Thus, the approach relies on a normal copula. This gives a total value for the portfolio and a distribution of credit losses over an annual horizon.

These simulations can also be used to compute correlations among default events. Because defaults are much less common than rating changes, the correlation is typically much less than the correlation between asset values. CreditMetrics reports that asset correlations in the range of 40% to 60% will typically translate into default correlations of 2% to 4%.[1]

Another drawback of this approach is that it does not integrate credit and market risk. Losses are generated only by changes in credit states, not by market movements. There is no uncertainty over market exposures. For swaps, for instance, the exposure on the target date is taken from the expected exposure. Bonds are revalued using today's forward rate and current credit spreads, applied to the credit rating on the horizon. So there is no interest rate risk.

23.4.3 CreditRisk+

CreditRisk+ was made public by Credit Suisse in October 1997. The approach is drastically different from CreditMetrics. It is based on a purely actuarial approach derived from the property insurance literature.

CreditRisk+ is a default mode (DM) model rather than a mark-to-market (MTM) model. Only two states of the world are considered—default and no-default.

The model starts with an assumption of a large number of identical loans n with independent default probability p. The total loss, $x = \sum_{i=1}^{n} b_i$, then follows a binomial distribution, which can be approximated by a Poisson distribution with intensity $\lambda = np$:

$$f(x) = e^{-\lambda} \frac{\lambda^x}{x!} \tag{23.12}$$

Default correlations are introduced by assuming that the intensity itself is random. A high draw in λ then increases the probability of default for each obligor. This default intensity can also be time-varying, in which case it is modeled as a function of factors that change over time. CreditRisk+ accounts for variability in default rates by dividing the portfolio into homogeneous sectors within which obligors share the same systematic risk factor.

The other component of the approach is the severity of losses. This is roughly modeled by sorting assets by severity bands, say loans around \$20,000 for the first band, \$40,000 for the second band, and so on. A distribution of losses is then obtained for each band. These distributions are combined across bands to generate an overall distribution of default losses.

[1] This result, however, is driven by the joint normality assumption, which is not totally realistic. Other distributions can generate a greater likelihood of simultaneous defaults. This is obviously of major importance for credit portfolios.

The method provides a quick analytical solution to the distribution of credit losses with minimal data inputs. As with CreditMetrics, however, there is no uncertainty over market exposures.

23.4.4 Moody's KMV

Moody's KMV provides forecasts of estimated default frequencies (EDFs) for approximately 30,000 public firms globally.[2] Much of its technology is considered proprietary and is unpublished.

The model is an application of the Merton approach, which views the firm's equity E as a call option on the firm's assets

$$E = c(A, K, r, \sigma_A, \tau) \tag{23.13}$$

In practice, KMV defines the floor K as the value of all short-term liabilities (one year and under) plus half the book value of all long-term debt. The value of assets is taken as the market value of equity plus the book value of all debt $A = nS + D$.

As seen in Section 20.2.2, this equation has to be iteratively estimated from observable variables, in particular the stock price S and its volatility σ_S to get the asset volatility. KMV computes a normalized **distance to default** (DD), which is essentially the distance between the current value of assets and the boundary point. Suppose, for instance, that $A = \$100$ million, $K = \$80$ million, and $\sigma_A = \$10$ million. We have

$$\mathrm{DD} = z = \frac{A - K}{\sigma_A} = \frac{\$100 - \$80}{\$10} = 2 \tag{23.14}$$

The main drivers of DD are (1) the level of the stock price, (2) the amount of leverage, and (3) the volatility of asset value. Lower stock prices, higher leverage, and higher asset volatility will decrease the DD measure.

In the final step, KMV uses this information to report an estimated default frequency (EDF), or default probability. If we assume normally distributed returns, for example, the probability of a standard normal variate z falling below -2 is about PD $= 2.3\%$. In practice, the EDFs are calibrated to actual default data, which gives objective (as opposed to risk-neutral) probabilities of default.

KMV generates default correlations between obligors directly from their equity prices, unlike CreditMetrics. First, returns on asset values are computed from changes in equity and debt values for the obligor. Second, these returns are regressed against a set of macroeconomic factors, country, and industry indices. Finally, this factor model is used to generate joint random variables representing obligor asset values, again using the industry standard normal copula.

The strength of this approach is that it relies on what is perhaps the best market data for a company—its stock price. Thus, it works best for *public firms*.

[2] KMV was founded by S. Kealhofer, J. McQuown, and O. Vasicek (hence the abbreviation KMV) to provide credit risk services. KMV started as a private firm based in San Francisco in 1989 and was acquired by Moody's in April 2002.

KMV also provides a model for *private companies*, which is based on accounting data for the firm and the industry as well as equity information, but only for public firms in the same industry. As expected, EDFs for private companies are considerably less accurate.

23.4.5 Credit Portfolio View

The last model we consider is **Credit Portfolio View** (CPV), published by the consulting firm McKinsey in 1997. The focus of this top-down model is on the effect of macroeconomic factors on portfolio credit risk.

This approach models loss distributions from the number and size of credits in subportfolios, typically consisting of customer segments. Instead of considering fixed transition probabilities, this model conditions the default probability on the state of the economy, allowing increases in defaults during recessions. The default probability p_t at time t is driven by a set of macroeconomic variables x^k for various countries and industries through a linear combination called y_t. The functional relationship to y_t, called the *logit model*, ensures that the probability is always between 0 and 1:

$$p_t = 1/[1 + \exp(y_t)], \qquad y_t = \alpha + \sum \beta^k x_t^k \tag{23.15}$$

Using a multifactor model, each debtor is assigned to a country, industry, and rating segment. Uncertainty in recovery rates is also factored in. The model uses numerical simulations to construct the distribution of default losses for the portfolio. While useful for modeling default probabilities conditioned on the state of the economy, this approach is mainly top-down and does not generate sufficient detail of credit risk for corporate portfolios.

23.4.6 Comparisons

The International Swaps and Derivatives Association (ISDA) conducted a comparative survey of credit risk models.[3] The empirical study consisted of three portfolios of one-year loans with a total notional of $66.3 billion each.

1. High-credit-quality, diversified portfolio (500 names)
2. High-credit-quality, concentrated portfolio (100 names)
3. Low-credit-quality, diversified portfolio (500 names)

The models are listed in Table 23.5 and include CreditMetrics, CreditRisk+, and two internal models, all with a one-year horizon and 99% confidence level. Also reported are the charges from the Basel I "standard" rules, which will be explained in a later chapter. Suffice it to say that these rules make no allowance for variation in credit quality or diversification effects. Instead, the capital charge is based on 8% of the loan notional.

[3] ISDA (1998), *Credit Risk and Regulatory Capital*, New York: ISDA.

TABLE 23.5 Capital Charges from Various Credit Risk Models

	Assuming Zero Correlation		
	Portfolio A	Portfolio B	Portfolio C
CreditMetrics	777	2,093	1,989
CreditRisk+	789	2,020	2,074
Internal model 1	767	1,967	1,907
Internal model 2	724	1,906	1,756
Basel I rules	5,304	5,304	5,304
	Assessing Correlations		
	Portfolio A	Portfolio B	Portfolio C
CreditMetrics	2,264	2,941	11,436
CreditRisk+	1,638	2,574	10,000
Internal model 1	1,373	2,366	9,654
Basel I rules	5,304	5,304	5,304

The top of the table examines the case of zero correlations. The Basel rules yield the same capital charge, irrespective of quality or diversification effects. The charge is also uniformly higher than most others, at $5,304 million, which is 8% of the notional.

Generally, the four credit portfolio models show remarkable consistency in capital charges. Portfolios A and B have the same credit quality, but B is more concentrated. Portfolio A has indeed lower CVAR, approximately $800 million against $2,000 million for B. This reflects the benefit from greater diversification. Portfolios A and C have the same number of names, but C has lower credit quality. This increases CVAR from around $800 million to $2,000 million.

The bottom panel assesses empirical correlations, which are typically positive. The Basel charges are unchanged, as expected because they do not account for correlations. Internal models show capital charges to be systematically higher than in the previous case. There is also more dispersion in results across models, however. It is interesting to see, in particular, that the economic capital charge for portfolio C, with low credit quality, is typically twice the Basel charge. Such results demonstrate that the Basel rules can lead to inappropriate credit risk charges. As a result, banks subject to these capital requirements may shift the risk profile to lower-rated credits until their economic capital is in line with regulatory capital. This shift to lower credit quality was certainly not an objective of the original Basel rules. This lack of sensitivity is what led to the new Basel Accord, which will be discussed in Chapter 29.

More recently, another study compared the capital charges from the new Basel II rules with three commercial models, taking great care to align parameters.[4] The base portfolio consists of $100 billion in loans to 3,000 obligors spread across different industries and countries, with an average credit rating of BBB.

[4] IACPM and ISDA (2006), *Convergence of Economic Capital Models*, New York: ISDA.

TABLE 23.6 Comparison of Credit Risk Models

	Expected Loss	Capital at 99.9%
KMV (PM)	563	3,791
CreditMetrics (CM)	562	3,533
CreditRisk+	564	3,662
Basel II	607	3,345

Table 23.6 shows the results in default mode. The Basel II rules, under the advanced approach, require economic capital of about $3.3 billion, close to 4% of the notional. The three commercial models give remarkably close results. The report concludes that "if assumptions are aligned, there is not much difference between the valuation methods from PM and CM." Of course, this is also because these models are based on the same joint density function, using a normal copula, and calibrated to the same historical data.

23.5 CONCLUSIONS

Portfolio credit risk models take market risk models one step further. Here again, the fundamental intuition is that diversification across obligors, regions, and industries should lower portfolio risk.

The problem with internal portfolio credit risk models, however, is their complexity. Unlike market risk, where the risk manager can observe a history of movements in risk factors, there is generally no history of default for a particular obligor. Hence, default probabilities have to be modeled indirectly. The problem is even more difficult for estimating default correlations, as well as the shape of the joint density of defaults. Prior to 2007, these models had not been tested over a full market cycle, which should include a recession. This is particularly important because downturns lead to higher default probabilities, higher default correlations, and lower recovery rates.[5]

Finally, it will be difficult to verify such models given that they generate measures of economic capital over a long horizon, typically one year, and at a very high confidence level, typically 99.9%. In contrast, market risk models produce a daily VAR at the 99% confidence level, which should produce on average two or three exceptions a year. Thus, backtests of credit risk VAR estimates are not feasible, unlike for market risk.

This explains why regulators had considerable doubts about the precision of these models and, as a result, did not allow commercial banks to use their internal portfolio models as the basis for their credit risk charge. Indeed the losses suffered during the recession that started in 2007 have been much greater than

[5]The academic literature has long emphasized the sensitivity of credit risk models to correlations between defaults. See Das, Duffie, Kapadia, and Saita (2007), Common Failings: How Corporate Defaults are Correlated, *Journal of Finance*; Jorion and Zhang (2009), Credit Contagion from Counterparty Risk, *Journal of Finance*.

worst-case scenarios. Even though considerable progress has been made in our understanding of credit risk, it is fair to conclude that much more work is needed to develop robust models.

EXAMPLE 23.8: FRM EXAM 2004—QUESTION 11

When determining the standard deviation of value due to credit quality changes for a single exposure, the CreditMetrics model uses three primary factors. Which of the following is *not* one of the factors used in this model?

a. Credit ratings
b. Seniority
c. Equity prices
d. Credit spreads

EXAMPLE 23.9: FRM EXAM 2002—QUESTION 129

A bank computes the distribution of its loan portfolio marked-to-market value one year from now using the CreditMetrics approach of computing values for rating transition outcomes using (1) a rating agency transition matrix, (2) current forward curves, and (3) correlations among rating transition outcomes derived from stock returns of the obligors. In computing firm-wide risk using this distribution of its loan portfolio, the bank is most likely to understate its risk because it ignores

a. The term structure of interest rates
b. Rating drift
c. Spread risk
d. The negative correlation between the Treasury rates and credit spreads

EXAMPLE 23.10: FRM EXAM 2003—QUESTION 92

KMV measures the normalized distance from default. How is this defined?

a. (Expected assets – Weighted debt) / (Volatility of assets)
b. Equity/ (Volatility of equity)
c. Probability of stock price falling below a threshold
d. Leverage times stock price volatility

EXAMPLE 23.11: FRM EXAM 2004—QUESTION 20

A firm's assets are currently valued at $500 million and its current liabilities are $300 million. The standard deviation of asset values is $80 million. The firm has no other debt. What will be the approximate distance to default using the KMV calculation?

a. 2 standard deviations
b. 2.5 standard deviations
c. 6.25 standard deviations
d. Cannot be determined

EXAMPLE 23.12: FRM EXAM 2007—QUESTION 59

You are given the following information about a firm. The market value of assets at time 0 is 1,000; at time 1 is 1,200. Short-term debt is 500; long-term debt is 300. The annualized asset volatility is 10%. According to the KMV model, what are the default point and the distance to default at time 1?

a. 800 and 3.33
b. 650 and 7.50
c. 650 and 4.58
d. 500 and 5.83

EXAMPLE 23.13: FRM EXAM 2006—QUESTION 69

Which of the following model(s) calculates the change in portfolio value due to rating migration of the underlying instruments?

a. CreditRisk+
b. CreditMetrics
c. KMV
d. Both a. and c.

EXAMPLE 23.14: FRM EXAM 2005—QUESTION 36

Which of the following credit risk models uses the option theoretic approach for modeling correlation between the credit risky assets?

a. CreditRisk+
b. CreditMetrics
c. KMV for public firms
d. Both CreditMetrics and KMV for public firms

23.6 IMPORTANT FORMULAS

Credit loss: $\sum_{i=1}^{N} b_i \times \text{CE}_i \times \text{LGD}_i$

Expected credit loss: ECL = Prob[default] $\times$ E[Credit exposure] $\times$ E[LGD]

Present value of expected credit losses (PVECL):

$$\text{PVECL} = \sum_t E[\text{CL}_t] \times \text{PV}_t = \sum_t [k_t \times \text{ECE}_t \times (1 - f)] \times \text{PV}_t$$

Approximation to PVECL: $\text{PVECL}_F = c_T \times \text{ECE} \times (1 - f) \times \text{PV}_T$

Credit VAR: CVAR = WCL − ECL

KMV's normalized distance from default: $z = (A - K)/\sigma_A$

Credit portfolio view's default probability: $p_t = 1/[1 + \exp(y_t)]$, $y_t = \alpha + \sum \beta^k x_t^k$

23.7 ANSWERS TO CHAPTER EXAMPLES

Example 23.1: Credit Provisions

c. Credit provisions should be made for actual and expected losses. Capital, however, is supposed to provide a cushion against unexpected losses based on VAR.

Example 23.2: FRM Exam 2002—Question 74

b. This is an example of right-way trade. To have lower credit risk, it would be preferable to engage in a trade where there is a lower probability of a default by the counterparty when the contract is in-the-money. This will happen if the counterparty enters a transaction to hedge an operating exposure. For instance, a gas producer has a natural operating exposure to gas. If the producer sells gas at a fixed price, the swap will lose money if the market price of gas goes up. In this situation, however, there is little risk of default because the producer is sitting on an inventory of gas. A trading firm or distributor could go bankrupt if the transaction loses money.

Example 23.3: FRM Exam 2003—Question 26

a. The one-year PD needs to be adjusted to the maturity of the loan, using $(1 - d^m)^T$, where d^m is computed from $(1 - d^m)^{12} = (1 - d)$.

Loan	PD to Maturity	Loss Given Default	EL
a.	0.50%	60%	0.301%
b.	0.68%	70%	0.473%
c.	0.50%	75%	0.376%
d.	0.75%	50%	0.375%

Example 23.4: FRM Exam 2007—Question 38

b. The survival rate over two years is $S_2 = (1 - 4\%)^2 = 92.16\%$, which implies a cumulative two-year default rate of 7.84%. Put differently, the first-year PD is 4%, then $(1 - 4\%)4\% = 3.84\%$. Multiplying by 200 and 40% gives 6.27.

Example 23.5: Credit VAR for One Bond

c. First, we have to transform the annual default probability into a monthly probability. Using $(1 - 2\%) = (1 - d)^{12}$, we find $d = 0.00168$, which assumes a constant probability of default during the year. Next, we compute the expected credit loss, which is $d \times \$1{,}000{,}000 = \$1{,}682$. Finally, we calculate the WCL at the 99.9% confidence level, which is the lowest number CL_i such that $P(\text{CL} \leq \text{CL}_i) \geq 99.9\%$. We have $P(\text{CL} = 0) = 99.83\%$; $P(\text{CL} \leq 1{,}000{,}000) = 100.00\%$. Therefore, the WCL is \$1,000,000, and the CVAR is \$1,000,000 − \$1,682 = \$998,318.

Example 23.6: Credit VAR for Two Bonds

d. As in the previous question, the monthly default probability is 0.0168. The following table shows the distribution of credit losses.

Default	Probability (p_i)	Loss L_i	$p_i L_i$	$1 - \sum p_i$
2 bonds	$d^2 = 0.00000282$	\$1,000,000	\$2.8	100.00000%
1 bond	$2d(1 - d) = 0.00335862$	\$500,000	\$1,679.3	99.99972%
0 bonds	$(1 - d)^2 = 0.99663854$	\$0	\$0.0	99.66385%
Total	1.00000000		\$1,682.1	

This gives an expected loss of \$1,682, the same as before. Next, \$500,000 is the WCL at a minimum 99.9% confidence level because the total probability of observing a number equal to or lower than this is greater than 99.9%. The CVAR is then \$500,000 − \$1,682 = \$498,318.

Example 23.7: FRM Exam 2005—Question 122

c. The credit VAR could be zero. For instance, assume that the PD is 0.003. The joint probability of no default is then $(1 - 0.003)(1 - 0.003) = 99.4\%$. Because this is greater than the 99% confidence level, the worst loss is zero. The expected loss, however, would be 0.3% assuming zero recovery, which is greater than VAR.

Example 23.8: FRM Exam 2004—Question 11

c. CreditMetrics uses credit ratings, the transition matrix, recovery rates, and LGD for various seniority, but not equity prices for the obligor.

Example 23.9: FRM Exam 2002—Question 129

c. CreditMetrics ignores spread risk. It does account for ratings drift and the term structure of interest rates, albeit not their volatility.

Example 23.10: FRM Exam 2003—Question 92

a. The distance-to-default measure is a standardized variable that measures how much the value of firm assets exceeds the liabilities.

Example 23.11: FRM Exam 2004—Question 20

b. Using Equation (23.14), the DD is $(500 - 300)/80 = 2.5$ standard deviations.

Example 23.12: FRM Exam 2007—Question 59

c. The default point is given by short-term liabilities plus half of long-term liabilities, which is $500 + 300/2 = 650$. The distance to default at point 1 is $(V - K)/\sigma_V = (1{,}200 - 650)/(1{,}200 \times 0.10) = 4.58$.

Example 23.13: FRM Exam 2006—Question 69

b. Only CreditMetrics uses the rating migration. KMV uses the distance to default. CreditRisk+ uses random variables drawn from a Poisson distribution.

Example 23.14: FRM Exam 2005—Question 36

c. KMV estimates default probabilities using the Merton approach based on the company's stock price.

PART

Six

Legal, Operational, and Integrated Risk Management

CHAPTER 24

Operational Risk

The financial industry has developed standard methods to measure and manage market and credit risks. The industry is turning next to operational risk, which has proved to be an important cause of financial losses. Indeed, most company-specific financial disasters can be attributed to a combination of market and credit risk along with some failure of controls, which is a form of operational risk.

As in the case of market and credit risk, the financial industry is being pushed in the direction of better control of operational risk by bank regulators. For the first time, the Basel Committee has established capital charges for operational risk, in exchange for lowering them on market and credit risk. This new charge, which is explained in Chapter 29, would constitute approximately 12% of the total capital requirement.[1] As a result, this is forcing the banking industry to pay close attention to operational risk.

As with market and credit risk, the management of operational risk follows a sequence of logical steps: (1) identification, (2) assessment, (3) monitoring, and (4) control or mitigation.

Historically, operational risk has been managed by internal control mechanisms within business lines, supplemented by the audit function. The industry is now starting to use specific structures and control processes specifically tailored to operational risk.

To introduce operational risk, Section 24.1 summarizes lessons from well-known financial disasters. Given this information, Section 24.2 turns to definitions of operational risk. Various measurement approaches are discussed in Section 24.3. Finally, Section 24.4 shows how to use the distribution of operational losses to manage this risk better and offers some concluding comments.

24.1 THE IMPORTANCE OF OPERATIONAL RISK

The Basel Committee recently reported that "[a]n informal survey . . . highlights the growing realization of the significance of risks other than credit and market risks, such as operational risk, which have been at the heart of some important banking problems in recent years." These problems are described in case histories next.

[1] See Basel Committee on Banking Supervision (2003), *Sound Practices for the Management and Supervision of Operational Risk*, Basel: BIS.

24.1.1 Case Histories

- *January 2008—SocGen (4.9 billion euros loss).* A rogue trader, Jerome Kerviel, systematically deceives systems, taking unauthorized positions worth up to 49 billion euros in stock index futures. The bank has enough capital to absorb the loss but its reputation is damaged.
- *February 2002—Allied Irish Bank ($691 million loss).* A rogue trader, John Rusnack, hides three years of losing trades on the yen/dollar exchange rate at the U.S. subsidiary. The bank's reputation is damaged.
- *March 1997—NatWest ($127 million loss).* A swaption trader, Kyriacos Papouis, deliberately covers up losses by mispricing and overvaluing option contracts. The bank's reputation is damaged. NatWest is eventually taken over by the Royal Bank of Scotland.
- *September 1996—Morgan Grenfell Asset Management ($720 million loss).* A fund manager, Peter Young, exceeds his guidelines, leading to a large loss. Deutsche Bank, the German owner of MGAM, agrees to compensate the investors in the fund.
- *June 1996—Sumitomo ($2.6 billion loss).* A copper trader amasses unreported losses over three years. Yasuo Hamanaka, known as "Mr. Five Percent," after the proportion of the copper market he controlled, is sentenced to prison for forgery and fraud. The bank's reputation is severely damaged.
- *September 1995—Daiwa ($1.1 billion loss).* A bond trader, Toshihide Igushi, amasses unreported losses over 11 years at the U.S. subsidiary. The bank is declared insolvent.
- *February 1995—Barings ($1.3 billion loss).* Nick Leeson, a derivatives trader, amasses unreported losses over two years. Barings goes bankrupt.
- *October 1994—Bankers Trust ($150 million loss).* The bank becomes embroiled in a high-profile lawsuit with a customer that accuses it of improper selling practices. Bankers settles, but its reputation is badly damaged. It is later bought out by Deutsche Bank.

Many of these spectacular losses can be traced to a **rogue trader**, or a case of internal fraud. These failures involve a mix of market risk and operational risk, i.e., failure to supervise properly. It should be noted that the cost of these events has been quite high. They led to large direct monetary losses, sometimes even to bankruptcy. In addition to these direct costs, banks often suffered large indirect losses due to reputational damage.

24.1.2 Business Lines

These failures have occurred across a variety of business lines. Some are more exposed than others to market risk or credit risk. All have some exposure to operational risk, however.

Commercial banking is exposed mainly to credit risk, less so to operational risk, and least to market risk. Investment banking, trading, and treasury management have greater exposure to market risk. On the other hand, business lines such

TABLE 24.1 Examples of Operational Risks

Type of Risk	Definition	Market Bank	Credit Bank
Operations risk	Losses due to complex systems and processes	High risk	Medium risk
Ops. settlement risk	Lost interest/fines due to failed settlements	High risk	Low risk
Model risk	Losses due to imperfect model or data	High risk	Low risk
Fraud risk	Reputational/financial damage due to fraud	High risk	Low risk
Misselling risk	Losses due to unsuitable sales	Medium risk	Medium risk
Legal risk	Reputational/financial damage due to fraud	High risk	Medium risk

Source: Financial Services Authority (1999), "Allocating Regulatory Capital for Operational Risk," London: FSA.

as retail brokerage and asset management are exposed primarily to operational risk. Asset managers assume no direct market risk since they act as agents for the investors. If they act in breach of guidelines, however, they may be liable to clients for their losses, which represents operational risk.

Table 24.1 presents a partial list of risks for market banks that are primarily involved in trading, and credit banks that specialize in lending activities. The table shows that different lines of business are characterized by very different exposures to the listed risks. Credit banks deal with relatively standard products, such as mortgages, with little trading. Hence, they have medium operations risk and low operations settlement risk. This is in contrast with trading banks, with constantly changing products and large trading volume, for which both risks are high. Trading banks also have high model risk, because of the complexity of products, and high fraud risk, because of the autonomy given to traders. In contrast, these two risks are low for credit banks.

For trading banks that deal with so-called sophisticated investors, misselling risk has low probability but high value; hence, it is a medium risk. A good example is Merrill Lynch settling with Orange County for about $400 million following allegations that the broker had sold the county unsuitable investments. For credit banks that deal with retail investors, this risk has higher probability but lower value: hence, it is a medium risk. Legal risks are high for market banks and medium for credit banks due to the more litigious environment of corporations relative to retail investors.

24.2 IDENTIFYING OPERATIONAL RISK

One could argue that operational risk has no clear-cut definition, unlike market risk and credit risk. There was a long debate as to the proper definition of operational risk, or even whether it makes sense to attempt to measure it.

After much industry consultation, the Basel Committee has settled on a definition that is becoming an industry standard. Operational risk is defined as

the risk of loss resulting from inadequate or failed internal processes, people and systems, or from external events

This includes the usual internal business events but also external events such as external fraud, security breaches, regulatory effects, or natural disasters. It includes legal risk, which arises when a transaction proves unenforceable in law. This definition, however, excludes strategic and reputational risk, which would be very difficult to measure, anyway.

The British Bankers' Association provides further detail for this definition. Table 24.2 breaks down operational risk into categories of **people risk, process risk, system risk,** and **external risk.** Among these, a notable risk for complex products is **model risk,** which is due to the use of wrong models for valuing and hedging assets. This is an internal risk that combines lack of knowledge (people) with product complexity/valuation errors (process) and perhaps programming errors (systems).

TABLE 24.2 Operational Risk Classification

Internal Risks		
People	**Processes**	**Systems**
Employee collusion/fraud	Accounting error	Data quality
Employee error	Capacity risk	Programming errors
Employee misdeed	Contract risk	Security breach
Employer liability	Misselling/suitability	Strategic risks (platform/suppliers)
Employment law	Product complexity	System capacity
Health and safety	Project risk	System compatibility
Industrial action	Reporting error	System delivery
Lack of knowledge/skills	Settlement/payment error	System failure
Loss of key personnel	Transaction error	System suitability
	Valuation error	

External Risks	
External	**Physical**
Legal	Fire
Money laundering	Natural disaster
Outsourcing	Physical security
Political	Terrorism
Regulatory	Theft
Supplier risk	
Tax	

Source: British Bankers' Association survey.

EXAMPLE 24.1: FRM EXAM 2004—QUESTION 39

Which of the following is *not* a type of operational risk as defined by Basel II?

a. Human error and internal fraud
b. Destruction by fire or other external catastrophes
c. Damaged reputation due to a failed merger
d. Failure or breakdown in internal control processes

EXAMPLE 24.2: FRM EXAM 2002—QUESTION 133

Which one of the following cases or events can be considered as resulting from operational risk?

a. A bank reports losses on a diversified portfolio of stocks during the stock market decline.
b. The bank becomes embroiled in a high-profile lawsuit with a customer that accuses it of improper selling practices.
c. The bank reports the loss of $1.5 billion due to rises in interest rates.
d. A U.S. investor makes a loss as the yen depreciates relative to the dollar.

EXAMPLE 24.3: FRM EXAM 2003—QUESTION 65

Which of these outcomes is *not* associated with an operational risk process?

a. The sale of call options is being booked as a purchase.
b. A monthly volatility is inputted in a model that requires a daily volatility.
c. A loss is incurred on an option portfolio because ex post volatility exceeded expected volatility.
d. A volatility estimate is based on a time-series that includes a price that exceeds the other prices by a factor of 100.

EXAMPLE 24.4: FRM EXAM 2007—QUESTION 56

All the following are operational risk loss events, *except*:

a. An individual shows up at a branch presenting a check written by a customer for an amount substantially exceeding the customer's low checking account balance. When the bank calls the customer to ask him for the funds, the phone is disconnected and the bank cannot recover the funds.
b. A bank, acting as a trustee for a loan pool, receives less than the projected funds due to delayed repayment of certain loans.
c. During an adverse market movement, the computer network system becomes overwhelmed, and only intermittent pricing information is available to the bank's trading desk, leading to large losses as traders become unable to alter their hedges in response to falling prices.
d. A loan officer inaccurately enters client financial information into the bank's proprietary credit risk model.

EXAMPLE 24.5: FRM EXAM 2007—QUESTION 139

The risk of the occurrence of a significant difference between the mark-to-model value of a complex and/or illiquid instrument and the price at which the same instrument is revealed to have traded in the market is referred to as:

a. Liquidity risk
b. Dynamic risk
c. Model risk
d. Mark-to-market risk

24.3 ASSESSING OPERATIONAL RISK

Once identified, operational risk should be measured, or rather "assessed" if it is less amenable to precise quantification than market or credit risk. Various approaches can be broadly classified into top-down models and bottom-up models.

24.3.1 Comparison of Approaches

Top-down models attempt to measure operational risk at the broadest level, that is, using firm-wide or industry-wide data. Results are then used to determine the amount of capital that needs to be set aside as a buffer against this risk. This capital is allocated to business units.

Bottom-up models start at the individual business unit or process level. The results are then aggregated to determine the risk profile of the institution. The main benefit of bottom-up models is that they lead to a better understanding of the causes of operational losses, as in the case of VAR-based market risk systems.

Tools used to manage operational risk can be classified into six categories:

1. **Audit oversight.** This consists of reviews of business processes by an external audit department.
2. **Critical self-assessment.** Each business unit identifies the nature and degree of operational risk. These *subjective* evaluations include expected frequency and severity of losses, as well as a description of how risk is controlled. The tools used for this type of process include checklists, questionnaires, and facilitated workshops. The results are then aggregated, in a bottom-up approach.
3. **Key risk indicators.** These consist of simple measures that provide an indication of whether risks are changing over time.

   These *early warning signs* can include audit scores, staff turnover, trade volumes, and so on. The assumption is that operational risk events are more likely to occur when these indicators increase. These *objective* measures allow the risk manager to forecast losses through the application of regression techniques, for example.
4. **Earnings volatility.** This approach consists of taking a time series of earnings, after stripping the effect of market and credit risk, and computing its volatility. This measure is simple to use but has numerous problems. This risk measure also includes fluctuations due to business and macroeconomic risks, which fall outside operational risk. Also, such a measure is backward-looking and does not account for improvement or degradation in the quality of controls.
5. **Causal networks.** These describe how losses can occur from a cascade of different causes. Causes and effects are linked through conditional probabilities. Simulations are then run on the network, generating a distribution of losses. Such bottom-up models improve the understanding of losses since they focus on drivers of risk. The process is explained in the appendix.
6. **Actuarial models.** These combine the distribution of frequency of losses with their severity distribution to produce an *objective* distribution of losses due to operational risk. These can be either bottom-up or top-down models.

EXAMPLE 24.6: FRM EXAM 2003—QUESTION 50

Which of the following is a weakness of the top-down approach to measuring operational risk?

a. It fails to consider historical information.
b. You cannot use earnings volatility as an indicator of risk potential in this approach.
c. Information on specific sources of risk is not provided.
d. It is based on the specific mapping of business units and not the overall organization.

24.3.2 Actuarial Models

Actuarial models estimate the objective distribution of losses from historical data and are widely used in the insurance industry. Such models combine two distributions: loss frequencies and loss severities. The **loss frequency distribution** describes the number of loss events over a fixed interval of time. The **loss severity distribution** describes the size of the loss once it occurs.

Loss severities can be tabulated from historical data, for instance, measures of the loss severity y_k, at time k. These measures can be adjusted for inflation and some measure of current business activity. Define P_k as the consumer price index at time k and V_k as a business activity measure such as the number of trades. We could assume that the severity is proportional to the volume of business V and to the price level. The *scaled* loss is measured as of time t as

$$x_t = y_k \times \frac{P_t}{P_k} \times \frac{V_t}{V_k} \tag{24.1}$$

Loss severity distributions have very long tails, representing the possibility of very large losses. Ideally, they should include internal and external data.

Next, define the loss frequency distribution by the variable n, which represents the number of occurrences of losses over the period. The density function is

$$\text{p.d.f. of loss frequency} = f(n), \quad n = 0, 1, 2, \ldots \tag{24.2}$$

If x (or X) is the loss severity when a loss occurs, its density is

$$\text{p.d.f. of loss severity} = g(x \mid n = 1), \quad x \geq 0 \tag{24.3}$$

Finally, the total loss over the period is given by the sum of individual losses over a random number of occurrences:

$$S_n = \sum_{i=1}^{n} X_i \tag{24.4}$$

Table 24.3 provides a simple example of two such distributions. Our task is now to combine these two distributions into one—that of total losses over the period.

TABLE 24.3 Sample Loss Frequency and Severity Distributions

Frequency Distribution		Severity Distribution	
Probability	Frequency	Probability	Severity
0.6	0	0.5	\$1,000
0.3	1	0.3	\$10,000
0.1	2	0.2	\$100,000
Expectation	0.5	Expectation	\$23,500

Assuming that the frequency and severity of losses are independent, the two distributions can be combined into a distribution of aggregate loss through a process known as convolution. **Convolution** can be implemented, for instance, through tabulation. **Tabulation** consists of systematically recording all possible combinations with their associated probabilities and is illustrated in Table 24.4. Generally, convolution must be implemented by numerical methods, as there are too many combinations of variables for a systematic tabulation.

We start with the obvious case, no loss, which has probability 0.6. Next, we go through all possible realizations of one loss only. From Table 24.3, we see that a loss of \$1,000 can occur with total probability of $P(n = 1) \times P(x = \$1{,}000) = 0.3 \times 0.5 = 0.15$. Similarly, for one-time losses of \$10,000 and \$100,000, the probabilities are 0.09 and 0.06, respectively. We then go through all occurrences of two losses, which can result from many different combinations. For instance, a loss of \$1,000 can occur twice, for a total of \$2,000, with a probability of $0.1 \times 0.5 \times 0.5 = 0.025$. We can have a loss of \$1,000 and \$10,000, for a total of \$11,000, with probability $0.1 \times 0.5 \times 0.3 = 0.015$. We repeat these steps until we exhaust all combinations.

TABLE 24.4 Tabulation of Loss Distribution

Number of Losses	First Loss	Second Loss	Total Loss	Probability
0	0	0	0	0.600
1	1,000	0	1,000	0.150
1	10,000	0	10,000	0.090
1	100,000	0	100,000	0.060
2	1,000	1,000	2,000	0.025
2	1,000	10,000	11,000	0.015
2	1,000	100,000	101,000	0.010
2	10,000	1,000	11,000	0.015
2	10,000	10,000	20,000	0.009
2	10,000	100,000	110,000	0.006
2	100,000	1,000	101,000	0.010
2	100,000	10,000	110,000	0.006
2	100,000	100,000	200,000	0.004

Sorted Losses	Cumulative Probability
0	60.0%
1,000	75.0%
2,000	77.5%
10,000	86.5%
11,000	89.5%
20,000	90.4%
100,000	96.4%
101,000	98.4%
110,000	99.6%
200,000	100.0%

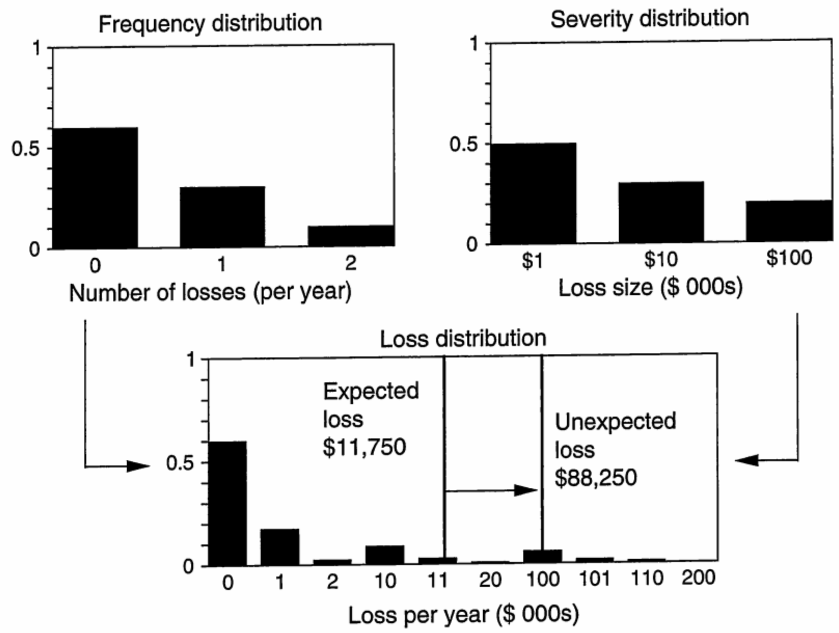

FIGURE 24.1 Construction of the Loss Distribution

The resulting distribution is displayed in Figure 24.1, and in the lower panel of Table 24.4. As usual for operational risk, losses are recorded as positive values. It is interesting to note that the very simple distributions in Table 24.3, with only three realizations, create a complex loss distribution. We can compute the expected loss, which is simply the product of expected values for the two distributions, or $E[S] = E[N] \times E[X] = 0.5 \times \$23{,}500 = \$11{,}750$. Risk management, however, is about the unexpected. So, the risk manager should also report the lowest number such that the probability is greater than 95%. This is $100,000, with a probability of 96.4%. Hence, the unexpected loss is $\$100{,}000 - \$11{,}750 = \$88{,}250$. If **operational VAR** must include the expected loss, this is simply $100,000. This is the default measure under Basel II, which measures VAR at the 99.9% level of confidence over one year.

EXAMPLE 24.7: FRM EXAM 2007—QUESTION 138

The severity distribution of operational losses usually has the following shape:

a. Symmetrical with short tails
b. Long-tailed to the right
c. Uniform
d. Symmetrical with long tails

EXAMPLE 24.8: FRM EXAM 2000—QUESTION 64

Which statement about operational risk is *true*?

a. Measuring operational risk requires estimating both the probability of an operational loss event and the potential size of the loss.
b. Measurement of operational risk is well developed, given the general agreement among institutions about the definition of this risk.
c. The operational risk manager has the primary responsibility for management of operational risk.
d. Operational risks are clearly separate from credit and market risks.

EXAMPLE 24.9: FRM EXAM 2007—QUESTION 33

Suppose you are given the following information about the operational risk losses at your bank. What is the estimate of the VAR at the 95% confidence level, including expected loss?

Frequency Distribution	
Probability	Number
0.5	0
0.3	1
0.2	2

Severity Distribution	
Probability	Loss
0.6	USD 1,000
0.3	USD 10,000
0.1	USD 100,000

a. USD 100,000
b. USD 101,000
c. USD 200,000
d. USD 110,000

EXAMPLE 24.10: FRM EXAM 2006—QUESTION 118

Which of the following statements about the differences between market and operational Value-at-Risk at financial institutions are correct?

I. The distribution of operational risk events must include sufficient mass in the extreme tail, making the assumption of a lognormal distribution invalid.
II. The typical time horizon of market VAR calculations is one day, whereas the typical time horizon of operational VAR calculations is one year.
III. Since prices are sufficiently available for liquid assets at all times, the market risk of liquid assets can be modeled using continuous distributions, but the nature of operational risk events requires using discrete distributions.
IV. Market VAR requires a higher confidence level than operational VAR.

a. I, II, and III
b. I, II, and IV
c. I, II, III, and IV
d. III and IV

24.4 MANAGING OPERATIONAL RISK

24.4.1 Capital Allocation and Insurance

Like market VAR, the distribution of operational losses can be used to estimate expected losses as well as the amount of capital required to support this financial risk. Figure 24.2 highlights important attributes of a distribution of losses due to operational risk.

The **expected loss** represents the size of operational losses that should be expected to occur. Typically, this is dominated by high-frequency, low-severity events. This type of loss is generally absorbed as an ongoing cost and managed through internal controls. Such losses are rarely disclosed.

The **unexpected loss** represents the deviation between the quantile loss at some confidence level and the expected loss. Typically, this represents lower-frequency, higher-severity events. This type of loss is generally offset against capital reserves or transferred to an outside insurance company, when available. Such losses are sometimes disclosed publicly but often with little detail.

The **stress loss** represents a loss in excess of the unexpected loss. By definition, such losses are very infrequent but extremely damaging to the institution. The Barings bankruptcy can be attributed, for instance, in large part to operational risk. This type of loss cannot be easily offset through capital allocation, as it would require too much capital. Ideally, it should be transferred to an insurance company. Due to their severity, such losses are disclosed publicly.

However, purchasing insurance is no panacea. The insurance payment would have to be made very quickly and in full. The bank could fail while waiting for payment or arguing over the size of compensation. In addition, the premium may be very high. This is because once the insurance is acquired, the purchaser has less

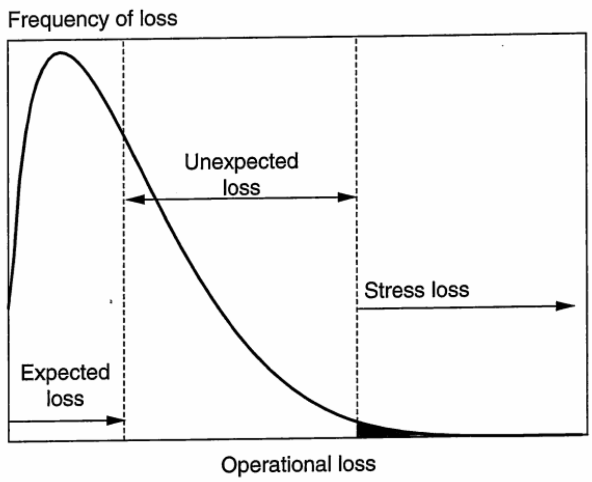

FIGURE 24.2 Distribution of Operational Losses

incentive to control losses. This problem is called **moral hazard.** The insurer will be aware of this and will increase the premium accordingly. The premium may also be high because of the **adverse selection** problem. This describes a situation where banks vary in the quality of their controls. Banks with poor controls are more likely to purchase insurance than banks with good controls. Because the insurance company does not know what type of bank it is dealing with, it will increase the average premium. Insurance with a **deductible** amount, i.e., where the bank would have to share the first layer of losses, only provides a partial solution to these problems.

24.4.2 Mitigating Operational Risk

The approach so far has been to take operational risk as given. Such measures are extremely useful because they highlight the size of losses due to operational risk. Armed with this information, the institution can then decide whether it is worth spending resources on decreasing operational risk.

Say that a bank is wondering whether to install a **straight-through processing** system, which automatically captures trades in the front office and transmits them to the back office. Such a system eliminates manual intervention and the potential for human errors, thereby decreasing losses due to operational risk. The bank should purchase the system if its cost is less than its operational risk benefit.

More generally, reduction of operational risk can occur in terms of the frequency of losses and/or the size of losses when they occur. Operational risk is also contained by a firm-wide risk management framework. In a later chapter, we will discuss *best practices* in risk management, which are designed to provide some protection against operational risk.

Consider, for instance, a transaction in a plain-vanilla, five-year interest rate swap. This simple instrument generates a large number of cash flows, each of which has the potential for errors. At initiation, the trade needs to be booked and confirmed with the counterparty. It must be valued so that a P&L can be attributed to the trading unit. With biannual payments, the swap will generate 10 cash flows along with 10 rate resets and net payment computations. These payments need to be computed with absolute accuracy—that is, to the last cent. Errors can range from minor issues, such as paying a day late, to major problems, such as failure to hedge or fraudulent valuation by the trader.

The swap will also create some market risk, which may need to be hedged. The position needs to be transmitted to the market risk management system, which will monitor the total position and risk of the trader and of the institution as a whole. In addition, the current and potential credit exposure must be regularly measured and added to all other trades with the same counterparty. Errors in this risk measurement process can lead to excessive exposure to market and/or credit risk.

Operational risk can be minimized in a number of ways.[2] Internal control methods consist of

- *Separation of functions*. Individuals responsible for committing transactions should not perform clearance and accounting functions.
- *Dual entries*. Entries (inputs) should be matched from two different sources—that is, the trade ticket and the confirmation by the back office.
- *Reconciliations*. Results (outputs) should be matched from different sources—for instance, the trader's profit estimate and the computation by the middle office.
- *Tickler systems*. Important dates for a transaction (e.g., settlement and exercise dates) should be entered into a calendar system that automatically generates a message before the due date.
- *Controls over amendments*. Any amendment to original deal tickets should be subject to the same strict controls as original trade tickets.

External control methods consist of

- *Confirmations*. Trade tickets need to be confirmed with the counterparty, which provides an independent check on the transaction.
- *Verification of prices*. To value positions, prices should be obtained from external sources. This implies that an institution should have the capability of valuing a transaction in-house before entering it.
- *Authorization*. The counterparty should be provided with a list of personnel authorized to trade, as well as a list of allowed transactions.
- *Settlement*. The payment process itself can indicate if some of the terms of the transaction have been incorrectly recorded—for instance, if the first cash payments on a swap are not matched across counterparties.
- *Internal and external audits*. These examinations provide useful information on potential weakness areas in the organizational structure or business process.

24.4.3 Conceptual Issues

The management of operational risk is beset by conceptual problems. First, unlike market and credit risk, operational risk is largely internal to financial institutions. Because institutions are understandably reluctant to advertise their mistakes, it is more difficult to collect data on operational losses. Another problem is that losses may not be directly applicable to another institution, as they were incurred under possibly different business profiles and internal controls. Internal data also has survival bias, because they will not contain cases of losses that could bankrupt the institution.

Second, market and credit risk can be conceptually separated into exposures and risk factors. Exposures can be easily measured and controlled. In contrast, the

[2] See Brewer (1997), Minimizing Operations Risk, in R. Schwartz & C. Smith (eds.), *Derivatives Handbook*, New York: Wiley.

link between risk factors and the likelihood and size of operational losses is not so easy to establish. Here, the line of causation runs through internal controls.

Third, very large operational losses, which can threaten the stability of an institution, are relatively rare (thankfully so). This leads to a very small number of observations in the tails. This "thin tails" problem makes it very difficult to come up with a robust "value for operational risk" at a high confidence level. As a result, there is still some skepticism as to whether operational risk can be subject to the same quantification as market and credit risks.

EXAMPLE 24.11: FRM EXAM 2002—QUESTION 102

Capital is used to protect the bank from which of the following risks?

a. Risks with an extreme financial impact
b. High-frequency low-loss events
c. Low-frequency risks with significant financial impact
d. High-frequency uncorrelated events

EXAMPLE 24.12: FRM EXAM 2001—QUESTION 49

Which of the terms below is used in the insurance industry to refer to the effect of a reduction in the control of losses by an individual who is insured because of the protection provided by insurance?

a. Control trap
b. Moral hazard
c. Adverse selection
d. Control hazard

EXAMPLE 24.13: FRM EXAM 2003—QUESTION 48

Which of the options below does *not* describe a problem faced by banks when purchasing insurance as a hedge against operational risk?

a. The fact that the loss reimbursement period can take several years
b. The credit rating of insurers
c. The different perspective of operational risk between banks and insurers
d. Not having an operational VAR

EXAMPLE 24.14: FRM EXAM 2005—QUESTION 48

Insurance is an effective tool to transfer which of types of operational risks?

a. High frequency, low severity
b. Low frequency, high severity
c. Operational losses whose magnitude is affected by the actions of the company
d. Operational losses for which insurance companies only sell policies with low limits

EXAMPLE 24.15: FRM EXAM 2005—QUESTION 52

Which of the following statements are valid about hedging operational risk?

I. A primary disadvantage of insurance as an operational risk management tool is the limitation of policy coverage.
II. If an operational risk hedge works properly, a firm will avoid damage to its reputation from a high-severity operational risk event.
III. While all insurance contracts suffer from the problem of moral hazard, deductibles help reduce this problem.
IV. Catastrophe (cat) bonds allow a firm to hedge operational risks associated with natural disasters.

a. I, III, and IV only
b. I, II, and IV only
c. II and III only
d. III and IV only

24.5 ANSWERS TO CHAPTER EXAMPLES

Example 24.1: FRM Exam 2004—Question 39

c. Damaged reputation due to a failed merger is a business risk. Also, reputational risk is not a type of operational loss.

Example 24.2: FRM Exam 2002—Question 133

b. Answers a., c., and d. correspond to the market risk of stocks, fixed-income securities, and currencies, respectively. Lawsuits, on the other hand, are part of operational risk.