and Ecuador (failure to pay). In addition, other events sometimes included are

- **Downgrading**, which means the credit rating is lower than previously, or is withdrawn
- **Currency inconvertibility**, which means the imposition of exchange controls or other currency restrictions by a governmental or associated authority
- **Governmental action**, which means either (1) declarations or actions by a government or regulatory authority that impair the validity of the obligation, or (2) the occurrence of war or other armed conflict that impairs the functioning of the government or banking activities

Ideally, the industry should agree on a common set of factors defining a credit event, to minimize the possibility of disagreements and costly legal battles that create uncertainty for everybody. The ISDA definitions are designed to minimize **legal risk** by precisely wording the definition of credit event.

Even so, unforeseen situations sometimes develop. For example, there have been differences of opinion as to whether a bank debt restructuring constitutes a credit event, as in the recent cases of Conseco, Xerox, and Marconi.

Another notable situation is that of Argentina, which represents the largest sovereign default recorded so far, in terms of external debt. Argentina announced in November 2001 a restructuring of its local debt that was more favorable to itself. Some holders of credit default swaps argued that this was a "credit event," since the exchange was coerced, and that they were entitled to payment. Swap sellers disagreed. This became an unambiguous default, however, when Argentina announced in December it would stop paying interest on its $135 billion foreign debt. Nonetheless, the situation was unresolved for holders of credit swaps that expired just before the official default.

EXAMPLE 19.1: DEFINITION OF A CREDIT EVENT

Which of the following events is *not* a "credit event"?

a. Bankruptcy
b. Calling back a bond
c. Downgrading
d. Default on payments

19.2 DEFAULT RATES

19.2.1 Credit Ratings

A **credit rating** is an "evaluation of creditworthiness" issued by a credit rating agency (CRA). The major U.S. bond rating agencies are Moody's Investors

Services, Standard & Poor's (S&P), and Fitch. More technically, a credit rating is defined by Moody's as an

> *opinion of the future ability, legal obligation, and willingness of a bond issuer or other obligor to make full and timely payments on principal and interest due to investors.*

Table 19.1 presents the interpretation of various credit ratings issued by Moody's and S&P. These ratings correspond to long-term debt; other ratings apply to short-term debt. Generally, the two agencies provide similar ratings for the same issuer.

Ratings are divided into the following:

- **Investment grade,** that is, at and above BBB for S&P and Baa for Moody's
- **Speculative grade,** or **below investment grade,** for the rest

Each letter is known as a class. In addition, the CRA use modifiers, also called **notches.** For instance, S&P subdivides the BBB category into BBB+, BBB, and BBB−. For Moody's, the equivalent ratings are Baa1, Baa2, and Baa3.

These ratings represent objective (or actuarial) probabilities of default.[1] Indeed, the agencies have published studies that track the frequency of bond defaults, classified by initial ratings for different horizons. These frequencies can be used to convert ratings to default probabilities.

The agencies use a number of criteria to decide on the credit rating, including various accounting ratios. Table 19.2 presents median value for selected accounting ratios for U.S. industrial corporations. The first column (under "leverage") shows that the ratio of total debt to total capital (debt plus book equity, or assets)

TABLE 19.1 Classification by Credit Ratings

Explanation	Standard & Poor's	Moody's Services
Investment grade:		
Highest grade	AAA	Aaa
High grade	AA	Aa
Upper medium grade	A	A
Medium grade	BBB	Baa
Speculative grade:		
Lower medium grade	BB	Ba
Speculative	B	B
Poor standing	CCC	Caa
Highly speculative	CC	Ca
Lowest quality, no interest	C	C
In default	D	

Modifiers: A+, A, A−, and A1, A2, A3

[1] In fact, the ratings measure the probability of default (PD) for S&P and the joint effect of PD×LGD for Moody's, where LGD is the proportional loss given default.

TABLE 19.2 S&P's Industrial Financial Ratios Across Ratings (2005 to 2007 Averages)

Rating	Leverage: (Percent) Total Debt/Capital	Cash Flow Coverage: (Multiplier) EBITDA/Interest	EBIT/Interest
AAA	12	32.0	26.2
AA	35	19.5	16.4
A	37	13.5	11.2
BBB	45	7.8	5.8
BB	53	4.8	3.4
B	73	2.3	1.4
CCC	99	1.1	0.4

varies systematically across ratings. Highly rated companies have low leverage ratios, 12% for AAA firms. In contrast, BB-rated (just below investment grade) companies have a leverage ratio of 53%. This implies a capital-to-equity leverage ratio of 2.1 to 1.[2]

The right-hand panel (under "cash flow") also shows systematic variations in a measure of free cash flow divided by interest payments. This represents the number of times the cash flow can cover interest payments. Focusing on earnings before interest and taxes (EBIT), AAA-rated companies have a safe cushion of 26.2, whereas BB-rated companies have coverage of only 3.4.

A related model for bankruptcy prediction is the **multiple discriminant analysis** (MDA), such as the **z-score** model developed by Altman.[3] MDA constructs a linear combination of accounting data that provides the best fit with the observed states of default and non-default for the sample firms.

The variables used in the z-score are (1) working capital over total assets, (2) retained earnings over total assets, (3) EBIT over total assets, (4) market value of equity over total liabilities, and (5) net sales over total assets. Lower scores indicate a higher likelihood of default. Each variable enters with a positive sign, meaning that an increase in each of these variables decreases the probability of bankruptcy.

EXAMPLE 19.2: FRM EXAM 2003—QUESTION 100

What is the lowest tier of an investment grade credit rating by Moody's?

a. Baa1
b. Ba1
c. Baa3
d. Ba3

[2] Defining D, E as debt and equity, the debt-to-asset ratio is $D/(D+E) = 53\%$. We then have an asset-to-equity ratio of $(D+E)/E = [D/E]+1 = [D/(D+E)]/[E/(D+E)]+1 = 53\%/(1-53\%)+1 = 2.1$

[3] Altman, E. (1968), Financial Ratios, Discriminant Analysis and the Prediction of Corporate Bankruptcy, *Journal of Finance* 23, 589–609.

EXAMPLE 19.3: FRM EXAM 2005—QUESTION 86

You are considering an investment in one of three different bonds. Your investment guidelines require that any bond you invest in carry an investment-grade rating from at least two recognized bond rating agencies. Which, if any, of the bonds listed below would meet your investment guidelines?

a. Bond A carries an S&P rating of BB and a Moody's rating of Baa.
b. Bond B carries an S&P rating of BBB and a Moody's rating of Ba.
c. Bond C carries an S&P rating of BBB and a Moody's rating of Baa.
d. None of the above.

EXAMPLE 19.4: FRM EXAM 2002—QUESTION 110

If Moody's and S&P are equally good at rating bonds, the average default rate on BB bonds by S&P will be lower than the average default rate on bonds rated by Moody's as

a. Baa3
b. Ba1
c. Ba
d. Ba3

19.2.2 Historical Default Rates

Tables 19.3 and 19.4 display historical default rates as reported by Moody's and Standard and Poor's, respectively. These describe the proportion of firms that default, $\bar{X}$, which is a statistical estimate of the true default probability:

$$E(\bar{X}) = p \tag{19.1}$$

For example, borrowers with an initial Moody's rating of Baa experienced an average default rate of 0.29% over the next year. Similar rates are obtained for S&P's BBB-rated credits, who experienced an average 0.23% default rate over the next year. On the other hand, A-rated firms experience a default rate around 0.07% over the next year. Firms at or below Caa have a default rate of 13.73%. Higher ratings are associated with lower default rates. As a result, this information could be used to derive estimates of default probability for an initial rating class.

In addition, the tables show that the cumulative default rate increases sharply with the horizon, for a given initial credit rating. The default rate for Baa firms

TABLE 19.3 Moody's Cumulative Default Rates (Percent), 1920–2007

Rating	Year 1	2	3	4	5	6	7	8	9	10
Aaa	0.00	0.00	0.02	0.08	0.16	0.26	0.37	0.53	0.70	0.90
Aa	0.06	0.18	0.29	0.45	0.70	1.01	1.34	1.65	1.95	2.29
A	0.07	0.24	0.50	0.81	1.12	1.45	1.80	2.13	2.50	2.90
Baa	0.29	0.85	1.56	2.34	3.14	3.94	4.71	5.48	6.28	7.06
Ba	1.34	3.20	5.32	7.49	9.59	11.56	13.36	15.11	16.73	18.44
B	4.05	8.79	13.49	17.72	21.43	24.66	27.59	30.04	32.15	33.93
Caa-C	13.73	22.46	29.03	33.92	37.64	40.58	42.87	44.92	47.00	48.98
Inv.	0.14	0.43	0.81	1.23	1.69	2.16	2.63	3.09	3.58	4.08
Spec.	3.59	7.24	10.75	13.92	16.71	19.18	21.37	23.34	25.11	26.83
All	1.41	2.88	4.32	5.63	6.80	7.85	8.80	9.67	10.48	11.28

Rating	Year 11	12	13	14	15	16	17	18	19	20
Aaa	1.07	1.21	1.36	1.41	1.45	1.53	1.62	1.68	1.77	1.83
Aa	2.68	3.10	3.51	3.93	4.25	4.49	4.68	4.88	5.09	5.27
A	3.34	3.77	4.15	4.50	4.92	5.28	5.56	5.81	6.08	6.33
Baa	7.80	8.54	9.24	9.89	10.44	11.01	11.53	12.00	12.44	12.91
Ba	20.00	21.52	23.04	24.34	25.51	26.64	27.81	28.91	29.85	30.78
B	35.64	37.26	38.69	40.08	41.40	42.68	43.73	44.52	45.07	45.38
Caa-C	50.99	53.07	55.05	57.11	59.12	60.98	62.63	64.20	65.68	67.13
Inv.	4.58	5.09	5.57	6.00	6.42	6.79	7.12	7.41	7.71	8.00
Spec.	28.44	30.00	31.50	32.87	34.13	35.35	36.52	37.57	38.46	39.28
All	12.05	12.80	13.50	14.15	14.74	15.29	15.79	16.25	16.66	17.06

increases from 0.29% over one year to 7.06% over the following 10 years. Because these are cumulative default rates, the number must necessarily increase with the horizon. For investment-grade credits, however, the increase is more than proportional with the horizon. The ratio is $7.06/0.29 = 24$ for Baa-rated credits, which is more than 10. In contrast, the ratio for B-rated credits is $33.93/4.05 = 8$. For speculative-grade credits, the increase is less than proportional with the horizon.

One problem with such historical information, however, is the relative paucity of data in some cells. There are few defaults for highly-rated firms over a one-year horizon. For example, the one-year default rate for Aa firms is 0.06%. This corresponds to very few defaults. From 1939 to 2007, only two companies initially rated Aa defaulted during the following year, which happened in 1989. Changing the sample period or having another number of defaults could have a substantial effect on this average default rate.

In addition, the sample size decreases as the horizon lengthens. For instance, S&P reports default rates for horizons up to 15 years using data from 1981 to 2007. The one-year default rate is an average using 27 periods—that is, 1981, 1982, and so on to 2007. For 15-year horizons, however, the first period is 1981

TABLE 19.4 S&P's Cumulative Global Default Rates (Percent), 1981–2007

	Year														
Rating	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
AAA	0.00	0.00	0.09	0.18	0.28	0.41	0.48	0.59	0.63	0.67	0.67	0.67	0.67	0.73	0.79
AA	0.01	0.05	0.09	0.19	0.29	0.40	0.52	0.62	0.71	0.81	0.91	0.99	1.09	1.17	1.21
A	0.06	0.16	0.29	0.45	0.64	0.85	1.11	1.32	1.53	1.76	1.95	2.11	2.26	2.39	2.61
BBB	0.23	0.65	1.13	1.75	2.38	2.98	3.47	3.96	4.42	4.89	5.37	5.75	6.22	6.68	7.20
BB	1.00	2.93	5.19	7.36	9.30	11.19	12.72	14.05	15.27	16.24	17.13	17.87	18.51	18.96	19.43
B	4.57	10.06	14.72	18.39	21.08	23.19	24.94	26.37	27.55	28.74	29.80	30.70	31.61	32.47	33.26
CCC	25.59	34.06	39.04	41.86	44.50	45.62	46.67	47.25	48.86	49.76	50.50	51.26	51.87	52.50	52.50
Inv.	0.10	0.30	0.52	0.81	1.11	1.42	1.69	1.95	2.19	2.44	2.66	2.85	3.05	3.24	3.47
Spec.	4.11	8.11	11.66	14.57	16.90	18.84	20.45	21.79	23.01	24.08	25.05	25.87	26.64	27.30	27.90
All	1.45	2.91	4.21	5.33	6.26	7.06	7.73	8.30	8.81	9.29	9.72	10.08	10.44	10.76	11.09

to 1995, the second is 1982 to 1996, and so on until the last period of 1993 to 2007. The average therefore uses only 13 periods, which is a much shorter sample. The data is also overlapping and therefore not independent. So, omitting or adding a few borrowers can drastically alter the reported default rate.

This can lead to inconsistencies in the tables. For instance, the default rate for CCC obligors is the same, at 52.50%, from year 14 to 15. This implies that there is no further risk of default after 14 years, which is an unrealistic implication. Also, when the categories are further broken down into modifiers, default rates sometimes do not decrease monotonically as the rating increases, which is probably a small-sample effect.

We can try to assess the accuracy of these default rates by computing their standard error. Consider for instance the default rate over the first year for AA-rated credits, which averaged out to $\bar{X} = 0.01\%$ in this S&P sample. Assume that this was taken out of a total of about $N = 10{,}000$ independent observations. The variance of the average is, from the distribution of a binomial process,

$$V(\bar{X}) = \frac{p(1-p)}{N} \tag{19.2}$$

which gives a standard error of about 0.01%. This is on the same order as the average of 0.01%, indicating that there is substantial imprecision in this average default rate. So we could not really distinguish an AA credit from an AAA credit.

The problem is made worse with lower sample sizes, which is the case in non-U.S. markets or when the true p is changing over time. For instance, if we observe a 5% default rate among 100 observations, the standard error becomes 2.2%, which is very large. Therefore, a major issue with credit risk is that estimation of default rates for low-probability events can be very imprecise.

19.2.3 Cumulative and Marginal Default Rates

The default rates reported in Tables 19.3 and 19.4 are **cumulative default rates** for an initial credit rating; that is, they measure the total frequency of default *at any time* between the starting date and year T. It is also informative to measure the **marginal default rate**, which is the frequency of default *during* year T.

The default process is illustrated in Figure 19.1. Here, d_1 is the marginal default rate during year 1, and d_2 is the marginal default rate during year 2. To default during the second year, the firm must have survived the first year and defaulted in the second. Thus, the probability of defaulting in year 2 is given by $(1-d_1)d_2$. The cumulative probability of defaulting up to year 2 is then $C_2 = d_1 + (1-d_1)d_2$. Subtracting and adding one, this is also $C_2 = 1 - (1-d_1)(1-d_2)$, which perhaps has a more intuitive interpretation, as this is 1 minus the probability of surviving the entire period.

More formally,

- $m[t+N \mid R(t)]$ is the number of issuers rated R at the end of year t that default in year $T = t + N$

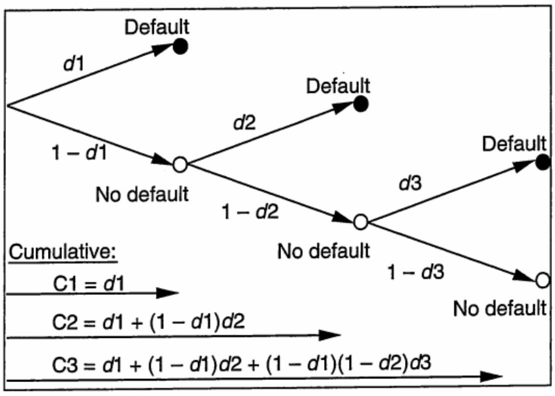


FIGURE 19.1 Sequential Default Process

- $n[t + N \mid R(t)]$ is the number of issuers rated R at the end of year t that have not defaulted by the beginning of year $t + N$

Marginal Default Rate during Year T This is the proportion of issuers initially rated R at initial time t that default in year T, relative to the remaining number at the beginning of the same year T:

$$d_N(R) = \frac{m[t + N \mid R(t)]}{n[t + N \mid R(t)]}$$

Survival Rate This is the proportion of issuers initially rated R that will not have defaulted by T:

$$S_N(R) = \Pi_{i=1}^{N}(1 - d_i(R)) \tag{19.3}$$

Marginal Default Rate from Start to Year T This is the proportion of issuers initially rated R that defaulted in year T, relative to the initial number in year t. For this to happen, the issuer will have survived until year $t + N - 1$, and then default the next year. Hence, this is:

$$k_N(R) = S_{N-1}(R)d_N(R) \tag{19.4}$$

Cumulative Default Rate This is the proportion of issuers rated R that defaulted at any point until year T:

$$C_N(R) = k_1(R) + k_2(R) + \cdots + k_N(R) = 1 - S_N(R) \tag{19.5}$$

Average Default Rate We can express the total cumulative default rate as an average, per period default rate d, by setting

$$C_N = 1 - \Pi_{i=1}^{N}(1 - d_i) = 1 - (1 - d)^N \tag{19.6}$$

As we move from annual to semiannual and ultimately continuous compounding, the average default rate becomes

$$C_N = 1 - (1 - d^a)^N = 1 - (1 - d^s/2)^{2N} \rightarrow 1 - e^{-d^c N} \tag{19.7}$$

where d^a, d^s, d^c are default rates using annual, semiannual, and continuous compounding. This is equivalent to the various definitions for the compounding of interest.

Example: Computing Cumulative Default Probabilities

Consider a B-rated firm that has default rates of $d_1 = 5\%$, $d_2 = 7\%$. Compute the cumulative default probabilities.

Answer

In the first year, $k_1 = d_1 = 5\%$. After one year, the survival rate is $S_1 = 0.95$. The probability of defaulting in year 2 is then $k_2 = S_1 \times d_2 = 0.95 \times 0.07 = 6.65\%$. After two years, the survival rate is $(1 - d_1)(1 - d_2) = 0.95 \times 0.93 = 0.8835$. Thus, the cumulative probability of defaulting in years 1 and 2 is $5\% + 6.65\% = 11.65\%$.

Based on this information, we can map these *forward*, or marginal, default rates from cumulative default rates for various credit ratings. Figure 19.2, for instance, displays cumulative default rates reported by Moody's in Table 19.3. The corresponding marginal default rates are plotted in Figure 19.3.

It is interesting to see that the marginal probability of default increases with maturity for initial high credit ratings, but decreases for initial low credit ratings. The increase is due to a mean reversion effect. The fortunes of an Aaa-rated firm can only stay the same at best, and often will deteriorate over time. In contrast, a B-rated firm that has survived the first few years must have a decreasing probability of defaulting as time goes by. This is a survival effect.

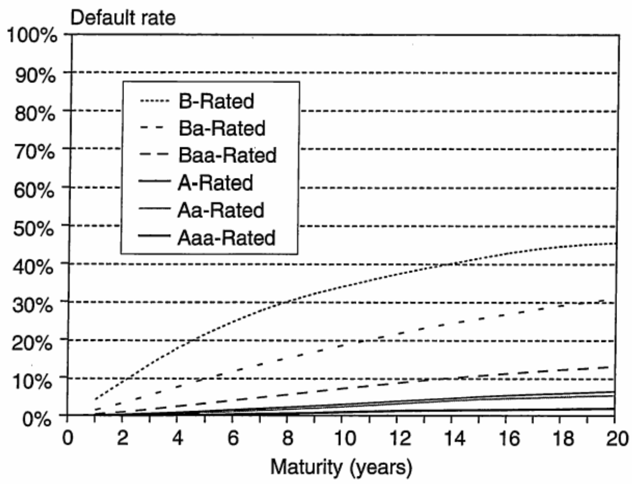


FIGURE 19.2 Moody's Cumulative Default Rates, 1920–2007

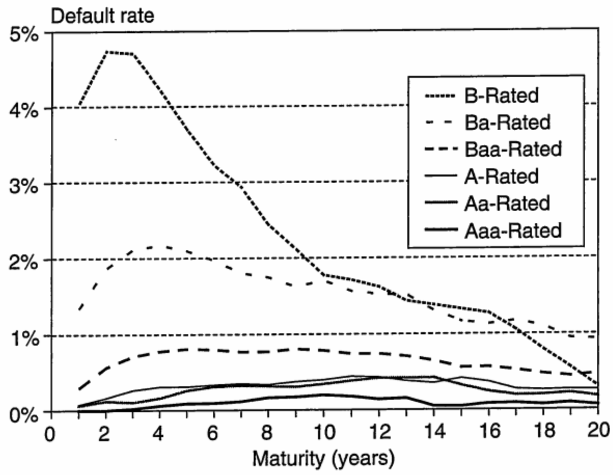


FIGURE 19.3 Moody's Marginal Default Rates, 1920–2007

EXAMPLE 19.5: FRM EXAM 2004—QUESTION 1

Company ABC was incorporated on January 1, 2004. It has an expected annual default rate of 10%. Assuming a constant quarterly default rate, what is the probability that company ABC will *not* have defaulted by April 1, 2004?

a. 2.40%
b. 2.50%
c. 97.40%
d. 97.50%

EXAMPLE 19.6: FRM EXAM 2002—QUESTION 77

If the default probability for an A-rated company over a three-year period is 0.30%, then the most likely probability of default for this company over a six-year period is:

a. 0.30%
b. Between 0.30% and 0.60%
c. 0.60%
d. Greater than 0.60%

EXAMPLE 19.7: FRM EXAM 2006—QUESTION 21

What is the survival rate at the end of three years if the annual default probabilities are 8%, 12%, and 15% in the first, second, and third years, respectively?

a. 68.8%
b. 39.1%
c. 99.9%
d. 65.0%

EXAMPLE 19.8: FRM EXAM 2004—QUESTION 14

A corporate bond will mature in three years. The marginal probability of default in year one is 3%. The marginal probability of default in year two is 4%. The marginal probability of default in year three is 6%. What is the cumulative probability that default will occur during the three-year period?

a. 12.47%
b. 12.76%
c. 13%
d. 13.55%

EXAMPLE 19.9: FRM EXAM 2000—QUESTION 34

What is the difference between the marginal default probability and the cumulative default probability?

a. Marginal default probability is the probability that a borrower will default in any given year, whereas the cumulative default probability is over a specified multiyear period.
b. Marginal default probability is the probability that a borrower will default due to a particular credit event, whereas the cumulative default probability is for all possible credit events.
c. Marginal default probability is the minimum probability that a borrower will default, whereas the cumulative default probability is the maximum probability.
d. Both a. and c. are correct.

19.2.4 Transition Probabilities

As we have seen, the measurement of long-term default rates can be problematic with small sample sizes. The computation of these default rates can be simplified by assuming a Markov process for the ratings migration, described by a transition matrix. **Migration** is a discrete process that consists of credit ratings changing from one period to the next.

The **transition matrix** gives the probability of moving to one rating conditional on the rating at the beginning of the period. The usual assumption is that these moves follow a **Markov process**, or that migrations across states are independent from one period to the next.[4] This type of process exhibits *no carry-over effect.* More formally, a **Markov chain** describes a stochastic process where the conditional distribution, given today's value, is constant over time. Only present values are relevant.

Table 19.5 gives an example of a simplified transition matrix for four states, A, B, C, D, where the last represents default. Consider a company in year 0 in the B category. The company could default

- In year 1, with probability $D[t_1 \mid B(t_0)] = P(D_1 \mid B_0) = 3\%$.
- In year 2, after going from B to A in the first year, then A to D in the second; or from B to B, then to D; or from B to C, then to D. The default probability is $P(D_2 \mid A_1)P(A_1) + P(D_2 \mid B_1)P(B_1) + P(D_2 \mid C_1)P(C_1) = 0.00 \times 0.02 + 0.03 \times 0.93 + 0.23 \times 0.02 = 3.25\%$.

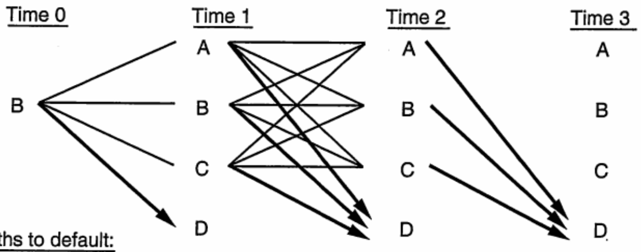
The cumulative probability of default over the two years is then $3\% + 3.25\% = 6.25\%$. Figure 19.4 illustrates the various paths to default in years 1, 2, and 3.

The advantage of using this approach is that the resulting data are more robust and consistent. For instance, the 15-year cumulative default rate obtained this way will always be greater than the 14-year default rate.

TABLE 19.5 Credit Ratings Transition Probabilities

State	Ending				Total
Starting	A	B	C	D	Prob.
A	0.97	0.03	0.00	0.00	1.00
B	0.02	0.93	0.02	0.03	1.00
C	0.01	0.12	0.64	0.23	1.00
D	0	0	0	1.00	1.00

[4] There is some empirical evidence, however, that credit downgrades are not independent over time but instead display a momentum effect.



Paths to default:

Time 1	Time 2	Time 3
B → D 0.03 = 0.0300	B → A → D 0.02 × 0.00 = 0.0000	B → A → A → D 0.02 × 0.97 × 0.00 = 0.0000
	B → B → D 0.93 × 0.03 = 0.0279	B → A → B → D 0.02 × 0.03 × 0.03 = 0.0000
	B → C → D 0.02 × 0.23 = 0.0046	B → A → C → D 0.02 × 0.00 × 0.23 = 0.0000
		B → B → A → D 0.93 × 0.02 × 0.00 = 0.0000
		B → B → B → D 0.93 × 0.93 × 0.03 = 0.0259
		B → B → C → D 0.93 × 0.02 × 0.23 = 0.0043
		B → C → A → D 0.02 × 0.00 × 0.00 = 0.0000
		B → C → B → D 0.02 × 0.12 × 0.03 = 0.0001
		B → C → C → D 0.02 × 0.64 × 0.23 = 0.0029

	Time 1	Time 2	Time 3
Default prob:	0.0300	0.0325	0.0333
Cumulative:	0.0300	0.0625	0.0958

FIGURE 19.4 Paths to Default

EXAMPLE 19.10: FRM EXAM 2005—QUESTION 105

A rating transition table includes sufficient information to find all but the following item:

a. The likelihood that an AA-rated firm will fall to a BB rating over five years
b. The price of a bond that has been downgraded to BB from BBB
c. The probability of default on a B-rated bond
d. The probability that a high-yield bond will be upgraded to investment grade

EXAMPLE 19.11: FRM EXAM 2007—QUESTION 51

Fitch ratings provides a table indicating that the number of A-rated issuers (1) migrating to AAA is 2, (2) to AA is 5, (3) staying at A is 40, (4) migrating to BBB is 2, and (5) going into default is 3.

Based on this information, what is the probability that an issue with a rating of A at the beginning of the year will be downgraded by the end of the year?

a. 13.46%
b. 13.44%
c. 9.62%
d. 3.85%

EXAMPLE 19.12: FRM EXAM 2003—QUESTION 59

Given the following ratings transition matrix, calculate the two-period cumulative probability of default for a B credit.

Rating at beginning of period	Rating at end of period			
	A	B	C	Default
A	0.95	0.05	0.00	0.00
B	0.03	0.90	0.05	0.02
C	0.01	0.10	0.75	0.14
Default	0.00	0.00	0.00	1.00

a. 2.0%
b. 2.5%
c. 4.0%
d. 4.5%

19.2.5 Time Variation in Default Probabilities

Defaults are also correlated with economic activity. Moody's, for example, has compared the annual default rate to the level of industrial production since 1920. Moody's reports a marked increase in the default rate in the 1930s at the time of the Great Depression and during the recent recessions. These default rates, however, do not control for structural shifts in the credit quality. In recent years, many issuers came to the market with a lower initial credit rating than in the past. This should lead to more defaults even with a stable economic environment.

To control for this effect, Figure 19.5 plots the default rate for investment-grade and speculative credits over the years 1981 to 2006. As expected, the default rate of investment-grade bonds is very low. More interestingly, however, it displays minimal variation through time. We do observe, however, significant variation in the default rate of speculative-grade credits, which peaks during the recessions that started in 1981, 1990, and 2001. Thus, economic activity significantly affects the frequency of defaults. This effect is most marked for speculative-grade bonds.

19.3 RECOVERY RATES

Credit risk also depends on the **loss given default** (LGD). This can be measured as 1 minus the **recovery rate**, or fraction recovered after default.

19.3.1 The Bankruptcy Process

Normally, default is a state that affects all obligations of an issuer equally, especially when accompanied by a bankruptcy filing. In most countries, a formal

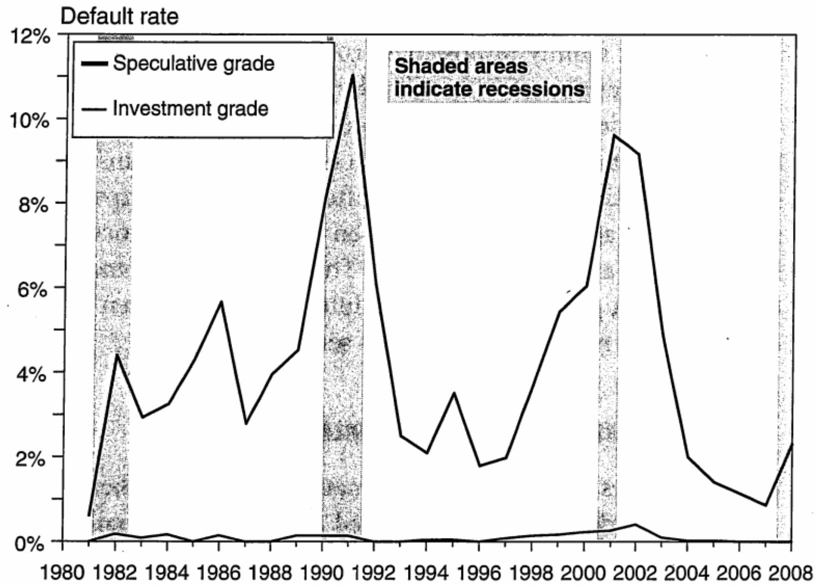


FIGURE 19.5 Time Variation in Defaults (from S&P)

bankruptcy process provides a centralized forum for resolving all the claims against the corporation. The bankruptcy process creates a **pecking order** for a company's creditors. This spells out the order in which creditors are paid, thereby creating differences in the recovery rate across creditors. Within each class, however, creditors should be treated equally.

In the United States, firms that are unable to make required payments can file for either **Chapter 7** bankruptcy, which leads to the liquidation of the firm's assets, or **Chapter 11** bankruptcy, which leads to a reorganization of the firm during which the firm continues to operate under court supervision.

Under Chapter 7, the proceeds from liquidation should be divided according to the **absolute priority rule**, which states that payments should be made first to claimants with the highest priority.

Table 19.6 describes the pecking order in bankruptcy proceedings. At the top of the list are **secured creditors**, who, because of their property right, are paid

TABLE 19.6 Pecking Order in U.S. Federal Bankruptcy Law

Seniority	Type of Creditor
Highest (paid first)	1. Secured creditors (up to the extent of secured collateral)
	2. Priority creditors
	• Firms that lend money during bankruptcy period
	• Providers of goods and services during bankruptcy period (e.g., employees, lawyers, vendors)
	• Taxes
	3. General creditors
	• Unsecured creditors before bankruptcy
Lowest (paid last)	• Shareholders

to the fullest extent of the value of their collateral. Then come **priority creditors**, which consist mainly of post-bankruptcy creditors. Finally, **general creditors** can be paid if funds remain after distribution to others.

Similar rules apply under Chapter 11. In this situation, the firm must submit a **reorganization plan**, which specifies new financial claims to the firm's assets. The absolute priority rule, however, is often violated in Chapter 11 settlements. Junior debt holders and stockholders often receive some proceeds even though senior shareholders are not paid in full. This is allowed to facilitate timely resolution of the bankruptcy and to avoid future lawsuits. Even so, there remain sharp differences in recovery rates across seniority.

19.3.2 Estimates of Recovery Rates

Recovery rates are commonly estimated from the market prices of defaulted debt shortly after default. This is viewed as the best estimate of the future recovery and takes into account the value of the firm's assets, the estimated cost of the bankruptcy process, and various means of payment (e.g., using equity to pay bondholders), discounted into the present.

The recovery rate has been shown to depend on a number of factors:

- *The status or seniority of the debtor.* Claims with higher seniority have higher recovery rates. More generally, a greater **debt cushion**, or the percentage of total company debt below the instrument, also leads to higher recovery rates.
- *The state of the economy.* Recovery rates tend to be higher (lower) when the economy is in an expansion (recession).
- *The obligor's characteristics.* Recovery rates tend to be higher when the borrower's assets are tangible and when the previous rating was high. Utilities have more **tangible assets**, such as power-generating plants, than other industries and consequently have higher recovery rates. Also, companies with greater interest coverage, as measures by higher credit ratings, typically have higher recovery rates.
- *The type of default.* Distressed exchanges, as opposed to bankruptcy proceedings, usually lead to higher recovery rates. Unlike a bankruptcy proceeding, which causes all debts to go into default, a distressed exchange only involves the instruments that have defaulted.

Ratings can also include the loss given default. The same borrower may have various classes of debt, which may have different credit ratings due to the different level of protection. If so, debt with lower seniority should carry a lower rating.

Table 19.7 displays recovery rates for corporate debt, from Moody's. The average recovery rate for senior unsecured debt is around $f = 37\%$. Derivative instruments rank as senior unsecured creditors and should have the same recovery rates as senior unsecured debt.

Bank loans are usually secured and therefore have higher recovery rates, typically around 60%. As expected, subordinated bonds have the lowest recovery rates, typically around 20% to 30%.

TABLE 19.7 Moody's Recovery Rates for Global Corporate Debt (Percent)

Priority	Count	Mean	S.D.	Min.	10th.	Median	90th.	Max.
All Bank Loans	310	61.6	23.4	5.0	25.0	67.0	90.0	98.0
Equipment Trust	86	40.2	29.9	1.5	10.6	31.0	90.0	103.0
Sr. Secured	238	53.1	26.9	2.5	10.0	34.0	82.0	125.0
Sr. Unsecured	1,095	37.4	27.2	0.3	7.0	30.0	82.2	122.6
Sr. Subordinated	450	32.0	24.0	0.5	5.0	27.0	66.5	123.0
Subordinated	477	30.4	21.3	0.5	5.0	27.1	60.0	102.5
Jr. Subordinated	22	23.6	19.0	1.5	3.8	16.4	48.5	74.0
All Bonds	2,368	36.8	26.3	0.3	7.5	30.0	80.0	125.0

Source: Adapted from Moody's, based on 1982–2002 defaulted bond prices

There is, however, much variation around the average recovery rates. The table reports not only the average value but also the standard deviation, minimum, maximum, and 10th and 90th percentiles. Recovery rates vary widely. In addition, recovery rates are negatively related to default rates. During years with more bond defaults, prices after default are more depressed than usual. This correlation creates bigger losses, which extends the tail of the credit loss distribution. In practice, the distribution of recovery rates is often modeled with a beta distribution, which has an argument ranging from 0 to 1.

The legal environment is also a main driver of recovery rates. Differences across national jurisdictions cause differences among recovery rates. Table 19.8 compares mean recovery rates across Europe and North America. Recovery rates are significantly higher in the United States than in Europe.

Using trading prices of debt shortly after default as estimates of recovery is convenient because the bankruptcy process can be slow, often taking years. Computing the total value of payments to debt holders can also be complicated, and should take into account the time value of money.

The evidence, however, is that trading prices are on average lower than the discounted recovery rate, as shown in Table 19.9. The average discounted recovery

TABLE 19.8 Moody's Mean Recovery Rates (Percent): Europe and North America

Instrument	Europe	North America
Bank loans	47.6	61.7
Bonds		
Senior secured	52.2	52.7
Senior unsecured	25.6	37.5
Senior subordinated	24.3	32.1
Subordinated	13.9	31.3
Junior subordinated	NA	24.5
All bonds	28.4	35.3
Preferred stock	3.4	10.9
All instruments	27.6	35.9

Source: Adapted from Moody's, from 1982–2002 defaulted bond prices

TABLE 19.9 S&P's Recovery Rates for Corporate Debt (Percent)

Instrument	Trading Prices 15–45 days	Discounted Recovery
Bank loans	58.0	81.6
Senior secured bonds	48.6	67.0
Senior unsecured bonds	34.5	46.0
Senior subordinated bonds	28.4	32.4
Subordinated bonds	28.9	31.2

Source: Adapted from S&P, from 1988–2002 defaulted debt

rate is systematically higher than the indication given by trading prices. This could be due to different clienteles for the two markets, or to a risk premium in trading prices. In other words, trading prices may be artificially depressed because investors want to get risk of defaulted securities in their portfolio. If so, this creates an interesting trading opportunity. Buying the defaulted debt and working through the recovery process should create value. Indeed, this largely explains the existence of the hedge fund category called **distressed securities funds.** Such funds invest in selected distressed securities and benefit from their subsequent increase in value.

EXAMPLE 19.13: FRM EXAM 2005—QUESTION 74

Moody's estimates the average recovery rate for senior unsecured debt to be nearest to

a. 20%
b. 40%
c. 60%
d. 80%

EXAMPLE 19.14: FRM EXAM 2002—QUESTION 123

The recovery rate on credit instruments is defined as one minus the loss rate. The loss rate can be significantly influenced by the volatility of the value of a firm's assets before default. All other things being equal, in the event of a default, which type of company would we expect to have the highest recovery rate?

a. A trading company active in volatile markets
b. An Internet merchant of trendy consumer products
c. An asset-intensive manufacturing company
d. A highly leveraged hedge fund

19.4 ASSESSING CORPORATE AND SOVEREIGN RATING

19.4.1 Corporate Ratings

Rating agencies expend considerable effort and financial resources in coming up with publicly available credit ratings. As explained in Table 19.2, the primary inputs for the credit rating process are accounting variables such as balance sheet leverage and debt coverage. The weight assigned to these variables may change if their informativeness changes over time (i.e., if earnings management is suspected).

By nature, however, accounting information is backward looking. The economic prospects of a company are also crucial for assessing credit risk. These include growth potential, market competition, and exposure to financial risk factors. Rating agencies also have access to private information, including meetings with management during which they might be provided with confidential information.

Ratings agencies also need to account for structural differences across countries. These could arise because of a number of factors:

- *Differences in financial stability across countries.* Countries differ in terms of financial market structures and government policies. The mishandling of economic policy can turn, for instance, what should be a minor devaluation into a major problem, leading to a recession.
- *Differences in legal systems.* The protection accorded to creditors can vary widely across countries, some of which have not yet established a bankruptcy process.

In theory, ratings provided by credit rating agencies are supposed to be *consistent* across countries and industrial sectors. In other words, they should take into account such variations and represent the same probability of default.

Finally, credit ratings are supposed to look **through the cycle**, where a business cycle typically covers several years. This means that the rating should not depend on the current position in the business cycle. There is no point in assigning a high rating to a company enjoying peak prosperity if that performance is expected to be temporary, for instance due to high consumer demand that will revert soon to a long-run average. Figure 19.6 illustrates how corporate performance depends on the cycle. In theory, ratings should be constant through time.

In practice, a down cycle can have a lasting impact on credit quality, in extreme cases leading to default. As a result, actual ratings may be affected by the business cycle, in particular for speculative-grade firms.

Ratings less sensitive to cyclical factors should be more stable over time. This should reduce the **procyclical effect** of capital charges based on external credit ratings. Lower credit ratings during a recession would lead to higher capital charges, precisely after banks have suffered credit losses, which would force them to raise additional equity and to reduce lending, which would aggravate the recession. This effect is further discussed in Chapter 29. On the other hand, the practice of rating

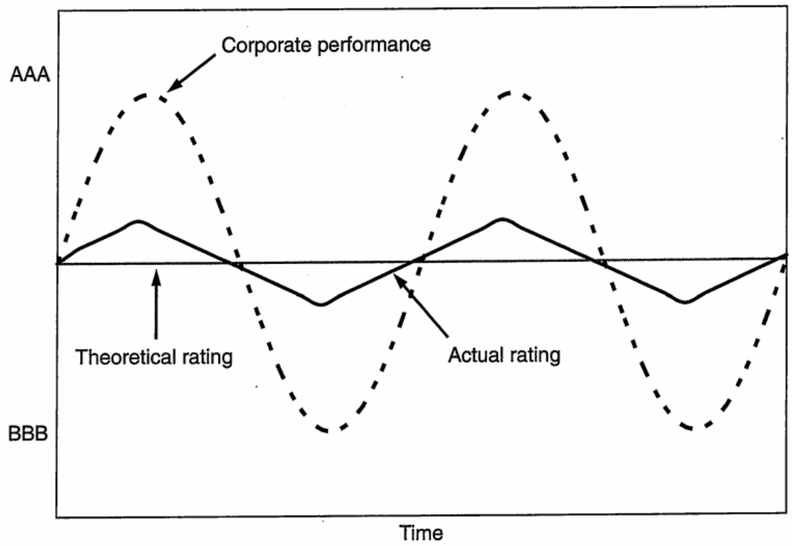


FIGURE 19.6 Business Cycle and Credit Ratings

through the cycle implies that credit ratings may underestimate the probabilities of default during a recession, and conversely during an expansion.

19.4.2 Sovereign Ratings

Rating agencies have only recently started to rate sovereign bonds. In 1975, S&P only rated seven countries, all of which were investment grade. By 1990, the pool had expanded to 31 countries, of which only nine were from emerging markets. Now, S&P rates approximately 120 countries. The history of default, however, is even more sparse. As a result, it is difficult to generalize from a very small sample.

Assessing credit risk for sovereign nations is significantly more complex than for corporates. When a corporate borrower defaults, legal action can be taken by the creditors. For instance, an unsecured creditor can file an action against a debtor and have the defendant's assets seized under a "writ of attachment." This creates a **lien** on its assets, or a claim on the assets as security for the payment of the debt. In contrast, it is impossible to attach the domestic assets of a sovereign nation. As a result, recovery rates on sovereign debt are usually lower than recovery rates on corporate debt. Thus, sovereign credit evaluation involves not only **economic risk** (the ability to repay debts when due), but also **political risk** (the willingness to pay). Some countries have unilaterally repudiated their debt, i.e., refused to make payments even when they had the ability to do so.

Sovereign credit ratings also differ depending on whether the debt is **local currency debt** or **foreign currency debt.** Table 19.10 displays the factors involved in local and foreign currency ratings.

Political risk factors (e.g., degree of political consensus, integration in global trade and financial system, and internal or external security risk) play an important part in sovereign credit risk. Countries with greater political stability have higher credit ratings.

TABLE 19.10 Credit Ratings Factors

Categories	Local Currency	Foreign Currency
Political risk	x	x
Income and economic structure	x	x
Economic growth prospects	x	x
Fiscal flexibility	x	x
Public debt burden	x	x
Contingent liabilities	x	x
Monetary flexibility	x	x
External liquidity		x
External debt burden		x

The second group of factors includes income and economic structure, and economic growth prospects. Countries that are richer or growing faster tend to have higher credit ratings.

The third group of factors includes fiscal flexibility, the public debt burden, and contingent liabilities. Countries with lower budget deficits and lower debt amounts in relation to the size of their economy tend to have higher credit ratings. The evaluation of debt, however, must also include public sector enterprises where a default would require government funding, as well as the financial sector, where the government may have to inject funds to ensure stability.

The fourth group includes monetary flexibility. High rates of inflation typically reflect economic mismanagement and are associated with political instability. Countries with high inflation rates tend to have low credit ratings.

All of the previous factors affect both *local currency debt* and *foreign currency debt*. Foreign currency debt is also affected by external liquidity and the external debt burden. External liquidity is assessed by balance of payment flows. Countries with large current account deficits, reflecting excess imports, tend to have low credit ratings, especially when financed by volatile capital inflows. In particular, the ratio of external interest payments to exports is closely watched. The maturity profile of flows is also important. In the case of the 1997 Asian crisis, rating agencies seem to have overlooked other important aspects of creditworthiness, such as the currency and maturity structure of national debt. Too many Asian creditors had borrowed short-term in dollars to invest in the local currency, which created a severe liquidity problem when their currency devalued. Finally, the external debt burden is assessed by the international investment position of a country (that is, public and private external debt) as well as the stock of foreign currency reserves.

Because local currency debt is backed by the taxation power of the government, local-currency debt is considered to have less credit risk than foreign-currency debt. Table 19.11 displays local- and foreign-currency debt ratings for a sample of countries. Ratings for foreign-currency debt are the same, or generally only one notch below, those of local currency debt. Similarly, sovereign debt is typically rated higher than corporate debt in the same country. Governments can repay foreign-currency debt, for instance, by controlling capital flows or seizing foreign-currency reserves.

TABLE 19.11 S&P's Sovereign Credit Ratings, December 2008

Issuer	Local Currency	Foreign Currency
Argentina	B−	B−
Australia	AAA	AAA
Belgium	AA+	AA+
Brazil	BBB+	BBB−
Canada	AAA	AAA
China	A+	A+
France	AAA	AAA
Germany	AAA	AAA
Hong Kong	AA+	AA+
India	BBB−	BBB−
Italy	A+	A+
Japan	AA	AA
South Korea	A+	A
Mexico	A+	BBB+
Netherlands	AAA	AAA
Russia	BBB+	BBB
South Africa	A+	BBB+
Spain	AAA	AAA
Switzerland	AAA	AAA
Taiwan	AA−	AA−
Thailand	A	BBB+
Turkey	BB	BB−
United Kingdom	AAA	AAA
United States	AAA	AAA

Overall, sovereign debt ratings are considered less reliable than corporate ratings. Indeed, bond spreads are generally greater for sovereigns than corporate issuers. There are also greater differences in sovereign ratings across agencies than for corporates. Thus, the evaluation of sovereign credit risk is a much more subjective process than for corporates.

EXAMPLE 19.15: FRM EXAM 2005—QUESTION 79

In the context of evaluating sovereign risk, which of the following statements is *incorrect*?

a. Bankruptcy law does not typically protect investors from sovereign risk.
b. Debt repudiation is a postponement of all current and future foreign debt obligations of a borrower.
c. Debt rescheduling occurs when a group of creditors declares a moratorium on debt obligations and seeks to reschedule terms.
d. Sovereign risk can be a cause of default in a non-governmental borrower of high credit quality.

EXAMPLE 19.16: FRM EXAM 2006—QUESTION 126

The 1997 Thai default was unusual compared to past sovereign defaults because:

a. The country repudiated its debt, whereas most defaults are reschedulings.
b. The country had a low inflation level, whereas most previous defaults had high inflation, largely as the result of fiscal deficits.
c. The country had a strong banking system, whereas most previous defaults arose from weakness in the financial intermediation arena.
d. The country was a strong exporter prior to the crisis, whereas most defaulting countries were net importers.

19.4.3 The Role of Credit Rating Agencies

Credit rating agencies play an important role in financial markets. They provide widely available summary information about the credit risk of a wide range of instruments and obligors. They help mitigate the asymmetry of information between borrowers and investors. This is especially useful for small investors, who do not have the time nor the resources for detailed credit analyses.

Investors use these credit ratings to assess credit risk and to comply with investment guidelines and regulations. Sell-side firms such as broker–dealers use them to determine the amount of collateral to hold against credit exposure.

The role of credit ratings is also officially recognized. Indeed, credit rating agencies registered with the Securities and Exchange Commission (SEC) are known as **Nationally Recognized Statistical Rating Organizations** (NSRO). Several regulations at the federal and state levels, for example, explicitly use ratings from NSROs. Most recently, the Basel II rules for commercial banks, which will be explained in more detail in Chapter 29, include capital charges that depend on external credit ratings.

Credit rating agencies, however, are beset by conflicts of interest. Even though their rating is viewed as an "opinion" to the reader, they are paid directly by the firms that they rate. Thus, on the one hand, they may have an incentive to *maximize profits* by providing easy ratings to many client firms. On the other hand, they have a countervailing incentive to *protect their reputation* as a neutral third party that provides independent and objective advice. If their ratings became worthless, they would quickly lose their business franchise.

Credit rating agencies argue that there is no strong incentive to accommodate the preferences of bond issuers because each single issuer represents a small fraction of revenues. For a long time, these countervailing incentives were delicately balanced and the system seemed to work well.

In recent years, however, credit rating agencies have expanded into rating the different tranches of structured credit products. This was a booming market, with complex products that will be described in Chapter 22. The business of rating structured products was highly profitable, much more so than before. Credit rating agencies were also involved in the design of the products, for instance offering advice as to the width of various tranches to achieve the desired rating. This was quite a step from giving an opinion on the default risk of a bond issued by a corporate entity. The banks that designed these structured products could also "shop around" the three major credit rating agencies, giving their business to the most accommodating one. These pressures seem to have led to a marked relaxation of credit standards. Credit rating agencies gave very high grades to securities that quickly went bad when subprime-backed debt started to default in 2007.

Investors, burned by these losses, lost faith in the quality of credit ratings and withdrew from credit markets. This put in motion the sequence of events that led to the recent credit crisis.

The credit rating agencies are widely viewed as having played a role in this crisis. As a result, the SEC strengthened the regulation of CRAs. It now prohibits them from providing both ratings and advice on how to structure securities. Agencies are also now required to provide information on the history of upgrades and downgrades over time for each asset class. In addition, EU regulators made it clear that self-regulation was no longer viewed as adequate and that further regulation was necessary.

19.5 IMPORTANT FORMULAS

Credit ratings by Standard & Poor's (modifiers, +, −):
 AAA, AA, A, BBB (investment grade); BB, B, CCC and below (speculative grade)

Credit ratings by Moody's (modifiers, 1, 2, 3):
 Aaa, Aa, A, Baa (investment grade); Ba, B, Caa and below (speculative grade)

Default rate $\bar{X}$ mean and variance: $E(\bar{X}) = p,\ \ V(\bar{X}) = \frac{p(1-p)}{N}$

Marginal default rate for firm initially rated R during year $T = t + N$:
 $d_N(R) = \frac{m[t+N]}{n[t+N]}$

Survival rate for N years: $S_N(R) = \Pi_{i=1}^{N}(1 - d_i(R))$

Marginal default rate from start to year T: $k_N(R) = S_{N-1}(R)d_N(R)$

Cumulative default rate: $C_N(R) = k_1(R) + k_2(R) + \cdots + k_N(R) = 1 - S_N(R)$

Average default rate, d: $C_N = 1 - \Pi_{i=1}^{N}(1 - d_i) = 1 - (1 - d)^N$

19.6 ANSWERS TO CHAPTER EXAMPLES

Example 19.1: Definition of a Credit Event

b. Calling back a bond occurs when the borrower wants to refinance its debt at a lower cost, which is not a credit event.

Example 19.2: FRM Exam 2003—Question 100

c. Baa3 is the lowest investment-grade rating for Moody's.

Example 19.3: FRM Exam 2005—Question 86

c. The lowest investment-grade ratings are BBB and Baa.

Example 19.4: FRM Exam 2002—Question 110

d. The BB rating by S&P is similar to a Ba rating by Moody's. A BB bond will have lower default rate than a bond rated lower. Hence, the answer is the next lower rating category by Moody's.

Example 19.5: FRM Exam 2004—Question 1

c. The probability of survival for one year is $S_1 = (1 - d) = (1 - d^Q)^4$. This gives a probability of surviving the first quarter of $(1 - d^Q) = (1 - 0.10)^{1/4} = 0.974$.

Example 19.6: FRM Exam 2002—Question 77

d. The marginal default rate increases with maturity. So, this could be for example 0.50% over the last three years of the six-year period. This gives a cumulative default probability greater than 0.60%.

Example 19.7: FRM Exam 2006—Question 21

a. The survival rate is $S_3 = (1 - d_1)(1 - d_2)(1 - d_3) = (1 - 0.08)(1 - 0.12)(1 - 0.15) = 68.8\%$.

Example 19.8: FRM Exam 2004—Question 14

b. This is one minus the survival rate over three years: $S_3(R) = (1 - d_1)(1 - d_2)(1 - d_3) = (1 - 0.03)(1 - 0.04)(1 - 0.06) = 0.8753$. Hence, the cumulative default rate is 0.1247.

Example 19.9: FRM Exam 2000—Question 34

a. The marginal default rate is the probability of defaulting over the next year, conditional on having survived to the beginning of the year.

Example 19.10: FRM Exam 2005—Question 105

b. A rating transition table has probabilities of changing from one rating to another over one year, which can be extrapolated over several years. Hence, statement a. is correct. This also include default, hence statement c. is correct. The probabilities can be used to group ratings, hence, statement d. is correct. Transitions matrices have no information about prices, so answer b. is the correct one.

Example 19.11: FRM Exam 2007—Question 51

c. This is given by the ratio of entries to BBB and D, which is $2 + 3$ over the total of 52, which is 0.096.

Example 19.12: FRM Exam 2003—Question 59

d. B can go into default the first year, with probability of 0.02. Or it could go to A then D, with probability of $0.03 \times 0.00 = 0$. Or it could go to B then D, with probability of $0.90 \times 0.02 = 0.018$. Or it could go to C then D, with probability of $0.05 \times 0.14 = 0.007$. The total is 0.045.

Example 19.13: FRM Exam 2000—Question 58

a. The recovery rate on loans is typically higher than that on bonds. Hence, the credit rating, if it involves both probability of default and recovery, should be higher for loans than for bonds.

Example 19.14: FRM Exam 2002—Question 123

c. The recovery rate is higher when the assets of the firm in default consist of tangible assets that can be resold easily. More volatile assets mean that there is a greater probability of a fall in market value upon liquidation. So, the tangible assets of a manufacturing company is the best answer.

Example 19.15: FRM Exam 2005—Question 79

b. Statements a., c., and d. are all correct. Debt repudiation is a cancellation, not a postponement, so b. is incorrect.

Example 19.16: FRM Exam 2006—Question 126

b. Thailand did not repudiate its debt, so a. is false. Thailand had a weak banking system and suffered from a poor allocation of credit, so c. is false. Thailand had strong imports, so d. is false. Thailand had low inflation but a very large current account deficit finance by capital inflows that eventually led to a credit crisis.

CHAPTER 20

Measuring Default Risk from Market Prices

The previous chapter discussed how to quantify credit risk from categorization into credit risk ratings. Based on these external ratings, we can forecast credit losses from historical default rates and recovery rates.

Credit risk can also be assessed from market prices of securities whose values are affected by default. These include corporate bonds, equities, and credit derivatives. In principle, these should provide more up-to-date and accurate measures of credit risk because financial markets have access to a very large amount of information and are forward-looking. Agents also have very strong financial incentives to impound this information in trading prices. This chapter shows how to infer default risk from market prices.

Section 20.1 will show how to use information about the market prices of credit-sensitive bonds to infer default risk. In this chapter, we will call defaultable debt interchangeably "credit-sensitive," "corporate," and "risky" debt. Here *risky* refers to credit risk and not market risk. We show how to break down the yield on a corporate bond into a default probability, a recovery rate, and a risk-free yield.

Section 20.2 turns to equity prices. The advantage of using equity prices is that they are much more widely available and of much better quality than corporate bond prices. We show how equity can be viewed as a call option on the value of the firm and how a default probability can be inferred from the value of this option. This approach also explains why credit positions are akin to short positions in options and are characterized by distributions that are skewed to the left. Chapter 22 will discuss credit derivatives, which can also be used to infer default risk.

20.1 CORPORATE BOND PRICES

To assess the credit risk of a transaction with a counterparty, consider **credit-sensitive** bonds issued by the same counterparty. We assume that default is a state that affects all obligations equally.

20.1.1 Spreads and Default Risk

Assume for simplicity that the bond makes only one payment of \$100 in one period. We can compute a market-determined yield y^* from the price P^* as

$$P^* = \frac{\$100}{(1+y^*)} \tag{20.1}$$

This can be compared with the risk-free yield over the same period y.

The payoffs on the bond can be described by a simplified default process, which is illustrated in Figure 20.1. At maturity, the bond can be in default or not. Its value is \$100 if there is no default and $f \times \$100$ if default occurs, where f is the fractional recovery. We define π as the default rate over the period. How can we value this bond?

Using **risk-neutral pricing,** the current price must be the mathematical expectation of the values in the two states, discounting the payoffs at the risk-free rate. Hence,

$$P^* = \frac{\$100}{(1+y^*)} = \left[\frac{\$100}{(1+y)}\right] \times (1-\pi) + \left[\frac{f \times \$100}{(1+y)}\right] \times \pi \tag{20.2}$$

Note that the discounting uses the risk-free rate y because there is no risk premium with risk-neutral valuation. After rearranging terms,

$$(1+y) = (1+y^*)[1-\pi(1-f)] \tag{20.3}$$

which implies a default probability of

$$\pi = \frac{1}{(1-f)}\left[1 - \frac{(1+y)}{(1+y^*)}\right] \tag{20.4}$$

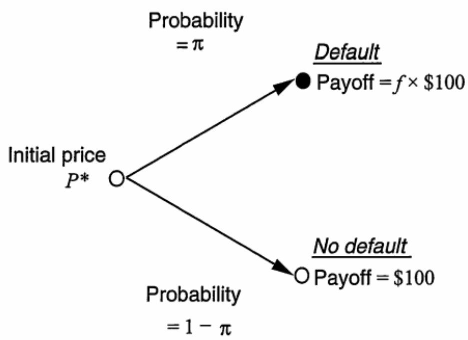


FIGURE 20.1 A Simplified Bond Default Process

Assuming that yields and default probabilities are small, and dropping second-order terms, this simplifies to

$$y^* \approx y + \pi(1 - f) \tag{20.5}$$

This equation shows that the credit spread $y^* - y$ measures credit risk. More specifically, it includes the probability of default, π, times the loss given default, $(1 - f)$. This makes sense because there is no potential credit loss either if the default probability is zero or if the loss given default is zero.

Let us now consider multiple periods, which number T. We compound interest rates and default rates over each period. In other words, π^a is now the *average* annual default rate. Assuming one payment only, the present value is

$$P^* = \frac{\$100}{(1+y^*)^T} = \left[\frac{\$100}{(1+y)^T}\right] \times (1-\pi^a)^T + \left[\frac{f \times \$100}{(1+y)^T}\right] \times [1-(1-\pi^a)^T] \tag{20.6}$$

which can be written as

$$(1+y)^T = (1+y^*)^T\{(1-\pi^a)^T + f[1-(1-\pi^a)^T]\} \tag{20.7}$$

Unfortunately, this does not simplify easily. Alternatively, using the cumulative default probability,

$$\frac{1}{(1+y^*)^T} = \left[\frac{1}{(1+y)^T}\right] \times (1-\pi) + \left[\frac{f \times 1}{(1+y)^T}\right] \times [1-(1-\pi)] \tag{20.8}$$

or

$$\frac{1}{(1+y^*)^T} = \frac{1}{(1+y)^T} \times [1 - \pi(1-f)] \tag{20.9}$$

for which a very rough approximation is

$$y^* \approx y + (\pi/T)(1 - f) \tag{20.10}$$

When we have risky bonds of various maturities, they can be used to compute default probabilities for different horizons. If we have two periods, for example, we could use Equation (20.3) to find the probability of defaulting over the first period, π_1, and Equation (20.7) to find the annualized, or average, probability of defaulting over the first two periods, π_2. As we saw in the previous chapter, the marginal probability of defaulting in the second period, d_2, is given by solving

$$(1-\pi_2)^2 = (1-\pi_1)(1-d_2) \tag{20.11}$$

This enables us to recover a term structure of forward default probabilities from a sequence of zero-coupon bonds. In practice, if we have access to only coupon-paying bonds, the computation becomes more complicated because we need to consider the payments in each period with and without default.

20.1.2 Risk Premium

It is worth emphasizing that the preceding approach assumed risk-neutrality. As in the methodology for pricing options, we assumed that the value of any asset grows at the risk-free rate and can be discounted at the same risk-free rate. Thus the probability measure π is a risk-neutral measure, which is not necessarily equal to the objective, physical probability of default.

Defining this objective probability as π' and the discount rate as y', the current price can be also expressed in terms of the true expected value discounted at the risky rate y':

$$P^* = \frac{\$100}{(1+y^*)} = \left[\frac{\$100}{(1+y')}\right] \times (1-\pi') + \left[\frac{f \times \$100}{(1+y')}\right] \times \pi' \tag{20.12}$$

Equation (20.4) allows us to recover a risk-neutral default probability only. More generally, if investors require some compensation for bearing credit risk, the credit spread will include a risk premium, rp:

$$y^* \approx y + \pi'(1-f) + \text{rp} \tag{20.13}$$

To be meaningful, this risk premium must be tied to some measure of bond riskiness as well as investor risk aversion. In addition, this premium may incorporate a **liquidity premium** and tax effects.[1]

KEY CONCEPT

The yield spread between a corporate bond and an otherwise identical bond with no credit risk reflects the expected actuarial loss, or annual default rate times the loss given default, plus a risk premium.

Example: Deriving Default Probabilities

We wish to compare a 10-year U.S. Treasury strip and a 10-year zero issued by International Business Machines (IBM), which is rated A by S&P and Moody's. The respective yields are 6% and 7%, using semiannual compounding. Assuming

[1] For a decomposition of the yield spread into risk premium effects, see Elton, E., M. Gruber, D. Agrawal, and C. Mann (2001), Explaining the Rate Spread on Corporate Bonds, *Journal of Finance*. The authors find a high risk premium, which is related to common risk factors from the stock market. Part of the risk premium is also due to tax effects. Because Treasury coupon payments are not taxable at the state level (for example, New York state), investors are willing to accept a lower yield on Treasury bonds, which increases the corporate yield spread.

that the recovery rate is 45% of the face value, what does the credit spread imply for the probability of default?

Equation (20.9) shows that $\pi(1 - f) = 1 - (1 + y/200)^{20}/(1 + y^*/200)^{20} = 0.0923$. Hence, $\pi = 9.23\%/(1 - 45\%) = 16.8\%$. Therefore, the cumulative (risk-neutral) probability of defaulting during the next 10 years is 16.8%. This number is rather high compared with the historical record for this risk class. Table 19.3 shows that Moody's reports a historical 10-year default rate for A credits around 3% only.

If these historical default rates are used as the future probability of default, the implication is that a large part of the credit spread reflects a risk premium. For instance, assume that 80 basis points out of the 100-basis-point credit spread reflects a risk premium. We change the 7% yield to 6.2% and find a probability of default of 3.5%. This is more in line with the actual default experience of such issuers.

EXAMPLE 20.1: FRM EXAM 2007—QUESTION 77

The risk-free rate is 5% per year and a corporate bond yields 6% per year. Assuming a recovery rate of 75% on the corporate bond, what is the approximate market implied one-year probability of default of the corporate bond?

a. 1.33%
b. 4.00%
c. 8.00%
d. 1.60%

EXAMPLE 20.2: FRM EXAM 2007—QUESTION 48

The spread on a one-year BBB-rated bond relative to the risk-free Treasury of similar maturity is 2%. It is estimated that the contribution to this spread by all noncredit factors (e.g., liquidity risk, taxes) is 0.8%. Assuming the loss given default rate for the underlying credit is 60%, what is, approximately, the implied default probability for this bond?

a. 3.33%
b. 5.00%
c. 3.00%
d. 2.00%

EXAMPLE 20.3: FRM EXAM 2002—QUESTION 96

A loan of $10 million is made to a counterparty whose expected default rate is 2% per annum and whose expected recovery rate is 40%. Assuming an all-in cost of funds of LIBOR for the lender, what would be the fair price for the loan?

a. LIBOR + 120 bp
b. LIBOR + 240 bp
c. LIBOR − 120 bp
d. LIBOR + 160 bp

20.1.3 The Cross-Section of Yield Spreads

We now turn to actual market data. Figure 20.2 illustrates a set of par yield curves for various credits as of December 1998. For reference, the spreads are listed in Table 20.1. The curves are sorted by credit rating, from AAA to B, using S&P's ratings.

These curves bear a striking resemblance to the cumulative default rate curves reported in the previous chapter. They increase with maturity and with lower credit quality.

The lowest curve is the Treasury curve, which represents risk-free bonds. Spreads for AAA credits are low, starting at 46 bp at short maturities and

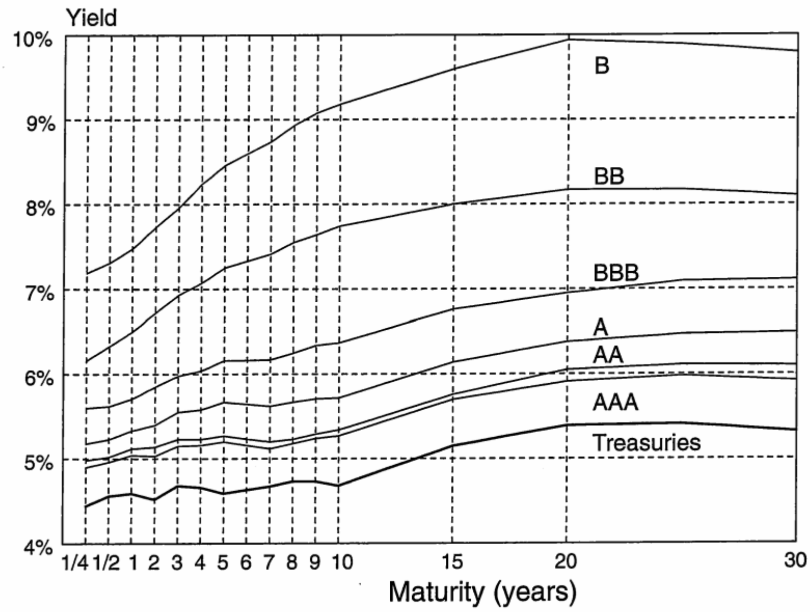


FIGURE 20.2 Yield Curves for Different Credits

TABLE 20.1 Credit Spreads

Maturity (Years)	Credit Rating AAA	AA	A	BBB	BB	B
3M	46	54	74	116	172	275
6M	40	46	67	106	177	275
1	45	53	74	112	191	289
2	51	62	88	133	220	321
3	47	55	87	130	225	328
4	50	57	92	138	241	358
5	61	68	108	157	266	387
6	53	61	102	154	270	397
7	45	53	95	150	274	407
8	45	50	94	152	282	420
9	51	56	98	161	291	435
10	59	66	104	169	306	450
15	55	61	99	161	285	445
20	52	66	99	156	278	455
30	60	78	117	179	278	447

increasing to 60 bp at longer maturities. Spreads for B credits are much wider; they also increase faster, from 275 to 450. Finally, note how close together the AAA and AA spreads are, in spite of the fact that default probabilities approximately double from AAA to AA. The transition from Treasuries to AAA credits most likely reflects other factors, such as liquidity and tax effects, rather than actuarial credit risk.

The previous sections showed that we could use information in corporate bond yields to make inferences about credit risk. Indeed, bond prices represent the best assessment of traders, or real "bets," on credit risk. Thus, we would expect bond prices to be the best predictors of credit risk and to outperform credit ratings. To the extent that agencies use public information to form their credit ratings, this information should be subsumed into market prices. Bond prices are also revised more frequently than credit ratings. As a result, movements in corporate bond prices tend to *lead* changes in credit ratings.

EXAMPLE 20.4: FRM EXAM 2002—QUESTION 81

Which of the following is true?

- **a.** Changes in bond spreads tend to lead changes in credit ratings.
- **b.** Changes in bond spreads tend to lag changes in credit ratings.
- **c.** Changes in bond spreads tend to occur at the exact same time as changes in credit ratings.
- **d.** There is absolutely no perceived general relationship in the timing of changes in bond spreads and changes in credit ratings.

EXAMPLE 20.5: TERM STRUCTURE OF CREDIT SPREADS

Suppose XYZ Corporation has two bonds paying semiannually according to the following table:

Remaining Maturity	Coupon (sa 30/360)	Price	T-Bill Rate (Bank Discount)
6 months	8.0%	99	5.5%
1 year	9.0%	100	6.0%

The recovery rate for each in the event of default is 50%. For simplicity, assume that each bond will default only at the end of a coupon period. The market-implied risk-neutral probability of default for XYZ Corporation is

a. Greater in the first six-month period than in the second
b. Equal between the two coupon periods
c. Greater in the second six-month period than in the first
d. Cannot be determined from the information provided

20.1.4 Time Variation in Credit Spreads

Credit spreads reflect potential losses caused by default risk, and perhaps a risk premium. Some of this default risk is specific to the issuer and requires a detailed analysis of its prospective financial condition. Part of this risk, however, can be attributed to common credit risk factors. These common factors are particularly important, as they cannot be diversified away in a large portfolio of credit-sensitive bonds.

First among these factors are general economic conditions. Economic growth is negatively correlated with credit spreads. When the economy slows down, more companies are likely to have cash-flow problems and to default on their bonds.

Figure 20.3 compares the speculative-grade default rate from Moody's and the Baa-Treasury credit spread. Shaded areas indicate periods of recession. The graph shows that both default rates and credit spreads tend to increase around recessions. Because spreads are forward-looking, however, they tend to lead default rates, which peak after recessions. Also, the effect of the 2007–2008 credit crisis is apparent from the unprecedented widening of credit spreads. These reflect a combination of higher risk aversion and anticipation of very high default rates.

Volatility is also a factor. In a more volatile environment, investors may require larger risk premiums, thus increasing credit spreads. When this happens, liquidity may also dry up. Investors may then require a greater credit spread in order to hold increasingly illiquid securities.

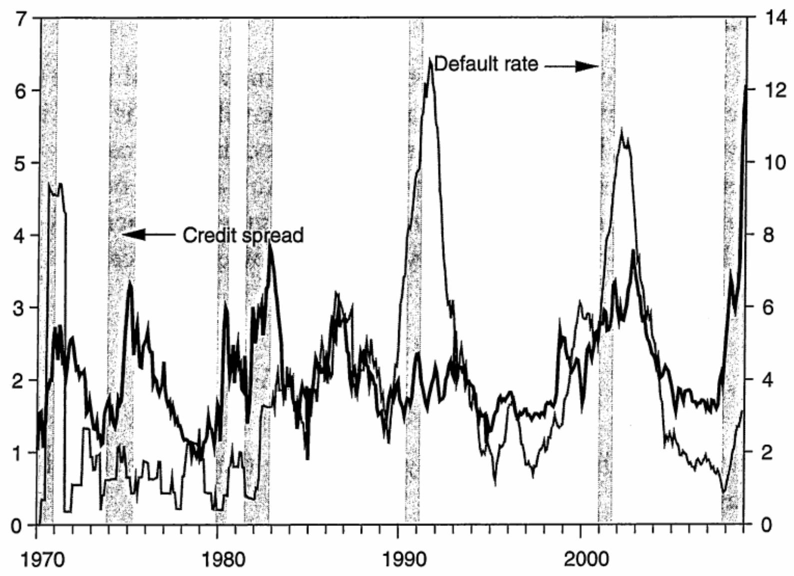


FIGURE 20.3 Default Rates and High Yield Spreads

Finally, volatility has another effect through an option channel. Corporate bond indices include many callable bonds, unlike Treasury indices. The buyer of a callable bond requires a higher yield in exchange for granting the call option. Higher volatility should increase the value of this option and therefore this yield, all else equal. Thus, credit spreads directly increase with volatility.

20.2 EQUITY PRICES

The credit spread approach, unfortunately, is only useful when there is good bond market data. The problem is that this is rarely the case, for a number of reasons.

- Many countries do not have a well-developed corporate bond market. As Table 7.2 has shown, the United States has by far the largest corporate bond market in the world. This means that other countries have much fewer outstanding bonds and a much less active market.
- The counterparty may not have an outstanding publicly traded bond or if so, the bond may contain other features such as a call that make it more difficult to interpret the yield.
- The bond may not trade actively and instead reported prices may simply be **matrix prices**, that is, interpolated from yields on other issuers.

An alternative is to turn to default risk models based on stock prices, because equity prices are available for a larger number of companies and because equities are more actively traded than corporate bonds. The Merton (1974) model views equity as akin to a call option on the assets of the firm, with an exercise price given by the face value of debt.

20.2.1 The Merton Model

To simplify to the extreme, consider a firm with total value V that has one bond due in one period with face value K. If the value of the firm exceeds the promised payment, the bond is repaid in full and stockholders receive the remainder. However, if V is less than K, the firm is in default and the bondholders receive V only. The value of equity goes to zero. We assume that there are no transaction costs and that the absolute-priority rule is followed. Hence, the value of the stock at expiration is

$$S_T = \text{Max}(V_T - K, 0) \tag{20.14}$$

Because the bond and equity add up to the firm value, the value of the bond must be

$$B_T = V_T - S_T = V_T - \text{Max}(V_T - K, 0) = \text{Min}(V_T, K) \tag{20.15}$$

The current stock price, therefore, embodies a forecast of default probability in the same way that an option embodies a forecast of being exercised. Figures 20.4 and 20.5 describe how the value of the firm can be split up into the bond and stock values.

Note that the bond value can also be described as

$$B_T = K - \text{Max}(K - V_T, 0) \tag{20.16}$$

In other words, a long position in a risky bond is equivalent to a long position in a risk-free bond plus a short put option, which is really a credit derivative.

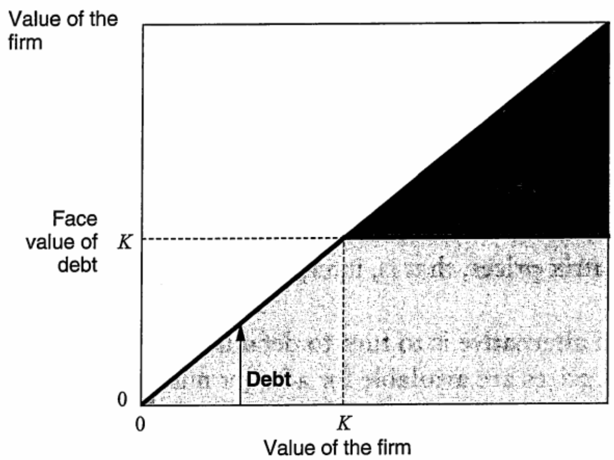


FIGURE 20.4 Equity as an Option on the Value of the Firm

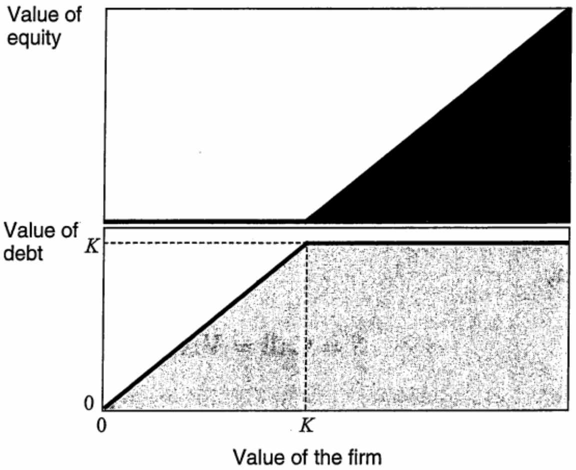


FIGURE 20.5 Components of the Value of the Firm

KEY CONCEPT

Equity can be viewed as a call option on the firm value with strike price equal to the face value of debt. Corporate debt can be viewed as risk-free debt minus a put option on the firm value.

This approach is particularly illuminating because it demonstrates that corporate debt has a payoff akin to a short position in an option, explaining the left skewness that is characteristic of credit losses. In contrast, equity is equivalent to a long position in an option due to its **limited-liability feature**. In other words, investors can lose no more than their equity investment.

20.2.2 Pricing Equity and Debt

To illustrate, we proceed along the lines of the usual Black–Scholes (BS) framework, assuming that the firm value follows the geometric Brownian motion process:

$$dV = \mu V\, dt + \sigma V\, dz \tag{20.17}$$

If we assume that markets are frictionless and that there are no bankruptcy costs, the value of the firm is simply the sum of the firm's equity and debt: $V = B + S$.

To price a claim on the value of the firm, we need to solve a partial differential equation with appropriate boundary conditions. The corporate bond price is obtained as

$$B = F(V, t), \quad F(V, T) = \mathrm{Min}[V, B_F] \tag{20.18}$$

where $B_F = K$ is the face value of the bond to be repaid at expiration, or the strike price.

Similarly, the equity value is

$$S = f(V, t), \qquad f(V, T) = \text{Max}[V - B_F, 0] \tag{20.19}$$

Stock Valuation With no dividend, the value of the stock is given by the BS formula,

$$S = \text{Call} = VN(d_1) - Ke^{-r\tau}N(d_2) \tag{20.20}$$

where $N(d)$ is the cumulative distribution function for the standard normal distribution, and

$$d_1 = \frac{\ln(V/Ke^{-r\tau})}{\sigma\sqrt{\tau}} + \frac{\sigma\sqrt{\tau}}{2}, \qquad d_2 = d_1 - \sigma\sqrt{\tau}$$

where $\tau = T - t$ is the time to expiration, r the risk-free interest rate, and σ the volatility of *asset* value. The option value depends on two factors, $x = Ke^{-r\tau}/V$ and $\sigma\sqrt{\tau}$. The first factor is the debt/value ratio and is inversely related to leverage, which can be written as $l = V/(V - Ke^{-r\tau})$. The value of the stock increases as the volatility σ increases and as x decreases, or when leverage increases.

Firm Volatility Note that this application is different from the BS model, where we plug in the value of V and of its volatility, $\sigma = \sigma_V$, and solve for the value of the call. Here we observe the market value of the firm S and the equity volatility σ_S and must infer the values of V and its volatility so that Equation (20.20) is satisfied. This can only be done iteratively. Defining $\Delta = N(d_1)$ as the hedge ratio, we have

$$dS = \frac{\partial S}{\partial V}dV = \Delta dV \tag{20.21}$$

Defining σ_S as the volatility of (dS/S), we have $(\sigma_S S) = \Delta(\sigma_V V)$ and

$$\sigma_V = (1/\Delta)\,\sigma_S(S/V) \tag{20.22}$$

Bond Valuation Next, the value of the bond is given by $B = V - S$, or

$$B = Ke^{-r\tau}N(d_2) + V[1 - N(d_1)] \tag{20.23}$$

$$B/Ke^{-r\tau} = [N(d_2) + (V/Ke^{-r\tau})N(-d_1)] \tag{20.24}$$

Hence, the value of the bond is related to the firm by

$$dB = \frac{\partial B}{\partial V}dV = N(-d_1)dV \tag{20.25}$$

Equations (20.21) and (20.25) are sometimes used for **capital arbitrage** trades, which involve buying and selling different types of claims on the firm, using these hedge ratios to try to minimize risk—that is, assuming the model is correct.

The bond price can be expressed in terms of the annualized credit spread s

$$B = Ke^{-(r+s)\tau} \tag{20.26}$$

which gives

$$s = -(1/\tau)[N(d_2) + (1/x)N(-d_1)] \tag{20.27}$$

The value of the bond decreases when volatility increases and when leverage increases. The spread moves conversely. Equation (20.27) can create rich patterns in the term structure of credit spreads. For firms with low leverage and volatility, spreads are low for near maturities and increase with maturity. Firms with high leverage have high spreads, with a term structure that can have a negative slope.

Risk-Neutral Dynamics of Default In the Black–Scholes model, $N(d_2)$ is also the probability of exercising the call, or that the bond will not default. Conversely, $1 - N(d_2) = N(-d_2)$ is the risk-neutral probability of default.

Pricing Credit Risk At maturity, the credit loss is the value of the risk-free bond minus the corporate bond, $\text{CL} = B_F - B_T$. At initiation, the expected credit loss (ECL) is

$$\begin{aligned} B_F e^{-r\tau} - B &= Ke^{-r\tau} - \{Ke^{-r\tau}N(d_2) + V[1 - N(d_1)]\} \\ &= Ke^{-r\tau}[1 - N(d_2)] - V[1 - N(d_1)] \\ &= Ke^{-r\tau}N(-d_2) - VN(-d_1) \\ &= N(-d_2)[Ke^{-r\tau} - VN(-d_1)/N(-d_2)] \end{aligned}$$

This decomposition is quite informative. Multiplying by the future value factor $e^{r\tau}$ shows that the ECL at maturity is

$$\text{ECL}_T = N(-d_2)[K - Ve^{r\tau}N(-d_1)/N(-d_2)] = p \times [\text{Exposure} \times \text{LGD}] \tag{20.28}$$

This involves two terms. The first is the probability of default, $N(-d_2)$. The second, between brackets, is the loss when there is default. This is obtained as the face value of the bond K minus the recovery value of the loan when in default, $Ve^{r\tau}N(-d_1)/N(-d_2)$, which is also the expected value of the firm in the state of default. Note that the recovery rate is endogenous here, as it depends on the value of the firm, time, and debt ratio.

Credit Option Valuation This approach can also be used to value the put option component of the credit-sensitive bond. This option pays $K - B_T$ in case of default.

A portfolio with the bond plus the put is equivalent to a risk-free bond $Ke^{-r\tau} = B +$ put. Hence, using Equation (20.23), the credit put should be worth

$$\text{Put} = Ke^{-r\tau} - \{Ke^{-r\tau}N(d_2) + V[1 - N(d_1)]\} = -V[N(-d_1)] + Ke^{-r\tau}[N(-d_2)] \tag{20.29}$$

This will be used later in the chapter on credit derivatives.

EXAMPLE 20.6: FRM EXAM 2001—QUESTION 14

To what sort of option on the counterparty's assets can the current exposure of a credit-risky position better be compared?

a. A short call
b. A short put
c. A short knock-in call
d. A binary option

20.2.3 Applying the Merton Model

These valuation formulas can be used to recover, given the current value of equity and of nominal liabilities, the value of the firm and its probability of default. Figure 20.6 illustrates the evolution of the value of the firm. The firm defaults if this value falls below the liabilities at the horizon. We measure this risk-neutral probability by $N(-d_2)$.

In practice, default is much more complex than depicted here. We would have to collect information about all the liabilities of the company, as well as their

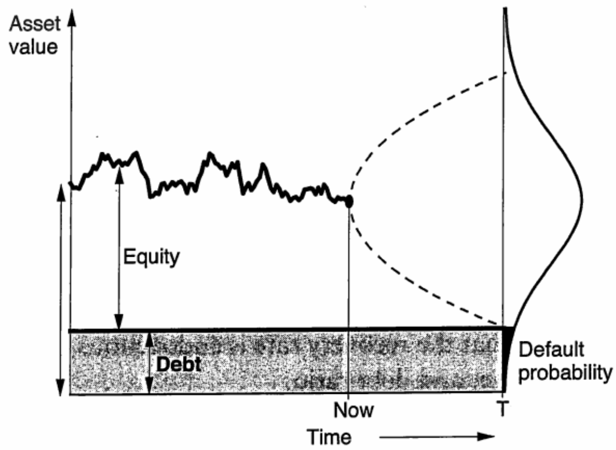


FIGURE 20.6 Default in the Merton Model

maturities. Default can also occur with coupon payments. So instead of default on a target date, we could measure default probability as a function of the distance relative to a moving floor that represents liabilities. This is essentially the approach undertaken by **KMV Corporation**, now part of Moody's, which sells **estimated default frequencies** (EDFs) for global firms. The approach will be explained in Chapter 23.

The Merton approach has many advantages. First, it relies on the prices of equities, which are more actively traded than bonds. Second, correlations between equity prices can generate correlations between defaults, which would be otherwise difficult to measure. Perhaps the most important advantage of this model is that it generates movements in EDFs that seem to *lead* changes in credit ratings.

Figure 20.7 displays movements in EDFs and credit rating for WorldCom, using the same vertical scale. WorldCom went bankrupt on July 21, 2002. With $104 billion in assets, this was America's largest bankruptcy ever. The agency rating was BBB until April 2002. It gave no warning of the impending default. In contrast, starting one year before the default, the EDF began to move up. In April, it reached 20%, presaging bankruptcy.

These models have disadvantages as well. The first limitation of the model is that it cannot be used to price sovereign credit risk, as countries obviously do not have a stock price. This is a problem for credit derivatives, where a large share of the market consists of sovereign risks.

A more fundamental drawback is that it relies on a static model of the firm's capital and risk structure. The debt level is assumed to be constant over the horizon. Similarly, the model cannot handle new injections of equity, which protect

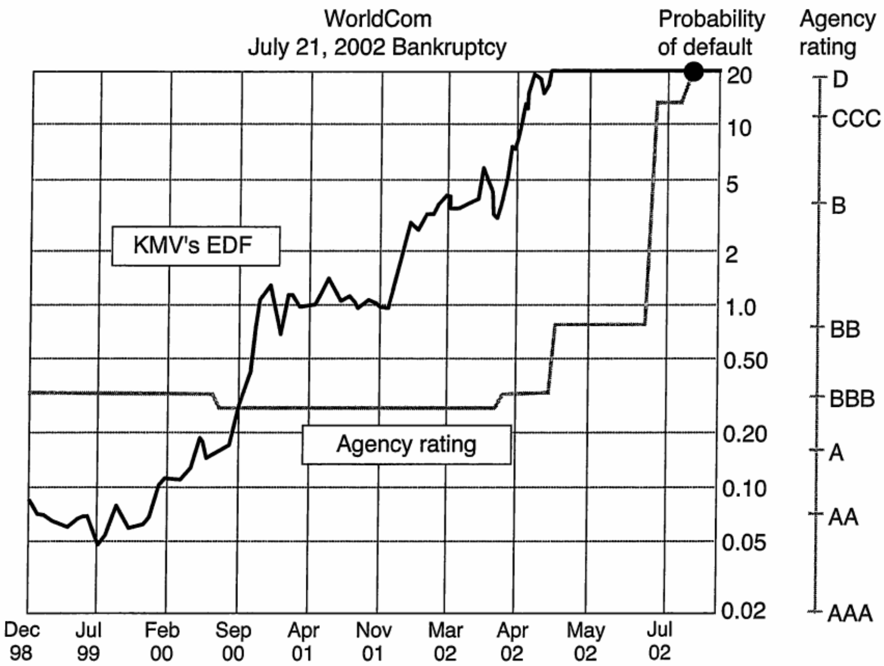


FIGURE 20.7 KMV's EDF and Credit Rating

existing debt holders. Also, the model needs to be expanded to a more realistic setting where debt matures at various points in time, which is not an obvious extension.

Another problem is that management could undertake new projects that increase not only the value of equity but also its volatility, thereby increasing the credit spread. This runs counter to the fundamental intuition of the Merton model, which is that, all else equal, a higher stock price reflects a lower probability of default and hence should be associated with a smaller credit spread.

Finally, this class of models fails to explain the magnitude of credit spreads we observe on credit-sensitive bonds. Recent work has attempted to add other sources of risk, such as interest rate risk, but still falls short of explaining these spreads. Thus these models are most useful in tracking *changes* in EDFs over time. Indeed, KMV calibrates the risk-neutral default probabilities to actual default data.

20.2.4 Example

It is instructive to work through a simplified example. Consider a firm with assets worth $V = \$100$ and with volatility $\sigma_V = 20\%$. In practice, one would have to start from the observed stock price and volatility and iterate to find σ_V.

The horizon is $\tau =$ one year. The risk-free rate is $r = 10\%$ using continuous compounding. We assume a leverage factor such that $x = 0.9$, which implies a face value of $K = \$99.46$ and a risk-free current value of $Ke^{-r\tau} = \$90$.

Working through the Merton analysis, one finds that the current stock price should be $S = \$13.59$. Hence, the current bond price is

$$B = V - S = \$100 - \$13.59 = \$86.41$$

which implies a yield of $\ln(K/B)/\tau = \ln(99.46/86.41) = 14.07\%$, or a yield spread of 4.07%. The current value of the credit put is then

$$P = Ke^{-r\tau} - B = \$90 - \$86.41 = \$3.59$$

The analysis also generates values for $N(d_2) = 0.6653$ and $N(d_1) = 0.7347$. Thus, the *risk-neutral* probability of default is EDF $= N(-d_2) = 1 - N(d_2) =$ 33.47%. Note that this could differ from the *actual* or *objective* probability of default since the stock could very well grow at a rate that is greater than the risk-free rate of 10%.

Finally, let us decompose the expected loss at expiration from Equation (20.28), which gives

$$\begin{aligned} N(-d_2)[K - Ve^{r\tau}N(-d_1)/N(-d_2)] &= 0.3347 \times [\$99.46 - \$110.56 \times 0.2653/0.3347] \\ &= 0.3347 \times [\$11.85] = \$3.96 \qquad (20.30) \end{aligned}$$

This combines the probability of default with the expected loss upon default, which is \$11.85. This future expected credit loss of \$3.96 must also be the future value of the credit put, or $\$3.59e^{r\tau} = \$3.96$.

Note that the model needs very high leverage, here $x = 90\%$, to generate a reasonable credit spread of 4.07%. This implies a debt-to-equity ratio of $0.9/0.1 = 900\%$, which is unrealistically high for this type of spread.

With lower leverage, say $x = 0.7$, the credit spread shrinks rapidly, to 0.36%. At $x = 50\%$ or below, the predicted spread goes to zero. As this leverage would be considered normal, the model fails to reproduce the size of observed credit spreads. Perhaps it is most useful for tracking time variation in estimated default frequencies.

EXAMPLE 20.7: FRM EXAM 2002—QUESTION 97

Among the following variables, which one is the main driver of the probability of default in the KMV model?

a. Stock prices
b. Bond prices
c. Bond yield
d. Loan prices

EXAMPLE 20.8: FRM EXAM 2003—QUESTION 41

Which of the following is *not* a limitation of KMV's EDF model?

a. It is difficult to price sovereign credit risk since asset values and volatility are not directly observable.
b. EDFs are biased by periods of high or low defaults.
c. Takes a simplified view of the capital structure of a firm.
d. The model often fails to explain real-world credit spreads.

EXAMPLE 20.9: FRM EXAM 2005—QUESTION 108

The KMV model produces a measure called Expected Default Frequency. Which of the following statements about this variable is correct?

a. It decreases when the leverage of the firm falls.
b. It increases when the stock price of the firm has been rising.
c. It is the risk-neutral probability of default from Merton's model.
d. It tells investors how the default risk of a bond is correlated with the default risk of other bonds in the portfolio.

EXAMPLE 20.10: FRM EXAM 2007—QUESTION 82

Using the Merton model, the value of the debt increases if all other parameters are fixed and

I. The value of the firm decreases.
II. The riskless interest rate decreases.
III. Time to maturity increases.
IV. The volatility of the firm value decreases.

a. I and II only
b. I and IV only
c. II and III only
d. II and IV only

EXAMPLE 20.11: FRM EXAM 2005—QUESTION 134

You have a large position of bonds of firm XYZ. You hedge these bonds with equity using Merton's debt valuation model. The value of the debt falls unexpectedly, but the value of equity does not fall, so you make a loss. Consider the following statements:

I. Interest rates increased.
II. Volatility fell.
III. Volatility increased.
IV. A liquidity crisis increased the liquidity component of the credit spreads.

Which statements are possible explanations for why your hedge did not work out?

a. I and II only
b. I and III only
c. I, III, and IV only
d. III and IV only

20.3 IMPORTANT FORMULAS

Implied default probability, 1 period: $(1 + y) = (1 + y^*)[1 - \pi(1 - f)]$

Approximation of implied default probability: $y^* \approx y + \pi(1 - f)$

Implied default probability, T period:

$$(1 + y)^T = (1 + y^*)^T\{(1 - \pi)^T + f[1 - (1 - \pi)^T]\}$$

Approximation of physical default probability: $y^* \approx y + \pi'(1 - f) + \text{rp}$
Merton model for stock price: $S_T = \text{Max}(V_T - K, 0)$
Merton model for bond price: $B_T = V_T - S_T = \text{Min}(V_T, K)$
Stock valuation: $S = \text{Call} = VN(d_1) - Ke^{-r\tau}N(d_2)$
Firm value and stock volatility: $\sigma_V = (1/\Delta)\,\sigma_S(S/V)$
Bond valuation:
$B = \text{Risk-free bond} - \text{Put}$, $B/Ke^{-r\tau} = [N(d_2) + (V/Ke^{-r\tau})N(-d_1)]$
Risk-neutral PD: $1 - N(d_2) = N(-d_2)$
Credit default swap, or put option:
$\text{Put} = Ke^{-r\tau} - \{Ke^{-r\tau}N(d_2) + V[1 - N(d_1)]\} = -V[N(-d_1)] + Ke^{-r\tau}[N(-d_2)]$

20.4 ANSWERS TO CHAPTER EXAMPLES

Example 20.1: FRM Exam 2007—Question 77

b. The spread is $7 - 6 = 1\%$. Dividing by the loss given default of $(1 - f) = 0.25$, we get $\pi = (y^* - y)/(1 - f) = 4\%$.

Example 20.2: FRM Exam 2007—Question 48

d. The part of the spread due to expected credit losses is $2.00 - 0.80 = 1.20\%$. Dividing by the LGD of $(1 - f) = 0.65$, we get 2%.

Example 20.3: FRM Exam 2002—Question 96

a. The credit spread should be $y^* - y = \pi(1 - f)$. Thus, $\pi(1 - f) = 2\%(1 - 40\%) = 1.2\%$. The spread over LIBOR should be 120 bp.

Example 20.4: FRM Exam 2002—Question 81

a. Changes in market prices, including bond spreads, tend to lead to changes in credit ratings. This is because market prices reflect all publicly available information about a company.

Example 20.5: Term Structure of Credit Spreads

a. First, we compute the current yield on the six-month bond, which is selling at a discount. We solve for y^* such that $99 = 104/(1 + y^*/200)$ and find $y^* = 10.10\%$. Thus the yield spread for the first bond is $10.1 - 5.5 = 4.6\%$. The second bond is at par, so the yield is $y^* = 9\%$. The spread for the second bond is $9 - 6 = 3\%$. The default rate for the first period must be greater. The recovery rate is the same for the two periods, so it does not matter for this problem.

Example 20.6: FRM Exam 2001—Question 14

b. The lender is short a put option, since exposure exists only if the value of assets falls below the amount lent.

Example 20.7: FRM Exam 2002—Question 97

a. Stock prices are the main driver of KMV's estimated default frequency (EDF), because they drive the value of equity. These models also use the volatility of asset values and the value of liabilities.

Example 20.8: FRM Exam 2003—Question 41

b. Answer a. is a limitation because there is no asset value for sovereign debt. Answer c. is also a limitation because this is a simple model. Answer d. is also a limitation as this model generates spreads that are too low. Finally, b. is not a limitation because stock prices pick up time variation in default rates, unlike credit ratings, which are "through the cycle" and thus less responsive to temporary changes in default rates.

Example 20.9: FRM Exam 2005—Question 108

a. The EDF, similarly to the risk-neutral PD, decreases when the stock prices goes up, when the leverage goes down, or when the volatility goes down. It is a transformation of the PD from a Merton-type model. The KMV framework can be extended to finding correlations, but the EDF is not sufficient.

Example 20.10: FRM Exam 2007—Question 82

b. The value of credit-sensitive debt is $B = Ke^{-(r+s)\tau}$. This increases (1) if the risk-free interest rate decreases, or (2) if the credit spread decreases, or (3) if the maturity decreases. The credit spread decreases if the value of the firm goes up, if the leverage goes down, or if the volatility goes down. Hence, the value of debt increases if the riskless rate decreases or if the volatility decreases.

Example 20.11: FRM Exam 2005—Question 134

b. We need to identify shocks that decrease the value of debt but not that of equity. An increase in the risk-free rate will decrease the value of the debt but not the equity (because this decreases leverage). An increase in volatility will have the opposite effect on debt and equity. Finally, a liquidity crisis cannot explain the divergent behavior, because, as we have seen during 2008, it would affect both corporate bonds and equity adversely.

CHAPTER 21
Credit Exposure

Credit exposure is the amount at risk during the life of the financial instrument. Upon default, it is called **exposure at default** (EAD). When banking simply consisted of making loans, exposure was essentially the face value of the loan. In this case, the exposure is the notional amount and is fixed.

Since the development of the swap markets, the measurement of credit exposure has become more complicated. This is because swaps, like most derivatives, have an up-front value that is much smaller than the notional amount. Indeed, the initial value of a swap is typically zero, which means that at the outset, there is no credit risk because there is nothing to lose.

As the swap contract matures, however, it can turn into a positive or negative value. The asymmetry of bankruptcy treatment is such that a credit loss can only occur if the instrument has positive value, or is a claim against the defaulted counterparty. Thus, the credit exposure is the value of the asset if it is positive, like an option.

This chapter turns to the quantitative measurement of credit exposure. Section 21.1 describes the general features of credit exposure for various types of financial instruments, including loans or bonds, guarantees, credit commitments, repos, and derivatives. Section 21.2 shows how to compute the distribution of credit exposure and gives detailed examples of exposures of interest rate and currency swaps. Section 21.3 discusses exposure modifiers, or techniques that have been developed to reduce credit exposure. It shows how credit risk can be controlled by marking to market, margins, position limits, recouponing, and netting agreements. For completeness, Section 21.4 includes credit risk modifiers such as credit triggers and time puts, which also control default risk.

21.1 CREDIT EXPOSURE BY INSTRUMENT

Credit exposure is the positive part of the value of the asset at various points during its life. In particular, the **current exposure** is the value of the asset at the current time V_t if positive:

$$\mathrm{CE}_t = \mathrm{Max}(V_t, 0) \tag{21.1}$$

The **potential exposure** represents the exposure on some future date, or sets of dates. Based on this definition, we can characterize the exposure of a variety of financial instruments. The measurement of current and potential exposure also motivates regulatory capital charges for credit risk, which are explained in Chapter 29.

21.1.1 Loans or Bonds

Loans or **bonds** are balance sheet assets whose current and potential exposure basically is the notional, or amount loaned or invested. To be more precise, this should be the market value of the asset given current interest rates, but, as we will show, this is not very far from the notional. The exposure is also the notional for **receivables** and **trade credits**, as the potential loss is the amount due.

21.1.2 Guarantees

Guarantees are off–balance sheet contracts whereby the bank has underwritten, or agrees to assume, the obligations of a third party. The exposure is the notional amount, because this will be fully drawn when default occurs. By nature, guarantees are **irrevocable**, that is, unconditional and binding, whatever happens.

An example of a **guarantee** is a contract whereby bank A makes a loan to client C only if it is guaranteed by bank B. Should C default, B is exposed to the full amount of the loan. Another example is an **acceptance**, whereby a bank agrees to pay the face value of a bill at maturity. Alternatively, **standby facilities**, or **financial letters of credit**, provide a guarantee to a third party of the making of a payment should the obligor default.

21.1.3 Commitments

Commitments are off–balance sheet contracts whereby the bank commits to a future transaction that may result in creating a credit exposure at a *future* date. For instance, a bank may provide a **note issuance facility** whereby it promises a minimum price for notes regularly issued by a borrower. If the notes cannot be placed at the market at the minimum price, the bank commits to buy them at a fixed price. Such commitments have less risk than guarantees because it is not certain that the bank will have to provide backup support.

It is also useful to distinguish between **irrevocable commitments**, which are unconditional and binding on the bank, and **revocable commitments**, where the bank has the option to revoke the contract should the counterparty's credit quality deteriorate. This option substantially decreases the credit exposure.

21.1.4 Swaps or Forwards

Swaps or **forwards** contracts are off–balance sheet items that can be viewed as irrevocable commitments to purchase or sell some asset on prearranged terms. The current and potential exposure will vary from zero to a large amount depending on movements in the driving risk factors. Similar arrangements are **sale–repurchase agreements** (repos), whereby an institution sells an asset to another in exchange for a promise to buy it back later.

21.1.5 Long Options

Options are off–balance sheet items that may create credit exposure. The current and potential exposure also depends on movements in the driving risk factors. Here there is no possibility of negative values because options always have positive value, or zero value at worst: $V_t \geq 0$.

21.1.6 Short Options

Unlike long options, the current and potential exposure for short options is zero because the bank writing the option can incur only a negative cash flow, assuming the option premium has been fully paid.

Exposure also depends on the features of any embedded option. With an American option, for instance, the holder of an in-the-money swap may want to exercise early if the credit rating of its counterparty starts to deteriorate. This decreases the exposure relative to an equivalent European option.

EXAMPLE 21.1: FRM EXAM 2006—QUESTION 95

A credit loss on market-driven instruments such as swaps and forwards arises if:

a. Market rates move in your favor.
b. Market rates move against you.
c. Market rates move against you and the counterparty defaults.
d. Market rates move in your favor and the counterparty defaults.

EXAMPLE 21.2: FRM EXAM 2002—QUESTION 93

Which transaction does *not* result in a long-term credit risk for party A?

a. Party A makes an unsecured loan to party B.
b. Party A is a fixed-price receiver in an interest rate swap from party B.
c. Party A buys a call option on September wheat from party B.
d. Party A sells a put option on the S&P 500 index to party B.

EXAMPLE 21.3: FRM EXAM 2006—QUESTION 117

Which of the following will have the greatest potential credit exposure?

a. Long 3,000 ounces of gold for delivery in one year
b. Long 3,000 ounces of gold for delivery in two years
c. Short 3,000 ounces of gold for delivery in two years
d. Selling an at-the-money call option on 10,000 ounces of gold for delivery in two years

EXAMPLE 21.4: FRM EXAM 2004—QUESTION 8

Your company has reached its credit limit to Ford but Ford is insisting that your firm provide them some increased protection in the event a major project they are undertaking results in some unforeseen liability. Ignoring settlement risk and assuming option premiums are paid immediately at the time of the transaction, which of these strategies will *not* give rise to increased credit exposure to Ford?

a. Selling a costless collar to Ford
b. Buying an option from Ford
c. Selling an option to Ford
d. None of the above

EXAMPLE 21.5: FRM EXAM 2001—QUESTION 84

If a counterparty defaults before maturity, which of the following situations will cause a credit loss?

a. You are short euros in a one-year euro/USD forward FX contract, and the euro has appreciated.
b. You are short euros in a one-year euro/USD forward FX contract, and the euro has depreciated.
c. You sold a one-year OTC euro call option, and the euro has appreciated.
d. You sold a one-year OTC euro call option, and the euro has depreciated.