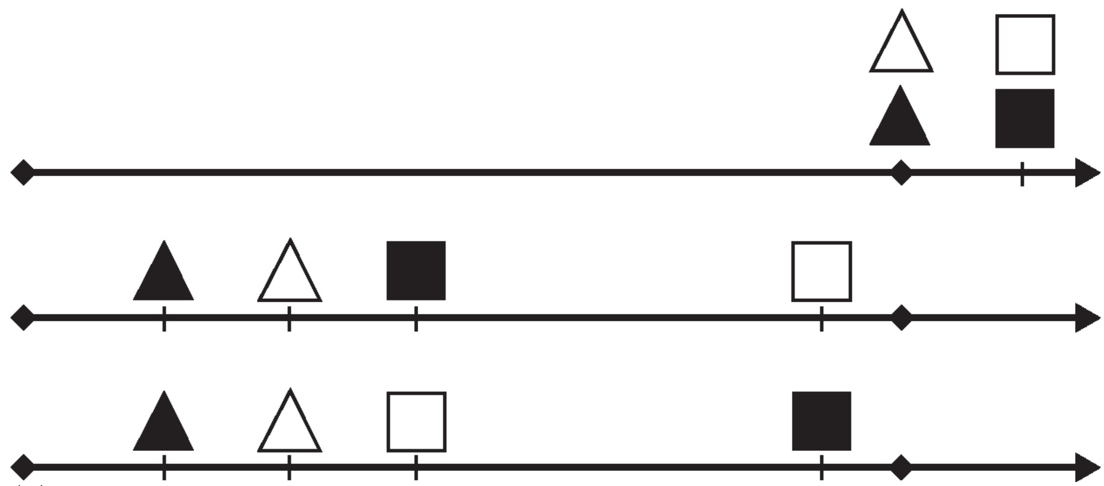


(c) Examples: Simultaneous moves

FIGURE 6.5 Strategic Order of Moves Depends on the Chronological Order in Which Players' Moves Become Irreversible and Observable

---

The second key question in determining the strategic order of moves is: When does each player's move become observable to the other player? Suppose that Row's move is the first to become irreversible. If Row's move also becomes observable before Column's move becomes irreversible, then Column is able to choose her move *in reaction* to Row's choice. The game has sequential moves, with Row as first mover. On the other hand, if Row's move is the first to become irreversible but remains unobservable until after Column has moved, then Column must make her choice in ignorance of Row's move. The game has simultaneous moves.

Figure 6.5 provides several illustrative examples, depicting the players' moments of irreversibility as triangles and moments of observability as squares on a chronological timeline. In this visualization, a game has sequential moves whenever one player's triangle and square both appear to the left of the other player's triangle and square, as in Figures 6.5a, which shows Row as the first mover, and 6.5b, which shows Column as the first mover. For any other configuration—including the three examples shown in Figure 6.5c—the game has simultaneous moves.

Consider the pure coordination game between Sherlock Holmes and Dr. Watson that we considered in Chapter 4. As both dashed off on separate errands, Holmes shouted that they should rendezvous at "four o'clock at our meeting place," but both men realized, in retrospect, that “our meeting place” could potentially mean St. Bart’s (a hospital) or Simpson’s (a restaurant). Because of the distance between the two locations and travel time across the city of London, Holmes and Watson each had to commit themselves to just one of these two destinations well before 4:00 p.m. For instance, perhaps Watson had to make his choice at 3:45 p.m., while Holmes made his choice at 3:50 p.m. Watson moved first from a chronological point of view, but because Holmes had no way of observing Watson’s choice until 4:00 p.m. (when Watson would either be in the same place as Holmes or not), the game had simultaneous moves. The order-of-moves visualization for this game is shown in Figure 6.6.

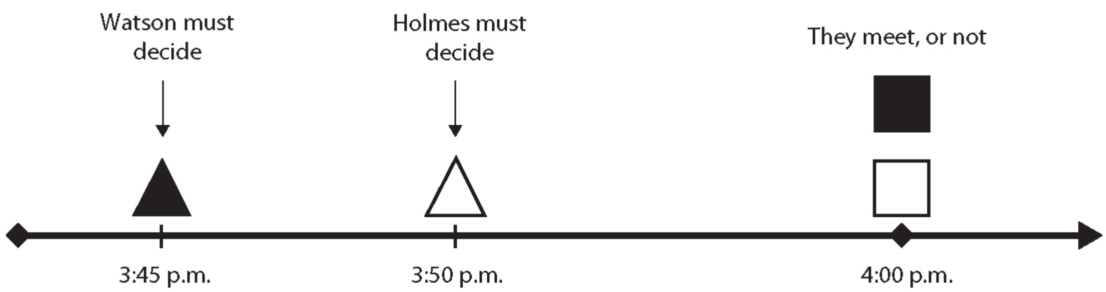


Figure 6.6 Order-of-Moves Visualization for Holmes and Watson Coordination Game

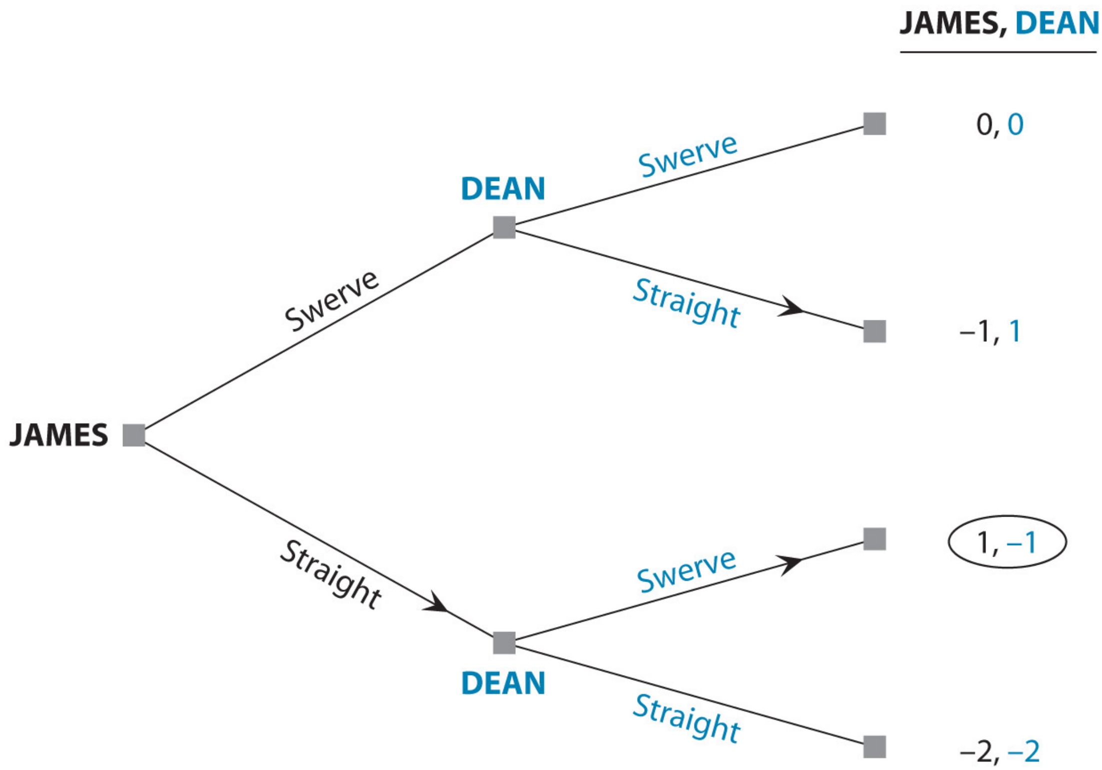
B. Moving First versus Moving Second

I. GAMES WITH A FIRST-MOVER ADVANTAGEThere is a *first-mover advantage* in a sequential-move game if each player gets a better outcome for herself in the rollback equilibrium of the game when she moves first than she gets in the rollback equilibrium of the game when she moves second.

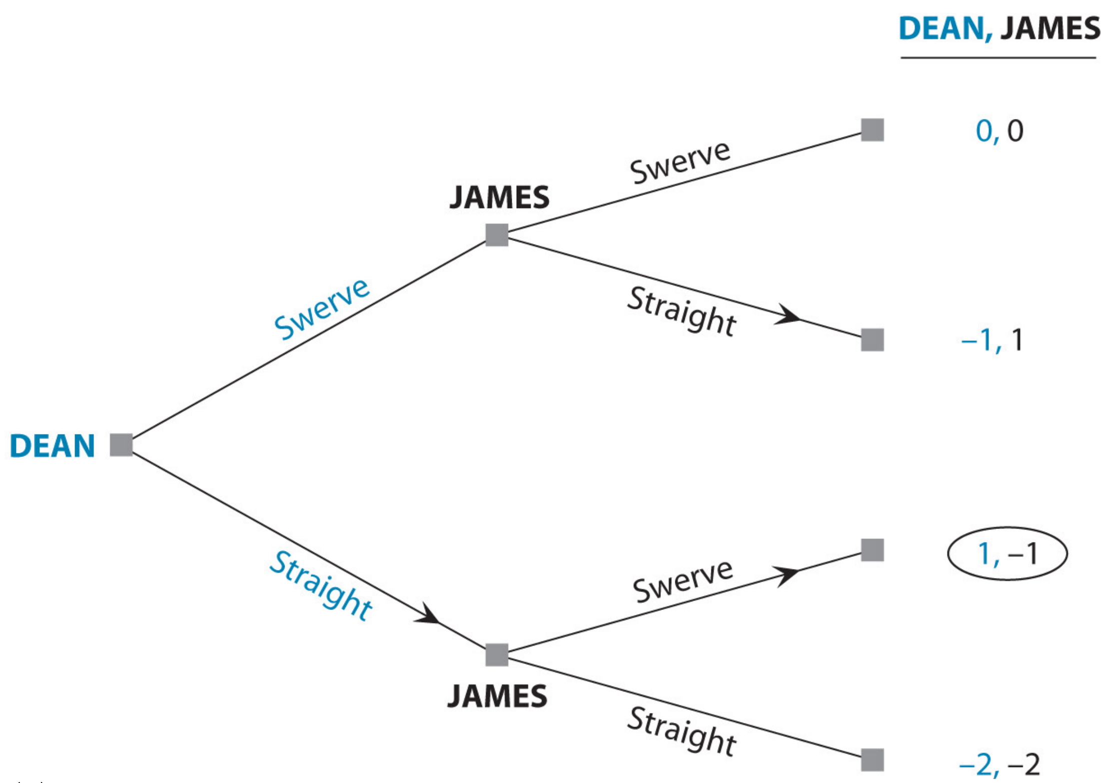
Consider the game of chicken from Chapter 4. We reproduce this game's strategic form in Figure 6.7a along with two extensive forms, one for each possible ordering of play, in Figures 6.7b and 6.7c. If the game has sequential moves, there is a unique rollback equilibrium in which the first mover goes straight (is "tough") and the second mover swerves (is "chicken"). Note that the first mover gets his best outcome while the second mover does not. Hence, there is a first-mover advantage in this game.

(a) Simultaneous play

		DEAN	
		Swerve (Chicken)	Straight (Tough)
JAMES	Swerve (Chicken)	0, 0	−1, 1
	Straight (Tough)	1, −1	−2, −2



(b) Sequential play: James moves first



(c) Sequential play: Dean moves first

Figure 6.7 Chicken in Simultaneous-Play and Sequential-Play Versions

---

In the game of chicken, moving first is best for each player, but interestingly, moving simultaneously may be even worse than moving last. In Chapter 7, we will return to this game and show that in the simultaneous-move version of the game, there is an equilibrium in mixed strategies in which each player has an expected payoff of -0.5, worse than what each player gets by moving last.

II. GAMES WITH A SECOND-MOVER ADVANTAGEThere is a *second-mover advantage* in a sequential-move game if each player gets a better outcome for herself in the rollback equilibrium of the game when she moves second than in the rollback equilibrium of the game when she moves first.

Consider the tennis-point game described in Chapter 4. Recall that in that game, Evert is planning the location of her passing shot while Navratilova considers where to cover. That simultaneous-move version of the game assumed that both players were skilled at disguising their intended moves until the very last moment, so that they moved at essentially the same time. If Evert's movement as she goes to hit the ball belies her shot intentions, however, then Evert's moments of irreversibility and observability both occur before Navratilova makes her move. Navratilova can react, and becomes the second mover in the game. In the same way, if Navratilova leans toward the side that she intends to cover before Evert actually hits her return, then Evert is the second mover.

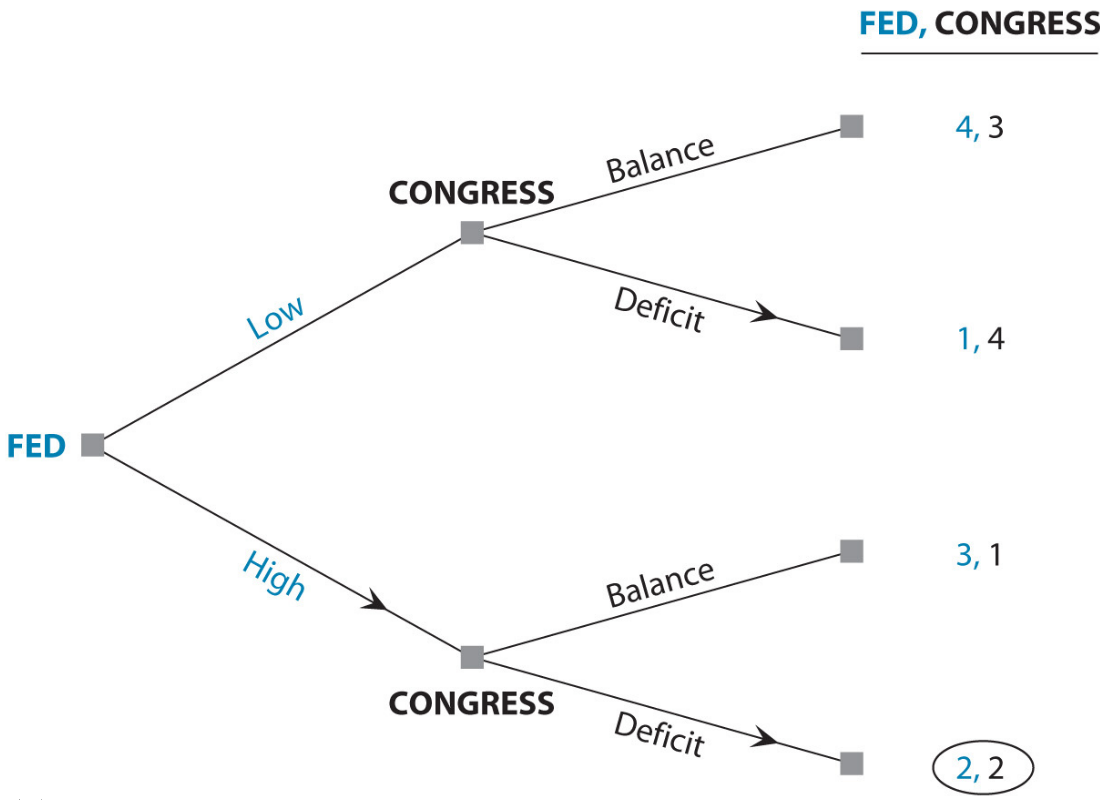
The simultaneous-move version of this game has no equilibrium in pure strategies. In each ordering of the sequential-move version, however, there is a unique rollback equilibrium outcome. There are different outcomes, however, depending on who moves first. If Evert moves first, then Navratilova chooses to cover whichever direction Evert chooses, and Evert should opt for a down-the-line shot. Each player is expected to win the point half the time in this equilibrium. If the order is reversed, Evert chooses to send her shot in the opposite direction from that which Navratilova covers; so Navratilova should move to cover crosscourt. In this case, Evert is expected to win the point 80% of the time. The second mover does better by being able to respond optimally to the opponent's move. (You should be able to draw game trees similar to those in Figure 6.7b and 6.7c to illustrate this point.)

We return to the simultaneous-move version of this game in Chapter 7, where we show that it does have a Nash equilibrium in mixed strategies. In that equilibrium, Evert succeeds, on average, 62% of the time. Her success rate in the mixed-strategy equilibrium of the simultaneous-move game is thus better than the 50% that she gets by moving first, but is worse than the 80% that she gets by moving second.

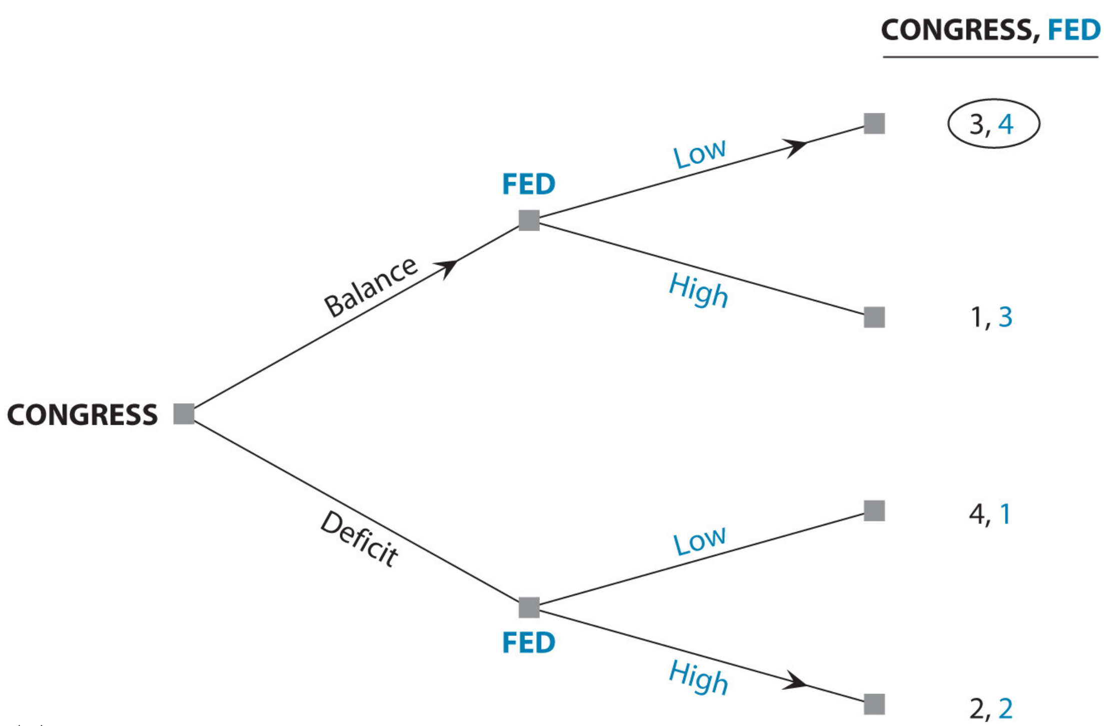
III. GAMES IN WHICH THE PLAYERS' MOVE-ORDER PREFERENCES ARE ALIGNEDIn some games, both players prefer the same ordering of moves. For instance, Row and Column may both prefer that Row move first. In such games, there is neither a first-mover advantage nor a second-mover advantage.

(a) Simultaneous moves

		FEDERAL RESERVE	
		Low interest rates	High interest rates
CONGRESS	Budget balance	3, 4	1, 3
	Budget deficit	4, 1	2, 2



(b) Sequential moves: Fed moves first



(c) Sequential moves: Congress moves first

Figure 6.8 Three Versions of the Fiscal–Monetary Policy Game

Consider the game of fiscal and monetary policies played by Congress and the Federal Reserve. We studied the simultaneous-move version of this game in Chapter 4. We reproduce the game's ordinal payoff table (Figure 4.5) as Figure 6.8a and show the two sequential-move versions as Figure 6.8b and 6.8c. For brevity, we write the strategies in the sequential-move game trees as Balance and Deficit, instead of Budget Balance and Budget Deficit, for Congress and as High and Low, instead of High Interest Rates and Low Interest Rates, for the Fed.

In the simultaneous-move version, Congress has a dominant strategy (Deficit), and the Fed, knowing this, chooses High, resulting in the second-worst outcome (ordinal payoff 2) for both players. The same outcome arises in the unique rollback equilibrium of the sequential-move version of the game in which the Fed moves first. The Fed foresees that, whether it chooses High or Low, Congress will respond with Deficit. High is therefore the better choice for the Fed, yielding an ordinal payoff of 2 instead of 1.

But the sequential-move version in which Congress moves first is different. Now Congress foresees that if it chooses Deficit, the Fed will respond with High, whereas if it chooses Balance, the Fed will respond with Low. Of these two possible outcomes, Congress prefers the latter, as it yields a payoff of 3 instead of 2. Therefore, the rollback equilibrium with this ordering of moves is for Congress to choose a balanced budget and the Fed to respond with low interest rates. The resulting ordinal payoffs, 3 for Congress and 4 for the Fed, are better for both players than when moves are simultaneous or when the Fed moves first.

A surprising aspect of the rollback equilibrium when Congress moves first is that Congress plays Balance, its dominated strategy, rather than Deficit, its dominant strategy. How can this be? The fact that Balance is dominated means that, for any *fixed* interest-rate policy by the Fed, Congress would always prefer Deficit over Balance. Because Congress is the first mover, however, the Fed's interest-rate policy is not fixed, but is dependent on what Congress chooses to do—the Fed will "reward" Congress with low interest rates only if Congress chooses to balance the budget. To change the Fed's behavior and get that reward, Congress is willing to play its dominated strategy, something it would never do if the game had simultaneous moves.

The distinction between dominant and superdominant strategies, discussed originally in Section 4 of Chapter 4, also plays a role here. Deficit is a dominant strategy for Congress and, as such, would be its equilibrium strategy if the game had simultaneous moves. Deficit is not a superdominant strategy, however, because the outcome with a deficit and high interest rates is worse for Congress than the one with a balanced budget and low interest rates. Consequently, when Congress, as first mover, looks ahead to the Fed's choices and sees (Deficit, High) and (Balance, Low) as the only possible outcomes of the game, Congress will choose to balance the budget.

If Congress and the Fed could choose the order of moves in the game, they would agree that Congress should move first. The outcome with that order of moves (a balanced budget and low interest rates) is better for both players than the outcome that occurs under simultaneous moves or when the Fed moves first (a budget deficit and high interest rates). Indeed, when budget deficits and inflation threaten, chairs of the Federal Reserve, in testimony before various congressional committees, often offer such deals: They promise to respond to fiscal discipline by lowering interest rates. But it is often not enough to make a verbal deal with the other player. The technical requirements of a first move—namely, that it be observable to the second mover and irreversible thereafter—must be satisfied. In the context of fiscal and monetary policies, it is fortunate that the legislative process of setting fiscal policy in the United States is both very visible and very slow, whereas monetary policy can be changed quickly in a meeting of the Federal Reserve Board. Therefore, the sequential-play scenario where Congress moves first and the Fed moves second is quite realistic.

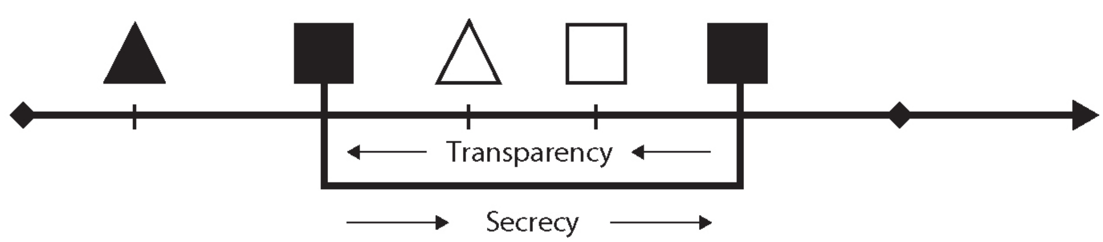
IV. GAMES IN WHICH ORDER OF MOVES DOESN'T MATTERIn some games, equilibrium outcomes are the same no matter what the order of moves. This will always be true whenever one of the players has a superdominant strategy or both players have a dominant strategy.

Consider the prisoners' dilemma game in Chapter 4, in which a husband and wife are being questioned regarding their roles in a crime. The Nash equilibrium of that simultaneous-move game is for each player to confess (that is, to defect from cooperating with the other). But what would happen if one spouse made an irreversible and observable choice before the other chose at all? Using rollback on a game tree similar to that in Figure 6.7b (which you can draw on your own as a check of our analysis), one can show that the second player does best to confess if the first has already confessed (for a payoff of 10 years rather than 25 years in jail), and the second player also does best to confess if the first has denied (1 year rather than 3 years in jail). Given these choices by the second player, the first player does best to confess (10 years rather than 25 years in jail). The equilibrium entails 10 years of jail for both spouses regardless of which one moves first. Thus, the equilibrium is the same in all three versions of this game!

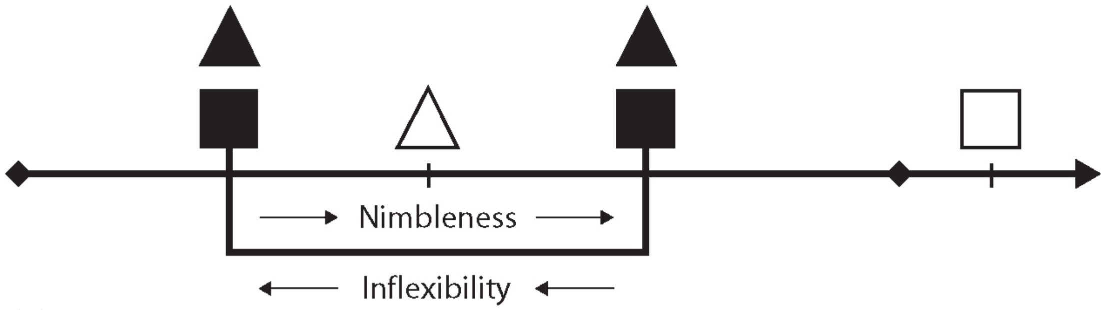
C. How One Player Can Change the Order of Moves

A player can change the *strategic* order of moves by taking steps to change the *chronological* order in which moves become irreversible and/or observable. Whether or how a player would choose to change the order of moves depends on her payoffs for the possible outcomes of the game. Here we highlight some especially simple ways that a player can alter the order of moves and discuss some circumstances in which she might want to do so. We will discuss a wider range of game-changing tactics, especially so-called strategic moves, in more depth in Chapter 8.

I. TRANSPARENCY VERSUS SECRECYSuppose that Row has to make her move far in advance and hence makes her irreversible move first in chronological time, as shown in Figure 6.9a. If Row is *transparent* in making her move, so that Column can easily observe it, Row can ensure that the game has sequential moves and that she is the first mover. In this case (illustrated by the leftmost black square in the figure), Row' s move is observable before Column' s move becomes irreversible. On the other hand, if Row is *secretive* with her move, so that Column cannot observe it, Row' s move becomes observable after Column' s move becomes irreversible (illustrated by the rightmost black square in the figure), and the game has simultaneous moves. Whether Row prefers to be transparent or secretive depends on whether Row gets a better payoff in the game when moving first or when moving simultaneously with Column. For example, in our tennis-point example, each player would want to be secretive, but in a game of chicken, a player choosing Tough would want to be transparent. (A player choosing Swerve in Chicken may want to be secretive in the hope that the other would swerve also.)



(a) Transparency allows Row to move first rather than simultaneously



(b) Nimbleness allows Row to move simultaneously rather than first

Figure 6.9 How Row Can Change the Order of Moves

II. NIMBLENESS VERSUS INFLEXIBILITYSuppose that Column moves secretly, so that her move is not observable until Row's move is complete, while Row's move is observable to Column as soon as it becomes irreversible. Row, however, might have the ability—call it *nimbleness*—to wait for some period of time before making her move irreversible (and also observable). This possibility is illustrated in Figure 6.9b. With nimbleness, Row can wait as long as possible before making her move, ensuring that the game has simultaneous moves. (When Row is nimble, Column's move is the first to become irreversible, but since Column moves secretly, her move does not become observable until after Row's move becomes irreversible.) If Row cannot be nimble, we say she is *inflexible*, and her inflexibility requires that she move right away, before Column, which makes her the first mover. Whether Row wants to be nimble or inflexible therefore depends on whether she gets a better payoff when moves are simultaneous or when she moves first.

Consider a game of chicken played by two firms, each eyeing a new market that has room for only one firm to operate profitably. Each firm would like to be the first mover because it could then deter the other firm's entrance while entering profitably itself. Both firms will naturally jockey to make their own moves as inflexible and transparent as possible, but this may be easier for some firms than for others. Suppose that one of the firms is traded on the New York Stock Exchange, while the other is privately held. Because publicly traded firms in the United States must submit quarterly audited financial reports that give investors (and competitors) a window into their operations, they necessarily have high transparency. Moreover, investors demand information about publicly traded firms, and analysts’ scrutiny of these firms’ activities further increases transparency. Such scrutiny could possibly make publicly traded firms more inflexible as well. If the CEO of a publicly traded firm goes on the record in the business press saying that entering the new market is essential to the firm’s long-term growth, she knows that if she were to then back down and cancel her plan to enter that market, investors would swiftly punish her company’s stock. To avoid such a reaction, she is effectively forced to go through with the announced strategy. The fact that publicly held firms must disclose so much information is often viewed as a disadvantage. However, in this context, disclosure requirements benefit the CEO and her firm by allowing it to seize the first-mover advantage.

Glossary

strategic order

The order of moves from a game-theoretic point of view, determined by considerations of observability and irreversibility. It may differ from the chronological order of actions and, in turn, determine whether the game has sequential or simultaneous moves.

irreversible

Cannot be undone by a later action. In a sequential-move game, the first mover's action must be irreversible and *observable* before the second mover's action is irreversible.

observable

Known to other players before they make their responding actions. Together with irreversibility, this is an important condition for a game to be sequential-move.

3 ALTERNATIVE METHODS OF ANALYSIS

Game trees are the natural way to display sequential-move games, and payoff tables are the natural representation of simultaneous-move games. However, each technique can be adapted to the other type of game. Here we show how to translate the information contained in one illustration to an illustration of the other type. In the process, we develop some new ideas that will prove useful in subsequent analysis of games.

A. Illustrating Simultaneous-Move Games Using Game Trees

Consider the game of the passing shot in tennis, originally described in Chapter 4, where the action is so quick that moves are truly simultaneous. But suppose we want to show this game in extensive form—that is, by using a tree, rather than a table as in Figure 4.17. We show how this can be done in Figure 6.10.

To draw the tree, we must choose one player—say, Evert—to make her choice at the initial node of the tree. The branches for her two choices, a down-the-line shot (DL) and a crosscourt shot (CC), then end in two action nodes, at each of which Navratilova makes her choice. However, because the moves are actually simultaneous, Navratilova must choose without knowing what Evert has picked. That is, she must choose without knowing whether she is at the node following Evert’ s DL branch or the one following Evert’ s CC branch. Our tree must in some way show this lack of information on Navratilova’ s part.

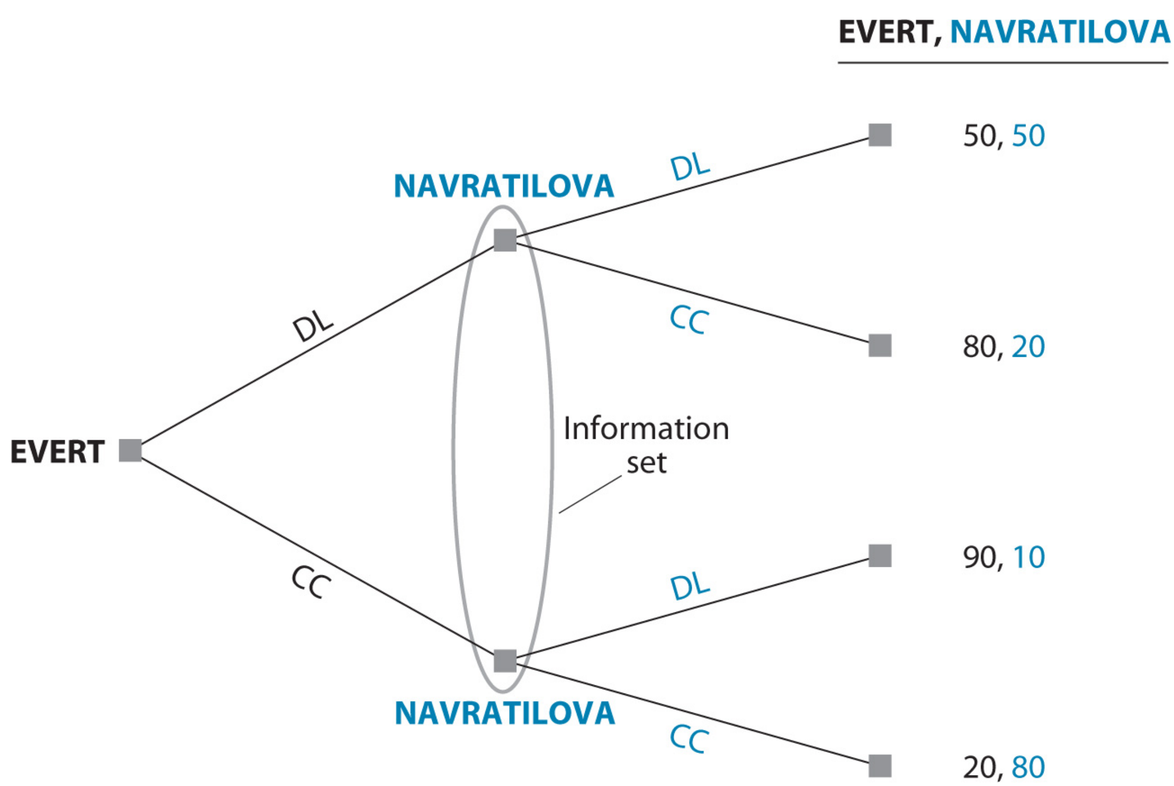


Figure 6.10 Simultaneous-Move Tennis-Point Game Shown in Extensive Form

---

We illustrate Navratilova's strategic uncertainty about the node at which her decision is being made by drawing an oval or balloon to surround the two possible nodes. (An alternative is to connect the nodes with a dotted line; the line is dotted to distinguish it from the solid lines that represent the branches of the tree.) The nodes within this oval constitute an information set for the player who makes a move there. An information set consisting of more than one node indicates the presence of imperfect information for the player; she cannot distinguish between the nodes in the set given the available information (because she cannot observe the other player's move before making her own). As such, her choice of strategies from within the information set must specify the same move at all the nodes contained in it. That is, Navratilova must choose either DL at both the nodes in this information set or CC at both of them. She cannot choose DL at one and CC at the other.

Accordingly, we must adapt our definition of *strategy*. In Chapter 3, we clarified that a *complete* strategy must specify the move that a player would make at each *node* where the rules of the game specify that it is her turn to move. We should now more accurately redefine a complete strategy as specifying the move that a player would make at each *information set* at whose nodes the rules of the game specify that it is her turn to move.

The concept of an information set is also relevant when a player faces external uncertainty about some condition other than another player's moves that affects his decision. For example, a farmer planting a crop is uncertain about the weather during the growing season, although he knows the probabilities of various alternative possibilities from past experience or meteorological forecasts. We can regard the weather as a random choice by an outside player, Nature, that has no payoffs but merely chooses according to known probabilities.[3] We can then enclose the various nodes corresponding to Nature's moves in an information set for the farmer, constraining the farmer's choice to be the same at all of these nodes. Figure 6.11 illustrates this situation.

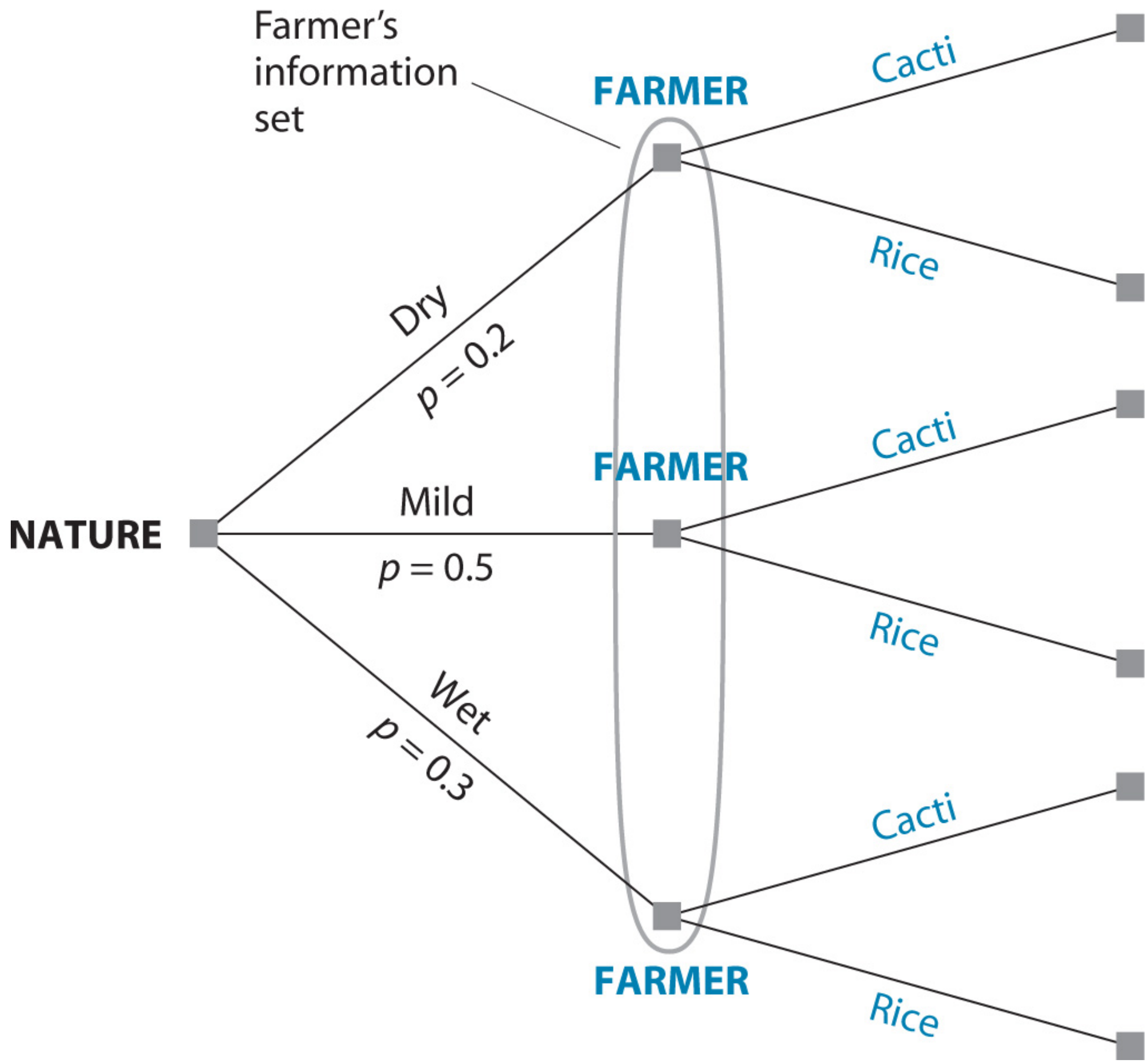


Figure 6.11 Nature and Information Sets

Using the concept of an information set, we can formalize the concepts of perfect and imperfect information in a game, which we introduced in Section 2.D of Chapter 2. A game has perfect information if it has neither strategic nor external uncertainty, which will happen if it has no information sets enclosing two or more nodes. Thus, a game has perfect information if all of its information sets consist of singleton nodes.

Although this representation is conceptually simple, it does not provide any simpler way of solving a simultaneous-move game. Therefore, we use it only occasionally, where it conveys some point more simply than any alternative. Some examples of game trees using information sets can be found in Chapters 9 and 13.

B. Showing and Analyzing Sequential-Move Games Using Game Tables

Consider now the sequential-move game of fiscal and monetary policy from Figure 6.8c, in which Congress has the first move. Suppose we want to show this game in normal or strategic form—that is, by using a game table. The rows and the columns of the table show the strategies of the two players. We must therefore begin by specifying those strategies.

For Congress, the first mover, listing the strategies is easy. There are just two possible moves—Balance and Deficit—and they are also the two strategies. For the Fed—the second mover—matters are more complex. Remember that a strategy is a complete plan of action, specifying the moves to be made at each node where it is a player’s turn to move. Because the Fed gets to move at two nodes (and because we are supposing that this game actually has sequential moves, so that the two nodes are not combined into one information set) and can choose either Low or High at each node, it has four strategies. These strategies are (1) Low if Balance, High if Deficit (we can write this as “L if B, H if D” for short); (2) High if Balance, Low if Deficit (“H if B, L if D” for short); (3) Low always; and (4) High always.

We show the resulting two-by-four payoff matrix in Figure 6.12. The last two columns are no different from those in the two-by-two payoff matrix for this game under simultaneous-move rules (Figure 6.8a). If the Fed chooses a strategy in which it always makes the same move, it is just as if the Fed were moving without taking into account what Congress had done—it is as if the players’ moves were simultaneous. But calculation of the payoffs for the first two columns, where the Fed’s second move does depend on Congress’s first move, needs some care.

To illustrate, let’s first consider the cell in the first row and the second column. Here, Congress chooses Balance and the Fed chooses H if B, L if D. Given Congress’s choice, the Fed’s actual choice under this strategy is High. Then the payoffs are those for the Balance and High combination—namely, 1 for Congress and 3 for the Fed.

Best-response analysis quickly shows that the game has two pure-strategy Nash equilibria, which we show by shading the cells gray. One is in the top-left cell, where Congress's strategy is Balance and the Fed's is L if B, H if D, so that the Fed's actual choice is L. This outcome is just the rollback equilibrium of the sequential-move game. But there is another Nash equilibrium in the bottom-right cell, where Congress chooses Deficit and the Fed chooses High always. As in any Nash equilibrium, neither player has a clear reason to deviate from the strategy that leads to this outcome. Congress would do worse by switching to Balance, and the Fed could do no better by switching to any of its other three strategies, although it could do just as well with L if B, H if D.

This sequential-move game, when analyzed in its extensive form using the game tree in Figure 6.8c, led to a single rollback equilibrium. But when analyzed in its normal or strategic form using the game table in Figure 6.12, it has two Nash equilibria. What is going on?

		FED			
		L if B, H if D	H if B, L if D	Low always	High always
CONGRESS	Balance	3, 4	1, 3	3, 4	1, 3
	Deficit	2, 2	4, 1	4, 1	2, 2

You may need to scroll left and right to see the full figure.

FIGURE 6.12 Sequential-Move Game of Fiscal and Monetary Policy in Strategic Form

---

The answer lies in the different nature of the logic of Nash equilibrium and rollback analyses. Nash equilibrium requires that neither player have a reason to deviate from her strategy, given the strategy of the other player. However, rollback does not take the strategies of later movers as given. Instead, it asks what would be optimal to do if the opportunity to move actually arose.

In our example, the Fed’ s strategy of High always does not satisfy the criterion of being optimal if the opportunity to move actually arose. If Congress chose Deficit, then High would indeed be Fed’ s optimal response. However, if Congress chose Balance, and the Fed had to respond, it would want to choose Low, not High. So High always does not describe the Fed’ s optimal response in all possible configurations of play and cannot be a rollback equilibrium. But the logic of Nash equilibrium does not impose such a test, instead regarding the Fed’ s High always as a strategy that Congress could legitimately take as given. If it did so, then Deficit would be Congress’ s best response. And, conversely, High always is one of the Fed’ s best responses to Congress’ s Deficit (although it is tied with L if B, H if D). Thus, the pair of strategies Deficit and High always are mutual best responses and constitute a Nash equilibrium, although they do not constitute a rollback equilibrium.

We can therefore think of rollback as a further test, supplementing the requirements of Nash equilibrium and helping to select from among multiple Nash equilibria in the strategic form. In other words, rollback is a refinement of the Nash equilibrium concept.

To state this idea somewhat more precisely, recall the concept of a subgame. At any node of the full game tree, we can think of the part of the game that begins there as a subgame. In fact, as successive players make their choices, the play of the game moves along a succession of nodes, and each move can be thought of as starting a subgame. The equilibrium derived by using rollback corresponds to one particular succession of choices in each subgame and gives rise to the equilibrium path of play. Certainly, other paths of play are consistent with the rules of the game. We call these other paths off-equilibrium paths, and we call any subgames that arise along these paths off-equilibrium subgames.

With this terminology, we can now say that the equilibrium path of play is itself determined by the players' expectations of what would happen if they chose a different action—if they moved the game to an off-equilibrium path and started an off-equilibrium subgame. Rollback requires that all players make their best choices in *every* subgame of the larger game, whether or not the subgame lies along the path to the ultimate equilibrium outcome.

Because strategies are complete plans of action, a player's strategy must specify what she will do in each eventuality, at each and every node of the game, whether on or off the equilibrium path of play, where it is her turn to act. When she reaches a particular decision node, only the plan of action starting there—namely, the part of her full strategy that pertains to the subgame starting at that node—is pertinent. This part of her strategy is called the continuation of the strategy for that subgame. Rollback requires that the optimal (equilibrium) strategy be such that its continuation in every subgame is optimal for the player whose turn it is to act at that node, whether or not the node and the subgame lie on the equilibrium path of play.

Let's return to the fiscal–monetary policy game, with Congress moving first, and consider the second Nash equilibrium, in the lower-right corner of Figure 6.12, that arises in its strategic form. Here, Congress chooses Deficit and the Fed chooses High. On the equilibrium path of play, High is indeed the Fed's best response to Deficit. Congress's choice of Balance, however, would be the start of an off-equilibrium path. That path leads to a node where a rather trivial subgame starts—namely, a decision by the Fed. The Fed's purported equilibrium strategy High always asks it to choose High in this subgame. But that choice is not optimal; this second Nash equilibrium is specifying a nonoptimal choice for an off-equilibrium subgame.

In contrast, the equilibrium path of play for the Nash equilibrium in the upper-left corner of Figure 6.12 is for Congress to choose Balance and the Fed to follow with Low. Here, the Fed would be responding optimally on the equilibrium path of play. The off-equilibrium path would have Congress choosing Deficit, and the Fed, given its strategy of L if B, H if D, would follow with High. It is optimal for the Fed to respond to Deficit with High, so that choice remains optimal off the equilibrium path as well as on it.

The requirement that continuation of a strategy remain optimal under all circumstances is important because the equilibrium path itself is the result of players' thinking strategically about what would happen if they did something different. A later player may try to achieve an outcome that she would prefer by threatening that certain actions by the first mover will be met with dire responses, or by promising that certain other actions will be met with kindly responses. But the first mover will be skeptical of such threats and promises. The only way to remove that doubt is to check whether the stated responses would actually be optimal for the later player. If those responses would not be optimal, then the threats or promises have no credibility, and those responses will not be observed along the equilibrium path of play.

The equilibrium found by using rollback is called a subgame-perfect equilibrium (SPE). It is a set of strategies (complete plans of action), one for each player, such that, at every node of the game tree, whether or not that node lies along the equilibrium path of play, the continuation of the same strategy in the subgame starting at that node is optimal for the player who takes the action there. More simply, an SPE requires players to use strategies that constitute a Nash equilibrium in every subgame of the larger game.

In fact, as a rule, in games with finite trees and perfect information, where players can observe every previous action taken by all other players so that there are no information sets containing multiple nodes, rollback finds the unique (except for trivial and exceptional cases of ties) subgame-perfect equilibrium of the game. Consider: If you look at any subgame that begins at the last decision node for the last player who moves, the best choice for that player is the one that gives her the highest payoff. But that is precisely the action chosen with the use of rollback. As players move backward through the game tree, rollback eliminates all unreasonable strategies, including noncredible threats or promises, so that the collection of actions ultimately selected is the SPE. Therefore, for the purposes of this book, subgame perfectness is just a fancy name for rollback. At more advanced levels of game theory, where games include complex information structures and information sets, subgame-perfectness becomes a richer notion.

Endnotes

- Some people believe that Nature is actually a malevolent player who plays a zero-sum game with us, so that its payoffs are higher when ours are lower. For example, we believe it is more likely to rain if we have forgotten to bring an umbrella. We understand such thinking, but it does not have real statistical support. Return to reference 3

Glossary

information set
: A set of nodes among which a player is unable to distinguish when taking an action. Thus, his strategies are restricted by the condition that he should choose the same action at all points of an information set. For this, it is essential that all the nodes in an information set have the same player designated to act, with the same number and similarly labeled branches emanating from each of these nodes.

off-equilibrium path
: A path of play that does not result from the players' choices of strategies in a subgame-perfect equilibrium.

off-equilibrium subgame
: A subgame starting at a node that does not lie on the equilibrium path of play.

continuation
: The continuation of a strategy from a (noninitial) node is the remaining part of the plan of action of that strategy, applicable to the subgame that starts at this node.

credibility
: A strategy is credible if its continuation at all nodes, on or off the equilibrium path, is optimal for the subgame that starts at that node.

subgame-perfect equilibrium (SPE)
: A configuration of strategies (complete plans of action) such that their continuation in any subgame remains optimal (part of a rollback equilibrium), whether that subgame is on- or off-equilibrium. This ensures credibility of all the strategies.

4 THREE-PLAYER GAMES

We have restricted the discussion so far in this chapter to games with two players and two moves each. But the same methods also work for some larger and more general examples. We now illustrate this by using the street-garden game introduced in Chapter 3. Specifically, we (1) change the rules of the game from sequential to simultaneous moves, and then (2) keep the moves sequential but show and analyze the game in its strategic form. First, we reproduce the tree for the original sequential-move game (Figure 3.5) as Figure 6.13 here and remind you of its rollback equilibrium.

The equilibrium strategy of the first mover (Emily) is simply one move, “Don’t contribute.” The second mover (Nina) chooses from among 4 possible strategies (with a choice of two responses at each of two nodes) and chooses the strategy “Choose Don’t contribute (D) if Emily has chosen Contribute, and choose Contribute (C) if Emily has chosen Don’t contribute,” or, more simply, “D if C, C if D,” or even more simply, “DC.” The third mover (Talia) has 16 available strategies (with a choice of two responses at each of four nodes), and her equilibrium strategy is “Choose D following Emily’s C and Nina’s C, C following their CD, C following their DC, and D following their DD,” or “DCCD” for short.

Remember, too, the reason for these choices. The first mover has the opportunity to choose Don’t, knowing that the other two will recognize that a pleasant garden won’t be forthcoming unless they contribute, and knowing that they prefer a pleasant garden strongly enough that they will contribute.

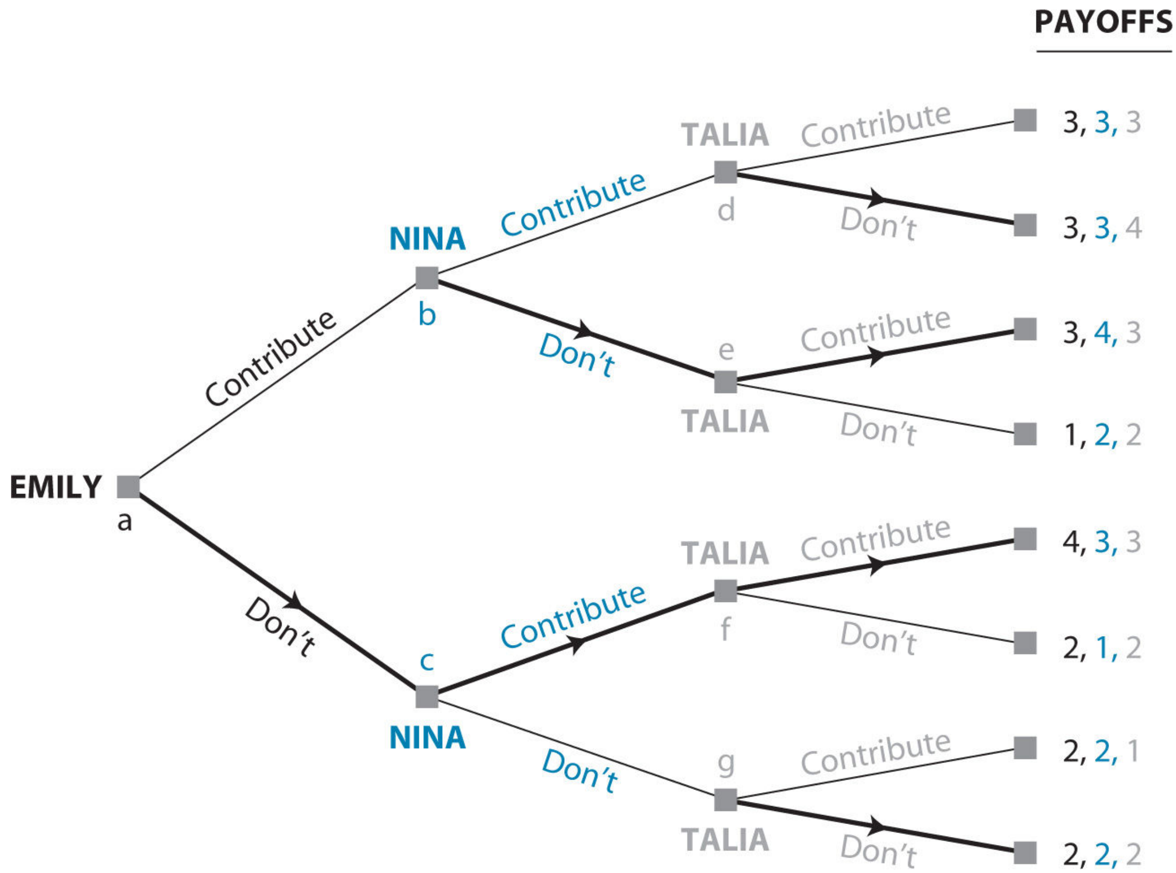


Figure 6.13 The Street-Garden Game with Sequential Moves

Now we change the rules of the game to make it a simultaneous-move game. (In Chapter 4, we solved a simultaneous-move version with somewhat different payoffs; here we keep the payoffs the same as in Chapter 3.) The payoff matrix is shown in Figure 6.14. Best-response analysis shows easily that there are four Nash equilibria.

In three of the Nash equilibria of the simultaneous-move game, two players contribute, while the third does not. These equilibria are similar to the rollback equilibrium of the sequential-move game. In fact, each one corresponds to the rollback equilibrium of the sequential-move game with a particular order of play. Further, any given order of play in the sequential-move version of this game leads to the same simultaneous-move payoff table.

But there is also a fourth Nash equilibrium here, where no one contributes. Given the specified strategies of the other two—namely, Don’t contribute—any one player is powerless to bring about the pleasant garden and therefore chooses not to contribute as well. Thus, in the change from sequential to simultaneous moves, the first-mover advantage has been lost. Multiple equilibria arise, only one of which retains the original first mover’s high payoff.

Next we return to the sequential-move version—Emily first, Nina second, and Talia third—but show the game in its normal or strategic form. In the sequential-move game, Emily has 2 pure strategies, Nina has 4, and Talia has 16, so this means constructing a payoff table that is 2 × 4 × 16. With the use of the same conventions that we used for three-player tables in Chapter 4, this particular game would require a table with 16 “pages” of two-by-four payoff tables. That would look too messy. so we opt instead for a reshuffling of the players. Let Talia be the Row player, Nina be the Column player, and Emily be the Page player. Then “all” that is required to illustrate this game is the 16 × 4 × 2 game table shown in Figure 6.15. The order of payoffs still corresponds to our earlier convention in that they are listed in the order Row player, Column player, Page player; in our example, that means the payoffs are now listed in the order Talia, Nina, and Emily.

TALIA chooses:

Contribute

		NINA	
		Contribute	**Don’t**
EMILY	Contribute	3, 3, 3	3, 4, 3
	Don’t	4, 3, 3	2, 2, 1

You may need to scroll left and right to see the full figure.

Don’t Contribute

		NINA	

		NINA	
		Contribute	Don’t
EMILY	Contribute	3, 3, 1	1, 2, 2
	Don’t	2, 1, 2	2, 2, 2

You may need to scroll left and right to see the full figure.

FIGURE 6.14 The Street-Garden Game with Simultaneous Moves

As in the fiscal–monetary policy game between Congress and the Fed, there are multiple Nash equilibria in the simultaneous-move street-garden game. (In Exercise S9, we ask you to find them all.) But there is only one subgame-perfect equilibrium, corresponding to the rollback equilibrium found in Figure 6.13. Although best-response analysis does find all the Nash equilibria, successive elimination of dominated strategies can reduce the number of reasonable equilibria for us here. This process works because elimination identifies those strategies that include noncredible components (such as High always for the Fed in Section 3.B). As it turns out, such elimination can take us all the way to the unique subgame-perfect equilibrium.

In Figure 6.15, we start with Talia and eliminate all of her (weakly) dominated strategies. This step eliminates all but the strategy listed in the eleventh row of the table, DCCD, which we have already identified as Talia’s rollback equilibrium strategy. Elimination can continue with Nina, for whom we must compare outcomes from strategies across both pages of the table. To compare her CC to CD, for example, we look at the payoffs associated with CC in *both pages* of the table and compare these payoffs with the similarly identified payoffs for CD. For Nina, the elimination process leaves only her strategy DC; again, this is the rollback equilibrium strategy found for her above. Finally, Emily has only to compare the two remaining cells associated with her choice of Don’t and Contribute; she gets the highest payoff when she chooses Don’t and so makes that choice. As before, we have identified her rollback equilibrium strategy.

The unique subgame-perfect outcome in the game table in Figure 6.15 thus corresponds to the cell associated with the rollback equilibrium strategies for each player. Note that the process of successive elimination that leads us to this subgame-perfect equilibrium is carried out by considering the players in reverse order of the actual play of the game. This order conforms to the order in which player actions are considered in rollback analysis and therefore allows us to eliminate exactly those strategies, for each player, that are not consistent with rollback. In so doing, we eliminate all the Nash equilibria that are not subgame-perfect.

EMILY								
	Contribute				Don’t			
	NINA				NINA			
TALIA	CC	CD	DC	DD	CC	CD	DC	DD
CCCC	3, 3, 3	3, 3, 3	3, 4, 3	3, 4, 3	3, 3, 4	1, 2, 2	3, 3, 4	1, 2, 2
CCCD	3, 3, 3	3, 3, 3	3, 4, 3	3, 4, 3	3, 3, 4	2, 2, 2	3, 3, 4	2, 2, 2
CCDC	3, 3, 3	3, 3, 3	3, 4, 3	3, 4, 3	2, 1, 2	1, 2, 2	2, 1, 2	1, 2, 2
CDCC	3, 3, 3	3, 3, 3	3, 2, 1	2, 2, 1	3, 3, 4	1, 2, 2	3, 3, 4	1, 2, 2
DCCC	4, 3, 3	4, 3, 3	3, 4, 3	3, 4, 3	3, 3, 4	1, 2, 2	3, 3, 4	1, 2, 2

CCDD	3, 3, 3	3, 3, 3	3, 4, 3	3, 4, 3	2, 1, 2	2, 2, 2	2, 1, 2	2, 2, 2
CDDC	3, 3, 3	3, 3, 3	2, 2, 1	2, 2, 1	2, 1, 2	1, 2, 2	2, 1, 2	1, 2, 2
DDCC	4, 3, 3	4, 3, 3	2, 2, 1	2, 2, 1	3, 3, 4	1, 2, 2	3, 3, 4	1, 2, 2
CDCD	3, 3, 3	3, 3, 3	2, 2, 1	2, 2, 1	3, 3, 4	2, 2, 2	3, 3, 4	2, 2, 2
DCDC	4, 3, 3	4, 3, 3	3, 4, 3	3, 4, 3	2, 1, 2	1, 2, 2	2, 1, 2	1, 2, 2
DCCD	4, 3, 3	4, 3, 3	3, 4, 3	3, 4, 3	3, 3, 4	2, 2, 2	3, 3, 4	2, 2, 2
CDDD	3, 3, 3	3, 3, 3	2, 2, 1	2, 2, 1	2, 1, 2	2, 2, 2	2, 1, 2	2, 2, 2
DCDD	4, 3, 3	4, 3, 3	3, 4, 3	3, 4, 3	2, 1, 2	2, 2, 2	2, 1, 2	2, 2, 2
DDCD	4, 3, 3	4, 3, 3	2, 2, 1	2, 2, 1	3, 3, 4	2, 2, 2	3, 3, 4	2, 2, 2
DDDC	4, 3,	4, 3,	2, 2,	2, 2,	2, 1,	1, 2,	2, 1,	1, 2,

	3	3	1	1	2	2	2	2
DDDD	4, 3, 3	4, 3, 3	2, 2, 1	2, 2, 1	2, 1, 2	2, 2, 2	2, 1, 2	2, 2, 2

FIGURE 6.15 The Street-Garden Game in Strategic Form

SUMMARY

Many games include multiple components, some of which entail simultaneous play and others of which entail sequential play. In two-stage (and multistage) games, a “tree house” can be used to illustrate the game; this construction allows the identification of the different stages of play and the ways in which those stages are linked together. Full-fledged games that arise in later stages of play are called *subgames* of the full game. Players’ actions in each subgame are determined by the *continuation* of their strategies for that subgame.

The *strategic order* of moves in a game is determined by when each player’s move becomes irreversible and observable. If the Row player’s move becomes irreversible and observable before the Column player’s move becomes irreversible, the game has *sequential moves* with Row as first mover, and vice versa. On the other hand, if each player’s move becomes irreversible before the other player’s move becomes observable, the game has *simultaneous moves*. Players can influence the order of moves by making a move in a transparent or secretive way or by becoming more or less nimble or inflexible in the making of a move.

Changing the order of moves may or may not alter the equilibrium outcome of a game. Simultaneous-move games that are changed to make moves sequential may have the same outcome (if both players have dominant strategies), may have a first-mover or second-mover advantage, or may lead to better outcomes for both players. The sequential-move version of a simultaneous-move game will generally have a unique rollback equilibrium even if the simultaneous-move version has no equilibrium or multiple equilibria. Similarly, a sequential-move game that has a unique rollback equilibrium may have several Nash equilibria when the rules are changed to make it a simultaneous-move game.

Simultaneous-move games can be illustrated using a game tree by collecting decision nodes in *information sets* when players must make decisions without knowing at which specific node they find themselves. Similarly, sequential-move games can be illustrated using a game table; in this case, each player's full set of strategies must be carefully identified. Solving a sequential-move game from its strategic form may lead to many possible Nash equilibria. The number of potential equilibria can be reduced by using the criterion of *credibility* to eliminate some strategies as possible equilibrium strategies. This process leads to the *subgame-perfect equilibrium (SPE)* of the sequential-move game. These solution processes also work for games with additional players.

KEY TERMS

continuation (196)

credibility (196)

information set (192)

irreversible (182)

observable (182)

off-equilibrium path (195)

off-equilibrium subgame (195)

subgame (180)

subgame-perfect equilibrium (SPE) (196)

strategic order (182)

Glossary

continuation
: The continuation of a strategy from a (noninitial) node is the remaining part of the plan of action of that strategy, applicable to the subgame that starts at this node.

credibility
: A strategy is credible if its continuation at all nodes, on or off the equilibrium path, is optimal for the subgame that starts at that node.

information set
: A set of nodes among which a player is unable to distinguish when taking an action. Thus, his strategies are restricted by the condition that he should choose the same action at all points of an information set. For this, it is essential that all the nodes in an information set have the same player designated to act, with the same number and similarly labeled branches emanating from each of these nodes.

irreversible
: Cannot be undone by a later action. In a sequential-move game, the first mover's action must be irreversible and *observable* before the second mover's action is irreversible.

observable
: Known to other players before they make their responding actions. Together with irreversibility, this is an important condition for a game to be sequential-move.

off-equilibrium path
: A path of play that does not result from the players' choices of strategies in a subgame-perfect equilibrium.

off-equilibrium subgame
: A subgame starting at a node that does not lie on the equilibrium path of play.

subgame

A game comprising a portion or remnant of a larger game, starting at a noninitial node of the larger game.

subgame-perfect equilibrium (SPE)

A configuration of strategies (complete plans of action) such that their continuation in any subgame remains optimal (part of a rollback equilibrium), whether that subgame is on- or off-equilibrium. This ensures credibility of all the strategies.

strategic order

The order of moves from a game-theoretic point of view, determined by considerations of observability and irreversibility. It may differ from the chronological order of actions and, in turn, determine whether the game has sequential or simultaneous moves.

SOLVED EXERCISES

1. Consider the simultaneous-move tennis-point game with two players that has no Nash equilibrium in pure strategies, illustrated in Figure 4.17. If the game were transformed into a sequential-move game, would you expect that game to exhibit a first-mover advantage, a second-mover advantage, or neither? Explain your reasoning.
2. A Rebel Force of guerilla fighters seeks to inflict damage on a Conventional Army of the government, while the Conventional Army would like to destroy the Rebel Force. The two sides play a game in which each must decide whether to locate their forces in the Hills or in the Valley. The Rebels can inflict the most damage from the Valley, but if the Conventional Army is also in the Valley, it will be able to force the Rebels into open combat, where they are most vulnerable. By contrast, the Rebel Force is safer in the Hills, although it is also less capable of inflicting damage from that location. The ordinal payoff matrix for this game is shown below.

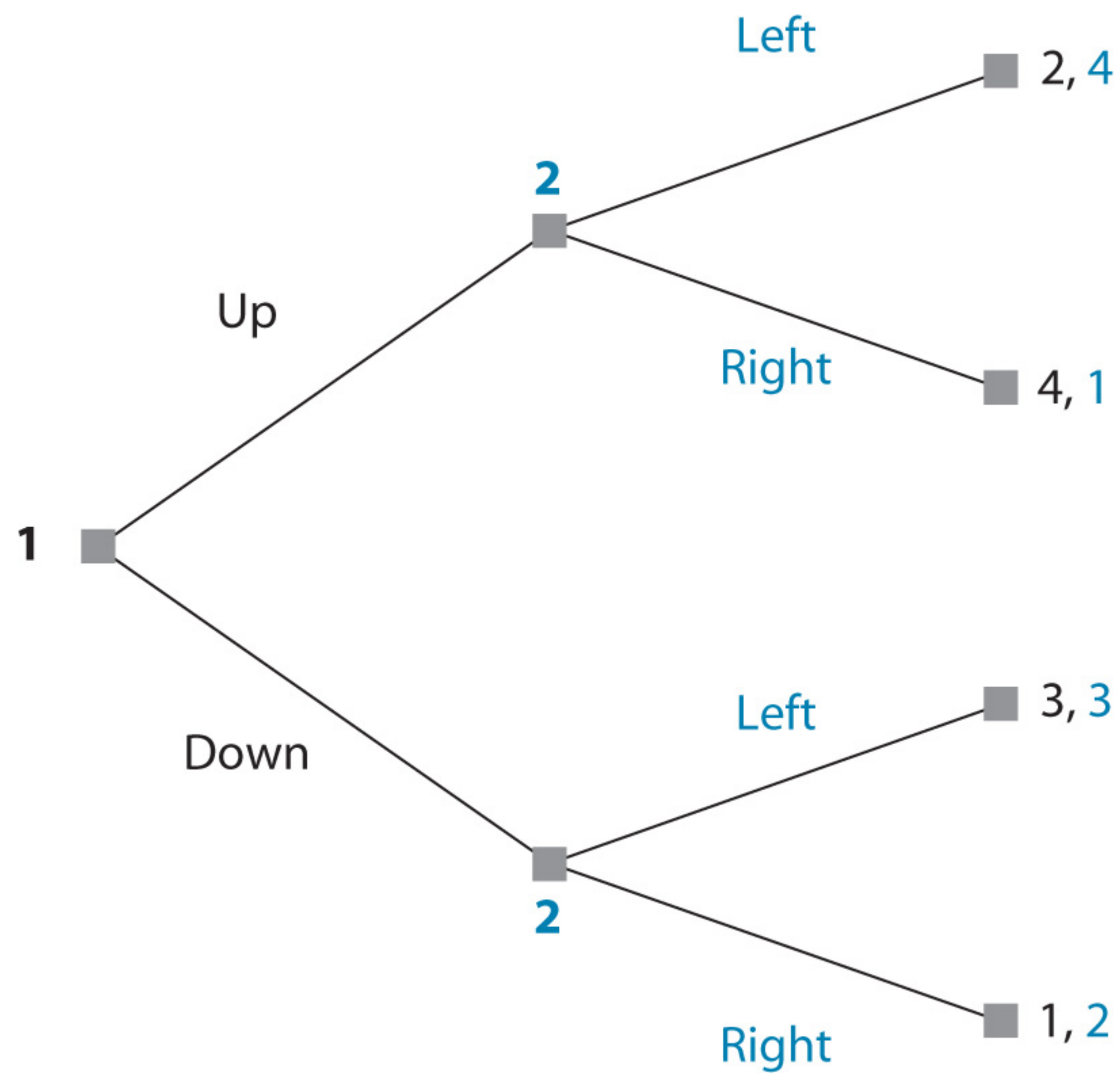
		CONVENTIONAL ARMY	
		Valley	Hills
REBEL FORCE	Valley	1, 4	4, 1
	Hills	3, 2	2, 3

---

1. Is this a zero-sum game? Explain your answer. Hint: Remember that a zero-sum game is one in which any change in outcome that makes one player better off must make the other player worse off. To show that a game is *not* zero-sum, it suffices to identify one outcome that is better for both players than some other outcome.
2. Identify all pure-strategy Nash equilibria of this game (or explain why no such equilibrium exists) in the case when the Rebel Force and Conventional Army move simultaneously.
3. Draw the game tree for the case when the Rebel Force moves first. What are the rollback equilibrium strategies and outcome?
4. Draw the game tree for the case when the Conventional Army moves first. What are the rollback equilibrium strategies and

    outcome?
    5. Does the order of moves matter in this game? If so, does the game have a first-mover advantage, a second-mover advantage, or do both players prefer the same ordering of moves?
    6. In the 1961 military handbook *Guerrilla Warfare*, Marxist revolutionary Che Guevara wrote, “The fundamental characteristic of a guerrilla band is mobility.” Discuss how the mobility, and therefore the nimbleness, of guerrilla forces might affect the possible ways in which moves could be ordered in this game. In particular, consider whether all the move orders analyzed in parts (b), (c), and (d) would actually be possible with a very mobile Rebel Force.

3. Consider the game represented by the game tree below. The first mover, Player 1, may move either Up or Down, after which Player 2 may move either Left or Right. Payoffs for the possible outcomes are shown in the game tree below. Show this game in strategic form. Then find all the pure-strategy Nash equilibria in the game. If there are multiple equilibria, indicate which one is subgame-perfect. For those equilibria that are not subgame-perfect, identify the reason (the source of their lack of credibility).



---

4. Consider the Airbus–Boeing game in Exercise S4 in Chapter 3. Show that game in strategic form and locate all Nash equilibria. Which one of the equilibria is subgame-perfect? For those equilibria that are not subgame-perfect, identify the reason.
5. Return to the two-player game tree in part (a) of Exercise S2 in Chapter 3.
    1. Draw the game in strategic form, making Scarecrow the Row player and Tinman the Column player.
    2. Find the Nash equilibrium.
6. Return to the two-player game tree in part (b) of Exercise S2 in Chapter 3.
    1. Draw the game in strategic form. (Hint: Refer to your answer to Exercise S2 in Chapter 3.) Find all Nash equilibria. There will be many.

2. For those equilibria that you found in part (a) that are not subgame-perfect, identify the reason.

7. Return to the three-player game tree in part (c) of Exercise S2 in Chapter 3.
   1. Draw the game table. Make Scarecrow the Row player, Tinman the Column player, and Lion the Page player. (Hint: Refer to your answer to Exercise S2 in Chapter 3.) Find all Nash equilibria. There will be many.
   2. For those equilibria that you found in part (a) that are not subgame-perfect, identify the reason.
8. Consider a simplified baseball game played between a pitcher and a batter. The pitcher chooses between throwing a fastball or a curve, while the batter chooses which pitch to anticipate. The batter has an advantage if he correctly anticipates the type of pitch. In this constant-sum game, the batter's payoff is the probability that he will get a base hit. The pitcher's payoff is the probability that the batter will fail to get a base hit, which is simply 1 minus the payoff for the batter. There are four potential outcomes:
   1. If the pitcher throws a fastball, and the batter guesses fastball, the probability of a hit is 0.300.
   2. If the pitcher throws a fastball, and the batter guesses curve, the probability of a hit is 0.200.
   3. If the pitcher throws a curve, and the batter guesses curve, the probability of a hit is 0.350.
   4. If the pitcher throws a curve, and the batter guesses fastball, the probability of a hit is 0.150.

   Suppose that the pitcher is "tipping" his pitches—which means that the pitcher is holding the ball, positioning his body, or doing something else in a way that reveals to the batter which pitch he is going to throw. For our purposes, this means that the pitcher–batter game is a sequential-move game in which the pitcher announces his pitch choice before the batter has to choose his strategy.

   1. Draw a game tree for this situation.
   2. Suppose that the pitcher knows he is tipping his pitches but can't stop himself from doing so. Thus, the pitcher and batter are playing the game you just drew. Find the rollback equilibrium of this game.
   3. Now change the timing of the game so that the batter has to reveal his action (perhaps by altering his batting stance)

before the pitcher chooses which pitch to throw. Draw the game tree for this situation, and find the rollback equilibrium.

Now assume that tipping by each player occurs so quickly that neither player can react to them, so that the game is in fact simultaneous.

1. (d) Draw a game tree to represent this simultaneous-move game, indicating information sets where appropriate.
2. (e) Draw the game table for the simultaneous-move game. Is there a Nash equilibrium in pure strategies? If so, what is it?

9. The street-garden game analyzed in Section 4 of this chapter has a 16 × 4 × 2 game table when the sequential-move version of the game is expressed in strategic form, as in Figure 6.15. There are *many* Nash equilibria to be found in this table.
   1. Use best-response analysis to find all Nash equilibria in Figure 6.15.
   2. Identify the subgame-perfect equilibrium from among your set of all Nash equilibria. Other equilibrium outcomes look identical to the subgame-perfect one—they entail the same payoffs for each of the three players—but they arise after different combinations of strategies. Explain how this can happen. Describe the credibility problems that arise in the equilibria that are not subgame-perfect.
10. Figure 6.1 represents the two-stage game between CrossTalk and GlobalDialog with a combination of tables and trees. Instead, represent the entire two-stage game with a single, very large game tree. Be careful to label which player makes the decision at each node, and remember to indicate information sets between nodes where necessary.
11. Recall the mall location game in Exercise S9 in Chapter 3. That three-player sequential-move game has a game tree that is similar to the one for the street-garden game shown in Figure 6.13.
   1. Draw the tree for the mall location game. How many strategies does each store have?
   2. Illustrate the game in strategic form and find all pure-strategy Nash equilibria in the game.
   3. Use successive elimination of dominated strategies to find the subgame-perfect equilibrium. (Hint: Reread the last two paragraphs of Section 4 in this chapter.)

12. The rules of the mall location game, analyzed in Exercise S11 above, specify that when all three stores request space in Urban Mall, the two bigger (more prestigious) stores get the available spaces. The original version of the game also specifies that the firms move sequentially in requesting mall space.
    1. Suppose that the three firms make their location requests simultaneously. Draw the payoff table for this version of the game and find all Nash equilibria. Which one of these equilibria do you think is most likely to be played in the real world? Explain.

    Now suppose that when all three stores simultaneously request Urban Mall, the two spaces are allocated by lottery, giving each store an equal chance of getting into Urban Mall. With such a system, each would have a two-thirds probability (or a 66.67% chance) of getting into Urban Mall when all three have requested space there, and a one-third probability (33.33% chance) of being alone in Rural Mall.

    1. (b) Draw the game table for this new version of the simultaneous-move mall location game. Find all Nash equilibria of the game. Which one of these equilibria do you think is most likely to be played in the real world? Explain.
    2. (c) Compare and contrast the equilibria found in part (b) with the equilibria found in part (a). Did you get the same Nash equilibria? Why or why not?
13. Return to the game between Monica and Nancy in Exercise S10 in Chapter 5. Assume that Monica and Nancy choose their effort levels sequentially instead of simultaneously. Monica commits to her choice of effort level first, and on observing this decision, Nancy commits to her own effort level.
    1. What is the subgame-perfect equilibrium of the game where the joint profits are $4m + 4n + mn$, the costs of their efforts to Monica and Nancy are m^2 and n^2, respectively, and Monica commits to an effort level first?
    2. Compare the payoffs to Monica and Nancy with those found in Exercise S10 in Chapter 5. Does this game have a first-mover or a second-mover advantage? Explain.
14. Extending Exercise S13, Monica and Nancy need to decide which (if either) of them will commit to an effort level first. To do this, each of them simultaneously writes on a separate slip of paper whether or not she will commit first. If they both write Yes or

they both write No, they choose effort levels simultaneously, as in Exercise S10 in Chapter 5. If Monica writes Yes and Nancy writes No, then Monica commits to her move first, as in Exercise S13 above. If Monica writes No and Nancy writes Yes, then Nancy commits to her move first.

1. Use the payoffs to Monica and Nancy in Exercise S13, as well as in Exercise S10 in Chapter 5, to construct the game table for the first-stage paper-slip decision game. (Hint: Note the symmetry of the game.)
2. Find the pure-strategy Nash equilibria of this first-stage game.

UNSOLVED EXERCISES

1. Consider a game in which there are two players, A and B. Player A moves first and chooses either Up or Down. If A chooses Up, the game is over, and each player gets a payoff of 2. If A chooses Down, then B gets a turn and chooses between Left and Right. If B chooses Left, both players get 0; if B chooses Right, A gets 3 and B gets 1.
   1. Draw the tree for this game and find the subgame-perfect equilibrium.
   2. Show this sequential-move game in strategic form, and find all Nash equilibria. Which is or are subgame-perfect? Which is or are not? If any are not, explain why.
   3. What method could be used to find the subgame-perfect equilibrium using the strategic form of the game? (Hint: Refer to the last two paragraphs of Section 4 in this chapter.)
2. A Monopolist faces potential competition from an Entrant in the undifferentiated product market for computer memory disks. The Monopolist currently operates one factory and must decide whether or not to build a second factory. The Entrant must similarly decide whether to build its first factory. A factory would cost either firm \$1.5 billion to build. Total (gross) profit—after subtracting production costs, but not factory construction costs—depends on how many factories have been built and is divided between the two firms in proportion to how many factories they operate. In particular, total (gross) profit takes the form $\Pi(Q) = Q(4 - Q)$, where Q is the total number of factories. Each firm's net profit in each of the four possible outcomes (after accounting for the cost of building factories) is shown in the payoff matrix below, expressed in billions of dollars. (You can verify these payoffs if you so desire. Recall that the Monopolist already has one factory and that each new factory costs \$1.5 billion.)

	ENTRANT	
	Build	Don't Build

You may need to scroll left and right to see the full figure.

		ENTRANT	
		Build	Don’t Build
MONOPOLIST	Build	0.5, −0.5	2.5, 0
	Don’t Build	2, 0.5	3, 0
You may need to scroll left and right to see the full figure.			

---

1. Is this a zero-sum game? Explain your answer. (For a hint, see Exercise S2 above.)
2. Identify all pure-strategy Nash equilibria of this game (or explain why no such equilibrium exists) in the case when the Monopolist and Entrant move simultaneously.
3. Draw the game tree for the case when the Monopolist moves first. What are the rollback equilibrium strategies and outcome?
4. Draw the game tree for the case when the Entrant moves first. What are the rollback equilibrium strategies and outcome?
5. Does the order of moves matter in this game? If so, does the game have a first-mover advantage, a second-mover advantage, or do both players prefer the same ordering of moves?
6. You serve on the Board of Directors of the Entrant firm. At a board meeting, another director makes the following argument: “$1.5 billion is an enormous investment and we don’t want to enter the market if the Monopolist is going to be building another factory. We need to wait to see whether they are going to be building a second factory before deciding what to do.” How would you use game theory to respond to this argument?

3. Return to the two-player game tree in part (a) of Exercise U2 in Chapter 3.
    1. Write the game in strategic form, making Albus the Row player and Minerva the Column player. Find all Nash equilibria.
    2. For those equilibria you found in part (a) of this exercise that are not subgame-perfect, identify the reason.
4. Return to the two-player game tree in part (b) of Exercise U2 in Chapter 3.
    1. Write the game in strategic form. Find all Nash equilibria.
    2. For those equilibria you found in part (a) that are not subgame-perfect, identify the reason.

5. Return to the two-player game tree in part (c) of Exercise U2 in [Chapter 3].
   1. Draw the game table. Make Albus the Row player, Minerva the Column player, and Severus the Page player. Find all Nash equilibria.
   2. For those equilibria you found in part (a) that are not subgame-perfect, identify the reason.
6. Consider the cola industry, in which Coke and Pepsi are the two dominant firms. (To keep the analysis simple, just forget about all the others.) The market size is \$8 billion. Each firm can choose whether to advertise. Advertising costs \$1 billion for each firm that chooses it. If one firm advertises and the other doesn't, then the former firm captures the whole market. If both firms advertise, they split the market 50:50 and pay for the advertising. If neither advertises, they split the market 50:50 but without the expense of advertising.
   1. Draw the payoff table for this game, and find the equilibrium when the two firms move simultaneously.
   2. Draw the game tree for this game (assuming that it is played sequentially), with Coke moving first and Pepsi following.
   3. Is either equilibrium in parts (a) and (b) better from the joint perspective of Coke and Pepsi? How could the two firms do better?
7. Along a stretch of a beach are 500 children in five clusters of 100 each. (Label the clusters A, B, C, D, and E, in that order.) Two ice-cream vendors are deciding simultaneously where to locate. Each vendor must choose the exact location of one of the clusters.

   If there is a vendor in a cluster, all 100 children in that cluster will buy an ice cream. For clusters without a vendor, 50 of the 100 children are willing to walk to a vendor who is one cluster away, only 20 are willing to walk to a vendor two clusters away, and no children are willing to walk the distance of three or more clusters. The ice cream melts quickly, so the walkers cannot buy for the nonwalkers.

   If the two vendors choose the same cluster, each will get a 50% share of the total demand for ice cream. If they choose different clusters, then those children (locals or walkers) for whom one vendor is closer than the other will go to the closer one, and those for whom the two are equidistant will be split 50:50 between them. Each vendor seeks to maximize her sales.

    1. Construct the five-by-five payoff table for the vendor location game. The following list will give you a start and a check on your calculations.
        1. If both vendors choose to locate at A, each sells 85 units.
        2. If the first vendor chooses B and the second chooses C, the first sells 150 and the second sells 170.
        3. If the first vendor chooses E and the second chooses B, the first sells 150 and the second sells 200.
    2. Eliminate dominated strategies as far as possible.
    3. In the remaining table, locate all pure-strategy Nash equilibria.
    4. If the game is altered to one with sequential moves, where the first vendor chooses her location first and the second vendor follows, what are the locations and the sales that result from the subgame-perfect equilibrium? How does the change in the timing of moves here help players resolve the coordination problem in part (c)?
8. Return to the game among the three lions in the Roman Colosseum in Exercise S8 in Chapter 3.
    1. Draw this game in strategic form. Make Lion 1 the Row player, Lion 2 the Column player, and Lion 3 the Page player.
    2. Find all Nash equilibria for the game. How many did you find?
    3. You should have found Nash equilibria that are not subgame-perfect. For each of those equilibria, which lion is making a noncredible threat? Explain.
9. Assume that the mall location game (from Exercises S9 in Chapter 3 and S11 in this chapter) is now played sequentially, but with a different order of play: Big Giant, then Titan, then Frieda's.
    1. Draw the new game tree.
    2. What is the subgame-perfect equilibrium of the game? How does it compare with the subgame-perfect equilibrium for Exercise S9 in Chapter 3?
    3. Now draw the strategic form for this new version of the game.
    4. Find all Nash equilibria of the game. How many are there? How does this number compare with the number of equilibria from Exercise S11 in this chapter?
10. Return to the game between Monica and Nancy in Exercise U10 in Chapter 5. Assume that Monica and Nancy choose their effort levels sequentially instead of simultaneously. Monica commits to her choice of effort level first. On observing this decision, Nancy commits to her own effort level.

1. What is the subgame-perfect equilibrium of the game where the joint profits are $5m + 4n + mn$, the costs of their efforts to Monica and Nancy are m^2 and n^2, respectively, and Monica commits to an effort level first?
2. Compare the payoffs to Monica and Nancy with those found in Exercise U10 in Chapter 5. Does this game have a first-mover or second-mover advantage?
3. Using the same joint profit function as in part (a), find the subgame-perfect equilibrium for the game where *Nancy* must commit first to an effort level.

11. Extending Exercise U10, Monica and Nancy need to decide which (if either) of them will commit to an effort level first. To do this, each of them simultaneously writes on a separate slip of paper whether or not she will commit first. If they both write Yes or they both write No, they choose effort levels simultaneously, as in Exercise U10 in Chapter 5. If Monica writes Yes and Nancy writes No, they play the game in part (a) of Exercise U10 above. If Monica writes No and Nancy writes Yes, they play the game in part (c).
    1. Use the payoffs to Monica and Nancy in parts (b) and (c) in Exercise U10 above, as well as those in Exercise U10 in Chapter 5, to construct the game table for the first-stage paper-slip decision game.
    2. Find the pure-strategy Nash equilibria of this first-stage game.
12. In the faraway town of Saint James, two firms, Bilge and Chem, compete in the soft-drink market (Coke and Pepsi aren't in this market yet). They sell identical products, and since their good is a liquid, they can easily choose to produce fractions of units. Since they are the only two firms in this market, the price of the good (in dollars), P, is determined by $P = (30 - Q_B - Q_C)$, where Q_B is the quantity produced by Bilge and Q_C is the quantity produced by Chem (each measured in liters). At this time, both firms are considering whether to invest in new bottling equipment that will lower their operating costs.
    1. If firm j decides *not* to invest, its total cost will be $C_j = Q_j^2/2$, where j stands for either B (Bilge) or C (Chem).
    2. If firm j decides to invest, its total cost will be $C_j = 20 + Q_j^2/6$. This new cost function reflects the cost of investing in the new machines (20) as well as the lower operating costs associated with those machines.

The two firms make their investment choices simultaneously, but the payoffs in this investment game depend on the subsequent duopoly games that arise. The game is thus really a two-stage game: decide whether to invest, and then play a duopoly game.

1. Suppose both firms decide to invest. Write the profit functions in terms of Q_B and Q_C for the two firms. Use these to find the Nash equilibrium of the quantity-setting game. What are the equilibrium quantities and profits for both firms? What is the market price?
2. Now suppose both firms decide not to invest. What are the equilibrium quantities and profits for both firms? What is the market price?
3. Now suppose that Bilge decides to invest, and Chem decides not to invest. What are the equilibrium quantities and profits for both firms? What is the market price?
4. Draw the two-by-two game table for the first-stage investment game between the two firms. Each firm has two strategies: Invest and Don’t Invest. The payoffs are simply the profits found in parts (a), (b), and (c). (Hint: Note the symmetry of the game.)
5. What is the subgame-perfect equilibrium of the overall two-stage game?

13. Two French aristocrats, Chevalier Chagrin and Marquis de Renard, fight a duel. Each has a pistol loaded with one bullet. They start 10 steps apart and walk toward each other at the same pace, 1 step at a time. After each step, either may fire his gun. When one shoots, the probability of scoring a hit depends on the distance. After k steps it is $k/5$, so it rises from 0.2 after the first step to 1 (certainty) after 5 steps, at which point the two are right up against each other. If one player fires and misses while the other has yet to fire, the walk must continue even though the bulletless one now faces certain death; this rule is dictated by the code of the aristocracy. Each player gets a payoff of −1 if he himself is killed and 1 if the other is killed. If neither or both are killed, each gets 0.

    This is a game with five sequential steps and simultaneous moves (shoot or don’t shoot) at each step. Find the rollback (subgame-perfect) equilibrium of this game.

Hint: Begin at step 5, when the duelists are right up against each other. Set up the two-by-two table for the simultaneous-move game at this step, and find the Nash equilibrium. Now move back to step 4, where the probability of scoring a hit is 4/5, or 0.8, for each. Set up the two-by-two table for the simultaneous-move game at this step, correctly specifying in the appropriate cell what happens in the future. For example, if one shoots and misses, but the other does not shoot, then the other will wait until step 5 and score a sure hit. If neither shoots, then the game will go to the next step, for which you have already found the equilibrium. Using all this information, find the payoffs in the two-by-two table of step 4, and find the Nash equilibrium at this step. Work backward in the same way through the rest of the steps to find the Nash equilibrium of the full game.

14. Describe an example of business competition that is similar in structure to the duel in Exercise U13.

7 ■ Simultaneous-Move Games: Mixed Strategies

IN OUR STUDY of simultaneous-move games in Chapter 4, we came across some games that the solution methods described there could not solve—games with no Nash equilibria in pure strategies. To predict outcomes for such games, we need to extend our concepts of *strategy* and *equilibrium* to allow for the possibility that players may use strategies in which they make *random* choices among the actions available to them, more commonly known as *mixed strategies*.

Consider the tennis-point game from the end of Chapter 4. This game is zero-sum; the interests of the two tennis players are exactly opposite. Evert wants to hit her passing shot to whichever side—down the line (DL) or crosscourt (CC)—is not covered by Navratilova, whereas Navratilova wants to cover the side to which Evert hits her shot. In other words, Evert wants her choice to differ from Navratilova's, and Navratilova wants her choice to coincide with Evert's. In Chapter 4, we pointed out that in such a situation, any systematic choice by Evert will be exploited by Navratilova to her own advantage and therefore to Evert's disadvantage. Conversely, Evert can exploit any systematic choice by Navratilova. To avoid being thus exploited, each player wants to keep the other guessing, which can be done by acting unsystematically or randomly.

However, randomness doesn't mean choosing each shot half the time or alternating between the two. The latter would itself be a systematic action open to exploitation. A 60:40 or 75:25 random mix may be better than a 50:50 mix depending on the situation. In this chapter, we develop methods for calculating the best mix and discuss how well game theory helps us understand actual play in zero-sum games with equilibria in mixed strategies.

Our method for calculating the best mix can also be applied to non-zero-sum games. However, in such games, the players' interests can partially coincide, so when player B exploits A's systematic choice to her own advantage, it is not necessarily to A's disadvantage. Therefore, the logic of keeping the other player guessing is weaker, or even absent altogether, in non-zero-sum games. We will discuss whether and when mixed-strategy equilibria make sense in such games.

We start this chapter with a discussion of mixed strategies in two-by-two games and describe the most direct method for calculating best responses and finding a mixed-strategy equilibrium. Many of the concepts and methods we develop in Section 2 continue to be valid in more general games, and Sections 6 and 7 extend these methods to games where players may have more than two pure strategies. We conclude with some general observations about how to mix strategies in practice and some evidence on whether mixing is observed in reality.

1 WHAT IS A MIXED STRATEGY?

When players choose to act unsystematically, they choose from among the pure strategies available to them in some random way. Consider the tennis-point game discussed in Section 8 of Chapter 4. In that game, Martina Navratilova and Chris Evert each choose from two initially given pure strategies: down the line (DL) and crosscourt (CC). We call a random mixture of these two pure strategies a *mixed strategy*.

Such mixed strategies cover a continuous range of possibilities. At one extreme, DL could always be chosen (probability 1), meaning that CC is never chosen (probability 0); this “mixture” is simply the pure strategy DL. At the other extreme, DL could be chosen with probability 0 and CC with probability 1; this “mixture” is the same as the pure strategy CC. In between is a continuous range of possibilities: DL chosen with probability 75% (0.75) and CC with probability 25% (0.25); or both chosen with probabilities 50% (0.5) each; or DL with probability ⅓ (0.33 . . .) and CC with probability ⅔ (0.66 . . .); and so on.[1]

The payoff from a mixed strategy is the probability-weighted average of the payoffs from its constituent pure strategies. For example, in the tennis-point game, suppose that Navratilova were to play the pure strategy DL and Evert were to play a mixture of 75% DL and 25% CC. Against Navratilova’s DL, Evert’s payoff from DL is 50, and her payoff from CC is 90. On average, Evert’s mixture (0.75 DL, 0.25 CC) therefore yields $0.75 \times 50 + 0.25 \times 90 = 37.5 + 22.5 = 60$. This result is Evert’s expected payoff from this particular mixed strategy.[2]

The probability of choosing one or the other pure strategy is a continuous variable that ranges from 0 to 1. Therefore, mixed strategies are just special kinds of continuously variable strategies like those we studied in Chapter 5. Each pure strategy is an extreme special case where the probability of choosing that pure strategy equals 1.

The notion of Nash equilibrium also extends easily to include mixed strategies. In this case, Nash equilibrium is defined as the list of mixed strategies, one for each player, such that the choice of each is her best choice, in the sense of yielding the highest expected payoff for her, given the mixed strategies of the other players. Allowing for mixed strategies in a game solves the problem of nonexistent Nash equilibria, which we encountered for pure strategies, automatically and almost entirely. Nash's celebrated theorem shows that, under very general circumstances (which are broad enough to cover all the games that we meet in this book and many more besides), a Nash equilibrium in mixed strategies exists.

At this broadest level, therefore, incorporating mixed strategies into our analyses does not entail anything different from the general theory of continuous strategies developed in Chapter 5. However, the special case of mixed strategies raises several special conceptual and methodological issues that deserve separate study.

Endnotes

- When a chance event has just two possible outcomes, people often speak of the odds in favor of or against one of the outcomes. If the two possible outcomes are labeled A and B, and the probability of A is p, so that the probability of B is $(1 - p)$, then the ratio $p/(1 - p)$ gives the odds in favor of A, and the reverse ratio $(1 - p)/p$ gives the odds against A. Thus, when Evert chooses CC with probability 0.25 (25%), the odds against her choosing CC are 3 to 1, and the odds in favor of it are 1 to 3. This terminology is often used in betting contexts, so those of you who misspent your youth in that way will be more familiar with it. However, this usage does not readily extend to situations in which three or more outcomes are possible, so we avoid it here. Return to reference 1
- Game theory assumes that players will calculate and try to maximize their expected payoffs when probabilistic mixtures of strategies or outcomes are included. It is important to remember that the word *expected* in *expected payoff* is a technical term from probability and statistics theory. It merely denotes a probability-weighted average. It does not refer to the payoff that a player should expect, in the sense of regarding it as her right or entitlement. Return to reference 2

Glossary

expected payoff
: The probability-weighted average (statistical mean or expectation) of the payoffs of one player in a game, corresponding to all possible realizations of a random choice of nature or mixed strategies of the players.

2 MIXING MOVES

We begin our analysis of games with mixed-strategy equilibria using the tennis-point example from Chapter 4 that we reintroduced above. This game does not have a Nash equilibrium in pure strategies. In this section, we show how extending the set of strategies to include mixed strategies remedies this deficiency, and we interpret the resulting equilibrium as one in which each player keeps the other guessing. In Figure 7.1, we reproduce the payoff matrix of Figure 4.17. For extra clarity, we also indicate Navratilova's payoff numbers (50, 20, 10, and 80) in blue whenever they appear in the text for the remainder of this chapter. We adopt this number-coloring convention as a way of helping you to understand the origin of the numbers in the equations that follow and so that you can more easily reproduce the analysis yourself.

		NAVRATILOVA	
		DL	CC
EVERT	DL	50, 50	80, 20
	CC	90, 10	20, 80

FIGURE 7.1 No Equilibrium in Pure Strategies

A. The Benefit of Mixing

In the tennis-point game illustrated in Figure 7.1, if Evert always chooses DL, Navratilova will then cover DL and hold Evert's payoff down to 50. Similarly, if Evert always chooses CC, Navratilova will choose to cover CC and hold Evert down to 20. If Evert can choose only one of her two basic (pure) strategies and Navratilova can predict that choice, Evert's better (or less bad) pure strategy will be DL, which will yield her a payoff of 50.

But suppose Evert is not restricted to using only pure strategies and can choose a mixed strategy, perhaps one in which her probability of playing DL on any one occasion is 75%, or 0.75; this makes her probability of playing CC 25%, or 0.25. Using the method outlined in Section 1, we can calculate *Navratilova*'s expected payoff against this mixture as

$0.75 \times 50 + 0.25 \times 10 = 37.5 + 2.5 = 40$ if she covers DL, and

$0.75 \times 20 + 0.25 \times 80 = 15 + 20 = 35$ if she covers CC.

If Evert chooses this 75–25 mix, the expected payoffs show that Navratilova can best exploit it by covering DL.

When Navratilova chooses DL to best exploit Evert's 75–25 mix, her choice works to Evert's disadvantage because this is a zero-sum game. Evert's expected payoffs are

$0.75 \times 50 + 0.25 \times 90 = 37.5 + 22.5 = 60$ if Navratilova covers DL, and

$0.75 \times 80 + 0.25 \times 20 = 60 + 5 = 65$ if Navratilova covers CC.

By choosing DL, Navratilova holds Evert down to 60 rather than 65. But notice that Evert's payoff with the mixture is still better than the 50 she would get by playing purely DL or the 20 she would get by playing purely CC.[3]

This 75–25 mix, while improving Evert's expected payoff relative to her pure strategies, does leave her strategy open to some exploitation by Navratilova. By choosing to cover DL, Navratilova can hold Evert down to a lower expected payoff than when she chooses to cover CC. Ideally, Evert would like to find a mix that would be exploitation-proof—a mix that would leave Navratilova no obvious choice of pure strategy to use against it. Evert's exploitation-proof mixture must have the property that Navratilova gets the same expected payoff against it by covering DL as by covering CC; that is, it must keep Navratilova indifferent between her two pure strategies. This property, called the opponent's indifference property, is the key to mixed-strategy equilibria in non-zero-sum games, as we will see later in this chapter.

Finding the exploitation-proof mixture requires taking a more general approach to describing Evert's mixed strategy so that we can solve algebraically for the appropriate probabilities. In this approach, we denote the probability of Evert choosing DL with the algebraic symbol p, so the probability of her choosing CC is $1 - p$. We refer to this mixture as Evert's p-mix for short.

Against the p-mix, Navratilova's expected payoffs are

$50p + 10(1 - p)$ if she covers DL, and

$20p + 80(1 - p)$ if she covers CC.

For Evert's strategy—her p-mix—to be exploitation-proof, these two expected payoffs for Navratilova should be equal. That implies $50p + 10(1 - p) = 20p + 80(1 - p)$; or $30p = 70(1 - p)$; or $100p = 70$; or $p = 0.7$. Thus, Evert's exploitation-proof mix uses DL with probability 70% and CC with probability 30%. With these probabilities, Navratilova gets the same expected payoff from each of her pure strategies and therefore cannot exploit any one of them to her advantage (or to Evert's disadvantage in this zero-sum game). And Evert's expected payoff from this mixed strategy is

$50 \times 0.7 + 90 \times 0.3 = 35 + 27 = 62$ if Navratilova covers DL, and also

$80 \times 0.7 + 20 \times 0.3 = 56 + 6 = 62$ if Navratilova covers CC.

This expected payoff is better than the 50 that Evert would get if she used the pure strategy DL and better than the 60 she would get from the 75–25 mixture. We now know this mixture is exploitation-proof, but is it Evert’s optimal or equilibrium mixture?

B. Best Responses and Equilibrium

To find the equilibrium mixtures in this game, we return to the method of best-response analysis originally described in Chapter 4 and extended to games with continuous strategies in Chapter 5. Our first task is to identify Evert's best response to—her best choice of p for—each of Navratilova's possible strategies. Since those strategies can also be mixed, they are similarly described by the probability with which Navratilova covers DL. We label this probability q, so $1 - q$ is the probability that Navratilova covers CC. We refer to Navratilova's mixed strategy as her q-mix, and we now look for Evert's best choice of p for each of Navratilova's possible choices of q.

Using Figure 7.1, we see that Evert's p-mix gets her the expected payoff

$50p + 90(1 - p)$ if Navratilova covers DL, and

$80p + 20(1 - p)$ if Navratilova covers CC.

Therefore, against Navratilova's q-mix, Evert's expected payoff is

$$[50p + 90(1 - p)]q + [80p + 20(1 - p)](1 - q).$$

Rearranging the terms, Evert's expected payoff becomes

$$[50q + 80(1 - q)]p + [90q + 20(1 - q)](1 - p)$$

$$= [90q + 20(1 - q)] + [50q + 80(1 - q) - 90q - 20(1 - q)]p$$

$$= [20 + 70q] + [60 - 100q]p,$$

and we use this expected payoff to help us find Evert's best-response values of p.

We are trying to identify the p that maximizes Evert's payoff at each value of q, so the key question is how her expected payoff varies with p. What matters is the coefficient on p: $[60 - 100q]$. Specifically, it matters whether that coefficient is positive (in which case Evert's expected payoff increases as p increases) or negative (in which case Evert's expected payoff decreases as p increases). Clearly, the sign of the coefficient depends on q, the critical value of q being the one that makes $60 - 100q = 0$. That q value is 0.6.

Thus, when Navratilova's $q < 0.6$, $[60 - 100q]$ is positive, Evert's expected payoff increases as p increases, and her best choice is $p = 1$, or the pure strategy DL. Similarly, when Navratilova's $q > 0.6$, Evert's best choice is $p = 0$, or the pure strategy CC. If Navratilova's $q = 0.6$, Evert gets the same expected payoff regardless of p, and any mixture between DL and CC is just as good as any other; any p from 0 to 1 can be a best response. We summarize our results for this game for future reference:

If $q < 0.6$, the best response is $p = 1$ (pure DL).

If $q = 0.6$, any p-mix is a best response.

If $q > 0.6$, the best response is $p = 0$ (pure CC).

These expressions should confirm your intuition that when q is low (Navratilova is sufficiently unlikely to cover DL), Evert should choose DL, and when q is high (Navratilova is sufficiently likely to cover DL), Evert should choose CC. The exact sense of *sufficiently*, and therefore the switching point $q = 0.6$, depends, of course, on the specific payoffs in this particular example.

We pointed out earlier that mixed strategies are simply a special kind of continuous strategy, in which the probability is the continuous variable. Now we have found Evert's best p corresponding to each of Navratilova's choices of q. In other words, we have found Evert's best-response rule, and we can graph it exactly as we did for the games in Chapter 5.

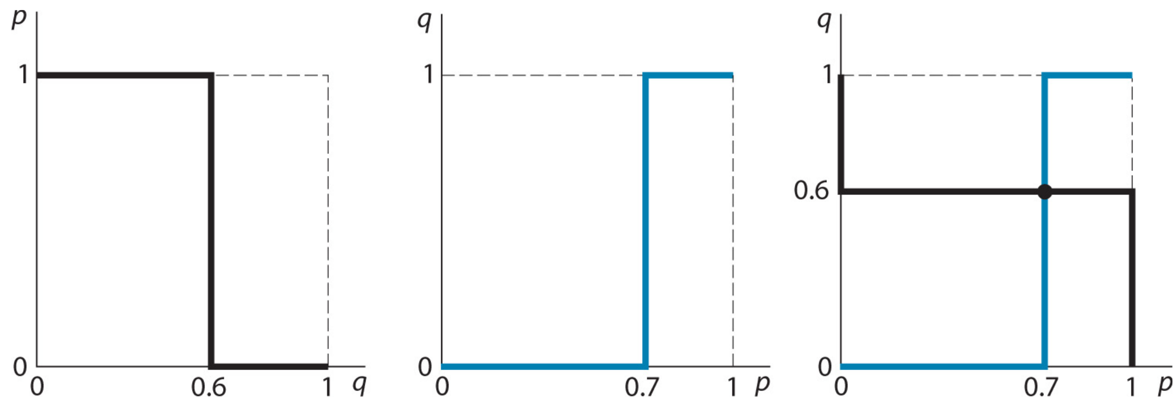


Figure 7.2 Best Responses and Equilibrium in the Tennis-Point Game

---

We show this graph in the left-hand panel of Figure 7.2, with q on the horizontal axis and p on the vertical axis. Both variables are probabilities, limited to the range from 0 to 1. For $q < 0.6$, p is at its upper limit of 1; for $q > 0.6$, p is at its lower limit of 0. At $q = 0.6$, all values of p between 0 and 1 are equally "best" for Evert; therefore, the best-response curve is the vertical line between 0 and 1. This is a new flavor of best-response curve; unlike the steadily rising or falling lines or curves of Chapter 5, it is flat over two intervals of q and vertical at the point where those two intervals meet. But conceptually, it is just like any other best-response curve.

Navratilova's best-response rule—her best q-mix corresponding to each of Evert's p-mixes—can be calculated and graphed in the same way; we leave this for you to do so you can consolidate your understanding of the idea and the algebra. You should also check that your intuition regarding Navratilova's choices is consistent with what the calculations indicate, as we did for Evert's. We simply state the result here:

If $p < 0.7$, the best response is $q = 0$ (pure CC).

If $p = 0.7$, any q-mix is a best response.

If $p > 0.7$, the best response is $q = 1$ (pure DL).

This best-response rule for Navratilova is graphed in the middle panel of Figure 7.2.

The right-hand panel of Figure 7.2 combines the left-hand and middle panels by reflecting the left graph across the diagonal ($p = q$ line) so that p is on the horizontal axis and q on the vertical axis, then superimposing this graph on the middle graph. Now the two best-response curves meet at exactly one point, where $p = 0.7$ and $q = 0.6$. Here, each player's mixture is the best response to the other's mixture, so the pair of mixtures constitutes a Nash equilibrium in mixed strategies.

This representation of best-response rules includes pure strategies as special cases corresponding to the extreme values of p and q. We can see that the best-response curves do not have any points in common on any of the edges of the graph where each value of p and q equals either 0 or 1; this shows us that the game does not have any pure-strategy equilibria, as we determined directly in Section 8 of Chapter 4.[4] The mixed-strategy equilibrium in this example is the unique Nash equilibrium of the game.

You can also calculate Navratilova's exploitation-proof choice of q using the same method that we used in Section 2.A for finding Evert's exploitation-proof p. You will get the answer $q = 0.6$. Thus, the two exploitation-proof choices are indeed best responses to each other, and they are the Nash equilibrium mixtures for the two players.

In fact, if all you want to do is to find a mixed-strategy equilibrium of a zero-sum game where each player has just two pure strategies, you don't have to go through the detailed construction of best-response curves, graph them, and look for their intersection. You can write down the equations from Section 2.A for each player's exploitation-proof mixture and solve them. If both probabilities fall between 0 and 1, you have found what you want. If the solution includes a probability that is negative, or greater than 1, then the game does not have a mixed-strategy equilibrium; you should go back and look for a pure-strategy equilibrium. In Sections 6 and 7 we examine solution techniques for games where a player has more than two pure strategies.

Endnotes

- Not every mixed strategy will perform better than its constituent pure strategies. For example, if Evert chooses a 50–50 mixture of DL and CC, Navratilova can hold Evert's expected payoff down to 50, exactly the same as Evert would get from pure DL. And a mixture that attaches a probability of less than 30% to DL will be worse for Evert than pure DL. We suggest that you verify these statements as a useful exercise to acquire the skill of calculating expected payoffs and comparing strategies. Return to reference 3
- If, in some game you are trying to solve, the best-response curves meet on an edge (but not exactly at a corner) of the graph, one player uses a pure strategy in equilibrium and the other mixes; if the best-response curves meet at a corner of the graph, both players use pure strategies in equilibrium. Return to reference 4

Glossary

opponent's indifference property
: An equilibrium mixed strategy of one player in a two-person game has to be such that the other player is indifferent among all the pure strategies that are actually used in her mixture.

3 NASH EQUILIBRIUM AS A SYSTEM OF BELIEFS AND RESPONSES

When the moves in a game are simultaneous, neither player can respond to the other's actual choice. Instead, each takes her best action in light of what she thinks the other might be choosing to do at that instant. In Chapter 4, we called such thinking a player's belief about the other's strategy choice. We then interpreted Nash equilibrium as a configuration where such beliefs are correct, so that each player chooses her best response to the actual actions of the other. This concept proved useful for understanding the structures and outcomes of many important types of games, most notably the prisoners' dilemma, coordination games, and games of chicken.

In Chapter 4, however, we considered only pure-strategy Nash equilibria. Therefore, a hidden assumption went almost unremarked—namely, that each player was sure or confident in her belief that the other would choose a particular pure strategy. Now that we are considering mixed strategies, the concept of belief requires a corresponding reinterpretation.

Players may be unsure about what others might be doing. In the coordination game in Chapter 4, in which Sherlock Holmes wanted to meet Dr. Watson, he might be unsure whether his partner would go to St. Bart's or Simpson's, and his belief might be that there was a 50–50 chance that Watson would go to either one. And in the tennis-point example, Evert might recognize that Navratilova was trying to keep her (Evert) guessing and would therefore be unsure which of the available actions Navratilova would play. In Chapter 2, Section 2, we labeled this concept *strategic uncertainty*, and in Chapter 4, we mentioned that such uncertainty can give rise to mixed-strategy equilibria. Now we develop this idea more fully.

It is important, however, to distinguish between being unsure and having incorrect beliefs. For example, in the tennis-point example, Navratilova cannot be sure of what action Evert is choosing on any one occasion. But she can still have correct beliefs about Evert's mixture—namely, about the probabilities with which Evert chooses each of her two pure strategies. Having correct beliefs about mixed strategies means knowing or calculating or guessing the correct probabilities with which the other player chooses each of her underlying basic or pure actions. In our example, it turned out that Evert's equilibrium mixture was 70% DL and 30% CC. If Navratilova believes that Evert will play DL with 70% probability and CC with 30% probability, then her belief, although uncertain, will be correct in equilibrium.

Thus, we have an alternative and mathematically equivalent way to define Nash equilibrium in terms of beliefs: Each player forms beliefs about the probabilities with which the other is choosing his actions and chooses her own best response to those probabilities. A Nash equilibrium in mixed strategies occurs when those beliefs are correct in the sense just explained.

In the next section, we consider mixed strategies and their Nash equilibria in non-zero-sum games. In such games, there is no general reason that one player's pursuit of her own interests should work against her opponent's interests. Therefore, it is not generally the case that she would want to conceal her intentions from her opponent, and there is no general argument in favor of keeping her opponent guessing. However, because moves are simultaneous, each player may still be unsure of what action the other is taking, and may therefore have uncertain beliefs that make her unsure about how she should act. This situation can lead to mixed-strategy equilibria, and their interpretation in terms of subjectively uncertain but correct beliefs proves particularly important.

4 MIXING IN NON-ZERO-SUM GAMES

The same mathematical method used to find mixed-strategy equilibria in zero-sum games—namely, solution of equations derived from the opponent's indifference property—can be applied to non-zero-sum games as well, and it can reveal mixed-strategy equilibria in some of them. However, in such games, the players' interests may coincide to some extent. Therefore, the fact that one player will exploit her opponent's systematic choice of strategy to her advantage need not work out to her opponent's disadvantage, as was the case with zero-sum interactions. In a coordination game of the kind we studied in Chapter 4, for example, the players are better able to coordinate if each can rely on the other's acting systematically; random actions only increase the risk of coordination failure. As a result, mixed-strategy equilibria have a weaker rationale, and sometimes no rationale at all, in non-zero-sum games. Here, we examine mixed-strategy equilibria in some prominent non-zero-sum games and discuss their relevance, or lack thereof.

A. Will Holmes Meet Watson? Assurance, Pure Coordination, and Battle of the Sexes

We illustrate mixing in non-zero-sum games by using the assurance version of the meeting coordination game between Holmes and Watson. For your convenience, we reproduce its game table (Figure 4.14) here as Figure 7.3. We consider the game from Watson's perspective first. If Watson is confident that Holmes will go to St. Bart's, he, too, should go to St. Bart's. If Watson is confident that Holmes will go to Simpson's, so should he. But if Watson is unsure about Holmes's choice, what is his own best choice?

To answer this question, we must give a more precise meaning to the uncertainty in Holmes's mind. (The technical term for uncertainty in a player's mind, in the theory of probability and statistics, is *subjective uncertainty*. In the context where the uncertainty is about another player's action in a game, it is also strategic uncertainty, which we discussed in Chapter 2, Section 2.D.) We gain precision by stipulating the probability with which Watson thinks Holmes will choose one meeting place or the other. The probability of Holmes choosing St. Bart's can be any real number between 0 and 1 (that is, between 0% and 100%). We cover all possible cases by using algebra, letting the symbol p denote the probability (in Watson's mind) that Holmes will choose St. Bart's; the variable p can take on any real value between 0 and 1. Then $(1 - p)$ is the probability (again, in Watson's mind) that Holmes will choose Simpson's. In other words, we describe Watson's strategic uncertainty as follows: He thinks that Holmes is using a mixed strategy, mixing the two pure strategies, St. Bart's and Simpson's, in proportions or probabilities p and $(1 - p)$, respectively. We call this mixed strategy Holmes’s p-mix, even though for the moment it is purely an idea in Watson’s mind.

		WATSON	
		St. Bart’s	Simpson’s
HOLMES	St. Bart’s	1, 1	0, 0
	Simpson’s	0, 0	2, 2

FIGURE 7.3 Assurance

Given his uncertainty, Watson can calculate the expected payoffs from his own actions when they are played against his belief about Holmes’s p-mix. If Watson chooses St. Bart’s, it will yield him $1 \times p + 0 \times (1 - p) = p$. If he chooses Simpson’s, it will yield him $0 \times p + 2 \times (1 - p) = 2 \times (1 - p)$. When p is high, $p > 2(1 - p)$; so if Watson is fairly sure that Holmes is going to St. Bart’s, then he does better by also going to St. Bart’s. Similarly, when p is low, $p < 2(1 - p)$; if Watson is fairly sure that Holmes is going to Simpson’s, then he does better by going to Simpson’s. If $p = 2(1 - p)$, or $3p = 2$, or $p = 2/3$, the two choices give Watson the same expected payoffs. Therefore, if he believes that $p = 2/3$, he might be unsure about his own choice, so he might dither between the two.

Holmes can figure this out, and that makes him unsure about Watson’s choice. Thus, Holmes also faces subjective strategic uncertainty. Suppose, in Holmes’s mind, Watson will choose St. Bart’s with probability q and Simpson’s with probability $(1 - q)$. Similar reasoning shows that Holmes should choose St. Bart’s if he believes Watson’s $q > 2/3$ and Simpson’s if he believes $q < 2/3$. If Holmes believes $q = 2/3$, he will be indifferent between the two actions and unsure about his own choice.

Now we have the basis for a mixed-strategy equilibrium with $p = 2/3$ and $q = 2/3$. In such an equilibrium, these p and q values are simultaneously the actual mixture probabilities and the subjective beliefs of each player about the other's mixture probabilities. The correct beliefs sustain each player's own indifference between the two pure strategies and therefore each player's willingness to mix the two. This situation matches exactly the concept of a Nash equilibrium as a system of self-fulfilling beliefs and responses, as described in Section 3.

The key to finding the mixed-strategy equilibrium is that Watson is willing to mix his two pure strategies only if his subjective uncertainty about Holmes's choice is just right—that is, if the value of p in Holmes's p-mix is just right. Algebraically, this idea is borne out by solving for the equilibrium value of p by using the equation $p = 2(1 - p)$, which ensures that Watson gets the same expected payoff from each of his two pure strategies when each is matched against Holmes's p-mix. When the equation holds in equilibrium, it is as if Holmes's mixture probabilities are doing the job of keeping Watson indifferent. We emphasize *as if* because in this game, Holmes has no reason to keep Watson indifferent; that outcome is merely a property of the equilibrium. Still, the general idea is worth remembering: In a mixed-strategy Nash equilibrium, each person's mixture probabilities keep the other player indifferent between his pure strategies. We derived the opponent's indifference property in the discussion of zero-sum games above, and now we see that it remains valid even in non-zero-sum games.

However, the mixed-strategy equilibrium has some very undesirable properties in the assurance game. First, it yields both players rather low expected payoffs. Watson's expected payoffs from his two actions, p and $2(1 - p)$, both equal $2/3$ when $p = 2/3$. Similarly, Holmes's expected payoffs against Watson's equilibrium q-mix for $q = 2/3$ are also both

⅔. Thus, each player gets ⅔ in the mixed-strategy equilibrium. In Chapter 4, we found two pure-strategy equilibria for this game; even the worse of them (both choosing St. Bart's) yields the players 1 each, and the better one (both choosing Simpson's) yields them 2 each.

The reason the two players fare so badly in the mixed-strategy equilibrium is that when they choose their actions independently and randomly, they create a significant probability of going to different places; when that happens, they do not meet, and each gets a payoff of 0. Holmes and Watson fail to meet if one goes to St. Bart's and the other goes to Simpson's, or vice versa. The probability of this happening when both are using their equilibrium mixtures is $2 \times (⅔) \times (⅓) = 4/9$.[5] Similar problems exist in the mixed-strategy equilibria of most non-zero-sum games.

A second undesirable property of the mixed-strategy equilibrium here is that it is very unstable. If either player departs ever so slightly from the exact values $p = ⅔$ or $q = ⅔$, the best choice of the other tips to one pure strategy. Once one player chooses a pure strategy, then the other also does better by choosing the same pure strategy, and play moves to one of the two pure-strategy equilibria. This instability of mixed-strategy equilibria is also common to many non-zero-sum games. However, some important non-zero-sum games do have mixed-strategy equilibria that are not so fragile. One example, considered later in this chapter and in Chapter 12, is the mixed-strategy equilibrium in the game of chicken, which has an interesting evolutionary interpretation.

Given this analysis of the mixed-strategy equilibrium in the assurance version of the meeting coordination game, you can now probably guess the mixed-strategy equilibria for the related non-zero-sum meeting games. In the pure-coordination version (see Figure 4.13), the payoffs from meeting in the two places are the same, so the mixed-strategy equilibrium will have p = ½ and q = ½. In the battle-of-the-sexes version (see Figure 4.15), Watson prefers to meet at Simpson's because his payoff is 2 rather than the 1 that he gets from meeting at St. Bart's. Watson's decision hinges on whether his subjective probability of Holmes's going to St. Bart's is greater than or less than ⅔. (Watson's payoffs here are similar to those in the assurance version, so the critical p is the same.) Holmes prefers to meet at St. Bart's, so his decision hinges on whether his subjective probability of Watson's going to St. Bart's is greater than or less than ⅓. Therefore, the mixed-strategy Nash equilibrium again has p = ⅔ and q = ⅓.

		DEAN	
		Swerve (Chicken)	Straight (Tough)
JAMES	Swerve (Chicken)	0, 0	−1, 1
	Straight (Tough)	1, −1	−2, −2

FIGURE 7.4 Chicken

B. Will James Meet Dean? Chicken

The non-zero-sum game of chicken also has a mixed-strategy equilibrium that can be found using the method developed above, although its interpretations are slightly different. Recall that this is a game between James and Dean, who are trying to *avoid* a meeting; the game table, originally introduced in Figure 4.16, is reproduced here as Figure 7.4.

If we introduce mixed strategies into this game, James's p-mix will entail a probability p of swerving and a probability $1 - p$ of going straight. Against that p-mix, Dean gets expected payoffs of $0 \times p - 1 \times (1 - p) = p - 1$ if he chooses Swerve and $1 \times p - 2 \times (1 - p) = 3p - 2$ if he chooses Straight. Comparing the two, we see that Dean does better by choosing Swerve when $p - 1 > 3p - 2$, or when $2p < 1$, or when $p < ½$; that is, when p is low and James is more likely to choose Straight. Conversely, when p is high and James is more likely to choose Swerve, then Dean does better by choosing Straight. If James's p-mix has p exactly equal to ½, then Dean is indifferent between his two pure actions; he is therefore equally willing to use a mixture of the two. Similar analysis of the game from James's perspective when considering his options against Dean's q-mix yields the same results. Therefore, $p = ½$ and $q = ½$ is a mixed-strategy equilibrium of this game.

The properties of this equilibrium have some similarities to, but also some differences from, the mixed-strategy equilibrium of the Holmes–Watson assurance game. Here, each player's expected payoff in the mixed-strategy equilibrium is low (−½). This is bad for James and Dean, as was the case with the expected payoff of ⅔ for Holmes and Watson in the assurance game, but in this game of chicken, the mixed-strategy equilibrium payoff is not worse for both players than either of the two pure-strategy equilibrium payoffs. In fact, because player interests are somewhat opposed here, each player will do strictly better in the mixed-strategy equilibrium than in the pure-strategy equilibrium that entails his choosing Swerve.

This mixed-strategy equilibrium is again unstable, however. If James increases his probability of choosing Straight to just slightly above ½, this change tips Dean's choice to pure Swerve. Then (Straight, Swerve) becomes the pure-strategy equilibrium. If James instead lowers his probability of choosing Straight slightly below ½, Dean chooses Straight, and the game goes to the other pure-strategy equilibrium.[6]

In this section, we found mixed-strategy equilibria in several non-zero-sum games by solving the equations that come from the opponent's indifference property. We already know from Chapter 4 that these games also have equilibria in pure strategies. Best-response curves can give us a comprehensive picture, displaying all Nash equilibria at once. As you already know all of the equilibria from the two separate analyses, we do not spend time and space graphing the best-response curves here. We merely note that when there are two pure-strategy equilibria and one mixed-strategy equilibrium, as in the previous examples, you will find that the best-response curves cross in three different places, one for each of the Nash equilibria. We will also invite you to graph best-response curves for similar games in the exercises at the end of this chapter.

Endnotes

- The probability that each chooses St. Bart's in equilibrium is ⅔. The probability that each chooses Simpson's is ⅓. The probability that one chooses St. Bart's while the other chooses Simpson's is (⅔) × (⅓). But that can happen two different ways (once when Holmes chooses St. Bart's and Watson chooses Simpson's, and again when the choices are reversed) so the total probability of not meeting is 2 × (⅔) × (⅓). See the appendix to this chapter for more details on the algebra of probabilities. Return to reference 5
- In Chapter 12, we consider a different kind of stability—namely, evolutionary stability. The question in the evolutionary context is whether a stable mix of Straight and Swerve choosers can arise and persist in a population of chicken players. The answer is yes, and the proportions of the two types are exactly equal to the probabilities of playing each action in the mixed-strategy equilibrium. Thus, we derive a new and different motivation for that equilibrium in this game. Return to reference 6

5 GENERAL DISCUSSION OF MIXED-STRATEGY EQUILIBRIA

Now that we have seen how to find mixed-strategy equilibria in both zero-sum and non-zero-sum games, it is worthwhile to consider these equilibria in more detail. In particular, we highlight in this section some general properties of mixed-strategy equilibria and some initially counterintuitive aspects of such equilibria.

A. Weak Sense of Equilibrium

The opponent's indifference property described in Section 2 implies that in a mixed-strategy equilibrium, each player gets the same expected payoff from each of her two pure strategies, and therefore also gets the same expected payoff from any mixture between them. Thus, mixed-strategy equilibria are Nash equilibria only in a weak sense. When one player is choosing her equilibrium mix, the other has no positive reason to deviate from her own equilibrium mix. But she would not do any worse if she chose another mix or even one of her pure strategies. Each player is indifferent between her pure strategies and, indeed, any mixture of them, so long as the other player is playing her correct (equilibrium) mix.

This property may at first seem to undermine the basis for mixed-strategy Nash equilibria as a solution concept for games. Why should a player choose her appropriate mixture when the other player is choosing her own? Why not just do the simpler thing and choose one of her pure strategies? After all, the expected payoff is the same. The answer is that playing a pure strategy would give the other player an incentive to deviate from her own mixture.

For instance, in the tennis-point game played between Evert and Navratilova, imagine that Evert said to herself, "When Navratilova is choosing her best mix (q = 0.6), I get the same payoff from DL, CC, or any mixture. So why bother to mix; why don't I just play DL?" If Evert followed through on this plan, Navratilova would undoubtedly notice Evert's consistent play and switch herself to her pure strategy of covering DL, making Evert worse off than in the mixed-strategy equilibrium. Consequently, Evert has an incentive to mix in order to keep Navratilova guessing.

On the other hand, if Holmes chooses pure St. Bart’s in the assurance game, then Watson’s response (also playing pure St. Bart’s) gives both players a higher payoff than in the mixed-strategy equilibrium, in which both players go to St. Bart’s two-thirds of the time. Moreover, the outcome where both players always go to St. Bart’s is itself a pure-strategy Nash equilibrium. We will return to discuss this difference between the tennis-point game and the assurance game in Chapter 12, where we will introduce the concept of evolutionary stability. As we will show in that analysis, the mixed-strategy equilibrium of the tennis-point game is evolutionarily stable, but the mixed-strategy equilibrium of the assurance game is evolutionarily unstable.

B. Counterintuitive Changes in Mixture Probabilities with Changes in Payoffs

Games with mixed-strategy equilibria may exhibit some features that seem counterintuitive at first glance. The most interesting of them is the change in the equilibrium mixes that follows a change in the structure of a game’s payoffs. To illustrate, we return to Evert and Navratilova and their tennis-point game.

Suppose that Navratilova works on improving her skill at covering down the line to the point where Evert’s success when using her DL strategy against Navratilova’s covering DL drops from 50% to 30%. This improvement in Navratilova’s skill alters the payoff table from that in Figure 7.1. We present the new table in Figure 7.5.

		NAVRATILOVA	
		DL	CC
EVERT	DL	30, 70	80, 20
	CC	90, 10	20, 80

FIGURE 7.5 Changed Payoffs in the Tennis-Point Game

The only change from the table in Figure 7.1 has occurred in the upper-left cell of Figure 7.5, where our earlier 50 for Evert is now a 30 and the 50 for Navratilova is now a 70. This change in the payoff table does not lead to a game with a pure-strategy equilibrium because the players still have opposing interests: Navratilova still wants their choices to coincide, and Evert still wants their choices to differ. We still have a game in which mixing will occur.

But how will the equilibrium mixes in this new game differ from those calculated in Section 2? At first glance, many people would argue that Navratilova should cover DL more often now that she has gotten so much better at doing so. Thus, the assumption is that her equilibrium q-mix should be more heavily weighted toward DL, and her equilibrium q should be higher than the 0.6 calculated before.

But when we calculate Navratilova's q-mix that will keep Evert indifferent between her two pure strategies, we get $30q + 80(1 - q) = 90q + 20(1 - q)$, or $q = 0.5$. Thus, the actual equilibrium value for q, 50%, has exactly the opposite relation to the original q of 60% than what many people's intuition predicts.

Although our intuition seems reasonable, it misses an important aspect of the theory of strategy: the interaction between the two players. Evert reassesses her equilibrium mix after the change in payoffs, and Navratilova must take the new payoff structure *and* Evert's behavior into account when determining her new equilibrium mix. Specifically, because Navratilova is now so much better at covering DL, Evert uses CC more often in her mix. To counter that, Navratilova covers CC more often.

We can see this more explicitly by calculating Evert's new mixture. Her equilibrium p must equate Navratilova's expected payoff from covering DL, $70p + 10(1 - p)$, with her expected payoff from covering CC, $20p + 80(1 - p)$. So we have $70p + 10(1 - p) = 20p + 80(1 - p)$, or $90 - 60p = 20 + 60p$, or $120p = 70$. Thus, Evert's p must be 7/12, which is 0.583, or 58.3%. Comparing this new equilibrium p with the original 70% calculated in Section 2 shows that Evert has significantly decreased the number of times she sends her shot DL in response to Navratilova's improved skills. Evert has taken into account the fact that she is now facing an opponent with better DL coverage, and so she does better to play DL less frequently in her mixture. By virtue of this behavior, Evert makes it better for Navratilova to decrease the frequency of her DL play. Evert would now exploit any other choice of mix by Navratilova, particularly a mix heavily favoring DL.

So is Navratilova's skill improvement wasted? No, but we must judge it properly—not by how often one strategy or the other gets used, but by the resulting payoffs. When Navratilova uses her new equilibrium mix with $q = 0.5$, Evert's expected payoff from either of her pure strategies is $(30 \times 0.5) + (80 \times 0.5) = (90 \times 0.5) + (20 \times 0.5) = 55$. This is less than Evert's expected payoff of 62 in the original example. Thus, Navratilova's average expected payoff rises from 38 to 45, and she does benefit by improving her DL coverage.

Unlike the counterintuitive result that we saw when we considered Navratilova's strategic response to the change in payoffs, we see here that her response is absolutely intuitive when considered in light of her expected payoff. In fact, players' expected-payoff responses to changed payoffs can never be counterintuitive, although their strategic responses, as we have seen, can be.[7] The most interesting aspect of this counterintuitive outcome in players' strategic responses is the message that it sends to tennis players and to strategic game players more generally: Navratilova should improve her down-the-line coverage so that she does not have to use it so often.

Next, we present an even more general, and more surprising, result of changes in mixture probabilities. The opponent's indifference property means that each player's equilibrium mixture depends only on the other player's payoffs, not on her own. Consider the assurance game in Figure 7.3. Suppose Watson's payoff from meeting at Simpson's increases from 2 to 3, while all other payoffs remain unchanged. Now, against

Holmes's p-mix, Watson gets $1 \times p + 0 \times (1 - p) = p$ if he chooses St. Bart's, and $0 \times p + 3 \times (1 - p) = 3 - 3p$ if he chooses Simpson's. Watson is indifferent between his two pure strategies when $p = 3 - 3p$, or $4p = 3$, or $p = \frac{3}{4}$, compared with the value of two-thirds we found earlier for Holmes's p-mix in the original game. The calculation of Holmes's indifference condition is unchanged and yields $q = \frac{2}{3}$ for Watson's equilibrium strategy. The change in Watson's payoffs changes Holmes's mixture probabilities, not Watson's! In Exercise S13, you will have the opportunity to prove that this is true quite generally: A player's equilibrium mixing proportions do not change with his own payoffs, only with his opponent's payoffs.

C. Risky and Safe Choices in Zero-Sum Games

In sports, some strategies are relatively safe; they do not fail disastrously even if anticipated by the opponent, but they don’t do very much better when unanticipated. Other strategies are risky; they do brilliantly if the opponent is not prepared for them, but fail miserably if the opponent is ready. In American football, on third down with a yard to go, a run up the middle is safe, and a long pass is risky. An interesting question arises because in some third-and-one situations, there is more at stake than in others. For example, making the play from your opponent’s 10-yard line has a much greater effect on your chance of scoring than making the play from your own 20-yard line. The question is, when the stakes are higher, should you play the risky strategy more or less often than when the stakes are lower?

To make this question concrete, consider the success probabilities shown in Figure 7.6. (Note that, whereas in the tennis-point game we used percentages between 0 and 100, here we use probabilities between 0 and 1.) The offense’s safe play is the run; the probability of a successful first down is 0.6 if the defense anticipates a run versus 0.7 if the defense anticipates a pass. The offense’s risky play is the pass because its probability of success depends much more on what the defense does: Its probability of success is 0.8 if the defense anticipates a run, but only 0.3 if it anticipates a pass.

		DEFENSE EXPECTS	
		Run	Pass
OFFENSE PLAYS	Run	0.6	0.7

	DEFENSE EXPECTS		
		Run	Pass
	Pass	0.8	0.3

FIGURE 7.6 Probability of Offense's Success on Third Down with One Yard to Go

		DEFENSE	
		Run	Pass
OFFENSE	Run	0.6V, −0.6V	0.7V, −0.7V
	Pass	0.8V, −0.8V	0.3V, −0.3V

FIGURE 7.7 The Third-and-One Game

Suppose that when the offense succeeds with its play, it earns a payoff equal to V, and if the play fails, the payoff is 0. The payoff V could be some number of points, such as three for a field goal or seven for a touchdown. Alternatively, it could represent some amount of status or money that the team earns, perhaps $V = 100$ for succeeding in a game-winning play in an ordinary game or $V = 1{,}000{,}000$ for clinching victory in the Super Bowl.[8]

The payoff table for the game between Offense and Defense, illustrated in Figure 7.7, shows expected payoffs for each team. The expected payoff in each case is the weighted average of the success payoff V and the failure payoff 0. For example, the expected payoff to Offense for playing Run when Defense expects Run is $0.6 \times V + 0.4 \times 0 = 0.6V$. The zero-sum nature of the game means that Defense's payoff in the same cell is $-0.6V$. You can similarly compute the expected payoffs for each cell of the table to verify that the payoffs shown below are correct.

In the mixed-strategy equilibrium, Offense's probability p of choosing Run is determined by the opponent's indifference property. The correct p therefore satisfies

$$p[-0.6V] + (1 - p)[-0.8V] = p[-0.7V] + (1 - p)[-0.3V].$$

Notice that we can divide both sides of this equation by V to eliminate V entirely from the calculation for p. The equation becomes $-0.6p - 0.8(1 - p) = -0.7p - 0.3(1 - p)$, or $0.1p = 0.5(1 - p)$.[9] Solving this reduced equation yields $p = 5/6$, so Offense will play Run with high probability in its optimal mixture. This safer play is often called the *percentage play* because it is the normal play in such situations. The risky play (Pass) is played only occasionally to keep the opponent guessing or, in football commentators' terminology, "to keep the defense honest."

The interesting part of this result is that the expression for p is completely independent of V. That is, the theory says that you should mix the percentage play and the risky play in exactly the same proportions on a big occasion as you would on a minor occasion. This result runs against the intuition of many people. They think that the risky play should be engaged in less often when the occasion is more important. Throwing a long pass on third down with a yard to go may be fine on an ordinary Sunday afternoon in October, but doing so in the Super Bowl is too risky.

So which is right: theory or intuition? We suspect that readers will be divided on this issue. Some will think that the sports commentators are wrong about playing it safer in more important games and will be glad to have found a theoretical argument to refute their claims. Others will side with the commentators and argue that bigger occasions call for safer play. Still others may think that bigger risks should be taken when the prizes are bigger, but even they will find no support in the theory, which says that the size of the prize or the loss should make no difference to the mixture probabilities.

In many previous parts of this book where discrepancies between theory and intuition arose, we argued that the discrepancies were only apparent, that they were the result of failing to make the theory sufficiently general or rich enough to capture all the features of the situation that created the intuition, and that improving the theory removed the discrepancy. This situation is different: The problem is fundamental to the calculation of payoffs from mixed strategies as probability-weighted averages or expected payoffs. And almost all of existing game theory has this starting point.[10]

Endnotes

- For a general theory of the effect that changing the payoff in a particular cell of a payoff table has on the equilibrium mixture and the expected payoffs in equilibrium, see Vincent Crawford and Dennis Smallwood, "Comparative Statics of Mixed-Strategy Equilibria in Noncooperative Games," *Theory and Decision*, vol. 16 (May 1984), pp. 225–32. Return to reference 7
- Note that V is not necessarily a monetary amount. It can capture other relevant aspects of payoffs to the team, such as aversion to risk or loss (spelled out and used in greater detail in Chapters 9 and 14), effect of criticism by fans and journalists, and so on. Return to reference 8
- This result comes from the fact that we can eliminate V entirely from the opponent's indifference equation, so it does not depend on the particular success probabilities specified in Figure 7.6. The result is therefore quite general for mixed-strategy games where each payoff equals a success probability times a success value. Return to reference 9
- Vincent P. Crawford, "Equilibrium without Independence," *Journal of Economic Theory*, vol. 50, no. 1 (February 1990), pp. 127–54; and James Dow and Sergio Werlang, "Nash Equilibrium under Knightian Uncertainty," *Journal of Economic Theory*, vol. 64, no. 2 (December 1994), pp. 305–24, are among the few research papers that suggest alternative foundations for game theory. And our exposition of this problem in the first edition of this book inspired an article that uses such new methods on it: Simon Grant, Atsushi Kaji, and Ben Polak, "Third Down and a Yard to Go: Recursive Expected Utility and the Dixit-Skeath Conundrum," *Economic Letters*, vol. 73, no. 3 (December 2001), pp. 275–86. Unfortunately, the article uses concepts more advanced

than those available at the introductory level of this book. Return to reference 10

6 MIXING WHEN ONE PLAYER HAS THREE OR MORE PURE STRATEGIES

Our discussion of mixed strategies up to this point has been confined to games in which each player has available only two pure strategies and mixes between them. In many strategic situations, each player has available a larger number of pure strategies, and we should be ready to calculate equilibrium mixes for those cases as well. However, these calculations get complicated quickly. For truly complex games, we would turn to a computer to find the mixed-strategy equilibrium. But for some small games, it is possible to calculate equilibria by hand quite easily. The calculation process gives us a better understanding of how the equilibrium works than can be obtained just from looking at a computer-generated solution. Therefore, in this section and the next one, we solve some of these larger games.

In this section, we consider zero-sum games in which one of the players has two pure strategies, whereas the other player has more. In such games, we find that the player who has three (or more) pure strategies typically uses only two of them in equilibrium. The others do not figure in his mix; they have probabilities of 0. We must determine which strategies are used and which ones are not.[11]

Our example is that of the tennis-point game, augmented here by giving Evert a third type of passing shot. In addition to passing down the line or crosscourt, she now can consider using a lob (a slower, but higher and longer, passing shot). The equilibrium of the game depends on the payoffs of the lob against each of Navratilova’s two defensive choices. We begin with the case that is most likely to arise, then consider a coincidental or exceptional case.

A. A General Case

Evert now has three pure strategies in her repertoire: DL, CC, and Lob. We leave Navratilova with just two pure strategies, Cover DL or Cover CC. The payoff table for this new game can be obtained by adding a Lob row to Figure 7.1; the result is shown in Figure 7.8. We have assumed that Evert's payoffs from Lob are between the best and the worst she can get with DL and CC, and that they vary little with Navratilova's choices between DL and CC. We have shown not only the payoffs from all the pure strategies, but also those for Evert's three pure strategies against Navratilova's q-mix. [We do not show a row for Evert's p-mix because we don't need it. It would require two probabilities—say, p_1 for DL and p_2 for CC, and then the probability for Lob would be $(1 - p_1 - p_2)$. We will show you how to solve for equilibrium mixtures of this type in the following section.]

		NAVRATILOVA		
		DL	CC	q-mix
EVERT	DL	50, 50	80, 20	$50q + 80(1 - q)$, $50q + 20(1 - q)$
	CC	90, 10	20, 80	$90q + 20(1 - q)$, $10q + 80(1 - q)$
	Lob	70, 30	60, 40	$70q + 60(1 - q)$, $30q + 40(1 - q)$
You may need to scroll left and right to see the full figure.				

FIGURE 7.8 Payoff Table for the Tennis-Point Game with Lob

Technically, before we begin looking for a mixed-strategy equilibrium, we should verify that this game has no pure-strategy equilibrium. This is easy to do, however, so we leave it to you and turn to mixed strategies.

We will use the logic of best responses to consider Navratilova's optimal choice of q. In Figure 7.9, we show Evert's expected payoffs (success percentages) from playing each of her pure strategies DL, CC, and Lob as the q in Navratilova's q-mix varies over its full range from 0 to 1. The payoff lines are just graphs of Evert's payoff expressions in the right-hand column of Figure 7.8. For each q, if Navratilova were to choose that q-mix in equilibrium, Evert's best response would be to choose the strategy that gives her (Evert) the highest payoff. We show this set of best-response outcomes for Evert with the thicker line in Figure 7.9; in mathematical jargon, this is the *upper envelope* of the three payoff lines. Navratilova wants to choose her own best possible q—the q that makes her own payoff as high as possible (thereby making Evert's payoff as low as possible)—from this set of Evert's best responses.

To be more precise about Navratilova's optimal choice of q, we must calculate the coordinates of the kink points in the thicker line showing her worst-case (Evert's best-case) outcomes. The value of q at the leftmost kink in this line makes Evert indifferent between DL and Lob. That q must equate the two payoffs from DL and Lob when used against the q-mix. Setting those two expressions equal gives us $50q + 80(1 - q) = 70q + 60(1 - q)$, or $q = 20/40 = ½ = 50\%$. Evert's expected payoff at this point is $50 \times 0.5 + 80 \times 0.5 = 70 \times 0.5 + 60 \times 0.5 = 65$. At the second (rightmost) kink, Evert is indifferent between CC and Lob. Thus, the q value at this kink is the one that equates the CC and Lob payoff expressions. Setting $90q + 20(1 - q) = 70q + 60(1 - q)$, we find $q = 40/60 = ⅔ = 66.7\%$. Here, Evert's expected payoff is $90 \times 0.667 + 20 \times 0.333 = 70 \times 0.667 + 60 \times 0.333 = 66.67$. Therefore, Navratilova's best (or least bad) choice of q is at the left kink, namely, $q = 0.5$. Evert's expected payoff is 65, so Navratilova's is 35.

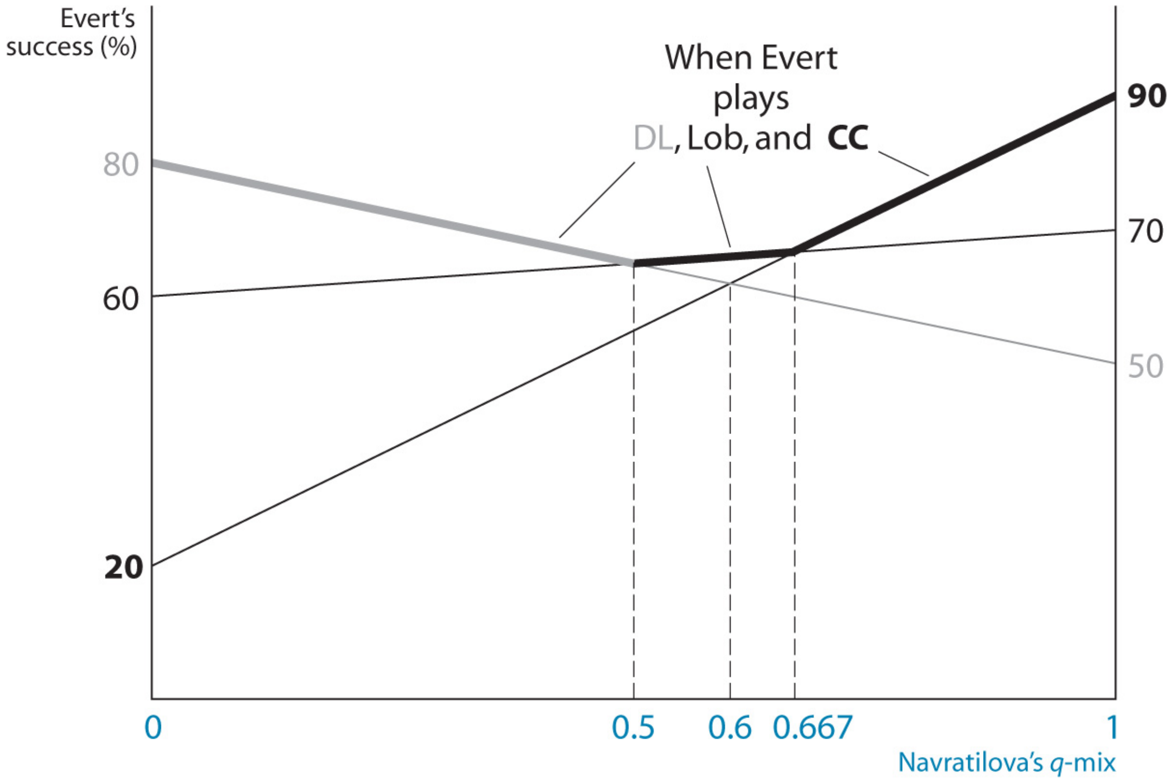


Figure 7.9 Graphical Solution for Navratilova's q-Mix

When Navratilova chooses $q = 0.5$, Evert is indifferent between DL and Lob, and either of these choices gives her a better payoff than does CC. Therefore, Evert will not use CC at all in equilibrium. CC will be an unused strategy in her equilibrium mix.

Now we can proceed with the equilibrium analysis as if this were a game with just two pure strategies for each player: DL and CC for Navratilova, and DL and Lob for Evert. We are back in familiar territory. Therefore, we leave the calculation to you and just tell you the result. Evert's optimal mixture in this game entails her using DL with probability 0.25 and Lob with probability 0.75. Evert's expected payoff from this mixture, against Navratilova's DL and CC, is $50 \times 0.25 + 70 \times 0.75 = 80 \times 0.25 + 60 \times 0.75 = 65$, as of course it should be.

We could not have started our analysis with this two-by-two game because we did not know in advance which of her three strategies Evert would not use. But we can be confident that in the general case, a player in Evert’s situation will mix between just two strategies rather than mixing among all three. If we consider all the possible combinations of values that the payoff numbers could take, the resulting three expected payoff lines would cross at a single point only in exceptional cases; generally, they will intersect pair by pair. Then the upper envelope has the shape that we see in Figure 7.9. Its lowest point is defined by the intersection of the payoff lines associated with two of the three strategies and these are the two strategies that will be used in the player’s equilibrium mix. The payoff from the third strategy lies below the intersection at this point, so the player choosing among the three strategies does not use that third one.

B. Exceptional Cases

The positions and intersections of the three lines of Figure 7.9 depend on the payoffs specified for the pure strategies. We chose the payoffs for that particular game to show a general configuration of payoff lines. But if the payoffs stand in other, very specific relationships to each other, we can get some exceptional configurations with different results. We describe the possibilities here, but leave it to you to redraw the graph for these cases.

First, if Evert’ s payoffs from Lob against Navratilova’ s DL and CC are equal, then the line for Lob is horizontal, and a whole range of *q*-values make Navratilova’ s mixture exploitation-proof. For example, if the two payoffs in the Lob row of the table in Figure 7.8 are 70 each, then it is easy to calculate that the left kink in a revised Figure 7.9 would be at $q = ⅓$ and the right kink at $q = 5/7$. For any q in the range from ⅓ to 5/7, Evert’ s best response is Lob, and we get an unusual equilibrium in which Evert plays a pure strategy and Navratilova mixes. Further, Navratilova’ s equilibrium mixture probabilities are indeterminate within the range from $q = ⅓$ to $q = 5/7$.

Second, if Evert’ s payoffs from Lob against Navratilova’ s DL and CC are lower than those of Figure 7.8 by just the right amounts (or those of Evert’ s other two strategies are higher by just the right amounts), all three lines can meet at one point. For example, if the payoffs of Evert’ s Lob are 66 and 56 against Navratilova’ s DL and CC, respectively, instead of 70 and 60, then for $q = 0.6$, Evert’ s expected payoff from Lob becomes $66 \times 0.6 + 56 \times 0.4 = 39.6 + 22.6 = 62$, the same as that from DL and CC when $q = 0.6$. So Evert is indifferent among all three of her strategies when $q = 0.6$ and is willing to mix among all three.

In this special case, Evert’ s equilibrium mixture probabilities are not fully determinate. Rather, a whole range of mixtures, including some where all three strategies are used, can do the job of keeping Navratilova indifferent between her DL and CC and therefore willing to mix. However, Navratilova must use a mixture with $q = 0.6$. If she does not, Evert's best response will be to switch to one of her pure strategies, and this will work to Navratilova's detriment. We do not dwell on the determination of the precise range over which Evert's equilibrium mixtures can vary, because this case can arise only for exceptional combinations of the payoff numbers and is therefore relatively unimportant.

Note that Evert's payoffs from using her Lob against Navratilova's DL and CC could be even lower than the values that make all three lines intersect at one point (for example, if the payoffs from Lob were 75 and 30 instead of 70 and 60 as in Figure 7.8). Then Lob is never the best response for Evert even though it is not dominated by either DL or CC. This case of Lob being dominated by a *mixture* of DL and CC is explained in the online appendix to this chapter, available at digital.wwnorton.com/gamesofstrategy5.

Endnotes

- Even when a player has only two pure strategies, he may not use one of them in equilibrium. The other player then generally finds one of his strategies to be better against the one that the first player does use. In other words, the equilibrium “mixtures” collapse to the special case of pure strategies. But when one or both players have three or more strategies, we can have a genuinely mixed-strategy equilibrium where some of the pure strategies go unused. Return to reference 11

7 MIXING WHEN BOTH PLAYERS HAVE THREE STRATEGIES

When we consider two-player games in which both players have three pure strategies and are considering mixing among all three, we need two variables to specify each mix.[12] The row player’s p-mix would put probability p_1 on her first pure strategy and probability p_2 on her second pure strategy. Then the probability of her using the third pure strategy would equal 1 minus the sum of the probabilities of the other two. The same would be true for the column player’s q-mix. So when both players have three strategies, we cannot find a mixed-strategy equilibrium without doing two-variable algebra. In many cases, however, such algebra is still manageable.

A. Full Mixtures of All Strategies

Consider a simplified representation of a penalty kick in soccer. Suppose a right-footed kicker has just three pure strategies: kick to the left, right, or center. (Left and right refer to the goalie's left or right. For a right-footed kicker, the most natural motion would send the ball to the goalie's right.) Then he can mix among these strategies, with probabilities denoted by p_L, p_R, and p_C, respectively. Any two of them can be taken to be the independent variables and the third expressed in terms of them. If p_L and p_R are the two independent variables, then $p_C = 1 - p_L - p_R$. The goalie also has three pure strategies—namely, move to the kicker's left (the goalie's own right), move to the kicker's right, or continue to stand in the center—and can mix among them with probabilities q_L, q_R, and q_C, two of which can be chosen independently.

We again use the opponent's indifference property to focus on the mixture probabilities for one player at a time. Each player's probabilities should be such that the other player is indifferent among all the pure strategies that constitute his mixture. This gives us a set of equations that can be solved for the mixture probabilities. In the soccer example, the kicker's (p_L, p_R) would satisfy two equations expressing the requirement that the goalie's expected payoff from moving to his left should equal that from moving to his right and that the goalie's expected payoff from moving to his right should equal that from staying at the center. (Then the equality of the expected payoffs from left and center follows automatically and is not a separate equation.) With more pure strategies, the number of probabilities to be solved for and the number of equations that they must satisfy also increase.

Figure 7.10 shows the game table for the interaction between Kicker and Goalie, with success percentages as payoffs for each player. (Unlike the evidence we present on European soccer later in this chapter, these are not real data, but similar rounded numbers meant to simplify calculations.) This is a zero-sum game in which the kicker wants to maximize his probability of successfully scoring a goal, and the goalie wants to minimize the probability of a goal. For example, if the kicker kicks to his left while the goalie moves to the kicker's left (the top-left corner cell), we suppose that the kicker still succeeds (in scoring) 45% of the time and the goalie therefore succeeds (in saving a goal) 55% of the time. But if the kicker kicks to his right and the goalie moves to the kicker's left, then the kicker has a 90% chance of scoring; we suppose a 10% probability that he might kick wide or too high, so the goalie is still successful 10% of the time. You can experiment with different payoff numbers that you think might be more appropriate.

		GOALIE		
		Left	Center	Right
KICKER	Left	45, 55	90, 10	90, 10
	Center	85, 15	0, 100	85, 15
	Right	95, 5	95, 5	60, 40

You may need to scroll left and right to see the full figure.

FIGURE 7.10 Soccer Penalty-Kick Game

It is easy to verify that the game has no equilibrium in pure strategies. So suppose the kicker is mixing with probabilities p_L, p_R, and $p_C = 1 - p_L - p_R$. For each of the goalie's pure strategies, this mixture yields the goalie the following payoffs:

Left: $55p_L + 15p_C + 5p_R = 55p_L + 15(1 - p_L - p_R) + 5p_R$

Center: $10p_L + 100p_C + 5p_R = 10p_L + 100(1 - p_L - p_R) + 5p_R$

Right: $10p_L + 15p_C + 40p_R = 10p_L + 15(1 - p_L - p_R) + 40p_R$.

The opponent's indifference rule says that the kicker should choose p_L and p_R so that all three of these expressions are equal in equilibrium.

Equating the Left and Right expressions and simplifying, we have $45p_L = 35p_R$, or $p_R = (9/7)p_L$. Next, equate the Center and Right expressions and simplify by using the link between p_L and p_R just obtained. This gives

$10p_L + 100[1 - p_L - (9p_L/7)] + 5(9p_L/7) = 10p_L + 15[1 - p_L - (9p_L/7)] + 40(9p_L/7)$, or $[85 + 120(9/7)]p_L = 85$, which yields $p_L = 0.355$.

Then we get $p_R = 0.355(9/7) = 0.457$, and finally $p_C = 1 - 0.355 - 0.457 = 0.188$. The goalie's payoff from any of his pure strategies against this mixture can then be calculated by using any of the preceding three payoff lines; the result is 24.6.

The goalie's mixture probabilities can be found by writing down and solving the equations for the kicker's indifference among his three pure strategies against the goalie's mixture. We will do this in detail for a slight variant of the same game in Section 7.B, so we omit the details here and just give you the answer: $q_L = 0.325$, $q_R = 0.561$, and $q_C = 0.113$. The kicker's payoff from any of his pure strategies when played against the goalie's equilibrium mixture is 75.4. That answer is, of course, consistent with the goalie's payoff of 24.6 that we calculated before.

Now we can interpret the findings. The kicker does better with his pure Right than his pure Left, both when the goalie guesses correctly (60 > 45) and when he guesses incorrectly (95 > 90). Therefore, the kicker chooses Right with the highest probability, and, to counter that, the goalie chooses Right with the highest probability, too. However, the kicker should not and does not choose his pure strategy Right; if he did so, the goalie would then choose his own pure strategy Right, too, and the kicker’ s payoff would be only 60, less than the 75.4 that he gets in the mixed-strategy equilibrium.

B. Equilibrium Mixtures with Some Strategies Unused

In the preceding equilibrium, the probabilities of using Center in the mix are quite low for each player. The (Center, Center) combination would result in a sure save, and the kicker would get a really low payoff—namely, 0. Therefore, the kicker puts a low probability on this choice. But then the goalie, too, should put a low probability on it, concentrating on countering the kicker's more likely choices. But if the kicker gets a sufficiently high payoff from choosing Center when the goalie chooses Left or Right, then the kicker will choose Center with some positive probability. If the kicker's payoffs in the Center row were lower, he might then choose Center with probability 0; if so, the goalie would similarly put probability 0 on Center. The game would be reduced to one with just two basic pure strategies, Left and Right, for each player.

We show such a variant of the soccer game in Figure 7.11. The only difference in payoffs between this variant and the original game of Figure 7.10 is that the kicker's payoffs from (Center, Left) and (Center, Right) have been lowered even further, from 85 to 70, causing the goalie's payoffs in these outcomes to rise from 15 to 30. That might be because this kicker has the habit of kicking too high and therefore missing the goal when aiming for the center. Let us try to calculate the equilibrium here by using the same methods as in Section 7.A. This time, we do it from the goalie's perspective: We try to find his mixture probabilities q_L, q_R, and q_C that would make the kicker indifferent among all three of his pure strategies when they are played against the goalie's mixture.

The kicker's payoffs from his pure strategies are

Left: $45q_L + 90q_C + 90q_R = 45q_L + 90(1 - q_L - q_R) + 90q_R = 45q_L + 90(1 - q_L)$,

Center: $70q_L + 0q_C + 70q_R = 70q_L + 70q_R$, and

Right: $95q_L + 95q_C + 60q_R = 95q_L + 95(1 - q_L - q_R) + 60\ q_R = 95(1 - q_R) + 60q_R$.

		GOALIE		
		Left	Center	Right
KICKER	Left	45, 55	90, 10	90, 10
	Center	70, 30	0, 100	70, 30
	Right	95, 5	95, 5	60, 40

You may need to scroll left and right to see the full figure.

FIGURE 7.11 Variant of Soccer Penalty-Kick Game

Equating the Left and Right expressions and simplifying, we have $90 - 45q_L = 95 - 35q_R$, or $35q_R = 5 + 45q_L$. Next, equate the Left and Center expressions and simplify to get $90 - 45q_L = 70q_L + 70q_R$, or $115q_L + 70q_R = 90$. Substituting for q_R from the first of these equations (after multiplying through by 2 to get $70q_R = 10 + 90q_L$) into the second yields $205q_L = 80$, or $q_L = 0.390$. Then, using this value for q_L in either of the equations gives $q_R = 0.644$. Finally, we use both of these values to obtain $q_C = 1 - 0.390 - 0.644 = -0.034$. Because probabilities cannot be negative, something has obviously gone wrong.

To understand what happens in this example, start by noting that Center is now a poorer strategy for the kicker than it was in the original version of the game, where his probability of choosing it was already quite low. But the logic of the opponent’s indifference property, expressed in the equations that led to the solution, means that the kicker has to be kept willing to use this poor strategy. That can happen only if the goalie is using his best counter to the kicker’s Center—namely, the goalie’s own Center—sufficiently infrequently. And in this example, that logic has to be carried so far that the goalie’s probability of Center has to become negative.

As pure algebra, the solution that we derived may be fine, but it violates the requirement of probability theory and real-life randomization that probabilities be nonnegative. The best that can be done in reality is to push the goalie’s probability of choosing Center as low as possible—namely, to zero. But that leaves the kicker unwilling to use his own Center. In other words, we get a situation in which each player is not using one of his pure strategies in his mixture—that is, each is using it with probability 0.

Can there then be an equilibrium in which each player is mixing between his two remaining strategies—namely, Left and Right? If we regard this reduced two-by-two game in its own right, we can easily find its mixed-strategy equilibrium. With all the practice that you have had so far, it is safe to leave the details to you and to state the result:

Kicker’s mixture probabilities: p_L = 0.4375, p_R = 0.5625.

Goalie’s mixture probabilities: q_L = 0.3750, q_R = 0.6250.

Kicker’s success percentage: 73.13.

Goalie’s success percentage:26.87.

We found this result by simply removing the two players’ Center strategies from consideration on intuitive grounds.

But we must make sure that it is a genuine equilibrium of the full three-by-three game. That is, we must check that neither player finds it desirable to bring in his third strategy, given the mixture of two strategies chosen by the other player.

When the goalie is choosing this particular mixture, the kicker’s payoff from pure Center is $0.375 \times 70 + 0.625 \times 70 = 70$. This payoff is less than the 73.13 that he gets from either his pure Left or pure Right, or any mixture between the two, so the kicker does not want to bring his Center strategy into play. When the kicker is choosing the two-strategy mixture with the preceding probabilities, the goalie’s payoff from pure Center is $0.4375 \times 10 + 0.5625 \times 5 = 7.2$. This number is (well) below the 26.87 that the goalie would get using his pure Left or pure Right, or any mixture of the two. Thus, the goalie does not want to bring his Center strategy into play either. The equilibrium that we found for the two-by-two game is indeed an equilibrium of the three-by-three game.

To allow for the possibility that some strategies may go unused in an equilibrium mixture, we must modify or extend the opponent’s indifference principle: Each player’s equilibrium mix should be such that the other player is indifferent among all the strategies *that are actually used in his equilibrium mix*. The other player is not indifferent between these and his unused strategies; he prefers the ones used to the ones unused. In other words, against the opponent’s equilibrium mix, all the strategies used in your own equilibrium mix should give you the same expected payoff, which in turn should be higher than what you would get from any of your unused strategies.

Which strategies will go unused in equilibrium? Answering that question requires much trial and error, as in our previous calculation, or leaving it all to a computer program, and once you have understood the concept, it is safe to do the latter. For the general theory of mixed-strategy equilibria when players can have any number of possible strategies, see the online appendix to this chapter.

Endnotes

- More generally, if a player has N pure strategies, then her mix has $(N - 1)$ independent variables, or “degrees of freedom of choice.” Return to reference 12

8 HOW TO USE MIXED STRATEGIES IN PRACTICE

There are several important things to remember when finding or using a mixed strategy in a zero-sum game. First, to use a mixed strategy effectively in such a game, a player needs to do more than calculate the percentages with which to use each of her actions. Indeed, in our tennis-point game, Evert cannot simply play DL seven-tenths of the time and CC three-tenths of the time by mechanically rotating seven shots down the line and three shots crosscourt. Why not? Because mixing your strategies is supposed to help you benefit from the element of surprise against your opponent. If you use a recognizable pattern of moves, your opponent is sure to discover it and exploit it to her advantage.

The lack of a pattern means that, after any history of choices, the probability of choosing DL or CC on the next turn is the same as it always was. That is, if a run of several successive DLs happens by chance, there is no sense in which CC is now "due" on the next turn. In practice, many people mistakenly think otherwise, and therefore they alternate their choices too much compared with what a truly random sequence of choices would require: They produce too few runs of identical successive choices. However, detecting a pattern from observed actions is a tricky statistical exercise that the opponents may not be able to perform while playing the game. As we will see in Section 9, analysis of data from grand-slam tennis finals found that servers alternated their serves too much, but that receivers were not able to detect and exploit this departure from true randomization.

The importance of avoiding predictability is clearest in ongoing interactions of a zero-sum nature. Because of the diametrically opposed interests of the players in such games, your opponent always benefits from exploiting your choice of action to the greatest degree possible. Thus, if you play the same game against each other on a regular basis, she will always be on the lookout for ways to break the code that you are using to randomize your moves. If she can do so, she has a chance to improve her payoffs in future plays of the game. But even in single-meet (sometimes called one-shot) zero-sum games, mixing remains beneficial because of the benefit of tactical surprise.

Predictability may come from a pattern in series of successive plays, or from certain quirks that act as a "tell" to your opponent. A wonderful example comes from top-level men's tennis.[13] For a long time, Andre Agassi found Boris Becker's serve unplayable. Finally, after watching hours of films, he figured out that the way Becker stuck his tongue slightly out of his lips indicated which way he would serve. And Agassi was smart enough not to use this knowledge all the time, which could make Becker suspicious enough to make him change his behavior. Agassi used Becker's tell against him only on important points.

Daniel Harrington, a winner of the World Series of Poker and the author, with Bill Robertie, of an excellent series of books on how to play in Texas Hold 'Em tournaments, notes the importance of randomizing your strategy in poker in order to prevent opponents from reading what cards you're holding and exploiting your behavior.[14] Because humans often have trouble being unpredictable, he gives the following advice about how to implement a mixture between the pure strategies of calling and raising:

> It's hard to remember exactly what you did the last four or five times a given situation appeared, but fortunately

you don't have to. Just use the little random number generator that you carry around with you all day. What's that? You didn't know you had one? It's the second hand on your watch. If you know that you want to raise 80 percent of the time with a premium pair in early position and call the rest, just glance down at your watch and note the position of the second hand. Since 80 percent of 60 is 48, if the second hand is between 0 and 48, you raise, and if it's between 48 and 60 you just call. The nice thing about this method is that even if someone knew exactly what you were doing, they still couldn't read you![15]

Of course, in using the second hand of a watch to implement a mixed strategy, it is important that your watch not be so accurate and synchronized that your opponent can use the same watch and figure out what you are going to do!

So far, we have assumed that you are interested in implementing a mixed strategy in order to avoid possible exploitation by your opponent. But if your opponent is not playing his equilibrium strategy, you may want to try to exploit his mistake. A simple example is illustrated using an episode of *The Simpsons* in which Bart and Lisa play a game of rock-paper-scissors with each other. (In Exercise S10, we give a full description of this three-by-three game and ask you to derive each player's equilibrium mixture.) Just before they choose their strategies, Bart thinks to himself, "Good ol' Rock. Nothing beats Rock," while Lisa thinks to herself, "Poor Bart. He always plays Rock." Clearly, Lisa's best response is the pure strategy Paper against this naive opponent; she need not use her equilibrium mix.

We have observed a more subtle example of exploitation when pairs of students play a best-of-100 version of the tennis-point game in this chapter. Like professional tennis players, many of our students switch strategies too often, apparently thinking that playing five DLs in a row doesn't look "random" enough. To exploit this behavior, a Navratilova player could predict that after playing three DLs in a row, an Evert player is likely to switch to CC, and she can exploit this by switching to CC herself. She should do this more often than if she were randomizing independently each round, but ideally not so often that the Evert player notices and starts learning to repeat her strategy in longer runs.

Finally, you must understand and accept the fact that the use of mixed strategies guards you against exploitation and gives the best possible expected payoff against an opponent who is making her best choices, but that it is only a probabilistic average. On particular occasions, you can get poor outcomes. For example, the long pass on third down with a yard to go, intended to keep the defense honest, may fail on any specific occasion. If you use a mixed strategy in a situation in which you are responsible to a higher authority, therefore, you may need to plan ahead for this possibility. You may need to justify your use of such a strategy ahead of time to your coach or your boss, for example. They need to understand why you have adopted your mixture and why you expect it to yield you the best possible payoff on average, even though it might yield an occasional low payoff as well. Even such advance planning may not work to protect your reputation, though, and you should prepare yourself for criticism in the face of a bad outcome.[16]

Endnotes

- See https://www.youtube.com/watch?v=3woPuCIk_d8. We thank David Reiley for this example. Return to reference 13
- Poker is a game of incomplete information because each player holds private information about her cards. While we do not analyze the details of such games until Chapter 9, they may involve mixed-strategy equilibria (called *semiseparating equilibria*) in which the random mixtures are specifically designed to prevent other players from using your actions to infer your private information. Return to reference 14
- Daniel Harrington and Bill Robertie, *Harrington on Hold ’Em: Expert Strategies for No-Limit Tournaments*, vol. 1, *Strategic Play* (Henderson, Nev.: Two Plus Two Publishing, 2004), p. 53. Return to reference 15
- In Super Bowl XLIX, played on February 1, 2015, the Seattle Seahawks trailed the New England Patriots by a score of 28–24 with only 26 seconds left. From the 1-yard line of the Patriots, the Seahawks threw the ball instead of letting their star running back, Marshawn Lynch, run up the middle. The pass was intercepted, and New England won the game. The play choice was mercilessly criticized in the media. But it was eminently justifiable on several levels, not just for the general principle of mixing, but also because the run would be expected, and Lynch had been stopped from the 1-yard line in some previous games. Return to reference 16

9 EVIDENCE ON MIXING

A. Zero-Sum Games

Early researchers who performed laboratory experiments were generally dismissive of mixed strategies. To quote Douglas Davis and Charles Holt, “Subjects in experiments are rarely (if ever) observed flipping coins, and when told ex post that the equilibrium involves randomization, subjects have expressed surprise and skepticism.” [17] When the predicted equilibrium entailed mixing two or more pure strategies, experimental results showed some subjects in a group pursuing one of the pure strategies and others pursuing another, but this does not constitute true mixing by an individual player. When subjects played zero-sum games repeatedly, individual players often chose different pure strategies over time. But they seemed to mistake alternation for randomization—that is, they switched their choices more often than true randomization would require.

Later laboratory research has found somewhat better evidence for mixing in zero-sum games. When subjects are allowed to acquire a lot of experience in these games, they do appear to learn mixing. However, departures from equilibrium predictions remain significant. Averaged across all subjects, the empirical probabilities are usually rather close to those predicted by equilibrium, but many individual subjects play proportions far from those predicted by equilibrium. To quote Colin Camerer, “The overall picture is that mixed equilibria do not provide bad guesses about how people behave, on average.” [18]

Evidence gathered from field observations on how people play zero-sum games in sports and other activities suggests that inexperienced and amateur players do sometimes make choices that conform to theoretical predictions, but not always. Professionals, especially top-ranked ones, who have a lot of experience and who play for substantial stakes conform quite closely to the theory of mixed-strategy equilibria. For example, a classic empirical analysis of tennis by Mark Walker and John Wooders examined the serve-and-return play of top-level players at Wimbledon, and a more recent study by Wooders, with two other coauthors, considered the same topic, but based its analysis on a newly available big-data set.[19] These authors modeled the interaction as a game with two players, the server and the receiver, in which each player has two pure strategies. The server can serve to the receiver's forehand or backhand, and the receiver can guess to which side the serve will go and move that way. Because serves are so fast at the top levels (especially in men's singles), the receiver cannot react after observing the actual direction of the serve; rather, the receiver must move in anticipation of the serve's direction. Thus, this game has simultaneous moves. Further, because the receiver wants to guess correctly and the server wants to wrong-foot the receiver, this interaction has a mixed-strategy equilibrium. The research statistically tested one important prediction of the theory: that the correct mix should be exploitation-proof, so that the server should win a point with the same probability whether he serves to the receiver's forehand or backhand.

The big-data set employed in the more recent paper comes from Hawk-Eye, the computerized ball-tracking system used at Wimbledon and other top professional tennis tournaments, and includes 500,000 serves. Each data point includes the precise trajectory and bounce point of the ball. Wooders and his coauthors combined these data with data on the rankings of the players involved in each interaction.[20] Their findings are striking. The *opponent's indifference property*, that the serving player should be indifferent about what direction to serve, is strongly supported in men's play, for both first and second serves. It is also borne out, although less strongly, in women's play.[21] Further, it is the receiver's play that can or cannot exploit any failure of optimization in the server's mix; therefore, it is remarkable that equalization occurs even against top receivers.

Correct mixing should be serially uncorrelated—one serve to the opponent's forehand should neither increase nor decrease the probability that the next serve will also be to the forehand. In practice, people exhibit too much alternation relative to this standard. Tennis players in this study did likewise, but the serial correlation was smaller for better players. And in any case, the excessive negative serial correlation was too small to be exploited.

		GOALIE	
		Left	Right
KICKER	Left	50	95
	Right	93	70

FIGURE 7.12 Soccer Penalty Kick Success Probabilities in European Major Leagues

---

As we showed in Section 8, penalty kicks in soccer are another excellent context in which to study mixed strategies. The advantage to analyzing penalty kicks is that one can actually observe the strategies of both the kicker and the goalie: not only where the kicker aims, but also in which direction the goalie dives. This means that one can compute the actual mixing probabilities and compare them with the theoretical prediction. The disadvantage, relative to tennis, is that no two players ever face each other more than a few times in a season. Instead of analyzing specific matchups of players, one must aggregate across all kickers and goalies in order to get enough data. Two studies using exactly this kind of data find firm support for predictions of the theory.

Using a large data set from professional soccer leagues in Europe, Ignacio Palacios-Huerta constructed the payoff table of the kicker’s average success probabilities shown in Figure 7.12.[22] Because the data include both right- and left-footed kickers, and therefore the natural direction of kicking differs between them, they refer to any kicker’s natural side as “Right.” (Kickers usually kick with the inside of the foot. A right-footed kicker naturally kicks to the goalie’s right and a left-footed kicker to the goalie’s left.) The choices are Left and Right for each player. When the goalie chooses Right, it means covering the kicker’s natural side.

Using the opponent’s indifference property, it is easy to calculate that the kicker *should* choose Left 38.3% of the time and Right 61.7% of the time. This mixture achieves a success rate of 79.6% no matter what the goalie chooses. The goalie *should* choose the probabilities of covering her Left and Right to be 41.7 and 58.3, respectively; this mixture holds the kicker down to a success rate of 79.6%.

What actually happens? Kickers choose Left 40.0% of the time, and goalies choose Left 41.3% of the time. These values are startlingly close to the theoretical predictions. The chosen mixtures are almost exploitation-proof. The kicker’s mix achieves a success rate of 79.0% against the goalie’s Left and 80% against the goalie’s Right. The goalie’s mix holds kickers down to 79.3% if they choose Left and 79.7% if they choose Right.

In an earlier paper, Pierre-André Chiappori, Timothy Groseclose, and Steven Levitt used similar data and found similar results.[23] They also analyzed the whole sequence of choices of each kicker and goalie and did not find too much alternation between Left and Right. One reason for this last result could be that most penalty kicks take place as isolated incidents across many games, by contrast with the rapidly repeated points of tennis, so players may find it easier to ignore what happened on the previous kick. Nevertheless, these findings suggest that behavior in soccer penalty kicks is even closer to true mixing than behavior in the tennis serve-and-return game.

With such strong empirical confirmation of the theory, one might ask whether the mixed-strategy skills that players learn in soccer carry over to other game contexts. One study indicated that the answer is yes (Spanish professional soccer players played exactly according to the equilibrium predictions in laboratory experiments with two-by-two and four-by-four zero-sum games). But a second study failed to replicate these results. That study examined American major-league soccer players as well as participants in the World Series of Poker (who, as noted in Section 8 above, also have professional reasons to prevent exploitation by mixing). It found that the professionals' behavior in abstract matrix games was just as far from equilibrium as that of students. As with the results on professional chess players that we discussed in Chapter 3, experience led these professional players to mix according to equilibrium theory in their jobs, but this experience did not automatically lead the players to equilibrium in new and unfamiliar games.[24]

B. Non-Zero-Sum Games

Laboratory experiments on mixed strategies in non-zero-sum games yield even more negative results than experiments involving mixing in zero-sum games. That is not surprising. As we have seen, the opponent's indifference property is a logical property of the equilibrium in zero-sum games. But in contrast, the players in a non-zero-sum game may have no positive or purposive reason to keep the other players indifferent. Thus, the reasoning underlying the mixture calculations is more difficult for players to comprehend and learn, and this shows up in their behavior.

In a group of experimental subjects playing a non-zero-sum game, we may see some pursuing one pure strategy and others pursuing another. This type of mixing in the population, although it does not fit the theory of mixed-strategy equilibria, does have an interesting evolutionary interpretation, which we examine in Chapter 12.

As we saw in Section 5.B, a player's mixture probabilities should not change when that player's own payoffs change. But in fact they do: Players tend to choose an action more often when their own payoffs from that action increase.[25] The players do change their actions from one round to the next in repeated trials with different partners, but not in accordance with equilibrium predictions. The overall conclusion is that you should interpret and use mixed-strategy equilibria in non-zero-sum games with, at best, considerable caution.

Endnotes

- Douglas D. Davis and Charles A. Holt, *Experimental Economics* (Princeton, N.J.: Princeton University Press, 1993), p. 99. Return to reference 17
- For a detailed account and discussion, see Chapter 3 of Colin F. Camerer, *Behavioral Game Theory* (Princeton, N.J.: Princeton University Press, 2003); the quote is from p. 468. Return to reference 18
- Mark Walker and John Wooders, "Minimax Play at Wimbledon," *American Economic Review*, vol. 91, no. 5 (December 2001), pp. 1521–38; Romain Gouriot, Lionel Page, and John Wooders, "Nash at Wimbledon: Evidence from Half a Million Serves," working paper, January 2018, available at http://www.johnwooders.com/papers/NashAtWimbledon.pdf. Return to reference 19
- They also develop and use a more powerful statistical test, but we do not go into those technicalities here. Return to reference 20
- The authors attribute the difference to the fact that serves in women's play are slower, so any departure from optimal mixing is easier to exploit. Return to reference 21
- See Ignacio Palacios-Huerta, "Professionals Play Minimax," *Review of Economic Studies*, vol. 70, no. 20 (2003), pp. 395–415. Return to reference 22
- Pierre-André Chiappori, Timothy Groseclose, and Steven Levitt, "Testing Mixed Strategy Equilibria When Players are Heterogeneous: The Case of Penalty Kicks in Soccer," *American Economic Review*, vol. 92, no. 4 (September 2002), pp. 1138–51. Return to reference 23
- The first study referenced is Ignacio Palacios-Huerta and Oskar Volij, "Experientia Docet: Professionals Play Minimax in Laboratory Experiments," *Econometrica*, vol.

76, no. 1 (January 2008), pp. 71–115. The second is Steven D. Levitt, John A. List, and David H. Reiley, "What Happens in the Field Stays in the Field: Exploring Whether Professionals Play Minimax in Laboratory Experiments," *Econometrica*, vol. 78, no. 4 (July 2010), pp. 1413–34. Return to reference 24

- Jack Ochs, "Games with Unique Mixed-Strategy Equilibria: An Experimental Study," *Games and Economic Behavior*, vol. 10, no. 1 (July 1995), pp. 202–17. Return to reference 25

SUMMARY

Zero-sum games in which one player prefers a coincidence of actions and the other prefers the opposite often have no Nash equilibrium in pure strategies. In these games, each player wants to be unpredictable and thus may use a mixed strategy, which specifies a probability distribution over her set of pure strategies. Each player’s equilibrium mixture probabilities are calculated using the *opponent’s indifference property*, which means that the opponent should get equal *expected payoffs* from all her pure strategies when facing the player’s equilibrium mix. Best-response-curve graphs can be used to show all mixed-strategy (as well as pure-strategy) equilibria of a game.

Non-zero-sum games can also have mixed-strategy equilibria that can be calculated from the opponent’s indifference property and illustrated using best-response curves. But here the motivation for keeping the opponent indifferent is weaker or missing; therefore such equilibria have less appeal and are often unstable.

Mixed strategies are a special case of continuous strategies but have additional properties that deserve separate study. Mixed-strategy equilibria can be interpreted as outcomes in which each player has correct beliefs about the probabilities with which the other player chooses from among her underlying pure actions. And mixed-strategy equilibria may have some counterintuitive properties when payoffs for players change.

If one player has three pure strategies and the other has only two, the player with three available strategies will generally use only two in her equilibrium mix. If both players have three (or more) pure strategies, equilibrium mixtures may put positive probability on all pure strategies or only a subset. All strategies that are actively used in the mixture yield equal expected payoffs against the opponent's equilibrium mix; all the unused ones yield lower expected payoffs. In these large games, equilibrium mixtures may also be indeterminate in some exceptional cases.

When using mixed strategies, players should remember that their system of randomization should not be predictable in any way. Most importantly, they should avoid excessive alternation of actions. Laboratory experiments show only weak support for the use of mixed strategies. But mixed-strategy equilibria give good predictions in many zero-sum situations in sports played by experienced professionals.

KEY TERMS

expected payoff (214)

opponent’s indifference property (216)

Glossary

expected payoff
: The probability-weighted average (statistical mean or expectation) of the payoffs of one player in a game, corresponding to all possible realizations of a random choice of nature or mixed strategies of the players.

opponent's indifference property
: An equilibrium mixed strategy of one player in a two-person game has to be such that the other player is indifferent among all the pure strategies that are actually used in her mixture.

SOLVED EXERCISES

1. Consider the following game:

		COLIN	
		Safe	Risky
ROWENA	Safe	4, 4	4, 1
	Risky	1, 4	6, 6

    1. Which other game does this game most resemble: tennis, assurance, or chicken? Explain.
    2. Find all of this game’ s Nash equilibria.
2. The following table illustrates the money payoffs associated with a two-person simultaneous-move game:

		COLIN	
		Left	Right
ROWENA	Up	1, 16	4, 6
	Down	2, 20	3, 40

    1. Find the Nash equilibrium in mixed strategies for this game.
    2. What are the players’ expected payoffs in this equilibrium?
    3. Rowena and Colin jointly get the most money when Rowena plays Down. However, in the equilibrium, she does not always play Down. Why not? Can you think of ways in which a more cooperative outcome could be sustained?
3. Recall Exercise S8 from Chapter 4, about an old lady looking for help crossing the street and two players simultaneously deciding whether to offer help. If you did

that exercise, you also found all pure-strategy Nash equilibria of the game. Now find the mixed-strategy equilibrium of the game.

4. Revisit the tennis-point game in Section 2.A of this chapter. Recall that the mixed-strategy Nash equilibrium found in that section had Evert playing DL with probability 0.7, while Navratilova played DL with probability 0.6. Now suppose that Evert injures herself later in the match, so that her DL shots are much slower and easier for Navratilova to defend against. The payoffs are now given by the following table:

		NAVRATILOVA	
		DL	CC
EVERT	DL	30, 70	60, 40
	CC	90, 10	20, 80

---

1. Relative to the game before her injury (see Figure 7.1), the strategy DL seems much less attractive to Evert. Would you expect Evert to play DL more, less, or the same amount in a new mixed-strategy equilibrium? Explain.
2. Find each player’s equilibrium mixture for this game. What is Evert’s expected payoff in equilibrium?
3. How do the equilibrium mixtures found in part (b) compare with those of the original game and with your answer to part (a)? Explain why each mixture has or has not changed.

5. Exercise S8 in Chapter 6 introduced a simplified version of baseball, and in part (e) you determined that the simultaneous-move game has no Nash equilibrium in pure strategies. This is because pitchers and batters have conflicting goals. Pitchers want to get the ball *past* batters, but batters want to *connect* with pitched balls. The game table is as follows:

		PITCHER	
		Throw fastball	Throw curve
BATTER	Anticipate fastball	0.30, 0.70	0.20, 0.80
	Anticipate curve	0.15, 0.85	0.35, 0.65

---

1. Find the mixed-strategy Nash equilibrium of this simplified baseball game.
2. What is each player's expected payoff for the game?
3. Now suppose that the pitcher tries to improve his expected payoff in the mixed-strategy equilibrium by slowing down his fastball, thereby making it more similar to a curve ball. This changes the payoff to the hitter in the upper-left cell from 0.30 to 0.25, and the pitcher's payoff adjusts accordingly. Can this modification improve the pitcher's expected payoff as desired? Explain your answer carefully and show your work. Also, explain *why* slowing the fastball can or cannot improve the pitcher's expected payoff in the game.

6. Undeterred by their experiences with the game of chicken so far (see Section 4.B), James and Dean decide to increase the excitement (and the stakes) by starting their cars farther apart. This way they can keep the crowd in suspense longer, and they'll be able to accelerate to even higher speeds before they may or may not be involved in a much more serious collision. The new game table thus has a higher penalty for collision.

		DEAN	
		Swerve	Straight
JAMES	Swerve	0, 0	−1, 1
	Straight	1, −1	−10, −10

1. What is the mixed-strategy Nash equilibrium for this more dangerous version of chicken? Do James and Dean play Straight more or less often than in the game shown in Figure 7.4?
2. What is the expected payoff to each player in the mixed-strategy equilibrium found in part (a)?
3. James and Dean decide to play this game of chicken repeatedly (say, in front of different crowds of reckless youths). Moreover, because they don't want to collide, they collude and alternate between the two pure-strategy equilibria. Assuming they play an even number of games, what is the average payoff to each of them when they collude in this way? Is this better or worse than what they can expect from playing the mixed-strategy equilibrium? Why?
4. After several weeks of not playing chicken as in part (c), James and Dean agree to play again. However, each of them has entirely forgotten which pure-strategy Nash equilibrium they played last time, and neither realizes this until they're revving their engines moments before starting the game. Instead of playing the mixed-strategy Nash equilibrium, each of them tosses a separate coin to decide which strategy to play. What are the expected payoffs to James and Dean when each mixes 50–50 in this way? How do these payoffs compare with their expected payoffs when they play their equilibrium mixtures? Explain why these payoffs are the same or different from those found in part (c).

7. Section 2.B illustrates how to graph best-response curves for the tennis-point game. Section 4.B notes that when there are multiple equilibria, they can be identified from multiple intersections of the best-response curves. For the battle-of-the-sexes game in Figure 4.15, graph the best responses of Holmes and Watson on a p–q coordinate plane. Label all Nash equilibria.
8. Consider the following game:

		COLIN	
		Yes	No
ROWENA	Yes	x, x	0, 1
	No	1, 0	1, 1

---

1. For what values of x does this game have a unique Nash equilibrium? What is that equilibrium?
2. For what values of x does this game have a mixed-strategy Nash equilibrium? With what probability, expressed in terms of x, does each player play Yes in this mixed-strategy equilibrium?
3. For the values of x found in part (b), is the game an example of an assurance game, a game of chicken, or a game similar to tennis? Explain.
4. Let $x = 3$. Graph the best-response curves of Rowena and Colin on a $p - q$ coordinate plane. Label all Nash equilibria in pure and mixed strategies.
5. Let $x = 1$. Graph the best-response curves of Rowena and Colin on a $p - q$ coordinate plane. Label all Nash equilibria in pure and mixed strategies.

9. Consider the following game:

		PROFESSOR PLUM		
		Revolver	Knife	Wrench
MRS. PEACOCK	Conservatory	1, 3	2, −2	0, 6
	Ballroom	3, 1	1, 4	5, 0
You may need to scroll left and right to see the full figure.				

---

1. Graph the expected payoffs from each of Professor Plum’s strategies as a function of Mrs. Peacock’s p-mix.

2. Over what range of p does Revolver yield a higher expected payoff for Professor Plum than Knife?
3. Over what range of p does Revolver yield a higher expected payoff than Wrench?
4. Which pure strategies will Professor Plum use in his equilibrium mixture? Why?
5. What is the mixed-strategy Nash equilibrium of this game?

10. Many of you will be familiar with the children’s game rock-paper-scissors. In rock-paper-scissors, two people simultaneously choose Rock, Paper, or Scissors, usually by putting their hands into the shape of one of the three choices. The game is scored as follows. A player choosing Scissors beats a player choosing Paper (because scissors cut paper). A player choosing Paper beats a player choosing Rock (because paper covers rock). A player choosing Rock beats a player choosing Scissors (because rock breaks scissors). If two players choose the same object, they tie. Suppose that each individual play of the game is worth 10 points. The following matrix shows the possible outcomes in the game:

		LISA		
		Rock	Scissors	Paper
BART	Rock	0, 0	10, −10	−10, 10
	Scissors	−10, 10	0, 0	10, −10
	Paper	10, −10	−10, 10	0, 0

You may need to scroll left and right to see the full figure.

1. Derive the mixed-strategy equilibrium of the rock-paper-scissors game.
2. Suppose that Lisa announced that she would use a mixture in which her probability of choosing Rock would be 40%, her probability of choosing Scissors

would be 30%, and her probability of choosing Paper would be 30%. What is Bart’s best response to this strategy choice by Lisa? Explain why your answer makes sense, given your knowledge of mixed strategies.

11. Recall the game between ice-cream vendors on a beach from Exercise U7 in Chapter 6. In that game, we found two asymmetric pure-strategy equilibria. There is also a symmetric mixed-strategy equilibrium in the game.
    1. Draw the five-by-five table for the game.
    2. Eliminate dominated strategies, and explain why they should not be used in the equilibrium.
    3. Use your answer to part (b) to help you find the mixed-strategy equilibrium of the game.
12. Suppose that the soccer penalty kick game of Section 7.A in this chapter is expanded to include a total of six distinct strategies for the kicker: to shoot high and to the left (HL), low and to the left (LL), high and in the center (HC), low and in the center (LC), high right (HR), and low right (LR). The goalie continues to have three strategies: to move to the kicker’s left (L) or right (R) or to stay in the center (C). The players’ success percentages are shown in the following table:

		GOALIE		
		L	C	R
KICKER	HL	0.50, 0.50	0.85, 0.15	0.85, 0.15
	LL	0.40, 0.60	0.95, 0.05	0.95, 0.05
	HC	0.85, 0.15	0, 0	0.85, 0.15
	LC	0.70, 0.30	0, 0	0.70, 0.30
	HR	0.85, 0.15	0.85, 0.15	0.50, 0.50
	LR	0.95, 0.05	0.95, 0.05	0.40, 0.60

You may need to scroll left and right to see the full figure.

In this problem, you will verify that the mixed-strategy equilibrium of this game entails the goalie using L and R each 42.2% of the time and C 15.6% of the time, while the kicker uses LL and LR each 37.8% of the time and HC 24.4% of the time.

1. Given the goalie’s proposed mixed strategy, compute the expected payoff to the kicker for each of his six pure strategies. (Use only three significant digits to keep things simple.)
2. Use your answer to part (a) to explain why the kicker’s proposed mixed strategy is a best response to the goalie’s proposed mixed strategy.
3. Given the kicker’s proposed mixed strategy, compute the expected payoff to the goalie for each of his three pure strategies. (Again, use only three significant digits to keep things simple.)
4. Use your answer to part (a) to explain why the goalie’s proposed mixed strategy is a best response to the kicker’s proposed mixed strategy.
5. Using your previous answers, explain why the proposed strategies are indeed a Nash equilibrium.
6. Compute the equilibrium payoff to the kicker.

13. (**Optional**) In Section 5.B, we demonstrated for the assurance game that changing Watson’s payoffs does not change his equilibrium mix—only Holmes’s payoffs determine Watson’s equilibrium mix. In this exercise, you will prove this as a general result for the mixed-strategy equilibria of all two-by-two games. Consider a general two-by-two non-zero-sum game with the payoff table shown below:

		COLIN	
		Left	Right
ROWENA	Up	a, A	b, B

	COLIN		
		Left	Right
	Down	c, C	d, D

---

1. Suppose the game has a mixed-strategy equilibrium. As a function of the payoffs in the table, solve for the probability that Rowena plays Up in equilibrium.
2. Solve for the probability that Colin plays Left in equilibrium.
3. Explain how your results show that each player's equilibrium mixture depends only on the other player's payoffs.
4. What conditions must be satisfied by the payoffs in order to guarantee that the game does indeed have a mixed-strategy equilibrium?

14. **(Optional)** Recall Exercise S15 of Chapter 4, which was based on the bar scene from the film *A Beautiful Mind*. Here we consider the mixed-strategy equilibria of that game when played by $n > 2$ young men.
    1. Begin by considering the symmetric case in which each of the n young men independently approaches the solitary blonde with some probability p. This probability is determined by the condition that each young man should be indifferent between the pure strategies Blonde and Brunette, given that everyone else is mixing. What is the condition that guarantees the indifference of each player? What is the equilibrium value of p in this game?
    2. There are also some asymmetric mixed-strategy equilibria in this game. In these equilibria, $m < n$ young men each approach the blonde with probability q, and the remaining $n - m$ young men approach the brunettes. What is the condition that guarantees that each of the m young men is indifferent, given what everyone else is doing? What condition must hold so that the remaining $n - m$ players don't want to

switch from the pure strategy of approaching a brunette? What is the equilibrium value of q in the asymmetric equilibrium?

UNSOLVED EXERCISES

1. In American football, the offense can either run the ball or pass the ball, whereas the defense can either anticipate (and prepare for) a run or anticipate (and prepare for) a pass. Assume that the expected payoffs (in yards) for the two teams on any given down are as follows:

		DEFENSE	
		Anticipate Run	Anticipate Pass
OFFENSE	Run	1, −1	5, −5
	Pass	9, −9	3, −3
You may need to scroll left and right to see the full figure.			

   1. Show that this game has no pure-strategy Nash equilibrium.
   2. Find the unique mixed-strategy Nash equilibrium of this game.
   3. Explain why the mixture used by the offense is different from the mixture used by the defense.
   4. How many yards is the offense expected to gain per down in equilibrium?
2. On the eve of a problem-set due date, a professor receives an e-mail from one of her students, who claims to be stuck on one of the problems after working on it for more than an hour. The professor would rather help the student if he has sincerely been working, but she would rather not render aid if the student is just fishing for hints. Given the timing of the request, she could simply pretend not to have read the e-mail until later. Obviously, the student would rather receive help

whether or not he has been working on the problem. But if help isn't coming, he would rather be working instead of slacking, since the problem set *is* due the next day. Assume the payoffs are as follows:

		STUDENT	
		Work and ask for help	Slack and fish for hints
PROFESSOR	Help student	3, 3	−1, 4
	Ignore e-mail	−2, 1	0, 0

You may need to scroll left and right to see the full figure.

---

1. What is the mixed-strategy Nash equilibrium of this game?
2. What is the expected payoff to each of the players?

3. Exercise S14 in Chapter 4 introduced the game Evens or Odds, which has no Nash equilibrium in pure strategies. It does have an equilibrium in mixed strategies.
   1. If Anne plays 1 (that is, she shows one finger) with probability p, what is the expected payoff to Bruce from playing 1, in terms of p? What is his expected payoff from playing 2?
   2. What level of p will make Bruce indifferent between playing 1 and playing 2?
   3. If Bruce plays 1 with probability q, what level of q will make Anne indifferent between playing 1 and playing 2?
   4. What is the mixed-strategy equilibrium of this game? What is the expected payoff of the game to each player?
4. Return again to the tennis rivals Evert and Navratilova, discussed in Section 2.A. Months later, they meet again

in a new tournament. Evert has healed from her injury (see Exercise S4), but during that same time Navratilova has worked very hard on improving her defense against DL serves. The payoffs are now as follows:

		NAVRATILOVA	
		DL	CC
EVERT	DL	25, 75	80, 20
	CC	90, 10	20, 80

---

1. Find each player's equilibrium mixture for this game.
2. What has happened to Evert's p-mix compared with the original game presented in Figure 7.1? Why?
3. What is Evert's expected payoff in equilibrium? Why is it different from her expected payoff in the original game in Figure 7.1?

5. Section 4.A of this chapter discussed mixing in the battle-of-the-sexes game between Holmes and Watson.
    1. What do you expect to happen to the equilibrium values of p and q found in Section 4.A if Watson decides he really likes Simpson's a lot more than St. Bart's, so that the payoffs in the (Simpson's, Simpson's) cell of Figure 7.3 are now (1, 3)? Explain your reasoning.
    2. Now find the new mixed-strategy equilibrium values of p and q. How do they compare with those of the original game?
    3. What is the expected payoff to each player in the new mixed-strategy equilibrium?
    4. Do you think Holmes and Watson might play the mixed-strategy equilibrium in this new version of the game? Explain why or why not.
6. Consider the following variant of chicken, in which James's payoff from being "tough" when Dean is "chicken" is 2, rather than 1:

		DEAN	
		Swerve	Straight
JAMES	Swerve	0, 0	−1, 1
	Straight	2, −1	−2, −2

---

1. Find the mixed-strategy equilibrium in this game, including the expected payoffs for the players.
2. Compare the results with those of the original game in Section 4.B of this chapter. Is Dean's probability of playing Straight (being tough) higher now than before? What about James's probability of playing Straight?
3. What has happened to the two players' expected payoffs? Are these differences in the equilibrium outcomes paradoxical in light of the new payoff structure? Explain how your findings can be understood in light of the opponent's indifference principle.

7. For the chicken game in Figure 7.4, graph the best responses of James and Dean on a $p-q$ coordinate plane. Label all Nash equilibria.
8. Consider the following game:

		COLIN			
		L	M	N	R
ROWENA	Up	1, 1	2, 2	3, 4	9, 3
	Down	2, 5	3, 3	1, 2	7, 1
You may need to scroll left and right to see the full figure.					

---

1. Find all pure-strategy Nash equilibria of this game.
2. Now find a mixed-strategy equilibrium of the game. What are the players' expected payoffs in the

equilibrium?

9. Consider a revised version of the game from Exercise S9:

		PROFESSOR PLUM		
		Revolver	Knife	Wrench
MRS. PEACOCK	Conservatory	1, 3	2, −2	0, 6
	Ballroom	3, 2	1, 4	5, 0
You may need to scroll left and right to see the full figure.				

---

1. Graph the expected payoffs from each of Professor Plum’s strategies as a function of Mrs. Peacock’s p-mix.
2. Which strategies will Professor Plum use in his equilibrium mixture? Why?
3. What is the mixed-strategy Nash equilibrium of this game?
4. Note that this game is only slightly different from the game in Exercise S9. How are the two games different? Explain why you intuitively think that the equilibrium outcome has changed from Exercise S9.

10. Consider a modified version of rock-paper-scissors in which Bart gets a bonus when he wins with Rock. If Bart picks Rock while Lisa picks Scissors, Bart wins twice as many points as when either player wins in any other way. The new payoff matrix is

		LISA		
		Rock	Scissors	Paper
BART	Rock	0, 0	20, −20	−10, 10
	Scissors	−10, 10	0, 0	10, −10
You may need to scroll left and right to see the full figure.				

	LISA		
	Rock	Scissors	Paper
Paper	10, −10	−10, 10	0, 0

You may need to scroll left and right to see the full figure.

---

1. What is the mixed-strategy equilibrium in this version of the game?
2. Compare your answer here with your answer for the mixed-strategy equilibrium in Exercise S10. How can you explain the differences in the equilibrium strategy choices?

11. Consider the following game:

		MACARTHUR		
		Air	Sea	Land
PATTON	Air	0, 3	2, 0	1, 7
	Sea	2, 4	0, 6	2, 0
	Land	1, 3	2, 4	0, 3

You may need to scroll left and right to see the full figure.

---

1. Does this game have a pure-strategy Nash equilibrium? If so, what is it?
2. Find a mixed-strategy equilibrium for this game.
3. Actually, this game has two mixed-strategy equilibria. Find the one you didn't find in part (b). (Hint: In one of these equilibria, one of the players plays a mixed strategy, whereas the other plays a pure strategy.)

12. The recalcitrant James and Dean are playing their more dangerous variant of chicken again (see Exercise S6). They've noticed that their payoff for being perceived as

tough varies with the size of the crowd. The larger the crowd, the more glory and praise each receives by driving straight when his opponent swerves. Smaller crowds, of course, have the opposite effect. Let $k > 0$ be the payoff for appearing tough. The game may now be represented as

		DEAN	
		Swerve	Straight
JAMES	Swerve	0, 0	−1, k
	Straight	k, −1	−10, −10

---

1. Expressed in terms of k, with what probability does each driver play Swerve in the mixed-strategy Nash equilibrium? Do James and Dean play Swerve more or less often as k increases?
2. In terms of k, what is the expected payoff to each player in the mixed-strategy Nash equilibrium found in part (a)?
3. At what value of k do both James and Dean mix 50–50 in the mixed-strategy equilibrium?
4. How large must k be for the average payoff to be positive under the alternating scheme discussed in part (c) of Exercise S6?

13. (**Optional**) Recall the game from Exercise S13 in Chapter 4, where Larry, Moe, and Curly can choose to buy tickets toward a prize worth \$30. We found six pure-strategy Nash equilibria in that game. In this problem, you will find a symmetric equilibrium in mixed strategies.
    1. Eliminate the weakly dominated strategy for each player. Explain why a player would never use this weakly dominated strategy in his equilibrium mixture.
    2. Find the equilibrium in mixed strategies.
14. (**Optional**) Exercises S4 and U4 demonstrate that in zero-sum games such as the Evert–Navratilova tennis-point game, changes in a player's payoffs can sometimes lead to unexpected or counterintuitive changes to her

equilibrium mixture. But what happens to her expected payoff in equilibrium? Consider the following general form of a two-player zero-sum game:

		COLIN	
		L	R
ROWENA	U	a, −a	b, −b
	D	c, −c	d, −d

Assume that there is no Nash equilibrium in pure strategies, and assume that *a*, *b*, *c*, and *d* are all greater than or equal to 0. Can an *increase* in any one of *a*, *b*, *c*, or *d* lead to a *lower* expected payoff for Rowena in equilibrium? If not, prove why not. If so, provide an example.

■ Appendix: Working with Probabilities

To calculate the expected payoffs and mixed-strategy equilibria of games in this chapter, we had to do some simple manipulation of probabilities. Some simple rules govern calculations involving probabilities. Many of you may be familiar with them, but we give a brief statement and explanation of the basics here by way of reminder or remediation, as appropriate. We also explain how to calculate expected values for random numerical amounts.

THE BASIC ALGEBRA OF PROBABILITIES

The basic intuition about the probability of an event comes from thinking about the frequency with which this event occurs by chance among a larger set of possibilities. Usually, any one element of this larger set is just as likely to occur by chance as any other, so finding the probability of the event in which we are interested is simply a matter of counting the elements corresponding to that event and dividing by the total number of elements in the whole large set.[26]

In any standard deck of 52 playing cards, for instance, there are four suits (clubs, diamonds, hearts, and spades) and 13 cards in each suit (ace through 10 and the face cards—jack, queen, king). We can ask a variety of questions about the likelihood that a card of a particular suit or value—or suit *and* value—might be drawn from this deck of cards: How likely are we to draw a spade? How likely are we to draw a black card? How likely are we to draw a 10? How likely are we to draw the queen of spades? and so on. We would need to know something about the calculation and manipulation of probabilities to answer such questions. If we had two decks of cards, one with blue backs and one with green backs, we could ask even more complex questions (“How likely are we to draw one card from each deck and have them both be the jack of diamonds?”), but we would still use the algebra of probabilities to answer them.

In general, a probability measures the likelihood of a particular event or set of events occurring. The likelihood that you will draw a spade from a deck of cards is just the probability of the event “drawing a spade.” Here, the larger set of possibilities has 52 elements—the total number of equally likely possibilities—and the event "drawing a spade" corresponds to a subset of 13 particular elements. Thus, you have 13 chances out of the 52 to get a spade, which makes the probability of getting a spade in a single draw equal to 13/52 = ¼= 25%. To see this another way, consider the fact that there are four suits of 13 cards each, so your chance of drawing a card from any particular suit is one out of four, or 25%. If you made 52 such draws (each time from a complete deck), you would not always draw exactly 13 spades; by chance, you might draw a few more or a few less. But the chance averages out over different such occasions—say, over different sets of 52 draws. Then the probability of 25% is the average of the frequencies of drawing spades in a large number of observations.[27]

The algebra of probabilities simply develops such ideas in general terms and obtains formulas that you can then apply mechanically instead of having to do the thinking from scratch every time. We will organize our discussion of these probability formulas around the types of questions that one might ask when drawing cards from a standard deck (or two: blue backed and green backed).[28] This method will allow us to provide both specific and general formulas for you to use later. You can use the card-drawing analogy to help you reason out other questions about probabilities that you encounter in other contexts. One other point to note: In ordinary language, it is customary to write probabilities as percentages, but the algebra requires that they be written as fractions or decimals; thus, instead of 25%, the mathematics works with 13/52, or 0.25. We will use one or the other, depending on the occasion; be aware that they mean the same thing.

A. The Addition Rule

The first questions that we ask are, If we draw one card from the blue deck, how likely are we to draw a spade? And how likely are we to draw a card that is not a spade? We already know that the probability of drawing a spade is 25% because we determined that earlier. But what is the probability of drawing a card that is not a spade? It is the same as the likelihood of drawing a club or a diamond or a heart instead of a spade. It should be clear that the probability in question should be larger than any of the individual probabilities of which it is formed; in fact, the probability is 13/52 (clubs) + 13/52 (diamonds) + 13/52 (hearts) = 0.75. The *or*s in our verbal interpretation of the question are the clue that the probabilities should be added together because we want to know the chances of drawing a card from any of those three suits.

We could more easily have found our answer to the second question by noting that, given that we draw a spade 25% of the time, not drawing a spade is what happens the other 75% of the time. Thus, the probability of drawing “not a spade” is 75% (100% – 25%), or, more formally, 1 – 0.25 = 0.75. As is often the case with probability calculations, the same result can be obtained here by two different routes, entailing different ways of thinking about the event for which we are trying to find the probability. We will see other examples of such alternative routes later in this appendix, where it will become clear that the different methods of calculation can sometimes require vastly different amounts of effort. As you develop experience, you will discover and remember the easy ways or shortcuts. In the meantime, be comforted by the knowledge that each of the different routes, when correctly followed, leads to the same final answer.

To generalize our preceding calculation, we note that, if you divide the set of events, *X*, in which you are interested into some number of subsets, *Y*, *Z*, . . . , none of which overlap (in mathematical terminology, such subsets are said to be disjoint), then the probabilities of each subset occurring must sum to the probability of the full set of events; if that full set of events includes all possible outcomes, then its probability is 1. In other words, if the occurrence of *X* requires the occurrence of *any one* of several disjoint *Y*, *Z*, . . . , then the probability of *X* is the sum of the separate probabilities of *Y*, *Z*, ... Using Prob(*X*) to denote the probability that *X* occurs and remembering the caveats on *X* (that it requires any one of *Y*, *Z*, . . .) and on *Y*, *Z*, . . . (that they must be disjoint), we can write the addition rule in mathematical notation as Prob(*X*) = Prob(*Y*) + Prob(*Z*) + . . . ;.

EXERCISE Use the addition rule to find the probability of drawing two cards, one from each of the two decks, such that the two cards have identical faces.

B. The Multiplication Rule

Now we ask, What is the likelihood that when we draw two cards, one from each of the two decks, both of them will be spades? This event occurs if we draw a spade from the blue deck *and* a spade from the green deck. The switch from *or* to *and* in our verbal interpretation of what we are looking for indicates a switch in mathematical operations from addition to multiplication. Thus, the probability of two spades, one from each deck, is the product of the probabilities of drawing a spade from each deck, or (13/52) × (13/52) = 1/16 = 0.0625, or 6.25%. Not surprisingly, we are much less likely to get two spades than we were in the previous section to get one spade. (Always check to make sure that your calculations accord in this way with your intuition regarding the outcome.)

In much the same way as the addition rule requires events to be disjoint, the multiplication rule requires that they be independent events: If we break down a set of events, *X*, into some number of subsets *Y*, *Z*, . . . , those subsets are independent if the occurrence of one does not affect the probability of the other. Our events—a spade from the blue deck and a spade from the green deck—satisfy this condition of independence; that is, drawing a spade from the blue deck does nothing to alter the probability of getting a spade from the green deck. If we were drawing both cards from the same deck, however, then after we had drawn a spade (with a probability of 13/52), the probability of drawing another spade would no longer be 13/52 (in fact, it would be 12/51); drawing one spade and then a second spade from the *same* deck are not independent events.

The formal statement of the multiplication rule tells us that, if the occurrence of *X* requires the simultaneous occurrence of *all* the several independent events Y, Z, . . . , then the probability of X is the *product* of the separate probabilities of Y, Z, . . . : $\text{Prob}(X) = \text{Prob}(Y) \times \text{Prob}(Z) \times \cdots$.

EXERCISE Use the multiplication rule to find the probability of drawing two cards, one from each of the two decks, and getting a red card from the blue deck and a face card from the green deck.

C. Expected Values

If a numerical amount (such as money winnings or rainfall) is subject to chance and can take on any one of n possible values X_1, X_2, . . . , X_n with respective probabilities p_1, p_2, . . . , p_n, then the expected value is defined as the weighted average of all its possible values using the probabilities as weights; that is, as $p_1X_1 + p_2X_2 + . . . + p_nX_n$. For example, suppose you bet on the toss of two fair coins. You win \$5 if both coins come up heads, \$1 if one shows heads and the other tails, and nothing if both come up tails. Using the rules for manipulating probabilities discussed earlier in this section, you can see that the probabilities of these events are, respectively, 0.25, 0.50, and 0.25. Therefore, your expected winnings are (0.25 × \$5) + (0.50 × \$1) + (0.25 × \$0) = \$1.75.

Endnotes

- When we say “by chance,” we simply mean that a systematic order cannot be detected in the outcome or that it cannot be determined by using available scientific methods of prediction and calculation. Actually, the motions of coins and dice are fully determined by laws of physics, and highly skilled people can manipulate decks of cards, but for all practical purposes, coin tosses, rolls of dice, or card shuffles are devices of chance that can be used to generate random outcomes. However, randomness can be harder to achieve than you think. For example, a perfect shuffle, where a deck of cards is divided exactly in half and then interleaved by dropping cards one at a time alternately from each, may seem a good way to destroy the initial order of the deck. But Stanford mathematician Persi Diaconis has shown that, after eight of the shuffles, the original order is fully restored. For the slightly imperfect shuffles that people carry out in reality, he finds that some order persists through six, but randomness suddenly appears on the seventh! See “How to Win at Poker, and Other Science Lessons,” *The Economist*, October 12, 1996. For an interesting discussion of such topics, see Deborah J. Bennett, *Randomness* (Cambridge, Mass.: Harvard University Press, 1998), chap. 6–9. Return to reference 26
- Bennett, *Randomness*, chap. 4 and 5, offers several examples of such calculations of probabilities. Return to reference 27
- If you want a more detailed exposition of the following addition and multiplication rules, as well as more exercises to practice these rules, we recommend David Freeman, Robert Pisani, and Robert Purves, *Statistics,*

4th ed. (New York: W. W. Norton & Company, 2007), chap. 13 – 14. Return to reference 28

Glossary

probability
: The probability of a random event is a quantitative measure of the likelihood of its occurrence. For events that can be observed in repeated trials, it is the long-run frequency with which it occurs. For unique events or other situations where uncertainty may be in the mind of a person, other measures are constructed, such as subjective probability.

disjoint
: Events are said to be disjoint if two or more of them cannot occur simultaneously.

addition rule
: If the occurrence of X requires the occurrence of *any one* of several disjoint Y, Z, . . . , then the probability of X is the sum of the separate probabilities of Y, Z, . . .

independent events
: Events Y and Z are independent if the actual occurrence of one does not change the probability of the other occurring. That is, the conditional probability of Y occurring given that Z has occurred is the same as the ordinary or unconditional probability of Y.

multiplication rule
: If the occurrence of X requires the simultaneous occurrence of *all* the several independent Y, Z, . . . , then the probability of X is the *product* of the separate probabilities of Y, Z, . . .

expected value
: The probability-weighted average of the outcomes of a random variable, that is, its statistical mean or expectation.

SUMMARY

The *probability* of an event is the likelihood of its occurrence by chance from among a larger set of possibilities. Probabilities can be combined by using some rules. The *addition rule* says that the probability of any one of a number of *disjoint* events occurring is the sum of the probabilities of these events. According to the *multiplication rule*, the probability that all of a number of *independent events* will occur is the product of the probabilities of these events. Probability-weighted averages are used to compute *expected values*, such as payoffs in games.

KEY TERMS

addition rule (261)

disjoint (261)

expected value (262)

independent events (262)

multiplication rule (262)

probability (260)

Glossary

addition rule
: If the occurrence of *X* requires the occurrence of *any one* of several disjoint *Y*, *Z*, . . . , then the probability of *X* is the sum of the separate probabilities of *Y*, *Z*, . . .

disjoint
: Events are said to be disjoint if two or more of them cannot occur simultaneously.

expected value
: The probability-weighted average of the outcomes of a random variable, that is, its statistical mean or expectation.

independent events
: Events Y and Z are independent if the actual occurrence of one does not change the probability of the other occurring. That is, the conditional probability of Y occurring given that Z has occurred is the same as the ordinary or unconditional probability of Y.

multiplication rule
: If the occurrence of *X* requires the simultaneous occurrence of *all* the several independent *Y, Z,* . . . , then the probability of *X* is the *product* of the separate probabilities of *Y, Z,* . . .

probability
: The probability of a random event is a quantitative measure of the likelihood of its occurrence. For events that can be observed in repeated trials, it is the long-run frequency with which it occurs. For unique events or other situations where uncertainty may be in the mind of a person, other measures are constructed, such as subjective probability.

PART THREE

Some Broad Classes of Strategies and Games

8 ■ Strategic Moves

SUPPOSE YOU ARE about to play a game of strategy. You can figure out that the outcome will yield you one of your lower payoffs. And you would like to do something to change the outcome of the game in your favor. Various devices, collectively labeled strategic moves, exist for this purpose. The design and use of strategic moves is often specific to each context, but some general principles can be culled from examples and from game-theoretic thinking. Those principles are the subject of this chapter.

The interaction among players when they are attempting to find strategic moves that can change a game they are about to play constitutes a game of its own. For conceptual clarity, we call this the *pregame*. In the pregame, you make declarations and take actions aimed at changing the upcoming game; we will often refer to this upcoming game simply as *the game*. To use strategic moves effectively, players need to ask (and answer) several key questions. We begin by laying out these questions and then use the answers to guide our subsequent analyses.

What is to be accomplished? If you could improve your outcome in the game just by changing your own action, your original action must not have been a Nash equilibrium choice. So, any strategic move you make must seek to change some *other* player's action from what she would otherwise choose in equilibrium—that is, to change the other player's *default action* to some *favored action* that is better for you.

What changes are feasible? The actual game has specified rules and payoffs. Pregame moves can change these, but only within certain limits. Legislatures often make their own procedural rules, and manipulating these rules to facilitate or hinder the passage of particular legislation is a well-recognized pregame. But if a football team has a weak passing offense, it cannot suddenly change the rules of football to forbid forward passes. As for payoffs, you may be able to alter your own payoffs for certain outcomes—for example, by entering into side bets about those outcomes or by laying your reputation on the line. But in most cases, you cannot directly change others' payoffs. The order of moves in the upcoming game may also limit what sort of strategic moves you can make. In each case, feasibility is an important prerequisite for making a strategic move.

What changes are credible? In many games you play, you would like the other player to believe that you have limited or expanded your own set of available actions, or that you have changed your payoffs in some way. Even when such changes are technically feasible, simply declaring that you have done so may not be credible. For instance, in the game of chicken, you would like the other player to believe that you are unable to swerve, since she will then have an incentive to swerve out of your way. But the other player may not believe you when you say that you cannot swerve. If you are dieting, but love desserts, you can counter temptation by vowing to send $1,000 to a charity every time you order dessert. But unless there is some external agent to observe any cheating and hold you to your resolve, your vow may not be credible. The question of *credibility*, a concept we first introduced in Chapter 6, is crucial to the effective use of strategic moves, and we will examine it in connection with all the moves we discuss in this chapter.

What kinds of strategic moves? A pregame move may specify that you will take some particular action in the upcoming game regardless of what the other player does in that game. For example, you might declare you will definitely go straight in the game of chicken in order to induce the other player to swerve. Or a pregame move may establish a rule for how you will respond to the other player's move in the upcoming game depending on what she does. For example, a firm might announce that it will match a rival’s price in order to induce the rival to set a high price (which is beneficial to both). We will show you how to think about both types of pregame moves in the following section, and we will provide examples illustrating how and when to use such moves in the remainder of the chapter.

Glossary

strategic move
: Action taken at a pregame stage that changes the strategies or the payoffs of the subsequent game (thereby changing its outcome in favor of the player making the move).

feasibility
: Possibility within the (unchangeable) physical and/or procedural restrictions that apply in a given game.

1 A CLASSIFICATION OF STRATEGIC MOVES

In this section, we focus our attention on two-player games, but the ideas involved are perfectly general, and we will occasionally illustrate them with more than two players. We will call the two players in our examples Ann and Bob. As described above, strategic moves can be declared in two ways in the pregame: They can specify an action that you will take in the upcoming game no matter what. Or they can specify a rule you will follow in the upcoming game in which your chosen actions depend on the choices made by the other player. The first type of strategic move is *unconditional*, and the second type is *conditional*. We set out the details of each type of move below.

A. Unconditional Strategic Moves

Suppose that, in the pregame, Ann were to declare, "In the upcoming game, I will take a particular action, X, no matter what you do." This unconditional strategic move is called a commitment. When Ann's commitment is credibly made, Bob will make his move taking into account that Ann's move is essentially already fixed. That is, he will assume that the subsequent game will unfold *as if* Ann is the first mover and has already observably and irreversibly chosen X. In this way, a credible commitment allows a player who would otherwise move simultaneously or move last to become the first mover, seizing any *first-mover advantage* that may exist in the game.

In the street-garden game of Chapter 3, three women play a sequential-move game in which each must decide whether to contribute toward the creation of a public flower garden on their street; two or more contributors are necessary for the creation of a pleasant garden. The rollback equilibrium entails the first player (Emily) choosing not to contribute while the other players (Nina and Talia) do contribute. By making a credible commitment in the pregame not to contribute in the upcoming game, however, Talia (or Nina) could alter the outcome of the game. Even though she does not get her turn to announce her decision until after Emily and Nina have made theirs public, Talia could let it be known that she has sunk all of her savings (and energy) into a large house-renovation project, so she has absolutely nothing left to contribute to the street garden. By doing so, Talia essentially commits herself not to contribute regardless of Emily's and Nina's decisions, before Emily and Nina make those decisions. In other words, Talia changes the game to one in which she is the first mover. You can easily check that the new rollback equilibrium entails Emily and Nina both contributing to the garden, and that the equilibrium payoffs are 3 to each of them but 4 to Talia—the same equilibrium outcome associated with the game when Talia moves first. Several additional detailed examples of commitments are provided later in this chapter.

B. Conditional Strategic Moves

Consider now the use of a conditional strategic move. Suppose Ann declares a response rule in the pregame, stating how she will respond to each of Bob's possible actions in the upcoming game. For this strategic move to be meaningful, Ann must be the second mover in the upcoming game, or arrange in the pregame to become the second mover. (Ann could, for example, manipulate the reversibility or observability of her or Bob's moves, as described in Section 2 of Chapter 6.)

Suppose Ann does move last in the upcoming game and hence could potentially use a conditional strategic move. In order to have any hope of changing Bob's behavior to her own advantage, Ann must change Bob's belief about how she is going to act as second mover in the upcoming game. In any effective strategic move, Ann's response rule must therefore specify at least one response that is different from what she would normally do in the absence of the strategic move. Thus, at least one of Ann's declared responses must *not* be a best response.

In addition, Ann's conditional strategic move must be designed to improve her outcome in the upcoming game. The strategic move should motivate Bob to take an action Ann prefers, called the favored action, rather than the default action that he would otherwise have taken in the rollback equilibrium of the upcoming game.

There are three ways in which Ann might be able to get Bob to change his decision with a conditional strategic move. First, she could declare that she will respond to the *favored* action not with her equilibrium best-response action, but instead in a way that gives Bob a relatively *high* payoff—and, often implicitly, that she will respond to the default action with her best response to that action. This is called *making a promise*; Ann's response rule rewards Bob for doing what she wants. Second, she could declare that she will respond to the *default* action not with her equilibrium best-response action, but instead in a way that gives Bob a relatively *low* payoff—and, often implicitly, that she will respond to the favored action with her best response to that action. This is called *making a threat*; Ann's response rule hurts Bob when he does not act the way she wants. Finally, Ann could declare that she will do *both*: She will respond to the favored action in a way that gives Bob a relatively high payoff, and she will respond to the default action in a way that gives Bob a relatively low payoff. This is called *making a combined threat and promise*; Ann's response rule both rewards Bob for doing what she wants and hurts him when he does not act the way she wants.

To keep things clear in the analysis that follows, we introduce some additional terminology here. Specifically, we give names to each piece of a response rule that constitutes a conditional strategic move. A specified response to the favored action that is not your best response and benefits the other player is called a promise, and a specified response to the default action that is not your best response and hurts the other player is called a threat. An affirmation is a specified response to the favored action that is your best response, and a warning is a specified response to the default action that is your best response. With this terminology, *making a threat* entails declaring a response rule that includes a threat and an affirmation and *making a promise* entails declaring a response rule that includes a promise and a warning. In practice, players often state aloud only the threat or the promise when making those moves, leaving the affirmation or warning implicit rather than explicit. Finally, *making a combined threat and promise* entails explicit statement of both a threat and promise (as the name suggests!).

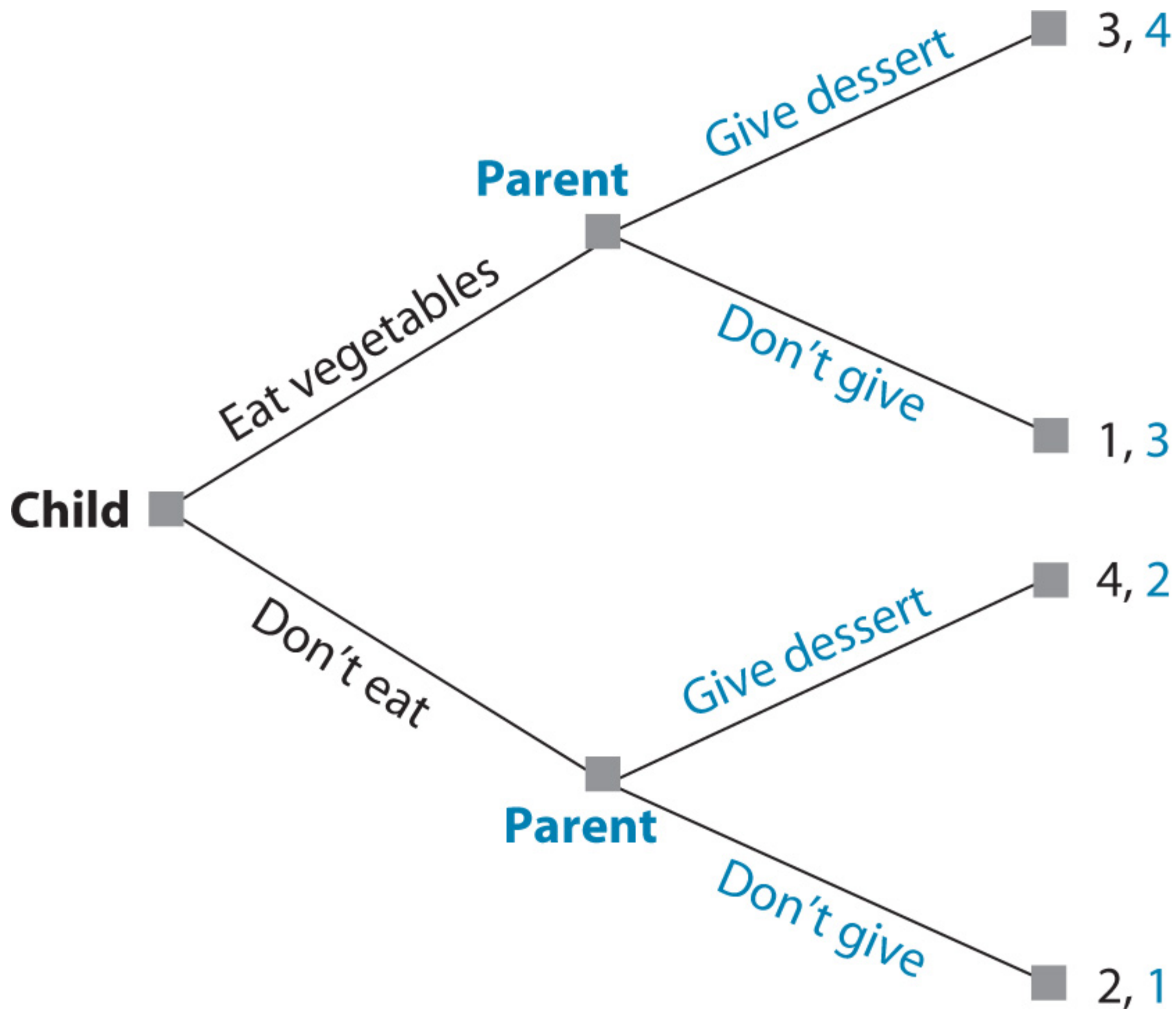


Figure 8.1 Parent–Child Dinner Game

We will provide several in-depth examples of the use of conditional strategic moves later in this chapter, but provide an additional example here to help clarify how to think about strategic moves and how to use the terminology we have just presented. Imagine a game between a parent and child that takes place at the dinner table. In the natural chronological order of moves, the child first decides whether to eat her vegetables, and then the parent decides whether to give the child dessert. So the parent moves last and can feasibly make a conditional strategic move. Figure 8. illustrates the game tree and shows how each player ranks the four possible outcomes. (As usual, 4 is best and 1 is worst.) Rollback analysis tells us what the default equilibrium outcome will be if the parent does nothing to change the game: The child refuses to eat the vegetables, knowing that the parent, unwilling to see the child hungry and unhappy, will give her the dessert. In the equilibrium outcome the child receives her highest payoff (4), but the parent receives one of his lower payoffs (2).

The parent can foresee this outcome and can try to alter it in his favor by making a conditional strategic move in the pregame. He might declare the response rule, “If you eat your vegetables, then I will serve dessert. But if you don’t eat them, then I won’t serve dessert.” More likely, he might say simply, “No dessert unless you eat your veggies!” Such a declaration is a pregame conditional strategic move that, if credibly made, fixes how the parent will react in the upcoming game. If the child believes the parent, this move alters the child’s rollback calculation. The child prunes that branch of the game tree in which the parent serves dessert after the child has not eaten her vegetables and assumes that she cannot achieve that outcome (which gives her payoff 4). This then induces the child to eat her vegetables, as she prefers the outcome in which she eats the vegetables and gets dessert (payoff 3) over the outcome in which she does not eat the vegetables and does not get dessert (payoff 2).

In this example, the parent induces the child to change her action in a way that benefits him. He gets her to change from her default action, Don’t eat, to his favored action, Eat vegetables, by convincing her that he will do something that he normally would not want to do (Don’t give in response to Don’t eat), something that would be harmful to her (because it reduces her payoff). In other words, the parent makes a threat by using a response rule that includes a threat (Don’t give in response to Don’t eat) and an implicit affirmation (Give in response to Eat).

And why should the child believe that the parent really will withhold dessert as he has threatened? After all, if she does not eat the vegetables, her parent will want to serve dessert anyway. This is a reminder that the parent will need to make his strategic move credible if it is to be effective. The parent must somehow motivate himself to follow through on the threatened action even though, normally, he would not want to do so. This process is called *establishing credibility* for the strategic move. For instance, the parent might achieve credibility by making his declaration in front of his other children, putting his authority as a parent on the line and thereby effectively compelling himself to follow through on withholding dessert if the child were to test his resolve. We will examine the issue of credibility in more detail below.

C. The Role of Feasibility, Framing, and Credibility

We conclude this section with some important additional remarks about the use of strategic moves.

First, consider the feasibility of conditional strategic moves. Whether it is feasible for Ann to make a threat or a promise depends on Bob’s payoffs in the game and, in particular, on whether there is anything she can do to make Bob worse off (in the case of making a threat) or better off (in the case of making a promise) than when she plays her best response. If Ann’s best response to the default action is already the most harmful thing that she can do (from Bob’s point of view), then there is no way for her to *threaten* anything further. Ann cannot feasibly make a threat in that type of game. Similarly, if Ann’s best response to the favored action is already the most beneficial thing that she can do (from Bob’s point of view), then there is no way for her to *promise* anything further. She cannot feasibly make a promise in that situation.

For instance, consider again the dinner game shown in Figure 8.1. The child’s default action is Don’t eat, but the parent wants the child to eat her veggies, so his favored action is Eat vegetables. In this game, the parent’s a dominant strategy is to give the child dessert, so his best-response actions are both Give dessert. Whenever the parent chooses a non‑best response action (Don’t give), he harms the child; there is no non‑best response action that rewards the child. Thus, the parent can potentially make a threat [“If you don’t eat your veggies, I will not give you dessert (and if you do eat them, I will give you dessert)”], but cannot make a promise. His best response of giving dessert when the child eats her veggies is already the best that he can do for her and there is nothing further he can promise.

Second, it sometimes matters how a conditional strategic move is phrased, or framed. When she makes a conditional strategic move, Ann's implied message to Bob can be interpreted as either "Don't take the default action" or "Take the favored action." These two interpretations are classified as, respectively, deterrence and compellence. Depending on the context or framing of the strategic situation, one or the other of these interpretations may be more pertinent, and if multiple strategic moves are available to achieve the same aim, it may be better achieved by one rather than another. Often the best choice depends on the time frame within which Ann wants to achieve her aim. We will illustrate and discuss these issues further in Section 4.B of this chapter.

Finally, because *all* strategic moves entail *not* playing one's best response under certain specific circumstances, establishing *credibility* for these moves is essential. To make effective use of commitments, threats, and promises, you must convince the other players that, if the relevant circumstances arise, you will follow through on your declaration and do something that you normally wouldn't want to do.

There are two basic ways to establish the credibility of a strategic move. First, you can remove the other moves that may tempt you from your own set of future choices. Second, you can change your own payoffs in such a way that it becomes truly optimal for you to act as you have declared you will. This can be done either by *increasing* your own payoff for making the stipulated move or by *reducing* your own payoff for making another move that would otherwise tempt you to deviate from your stipulated move. In the sections that follow, we will first elucidate the mechanics of each type of strategic move, taking as given that credibility can somehow be achieved. We will make some comments about credibility along the way, but will postpone our general analysis of credibility to the last two sections of this chapter.

Glossary

commitment
: An action taken at a pregame stage, stating what action you would take unconditionally in the game to follow.

response rule
: A rule that specifies how you will act in response to various actions of other players.

favored action
: The action that a player making a *strategic move* wants the other player to take, as opposed to the *default action*.

default action
: In the context of strategic moves, the action that the other player (the player not making a strategic move) will take in the absence of a strategic move, as opposed to the *favored action*.

promise
: A response to the *favored action* that benefits the other player and that is not a best response, as specified within a *strategic move*. It is only feasibly made by the second mover in a game. Credibility is required because the specified action is not a best response. The strategic move referred to as "making a promise" entails declaring both a promise and a warning, whereas "making a combined threat and promise" entails declaring both a promise and a threat.

threat
: A response to the default action that harms the other player and that is not a best response, as specified within a strategic move. It can only feasibly be made by the second mover in a game. Credibility is required because the specified action is not a best response. The strategic move referred to as "making a threat" entails declaring both a threat and an affirmation, whereas "making a combined threat and promise" entails declaring both a threat and a promise.

affirmation
: A response to the *favored action* that is a best response, as specified within a strategic move. Only the second mover in a game can feasibly make an affirmation. However, credibility

is not required since the specified action is already the player's best response. The strategic move referred to as "making a threat" entails declaring both a threat and an affirmation.

warning
: A response to the default action that is a best response, as specified within a strategic move. Only the second mover in a game can feasibly make a warning. However, credibility is not required since the specified action is already the player's best response. The strategic move referred to as "making a promise" entails declaring both a promise and a warning.

deterrence
: An attempt to induce the other player(s) to act to maintain the status quo.

compellence
: An attempt to induce the other player(s) to act to change the status quo in a specified manner.

2 COMMITMENTS

We studied the game of chicken in Chapter 4 and found two pure-strategy Nash equilibria: Each player prefers the equilibrium in which he goes straight and the other player swerves.[1] And we saw in Chapter 6 that if the game were to have sequential rather than simultaneous moves, the first mover would choose Straight, leaving the second to make the best of the situation by settling for Swerve rather than causing a crash. Now we can consider the same game from another perspective. Even if the game itself has simultaneous moves, if one player can make a strategic move—create a pregame in which he makes a credible declaration about his action in the upcoming game itself—then he can get the same advantage afforded a first mover—in this case, by making a commitment to act Tough (choose Straight).

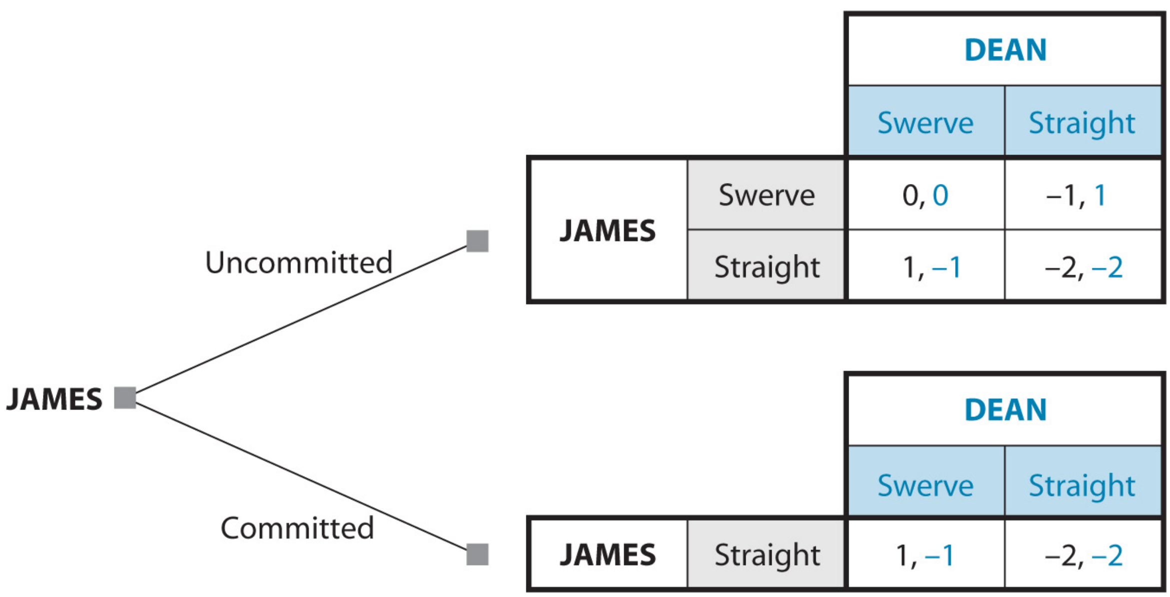


Figure 8.2 Chicken: Commitment by Restricting Freedom to Act

Although the point is simple, we outline the formal analysis here to help you develop your understanding and skill, which will be useful for later, more complex examples. Remember our two players, James and Dean. Suppose James is the one who has the opportunity to make a strategic move. Figure 8.2 shows the tree for the two-stage game. At the first stage (the pregame), James has to decide whether to make a commitment. Along the upper branch emerging from the first node, he does not make a commitment. At the second stage, the original simultaneous-move game is played. Its payoff table is the familiar one shown in Figure 4.16 and Figure 6.7. This second-stage game has multiple equilibria, and James gets his best payoff in only one of them. Along the lower branch emerging from the first node, James makes a commitment. Here, we interpret this commitment to mean giving up his freedom to act in such a way that Straight is the only action available to him at the second stage. Therefore, the second-stage game table has only one row for James, corresponding to his commitment to Straight. In this table, Dean's best action is Swerve, so the equilibrium outcome gives James his best payoff. Rollback analysis shows that James finds it optimal to make the commitment; this strategic move ensures his best payoff, while not committing leaves the matter uncertain.

How can James make this commitment credibly? Like any first move, the commitment move must be (1) irreversible and (2) observable before the other player makes his choice. People have suggested some extreme and amusing ideas for achieving irreversibility. James can disconnect the steering wheel of the car and throw it out the window so that Dean can see that James can no longer Swerve. Or James could just tie the wheel so that it could no longer be turned, but it would be more difficult to demonstrate to Dean that the wheel was truly tied and that the knot was not a trick one that could be undone quickly. These devices simply remove the Swerve option from the set of choices available to James in the second-stage game, leaving Straight as the only thing he can do.

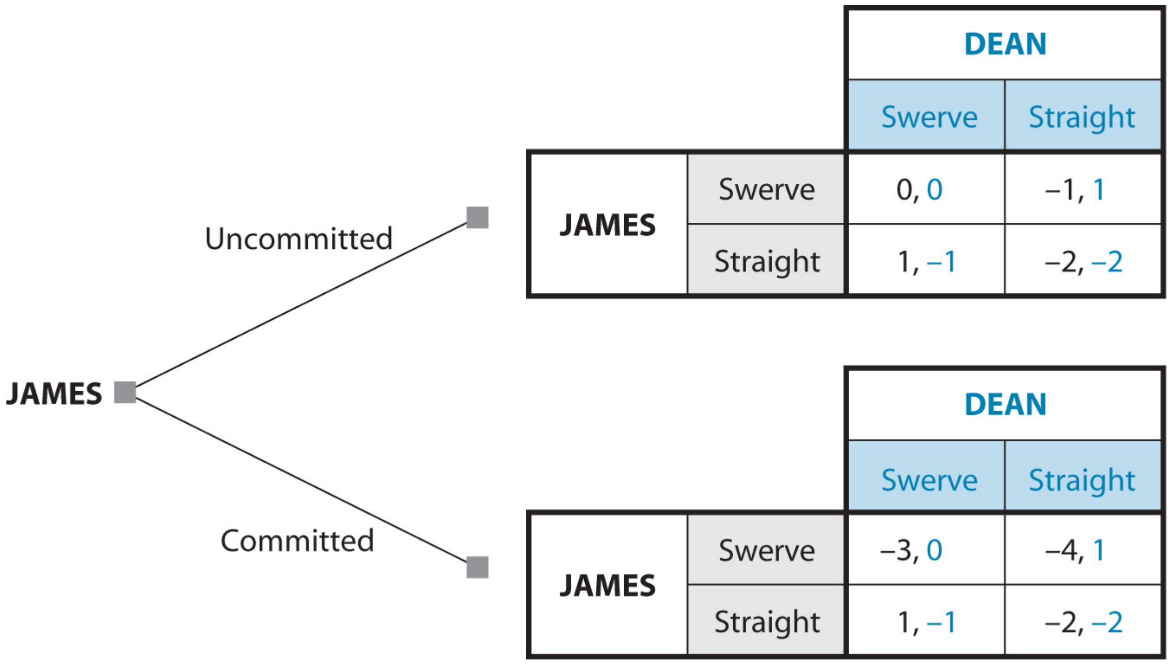


Figure 8.3 Chicken: Commitment by Changing Payoffs

More plausibly, if games of chicken are played every weekend, James can acquire a general reputation for toughness that acts as a guarantee of his action in any one game. In other words, James can alter his own payoff from swerving by subtracting an amount that represents the loss of his reputation. If this amount is large enough—say, 3—then the second-stage game when James has made the commitment has a different payoff table. The complete tree for this version of the game is shown in Figure 8.3.

Now, in the second stage with commitment, Straight has become truly optimal for James; in fact, it is his dominant strategy in that stage. Dean's optimal strategy is then Swerve. Looking ahead to this outcome at the first stage, James sees that he gets payoff 1 by making the commitment (changing his own stage 2 payoffs), while without the commitment, he cannot be sure of 1 and may do much worse. Thus, a rollback analysis shows that James should make the commitment.

Anyone can attempt to make a commitment. Success may depend both on the speed with which you can seize the first move and on the credibility with which you can make that move. If there are lags in observation, the two players may even make incompatible simultaneous commitments: Each disconnects his steering wheel and tosses it out the window just as he sees the other’s wheel come flying out, and the crash becomes unavoidable.

Even if one of the players can gain the advantage by making a commitment, the other player can defeat the first player’s attempt to do so. The second player could demonstrably remove his ability to “see” the other’s commitment—for example, by cutting off communication.

		STUDENT	
		Punctual	Late
TEACHER	Weak	4, 3	2, 4
	Tough	3, 2	1, 1

FIGURE 8.4 Payoff Table for Class Deadline Game

Games of chicken may be a 1950s anachronism, but our second example is perennial and familiar. In a class, the teacher’s enforcement of assignment deadlines can be Weak or Tough, and the students’ submissions can be Punctual or Late. Figure 8.4 shows this game in strategic form. The teacher does not like being tough; for her, the best outcome (with a payoff of 4) is that students are punctual even when she is weak; the worst (1) is that she is tough but students are still late. As for the two intermediate outcomes, she recognizes the importance of punctuality and ranks (Tough, Punctual) better than (Weak, Late). For the students, the best outcome is (Weak, Late), so that they can party all weekend without suffering any penalty for turning in their assignments late. (Tough, Late) is the worst for them, just as it is for the teacher. Of the intermediate outcomes, they prefer (Weak, Punctual) to (Tough, Punctual) because they have higher self-esteem if they can think that they acted punctually of their own volition rather than because of the threat of a penalty.[2]

If this game is played as a simultaneous-move game, or if the teacher moves second, Weak is dominant for the teacher, and the students choose Late. The equilibrium outcome is (Weak, Late), and the payoffs are (2, 4). But the teacher can achieve a better outcome by committing at the outset to the policy of Tough. We do not show a tree for this game, as we did in Figures 8.2 and 8.3, but that tree would be very similar to that for the preceding games of chicken, so we leave it for you to draw. Without a commitment, the second-stage game is as before, and the teacher gets a payoff of 2. However, when the teacher is committed to Tough, the students find it better to respond with Punctual at the second stage, and the teacher gets a payoff of 3. The teacher gains an advantage here by committing to tough enforcement, an action she would not normally take in simultaneous play. Making this strategic move changes the students’ expectations, and therefore their actions, in a way that benefits the teacher.

Once the students believe that the teacher is really committed to tough enforcement, they will choose to turn in their assignments punctually. If they test her resolve by being late, the teacher would like to make an exception for them, maybe with an excuse to herself, such as “just this once.” The existence of this temptation to shift away from a commitment is what makes its credibility problematic. Like any first move, the teacher’s commitment to toughness must be irreversible and observable to students before they make their own decisions whether to be punctual or late. The teacher must establish the ground rules of deadline enforcement right away, before any assignments are due, and the students must know these rules. In addition, the students must know that the teacher cannot, or at any rate will not, change her mind and make an exception for them. A teacher who leaves loopholes and provisions for incompletely specified emergencies is merely inviting imaginative excuses accompanied by fulsome apologies and assertions that “it won’t happen again.”

The teacher might also achieve credibility by hiding behind general university regulations, as this removes the Weak option from her set of available choices at stage 2. Or, as in the game of chicken, she might establish a reputation for toughness, changing the payoffs she would get from Weak by creating a sufficiently high cost of loss of reputation.

Endnotes

- We saw in Chapter 7, and will see again in Chapter 12, that the game has a third equilibrium, in mixed strategies, in which both players do quite poorly. Return to reference 1
- You may not regard these specific rankings of outcomes as applicable either to you or to your own teachers. We ask you to accept them for this example, whose main purpose is to convey some *general ideas* about commitment in a simple way. The same disclaimer applies to all the examples that follow. Return to reference 2

3 THREATS AND PROMISES

If you were free to do whatever you wanted, you would never follow through when making a threat or a promise, because, by definition, threats and promises specify responses that make you worse off. Nonetheless, threats and promises are powerful strategic tools that allow you to change others' behavior to your advantage. In this section, we provide three in-depth examples of threats and promises in action, along with some discussion of nuances associated with successfully implementing threats and promises in practice.

A. Making a Threat: U.S.–Japan Trade Relations

Our first example is based on economic relations between the United States and its trading partners in the decades after World War II. The United States believed in open markets, both as a general principle and because it hoped they would lead to faster economic growth in the free world and counter Soviet expansionism. Most other countries were *mercantilist*: They liked exports and disliked imports, wanting to restrict foreign access to their own markets, but wanting their producers to have unrestricted access to the large U.S. market. With only a slight caricature, we take Japan to be a representative of such countries at the time.

Figure 8.5 shows the payoff table for the U.S.–Japan trade game. For the United States, the best outcome (with a payoff of 4) comes when both countries' markets are open. This is partly because of its overall commitment to open markets and partly because of the benefits of trade with Japan itself: U.S. consumers get high-quality cars and consumer electronics, and U.S. producers can export their agricultural and high-tech products. Similarly, its worst outcome (payoff 1) occurs when both markets are closed. Of the two outcomes in which only one market is open, the United States prefers that its own market to be open because Japan's market is smaller, and loss of access to it is less important than the loss of access to Hondas and microchips.

		JAPAN	
		Open	Closed
UNITED STATES	Open	4, 3	3, 4
	Closed	2, 1	1, 2

FIGURE 8.5 Payoff Table for U.S.–Japan Trade Game

As for Japan, for the purpose of this example, we accept the protectionist, producer-oriented picture of "Japan, Inc." It gets its best outcome when the U.S. market is open and its own is closed; it gets its worst outcome when its own market is open and the U.S. market is closed; and of the other two outcomes, it prefers that both markets be open rather than that both be closed because its producers then have access to the much larger U.S. market.[3]

Note that each country has a dominant strategy: for the United States, an open market, and for Japan, a closed market. Thus, no matter what the order of moves—simultaneous, sequential with the United States first, or sequential with Japan first—the equilibrium outcome is (Open, Closed). Japan gets its best possible outcome (payoff 4) in this equilibrium and so has no need for strategic moves. The United States, however, gets only its second-best outcome (payoff 3). Can the United States employ a strategic move to get its best outcome in which both markets are open?

An unconditional commitment to open markets will not work to improve the U.S. payoff because Japan's best response will then be to keep its market closed. But suppose that the United States were to make the following threat: "We will close our market if you close yours." (As discussed previously, the explicit statement of only one half of a full conditional response rule generally implies a best-response action as the other half of the rule. In this case, the implied affirmation is, "We will keep our market open if you keep yours open." That action is the U.S. best response to the favored action of Open.) With the threat in place, the situation becomes the two-stage game shown in Figure 8.6. If the United States does not make the threat, the second stage is as before and leads to the equilibrium in which the U.S. market is open and it gets payoff 3, whereas Japan's market is closed and it gets payoff 4. If the United States does make the threat, then at the second stage, only Japan has freedom of choice. Along this branch of the tree, we show only Japan as an active player and write down the payoffs to the two parties: If Japan keeps its market closed, the United States will close its own, leading to payoff 1 for the United States and payoff 2 for

Japan. If Japan keeps its market open, then the United States will open its own, leading to payoff 4 for the United States and payoff 3 for Japan. Faced with (only) these two options, Japan will choose to open its market. Finally, reasoning backward to the very beginning of the two-stage game, the United States will choose to declare its threat, anticipating that Japan will respond by opening its market, and will get its best possible outcome.

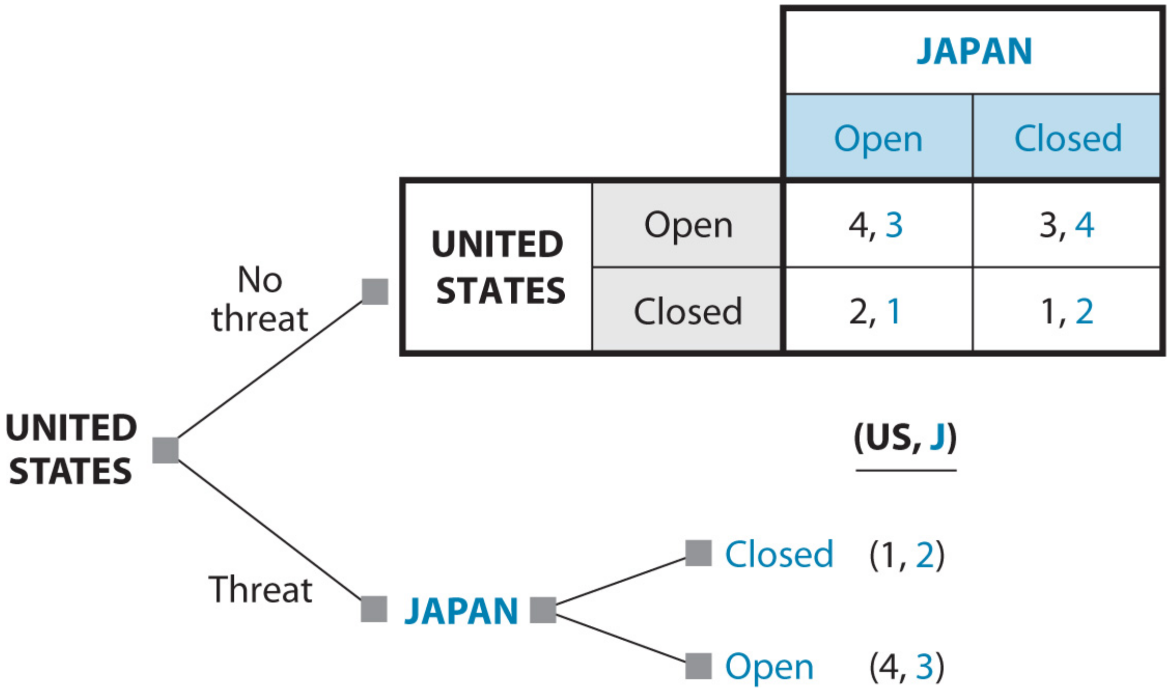


Figure 8.6 Game Tree for the U.S. – Japan Trade Game with Threat

---

Having described the mechanics of the threat, we now point out some of its important features, recapitulating and strengthening some points made earlier. First, notice that when the United States deploys its threat credibly, Japan doesn't follow its dominant strategy, Closed. Japan knows that the United States will take actions that depart from *its* dominant strategy and responds accordingly. Japan is looking at a choice between just two cells in the payoff table, the top-left and the bottom-right, and of those two, it prefers the former.

Next, notice that the credibility of the threat is problematic because, if Japan were to put it to the test by keeping its market closed, the United States would face the temptation to refrain from carrying out the threat. If the threatened action were the best U.S. response after the fact, there would have been no need to make the threat in advance (although the United States might have issued a *warning* just to make sure that Japan understood the situation). The strategic move has a special role exactly because it locks a player into doing something other than what it would have wanted to do after the fact.

How can the United States make its threat credible, then? One option is to enact a law that mandates the threatened action under the right circumstances. Such a law would remove the best-response action from the set of available choices at stage 2. Some reciprocity provisions in the World Trade Organization agreements have this effect, but the procedures involved are very slow and uncertain. Another possibility would be to delegate fulfillment of the threat to an agency such as the U.S. Commerce Department, which has been captured by U.S. producers who would like to keep U.S. markets closed so as to reduce the competitive pressure on their firms. This delegation would change the U.S. payoffs in the game—replacing the true U.S. payoffs with those of the Commerce Department—making the threatened action truly optimal. (A danger of this approach is that the Commerce Department might then retain a protectionist stance even if Japan opened its market. In that case, gaining credibility for the threat might cause the implied affirmation to become a *promise* that would then have to be made credible.)

Another key feature of a threat is that if it works to induce the other player to take the favored action, the player making the threat *does not have to carry out* the threatened action. Whether the threat is large or small is immaterial. All that matters is whether the threat is big enough to work. So it might seem that you should always use as big a threat as possible, just to make absolutely sure that the other player will have an incentive to comply. However, there is a danger in using supersized threats. Perhaps you have miscalculated or misunderstood the payoff structure, or maybe the threatened action could take place by mistake even if the other player performs the favored action, or perhaps the other player doesn't believe that you will actually follow through with such a mutually harmful action. For these reasons, it is wise to refrain from using threats that are more severe than necessary. For example, imagine that the United States were to threaten to pull out of its defensive alliances with Japan if Japan didn't buy its rice and semiconductors. Because the U.S.–Japan alliance is of such great military and geopolitical importance to the United States, many in Japan would doubt that the United States would ever follow through on such a threat, undermining its credibility and hence its effectiveness.

If the only available threat appears "too big," a player can reduce its size by making its fulfillment a matter of chance. A threat of this kind, which creates a risk, but not a certainty, of the bad outcome, is called *brinkmanship*. It is an extremely delicate and even dangerous variant of the conditional strategic move. We will discuss brinkmanship in greater detail in Section 5.B and illustrate its use during the Cuban missile crisis in Chapter 13.

Finally, observe that Japan gets a worse outcome when the United States deploys its threat than it would without the threat, so it would like to take strategic action to defeat or disable U.S. attempts to use the threat. For example, suppose its market is currently closed. Japan can agree to open its market in principle, but stall in practice, pleading unavoidable delays for assembling the necessary political consensus to legislate the market opening, then more delays for writing the necessary administrative regulations to implement the legislation, and so on. Because the United States does not want to go ahead with its threatened action, at each point it is tempted to accept the delay. Or Japan can claim that its domestic politics makes it difficult to open all markets fully; will the United States accept the outcome if Japan keeps just a few of its industries protected? It gradually expands this list, and at any point, the next small step is not enough cause for the United States to unleash a trade war. This device of defeating a threat by small steps, or "slice by slice," is called *salami tactics*. We will discuss this and other ways to undermine another’s strategic moves in Section 6 of this chapter.

		YVONNE’S BISTRO	
		20 (low)	26 (high)
XAVIER’S TAPAS	20 (low)	288, 288	360, 216
	26 (high)	216, 360	324, 324

FIGURE 8.7 Payoff Table for Restaurant Prisoners’ Dilemma (in Hundreds of Dollars per Month)

Our U.S.–Japan trade game example comes close to capturing the reality of the 1980s, when the United States was by far the dominant economy in the free world and, as leader, undertook the responsibility to prevent the world’s international trade system from falling into a spiral of protectionism like the one that caused so much harm in the Great Depression of the 1930s.[4] Now, with the economic rise of China and Europe, this leadership role has become much less important, and the United States has become more mercantilist. In the next edition of this book, we may have to re-work this example, reversing the roles!

B. Making a Promise: The Restaurant Pricing Game

Consider the restaurant pricing game of Chapter 5, simplified here to have only two possible prices: the jointly best price of \$26 or the Nash equilibrium price of \$20. We saw in Chapter 5 that this simplified version of the game is a prisoners' dilemma, in which both restaurants (Xavier's Tapas Bar and Yvonne's Bistro) have a dominant strategy—to price low—but both are better off when both price high. Each restaurant's profits can be calculated using the equations in Section 1 of Chapter 5; the results are shown in Figure 8.7. Without any strategic moves, the game has the usual equilibrium in dominant strategies in which both restaurants charge the low price of \$20, and both get lower profits than they would if they both charged the high price of \$26.

If either restaurant owner can make the credible promise, "I will charge a high price if you do" (and, implicitly, "I will charge a low price if you do"), the best joint outcome is achieved. For example, if Xavier makes the promise, then Yvonne knows that her choice of \$26 will be reciprocated, leading to the payoff shown in the lower-right cell of the table, and that her choice of \$20 will bring forth Xavier's best response—namely, matching her price of \$20—leading to the upper-left cell. Of the two payoffs, Yvonne prefers the first, and therefore chooses the high price.

The analysis can be done more properly by drawing a tree for the two-stage game in which Xavier has the choice of making or not making the promise at the first stage. We omit the tree, partly so that you can improve your understanding of the process by constructing it yourself and partly to show that such detailed analysis becomes unnecessary as one becomes familiar with the ideas involved.

The feasibility and credibility of Xavier’s promise is open to doubt. First, in order for Xavier to be able to respond to Yvonne’s move, he must arrange to move second in the pricing game, or Yvonne must arrange to move first. Assuming that the pricing game can be changed so that Yvonne moves first and Xavier moves second, Xavier’s promise will be feasible—but will it be credible? If Yvonne moves first and (irreversibly) sets her price high, Xavier will be tempted to renege on his promise and set his price low. Xavier must somehow convince Yvonne that he will not give in to the temptation to cheat—that is, to charge a low price when Yvonne charges a high price.

How can he do so? Perhaps Xavier can leave the pricing decision in the hands of a local manager, with clear written instructions to reciprocate with the high price if Yvonne charges the high price. Xavier can invite Yvonne to inspect these instructions, after which he can leave on a solo round-the-world sailing trip so that he cannot rescind them. (Even then, Yvonne may be doubtful—Xavier might secretly carry a telephone or a laptop computer on board.) This scenario is tantamount to removing the cheating action from the choices available to Xavier in the second-stage game.

Or Xavier can develop a reputation for keeping his promises, in business and in the community more generally. In a continuing interaction, the promise may work because reneging on the promise once could cause future cooperation to collapse. In essence, an ongoing relationship means splitting the game into smaller segments, in each of which the benefit of reneging is too small to justify the cost. In each such game, then, the payoff from cheating is altered by the cost of a collapse of future cooperation.[5]

Note that, as in the case of the U.S. threat to Japan, the promise “I will price high if you price high” is only half of Xavier’s response rule. The other half, as noted above, is “I will price low if you price low.” This statement is a *warning* (not a threat) because pricing low is already Xavier’s best response; it need not necessarily be made explicit.

Threats and promises differ in a crucially important way. When a threat succeeds in inducing the other player to take the favored action, the player making the threat does not have to carry out the threatened action and does not have to pay any price for having made the threat. On the other hand, when a promise is successful, the player making the promise always has to follow through and deliver what has been promised, which is costly. In the restaurant pricing example, the cost to Xavier of following through on his promise to match a high price is giving up the opportunity to undercut Yvonne and get the highest possible profit; in other instances where the promiser offers an actual gift or an inducement to the other player, the cost may be more tangible. A player making a promise therefore has a clear incentive to keep the cost of the promised action as small as possible.

		CHINA	
		Action	Inaction
UNITED STATES	Action	3, 3	2, 4
	Inaction	4, 1	1, 2

FIGURE 8.8 Payoff Table for U.S.–China Political Action Game

C. Making a Combined Threat and Promise: Joint U.S.–China Political Action

Finally, we consider an example in which a player benefits from making a combined threat and promise. In this example, the United States and China are contemplating whether to take action to compel North Korea to give up its nuclear weapons programs. Figure 8.8 is the payoff table for the United States and China when each must choose between action and inaction.

Each country would like the other to take on the whole burden of taking action against North Korea, so the top-right cell has the best payoff for China (4), and the bottom-left cell has the best payoff for the United States. The worst outcome (payoff 1) for the United States is the one in which no action is taken, because it finds the increased threat of nuclear war in that case to be unacceptable. For China, however, the worst outcome arises when it takes on the whole burden of action, because the costs of its action are so high. Both countries regard a joint action as the second-best outcome (payoff 3). The United States assigns a payoff of 2 to the situation in which it is the only country to act. And China assigns a payoff of 2 to the case in which no action is taken.

Without any strategic moves, this intervention game is dominance solvable. Inaction is the dominant strategy for China, and Action is the best response of the United States when China chooses Inaction. The equilibrium outcome is therefore the top-right cell, with payoffs of 2 for the United States and 4 for China. Because China gets its best outcome, it has no need for strategic moves. But the United States can try to do better.

Can the United States use a strategic move to improve on its equilibrium payoff—to get either its best or its second-best outcome rather than its second-worst? First, consider whether the United States can induce China to intervene alone in North Korea, the very best outcome for the United States (payoff 4). Unfortunately, no strategic move makes this possible. To see why, note that any strategic move ultimately gives China a choice between two outcomes, the one that follows from China choosing Action and the one that follows from China choosing Inaction. Intervening alone is China’s very worst outcome here (payoff 1), so there is no way to induce China to make that choice. When choosing between intervening alone and *any other outcome*, China will choose the other outcome.

Can the United States use a strategic move to achieve its second-best outcome, a joint intervention with China? Possibly, but only by declaring a response rule that combines a promise (“If you [China] intervene, then so will we [United States]”) and a threat (“If you don’t intervene, then neither will we”). This conditional strategic move, if credibly made, forces China to choose between the top-left cell (in which both intervene) and the bottom-right cell (in which neither intervenes) in Figure 8.8. Faced with this choice, China prefers to intervene, and the United States gets its second-best outcome.

Because the threat and the promise both entail taking actions that go against U.S. national interest, the United States has to make both the threat and the promise explicit and find a way to make them each credible. Usually such credibility has to be achieved by means of a treaty that covers the whole relationship, not just with agreements negotiated separately when each incident arises.

Endnotes

- Again, we ask you to accept this payoff structure as a vehicle for conveying some general ideas. You can experiment with the payoff tables to see what difference that would make to the roles of the two countries and the effectiveness of their strategic moves. Return to reference 3
- In Chapter 10, we offer some general analysis of how a big player can resolve a prisoners' dilemma through leadership. Return to reference 4
- In Chapter 10, we will investigate in greater detail the importance of repeated or ongoing relationships in attempts to reach the cooperative outcome in a prisoners' dilemma. Return to reference 5

4 SOME ADDITIONAL TOPICS

A. When Do Strategic Moves Help?

We have seen several examples in which a strategic move brings a better outcome to one player or another than the original game without such moves. What can be said in general about the desirability of such moves?

Making an unconditional strategic move—a commitment—is not always advantageous. This is especially obvious when the original game has a second-mover advantage. In that case, committing oneself to a move in advance is clearly a mistake, as doing so effectively makes one the first mover. In such situations, each player will do whatever he can to avoid being forced to make a commitment in the pregame.

Making a conditional strategic move, on the other hand, can never be disadvantageous. At the very worst, one can declare a response rule that would have been optimal after the fact. However, if such a move brings one an actual gain, it must be because one is choosing a response rule that in some eventualities specifies an action different from what one would find optimal at that later time. Thus, whenever a conditional strategic move brings a positive gain, it does so precisely when (one might say precisely because) its credibility is inherently questionable and must be achieved by some credibility device. We have mentioned some such devices in connection with each earlier example, and we will discuss the topic of achieving credibility in greater generality in the next section of this chapter.

What about the desirability of being on the receiving end of a strategic move? The answer depends on what type of strategic move is being made. It is never desirable to let another player threaten you. If a threat seems likely, you can gain by looking for a different kind of advance action—one that makes the threat less effective or less credible. We will consider some such actions in the final section of this chapter. It is, however, often desirable to let another player make promises to you. In fact, both players may benefit when one can make a credible promise, as in the prisoners' dilemma example of restaurant pricing earlier in this chapter, in which a promise achieved the cooperative outcome. Thus, it may be in the players' mutual interest to facilitate the making of promises in the pregame. Finally, it may or may not be desirable to let the other player make a commitment, as we saw in Chapter 6, Section 2.B. In the monetary–fiscal policy game shown in Figure 6.8, the Fed benefits when Congress is able to make a commitment; but in the game of chicken shown in Figure 6.7, each player gets their best outcome if they alone are able to make a commitment. Thus, in some cases you may benefit by enabling the other player to make a commitment, while in other cases you will be better off blocking his ability to do so.

B. Deterrence versus Compellence

In principle, a player making a threat or making a promise can use these moves to achieve either deterrence or compellence. Recall that deterrence entails sending the message, “Don’t take the default action,” and compellence entails sending the message, “Take the favored action.” These goals can seem very similar. Often there are only two possible actions, and not taking the default action is equivalent to taking the favored one. For example, a parent who wants a child who would normally take the default action of not studying to take the favored action of studying hard can promise a reward (a new mountain bike) for good performance in school or can threaten a punishment (a strict curfew the following term) if the performance is not sufficiently good. Similarly, a parent who wants a child not to let her room get messy, the child’s default action, can try either a reward (promise) or a punishment (threat) in order to get the child to take the favored action. Whether either of these moves is actually deterrent or compellent depends on how the move relates to some status quo in the interaction.

As a rule of thumb, the status quo in an upcoming game is defined by one of two things. The status quo describes either a tangible current state of affairs (the child’s bedroom is currently neat and tidy) or a player’s natural inclination to behave in a particular way (the child doesn’t like to study). Strategic moves that attempt to maintain the status quo are deterrent; those that attempt to change it are compellent. Thus, the promise to reward a child who keeps her room clean (or the threat to punish a child who lets her room get messy) is deterrent. And the promise to reward a child who studies (or the threat to punish a child who doesn’t) is compellent.

In practice, deterrence is better achieved by a threat and compellence by a promise, as we explain in more detail below. That means you might want to manipulate the status quo to your advantage if it is possible to do so. If you do not want to have to make the deterrent promise, “I will reward you for keeping your room clean,” because it isn’t obvious when you should follow through on your promise, you can wait until the child’s room is messy. (Presumably, you wouldn’t need to wait all that long.) Promising to reward the child for cleaning up a currently messy room is compellent and may be more easily put into effect.

Why is deterrence better achieved by making a threat? A deterrent threat can be passive—you don’t need to do anything so long as the other player doesn’t do what you are trying to deter. And it can be static—you don’t have to impose any time limit. Thus, you can set a trip wire and then leave things up to the other player. Consider again the parent who wants the child to keep her currently clean room in its pristine condition. The parent can threaten, “If you ever go to bed without cleaning your room, I will take away your screen privileges for a month.” Then the parent can sit back to wait and watch; only if the child acts contrary to the parent’s wishes does the parent have to act on her threat. Trying to achieve the same deterrence with a promise would require more complex monitoring and continual action: “At the end of each week in which I know that you kept your room tidy each day, I will give you $25.”

Compellence does not entail the passiveness of deterrence. Compellence must have a deadline or it is pointless—the other side can defeat your purpose by procrastinating or by eroding your threat in small steps (salami tactics). This makes a compellent threat harder to implement than a compellent promise. The parent who wants the child to study hard can simply say, “Each term at college that you get an average of B or better, I will give you $500.” The child will then take the initiative in showing the parent each time he has fulfilled the conditions. Trying to achieve the same thing by a threat— “Each term that your average falls below B, I will take away one of your electronic devices” —will require the parent to be much more vigilant and active. The child will postpone bringing home the grade report or will try to hide his devices.

We reiterate that you can change a threat into a promise, or deterrence into compellence, or vice versa, by changing the status quo. And you can use this change to your own advantage when making a strategic move. If you want to achieve compellence, try to choose a status quo such that what you do when the other player acts to comply with your demand becomes a reward, so that you are using a compellent promise. To give a rather dramatic example, a mugger can convert the threat, “If you don’t give me your wallet, I will take out my knife and cut your throat” into the promise, “Here is a knife at your throat; as soon as you give me your wallet I will take it away.” And if you want to achieve deterrence, try to choose a status quo such that, if the other player acts contrary to your wishes, what you do is a punishment, so that you are using a deterrent threat.

Glossary

credibility device
: A means by which a player acquires credibility, for instance, when declaring a promise or threat as part of a strategic move.

5 MAKING YOUR STRATEGIC MOVES CREDIBLE

We have emphasized the importance of the credibility of strategic moves throughout this chapter, and we have accompanied each example with some brief remarks about how credibility could be achieved in that particular context. Most devices for achieving credibility are indeed context specific, and there is a lot of art to discovering or developing such devices. Some general principles can help you organize your search.

We pointed out two broad approaches to credibility: (1) reducing your own future freedom of action in such a way that you have no choice but to carry out the action stipulated by your strategic move, and (2) changing your own future payoffs in such a way that it becomes optimal for you to do what you stipulate in your strategic move. We now elaborate some practical methods for implementing each of these approaches.

A. Reduce Your Freedom of Action

I. AUTOMATE YOUR RESPONSE Suppose that, in the pregame, you relinquish your ability to make decisions in the upcoming game and instead hand that authority over to a mechanical device, or similar procedure or mechanism, that is programmed to carry out your committed, threatened, or promised action under the appropriate circumstances. You demonstrate to the other player that you have done so. Then he will be convinced that you have no freedom to change your mind, and your strategic move will be credible (as long as he does not suspect that you control an override button to prevent a catastrophe!). The doomsday device, a nuclear explosive device that would detonate and contaminate the whole world's atmosphere if the enemy launched a nuclear attack, is the best-known example of this kind of mechanical device; it was popularized by the early 1960s movies *Fail Safe* and *Dr. Strangelove*.

II. DELEGATE YOUR DECISIONYou could delegate the power to act to another person, or agent, or to an organization that is required to follow certain pre-set rules or procedures. That is how the United States makes credible its threat to levy countervailing duties (retaliatory tariffs imposed to counter other countries' export subsidies). They are set by two agencies of the U.S. government—the Commerce Department and the International Trade Commission—whose operating procedures are laid down in the general trade laws of the country.

Your agent should not have his own objectives that defeat the purpose of your strategic move. For example, if one player delegates to an agent the task of inflicting threatened punishment, and the agent is a sadist who enjoys inflicting punishment, then he may act even when there is no reason to act—that is, even when the second player has complied. If the second player suspects this, then the threat loses its effectiveness, because his options amount to "damned if you do and damned if you don't."

As with automated response mechanisms like the doomsday device, delegation may not provide a complete guarantee of credibility because mandates can always be altered. In fact, the U.S. government has often set aside countervailing duties and reached other forms of agreement with other countries so as to prevent costly trade wars.

III. BURN YOUR BRIDGESMany invaders, from Xenophon in ancient Greece to William the Conqueror in England to Cortés in Mexico, are supposed to have deliberately cut off their own army's avenue of retreat to ensure that it would fight hard. Some of them literally burned bridges behind them, while others burned ships, but the device has become a cliché. Its most recent users in a military context may have been the Japanese kamikaze pilots in World War II, who carried only enough fuel to reach the U.S. naval ships into which they were to ram their airplanes. The principle even appears in the earliest known treatise on war, in a commentary attributed to Prince Fu Ch'ai: "Wild beasts, when they are at bay, fight desperately. How much more is this true of men! If they know there is no alternative they will fight to the death." [6]

Related devices are used in other high-stakes games. Although the member countries of Europe's Economic and Monetary Union (EMU) could have retained separate currencies and merely fixed the exchange rates among them, they adopted a common currency—the euro—precisely to make the process irreversible and thereby give themselves a much greater incentive to make the union a success. (In fact, it was the extent of the necessary commitment that kept some nations, Great Britain in particular, from agreeing to be part of the

EMU.) It would not be totally impossible to abandon the euro and go back to separate national currencies—but it would be very costly. And indeed, plans to leave the eurozone have been seriously discussed—but never implemented—in several member countries, most notably Greece, over the last decade. Like that of automated devices, the credibility of burning bridges is not an all-or-nothing matter, but one of degree.

IV. CUT OFF COMMUNICATIONIf you send the other player a message declaring your commitment to a particular action and at the same time cut off any means for her to communicate with you, then she cannot argue or bargain with you to reverse your action. The danger in cutting off communication is that if both players do so simultaneously, they may make mutually incompatible commitments that can cause great mutual harm. Additionally, cutting off communication is harder to do with a conditional strategic move, because you have to remain open to the one message that tells you whether the other player has complied and therefore whether you need to carry out your threat or promise. In this high-tech age, it is also quite difficult for a person to cut herself off from all contact.

But large teams or organizations can try variants of this device. Consider a labor union that makes its decisions at mass meetings of members. To convene such a meeting takes a lot of planning—reserving a hall, communicating with members, and so forth—and several weeks of time. A meeting is convened to decide on a wage demand. If management does not meet the demand in full, the union leadership is authorized to call a strike, and then it must call a new mass meeting to consider any counteroffer. This process puts management under a lot of time pressure in the bargaining; it knows that the union will not be open to communication for several weeks at a time. Here, we see that cutting off communication for extended periods can establish some degree of credibility, but not absolute credibility. The union’s device does not make communication totally impossible; it only creates several weeks of delay.

B. Change Your Payoffs

I. ESTABLISH A REPUTATION You can change your payoffs by acquiring a reputation for carrying out threats and delivering on promises. Such a reputation is most useful in a repeated game against the same player. It is also useful when playing different games against different players if each of those players can observe your actions in the games that you play with others. The circumstances favorable to the emergence of such a reputation are the same as those for achieving cooperation in the prisoners' dilemma, as we will see in Chapter 10, and for the same reasons. The greater the likelihood that the interaction will continue, and the greater the concern for the future relative to the present, the more likely the players will be to resist current temptations for the sake of future gains. The players will therefore be more willing to acquire and maintain reputations.

In technical terms, reputation is a device that links different games so that the payoffs of actions in one game are altered by the prospects of repercussions in other games. If you fail to carry out your threat or promise in one game, your reputation suffers, and you get a lower payoff in other games. Therefore, when you consider any one of these games, you should adjust your payoffs in that game to take into consideration such repercussions on your payoffs in the linked games.

The benefit of reputation in ongoing relationships explains why your regular car mechanic is less likely to cheat you by doing an unnecessary or excessively costly or shoddy repair than is a random garage that you go to in an emergency. But what would your regular mechanic actually stand to gain from acquiring this reputation if competition forced him to charge a price so low that he would no profit on any transaction? You pay indirectly for his integrity when he fixes your car—you have to be willing to let him charge you a little bit more than the rates that the cheapest garage in the area might advertise. The same reasoning explains why, when you are away from home, you might settle for the known quality of a restaurant chain instead of taking the risk of going to an unknown local restaurant. And a department store that expands into a new line of merchandise can use the reputation that it has acquired for its existing lines to promise its customers the same high quality in the new line.

In games where credible promises by one or both parties can bring mutual benefit, the players can cooperate in fostering the development of their reputations. But if the interaction ends at a known, specific time, there is always the problem of the endgame.

In the Middle East peace process that started in 1993 with the Oslo Accords, the early steps, in which Israel transferred some control over Gaza and small isolated areas of the West Bank to the Palestinian Authority and the latter accepted the existence of Israel and reduced its anti-Israel rhetoric and violence, continued well for a while. But as the final stages of the process approached, neither side trusted the other to deliver on its promises, and by 1998 the process stalled. Sufficiently attractive rewards could have come from the outside; for example, the United States or Europe could have promised both parties economic aid or expanded commerce to keep the process going. The United States offered Egypt and Israel large amounts of aid in this way to achieve the Camp David Accords in 1978. But such promises were not made in the more recent situation, and at the date of this writing, prospects for progress do not look bright.

II. DIVIDE THE GAME INTO SMALL STEPSSometimes a single game can be divided into a sequence of smaller games, thereby allowing reputation effects to come into play. In home construction projects, it is customary for the homeowner to pay the contractor in installments as the work progresses. In the Middle East peace process, Israel would never have agreed to a complete transfer of the West Bank to the Palestinian Authority in one fell swoop in return for a single promise to recognize Israel and stop its attacks. Proceeding in steps enabled the process to go at least part of the way. But this case again illustrates the difficulty of sustaining momentum as the endgame approaches.

III. USE TEAMWORKTeamwork is yet another way to embed one game in a larger game to enhance the credibility of strategic moves. It requires a group of players to monitor one another. If one fails to carry out a threat or a promise, the others are required to inflict punishment on him; failure to do so makes them, in turn, vulnerable to similar punishment by the others, and so on. Thus, a player’s payoffs in the larger game are altered in a way that makes it credible that each individual member’s actions will conform to the team’s norms of behavior.

Many universities have academic honor codes that act as credibility devices for students. Examinations are not proctored by the faculty; instead, students are required to report to a student committee if they see any cheating. That committee holds a hearing and hands out punishment, as severe as suspension for a year or outright expulsion, if it finds the accused student guilty of cheating. Students are very reluctant to place their fellow students in such jeopardy. To stiffen their resolve, such codes include the added twist that failure to report an observed infraction is itself an offense against the code. Even then, the general belief is that these codes work only imperfectly. A survey of 417 undergraduates at Princeton University in 2009 found that “one of every five respondents admitted to violating a professor’s rule for take-home assignments,” and that “of the 85 students who said they had become aware of another student violating the Honor Code, only four said they reported the infraction." [7]

IV. APPEAR IRRATIONALYour threat may lack credibility if other players know you are rational and that it is too costly for you to follow through with your threatened action. Therefore, those other players may believe that you will not carry out the threatened action if you are put to the test. You can counter this problem by appearing to be irrational so that they will believe that your payoffs are different from what they originally perceived. Apparent irrationality can thus turn into strategic rationality when the credibility of a threat is in question. Similarly, apparently irrational motives, such as honor or saving face, may make it credible that you will deliver on a promise even when tempted to renege.

The other player may see through such rational irrationality. Therefore, if you attempt to make your threat credible by claiming irrationality, he will not readily believe you. You will have to acquire a reputation for irrationality—for example, by acting irrationally in some related game. You could also use one of the strategies that we will discuss in Chapter 9 and do something that is a credible *signal* of irrationality (or delegate your decision to someone who is truly crazy) to achieve an equilibrium in which you can separate from the falsely irrational.

V. WRITE A CONTRACTYou can make it costly to yourself to fail to carry out a threat or to deliver on a promise by signing a contract under which you have to pay a sufficiently large sum in that eventuality. If such a contract is written with sufficient clarity that it can be enforced by a court or some outside authority, the resulting change in payoffs makes it optimal for you to carry out the stipulated action, and your threat or the promise becomes credible.

In the case of a promise, the other player can be the other party to the contract. It is in his interest that you deliver on the promise, so he will hold you to the contract if you fail to fulfill the promise. A contract to enforce a threat is more problematic. The other player does not want you to carry out the threatened action and will not want to enforce the contract, unless he gets some longer-term benefit in associated games by being subjected to a credible threat in this one. To implement a threat, the contract therefore has to include a third party. But when you bring in a third party and a contract merely to ensure that you will carry out your threat if put to the test, the third party does not actually benefit from your failure to act as stipulated. The contract thus becomes vulnerable to any renegotiation that would provide the third-party enforcer with some positive benefit. If the other player puts you to the test, you can say to the third party, “Look, I don’t want to carry out the threat. But I am being forced to do so by the prospect of the penalty in the contract, and you are not getting anything out of all this. Here is a sum of money in exchange for releasing me from the contract.” Thus, the contract itself is not credible; therefore, neither is the threat. The third party must have its own longer-term reasons for holding you to the contract, such as wanting to maintain its reputation, if the contract is to be renegotiation-proof and therefore credible.

Written contracts are usually more binding than verbal ones, but even verbal ones may constitute commitments. When George H. W. Bush said, “Read my lips; no new taxes” in the presidential campaign of 1988, the American public took this promise to be a binding contract; when Bush reneged on it in 1990, the public held that against him in the election of 1992.

VI. USE BRINKMANSHIPIn the U.S.–Japan trade game, we found that a threat can be too big to be credible. If a smaller but effective threat cannot be found in a natural way, the size of the large threat can be reduced to a credible level by making its fulfillment a matter of chance. The United States may not credibly be able to say to Japan, “If you don’t keep your markets open to U.S. goods, we will not defend you if the Russians or the Chinese attack you.” But it can credibly say, “If you don’t keep your markets open to U.S. goods, the relations between our countries will deteriorate, which will create the risk that, if you are faced with an invasion, Congress at that time will not sanction U.S. military involvement in defending you.” As mentioned earlier, such deliberate creation of risk is called brinkmanship. This is a subtle idea and difficult to put into practice. Brinkmanship is best understood by seeing it in operation, and the detailed case study of the Cuban missile crisis in Chapter 13 serves just that purpose.

In this section, we have described several devices for making one’s strategic moves credible and examined how well they work. In conclusion, we want to emphasize a point that runs through the entire discussion: Credibility in practice is not an all-or-nothing matter, but one of degree. Even though the theory is stark—rollback analysis shows either that a threat works or that it does not—the practical application of strategic moves must recognize that between these polar extremes lies a whole spectrum of possibility and probability.

Endnotes

- Sun Tzu, *The Art of War*, trans. Samuel B. Griffith (Oxford: Oxford University Press, 1963), p. 110. Return to reference 6
- The referenced survey was conducted by the *Daily Princetonian*, Princeton University's student-run newspaper. See Michelle Wu and Jack Ackerman, "In honor we trust?" *Daily Princetonian*, April 30, 2009, http://www.dailyprincetonian.com/article/2009/04/in-honor-we-trust, and "Editorial: We don't second that," *Daily Princetonian*, May 7, 2009, http://www.dailyprincetonian.com/article/2009/05/editorial-we-dont-second-that. Return to reference 7

Glossary

doomsday device
: An automaton that will under specified circumstances generate an outcome that is very bad for all players. Used for giving credibility to a severe threat.

reputation
: Relying on the effect on payoffs in future or related games to make threats or promises credible, when they would not have been credible in a one-off or isolated game.

rational irrationality
: Adopting a strategy that is not optimal after the fact, but serves a rational strategic purpose of lending credibility to a threat or a promise.

contract
: In this context, a way of achieving credibility for one's strategic move by entering into a legal obligation to perform the committed, threatened, or promised action in the specified contingency.

6 COUNTERING YOUR OPPONENT’S STRATEGIC MOVES

If your opponent can make a commitment or a threat that works to your disadvantage, then, before he actually does so, you may be able to make a strategic countermove of your own. You can do so by making his future strategic move less effective—for example, by removing its irreversibility or undermining its credibility. In this section, we examine some devices that can help achieve this purpose. Some are similar to the devices that the other side can use for its own purposes.

A. Appear Irrational

Apparent irrationality can work for the intended recipient of a commitment or a threat just as well as it does for the maker. If you are known to be so irrational that you will not give in to any threat and will accept the damage that befalls you when your opponent carries out that threat, then he may as well not make the threat in the first place, because having to carry it out will only end up hurting him, too. Everything that we said earlier about the difficulties of credibly convincing the other side of your irrationality holds true here as well.

B. Cut Off Communication

If you make it impossible for your opponent to convey to you the message that she has made a certain commitment or a threat, then your opponent will see no point in doing so. Thomas Schelling illustrates this possibility with the story of a child who is crying too loudly to hear his parent's threats.[8] Thus, it is pointless for the parent to make any strategic moves; communication has effectively been cut off.

C. Leave Escape Routes Open

If your opponent can benefit by burning bridges to prevent his own retreat, you can benefit by dousing those fires, or perhaps even by constructing new bridges or roads by which your opponent can retreat. This device was also known to the ancients. Sun Tzu said, “To a surrounded enemy, you must leave a way of escape.” The intent is not actually to allow the enemy to escape. Rather, “show him there is a road to safety, and so create in his mind the idea that there is an alternative to death. Then strike.”[9]

D. Undermine Your Opponent’s Motive to Uphold His Reputation

If the person threatening you says, “Look, I don’t want to carry out this threat, but I must because I want to maintain my reputation with others,” you can respond, “It is not in my interest to publicize the fact that you did not punish me. I am only interested in doing well in this game. I will keep quiet; both of us will avoid the mutually damaging outcome; and your reputation with others will stay intact.” Similarly, if you are a buyer bargaining with a seller and he refuses to lower his price on the grounds that “if I do this for you, I would have to do it for everyone else,” you can point out that you are not going to tell anyone else. This response may not work; the other player may suspect that you would tell a few friends, who would tell a few others, and so on.

E. Use Salami Tactics

Salami tactics are devices used to whittle down the other player’s threat in the way that a salami is cut—one slice at a time. You fail to comply with the other player’s wishes (whether for deterrence or compellence) to such a small degree that it is not worth the other player’s while to carry out the comparatively drastic and mutually harmful threatened action just to counter that small transgression. If that works, you transgress a little more, and a little more again, and so on.

You know this device perfectly well from your own childhood. Thomas C. Schelling[10] gives a wonderful description of the process:

> Salami tactics, we can be sure, were invented by a child. . . . Tell a child not to go in the water and he’ll sit on the bank and submerge his bare feet; he is not yet “in” the water. Acquiesce, and he’ll stand up; no more of him is in the water than before. Think it over, and he’ll start wading, not going any deeper. Take a moment to decide whether this is different and he’ll go a little deeper, arguing that since he goes back and forth it all averages out. Pretty soon we are calling to him not to swim out of sight, wondering whatever happened to all our discipline.

Salami tactics work particularly well against compellence because they can take advantage of the *time* dimension. When your parent tells you to clean up your room “or else,” you can put off the task for an extra hour by claiming that you have to finish your homework, then for a half day because you have to go to football practice, then for an evening because you can’t possibly miss *The Simpsons* on TV, and so on.

To counter the countermove of salami tactics, you must make a correspondingly graduated threat. There should be a scale of punishments that fits the scale of noncompliance or procrastination. This can also be achieved by gradually raising the risk of disaster, another application of brinkmanship.

Endnotes

- Thomas C. Schelling, *The Strategy of Conflict* (Oxford: Oxford University Press, 1960), p. 146. Return to reference 8
- Sun Tzu, *The Art of War*, pp. 109–10. Return to reference 9
- Thomas C. Schelling, *Arms and Influence* (New Haven, Conn.: Yale University Press, 1966), pp. 66–67. Return to reference 10

Glossary

salami tactics
: A method of defusing threats by taking a succession of actions, each sufficiently small to make it nonoptimal for the other player to carry out his threat.

SUMMARY

Actions taken by players to alter the outcome of an upcoming game in their favor are known as *strategic moves*. These pregame moves must be *observable* and *irreversible* to act as first moves. They must also be *feasible* within the rules and order of play in the upcoming game, and they must be credible if they are to have their desired effect of altering the equilibrium outcome of the game. *Commitment* is an unconditional strategic move used to seize a first-mover advantage when one exists. Such a move usually entails committing to a strategy that would not have been one’s equilibrium strategy in the original version of the game.

Conditional strategic moves entail declaring a response rule specifying one’s response to another player’s *default action* and *favored action*. A *threat* specifies a non-best response to the other player’s default action, while a *promise* specifies a non-best response to the other player’s favored action. When making a threat or a promise, players generally make only the threat or promise explicit, leaving the response to the other action implicit because it is a best response. The implied best response is called a *warning* (if it is a best response to the default action) or an *affirmation* (if it is a best response to the favored action).

Strategic moves may be designed either to *deter* rivals’ actions and preserve the status quo or to *compel* rivals’ actions and alter the status quo. Threats carry the possibility of mutual harm, but cost nothing if they work. Promises are costly only to the maker, but only if they are successful. Threats can be arbitrarily large, although excessive size compromises credibility, but promises are usually kept just large enough to be effective. If the implicit affirmation (or warning) that accompanies a threat (or promise) is not credible, players must *make a combined* *threat and promise* and see to it that both components are credible.

Credibility must be established for any strategic move. There are a number of general principles to consider in making moves credible and a number of specific *credibility devices* that can be used to acquire credibility. They generally work either by reducing the maker's future freedom to choose or by altering the maker's payoffs from future actions. Specific devices of this kind include establishing a *reputation*, using teamwork, demonstrating apparent irrationality, burning bridges, and making *contracts*, although the acquisition of credibility is often context specific. Similar devices exist for countering strategic moves made by rival players.

KEY TERMS

affirmation (270)

commitment (269)

compellence (273)

contract (291)

credibility device (285)

default action (270)

deterrence (273)

doomsday device (287)

favored action (270)

feasibility (268)

promise (270)

rational irrationality (291)

reputation (289)

response rule (269)

salami tactics (294)

strategic move (267)

threat (270)

warning (270)

Glossary

strategic move
: Action taken at a pregame stage that changes the strategies or the payoffs of the subsequent game (thereby changing its outcome in favor of the player making the move).

feasibility
: Possibility within the (unchangeable) physical and/or procedural restrictions that apply in a given game.

commitment
: An action taken at a pregame stage, stating what action you would take unconditionally in the game to follow.

response rule
: A rule that specifies how you will act in response to various actions of other players.

favored action
: The action that a player making a *strategic move* wants the other player to take, as opposed to the *default action*.

default action
: In the context of strategic moves, the action that the other player (the player not making a strategic move) will take in the absence of a strategic move, as opposed to the *favored action*.

promise
: A response to the *favored action* that benefits the other player and that is not a best response, as specified within a *strategic move*. It is only feasibly made by the second mover in a game. Credibility is required because the specified action is not a best response. The strategic move referred to as "making a promise" entails declaring both a promise and a warning, whereas "making a combined threat and promise" entails declaring both a promise and a threat.

threat

A response to the default action that harms the other player and that is not a best response, as specified within a strategic move. It can only feasibly be made by the second mover in a game. Credibility is required because the specified action is not a best response. The strategic move referred to as “making a threat” entails declaring both a threat and an affirmation, whereas “making a combined threat and promise” entails declaring both a threat and a promise.

affirmation

A response to the *favored action* that is a best response, as specified within a strategic move. Only the second mover in a game can feasibly make an affirmation. However, credibility is not required since the specified action is already the player’s best response. The strategic move referred to as “making a threat” entails declaring both a threat and an affirmation.

warning

A response to the default action that is a best response, as specified within a strategic move. Only the second mover in a game can feasibly make a warning. However, credibility is not required since the specified action is already the player’s best response. The strategic move referred to as “making a promise” entails declaring both a promise and a warning.

deterrence

An attempt to induce the other player(s) to act to maintain the status quo.

compellence

An attempt to induce the other player(s) to act to change the status quo in a specified manner.

credibility device

A means by which a player acquires credibility, for instance, when declaring a promise or threat as part of a strategic move.

doomsday device

An automaton that will under specified circumstances generate an outcome that is very bad for all players. Used for giving credibility to a severe threat.

[reputation]

Relying on the effect on payoffs in future or related games to make threats or promises credible, when they would not have been credible in a one-off or isolated game.

[rational irrationality]

Adopting a strategy that is not optimal after the fact, but serves a rational strategic purpose of lending credibility to a threat or a promise.

[contract]

In this context, a way of achieving credibility for one's strategic move by entering into a legal obligation to perform the committed, threatened, or promised action in the specified contingency.

[salami tactics]

A method of defusing threats by taking a succession of actions, each sufficiently small to make it nonoptimal for the other player to carry out his threat.

SOLVED EXERCISES

1. "One could argue that the size of a promise is naturally bounded, while in principle a threat can be arbitrarily severe so long as it is credible (and error free)." First, briefly explain why the statement is true. Despite the truth of the statement, players might find that an arbitrarily severe threat might not be to their advantage. Explain why the latter statement is also true.
2. For each of the following three games, answer these questions:
   1. What is the equilibrium if neither player can use any strategic moves?
   2. Can one player improve his payoff by using a strategic move [commitment, threat (with affirmation), promise (with warning), or combined threat and promise]? If so, which player makes what strategic move?

1.

		COLUMN	
		Left	Right
ROW	Up	0, 0	2, 1
	Down	1, 2	0, 0

You may need to scroll left and right to see the full figure.

2.

		COLUMN	
		Left	Right
ROW	Up	4, 3	3, 4
	Down	2, 1	1, 2

You may need to scroll left and right to see the full figure.

3.

		COLUMN	
		Left	Right

You may need to scroll left and right to see the full figure.

		COLUMN	
		Left	Right
ROW	Up	4, 1	2, 2
	Down	3, 3	1, 4

You may need to scroll left and right to see the full figure.

---

3. In the classic film *Mary Poppins*, the Banks children are players in a strategic game with a number of different nannies. In their view of the world, nannies are inherently harsh, and playing tricks on nannies is great fun. That is, they view themselves as playing a game in which the nanny moves first, showing herself to be either Harsh or Nice, and the children move second, choosing to be either Good or Mischievous. A nanny prefers to have Good children to take care of, but is also inherently Harsh, so she gets her highest payoff of 4 from (Harsh, Good) and her lowest payoff of 1 from (Nice, Mischievous), with (Nice, Good) yielding 3 and (Harsh, Mischievous) yielding 2. The children similarly most prefer to have a Nice nanny and to be Mischievous; they get their highest two payoffs when the nanny is Nice (4 if Mischievous, 3 if Good) and their lowest two payoffs when the nanny is Harsh (2 if Mischievous, 1 if Good).
   1. Draw the game tree for this game and find the subgame-perfect equilibrium in the absence of any strategic moves.
   2. In the film, before the arrival of Mary Poppins, the children write their own ad for a new nanny in which they state, “If you won’t scold and dominate us, we will never give you cause to hate us; we won’t hide your spectacles so you can’t see, put toads in your bed, or pepper in your tea.” Use the tree from part (a) to argue that this statement constitutes a promise. What would the outcome of the game be if the children keep their promise?
   3. What is the implied warning that goes with the promise in part (b)? Does the warning need to be made credible? Explain your answer.
   4. How could the children make the promise in part (b) credible?
   5. Is the promise in part (b) compellent or deterrent? Explain your answer by referring to the status quo in the game—namely, what would happen in the absence of the strategic move.

4. The following exercise is an interpretation of the rivalry between the United States and the Soviet Union for geopolitical influence during the 1970s and 1980s.[11] Each side has the choice of two strategies: Aggressive and Restrained. The Soviet Union wants to achieve world domination, so being Aggressive is its dominant strategy. The United States wants to prevent the Soviet Union from achieving world domination; it will match Soviet aggressiveness with aggressiveness, and restraint with restraint. Specifically, the payoff table is

		SOVIET UNION	
		Restrained	Aggressive
UNITED STATES	Restrained	4, 3	1, 4
	Aggressive	3, 1	2, 2

You may need to scroll left and right to see the full figure.

For each player, 4 is best and 1 is worst.

   1. Consider this game when the two countries move simultaneously. Find the Nash equilibrium.
   2. Next, consider three different and alternative ways in which the game could be played with sequential moves:
      1. The United States moves first, and the Soviet Union moves second.
      2. The Soviet Union moves first, and the United States moves second.
      3. The Soviet Union moves first, and the United States moves second, but the Soviet Union has a further move in which it can change its first move.

      For each case, draw the game tree and find the subgame-perfect equilibrium.

   3. What are the key strategic considerations for the two countries?
5. At a United Nations meeting during the Cuban missile crisis of October 1962, the U.S. ambassador to the U.N., Adlai Stevenson, challenged the Soviet ambassador, Valerian Zorin. Stevenson asked, “Do you . . . deny that the USSR has placed and is

placing . . . missiles and sites in Cuba?" Zorin replied, "I am not in an American courtroom, sir, and therefore I do not wish to reply." Stevenson retorted, "You are in the courtroom of world opinion right now. Yes or no? . . . I am prepared to wait for my answer until hell freezes over."

Comment on this interaction with reference to the theory of strategic moves. Consider specifically what Stevenson was trying to accomplish, what strategic move he was attempting to use, and whether the move was likely to work to achieve Stevenson's preferred outcome.

6. Tabloid favorite Kim Kardashian has to decide one night whether to party at L.A.'s hottest dance club. A paparazzo has to decide whether to stalk the club that night in the hope of taking a photograph of her. Kim wants most to party, but would also prefer not to be bothered by the paparazzo. The paparazzo wants to be at the club to take a photograph if Kim shows up, but would otherwise prefer to go elsewhere. The ordinal payoff matrix for this game is shown below.

		PAPARAZZO	
		Stalk	Don't stalk
KIM KARDASHIAN	Party	3, 4	4, 2
	Don't party	1, 1	1, 2

You may need to scroll left and right to see the full figure.

1. Identify all pure-strategy Nash equilibria of this game (or explain why no such equilibrium exists) in the case when Kim and the paparazzo move simultaneously.
2. Draw the game tree for the case when the paparazzo moves first. What are the rollback equilibrium strategies and outcome?
3. In the case when the paparazzo moves first, is it possible for Kim to achieve her best possible outcome by credibly declaring a strategic move in the pregame? If not, why not? And if so, what sort of strategic move would Kim use, and how might she phrase her declaration?

7. North Korea's leader, Kim Jong-un, must decide whether to keep or dismantle North Korea's nuclear weapons, while China must decide whether or not to provide economic aid to North Korea. China wants most for North Korea to dismantle, in order to avoid having a nuclear-armed neighbor, and has a dominant strategy, to provide aid, in order to prevent a humanitarian crisis on its southern border. For his part, Kim wants most to receive aid, but has a dominant strategy, Don't dismantle, in order to use his nukes to extract more concessions in the future. The ordinal payoff matrix for this game is shown below.

			CHINA
		Aid	Don't aid
NORTH KOREA [Kim Jong-un]	Dismantle	3, 4	1, 3
	Don't dismantle	4, 2	2, 1

---

1. Identify all pure-strategy Nash equilibria of this game (or explain why no such equilibrium exists) in the case when North Korea and China move simultaneously.
2. Draw the game tree for the case when North Korea moves first. What are the rollback equilibrium strategies and outcome?
3. In the case when North Korea moves first, is it possible for China to achieve its best possible outcome by credibly declaring a strategic move in the pregame? If not, why not? And if so, what sort of strategic move would China use, and how might it phrase its declaration?

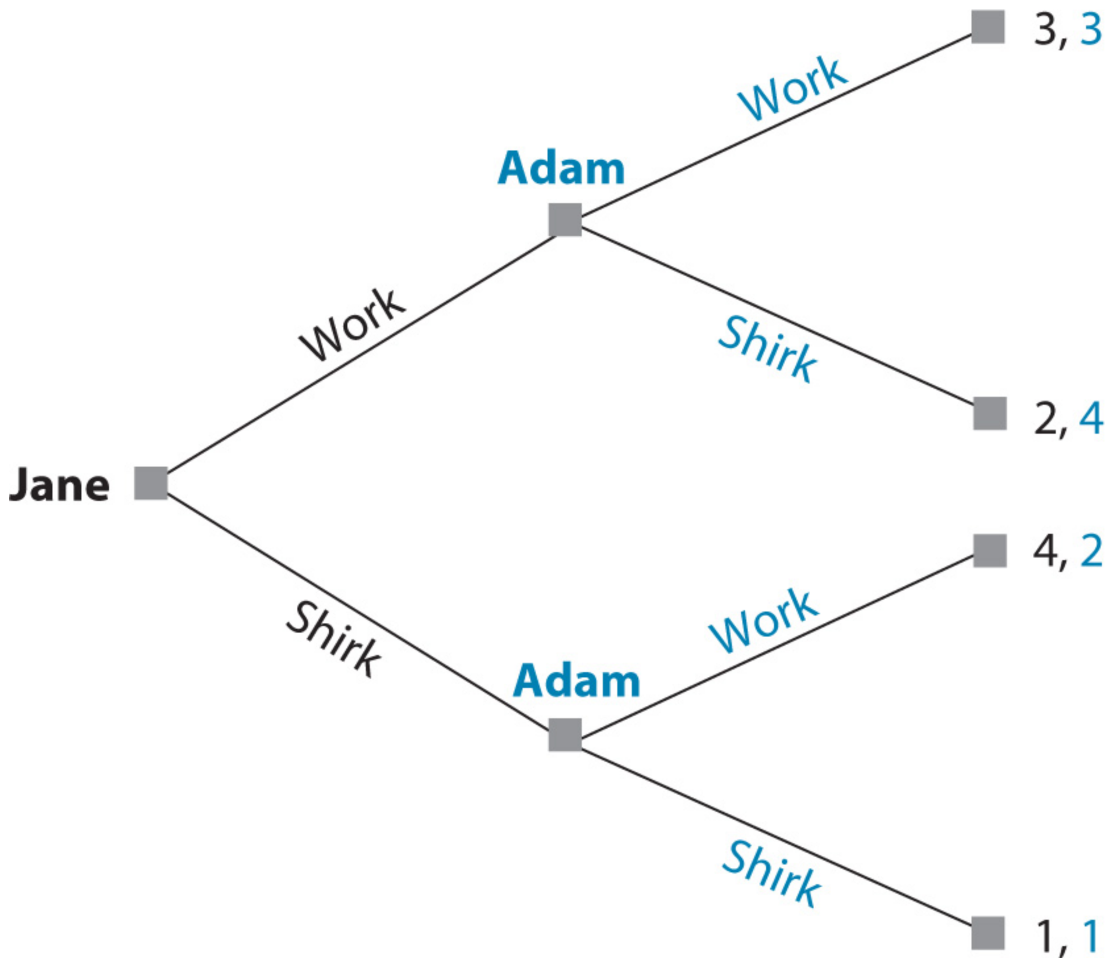
8. Dear Old Dad has just waved goodbye to his son, Student, who has enrolled at a prestigious private college, despite Dad's concern that the tuition is too much for the family to pay. Now at school, Student must decide whether to find a job or party during his free time, while Dad must decide whether to contribute toward Student's tuition (Pay) or save for retirement. The ordinal payoff matrix for this game is shown below. Student prefers to party, but most of all, wants Dad to pay. In particular, Student is willing to find a job if doing so will cause Dad to pay. For Dad, the best outcome is that in which Student finds a job and Dad can save, but if Student should choose to party, Dad would prefer to pay so that his son is not left with a ruinous debt-load. Further, should he pay, Dad prefers that his son find a job.

		DEAR OLD DAD	
		Pay	Save
STUDENT	Party	4, 2	2, 1
	Job	3, 3	1, 4

---

1. Identify all pure-strategy Nash equilibria of this game (or explain why no such equilibrium exists) in the case when Student and Dad move simultaneously.
2. Draw the game tree for the case when Student moves first. What are the rollback equilibrium strategies and outcome?
3. In the case when Student moves first, is it possible for Dad to achieve an outcome that is better than the rollback equilibrium outcome by credibly declaring a strategic move in the pregame? If not, why not? And if so, what sort of strategic move would Dad use, and how might he phrase his declaration?

9. Jane and Adam are teammates on a homework assignment that must be completed by noon tomorrow. It is currently midnight, and Adam is already asleep. He is an "early bird" who will wake up at 7:00 a.m., while Jane is a "night owl" who will stay awake until 5:00 a.m. and then sleep until after noon. Either of them can complete the assignment on their own (so that both get a good grade) with an hour of work. Both of them would prefer not to do this work, but most of all want to avoid a situation where no one does the work and both get a bad grade. Jane decides first whether to do the work between midnight and 5:00 p.m., and sends Adam an e-mail before going to sleep to tell him what she has done. Adam then decides whether to do the work himself between 7:00 a.m. and noon. The game tree for this game is shown below.



1. What are the rollback equilibrium strategies and outcome?
2. Is it possible for Adam to achieve his best possible outcome (payoff 4) by credibly declaring a strategic move in the pregame? If not, why not? And if so, what sort of strategic move would Adam use, and how might he phrase his declaration?

10. Consider the military game described in Exercise S2 in Chapter 6. The ordinal payoff matrix for that game between a Rebel Force and a Conventional Army is reproduced below. For the purposes of this exercise, assume that the high mobility of the Rebel Force makes it impossible for the Conventional Army to move last.

		CONVENTIONAL ARMY	
		Valley	Hills
REBEL FORCE	Valley	1, 4	4, 1
	Hills	3, 2	2, 3

1. Draw the game tree for this game when the Rebel Force moves second. What are the rollback equilibrium strategies and outcome?
2. Is it possible for the Rebel Force to achieve an outcome that is better than the rollback equilibrium outcome by credibly declaring a strategic move in the pregame? If not, why not? And if so, what sort of strategic move would it use, and how would it phrase its declaration?

UNSOLVED EXERCISES

1. In a scene from the movie *Manhattan Murder Mystery*, Woody Allen and Diane Keaton are at a hockey game in Madison Square Garden. She is obviously not enjoying herself, but he tells her, "Remember our deal. You stay here with me for the entire hockey game, and next week I will come to the opera with you and stay until the end." Later, we see them coming out of the Met into the deserted Lincoln Center Plaza while music is still playing inside. Keaton is visibly upset: "What about our deal? I stayed to the end of the hockey game, and so you were supposed to stay till the end of the opera." Allen answers, "You know I can't listen to too much Wagner. At the end of the first act, I already felt the urge to invade Poland." Comment on the strategic choices made here by using your knowledge of the theory of strategic moves and credibility.
2. Consider a game between a parent and a child. The child can choose to be good (G) or bad (B); the parent can punish the child (P) or not (N). The child gets enjoyment worth 1 from bad behavior, but hurt worth −2 from punishment. Thus, a child who behaves well and is not punished gets 0; one who behaves badly and is punished gets 1 − 2 = −1; and so on. The parent gets −2 from the child's bad behavior and −1 from inflicting punishment.
   1. Set up this game as a simultaneous-move game, and find the equilibrium.
   2. Next, suppose that the child chooses G or B first and that the parent chooses P or N after having observed the child's action. Draw the game tree and find the subgame-perfect equilibrium.
   3. Now suppose that before the child acts, the parent can declare any strategic move. What strategic move should the parent make in order to get his best possible outcome (G,N)? Draw the modified game tree for this sequential-move game when the parent has committed to this strategic move, and use this modified game tree to show why the child chooses to be good.
3. Thucydides's history of the Peloponnesian War has been expressed in game-theoretic terms by Professor William Charron of St. Louis University.[12] Athens had acquired a large empire of small coastal cities around the Aegean Sea because of its leadership role in

defending the Greek world from Persian invasions. Sparta, fearing Athenian power, was contemplating war against Athens. If Sparta decided against war, Athens would have to decide whether to retain or relinquish its empire. But Athens, in turn, feared that if it gave independence to the small cities, they could choose to join Sparta in a greatly strengthened alliance against Athens and receive very favorable terms from Sparta for doing so. Thus there are three players, Sparta, Athens, and the small cities, which move in that order. There are four possible outcomes, and the payoffs are as follows (4 being best):

Outcome	Sparta	Athens	Small cities
War	2	2	2
Athens retains empire	1	4	1
Small cities join Sparta	4	1	4
Small cities become independent	3	3	3

1. Draw the game tree and find the rollback equilibrium outcome. Is there another outcome that is better for all players?
2. What strategic move or moves could be used to attain the better outcome? Discuss the credibility of such moves.

4. It is possible to reconfigure the payoffs in the game in Exercise S3 so that the children’ s statement in their ad is a threat, rather than a promise.
    1. Redraw the tree from part (a) of Exercise S3 and fill in payoffs for both players so that the children’ s statement becomes a *threat*.
    2. Define the status quo in your game, and determine whether the threat is deterrent or compellent.
    3. Explain why the threatened action needs to be made credible, given your payoff structure.
    4. Explain why the implied warning does not need to be made credible.

5. Explain why the children would want to make a threat in the first place, and suggest a way in which they might make their threatened action credible.

5. Consider a research funding game in which two government agencies, the U.S. Department of Energy (DoE) and the Defense Advanced Research Projects Agency (DARPA), each decide which of two research projects ("batteries" or "solar") to fund. DoE has a dominant strategy, investing in batteries, but wants most for DARPA to invest in solar. DARPA prefers to invest in the same project as DoE, but wants most for DoE to invest in batteries. The ordinal payoff matrix for this game is shown below.

		DARPA	
		Batteries	Solar
DoE	Batteries	2, 4	4, 3
	Solar	1, 1	3, 2

---

1. Identify all pure-strategy Nash equilibria of this game (or explain why no such equilibrium exists) in the case when DoE and DARPA move simultaneously.
2. Draw the game tree for the case when DoE moves first. What are the rollback equilibrium strategies and outcome?
3. In the case when DoE moves first, is it possible for DARPA to achieve its best possible outcome by credibly declaring a strategic move in the pregame? If not, why not? And if so, what sort of strategic move would DARPA use, and how might it phrase its declaration?
4. Draw the game tree for the case when DARPA moves first. What are the rollback equilibrium strategies and outcome?
5. In the case when DARPA moves first, is it possible for DoE to achieve its best possible outcome by credibly declaring a strategic move in the pregame? If not, why not? And if so, what sort of strategic move would DoE use, and how might it phrase its declaration?

6. North Korea's leader Kim Jong-un plays a game with the United States: He decides whether to keep or dismantle his nuclear weapons, while the United States decides whether or not to provide him with economic aid. Like China in Exercise S7, the United States wants most for North Korea to dismantle its nukes, but unlike China, the United States has a dominant strategy, not to provide economic aid. (Assume that if North Korea dismantled

its nukes, the United States would then feel little incentive to aid North Korea's economic recovery.) The ordinal payoff matrix for this game is shown below.

		UNITED STATES (U.S.)	
		Aid	Don't aid
NORTH KOREA [Kim Jong-un]	Dismantle	3, 3	1, 4
	Don't dismantle	4, 1	2, 2

---

1. Identify all pure-strategy Nash equilibria of this game (or explain why no such equilibrium exists) in the case when North Korea and the United States move simultaneously.
2. Draw the game tree for the case when North Korea moves first. What are the rollback equilibrium strategies and outcome?
3. Draw the game tree for the case when the United States moves first. What are the rollback equilibrium strategies and outcome?
4. In the case when North Korea moves first, is it possible for the United States to achieve its best possible outcome (payoff 4) by credibly declaring a strategic move in the pregame? If not, why not? And if so, what sort of strategic move would the United States use, and how might it phrase its declaration?
5. In the case when North Korea moves first, is it possible for the United States to achieve its second-best possible outcome (payoff 3) by credibly declaring a strategic move in the pregame? If not, why not? And if so, what sort of strategic move would the United States use, and how might it phrase its declaration?

7. In 1981, Ronald Reagan was a newly elected president with tremendous popular support and a vision for tax reform. But whether he could get the support he needed in Congress to implement his vision depended on the game being played between Democrats and Republicans in Congress. In a pair of articles that year,[13] *New York Times* columnist (and economist) Leonard Silk laid out the essence of this game, with ordinal payoffs as shown below. The Democrats get their best outcome when they attack Reagan's vision and the Republicans compromise, because the Democrats can then claim credit for fiscal responsibility while

implementing their favored budget. The Republicans prefer to support Reagan completely no matter what Democrats do, and they get their best outcome when Reagan’s budget gets bipartisan support. When the Democrats attack while the Republicans hold firm, the result is a stalemate, and both parties lose. The Democrats would be willing to moderate their attack if the Republicans would compromise, in which case both parties would get their second-best outcomes.

		REPUBLICANS	
		Support Reagan completely	Compromise
DEMOCRATS	Mainly support Reagan	2, 4	3, 3
	Attack Reagan	1, 2	4, 1

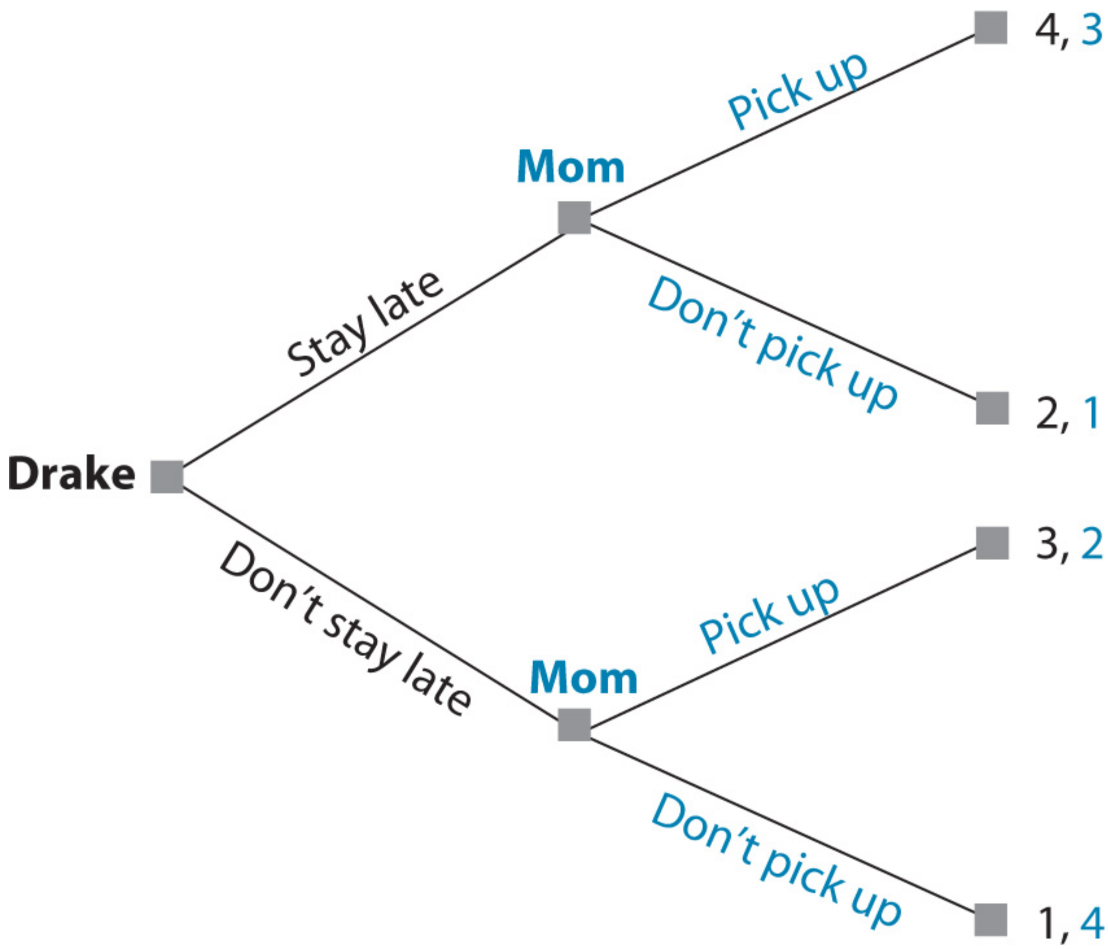
You may need to scroll left and right to see the full figure.

---

1. Does either player in this game have a dominant or superdominant strategy? Thoroughly explain your answer.
2. Identify all pure-strategy Nash equilibria of this game (or explain why no such equilibrium exists) in the case when Democrats and Republicans move simultaneously.
3. Draw the game tree for the case when Republicans move first. What are the rollback equilibrium strategies and outcome?
4. In the case when Republicans move first, is it possible for Democrats to achieve an outcome that is better than the rollback equilibrium outcome by credibly declaring a strategic move in the pregame? If not, why not? And if so, what sort of strategic move would the Democrats use, and how might they phrase their declaration?

8. Drake is a high-school student who lives in a tough urban neighborhood. Every day, Drake decides whether he will stay late after school and play basketball with his friends, and then calls his Mom, who decides whether to pick him up from school or ask him to get home another way—either by taking the bus (if he is not staying late) or by walking home (if he is staying late). Drake likes playing basketball, but most of all wants his Mom to pick him up, and least of all wants to walk home after staying late. For her part, Mom’s best outcome is when Drake takes the

bus home, but most of all, she does not want Drake to need to walk home. The game tree for this game is shown below.



1. What are the rollback equilibrium strategies and outcome?
2. Is it possible for Mom to achieve her best possible outcome (payoff 4) by credibly declaring a strategic move in the pregame? If not, why not? And if so, what sort of strategic move would Mom use, and how might she phrase her declaration?

9. Consider the market entry game described in Exercise U2 in Chapter 6. The payoff table for that game between a Monopolist and an Entrant is reproduced below. For the purposes of this exercise, assume that the Entrant moves first.

	ENTRANT	
	Build	Don' t build

You may need to scroll left and right to see the full figure.

		ENTRANT	
		Build	Don’t build
MONOPOLIST	Build	0.5, −0.5	2.5, 0
	Don’t build	2, 0.5	3, 0
You may need to scroll left and right to see the full figure.			

---

1. Draw the game tree for this game. What are the rollback equilibrium strategies and outcome?
2. Is it possible for the Monopolist (as second mover) to achieve its best possible outcome by credibly declaring a strategic move in the pregame? If not, why not? And if so, what sort of strategic move would the Monopolist use, and how might it phrase its declaration?

10. Write a brief description of a game in which you have participated that entailed strategic moves such as a commitment, threat, or promise, paying special attention to the essential aspect of credibility. Provide an illustration of the game if possible, and explain why the game that you describe ended as it did. Did the players use sound strategic thinking in making their choices?

Endnotes

- We thank political science professor Thomas Schwartz at UCLA for the idea for this exercise. Return to reference 11
- William C. Charron, “Greeks and Games: Forerunners of Modern Game Theory,” *Forum for Social Economics*, vol. 29, no. 2 (Spring 2000), pp. 1–32. Return to reference 12
- “Economic Scene,” *New York Times,* April 10, 1981, p. D2 and April 15, 1981, p. D2. This exercise is based on an example in *Thinking Strategically* by Avinash Dixit and Barry Nalebuff (New York: W.W. Norton, 1991), pp. 131–35. Return to reference 13

9■ Uncertainty and Information

IN CHAPTER 2, we mentioned different ways in which uncertainty (external and strategic) can arise in a game and ways in which players can have limited information about aspects of the game (imperfect and incomplete, and symmetrically or asymmetrically available to the players). We have already encountered and analyzed some of these concepts. Most notably, in simultaneous-move games, no player knows the actions other players are taking; this is a case of strategic uncertainty. In Chapter 6, we saw that strategic uncertainty gives rise to asymmetric and imperfect information because the different actions that could be taken by one player must be lumped into one information set for the other player. In Chapters 4 and 7, we saw how such strategic uncertainty is handled by having each player formulate beliefs about the others' actions (including beliefs about the probabilities with which different actions may be taken when mixed strategies are played) and by applying the concept of Nash equilibrium, in which such beliefs are confirmed. In this chapter, we focus on some further ways in which uncertainty and informational limitations arise in games.

We will begin this chapter by examining various strategies that individuals and societies can use for coping with the imperfect information generated by external uncertainty or risk. Recall that *external uncertainty* is uncertainty about matters outside any player's control, but affecting the payoffs of the game; weather is a simple example. We will describe the basic ideas behind the diversification, or spreading, of risk by an individual player and the pooling of risk by multiple players. Although these strategies can benefit everyone, the division of the total gains among the participants can be unequal; therefore, these situations contain a mixture of common interest and conflict.

We will then consider the informational limitations that often arise in games. Information in a game is *complete* only if all of the rules of the game—the strategies available to all players and the payoffs of each player as functions of the strategies of all players—are fully known by all players and, moreover, are common knowledge among them. By this exacting standard, most games in reality have *incomplete information*. Moreover, the incompleteness is usually *asymmetric*: Each player knows his own capabilities and payoffs much better than he knows those of other players. As we pointed out in Chapter 2, manipulation of information becomes an important dimension of strategy in such games. In this chapter, we will discuss when information can and cannot be communicated verbally in a credible manner. We will also examine other strategies designed to convey or conceal one's own information and to elicit another player's information. We spoke briefly of some such strategies—namely, screening and signaling—in Chapters 1 and 2; here, we study those strategies in more detail.

Of course, players in many games would also like to manipulate the actions of others. Managers would like their workers to work hard and well; insurance companies would like their policyholders to exert care to reduce the risk of an insured-against event occurring. If information were perfect, the actions of those other players would be observable. Workers' pay could be made contingent on the quality and quantity of their effort; payouts to insurance policyholders could be made contingent on the care they exercised. But in reality, these actions are difficult to observe; that creates a situation of imperfect asymmetric information, commonly called moral hazard. Thus, the players in these games have to devise various incentives to influence others' actions in the desired direction. We take up the design of such incentives, termed *mechanism design* or *incentive design*, in Chapter 14.

Information and its manipulation in games has been a topic of active research in recent decades. That research has shed new light on many previously puzzling matters in economics, such as the nature of incentive contracts, the organization of companies, markets for labor and for durable goods, government regulation of business, and myriad others.[1] More recently, political scientists have used the same concepts to explain phenomena such as the relationship of changes in tax and spending policy to elections, as well as the delegation of legislation to committees. These ideas have also spread to biology, where evolutionary game theory explains features such as the peacock's large and ornate tail as a signal. Perhaps even more importantly, you will recognize the role that signaling and screening play in your daily interactions with family, friends, teachers, coworkers, and so on, and you will be able to improve your strategies in these games.

Endnotes

- The pioneers of the theory of asymmetric information in economics have shared in two Nobel Prizes. George Akerlof, Michael Spence, and Joseph Stiglitz were recognized in 2001 for their work on signaling and screening, and Leo Hurwicz, Eric Maskin, and Roger Myerson in 2007 for their work on mechanism design. Return to reference 1

Glossary

moral hazard
: A situation of information asymmetry where one player’s actions are not directly observable to others.

1 STRATEGIES FOR DEALING WITH RISK

Imagine that you are a farmer subject to the vagaries of weather. If the weather is good for your crops, you will have an income of \$160,000. If it is bad for them, your income will be only \$40,000. The two possibilities are equally likely (with a probability of ½, or 0.5, or 50% each). Therefore, your average or expected income is \$100,000 (= ½ × 160,000 + ½ × 40,000), but there is considerable risk around this average value.

What can you do to reduce the risk to your income? You might try growing a crop that is less subject to the vagaries of weather, but suppose you have already done all such things that are under your individual control. Then you might be able to reduce the risk to your income further by getting someone else to accept some of that risk. Of course, you must give the other person something else in exchange. This quid pro quo usually takes one of two forms: a cash payment, or a mutual exchange or sharing of risk.

A. Sharing of Risk

Suppose you have a neighbor who faces a risk similar to yours, but gets good weather exactly when you get bad weather, and vice versa. (You live on opposite sides of an island, and rain clouds visit one side or the other, but not both.) In technical jargon, *correlation* is a measure of alignment between any two uncertain quantities—in this discussion, between one person's risk and another's. Thus, in this example, your neighbor's risk is perfectly negatively correlated with yours. The combined income of you and your neighbor is \$200,000, no matter what the weather: It is totally risk free. You can enter into a contract that gets each of you \$100,000 for sure: You promise to give him \$60,000 in years when you are lucky, and he promises to give you \$60,000 in years when he is lucky. You have eliminated your risks by combining them.

Currency swaps provide a good example of negative correlation of risk in real life. A U.S. firm exporting to Europe gets its revenues in euros, but it is interested in its dollar profits, which depend on the fluctuating euro-dollar exchange rate. Conversely, a European firm exporting to the United States faces similar uncertainty about its profits in euros. When the euro falls relative to the dollar, the U.S. firm's euro revenues convert into fewer dollars, and the European firm's dollar revenues convert into more euros. The opposite happens when the euro rises relative to the dollar. Thus, fluctuations in the exchange rate generate negatively correlated risks for the two firms. Both can reduce these risks by contracting for an appropriate swap of their revenues.

Even without such perfect negative correlation, risk sharing has some benefit. Return to your role as an island farmer, and suppose that you and your neighbor face risks that are independent from each other, as if the rain clouds could toss a separate coin to decide whether to visit each of you. Then there are four possible outcomes, each with a probability of ¼. The incomes you and your neighbor earn in these four cases are illustrated in Figure 9.1a. However, suppose the two of you were to make a contract to share risk; then your incomes would be those shown in Figure 9.1b. Although your average (expected) income in each table is $100,000, without the sharing contract, each of you would get $160,000 or $40,000 with probabilities of ½ each. With the contract, each of you would get $160,000 with probability ¼, $100,000 with probability ½, and $40,000 with probability ¼. Thus, for each of you, the contract has reduced the probabilities of the two extreme outcomes from ½ to ¼ and increased the probability of the middle outcome from 0 to ½. In other words, the contract has reduced the risk for each of you.

		NEIGHBOR	
		Lucky	Not
YOU	Lucky	160,000, 160,000	160,000, 40,000
	Not	40,000, 160,000	40,000, 40,000

(a) Without sharing

		NEIGHBOR	
		Lucky	Not
YOU	Lucky	160,000, 160,000	100,000, 100,000
	Not	100,000, 100,000	40,000, 40,000

(b) With sharing

FIGURE 9.1 Sharing Income Risk

In fact, as long as your incomes are not totally positively correlated—that is, as long as your luck and your neighbor's luck do not move in perfect tandem—you can both reduce your risks by sharing them. And if there are more than two of you with some degree of independence in your risks, then the law of large numbers makes possible even greater reduction in the risk of each. That is exactly what insurance companies do: By combining the similar but independent risks of many people, an insurance company is able to compensate any one of them when he suffers a large loss. It is also the basis of portfolio diversification: By dividing your wealth among many different assets with different kinds and degrees of risk, you can reduce your total exposure to risk.

However, such arrangements for risk sharing depend on observability of outcomes and enforcement of contracts. Otherwise, each farmer has the temptation to pretend to have suffered bad luck, or simply to renege on the deal and refuse to share when he has good luck. Similarly, an insurance company may falsely deny claims, but its desire to maintain its reputation for the sake of its ongoing business may check such reneging.

B. Paying to Reduce Risk

Now consider the possibility of trading risk for cash. Suppose you are the same farmer facing the same risk as before. But now your neighbor has a sure income of $100,000. You face a lot of risk, and he faces none. He may be willing to take a little of your risk for a price that is agreeable to both of you.

Suppose you come to an arrangement where the neighbor will give you $10,000 if your luck is bad, and you will give him $10,000 if your luck is good. Thus, your income will be $150,000 if your luck is good and $50,000 if your luck is bad; your neighbor’s income will be $90,000 and $110,000 in those respective events. The spread between your incomes in the two situations has decreased from $120,000 to $100,000—that is, by $20,000; the spread between your neighbor’s two incomes has increased from $0 to $20,000. Although the changes in the two spreads are equal, it is reasonable to suppose that an equal change in spreads is more of a concern when it starts from an already large base. Thus, the change will be less of a concern to your neighbor than to you. For sake of definiteness, suppose that you are willing to pay up to $3,000 up front to your neighbor to enter into such a contract with you; this is, in effect, a contract providing partial insurance for your risk, and the amount you pay up front is an insurance premium. Your neighbor is willing to enter into the contract for much less—say, only $250.[2] There is plenty of room between the neighbor’s willingness to accept and your willingness to pay; somewhere in this range, you two can negotiate a price and strike a deal that benefits you both.[3] At a premium close to $250, you reap almost all the gain from the deal; at a premium close to $3,000, your neighbor does.

Where the price actually settles depends on many things: whether there are many more farmers facing risks like yours than farmers with safe incomes like your neighbor’s (that is, the extent of competition that exists on the two sides of the deal); the attitudes toward risk on the two sides; and so on. If your “neighbor” is actually an insurance company, it can be nearly unconcerned about your risk because it is combining numerous such risks and because it is owned by well-diversified investors, for each of whom this business is only a small part of their total risk. And if insurance companies have to compete fiercely for business, the insurance market can offer you nearly complete insurance at a price that leaves almost all of the gain with you.

Common to all such arrangements is the idea that mutually beneficial deals can be struck whereby, for a suitable price, someone facing less risk takes some risk off the shoulders of someone else who faces more. In fact, the idea that a price and a market for risk exist is the basis for almost all of the financial arrangements in a modern economy. Stocks and bonds, as well as complex financial instruments such as derivatives, are just ways of spreading risk to those who are willing to bear it for the lowest asking price. Many people think these markets are purely forms of gambling. In a sense, they are. But those who start out with the least risk take the gambles, perhaps because they have already diversified in the way that we saw earlier. And the risk is sold or shed by those who are initially most exposed to it. This enables the latter to be more adventurous in their enterprises than they would be if they had to bear all the risk of those enterprises themselves. Thus, financial markets promote entrepreneurship by facilitating risk trading.

Here we have only considered the sharing of a given total risk. In practice, people may be able to take actions to reduce that total risk: A farmer can guard crops against frosts, and a car owner can drive carefully to reduce the risk of an accident. If such actions are not publicly observable, the game will be one of imperfect information, raising the problem of moral hazard that we mentioned in the introduction: People who are well insured will lack the incentive to reduce the risk they face. We will look at such problems, and the design of mechanisms to cope with them, in Chapter 14.

C. Manipulating Risk in Contests

The farmers in our example above faced risk due to the weather, not from any actions of their own or of other farmers. If the players in a game can affect the risk they or other players face, they can use such manipulation of risk strategically. A prime example is contests such as research and development races between companies to develop and market new information technology or biotech products; many sports contests have similar features.

The outcome of a sports event or related contest is determined by a mixture of skill and chance. You win if

Your skill + your luck > rival’s skill + rival’s luck

or

Your luck − rival’s luck > rival’s skill − your skill.

We denote the left-hand side of this equation with the symbol *L*, which measures your “luck surplus.” *L* is an uncertain magnitude; suppose its probability distribution is a normal curve, or bell curve, as illustrated by the black curve in Figure 9.2. At any point on the horizontal axis, the height of the curve represents the probability that *L* takes on that value. Thus, the area under this curve between any two points on the horizontal axis equals the probability that *L* lies between those points. Suppose your rival has more skill, so you are an underdog in the contest. Your “skill deficit,” which equals the difference between your rival’s skill and your skill, is therefore positive, as shown by the point *S*. You win if your luck surplus, *L*, exceeds your skill deficit, *S*. Therefore, the area under the curve to the right of the point *S*, which is shown by the gray hatching in Figure 9.2, represents your probability of winning. If you make the situation chancier, the bell curve will be flatter, like the blue curve in Figure 9.2, because the probability of relatively high and low values of *L* increases while the probability of moderate values decreases. Then the area under the curve to the right of *S* also increases. In Figure 9.2, the larger area under the flatter bell curve is shown by blue hatching. As the underdog, you should therefore adopt a strategy that flattens the curve to increase your probability of winning. Conversely, if you are the favorite, you should try to reduce the element of chance in the contest.

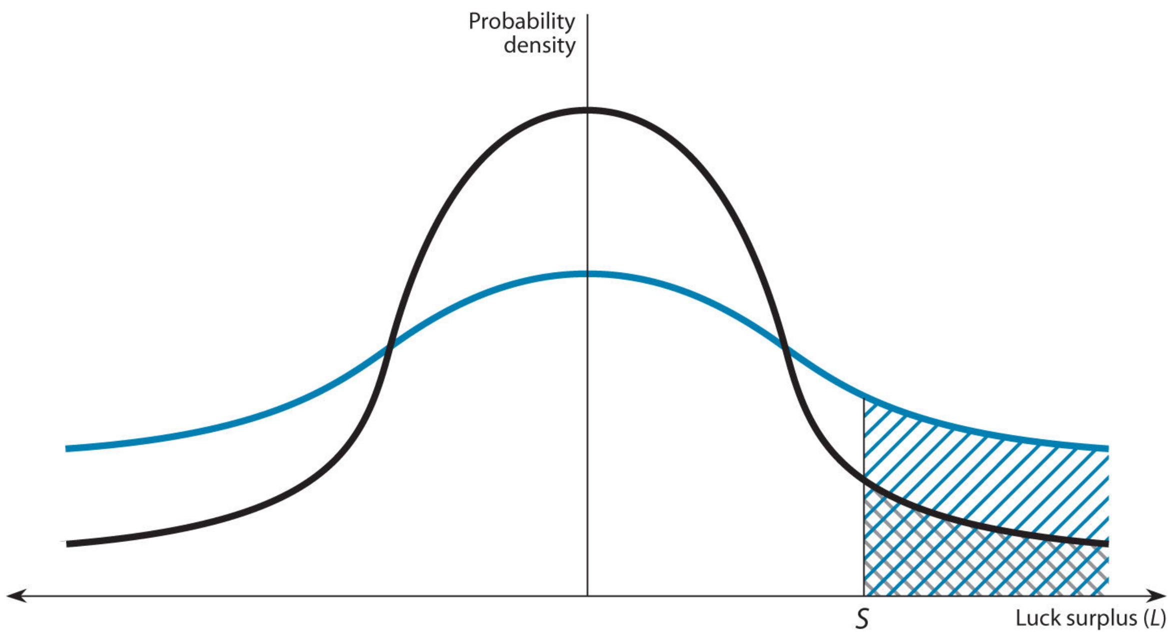


Figure 9.2 The Effect of Greater Risk on the Chances of Winning

Thus, we should see underdogs, or those who have fallen behind in a long race, try unusual or risky strategies, because it is their only chance to get level or ahead. In contrast, favorites, or those who have stolen a lead, will play it safe. Here is a practical piece of advice based on this principle: If you want to challenge someone who is a better player than you to a game of tennis, choose a windy day.

You may stand to benefit by manipulating not only the amount of risk in your strategy, but also the correlation between your risk and that of other players. The player who is ahead will try to choose a correlation as high and as positive as possible; then, whether his own luck is good or bad, the luck of his opponent will be the same, and his lead will be protected. Conversely, the player who is behind will try to find a risk as uncorrelated with that of his opponent as possible. It is well known that in a two-sailboat race, the boat that is behind should try to steer differently from the boat ahead, and the boat ahead should try to imitate all the tacks of the one behind.[4]

Endnotes

- Here we do not need the rigorous theories of risk aversion that yield such numbers for willingness to avoid risk, so we will refer interested readers to somewhat more advanced textbooks, such as Hal Varian, *Intermediate Microeconomics*, 9th ed. (New York, W. W. Norton, 2014), Chapter 12 . For any advanced students or teachers among the readers, these numbers come from an expected utility calculation with a square root utility function. Return to reference 2
- After it becomes known whether your luck is good or bad, one of you will have to make a unilateral payment to the other and may be tempted to renege. Therefore, it is important to have an external enforcement authority, or a repeated relationship between the two parties that keeps them honest, to sustain such deals. Return to reference 3
- Avinash Dixit and Barry Nalebuff, *Thinking Strategically* (New York: W. W. Norton, 1991), give a famous example of the use of this strategy in sailboat racing. For a more general theoretical discussion, see Luis Cabral, “R&D Competition When the Firms Choose Variance,” *Journal of Economics and Management Strategy*, vol. 12, no. 1 (Spring 2003), pp. 139–50. Return to reference 4

Glossary

negatively correlated
: Two random variables are said to be negatively correlated if, as a matter of probabilistic average, when one is above its expected value, the other is below its expected value.

positively correlated
: Two random variables are said to be positively correlated if, as a matter of probabilistic average, when one is above its expected value, the other is also above its expected value, and vice versa.

2 ASYMMETRIC INFORMATION: BASIC IDEAS

In many games, one player, or some of the players, may have the advantage of knowing with greater certainty what has happened or what will happen. Such advantages, or asymmetries of information, are common in actual strategic situations. At the most basic level, each player may know his own preferences or payoffs—for example, his risk tolerance in a game of brinkmanship, his patience in bargaining, or his peaceful or warlike intentions in international relations—quite well, but those of the other players much more vaguely. The same is true for a player's knowledge of his own innate characteristics (such as the skill of a job applicant or the riskiness of an applicant for auto or health insurance). And sometimes the actions available to one player—for example, the weaponry and readiness of a country for war—are not fully known to other players. Finally, some actual outcomes (such as the actual dollar value of loss to an insured homeowner in a flood or an earthquake) may be observed by one player, but not by others.

By manipulating what the other players know about your abilities and preferences, you can affect the equilibrium outcome of a game. Therefore, such manipulation of asymmetric information itself becomes a game of strategy. You might think that each player will always want to conceal the information he has and elicit information from the others, but that is not so. The better-informed player may want to do one of the following:

1. *Conceal information* or *reveal misleading information.* When mixing moves in a zero-sum game, you don't want the other player to see what you have done; for example, you bluff in poker to mislead others about your cards.

2. *Reveal selected information truthfully.* When you make a strategic move, you want others to see what you have done so that they will respond in the way you desire. For example, if you are in a tense situation but your intentions are not hostile, you want others to know this credibly, so that there will be no unnecessary fight.

Similarly, the less informed player may want to do one of the following:

1. *Elicit information* or *filter truth from falsehood.* An employer wants to find out the skill of a prospective employee or the effort of a current employee. An insurance company wants to know an applicant’s risk class, the amount of a claimant’s loss, and any contributory negligence by the claimant that would reduce its liability.
2. *Remain ignorant.* Being unable to know your opponent’s strategic move can immunize you against his commitments and threats. Top-level politicians or managers often benefit from having such “credible deniability.”

In most cases, words alone do not suffice to convey credible information; rather, actions speak louder than words. Yet even actions may not convey information credibly, if they are too easily imitated by others who do not possess the information in question. Therefore, less-informed players should pay closer attention to what a better-informed player does than to what he says. And knowing that the others will interpret his actions in this way, the better-informed player should in turn try to manipulate his actions for their information content.

When you are playing a strategic game, you may find that you have information that other players do not have. You may have information that is “good” (for yourself) in the sense that, if the other players knew this information, they would alter their actions in a way that would increase your payoff.

You know that you are a nonsmoker, for example, and should qualify for lower life-insurance premiums, or that you are a very highly skilled worker and should get paid a higher wage. You may also have "bad" information, whose disclosure would cause others to act in a way that would hurt you. You know that you are weak in math, for example, and don't deserve to be admitted to a prestigious graduate program, or that you have been dealt poor cards in a poker game and shouldn't win the hand. Of course, no matter whether your information is good or bad, you would like for others to believe that it is good. Therefore, you try to think of, and take, actions that will induce them to believe that your information is good. For example, you might provide medical records regarding your lung health to your insurance company or bluff in poker by betting as if you had an excellent hand. Such actions are called signals, and the strategy of using them is called signaling. Other players know that you can signal and will interpret your actions accordingly. So signaling and interpreting signals is a game, and how it plays out will be illustrated and analyzed in this chapter.

If other players know more than you do or take actions that you cannot directly observe, you can use strategies that reduce your informational disadvantage. You may be a life-insurance company and need to know the smoking habits of your clients, or a graduate school admissions officer seeking to identify the skills of your applicants. You want to do something to get your clients or applicants to reveal the information they have that can affect your payoffs. The strategy of making another player act so as to reveal his information is called screening, and specific methods used for this purpose are called screening devices.[5]

Because a player's private information often consists of knowledge of his own abilities or preferences, it is useful to think of players who come to a game possessing different private information as different types. When credible signaling works, less informed players will, in equilibrium, be able to infer the information of the more informed ones correctly from their actions; the law school, for example, will admit only the truly qualified applicants. Another way to describe the equilibrium outcome is to say that the different types are correctly revealed, or *separated*. Therefore, we call this equilibrium a separating equilibrium. In some cases, however, one or more types may successfully mimic the actions of other types, so that the less informed players cannot infer types from actions and cannot identify the different types; insurance companies, for example, may offer only one kind of life-insurance policy. In that case, we say the types are pooled together in equilibrium, and we call this equilibrium a pooling equilibrium. A third type of equilibrium, called a semiseparating equilibrium, occurs when one or more types mimic the actions of other types randomly, using a mixed strategy. When studying games of incomplete information, we will see that identifying the kind of equilibrium that occurs is of primary importance.

Endnotes

- A word of warning: In ordinary language, the word *screening* can have different meanings. The one used in game theory is that of testing or scrutinizing. Thus, a less-informed player uses screening to find out what a better-informed player knows. The alternative sense of *screening*—namely, concealing—is when a better-informed player who does not want to disclose his information through his actions chooses to mimic the behavior that would result from information he knows to be false. Such behavior leads to a *semiseparating* equilibrium, as discussed in Section 6.D. Return to reference 5

Glossary

signals
: Devices used for signaling.

signaling
: Strategy of a more-informed player to convey his “good” information credibly to a less-informed player.

screening
: Strategy of a less-informed player to elicit information credibly from a more-informed player.

screening devices
: Methods used for screening.

type
: Players who possess different private information in a game of asymmetric information are said to be of different types.

separating equilibrium
: A perfect Bayesian equilibrium in a game of asymmetric information, where the actions in the equilibrium reveal player type.

pooling equilibrium
: A perfect Bayesian equilibrium in a game of asymmetric information, where the actions in the equilibrium cannot be used to distinguish type.

semiseparating equilibrium
: A perfect Bayesian equilibrium in a game of asymmetric information, where the actions in the equilibrium convey some additional information about the players’ types, but some ambiguity about these types remains.

3 DIRECT COMMUNICATION, OR "CHEAP TALK"

The simplest way to convey information to others would seem to be to tell them; likewise, the simplest way to elicit information from them would seem to be to ask. But in a game of strategy, players should be aware that others may not tell the truth and, likewise, that their own assertions may not be believed by others. That is, the *credibility* of mere words may be questionable. It is a common saying that talk is cheap; indeed, direct communication has zero or negligible *direct* cost. However, it can *indirectly* affect the outcome and payoffs of a game by changing one player's beliefs about another player's actions or by influencing the selection of one equilibrium out of multiple equilibria. Direct communication that has no direct cost has come to be called *cheap talk* by game theorists, and the equilibrium achieved by using direct communication is termed a cheap talk equilibrium.

A. Perfectly Aligned Interests

Direct communication of information works well if the players' interests are well aligned. The assurance game first introduced in Chapter 4 provides the most extreme example of this. We reproduce its payoff table (Figure 4.14) here as Figure 9.3.

		WATSON	
		St. Bart's	Simpson's
HOLMES	St. Bart's	1, 1	0, 0
	Simpson's	0, 0	2, 2

FIGURE 9.3 Assurance Game

The interests of Sherlock Holmes and Dr. Watson are perfectly aligned in this game: They both want to meet, and both prefer meeting at Simpson's. The problem is that the game is played noncooperatively; they are making their choices independently, without knowledge of what the other is choosing. But suppose that Holmes is given an opportunity to send a message to Watson (or Watson is given an opportunity to ask a question and Holmes replies) before their choices are made. If Holmes's message (or reply; we will not keep repeating this) is "I am going to Simpson's," Watson has no reason to think he is lying.[6] If Watson believes Holmes, Watson should choose Simpson's, and if Holmes believes that Watson will believe him, it is equally optimal for Holmes to choose Simpson's, making his message truthful. Thus, direct communication very easily achieves the mutually preferable outcome. This is indeed the reason why, when we considered this game in Chapter 4, we had to construct a scenario in which such communication was infeasible.

Let us examine the outcome of allowing direct communication in the assurance game more precisely in game-theoretic terms. We have created a two-stage game. In the first stage, only Holmes acts, and his action is his message to Watson. In the second stage, the original simultaneous-move game is played. In the full two-stage game, we have a rollback equilibrium where the strategies (complete plans of action) are as follows. The second-stage action plans for both players are “If Holmes’ s first-stage message was ‘I am going to St. Bart’ s,’ then choose St. Bart’ s; if Holmes’ s first-stage message was ‘I am going to Simpson’ s,’ then choose Simpson’ s.” (Remember that players in sequential-move games must specify *complete* plans of action.) The first-stage action for Holmes is to send the message “I am going to Simpson’ s.” Verification that this is indeed a rollback equilibrium of the two-stage game is easy, and we leave it to you.

However, this equilibrium where cheap talk “works” is not the only rollback equilibrium of this game. Consider the following strategies: The second-stage action plan for each player is to go to St. Bart’ s regardless of Holmes’ s first-stage message; and Holmes’ s first-stage message can be anything. We can verify that this is also a rollback equilibrium. Regardless of Holmes’ s first-stage message, if one player is going to St. Bart’ s, then it is optimal for the other player to go there also. Thus, in each of the second-stage subgames that could arise—one after each of the two messages that Holmes could send—both choosing St. Bart’ s is a Nash equilibrium of the subgame. Then, in the first stage, Holmes, knowing his message is going to be disregarded, is indifferent about which message he sends.

The cheap talk equilibrium—where Holmes’ s message is not disregarded—yields higher payoffs, and we might normally think that it would be the one selected as a focal point. However, there may be reasons of history or culture that favor the other equilibrium. For example, for some reasons quite extraneous to this particular game, Holmes may have a reputation for being totally unreliable in following through on meetings, perhaps due to his absent-mindedness. Then people might generally disregard statements about meetings from him, and, knowing this to be the usual state of affairs, Watson might not believe this particular one.

Such problems exist in all communication games. They always have alternative equilibria where the communication is disregarded and therefore irrelevant. Game theorists call these babbling equilibria. Having noted that they exist, however, we will focus on cheap talk equilibria, where communication does have some effect.

B. Totally Conflicting Interests

The credibility of direct communication depends on the degree of alignment of players' interests. As a dramatic contrast with the assurance game example, consider a game where the players' interests are totally in conflict—namely, a zero-sum game. A good example is the tennis-point game in Figure 4.17; we reproduce its payoff matrix as Figure 9.4. Remember that the payoffs are Evert's success percentages. Remember, too, that this game has a Nash equilibrium only in mixed strategies (derived in Chapter 7); Evert's expected payoff in that equilibrium is 62.

Now suppose that we construct a two-stage game. In the first stage, Evert is given an opportunity to send a message to Navratilova. In the second stage, the original simultaneous-move game is played. What will be the rollback equilibrium?

		NAVRATILOVA	
		DL	CC
EVERT	DL	50, 50	80, 20
	CC	90, 10	20, 80

FIGURE 9.4 Tennis-Point Game

It should be clear that Navratilova will not believe any message she receives from Evert. For example, if Evert's message is "I am going to play DL," and Navratilova believes her, then Navratilova should choose to cover DL. But if Evert thinks that Navratilova will cover DL, then Evert's best choice is CC. At the next level of thinking, Navratilova should see through this and not believe the assertion of DL.

But there is more. Navratilova should not believe that Evert will do exactly the opposite of what she says either. Suppose Evert’s message is “I am going to play DL,” and Navratilova thinks, “She is just trying to trick me, and so I will take it that she will play CC.” This will lead Navratilova to choose to cover CC. But if Evert thinks that Navratilova will disbelieve her in this simple way, then Evert should choose DL after all. And Navratilova should see through this, too.

Thus, Navratilova’s disbelief should mean that she should totally disregard Evert’s message. Then the full two-stage game has only a babbling equilibrium. The two players’ actions in the second stage will be simply those of the original equilibrium, and Evert’s first-stage message can be anything. This is true of all zero-sum games.

C. Partially Aligned Interests

But what about games in which there is a mixture of conflict and common interest? Whether direct communication is credible in such games depends on how conflict and cooperation mix when players' interests are only partially aligned. Thus, we should expect to see both cheap talk and babbling equilibria in games of this type. More generally, the greater the alignment of interests, the more information should be communicable. We illustrate this intuition with an example.

Consider a situation that you may have already experienced or, if not, soon will when you start to earn and invest. When your financial adviser recommends an investment, she may be doing so as part of developing a long-term relationship with you for the steady commissions that your business will bring her, or she may be a fly-by-night operator who touts a loser, collects the up-front fee, and disappears. The credibility of her recommendation depends on what type of relationship you establish with her.

Suppose you want to invest \$100,000 in the asset recommended by your adviser, and that you can anticipate three possible outcomes. The asset could be a bad investment (B), leading to a 50% loss, or a payoff of −50 (measured in thousands of dollars). Or the asset could be a mediocre investment (M), yielding a 1% return, or a payoff of 1. Or, finally, it could be a good investment (G), yielding a 55% return, or a payoff of 55. If you choose to invest, you pay the adviser a 2% fee up front regardless of the performance of the asset; this fee gives your adviser a payoff of 2 and simultaneously lowers your payoff by 2. Your adviser will also earn 20% of any gain you make, leaving you with a payoff of 80% of the gain, but she will not have to share in any loss.

With no specialized knowledge related to the particular asset that has been recommended to you, you cannot judge which of the three outcomes is most likely. Therefore, you simply assume that all three possibilities—B, M, and G—are equally likely: that there is a one-third chance of each outcome occurring. In this situation, in the absence of any further information, you calculate your expected payoff from investing in the recommended asset as $[(⅓ \times -50) + (⅓ \times 0.8 \times 1) + (⅓ \times 0.8 \times 55)] - 2 = [⅓ \times (-50 + 0.8 + 44)] - 2 = [⅓ \times (-5.2)] - 2 = -1.73 - 2 = -3.73$. This calculation indicates an expected loss of \$3,730. Therefore, you do not make the investment, and your adviser does not get any fee. Similar calculations show that you would also choose not to invest, due to a negative expected payoff, if you believed the asset was definitely the B type, definitely the M type, or definitely any probability-weighted combination of the B and M types alone.

Your adviser is in a different situation. She has researched the investment and knows which of the three possibilities—B, M, or G—is the truth. We want to determine what she will do with her information—specifically whether she will truthfully reveal to you what she knows about the asset. We consider the various possibilities below, assuming that you update your belief about the asset's type based on the information you receive from your adviser. For this example, we assume that you simply believe what you are told: You assign probability 1 to the asset being the type stated by your adviser.[7]

I. SHORT-TERM RELATIONSHIPIf your adviser tells you that the recommended asset is type B, you will choose not to invest. Why? Because your expected payoff from that asset is −50, and investing will cost you an additional 2 (in fees to the adviser), for a final payoff of −52. Similarly, if she tells you the asset is M, you will also not invest. In that case, your expected payoff is 80% of the return of 1 minus the 2 in fees, for a total of −1.2. Only if the adviser tells you that the asset is G will you choose to invest. In this situation, your expected payoff is 80% of the 55 return less the 2 in fees, or 42.

What will your adviser do with her knowledge, then? If the truth is G, your adviser will want to tell you the truth in order to induce you to invest. But if she anticipates no long-term relationship with you, she will be tempted to tell you that the truth is G, even when she knows the asset is either M or B. If you decide to invest on the basis of her statement, she simply pockets her 2% fee and flees; she has no further need to stay in touch. Knowing that there is a possibility of getting bad advice, or false information, from an adviser with whom you will interact only once, you should ignore the adviser’s recommendation altogether. Therefore, in this asymmetric-information, short-term-relationship game, credible communication is not possible. The only equilibrium is the babbling one in which you ignore your adviser; there is no cheap talk equilibrium in this case.

II. LONG-TERM RELATIONSHIP: FULL REVELATIONNow suppose your adviser works for a firm that you have invested with for years; losing your future business may cost her her job. If you invest in the asset she recommends, you can compare its actual performance to your adviser’s forecast. That forecast could prove to have been wrong in a small way (the forecast was M and the truth is B, or the forecast was G and the truth is M) or in a large way (the forecast was G and the truth is B). If you discover such misrepresentations, your adviser and her firm lose your future business. They may also lose business from others if you bad-mouth them to friends and acquaintances. If the adviser attaches a cost to her loss of reputation, she is implicitly concerned about your possible losses, and therefore her interests are *partially aligned* with yours. Suppose the cost to her reputation of a small misrepresentation is 2 (the monetary equivalent of a $2,000 loss) and that of a large misrepresentation is 4 (a $4,000 loss). We can now determine whether the partial alignment of your interests with those of your adviser is sufficient to induce her to be truthful.

As we discussed earlier, your adviser will tell you the truth if the asset is G to induce you to invest. We need to consider her incentives when the truth is *not* G, when the asset is actually B or M. Suppose first that the asset is B. If your adviser truthfully reveals the asset's type, you will not invest, and she will not collect any fee, but she will also suffer no reputation cost: her payoff from reporting B when the truth is B is 0. If she tells you the asset is M (even though it is B), you still will not invest because your expected payoff is −1.2, as we calculated earlier. Then the adviser will still get 0, so she has no incentive to lie and tell you that a B asset is really M.[8] But what if she reports G? If you believe her and invest, she will get the up-front fee of 2, but she will also suffer the reputation cost of the large error, 4.[9] Her payoff from reporting G when the truth is B is negative; your adviser would do better to reveal B truthfully. Thus, in situations when the truth about the asset is G or B, the adviser's incentives are to reveal the type truthfully.

But what if the truth is M? Truthful revelation does not induce you to invest: The adviser's payoff is 0 from reporting M. If she reports G and you believe her, you invest. The adviser gets her fee of 2, plus the 20% of the 1 that is your return from M, and she also suffers the reputation cost of the small misrepresentation, 2. Her payoff is $2 + (0.2 \times 1) - 2 = 0.2 > 0$. Thus, your adviser does stand to benefit by falsely reporting G when the truth is M. Knowing this, you will not believe any report of G.

Because your adviser has an incentive to lie when the asset she is recommending is M, full information cannot be credibly revealed in this situation. The babbling equilibrium, where any report from the adviser is ignored, is still a possible equilibrium. But is it the *only* equilibrium here, or is some partial communication possible? The failure to achieve full revelation occurs because the adviser will misreport M as G, so suppose we lump those two possibilities together into one and label it "not-B." Thus, the adviser asks herself what she should report: "B or not-B?"[10] Now we can consider whether your adviser will choose to report truthfully in this case of partial communication.

III. LONG-TERM RELATIONSHIP: PARTIAL REVELATIONTo determine your adviser's incentives in the "B or not-B" situation, we need to figure out what inference you will draw from the report of not-B, assuming you believe it. Your *prior* (original) belief was that B, M, and G were equally likely, with probabilities ⅓ each. If you are told the asset is not-B, you are left with two possibilities, M and G. You regarded the two as equally likely originally, and there is no reason to change that assumption, so you now give each a probability of ½. These are your new, *posterior*, probabilities, conditioned on the information you receive from your adviser's report. With these probabilities, your expected payoff if you invest when the report is not-B is $[\frac{1}{2} \times (0.8 \times 1)] + [\frac{1}{2} \times (0.8 \times 55)] - 2 = 0.4 + 22 - 2 = 20.4 > 0$. This positive expected payoff is sufficient to induce you to invest when given a report of not-B.

Knowing that you will invest if you are told not-B, we can determine whether your adviser will have any incentive to lie. Will she want to tell you not-B even if the truth is B? When the asset is actually B and the adviser tells the truth (reports B), her payoff is 0, as we calculated earlier. If she reports not-B instead, and you believe her, she gets 2 in fees.[11] She also incurs the reputation cost associated with misrepresentation. Because you assume that M or G is equally likely on the basis of the not-B report, the expected value of the reputation cost in this case is ½ times the cost of 2 for small misrepresentation plus ½ times the cost of 4 for large misrepresentation: The expected reputation cost is then (½ × 2) + (½ × 4) = 3. Your adviser's net payoff from reporting not-B when the truth is B is 2 − 3 = −1. Therefore, she does not gain by making a false report to you. Because telling the truth is your adviser's best strategy here, a cheap talk equilibrium with credible *partial revelation* of information is possible.

The concept of the partial-revelation cheap talk equilibrium can be made more precise using the concept of a *partition*. Recall that you can anticipate three possible outcomes or events: B, M, and G. In our example, this set of events is divided, or partitioned, into distinct subsets, and your adviser then reports to you which subset contains the truth. (Of course, the verity of her report remains to be examined as part of the analysis.) Here, our three events are partitioned into two subsets, one consisting of the singleton B, and the other consisting of the pair of events {M, G}. In the partial-revelation equilibrium, these two subsets can be distinguished on the basis of the adviser's report, but the finer distinction between M and G—leading to the finest possible partition into three subsets, each consisting of a singleton—cannot be made. That finer distinction would be possible only in a case in which a full-revelation equilibrium exists.

We advisedly said earlier that a cheap talk equilibrium with credible partial revelation of information is *possible*. This game is one with multiple equilibria because the babbling equilibrium also remains possible. The configuration of strategies and beliefs where you ignore the adviser's report, and the adviser sends the same report (or even a random report) regardless of the truth, is still an equilibrium: Given each player's strategy, the other player has no reason to change her actions or beliefs. In the terminology of partitions, we can think of this babbling equilibrium as having the coarsest possible, and trivial, partition, with just one (sub)set {B, M, G} containing all three possibilities. In general, whenever you find a non-babbling equilibrium in a cheap talk game, there will also be at least one other equilibrium with a coarser or cruder partition of outcomes.

IV. MULTIPLE EQUILIBRIAAs an example of a situation in which coarser partitions are associated with additional equilibria, consider the case in which your adviser’s cost of reputation is higher than assumed above. Let the reputation cost be 4 (instead of 2) for a small misrepresentation of the truth and 8 (instead of 4) for a large misrepresentation. Our analysis above showed that your adviser will report G if the truth is G, and that she will report B if the truth is B. These results continue to hold. Your adviser wants you to invest when the truth is G, and she still gets the same payoff from reporting B when the truth is B as she does from reporting M in that situation. The higher reputation cost gives her even less incentive to falsely report G when the truth is B. So if the asset is either B or G, the adviser can be expected to report truthfully.

The problem for full revelation in our earlier example arose because of the adviser’s incentive to lie when the asset is M. With our earlier numbers, her payoff from reporting G when the truth is M was higher than that from reporting M truthfully. Will that still be true with the higher reputation costs?

Suppose the truth is M and the adviser reports G. If you believe her and invest in the asset, her expected payoff is 2 (her fee) + 0.2×1 (her share in the actual return from M) − 4 (her reputation cost) = $-1.8 < 0$. The truth would get her 0. She no longer has the temptation to exaggerate the quality of the asset. The outcome where she always reports the truth, and you believe her and act upon her report, is now a cheap talk equilibrium with full revelation. This equilibrium has the finest possible partition, consisting of three singleton subsets, {B}, {M}, and {G}.

There are also *three* other equilibria in this case, each with a coarser partition than the full-revelation equilibrium. Both two-subset situations—one with {B, M} and {G} and the other with {B} and {M, G}—and the babbling situation with {B, M, G} are all alternative possible equilibria. We leave it to you to verify this. Which one prevails depends on all the considerations addressed in Chapter 4 in our discussion of games with multiple equilibria.

The biggest practical difficulty associated with attaining a non-babbling equilibrium with credible information communication lies in the players' knowledge about the extent to which their interests are aligned. The extent of alignment of the two players' interests must be common knowledge between them. In the investment example, it is critical that you know from past interactions or other credible sources (for example, a contract) that the adviser has a large reputational concern in your investment outcome. If you did not know to what extent her interests were aligned with yours, you would be justified in suspecting that she was exaggerating the quality of an asset to induce you to invest for the sake of the fee she would earn immediately.

What happens when even richer messages are possible? For example, suppose that your adviser could report a number *g*, representing her estimate of the rate of growth of the asset price, and that *g* could range over a continuum of values. In this situation, as long as the adviser gets some extra benefit if you buy a bad stock that she recommends, she has some incentive to exaggerate *g*. Therefore, a full-revelation cheap talk equilibrium is no longer possible. But a partial-revelation cheap talk equilibrium may be possible. The continuous range of growth rates may be split into intervals—say, from 0% to 1%, from 1% to 2%, and so on—such that the adviser finds it optimal to tell you truthfully into which of these intervals the actual growth rate falls, and you find it optimal to accept this advice and take your optimal action on its basis. The higher the adviser’s valuation of her reputation, the finer the possible partition will be—for example, half-percentage points instead of whole or quarter-percentage points instead of half. However, we must leave further explanation of this idea to more advanced treatments of the subject.[12]

Endnotes

- This reasoning assumes that Holmes’s payoffs are as stated, and that this fact is common knowledge between the two. If Watson suspects that Holmes wants Watson to go to Simpson’s so Holmes can go to St. Bart’s to follow up privately on a different case, Watson’s strategy will be different! Analysis of games of asymmetric information thus depends on how many different possible “types” of players are actually conceivable. Return to reference 6
- In the language of probability theory, the probability you assign to a particular event after having observed, or heard, information or evidence about that event is known as the *posterior probability* of the event. You thus assign posterior probability 1 to the stated quality of the asset. *Bayes’ theorem*, which we explain in detail in the appendix to this chapter, provides a formal quantification of the relationship between prior and posterior probabilities. Return to reference 7
- We are assuming that if you do not invest in the recommended asset, you do not find out its actual return, so the adviser can suffer no reputation cost in that case. This assumption fits nicely with the general interpretation of “cheap talk.” No message has any direct payoff consequences for the sender; those consequences arise only if the receiver acts upon the information received in the message. Return to reference 8
- The adviser’s payoff calculation does not include a 20% share of your return here. The adviser knows the truth to be B and so knows you will incur a loss, in which she will not share. Return to reference 9
- Our apologies to William Shakespeare. Return to reference 10

- Again, the adviser's payoff calculation includes no portion of your gain because you will incur a loss: the truth is B, and the adviser knows the truth. Return to reference 11
- The seminal paper by Vincent Crawford and Joel Sobel, "Strategic Information Transmission," *Econometrica*, vol. 50, no. 6 (November 1982), pp. 1431–52, developed this theory of partial communication. An elementary exposition and survey of further work is in Joseph Farrell and Matthew Rabin, "Cheap Talk," *Journal of Economic Perspectives*, vol. 10, no. 3 (Summer 1996), pp. 103–18. Return to reference 12

Glossary

cheap talk equilibrium
: In a game where communication among players (which does not affect their payoffs directly) is followed by their choices of actual strategies, a cheap talk equilibrium is one where the strategies are chosen optimally given the players' interpretation of the communication, and the communication at the first stage is optimally chosen by calculating the actions that will ensue.

babbling equilibrium
: In a game where communication among players (which does not affect their payoffs directly) is followed by their choices of actual strategies, a babbling equilibrium is one where the strategies are chosen ignoring the communication, and the communication at the first stage can be arbitrary.

4 ADVERSE SELECTION, SIGNALING, AND SCREENING

A. Adverse Selection and Market Failure

In many games, one of the players knows something pertinent to the outcomes that the other players don’t know. An employer knows much less about the skills of a potential employee than does the employee himself; vaguer but important characteristics such as work attitude and collegiality are even harder to observe. An insurance company knows much less about the health or the driving skills of someone applying for medical or auto insurance than does the applicant. The seller of a used car knows a lot about the car from long experience; a potential buyer can at best get a little information by inspection.

In such situations, direct communication will not credibly signal information. Unskilled workers will claim to have skills to get higher-paid jobs; people who are bad risks will claim good health or driving habits to get lower insurance premiums; owners of bad cars will assert that their cars run fine and have given them no trouble in all the years they have owned them. The other parties to the transactions will be aware of the incentives to lie and will not trust information conveyed by words alone. There is no possibility of a cheap talk equilibrium of the type described in Section 3.

What if the less informed parties in these transactions have no way of obtaining the pertinent information at all? In other words, to use the terminology introduced in Section 2, suppose that no credible screening devices or signals are available. If an insurance company offers a policy that costs 5 cents for each dollar of coverage, then the policy will be especially attractive to people who know that their own risk (of illness or a car crash) exceeds 5%. Of course, some people who know their risk to be lower than 5% will still buy the insurance because they are risk averse. But the pool of applicants for this insurance policy will have a larger proportion of the high-risk people than the population as a whole. The insurance company will selectively attract an unfavorable, or adverse, group of customers. This phenomenon, which is very common in transactions involving asymmetric information, is known as adverse selection (a term that, in fact, originated within the insurance industry).

The potential consequences of adverse selection for market transactions were dramatically illustrated by George Akerlof in a paper that became the starting point of economic analysis of asymmetric-information situations and won him a Nobel Prize in 2001.[13] We use his example to introduce you to the effects that adverse selection may have.

B. The Market for “Lemons”

Think of the market in 2020 for a specific kind of used car—say, a 2017 Citrus. Suppose that in use, these cars have either proved to be largely trouble free and reliable or have had many things go wrong. The usual slang name for the latter type of car is “lemon,” so for contrast, let us call the former type “orange.”

Suppose that each owner of an orange Citrus values it at $12,500; he is willing to part with it for a price higher than this, but not for a lower price. Similarly, each owner of a lemon Citrus values it at $3,000. Suppose that potential buyers are willing to pay more than these values for each type. If a buyer could be confident that the car he was buying was an orange, he would be willing to pay $16,000 for it; if the car was a known lemon, he would be willing to pay $6,000. Since the buyers value each type of car more than do the original owners, it benefits everyone if all the cars are traded. The price for an orange can be anywhere between $12,500 and $16,000; that for a lemon anywhere between $3,000 and $6,000. For definiteness, we will suppose that there is a limited stock of such cars and a larger number of potential buyers. Then the buyers, competing with one another, will drive the price up to their full willingness to pay. The prices will be $16,000 for an orange and $6,000 for a lemon—if each type can be identified with certainty.

But information about the quality of any specific car is asymmetric between the two parties to the transaction. The owner of a Citrus knows perfectly well whether it is an orange or a lemon. Potential buyers don’t, and the owner of a lemon has no incentive to disclose the truth. For now, we confine our analysis to the private used-car market, in which laws requiring truthful disclosure are either nonexistent or hard to enforce. We also assume away any possibility that the potential buyer can observe something that tells him whether the car is an orange or a lemon; similarly, the car owner has no way to indicate the type of car he owns. Thus, for this example, we consider the effects of the information asymmetry alone, without allowing either side of the transaction to signal or screen.

When buyers cannot distinguish between oranges and lemons, there cannot be distinct prices for the two types in the market. Oranges and lemons, if sold at all, must sell at the same price p, which we will refer to as the "Citrus price." Whether efficient trade is possible under such circumstances will depend on the proportions of oranges and lemons in the population. We suppose that oranges are a fraction f of used Citruses and lemons the remaining fraction $(1 - f)$.

Even though buyers cannot verify the quality of an individual car, they can know the proportion of oranges in the Citrus population as a whole—for example, from newspaper reports—and we assume this to be the case. If all Citruses are being traded, a potential buyer will expect to get a random selection, with probabilities f and $(1 - f)$ of getting an orange and a lemon, respectively. The expected value of the car purchased is $16{,}000 \times f + 6{,}000 \times (1 - f) = 6{,}000 + 10{,}000 \times f$. He will buy such a car if its expected value exceeds the Citrus price; that is, if $6{,}000 + 10{,}000 \times f > p$.

Now consider the point of view of the seller, who knows whether his car is an orange or a lemon. The owner of a lemon is willing to sell it as long as the Citrus price exceeds its value to him; that is, if $p > 3{,}000$. But the owner of an orange requires $p > 12{,}500$. If this condition for an orange owner to sell is satisfied, so is the condition for a lemon owner to sell.

To meet the requirements for all buyers and sellers to want to make the trade, therefore, we need $6{,}000 + 10{,}000 \times f > p > 12{,}500$. If the fraction of oranges in the population satisfies $6{,}000 + 10{,}000 \times f > 12{,}500$, or $f > 0.65$, a Citrus price can be found that does the job; otherwise there cannot be efficient trade. If $6{,}000 + 10{,}000 \times f < 12{,}500$ (leaving out the exceptional and unlikely case where the two are equal), owners of oranges are unwilling to sell at the maximum price potential buyers are willing to pay. We then have adverse selection in the set of used cars put up for sale: No oranges will appear in the market at all. The potential buyers will recognize this, will expect to get a lemon, and will pay at most $6,000. The owners of lemons will be happy with this outcome, so lemons will sell. But the market for oranges will collapse completely due to the asymmetry of information. The outcome will be that bad cars drive out the good.

Because the lack of information makes it impossible to get a reasonable price for an orange, the owners of oranges will want a way to convince buyers that their cars are the good type. They will want to *signal* their type. The trouble is that the owners of lemons will want to pretend that their cars, too, are oranges, and to this end they can imitate most of the signals that owners of oranges might attempt to use. Michael Spence, who developed the concept of signaling, summarizes the problems facing our orange owners in his pathbreaking book on signaling:

> Verbal declarations are costless and therefore useless. Anyone can lie about why he is selling the car. One can offer to let the buyer have the car checked. The lemon owner can make the same offer. It's a bluff. If called, nothing is lost. Besides, such checks are costly. Reliability reports from the owner's mechanic are untrustworthy. The clever non-lemon owner might pay for the checkup but let the purchaser choose the inspector.

> The problem for the owner, then, is to keep the inspection cost down. Guarantees do not work. The seller may move to Cleveland, leaving no forwarding address.[14]

In reality, the situation is not as hopeless as Spence implies. People and firms that regularly sell used cars as a business can establish a reputation for honesty and profit from this reputation by charging a markup. (Of course, some used-car dealers are unscrupulous.) Some buyers are knowledgeable about cars; some buy from personal acquaintances and can therefore verify the history of the car they are buying. Or dealers may offer warranties, a topic we will discuss in more detail shortly. And in other markets, it is harder for bad types to mimic the actions of good types, so that credible signaling is possible. For a specific example of such a situation, consider the possibility that education can signal skill. If it is to do so, it must be hard for unskilled people to acquire enough education to be mistaken for highly skilled people. The key requirement for a signal to separate types is that it be sufficiently more costly for the truly unskilled to send the signal than for the truly skilled.

C. Signaling and Screening: Sample Situations

The basic idea of signaling or screening to convey or elicit information is very simple: Players of different *types* (that is, players possessing different information about their own characteristics or about the game and its payoffs more generally) should find it optimal to take different actions so that their actions truthfully reveal their types. Situations of information asymmetry, and signaling and screening strategies to cope with them, are ubiquitous. Here are some additional situations to which the methods of analysis developed throughout this chapter can be applied.

I. INSURANCEThe prospective buyers of an insurance policy vary in their risk classes, or their levels of riskiness to the insurer. For example, among the numerous applicants for an automobile collision insurance policy will be some drivers who are naturally cautious and others who are simply less careful. Each potential customer has a better knowledge of his or her own risk class than does the insurance company. Given the terms of any particular policy, the company will make less profit (or incur a greater loss) on the higher-risk customers. However, the higher-risk customers will be the ones who find the specified policy more attractive. Thus, the company will attract the less favorable group of customers, and we have a situation of adverse selection.[15] Clearly, the insurance company would like to distinguish between the risk classes. They can do so by using a screening device.

Suppose as an example that there are just two risk classes. The company can then offer two policies, from which any individual customer chooses one. The first has a lower premium (in units of cents per dollar of coverage), but covers a lower percentage of any loss incurred by the customer; the second has a higher premium, but covers a higher percentage, perhaps even 100%, of the customer's loss. (In the case of collision insurance, the loss represents the cost of having an auto body shop complete the needed repairs to the customer's car.) A higher-risk customer is more likely to suffer a loss and is therefore more willing to pay the higher premium to get more coverage. The company can then adjust the premiums and coverage ratios so that customers of the higher-risk type choose the high-premium, high-coverage policy and customers of the lower-risk type choose the lower-premium, lower-coverage policy. If there are more risk types, there have to be correspondingly more policies in the menu offered to prospective customers; with a continuous spectrum of risk types, there may be a corresponding continuum of policies.

Of course, this insurance company has to compete with other insurance companies for the business of each customer. That competition affects the premiums it can charge and the levels of coverage it can offer. Sometimes competition may even preclude the attainment of an equilibrium, as each offering can be undercut by another.[16] But the general idea behind differential premiums and policies for customers of different risk classes is valid and important.

II. WARRANTIESMany types of durable goods—cars, computers, washing machines—vary in their quality. Any company that has produced such a good will have a pretty good idea of its quality. But a prospective buyer will be much less informed. Can a company that knows its product to be of high quality signal this fact credibly to its potential customers?

The most obvious, and most commonly used, signal of high quality is a good warranty. The cost of providing a warranty is lower for a genuinely high-quality product because its producer is less likely to be called on to provide repairs or replacement than is a company with a shoddier product. Therefore, warranties can serve as credible signals of quality, and consumers are intuitively aware of this fact when they make their purchase decisions.

Typically in such situations, the signal has to be carried to excess in order to make it sufficiently costly to mimic. Thus, the producer of a high-quality car has to offer a sufficiently long or strong warranty to signal its quality credibly. This requirement is especially relevant for any company that is a relative newcomer to the market or one that does not have a previous reputation for offering high-quality products. Hyundai, for example, began selling cars in the United States in 1986, and for its first decade had a low-quality reputation. In the mid-1990s, it invested heavily in better technology, design, and manufacturing. To revamp its image, it offered the then-revolutionary 10-year, 100,000-mile warranty. Now it ranks with consumer groups as one of the better-quality automobile manufacturers.

III. PRICE DISCRIMINATIONThe buyers of most products are heterogeneous in terms of their willingness to pay, their willingness to devote time to searching for a better price, and so on. Companies would like to identify those potential customers with a higher willingness to pay and charge them one—presumably fairly high—price while offering selective good deals to those who are not willing to pay so much (as long as that willingness to pay still exceeds the cost of supplying the product). A company can successfully charge different prices to different types of customers by using screening devices to separate the types. We will discuss such strategies, known as price discrimination, in more detail in Chapter 14. Here we provide just a brief overview.

The example of discriminatory prices best known to most people comes from the airline industry. Business travelers are willing to pay more for their airline tickets than are tourists, often because at least some of the cost of the ticket is borne by the business traveler’ s employer. It would be illegal for airlines blatantly to identify each traveler’ s type and to charge different types different prices. But the airlines take advantage of the fact that tourists are more willing to commit to an itinerary well in advance, while business travelers need to retain flexibility in their plans. Therefore, airlines charge different prices for nonrefundable versus refundable fares and leave it to the travelers to choose their fare type. This pricing strategy is an example of *screening by self-selection*, which we will investigate more formally in Section 5 of this chapter. Other devices—advance purchase or Saturday night stay requirements; different classes of onboard service (first versus business versus coach)—serve the same screening purpose.

Price discrimination is not specific to high-priced products like airline tickets. Other discriminatory pricing schemes can be observed in many markets where product prices are considerably lower than those for air travel. Coffee and sandwich shops, for example, commonly offer “frequent buyer” discount cards. These cards effectively lower the price of coffee or a sandwich to the shop’ s regular customers. The idea is that regular customers are more willing to search for the best deal in the neighborhood, while visitors or occasional users would go to the first coffee or sandwich shop they see without spending the time necessary to determine whether any lower prices might be available. The higher regular price and “free 11th item” discount represent the menu of options from which the two types of customers select, thereby separating themselves by type.

IV. PRODUCT DESIGN AND ADVERTISINGCan an attractive, well-designed product exterior serve the purpose of signaling high quality? The key requirement for a credible signal is that its cost be sufficiently higher for a company trying to mimic high quality than for one that has a truly high-quality product. Typically, the cost of the product exterior is the same regardless of the innate quality that resides within. Therefore, the mimic would face no cost differential, and the signal would not be credible.

But such signals may have some partial validity. Exterior design is a fixed cost that is spread over the whole product run. Buyers do learn about quality from their own experience, from friends, and from reviews and comments in the media. These considerations indicate that a high-quality good can expect to have a longer market life and higher total sales. Therefore, the cost of an expensive exterior is spread over a larger volume, and adds less to the cost of each unit of the product, if that product is of higher innate quality. The firm is in effect making a statement: “We have a good product that will sell a lot. That is why we can afford to spend so much on its design. A fly-by-night firm would find this cost prohibitive for the few units it expects to sell before people find out its poor quality and don’t buy any more from it.” Even expensive, seemingly useless and uninformative product launch and advertising campaigns can have a similar signaling effect.[17]

Similarly, when you walk into a bank and see solid, expensive marble counters and plush furnishings, you may be reassured about the bank’s stability. However, for this particular signal to work, it is important that the building, furnishings, and decor be specific to the bank. If everything could easily be sold to other types of establishments and the space converted into a restaurant, then a fly-by-night operator could mimic a truly solid bank at no higher cost. In that situation, the signal would not be credible.

V. CRIMEThe world of organized crime is full of interactions that require signaling and screening. How do you know whether a prospective new member of your group is a police infiltrator? How do you know whether your partner in an illegal transaction will cheat you when your deal cannot be enforced in a court of law? People in that world have devised many strategies that exemplify some basic principles of games with asymmetric information. Here are just a few examples.[18]

To become a full member of a Mafia family, a candidate must "make his bones" (establish his bona fides or credibility) by committing a serious crime, often murder. More than just a test of toughness and ability, this requirement is a good screening device. Someone who truly wants to belong to the organization will go that far, but a police infiltrator will not. Infiltrators have been able to penetrate close to this level, however, by hanging out with criminals for a long time and mimicking their behavior, dress, and talk closely. Thus, Joseph Pistone was able to pass himself off as the Mafia hanger-on Donnie Brasco in the 1997 film of the same name because "it just was very hard for mobsters to think that, taken together, all the things he did and did not do were not near-perfect discriminating signals."[19]

The process of identifying potential business partners is risky and error-prone for criminals. The recipient of a signal may be a police informer or entrapper. Or he may be an innocent outsider who is horrified when he realizes the criminal intent of the signaler and goes to the police. The strategy for reducing these risks, in theory as well as in reality, is to use a multistage process, starting with a deliberately ambiguous signal. If the recipient reacts adversely, the signal can be explained away as meaning something different and innocent. If it is received favorably, a slightly less ambiguous signal is sent, and so on.

The 1951 British comedy *The Lavender Hill Mob* provides a brilliant example. Henry Holland (played by Alec Guinness) is a lowly bank clerk whose job is to supervise transport of large quantities of gold bars from one bank to another. He would like to steal a consignment and escape to live in luxury in an idyllic tropical spot. But he recognizes the difficulty of getting the bullion out of the country. At his lodging, he meets Alfred Pendlebury (Stanley Holloway), who runs a foundry where lead is melted to cast Eiffel Tower replica paperweights that are exported to France and sold as souvenirs at the actual tower. Holland coyly sounds out Pendlebury on the topic of partnering to steal the bullion by pondering aloud what it would take to do so successfully. As a last hint, he notes that "supposing one had the right sort of partner," one could get the gold to Europe "in the form of, shall we say, Eiffel Tower paperweights." Pendlebury remarks with a laugh, "By Jove, Holland, it's a good job we're both honest men." Holland, with a look of mock solemnity, replies, "It is indeed, Pendlebury." They understand each other's implied agreement and proceed to plan the job, but any allegation of criminality to that point remained deniable.[20]

Another excellent example of signaling comes from *Cogan's Trade*, a novel about Boston mafiosi. A character named Mark Trattman runs a high-stakes poker game under Mafia protection, and he arranges for it to be robbed in exchange for a cut of the proceeds. By the time the bosses find this out, the fuss has died down, and Trattman is well liked, so they do nothing. But then someone else gets the idea that if he robs the game, Trattman will be blamed again. The bosses detect the truth, and now they face a bigger problem: Their reputation as good protectors has been ruined and must be rebuilt. They need an effective signal, and to be credible in the standard game-theoretic sense, they must carry it to overkill—in this context, literally so. Even though Trattman is innocent this time, he has to be killed. Cogan, the up-and-coming enforcer, explains this very clearly to the godfather's consigliere[21]: "It's his responsibility. He did it before and he lied before and he fooled everybody, and I said . . . 'They should have whacked him out before.' . . . Now it happened again. . . . It's his responsibility for what the guys think. . . . Trattman did it before, [they think] Trattman did it again. . . . Trattman's gotta be hit." Of course, Cogan also "whacks out" the two actual perpetrators of the second robbery. First he forms a temporary alliance with one of them to set up the other, promising to spare him in exchange. The other fails to realize that the promise is not credible. Failing to check credibility, to see through cheap talk, and to figure out the proper rollback solution of a game can have life-and-death consequences!

VI. POLITICAL BUSINESS CYCLESIncumbent governments often increase spending to get the economy to boom just before an election, thereby hoping to attract more votes and win the election. But shouldn't rational voters see through this stratagem and recognize that as soon as the election is over, the government will be forced to retrench, perhaps setting off a recession? For pre-election spending to be an effective signal of type, there has to be some uncertainty in the voters' minds about the "competence type" of the government. The likely recession will create a political cost for the government, but this cost will be smaller if the government is competent in its handling of the economy. If the cost differential between competent and incompetent government types is large enough, a sufficiently high expenditure spike can credibly signal competence.[22]

Another similar example relates to inflation controls. Many countries at many times have suffered high inflation, and many governments have piously declared their intention to reduce the rate of inflation. Can a government that truly cares about price stability credibly signal its type? Yes. Governments can issue bonds protected against inflation. The interest rate on such a bond is automatically ratcheted up by the rate of inflation, or the capital value of the bond rises in proportion to the increase in the price level. Issuing government debt in this form is more costly to a government that embraces policies that lead to higher inflation because it has to make good on its contract to pay more interest or increase the value of its debt. Therefore, a government with genuinely anti-inflation preferences can issue inflation-protected bonds as a credible signal, separating itself from the inflation-loving type of government.

VII. EVOLUTIONARY BIOLOGYIn many species of birds, such as peafowl, the males (peacocks) have very elaborate and heavy plumage, which females (peahens) find attractive. One should expect the females to seek genetically superior males so that their offspring will be better equipped to survive to adulthood and to attract mates in their turn. But why does elaborate plumage indicate such desirable genetic qualities? One would think that such plumage might be a handicap, making the male bird more visible to predators (including human hunters) and less able to evade them. Why do females choose these seemingly handicapped males? The answer comes from the conditions for credible signaling. Although heavy plumage is indeed a handicap, it is less of a handicap to a male who is sufficiently genetically superior in qualities such as strength and speed. The weaker the male, the harder it will be for him to produce and maintain plumage of a given quality. Thus, it is precisely the heaviness of the plumage that makes it a credible signal of the male’s quality.[23]

VIII. AI AND COMPUTERS PLAYING SIGNALING GAMESIn Chapter 3, Section 5, we discussed how artificial intelligence has revolutionized the ability of computers to play games like chess and Go. However, the games we described there were all games of perfect and complete information. More recently, AI has progressed sufficiently to allow computers to play games involving hidden information and strategies like signaling or screening. By playing no-limit Texas Hold’em poker trillions of times against itself and then improving its strategies further in play against top humans, a computer AI program called Pluribus, designed by Noam Brown of Facebook and Tuomas Sandholm of Carnegie Mellon University, was able to defeat six of the world's top (human) poker players. During its learning process, Pluribus devised some previously unused strategies, such as dramatically upping the ante of small pots, that human pros are now mimicking.[24]

D. Experimental Evidence

The characterization of and solutions for equilibria in games with signaling and screening entail some subtle concepts and computations. Thus, in each case presented above, formal models must be carefully described in order to formulate reasonable and accurate predictions of player choices. In all such games, players must revise or update their beliefs about the probabilities of other players' types on the basis of their observations of those other players' actions. This updating requires an application of an algebraic formula called Bayes' theorem (also referred to as *Bayes' rule* or *Bayes' formula*), which is explained in the appendix to this chapter. We will carefully analyze an example of a game that requires this kind of updating in Section 6 of this chapter.

You can imagine, without going into any of the details in the appendix, that these probability-updating calculations are quite complex. Should we expect players to perform them correctly? There is ample evidence that people are very bad at performing calculations that include probabilities and are especially bad at updating probabilities on the basis of new information.[25] Therefore, we should be justifiably suspicious of equilibria that depend on the players' doing so.

Relative to this expectation, the findings of economists who have conducted laboratory experiments on signaling games are encouraging. Some surprisingly subtle refinements of *Bayesian Nash* and *perfect Bayesian equilibria* are successfully observed, even though these refinements require not only the updating of information by observing actions along the equilibrium path of play, but also the drawing of inferences from off-equilibrium actions that should never have been taken in the first place. However, the verdict of the experiments is not unanimous; much seems to depend on the precise details of the design of the experiment.[26]

Endnotes

- George Akerlof, "The Market for Lemons: Qualitative Uncertainty and the Market Mechanism," *Quarterly Journal of Economics*, vol. 84, no. 3 (August 1970), pp. 488–500. Return to reference 13
- A. Michael Spence, *Market Signaling: Information Transfer in Hiring and Related Screening Processes* (Cambridge, Mass.: Harvard University Press, 1974), pp. 93–94. The present authors apologize on behalf of Spence to any residents of Cleveland who may be offended by any unwarranted suggestion that that's where shady sellers of used cars go! Return to reference 14
- Here we are not talking about the possibility that a well-insured driver will deliberately exercise less care. That is moral hazard, and it can be mitigated using co-insurance schemes similar to those discussed here. For now, our concern is purely adverse selection, where some drivers are careful by nature, and others are equally uncontrollably spaced out and careless when they drive. Return to reference 15
- See Michael Rothschild and Joseph Stiglitz, "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information," *Quarterly Journal of Economics*, vol. 90, no. 4 (November 1976), pp. 629–49. Return to reference 16
- Kyle Bagwell and Gary Ramey, "Coordination Economies, Advertising, and Search Behavior in Retail Markets," *American Economic Review*, vol. 84, no. 3 (June 1994), pp. 498–517. Return to reference 17
- Probably the best serious scholar of this underworld (as opposed to numerous sensational writers) is sociologist Diego Gambetta. We highly recommend his *Codes of the Underworld: How Criminals Communicate* (Princeton, N.J.: Princeton University Press, 2009). Return to reference 18

- Gambetta, *Codes of the Underworld*, p. 22. See footnote 18. Return to reference 19
- Readers will surely recognize from their personal lives that the same graduated revelation of interest is the correct strategy when exploring the possibility of a romantic relationship. Return to reference 20
- George V. Higgins, *Cogan's Trade* (New York: Carroll and Graf, 1974), Chapter 8. Return to reference 21
- These ideas and the supporting evidence are reviewed by Alan Drazen in "The Political Business Cycle after 25 Years," in *NBER Macroeconomics Annual 2000*, ed. Ben S. Bernanke and Kenneth S. Rogoff (Cambridge, Mass.: MIT Press, 2001), pp. 75–117. Return to reference 22
- Matt Ridley, *The Red Queen: Sex and the Evolution of Human Behavior* (New York: Penguin, 1995), p. 148. Return to reference 23
- The exploits of Pluribus and its creators are detailed in "35 Innovators Under 35," *MIT Technology Review*, vol. 122, no. 4 (July/August 2019), p. 9; and "Bet on the Bot: AI Beats the Professionals at 6-Player Texas Hold 'Em," *All Things Considered*, NPR, WBUR, Boston, 11 July 2019. Return to reference 24
- Deborah J. Bennett, *Randomness* (Cambridge, Mass.: Harvard University Press, 1998), pp. 2–3 and Chapter 10. See also Paul Hoffman, *The Man Who Loved Only Numbers* (New York: Hyperion, 1998), pp. 233–40, for an entertaining account of how several probability theorists, as well as the brilliant and prolific mathematician Paul Erdös, got a very simple probability problem wrong and even failed to understand their error when it was explained to them. Return to reference 25
- Douglas D. Davis and Charles A. Holt, *Experimental Economics* (Princeton, N.J.: Princeton University Press, 1995), review and discuss these experiments in their Chapter 7. Return to reference 26

Glossary

adverse selection
: A form of information asymmetry where a player's type (available strategies, payoffs . . .) is his private information, not directly known to others.

Bayes' theorem
: An algebraic formula for estimating the probabilities of some underlying event by using knowledge of some consequences of it that are observed.

5 SIGNALING IN THE LABOR MARKET

Many of you expect that when you graduate, you will work for an elite firm in finance or computing. These firms have two kinds of jobs. One kind requires good quantitative and analytical skills and a high capacity for hard work, and offers high pay in return. The other kind of job is a semi-clerical, lower-skill, lower-paying job. Of course, you want the job with higher pay. You know your own qualities and skills far better than your prospective employer does. If you are highly skilled, you want your employer to know this about you, and he also wants to know. He can test and interview you, but what he can find out by these methods is limited by the available time and resources. You can tell him how skilled you are, but mere assertions about your qualifications are not credible. More objective evidence is needed, both for you to offer and for your employer to seek out.

What items of evidence can the employer seek, and what can you offer? Recall from Section 2 of this chapter that your prospective employer will use *screening devices* to identify your qualities and skills. You will use *signals* to convey your information about those same qualities and skills. Sometimes similar or even identical devices can be used for both signaling and screening.

In this instance, if you have selected (and passed) particularly tough and quantitative courses in college, your course choices can be credible evidence of your capacity for hard work in general and of your quantitative and logical skills in particular. Let us consider the role of course choice as a screening device.

A. Screening to Separate Types

To keep things simple, we approach this screening game using intuition and some algebra. Suppose college students are of just two types when it comes to the qualities most desired by employers: A (able) and C (challenged). Potential employers in finance or computing are willing to pay $160,000 a year to a type A and $60,000 to a type C. Other employment opportunities yield the A types a salary of $125,000 and the C types a salary of $30,000. These are the same numbers as in the Citrus car example in Section 4.B above, but multiplied by a factor of 10 better to suit the reality of the job-market example. And just as in the used-car example, where we supposed there was a fixed supply and numerous potential buyers, we suppose here that there are many potential employers who have to compete with one another for a limited number of job candidates, so they have to pay the maximum amount that they are willing to pay. Because employers cannot directly observe any particular job applicant's type, they have to look for other credible means to distinguish among them.[27]

Suppose the types differ in their tolerance for taking tough courses rather than easy ones in college. Each type must sacrifice some party time or other activities to take a tougher course, but this sacrifice is smaller or easier to bear for the A types than it is for the C types. Suppose the A types regard the cost of each such course as equivalent to $3,000 a year of salary, while the C types regard it as equivalent to $15,000 a year of salary. Can an employer use this differential to screen his applicants and tell the A types from the C types?

Consider the following hiring policy: Anyone who has taken a certain number, *n*, or more of tough courses will be regarded as an A and paid \$160,000, and anyone who has taken fewer than n will be regarded as a C and paid \$60,000. The aim of this policy is to create natural incentives whereby only the A types will take tough courses, and the C types will not. Neither wants to take more tough courses than he has to, so the choice is between taking n to qualify as an A or giving up and settling for being regarded as a C, in which case he may as well not take any of the tough courses and just coast through college.

To succeed, such a policy must satisfy two kinds of conditions. The first set of conditions requires that the policy give each type of job applicant the incentive to make the choice that the firm wants him to make. In other words, the policy should be compatible with the incentives of the applicants; therefore, conditions of this kind are called incentive-compatibility conditions. The second set of conditions ensures that, by making the incentive-compatible choice, the applicants get a better (at least, no worse) payoff than they would get from their alternative opportunities. In other words, the applicants should be willing to participate in the firm's offer; therefore, conditions of this kind are called participation conditions. We will develop these conditions in the labor market context now. Similar conditions will appear in other examples later in this chapter and again in Chapter 14, where we develop the general theory of incentive design.

I. INCENTIVE COMPATIBILITYThe criterion that the employer devises to distinguish an A from a C—here, the number of tough courses taken—should be sufficiently strict that the C types do not bother to meet it, but not so strict as to discourage even the A types from attempting it. The correct value of n must be such that the true C types prefer to settle for being revealed as such and getting \$60,000, rather than incurring the extra cost of imitating the A types' behavior.[28] That is, we need the policy to be incentive compatible for the C types, so

$$60{,}000 \geqslant 160{,}000 - 15{,}000n, \text{ or } 15n \geqslant 100, \text{ or } n \geqslant 6.67.$$

Similarly, the condition such that the true A types prefer to prove their type by taking n tough courses is

$$160{,}000 - 3{,}000n \geqslant 60{,}000, \text{ or } 3n \leqslant 100, \text{ or } n \leqslant 33.33.$$

These incentive-compatibility conditions or, equivalently, incentive-compatibility constraints, align a job applicant's incentives with the employer's desires, or make it optimal for the applicant to reveal the truth about his skill through his action. The n satisfying both constraints, because it is required to be an integer, must be at least 7 and at most 33.[29] The higher end of the range is not realistically relevant in this example, as an entire college program is typically 32 courses, but in other contexts it might matter.

What makes it possible to meet both conditions is that the two types have *sufficiently different* costs of taking tough courses: In particular, the "good" type that the employer wishes to identify has a sufficiently lower cost. When the constraints are met, the employer can devise a policy to which the two types will respond differently, thereby revealing their types. In this case, we get screening based on self-selection, which is an example of separation of types.

We did not assume here that the tough courses actually imparted any additional skills or work habits that might convert C types into A types. In our scenario, the tough courses serve only the purpose of identifying those who already possess these attributes. In other words, they have a pure screening function. In reality, education does increase productivity. But it also has the additional screening or signaling function of the kind described here. In our example, we found that education might be undertaken solely for the latter function; in reality, the corresponding outcome is that education is carried further than is justified by increased productivity alone. This extra education carries an extra cost—the cost of the information asymmetry.

II. PARTICIPATIONWhen the incentive-compatibility conditions for the two types of jobs in our firm are satisfied, the A types will take n tough courses and get a payoff of 160,000 − 3,000n, and the C types will take no tough courses and get a payoff of 60,000. For the applicants to be willing to make these choices instead of taking their alternative opportunities, the participation conditions must be satisfied as well. So we need

$$160{,}000 - 3{,}000n \geqslant 125{,}000, \text{ and } 60{,}000 \geqslant 30{,}000.$$

The C types' participation condition is trivially satisfied in this example (although that may not be the case in other examples); the A types' participation condition requires $n \leqslant 11.67$, or, since n must be an integer, $n \leqslant 11$. Here, any n that satisfies the A types' participation condition of $n \leqslant 11$ also satisfies their incentive compatibility condition of $n \leqslant 33$, so the latter condition becomes logically redundant.

The full set of conditions that are required to achieve separation of types and to attain a *separating equilibrium* in this labor market is then $7 \leqslant n \leqslant 11$. This restriction on possible values of n combines the incentive-compatibility condition for the C types and the participation condition for the A types. The participation condition for the C types and the incentive-compatibility condition for the A types in this example are automatically satisfied when the other conditions hold.

When the requirement of taking enough tough courses is used for screening, the A types bear the cost. Assuming that only the minimum number of tough courses needed to achieve separation is used—namely, $n = 7$—the cost to each A type has the monetary equivalent of $7 \times 3,000 = 21,000$. This is the cost, in this context, of the information asymmetry. It would not exist if an applicant's type could be directly and objectively identified. Nor would it exist if the population consisted solely of A types. The A types have to bear this cost because there are some C types in the population from whom they (or their prospective employers) seek to distinguish themselves.[30]

B. Pooling of Types

Rather than having the A types bear the cost of the information asymmetry, might it be better not to bother with the separation of types at all? With the separation, A types get a salary of $160,000 but suffer a cost, the monetary equivalent of $21,000, in taking the tough courses; thus, their net payoff is $139,000. And C types get a salary of $60,000. What happens to the two types if they are not separated?

If employers do not use screening devices, they have to treat every applicant as a random draw from the population and pay all the same salary. When there is no way to distinguish different types, we say that there is pooling of types, or simply pooling when the sense is clear. *Pooling of types* is therefore the opposite of *separation of types*, discussed earlier. In a competitive job market, the common salary under pooling will be the population average of what an applicant is worth to an employer, and this average will depend on the proportions of the types in the population. For example, if 65% of the population is type A and 35% is type C, then the common salary under pooling will be

$$0.65 \times 160{,}000 + 0.35 \times 60{,}000 = 125{,}000.$$

The A types will then prefer the situation with separation because it yields $139,000 instead of the $125,000 with pooling. But if the proportions are 80% A and 20% C, then the common salary with pooling will be $140,000, and the A types will be worse off under separation than they would be under pooling. The C types are always better off under pooling. The existence of the A types in the population means that the common salary with pooling will always exceed the C types' salary of $60,000 under separation.

However, even if both types prefer the outcome under pooling, it cannot be an equilibrium when many employers or applicants compete with one another in the screening or signaling process. Suppose the population proportions are 80–20 and there is an initial situation with pooling where both types are paid $140,000. An employer can announce that he will pay $144,000 for someone who takes just one tough course. Relative to the initial situation, the A types will find this offer worthwhile because their cost of taking the course is only $3,000 and it raises their salary by $4,000, whereas the C types will not find it worthwhile because their cost, $15,000, exceeds the benefit, $4,000. Because this particular employer selectively attracts the A types, each of whom is worth $160,000 to him but is paid only $144,000, he makes a profit by deviating from the pooling salary.

But his deviation starts a process of adjustment by competing employers, and that process causes the old pooling situation to collapse. As A types flock to work for him, the pool that remains available to other employers has a lower average quality, and eventually those employers cannot afford to pay $140,000 anymore. As the salary in the pool is lowered, the differential between that salary and the $144,000 offered by the deviating employer widens to the point where the C types also find it desirable to take that one tough course. But then the deviating employer must raise his requirement to two courses and must increase the salary differential to the point where two courses become too much of a burden for the C types, but the A types find it acceptable. Other employers who would like to hire some A types must use similar policies to attract them. This process continues until the job market reaches the separating equilibrium described earlier.

Even if the employers did not take the initiative to attract As rather than Cs, a type A earning $140,000 in a pooling situation might take a tough course, take his transcript to a prospective employer, and say, “I have a tough course on my transcript, and I am asking for a salary of \$144,000. This should be convincing evidence that I am type A; no type C would make you such a proposition.” Given the facts of the situation, the argument is valid, and the employer should find it very profitable to agree: The employee, being type A, will generate \$160,000 for the employer but get only \$144,000 in salary. Other A types can do the same. This starts the same kind of cascade that leads to the separating equilibrium. The only difference is in who takes the initiative. Now the type A workers choose to get the extra education as credible proof of their type; their actions constitute signaling rather than screening.

The general point is that, even though the outcome under pooling may be better for all, the employers and workers are playing a game in which pursuing their own individual interests leads them to the separating equilibrium. This game is like a prisoners’ dilemma with many players, and therefore there is something unavoidable about the cost of the information asymmetry.

C. Many Types

We have considered an example with only two types to be separated, but the analysis generalizes immediately. Suppose there are several types: A, B, C, . . . , ranked in an order that is at the same time decreasing in their worth to an employer and increasing in the costs of extra education. Then it is possible to set up a sequence of requirements for successively higher levels of education such that the very worst type needs none, the next-worst type needs the lowest level, the type third from the bottom needs the next higher level, and so on, and the types will self-select the level that identifies them.

To finish this discussion, we provide one further point, or perhaps a word of warning, regarding signaling. Suppose you are the informed party and have available an action that would credibly signal good information (information whose credible transmission would work to your advantage). If you fail to send that signal, you will be assumed to have bad information. In this respect, signaling is like playing chicken: If you refuse to play, you have already played and lost.

You should keep this in mind when you have the choice between taking a course for a letter grade or on a pass/fail basis. The whole population in the course spans the whole spectrum of grades; suppose the average is B. A student is likely to have a good idea of his own abilities. Those reasonably confident of getting an A+ have a strong incentive to take the course for a letter grade. Once they have made that choice, the average grade for the remaining students is less than B, say, B−, because the top end has been removed from the distribution. Now, among these students, those expecting an A have a strong incentive to choose the letter-grade option. That in turn lowers the average of the rest. And so on. Finally, the pass/fail option is chosen by only those anticipating Cs and Ds. A strategically smart reader of a transcript (a prospective employer or the admissions officer for a professional graduate school) will be aware that the pass/fail option will be selected mainly by students in the lower portion of the grade distribution; such a reader will therefore interpret a Pass as a C or a D, not as the class-wide average B.

Endnotes

- You may wonder whether the fact that the two types have different opportunities at other firms can be used to distinguish between them. For example, an employer may say, “Show me an offer of a job at $125,000, and I will accept you as type A and pay you $160,000.” However, such a competing offer can be forged or obtained in cahoots with someone else, so it is not reliable. Return to reference 27
- We require merely that the payoff from choosing the option intended for one’s type be at least as high as that from choosing a different option, not that it be strictly greater. However, it is possible to approach the outcome of this analysis as closely as one wants while maintaining a strict inequality, so nothing substantial hinges on this assumption. Return to reference 28
- If in some other context the corresponding choice variable is not required to be an integer—for example, if it is a sum of money or an amount of time—then a whole continuous range will satisfy both incentive-compatibility constraints. Return to reference 29
- In the terminology of economics, the C types in this example inflict a *negative external effect* on the A types. We will develop this concept in Chapter 11. Return to reference 30

Glossary

incentive-compatibility condition (constraint)

A constraint on an incentive scheme or screening device that makes it optimal for the agent (more-informed player) of each type to reveal his true type through his actions.

participation condition (constraint)

A constraint on an incentive scheme or a screening device that should give the more-informed player an expected payoff at least as high as he can get outside this relationship.

incentive-compatibility condition (constraint)

A constraint on an incentive scheme or screening device that makes it optimal for the agent (more-informed player) of each type to reveal his true type through his actions.

self-selection

Where different types respond differently to a screening device, thereby revealing their type through their own action.

separation of types

An outcome of a signaling or screening game in which different types follow different strategies and get the different payoffs, so types can be identified by observing actions.

pooling of types

An outcome of a signaling or screening game in which different types follow the same strategy and get the same payoffs, so types cannot be distinguished by observing actions.

pooling

Same as **pooling of types**.

6 EQUILIBRIA IN TWO-PLAYER SIGNALING GAMES

Our analysis so far in this chapter has covered the general concept of incomplete information as well as the specific strategies of screening and signaling; we have also seen some of the outcomes—separation and pooling—that can arise when these strategies are being used. We have seen how adverse selection could arise in a market where many car owners and buyers come together, and how signals and screening devices would operate in an environment where many employers and employees meet one another. However, we have not described and solved a game in which just two players with differential information confront each other. Here, we develop an example to show how that can be done using a game tree and payoff table as our tools of analysis. We will see that the outcome can be a *separating equilibrium*, a *pooling equilibrium*, or a third type, called a *semiseparating equilibrium*.

A. Basic Model and Payoff Structure

In this section, we analyze a game of market entry with asymmetric information. The players are two auto manufacturers, Tudor and Fordor. Tudor Auto Corporation currently enjoys a monopoly in the market for a particular kind of automobile—say, a nonpolluting, fuel-efficient compact car. An innovator, Fordor, has a competing concept and is deciding whether to enter the market. But Fordor does not know how tough a competitor Tudor will prove to be. Specifically, Tudor's production cost, unknown to Fordor, may be high or low. If it is high, Fordor can enter the market and compete profitably; if it is low, Fordor's entry and development costs will not be recouped by its subsequent operating profit, and it will incur a net loss if it enters.

The two firms interact in a sequential-move game. In the first stage of the game (period 1), Tudor sets a price (high or low, for simplicity), knowing that it is the only manufacturer in the market (that is, that it has a monopoly). In the next stage (period 2), Fordor makes its entry decision. Payoffs, or profits, are determined by the market price of a car relative to each firm's production costs and, for Fordor, its market entry and development costs as well.

Tudor, of course, would prefer that Fordor not enter the market. It might therefore try to use its price in the first stage of the game as a signal of its cost. A low-cost firm would charge a lower price than would a high-cost firm. Tudor might therefore hope that if it keeps its period 1 price low, Fordor will interpret this as evidence that Tudor's cost is low, and will stay out. (In later periods, once Fordor has given up and is out of the picture, Tudor could jack its price back up.) Just as a poker player might bet on a poor hand, hoping that the bluff will succeed and the opponent will fold, Tudor might try to bluff Fordor into staying out. Of course, Fordor, too, is a strategic player, and is aware of this possibility. The question is whether Tudor can bluff successfully in an equilibrium of this game. The answer depends on the probability that Tudor has a genuinely low production cost and on Tudor’s cost of bluffing. We consider different cases below and show the differing equilibria that result.

In all the cases, the per-unit costs and prices are expressed in thousands of dollars, and the numbers of cars sold are expressed in hundreds of thousands, so the profits are measured in hundreds of millions of dollars. This format will help us write the payoffs and tables in a relatively compact form that is easy to read. We calculate those payoffs using the same type of analysis that we used for the restaurant pricing game in Chapter 5, assuming that the underlying relationship between the price charged per car (P) and the quantity of cars demanded (Q) is given by $P = 25 - Q$.[31] To enter the market, Fordor must incur an up-front cost of 40 (this amount is in the same units as profits, or hundreds of millions, so the actual figure is \$4 billion) to build its plant, launch an ad campaign, and so on. If it enters the market, its cost for producing and delivering each of its cars to the market will always be 10 (thousand dollars).

Tudor could be either an old, lumbering firm with a high per-unit production cost of 15 or a nimble, efficient producer with a lower per-unit cost. To start, we suppose that the lower cost is 5; this cost is less than what Fordor can achieve. Later, in Sections 6.C and 6.D, we will investigate the effects of other cost levels. For now, suppose further that Tudor can achieve the lower per-unit cost with probability 0.4, or 40% of the time; therefore, it has the higher per-unit cost with probability 0.6, or 60% of the time.[32]

Fordor’s choices in the market entry game will depend on how much information it has about Tudor’s costs. We assume that Fordor knows Tudor’s two possible cost types—high (at 15) and low (at 5)—and therefore can calculate the profits associated with each type (as we do below). In addition, Fordor will form some belief about the probability that Tudor is the low-cost type. We are assuming that the structure of the game is common knowledge to both players. Therefore, although Fordor does not know the type of the specific Tudor it is facing, Fordor’s prior belief exactly matches the probability with which Tudor has the lower cost; that is, Fordor’ s belief is that the probability of facing a low-cost Tudor is 40%.

If Tudor’ s cost is high, at 15 (thousand), then under conditions of unthreatened monopoly in period 1, it will maximize its profit by pricing its car at 20 (thousand). At that price, it will sell 5 (hundred thousand) units and make a profit of 25 [= 5 × (20 − 15) hundred million, or $2.5 billion]. If Fordor enters and the two compete in period 2, then the Nash equilibrium of their duopoly game will yield a market price of 17 and operating profits of 3 to Tudor and 45 to Fordor. (In all cases, we have rounded to the nearest integer for simplicity.) Fordor’ s operating profit would exceed its up-front cost of entry (40), so Fordor would choose to enter and earn a net profit of 5 if it knew Tudor to be the high-cost type.

If Tudor’ s cost is low, at 5, then as a monopoly, it will price its car at 15, selling 10 units and making a profit of 100. In the second-stage equilibrium following the entry of Fordor, the market price of a car will be 13 and operating profits will be 69 for Tudor and 11 for Fordor (again, rounded to the nearest integer). The 11 profit would be less than Fordor’ s cost of entry of 40. Therefore, it would not enter, and would avoid incurring a loss of 29, if it knew Tudor to be the low-cost type.

B. Separating Equilibrium

If Tudor’ s cost is actually high, but it wants Fordor to think that its cost is low, Tudor must mimic the action of the low-cost type; that is, it must price its car at 15 in the first stage of the game. But that price equals its production cost in this case; Tudor will make a profit of 0. Will this sacrifice of initial profit give Tudor the benefit of scaring Fordor off and enjoying the benefits of being a monopoly in subsequent periods?

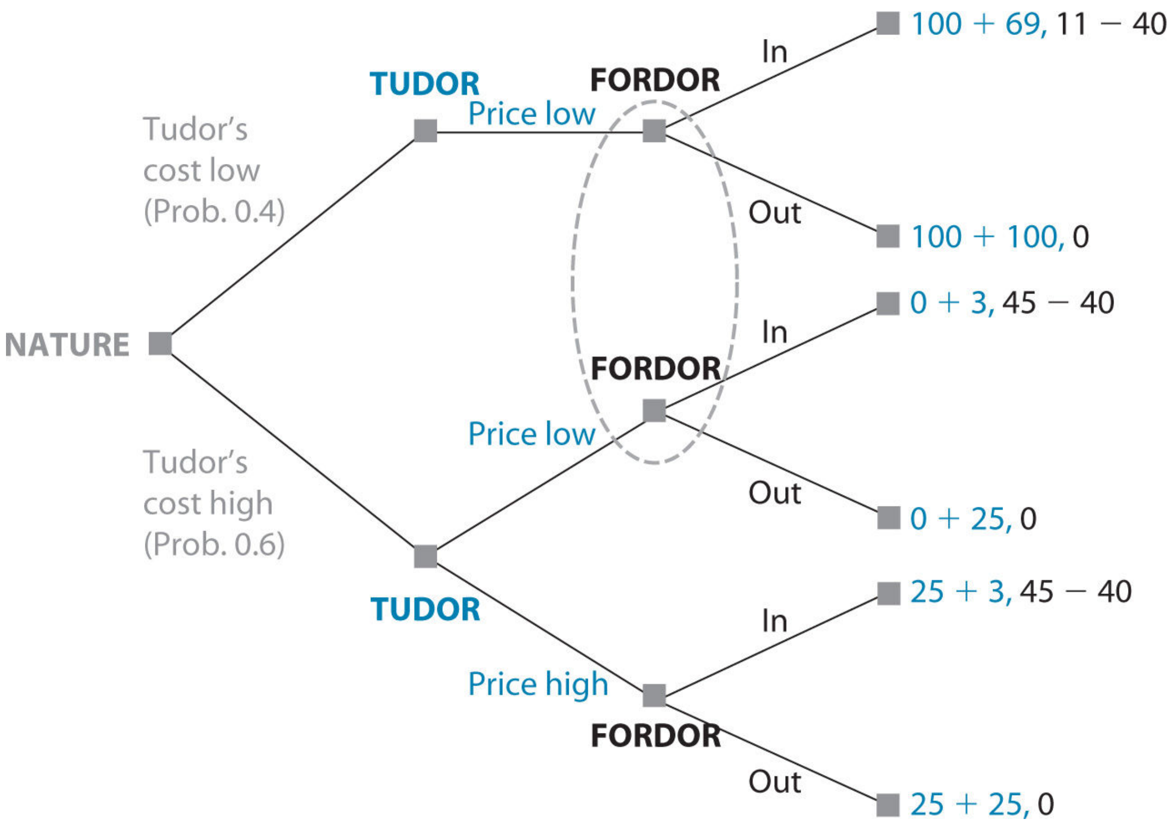


Figure 9.5 Extensive Form of Market Entry Game: Tudor’ s Low Cost Is 5

We show the full game in extensive form in Figure 9.5. Note that we use the fictitious player called Nature, as in Chapter 6, to choose Tudor’ s cost type at the start of the game. Then Tudor makes its pricing decision. We assume that if Tudor’ s cost is low, it will not choose a high price.[33] But if Tudor’ s cost is high, it may choose either the high price or, if it wants to bluff, the low price. Fordor cannot tell apart the two situations in which Tudor prices low; therefore, Fordor's two decision nodes following Tudor's choice of a low price are enclosed in one information set. Fordor must choose either In at both or Out at both.

At each terminal node, the first payoff entry (in blue) is Tudor's profit, and the second entry (in black) is Fordor's profit. Tudor's profit is added over two periods: the first period, when it is the sole producer, and the second period, when it may be a monopolist or a duopolist, depending on whether Fordor enters. Fordor's profit covers only the second period and is non-zero only when it has chosen to enter.

		FORDOR	
		Regardless (II)	Conditional (OI)
TUDOR	Bluff (LL)	169 × 0.4 + 3 × 0.6 = 69.4, −29 × 0.4 + 5 × 0.6 = −8.6	200 × 0.4 + 25 × 0.6 = 95, 0
	Honest (LH)	169 × 0.4 + 28 × 0.6 = 84.4, −29 × 0.4 + 5 × 0.6 = −8.6	200 × 0.4 + 28 × 0.6 = 96.8, 5 × 0.6 = 3

FIGURE 9.6 Strategic Form of Market Entry Game: Tudor's Low Cost Is 5

Using one step of rollback analysis, we see that Fordor will choose In at the bottom node where Tudor has chosen the high price, because 45 − 40 = 5 > 0. Therefore, we can prune the Out branch at that node. Then each player has just two strategies (complete plans of action). For Tudor, the strategies are Bluff, or choose the low price in period 1 at either cost (low-low, or LL in shorthand notation of the kind presented in Chapter 3); and Honest, or choose the low price in period 1 if its cost is low and the high price if its cost is high (LH). For Fordor, the two strategies are Regardless, or enter irrespective of Tudor's period 1 price (II, for In-In); and Conditional, or enter only if Tudor's period 1 price is high (OI).

We can now show the game in strategic (normal) form. Figure 9.6 shows each player's two possible strategies; payoffs in each cell are the expected profits for each firm, given the probability (40%) that Tudor's cost is low. You may find your calculation of these profits easier if you label the terminal nodes in the tree and determine which ones are relevant for each cell of the table.

This is a simple dominance-solvable game. For Tudor, Honest dominates Bluff. And Fordor's best response to Tudor's dominant strategy of Honest is Conditional. Thus (Honest, Conditional) is the only (subgame-perfect) Nash equilibrium of the game.

The equilibrium found in Figure 9.6 is a separating one. The two cost types of Tudor charge different prices in period 1. This action reveals Tudor's type to Fordor, which then makes its market entry decision appropriately.

The key to understanding why Honest is the dominant strategy for Tudor can be found in the comparison of its payoffs when Fordor plays Conditional. These are the outcomes when Tudor's bluff "works": Fordor enters if Tudor charges the high price in period 1 and stays out if Tudor charges the low price in period 1. If Tudor is truly the low-cost type, then its payoffs when Fordor plays Conditional are the same whether it chooses Bluff or Honest. But when Tudor is actually the high-cost type, the results are different.

If Fordor's strategy is Conditional and Tudor is the high-cost type, Tudor can bluff successfully. However, even a successful bluff will be too costly. If Tudor charged its best monopoly (Honest) price in period 1, it would make a profit of 25; the low bluffing price would reduce this period 1 profit drastically—in this instance, all the way to 0. The higher monopoly price in period 1 would encourage Fordor's entry and reduce period 2 profit for Tudor, from the monopoly level of 25 to the duopoly level of 3. But Tudor's period 2 benefit from charging the low bluffing price and keeping Fordor out (25 − 3 = 22) is less than the period 1 cost imposed by bluffing and giving up its monopoly profit (25 − 0 = 25). As long as there is some positive probability that Tudor is the high-cost type, then the benefits from choosing Honest will outweigh those from choosing Bluff, even when Fordor’s choice is Conditional.

If the low price were not so low, then a truly high-cost Tudor would sacrifice less by mimicking the low-cost type. In such a case, Bluff might be a more profitable strategy for a high-cost Tudor. We consider exactly this possibility next.

C. Pooling Equilibrium

Let us now suppose that the lower of the two possible production costs for Tudor is 10 per car instead of 5. With this cost change, the high-cost Tudor still makes a profit of 25 as a monopoly if it charges its profit-maximizing price of 20. But the low-cost Tudor now charges 17.5 as a monopoly (instead of 15) and makes a profit of 56. If the high-cost type mimics the low-cost type and also charges 17.5, its profit is now 19 (rather than the 0 it earned in this case before); the loss of profit from bluffing is now much smaller: 25 − 19 = 6, rather than 25. If Fordor enters, then the two firms' profits in their duopoly game are 3 for Tudor and 45 for Fordor if Tudor is the high-cost type (as in Section 6.B). If Tudor is the low-cost type and Fordor enters, the equilibrium market price of a car in the duopoly is 15 and profits are now 25 for each firm; in this situation, Fordor and the low-cost Tudor have identical production costs of 10.

Suppose again that the probability of Tudor being the low-cost type is 40% (0.4) and that Fordor's belief about the probability of Tudor being the low-cost type is correct. The tree for this new game is shown in Figure 9.7. Because Fordor will still choose In when Tudor prices High, the game again collapses to one in which each player has exactly two complete strategies; those strategies are the same ones we described in Section 6.B. The payoff table for the normal form of this game is then the one illustrated in Figure 9.8.

This is another dominance-solvable game. Here it is Fordor that has a dominant strategy, however; it will always choose Conditional. And given the dominance of Conditional, Tudor will choose Bluff. Thus, (Bluff, Conditional) is the unique (subgame-perfect) Nash equilibrium of this game. In all other cells of the table, one firm gains by deviating to its other action. We leave it to you to think about the intuitive explanations of why each of these deviations is profitable.

The equilibrium found using Figure 9.8 involves pooling. Both cost types of Tudor charge the same (low) price, and seeing this, Fordor stays out. When both cost types of Tudor charge the same price, observation of that price does not convey any information to Fordor. Its estimate of the probability of Tudor's being the low-cost type stays at 0.4, and it calculates its expected profit from entry to be $-3 < 0$, so it does not enter. Even though Fordor knows full well that Tudor is bluffing in equilibrium, the risk of calling its bluff is too great because the probability of Tudor's cost actually being low is sufficiently great.

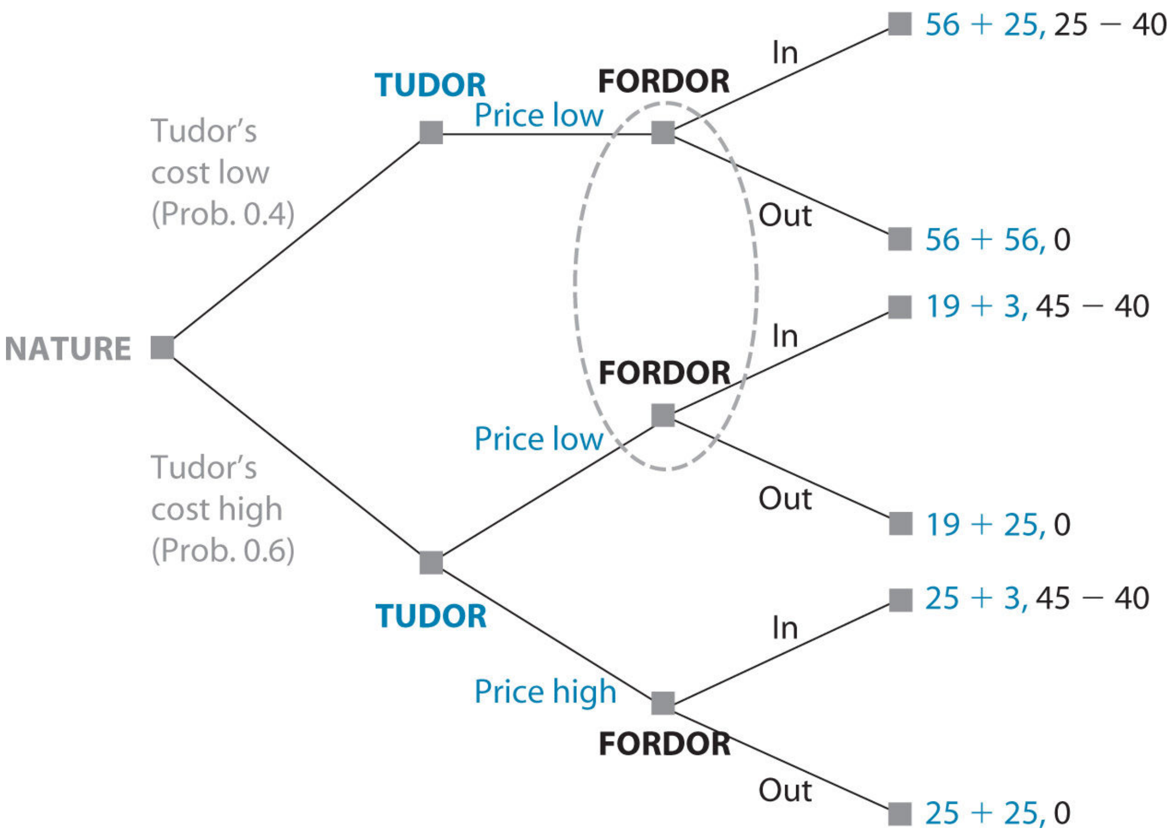


Figure 9.7 Extensive Form of Market Entry Game: Tudor's Low Cost Is 10

		FORDOR	
		Regardless (II)	Conditional (OI)
TUDOR	Bluff (LL)	81 × 0.4 + 22 × 0.6 = 45.6, -15 × 0.4 + 5 × 0.6 = -3	112 × 0.4 + 44 × 0.6 = 71.2, 0

	FORDOR	
	Regardless (II)	Conditional (OI)
Honest (LH)	81 × 0.4 + 28 × 0.6 = 49.2, −15 × 0.4 + 5 × 0.6 = −3	112 × 0.4 + 28 × 0.6 = 61.6, 5 × 0.6 = 3

FIGURE 9.8 Strategic Form of Market Entry Game: Tudor's Low Cost Is 10

What if this probability were smaller—say, 0.1—and Fordor was aware of this fact? If all the other numbers remain unchanged, then Fordor's expected profit from its Regardless strategy is $-15 \times 0.1 + 5 \times 0.9 = 4.5 - 1.5 = 3 > 0$. Then Fordor will enter no matter what price Tudor charges, and Tudor's bluff will not work. Such a situation results in a new kind of equilibrium, whose features we consider next.

D. Semiseparating Equilibrium

Here we consider the outcomes in the market entry game when Tudor's probability of achieving the low production cost of 10 is small, only 10% (0.1). All the cost, market price, and profit numbers are the same as in Section 6.C; only the probabilities have changed. Therefore, we do not show the game tree again; we show only the payoff table (Figure 9.9).

		FORDOR	
		Regardless (II)	Conditional (OI)
TUDOR	Bluff (LL)	$81 \times 0.1 + 22 \times 0.9 = 27.9$, $-15 \times 0.1 + 5 \times 0.9 = 3$	$112 \times 0.1 + 44 \times 0.9 = 50.8$, 0
	Honest (LH)	$81 \times 0.1 + 28 \times 0.9 = 33.3$, $-15 \times 0.1 + 5 \times 0.9 = 3$	$112 \times 0.1 + 28 \times 0.9 = 36.4$, $5 \times 0.9 = 4.5$

FIGURE 9.9 Strategic Form of Market Entry Game: Tudor's Low Cost Is 10 with Probability 0.1

In this new situation, the game illustrated in Figure 9.9 has no equilibrium in pure strategies. From (Bluff, Regardless), Tudor gains by deviating to Honest; from (Honest, Regardless), Fordor gains by deviating to Conditional; from (Honest, Conditional), Tudor gains by deviating to Bluff; and from (Bluff, Conditional), Fordor gains by deviating to Regardless. Once again, we leave it to you to think about the intuitive explanations of why each of these deviations is profitable.

So now we need to look for an equilibrium in mixed strategies. We suppose that Tudor mixes Bluff and Honest with probabilities p and $(1 - p)$, respectively. Similarly, Fordor mixes Regardless and Conditional with probabilities q and $(1 - q)$, respectively. Tudor's p-mix must keep Fordor indifferent between its two pure strategies of Regardless and Conditional; therefore, we need

$3p + 3(1 - p) = 0p + 4.5(1 - p)$, or $4.5(1 - p) = 3$, or

$1 - p = ⅔$, or $p = ⅓$.

And Fordor's q-mix must keep Tudor indifferent between its two pure strategies of Bluff and Honest; therefore, we need

$27.9q + 50.8(1 - q) = 33.3q + 36.4(1 - q)$, or $5.4q = 14(1 - q)$, or

$q = 14.4/19.8 = 16/22 = 0.727$.

The mixed-strategy equilibrium of the game, then, entails Tudor playing Bluff one-third of the time and Honest two-thirds of the time, while Fordor plays Regardless sixteen twenty-seconds of the time and Conditional six twenty-seconds of the time.

In this equilibrium, the two Tudor cost types are only partially separated. The low-cost Tudor always prices low in period 1, but the high-cost Tudor uses a mixed strategy and will charge the low price one-third of the time. If Fordor observes a high price in period 1, it can be sure that it is dealing with a high-cost Tudor; in that case, it will always enter. But if Fordor observes a low price, it does not know whether it faces a truly low-cost Tudor or a bluffing, high-cost Tudor. Then Fordor also plays a mixed strategy, entering 72.7% of the time. Thus, a high price conveys full information, but a low price conveys only partial information about Tudor's cost type. Therefore, this kind of equilibrium is labeled a *semiseparating equilibrium.*

		TUDOR'S PRICE		Sum of row
		Low	High	
TUDOR'S PRICE	Low	0.1	0	0.1
	High	0.9 × 1/3 = 0.3	0.9 × 2/3 = 0.6	0.9
Sum of column		0.4	0.6	
You may need to scroll left and right to see the full figure.				

FIGURE 9.10 Applying Bayes' Theorem to the Market Entry Game

To understand better the mixed strategies of each firm and the semiseparating equilibrium, consider how Fordor can use the partial information conveyed by Tudor's low price. If Fordor sees the low price in period 1, it will use this observation to update its belief about the probability that Tudor has a low cost; it does this updating using Bayes' theorem. We provide a thorough explanation of this theorem and its use in the appendix to this chapter; here, we simply apply the analysis found there to our market entry game. The table of calculations is shown as Figure 9.10; note that this table is similar to Figure 9A.1 in the appendix.

Figure 9.10 shows the possible Tudor cost types in the rows and the prices Fordor observes in the columns. The values in the cells represent the overall probability that a Tudor of the type shown in the corresponding row chooses the price shown in the corresponding column (incorporating Tudor's equilibrium mixture probability); the final row and column show the total probabilities of each Tudor cost type and of observing each price, respectively.

Using Bayes' theorem, when Fordor observes Tudor charging a low period 1 price, it will revise its belief about the probability of Tudor having a low cost by taking the probability that a low-cost Tudor is charging the low price (the 0.1 in the top-left cell) and dividing that by the total probability of the two Tudor cost types choosing the low price (0.4, the column sum in the left column). This calculation yields Fordor's updated belief that the probability of a low-cost Tudor is $0.1/0.4 = 0.25$. Then Fordor also updates its expected profit from entry to $-15 \times 0.25 + 5 \times 0.75 = 0$. Thus, Tudor's equilibrium mixture is exactly right for making Fordor indifferent between entering and not entering when it sees the low period 1 price. This outcome is exactly what is needed to keep Tudor willing to mix in the equilibrium.

The prior probability of a low-cost Tudor, 0.1, was too low to deter Fordor from entering. Fordor’s posterior probability of 0.25, revised after observing the low price in period 1, is higher. Why? Precisely because the high-cost Tudor is not always bluffing. If it were, then the low price would convey no information at all. Fordor’s posterior probability would equal 0.1 in that case, whereupon it would enter. But when the high-cost Tudor bluffs only sometimes, a low price is more likely to be indicative of low cost.

We have developed the equilibria in this market entry game in an intuitive way, but we now look back and think systematically about the nature of those equilibria. In each case, we first ensured that each player’s (and each type’s) strategy was optimal, given the strategies of everyone else; that is, we applied the concept of Nash equilibrium. Second, we ensured that players drew the correct inferences from their observations; this required a probability calculation using Bayes’ theorem, most explicitly in the case of a semiseparating equilibrium. The combination of concepts necessary to identify equilibria in such asymmetric information games justifies giving them the label Bayesian Nash equilibria. Finally, although this was a rather trivial part of this example, we did a little bit of rollback, or subgame-perfectness, reasoning. The use of rollback justifies calling such equilibria perfect Bayesian equilibria as well. Our example was a simple instance of all these equilibrium concepts; you will meet some of them again in slightly more sophisticated forms in later chapters, and in much fuller contexts in further studies of game theory.

Endnotes

- We do not supply the full calculations necessary to generate the profit-maximizing prices and the resulting firm profits in each case. You may do so on your own for extra practice, using the methods you learned in Chapter 5. Return to reference 31
- Tudor's probability of having the low per-unit cost could be denoted with an algebraic parameter, z. The equilibrium will be the same regardless of the value of z, as you will be asked to show in Exercise S4 at the end of this chapter. Return to reference 32
- This seems obvious: Why choose a price different from the profit-maximizing price? Charging the high price when you have low cost not only sacrifices some profit in period 1 (if the low-cost Tudor charges 20, its sales will drop by so much that it will make a profit of only 75 instead of the 100 it gets by charging 15), but also increases the risk of Fordor's entry and so lowers period 2 profits as well (competing with Fordor, the low-cost Tudor would have a profit of only 69 instead of the 100 it gets under monopoly). However, game theorists have found strange equilibria where a high period 1 price for Tudor is perversely interpreted as evidence of low cost, and they have applied great ingenuity in ruling out these equilibria. We leave out these complications, as we did in our analysis of cheap talk equilibria earlier, but refer interested readers to In-Koo Cho and David Kreps, "Signaling Games and Stable Equilibria," *Quarterly Journal of Economics*, vol. 102, no. 2 (May 1987), pp. 179–222. Return to reference 33

Glossary

Bayesian Nash equilibrium

A Nash equilibrium in an asymmetric information game where players use Bayes' theorem and draw correct inferences from their observations of other players' actions.

perfect Bayesian equilibrium (PBE)

An equilibrium where each player's strategy is optimal at all nodes given his beliefs, and beliefs at each node are updated using Bayes' rule in the light of the information available at that point including other players' past actions.

7 TWO-WAY ASYMMETRIC INFORMATION AND DYNAMIC CHICKEN

We have just analyzed a two-player sequential-move game in which one player had information about its own type (and therefore its true payoff structure) that the other player did not have. In that analysis, we showed that one player’s action at the first stage of the game could be used, in equilibrium, to convey information about its type to the other player or to hide that information (or, indeed, to do either with some probability). Here we extend our thinking about sequential-move games with private information in two ways, both in the context of the game of chicken that we introduced in Chapter 4. We first consider the possibility that the asymmetry of information in a two-player game could be two-way: Each player might know his own payoffs but lack information about the other’s. We then consider the effect of (two-way) asymmetric information on a sequential-move interaction in which the sequential nature of the game arises due to a timing factor. In chicken, a player’s actual choice may not be simply Straight or Swerve, but rather *when* to take one of those actions as the two cars get closer. To incorporate such possibilities, we introduce a dynamic version of chicken with private information. This model allows us to analyze the strategic move known as *brinkmanship* more thoroughly than was possible in Chapter 8.

		DEAN	
		Swerve	Straight
JAMES	Swerve	0, 0	−1, W_D
	Straight	W_J, −1	−2, −2

FIGURE 9.11 Chicken with Two-Way Private Information

A. Two-Way Asymmetric Information in Chicken

In our earlier study of the game of chicken, both in Chapter 4 and in Chapter 7, we assumed that our players, James and Dean, had complete information about the game they were playing. They were fully informed about each other's strategies and payoffs. In chicken, it is quite reasonable to imagine that the well-rounded certainty we assumed in Chapters 4 and 7 is unlikely. And the one-way asymmetry of information that we have assumed so far in this chapter is also unlikely. Instead, each player would be likely to know his own payoffs, but each would probably be somewhat unsure about the other's payoffs. We would then have a two-way asymmetry of information. Our first task here will be to consider how the game of chicken and its equilibria change when we allow for this two-way, or two-sided, private information.

Suppose that each player is unsure about the value the other places on "winning" (acting tough rather than being chicken, or choosing Straight rather than Swerve). In the version of chicken described in Chapters 4 and 7 and illustrated in Figures 4.16 and 7.4, that "winning payoff" was 1 for each player. Now let us denote that payoff with the algebraic symbol W.[34] The resulting payoff matrix is shown in Figure 9.11.

In Figure 9.11, W_J refers to James's winning payoff and W_D refers to Dean's winning payoff. However, when there can be no confusion, we will drop the J and D subscripts and refer simply to "his W" or "the other's W." We assume that each player knows his own winning payoff, but regards that of the other as unknown. However, each can guess the range and probabilities of the other's W. Suppose each player believes that the other's W is equally likely to be any value between 0 and 2. (That is, each believes that the other's W is distributed uniformly over [0, 2].) Then the probability that the other's W is less than 1, for example, is ½, and the probability that the other's W is less than any number x in the range is $x/2$.

You can form some intuition about the equilibrium strategies and outcomes that will arise in this version of the game without doing any calculations. If you were playing chicken and knew your *W* was close to 0, what choice would you make? Probably Swerve (or concede). And if you were playing and your *W* was close to 2, you would probably choose Straight (or act tough). Presumably you would make the same choices for any "low" or "high" *W*, with "low" and "high" depending on some cutoff or threshold value of *W*. If the other player thought the same way, each of you would, in equilibrium, choose Swerve or Straight by comparing your own *W* with the threshold value.

We can now conjecture that each player in this game will choose his equilibrium strategy by comparing his own value of *W* with a threshold value. Further, in equilibrium, that threshold value of *W*, which we call *W**, will be the same for both players. Here we calculate a precise value for *W** and, in the process, verify our conjecture about the equilibrium.

Consider the game from James's perspective. (The calculation for Dean is identical.) James figures that Dean will Swerve if his *W* is < *W**. As we described above, the probability of that happening is *W**/2. Otherwise, with probability

$$1-\left(\frac{W^*}{2}\right)=\frac{2-W^*}{2},$$

Dean will go Straight.

James's expected payoffs from his two possible actions are then

$$0\times\left(\frac{W^*}{2}\right)+(-1)\times\left(\frac{2-W^*}{2}\right)$$

from Swerve, and

$$W \times \left(\frac{W^*}{2} \right) + (-2) \times \left(\frac{2 - W^*}{2} \right)$$ from Straight.

James chooses Swerve whenever his expected payoff from Swerve exceeds that from Straight. Therefore, James should choose Swerve if

$$0 \times \left(\frac{W^*}{2} \right) + (-1) \times \left(\frac{2 - W^*}{2} \right) > W \times \left(\frac{W^*}{2} \right) + (-2) \times \left(\frac{2 - W^*}{2} \right)$$ or if

$$W \times \left(\frac{W^*}{2} \right) < \left(\frac{2 - W^*}{2} \right),$$ or

$$W < \left(\frac{2 - W^*}{W^*} \right).$$

The final statement verifies our intuition that James chooses Swerve when his W is low, in this case below $(2 - W^*)/W^*$. In order for James to prefer Swerve when $W < W^*$ and prefer Straight when $W > W^*$, it must be that James is indifferent between Swerve and Straight when $W = W^*$. This *indifference condition* gives us a formula for the threshold value, W^*:

$$W^* = \left(\frac{2 - W^*}{W^*} \right).$$

To solve for W^*, we rewrite the equation above to be

$$(W^*)^2 + W^* = 2.$$

One possible solution to this equation occurs at $W^* = 1$. And for $W^* > 0$ (which must be true here because W lies between 0 and 2), the left-hand side is an increasing function, so it can equal the right-hand side only once. The solution, that $W^* = 1$, is unique.[35]

Thus, each player in the chicken game with two-way private information has an unambiguous equilibrium strategy. Although each is uncertain about the other's W, he should concede (choose Swerve) if his own W is less than 1, but not concede (choose Straight) if it is greater than 1. (Any player with a W exactly equal to 1 is indifferent between the two choices and can flip a coin or otherwise randomize over the available options.)

B. Two-Way Asymmetric Information in a Large Population

There is another way to interpret the uncertainty facing the players here. Rather than thinking of the game as played between just two teens, James and Dean, consider the whole population of teenagers in their town as potential players. Each teen is naturally endowed with a different degree of toughness, so each has his own (known only to him) value of W. Assume that the distribution of all of their individual Ws is uniform over the range [0, 2].

At each of their evening encounters, the group of teens draws straws, and the two drawing the short straws are matched to play the version of chicken illustrated in Figure 9.11. Each player perceives the other as having an unknown W chosen randomly from the full range of values from 0 to 2 and, knowing his own W, chooses his optimal strategy as calculated above. Over many such pairwise matches occurring during a year or more of evening adventures, half of the teens who play (those with $W < 1$) choose Swerve and the other half (those with $W > 1$) choose Straight. (As above, those with W exactly equal to 1 can flip a coin to decide which action to choose.)

Interestingly, the fraction of players choosing Swerve in our large-population description of chicken (½) is the same as the probability of choosing Swerve in the mixed-strategy equilibrium for chicken with complete and symmetric information that we found in Chapter 7. The probabilities in the mixed-strategy equilibrium, then, can be interpreted alternatively as the fractions of a population choosing the different pure strategies when members of the population have heterogeneous, privately known payoffs and are randomly matched to play the game.

This interpretation of the probabilities in mixed-strategy equilibria, originally offered by John Harsanyi,[36] has come to be known by the somewhat misleading name *purification*. It is not the case that we are getting rid of mixed strategies, or that we are replacing them with pure strategies, but rather that we are thinking about them differently. Instead of thinking about the equilibrium probabilities as referring to particular actions for each player, we are thinking about them as referring to the observed fractions of a particular behavior when multiple games are played within large populations. We examine this interpretation of mixed strategies more explicitly in Chapter 12, on evolutionary games.

C. Dynamic Chicken: Wars of Attrition and Brinkmanship

Our previous analysis of chicken is too simplistic in another respect: The game is modeled as a simultaneous-move one in which each driver has a pure either-or choice of Straight or Swerve. In reality, as the cars are driving toward each other, the true choice is one of timing. Each player must decide *when* to swerve (if at all). The longer both drivers keep going straight, the higher the risk that the cars will get too close and a crash will occur even if one or both then choose to swerve. So we should model the game as if it were a sequential-move one, with a simultaneous-move chicken game played at each step of the sequential-move game. Such games are known as *dynamic games*. In dynamic chicken, if both players choose Straight at any given step, the game proceeds to the next step, with a higher risk of collision.

Analyzing chicken as a dynamic game with two-way private information opens up a whole new range of applications of greater importance than teenagers' contests. In a management–labor negotiation, the workers cannot be sure how much the company can afford to raise wages or benefits, and the company cannot be sure how determined the workers are to hold out or go on strike. As the talks proceed, tensions and tempers rise, making a miscalculation that inflicts economic damage on both parties more likely. Each has to decide how far to push this risk, so each is testing out the other's toughness.

Perhaps the most dramatic and dangerous instance of dynamic chicken (or "chicken in real time") was the Cuban missile crisis, which we will discuss in greater detail as a case study in Chapter 13. As the crisis unfolded over 13 days, the political and military stances of the U.S. and USSR created ever-increasing risk of nuclear war.

Such games, in which each player is testing the other's tolerance for risk while assessing how far to push its own, are often called wars of attrition. They also exemplify Thomas Schelling's strategy of *brinkmanship*, which we discussed briefly in Chapter 8. Recall the basic idea behind brinkmanship: If your threatened action would inflict an intolerably large cost on yourself, then your threat is not credible. However, that cost can be scaled down by making your threatened action probabilistic rather than guaranteed to occur. The use of such a probabilistic threat is what is termed brinkmanship. But that simple description of the strategic move leaves unanswered the question of how far to scale down your threat in your attempt to make the risk simultaneously tolerable to you but too big for the other player (so that he backs down and your threat works). The dynamic version of chicken provides a framework for determining how far to scale down your threat. Imagine starting at a greatly scaled-down level and gradually raising the risk (that is, raising the probability that you will follow through on your threat). If the opponent's risk-tolerance threshold is reached first, he concedes, and you win the game of brinkmanship. If your threshold arrives and your opponent is revealed to be tougher than you are, you concede, and your opponent wins. Of course, in the meantime there is a positive probability that the mutually damaging bad outcome happens (and both players fall off the "brink"). The concession thresholds are simply the levels where the risk of that bad outcome becomes too much to bear for each of the players.

Dynamic chicken is not a game in which just one player threatens the other, however. Rather, both are trying brinkmanship simultaneously. We present a simple analytical model of dynamic chicken here, with just two steps to the interaction. The math gets complicated with more steps, but the intuition emerges clearly from analyzing just two.

Consider a two-step chicken game, with simultaneous moves within each step but sequential moves from one step to the next. At each of the two steps, James and Dean each have the choice of whether to concede (choose Swerve). At the first step, if one or both concede, the game ends, with the payoffs as shown in Figure 9.11.

If neither concedes, the mutual disaster (a car crash) happens with some externally specified probability, in which case the game ends and each player gets a payoff of −2. To capture the idea of increasing risk inherent in the use of brinkmanship, we assume here that the probability of that disaster is proportional to the number of steps and increases linearly from one step to the next,. Thus, the probability of mutual disaster after the first step of a two-step game is ½.[37] With the remaining probability (also ½), the game goes on to the second step. At that step, if neither concedes, the calamity is sure to occur, and the payoff matrix is exactly as in Figure 9.11. The game starts with a uniform distribution of each player's W over the range [0, 2]. In the first step, some players, those with W values below some threshold, will concede. Then, if the game goes to the next step, only the players with W values above that threshold remain. Of course, this first-step threshold will be determined as part of the process of finding the solution to the game.

As usual in sequential-move games, we find the solution using rollback analysis. Start at the end of the game, with the second step. Denote the range of W values that remain at this stage as $[X, 2]$, where X represents the threshold of concession at the previous step. Its value will be determined in the process of finding the game's equilibrium. Let Y denote the concession threshold at the second step; its value will also be determined as we work toward a solution.

Then, as we did in the two-way private information calculations above, we consider the game from James's perspective. James knows that, in the second step of the game, Dean will concede if his W lies between X and Y, but act tough if his W is above Y. He then figures that the probability that Dean concedes (chooses Swerve) is $q = (Y - X)/(2 - X)$, where $Y - X$ is the range of possible W values at which Dean concedes and $2 - X$ is the range of all possible W values. Similarly, the probability that Dean acts tough (chooses Straight) is $1 - q = (2 - Y)/(2 - X)$. Therefore, James's expected payoffs from his own two choices are

$$0 \times \left(\frac{Y - X}{2 - X} \right) + (-1) \times \left(\frac{2 - Y}{2 - X} \right)$$ from conceding (Swerve), and

$$W \times \left(\frac{Y - X}{2 - X} \right) + (-2) \times \left(\frac{2 - Y}{2 - X} \right)$$ from acting tough (Straight).

Therefore, James should concede if

$$0 \times \left(\frac{Y - X}{2 - X} \right) + (-1) \times \left(\frac{2 - Y}{2 - X} \right) > W \times \left(\frac{Y - X}{2 - X} \right) + (-2) \times \left(\frac{2 - Y}{2 - X} \right),$$

---

which is equivalent to $$W \times \left(\frac{Y - X}{2 - X} \right) < \left(\frac{2 - Y}{2 - X} \right)$$

or, even more simply, $$W < \left(\frac{2 - Y}{Y - X} \right).$$

As before, this calculation verifies the intuition that a player with a low W should concede (choose Swerve). It also yields a formula for the threshold of concession in the second step, depending on the first-step threshold X; this is the value we called Y above. Thus,

$$Y = \left(\frac{2-Y}{Y-X} \right). \quad (9.1)$$

Now roll back to the first step, where the full range of *W* values, [0, 2], is in play. In this step, players with *W* values below *X* concede (Swerve). Here, James calculates the probability of Dean conceding as *X*/2, as in Section 7.A. above, and his own expected payoff if he concedes (Swerves) as

$$0 \times \left(\frac{X}{2} \right) + (-1) \times \left(\frac{2-X}{2} \right).$$

James's expected payoff if he does not concede (chooses Straight) is a little more complicated.[38] If Dean concedes, James gets his *W*. If Dean does not concede, the disaster happens with probability ½, and James gets −2. But there is also the chance, with probability ½, that the game continues to the second step. To determine James's expected payoff from not conceding, we need to know James's payoff at that second step. This calculation is most critical to the James who is at the threshold in the first step—that is, to the James whose *W*= *X*. Because *X* < *Y*, this James is sure to concede at the second step.[39] Therefore, we should use the second-step payoff from concession as James's payoff when the game continues. We have already calculated that payoff as

$$0 \times \left(\frac{Y-X}{2-X} \right) + (-1) \times \left(\frac{2-Y}{2-X} \right) = -\left(\frac{2-Y}{2-X} \right).$$

Putting all the pieces together shows that James's expected payoff from acting tough (choosing Straight) at the first step is

$$\frac{X}{2} \times W + \frac{2-X}{2} \times \left(\frac{1}{2} \times (-2) - \frac{1}{2} \times \left(\frac{2-Y}{2-X} \right) \right).$$

So James should concede at the first step if this expected payoff from Straight is smaller than his expected payoff from Swerve, calculated just above. James should concede if

$$\frac{X}{2} \times W + \frac{2-X}{2} \times \left(\frac{1}{2} \times (-2) - \frac{1}{2} \times \left(\frac{2-Y}{2-X} \right) \right) < -\left(\frac{2-X}{2} \right),$$

which, after canceling terms and simplifying, reduces to

$$W < \frac{2-Y}{2X}.$$

This final expression indicates that the threshold for concession at the first step of the game, which we called X above, satisfies

$$X = \frac{2-Y}{2X}. \quad (9.2)$$

To fully describe the equilibrium of the two-step game of chicken, we have to solve equations (9.1) and (9.2) for X and Y. In general, these equations depend on the payoff values and the probabilities of disaster at each step, so one would need to solve them numerically. (We provide such a numerical solution in our application of this analysis to the Cuban missile crisis in Chapter 13.) But in the two-step case, and with the specific numbers we have chosen here, we can find a simple, explicit solution.

Each of the equations for X and Y includes a $(2 - Y)$ term that allows us to write the equation for X as $(2 - Y) = (2X) \times X$ and the equation for Y as $(2 - Y) = Y \times (Y - X)$. Then it follows that

$$2 \times X^2 = Y \times (Y - X) = 2 - Y. \quad (9.3)$$

Consider the first equality in equation (9.3) and complete the square as follows:

$$2X^2 = Y^2 - YX, \text{ so } 2.25X^2 = Y^2 - YX + 0.25X^2 = (Y - 0.5X)^2.$$

Then taking the square root yields $1.5X = Y - 0.5X$, or $Y = 2X$. Substitute this expression for Y into the second equality in equation (9.3) and complete the square again to find

$$2 - 2X = 2X^2, \text{ or } X^2 + X = 1, \text{ so } X^2 + X + 0.25 = 1.25 \text{ or } (X + 0.5)^2 = 1.25.$$

Then the final solution for X and Y is

$$X = \sqrt{1.25} - 0.5 = 0.618,$$ and $Y = 1.236$.

The remaining numbers required to fully specify a solution to the two-step game follow easily. At the first step, the probability that each player concedes is 0.618/2 = 0.309. The probability that neither concedes is $(1 - 0.309)^2 = 0.477$. In the case that neither concedes in the first step, the calamity occurs with probability 0.477/0.5 = 0.2385. Otherwise (with probability 0.2385), the game goes to the second step. At that point, the distribution of the remaining Ws is uniform over [0.618, 2], and the resulting probability that each concedes is

$$\frac{1.236-0.618}{2-0.618}=0.447.$$

The probability that neither concedes (and the calamity then occurs) is $(1 - 0.447)^2 = 0.306$. Note that the calamity may occur at two times: after the first stage, which happens with probability 0.2385, or after the second stage, which happens with probability $0.2385 \times 0.306 = 0.0730$ (because the second stage is reached with probability 0.2385 and then calamity occurs with probability 0.306). Overall, then, the likelihood of calamity in the two-stage game is $0.2385 + 0.0730 = 0.3115$, or 31.15%.

Observe that the first-step threshold for W, beyond which a player would concede, is 0.618 here, but the threshold for concession in the one-step game above was 1.000. This result should feel intuitively reasonable. In the two-step game, there is only a 50% chance that the calamity occurs after both players act tough in the first step. But the second-step threshold for concession (1.236) is higher than that in the one-step game. Again, there is an intuitive explanation for the result. At the second step of the two-step game, even though the probability of mutual disaster rises to 1 should both players act tough, only the relatively tough players (those with $W > 0.618$) are left in the game!

Endnotes

- It is technically possible for a player's payoffs in all four cells to be private information and unknown to the other player. That level of uncertainty is too hard to handle mathematically, but analyzing the effect of uncertainty about just one of the other player's payoffs suffices to convey the implications of such limited information. To reinforce your understanding, you should try to do a similar analysis with private information about another of the payoff numbers. We provide an example of private information about the payoff associated with disaster (Straight, Straight) in Chapter 13. Return to reference 34
- Alternatively, we could complete the square. Then $(W^*)^2 + W^* + 0.25 = 2.25$, or $(W^* + 0.5)^2 = (1.5)^2$, so $W^* = 1.5 - 0.5 = 1$, and we reject the negative root as economically irrelevant. Return to reference 35
- Harsanyi shared the Nobel Prize in Economics with John Nash and Reinhard Selten in 1994. Return to reference 36
- In other applications, one might want the probability of mutual disaster to increase quadratically, exponentially, or in some other way that best fits the situations being considered. We provide an example of a "situation-specific" pattern for the probabilities of disaster in our application of dynamic chicken to the Cuban missile crisis in Chapter 13. Return to reference 37
- The calculation that follows is similar to the one for the CrossTalk–GlobalDialog game in Chapter 6, Section 1.A, where we calculated payoffs for some cells of the first-stage payoff matrix using the results of the second-stage equilibrium. Return to reference 38
- You may then wonder: If this James knows he is going to concede at the next step anyway, why doesn't he concede right away at the first step and get it over with? The answer, of course, is because of the possibility that Dean concedes at the first step, in which event James will get his W. The calculation we make takes into account all these

possibilities and their probabilities and payoffs. Return to reference 39

Glossary

dynamic chicken
: A game of chicken in which the choice to play Weak may be made at any time, the game ends as soon as either player chooses Weak, and the risk of the mutually worst outcome increases gradually over time if neither player has played Weak; a special case of the *war of attrition*.

war of attrition
: A contest between multiple players in which each player decides when to retreat, the victor is whoever remains the longest, and choosing to remain longer is costly for each player.

SUMMARY

When facing imperfect or incomplete information, game players with different attitudes toward risk or different amounts of information can engage in strategic behavior to control and manipulate the risk and information in a game. Players can reduce their risk with payment schemes or by sharing the risk with others, although the latter approach is complicated by *moral hazard* and *adverse selection*. Risk can sometimes be manipulated to a player’s benefit, depending on the circumstances within the game.

Players with private information may want to conceal or reveal that information, while those without that information may try to elicit it or avoid it. In some cases, mere words may be sufficient to convey information credibly, and then a *cheap talk equilibrium* can arise. The extent to which player interests are aligned plays an important role in achieving such equilibria. When the information content of a player’s words is ignored, the game has a *babbling equilibrium*.

More generally, specific actions taken by players convey information. *Signaling* works only if the signal action entails different costs to players of different *types*. To elicit information, a *screening device* that looks for a specific action may be required. A screening device works only if *it* induces others to reveal their *types* truthfully; there must be both *incentive compatibility* and *participation conditions (constraints).* At times, credible signaling or screening may not be possible; then the equilibrium can entail *pooling,* or there can be a complete collapse of the market or transaction for one of the types. Many examples of signaling and screening games can be found in ordinary situations such as the labor market or the provision of insurance.

In the equilibrium of a game with asymmetric information, players must not only use their best actions given their information, but must also draw correct inferences (update their information) by observing the actions of others. This type of equilibrium is known as a *Bayesian Nash equilibrium*. When the further requirement of optimality at all nodes (as in rollback analysis) is imposed, the equilibrium becomes a *perfect Bayesian equilibrium*. The outcome of such a game may entail pooling, separation, or semi-separation, depending on the specifics of the payoff structure and the specified updating processes used by players. In some parameter ranges, such games may have multiple types of perfect Bayesian equilibria. The evidence on players' abilities to achieve perfect Bayesian equilibria seems to suggest that, despite the difficult probability calculations necessary, such equilibria are often observed. Different experimental results appear to depend largely on the design of the experiment.

Some games of asymmetric information have multiple stages (are *dynamic*), and information is partially revealed over time as the game proceeds. A good example is the *war of attrition*, or *dynamic chicken*, where the strategic decision is when to swerve rather than whether to swerve, and the players are testing each other's levels of toughness.

KEY TERMS

adverse selection (326)

babbling equilibrium (319)

Bayesian Nash equilibrium (350)

Bayes' theorem (335)

cheap talk equilibrium (317)

dynamic chicken (354)

incentive-compatibility condition (constraint) (337)

moral hazard (309)

negatively correlated (310)

participation condition (constraint) (337)

perfect Bayesian equilibrium (350)

pooling (339)

pooling equilibrium (317)

pooling of types (339)

positively correlated (311)

screen (316)

screening device (316)

self-selection (338)

Glossary

moral_hazard
: A situation of information asymmetry where one player’s actions are not directly observable to others.

negatively_correlated
: Two random variables are said to be negatively correlated if, as a matter of probabilistic average, when one is above its expected value, the other is below its expected value.

positively_correlated
: Two random variables are said to be positively correlated if, as a matter of probabilistic average, when one is above its expected value, the other is also above its expected value, and vice versa.

signals
: Devices used for signaling.

signaling
: Strategy of a more-informed player to convey his “good” information credibly to a less-informed player.

screening
: Strategy of a less-informed player to elicit information credibly from a more-informed player.

screening_devices
: Methods used for screening.

type
: Players who possess different private information in a game of asymmetric information are said to be of different types.

separating_equilibrium
: A perfect Bayesian equilibrium in a game of asymmetric information, where the actions in the equilibrium reveal player type.

pooling_equilibrium

A perfect Bayesian equilibrium in a game of asymmetric information, where the actions in the equilibrium cannot be used to distinguish type.

semiseparating equilibrium

A perfect Bayesian equilibrium in a game of asymmetric information, where the actions in the equilibrium convey some additional information about the players' types, but some ambiguity about these types remains.

cheap talk equilibrium

In a game where communication among players (which does not affect their payoffs directly) is followed by their choices of actual strategies, a cheap talk equilibrium is one where the strategies are chosen optimally given the players' interpretation of the communication, and the communication at the first stage is optimally chosen by calculating the actions that will ensue.

babbling equilibrium

In a game where communication among players (which does not affect their payoffs directly) is followed by their choices of actual strategies, a babbling equilibrium is one where the strategies are chosen ignoring the communication, and the communication at the first stage can be arbitrary.

adverse selection

A form of information asymmetry where a player's type (available strategies, payoffs . . .) is his private information, not directly known to others.

Bayes' theorem

An algebraic formula for estimating the probabilities of some underlying event by using knowledge of some consequences of it that are observed.

incentive-compatibility condition (constraint)

A constraint on an incentive scheme or screening device that makes it optimal for the agent (more-informed player) of each type to reveal his true type through his actions.

participation condition (constraint)

A constraint on an incentive scheme or a screening device that should give the more-informed player an expected payoff at least as high as he can get outside this relationship.

self-selection

Where different types respond differently to a screening device, thereby revealing their type through their own action.

separation of types

An outcome of a signaling or screening game in which different types follow different strategies and get the different payoffs, so types can be identified by observing actions.

pooling of types

An outcome of a signaling or screening game in which different types follow the same strategy and get the same payoffs, so types cannot be distinguished by observing actions.

pooling

Same as **pooling of types**.

Bayesian Nash equilibrium

A Nash equilibrium in an asymmetric information game where players use Bayes' theorem and draw correct inferences from their observations of other players' actions.

perfect Bayesian equilibrium (PBE)

An equilibrium where each player's strategy is optimal at all nodes given his beliefs, and beliefs at each node are updated using Bayes' rule in the light of the information available at that point including other players' past actions.

dynamic chicken

A game of chicken in which the choice to play Weak may be made at any time, the game ends as soon as either player chooses Weak, and the risk of the mutually worst outcome increases gradually over time if neither player has played Weak; a special case of the *war of attrition*.

war of attrition

A contest between multiple players in which each player decides when to retreat, the victor is whoever remains the longest, and choosing to remain longer is costly for each player.

SOLVED EXERCISES

1. A local charity has been given a grant to serve free meals to the homeless in its community, but it is worried that its program might be exploited by nearby college students, who are always on the lookout for a free meal. Both a homeless person and a college student receive a payoff of 10 for a free meal. The cost of standing in line for the meal is $t^2/320$ for a homeless person and $t^2/160$ for a college student, where t is the amount of time in line measured in minutes. Assume that the staff of the charity cannot observe the true type of those coming for free meals.
   1. What is the minimum wait time t that will achieve separation of types?
   2. After a while, the charity finds that it can successfully identify and turn away college students half of the time. College students who are turned away receive no free meal and, further, incur a cost of 5 for their time and embarrassment. Will the partial identification of college students reduce or increase the answer in part (a)? Explain.
2. Consider the used-car market for the 2017 Citrus described in Section 4.B. There is now a surge in demand for used Citruses; buyers would now be willing to pay up to \$18,000 for an orange and \$8,000 for a lemon. All else remains identical to the example in Section 4.B.
   1. What price would buyers be willing to pay for a 2017 Citrus of unknown type if the fraction of oranges in the population, f, were 0.6?
   2. Will there be a market for oranges if $f = 0.6$? Explain.
   3. What price would buyers be willing to pay if f were 0.2?

4. Will there be a market for oranges if f = 0.2? Explain.
5. What is the minimum value of f such that the market for oranges does not collapse?
6. Explain why the increase in the buyers' willingness to pay changes the threshold value of f, where the market for oranges collapses.

3. Suppose electricians come in two types: competent and incompetent. Both types of electricians can get certified, but for the incompetent types, certification takes extra time and effort. Competent ones have to spend C months preparing for the certification exam; incompetent ones take twice as long. Certified electricians can earn 100 (thousand dollars) each year working on building sites for licensed contractors. Uncertified electricians can earn only 25 (thousand dollars) each year in freelance work; licensed contractors won't hire them. Each type of electrician gets a payoff equal to $\sqrt{S} - M,$ where S is the salary measured in thousands of dollars and M is the number of months spent getting certified. What is the range of values of C for which a competent electrician will choose to use certification as a signaling device but an incompetent one will not?
4. Return to the Tudor–Fordor example in Section 6.A, when Tudor's low per-unit production cost is 5. Let z be the probability that Tudor actually has a low per-unit cost.
   1. Rewrite the table in Figure 9.6 in terms of z.
   2. How many pure-strategy equilibria are there when z = 0? Explain.
   3. How many pure-strategy equilibria are there when z = 1? Explain.
   4. Show that the Nash equilibrium of this game is always a separating equilibrium for any value of z between 0 and 1 (inclusive).

5. Consider the Tudor–Fordor interaction again, but in a situation where Tudor has a low per-unit production cost of 6 (instead of 5 or 10 as in Section 6). If Tudor's cost is low (6), then it will earn 90 in a profit-maximizing monopoly. If Fordor enters the market, Tudor will earn 59 in the resulting duopoly, while Fordor earns 13. If Tudor's cost is actually high (that is, its per-unit cost is 15) and it prices its cars as if its cost were low (that is, as if it had a per-unit cost of 6), then it will earn 5 in a monopoly situation.
   1. Draw a game tree for this game equivalent to Figure 9.5 or 9.7 in the text, changing the appropriate payoffs.
   2. Write the normal form of this game, assuming that the probability that Tudor's cost is low is 0.4.
   3. What is the equilibrium of the game? Is it separating, pooling, or semiseparating? Explain why.
6. Felix and Oscar are playing a simplified version of poker. Each makes an initial bet of $8. Then each separately draws a card, which may be High or Low with equal probabilities. Each sees his own card, but not that of the other.

   Then Felix decides whether to Pass or to Raise (bet an additional $4). If he chooses to Pass, the two cards are revealed and compared. If the outcomes are different, the one who has the High card collects the whole pot. The pot has $16, of which the winner himself contributed $8, so his winnings are $8. The loser's payoff is −$8. If the outcomes are the same, the pot is split equally, and each gets his $8 back (payoff 0).

   If Felix chooses Raise, then Oscar has to decide whether to Fold (concede) or See (match with his own additional $4). If Oscar chooses Fold, then Felix collects the pot irrespective of the cards. If Oscar chooses See, then the cards are revealed and compared. The procedure at that point is the same as that in the preceding paragraph, but the pot is now bigger.

1. Show the game in extensive form. (Be careful about information sets.) If the game is instead written in the normal form, Felix has four strategies: (1) Pass always (PP for short), (2) Raise always (RR), (3) Raise if his own card is High and Pass if it is Low (RP), and (4) the other way round (PR). Similarly, Oscar has four strategies: (1) Fold always (FF), (2) See always (SS), (3) See if his own card is High and Fold if it is Low (SF), and (4) the other way round (FS).
2. Show that the table of payoffs to Felix is as follows:

		OSCAR			
		FF	SS	SF	FS
FELIX	PP	0	0	0	0
	RR	8	0	1	7
	RP	2	1	0	3
	PR	6	−1	1	4
You may need to scroll left and right to see the full figure.					

(In each case, you will have to take an expected value by averaging over the consequences for each of the four possible combinations of the card draws.)

3. Eliminate dominated strategies as far as possible. Find the mixed-strategy equilibrium in the remaining table and the expected payoff to Felix in the equilibrium.

    4. Use your knowledge of the theory of signaling and screening to explain intuitively why the equilibrium has mixed strategies.

7. Felix and Oscar are playing another simplified version of poker called Stripped-Down Poker. Both make an initial bet of $1. Felix (and only Felix) draws one card, which is either a King or a Queen with equal probability (there are four Kings and four Queens). Felix then chooses whether to Fold or to Bet. If Felix chooses to Fold, the game ends, and Oscar receives Felix’s $1 in addition to his own. If Felix chooses to Bet, he puts in an additional $1, and Oscar chooses whether to Fold or to Call.

   If Oscar Folds, Felix wins the pot (consisting of Oscar’s initial bet of $1 and $2 from Felix). If Oscar Calls, he puts in another $1 to match Felix’s bet, and Felix’s card is revealed. If the card is a King, Felix wins the pot ($2 from each of them). If it is a Queen, Oscar wins the pot.

    1. Show the game in extensive form. (Be careful about information sets.)
    2. How many strategies does each player have?
    3. Show the game in strategic form, where the payoffs in each cell reflect the expected payoffs given each player’s respective strategy.
    4. Eliminate dominated strategies, if any. Find the equilibrium in mixed strategies. What is the expected payoff to Felix in equilibrium?

8. Wanda works as a waitress and consequently has the opportunity to earn cash tips that are not reported by her employer to the Internal Revenue Service. Her tip income is rather variable. In a good year (G), she earns a high income, so her tax liability to the IRS is $5,000. In a bad year (B), she earns a low income, and her tax liability to the IRS is $0. The IRS knows that the probability of her having a good year is 0.6, and the

probability of her having a bad year is 0.4, but it doesn’t know for sure which outcome has resulted for her in any particular tax year.

In this game, first Wanda decides how much income to report to the IRS. If she reports high income (H), she pays the IRS $5,000. If she reports low income (L), she pays the IRS $0. Then the IRS has to decide whether to audit Wanda. If she reports high income, they do not audit, because they automatically know they’re already receiving the tax payment Wanda owes. If she reports low income, then the IRS can either audit (A) or not audit (N). When the IRS audits, it costs the IRS $1,000 in administrative costs, and it also costs Wanda $1,000 in the opportunity cost of the time she spends gathering bank records and meeting with the auditor. If the IRS audits Wanda in a bad year (B), then she owes nothing to the IRS, although she and the IRS have each incurred the $1,000 auditing cost. If the IRS audits Wanda in a good year (G), then she has to pay the $5,000 she owes to the IRS, and she and the IRS each incur the auditing cost.

1. Suppose that Wanda has a good year (G), but she reports low income (L). Suppose the IRS then audits her (A). What is the total payoff to Wanda, and what is the total payoff to the IRS?
2. Which of the two players has an incentive to bluff (that is, to give a false signal) in this game? What would bluffing consist of?
3. Show this game in extensive form. (Be careful about information sets.)
4. How many pure strategies does each player have in this game? Explain your reasoning.
5. Draw the payoff matrix for this game. Find all Nash equilibria. Identify whether the equilibria you find are separating, pooling, or semiseparating.
6. Let x equal the probability that Wanda has a good year. In the original version of this problem, we had

$x = 0.6$. Find a value of x such that Wanda always reports low income in equilibrium.

7. What is the full range of values of x for which Wanda always reports low income in equilibrium?

9. The design of a health-care system concerns matters of information and strategy at several points. The users—potential and actual patients—have better information about their own state of health, lifestyle, and so forth than the insurance companies can find out. The providers—doctors, hospitals, and so forth—know more about what the patients need than do either the patients themselves or the insurance companies. Doctors also know more about their own skills and efforts, and hospitals about their own facilities. Insurance companies may have some statistical information about outcomes of treatments or surgical procedures from their past records. But outcomes are affected by many unobservable and random factors, so the underlying skills, efforts, or facilities cannot be inferred perfectly from observation of the outcomes. The pharmaceutical companies know more about the efficacy of drugs than do the others. As usual, the parties do not have natural incentives to share their information fully or accurately with others. The design of the overall scheme must try to face these matters and find the best feasible solutions.

   Consider the relative merits of various payment schemes—fee for service versus capitation fees to doctors, comprehensive premiums per year versus payment for each visit for patients, and so forth—from this strategic perspective. Which are likely to be most beneficial to those seeking health care? To those providing health care? Think also about the relative merits of private insurance and coverage of costs from general tax revenues.

10. In a television commercial for a well-known brand of instant cappuccino, a gentleman is entertaining a lady

friend at his apartment. He wants to impress her and offers her cappuccino with dessert. When she accepts, he goes into the kitchen to make the instant cappuccino—simultaneously tossing take-out boxes into the trash and faking the noises made by a high-class (and expensive) espresso machine. As he is doing so, a voice comes from the other room: "I want to see the machine. . . ."

Use your knowledge of games of asymmetric information to comment on the actions of these two people. Pay attention to their attempts to use signaling and screening, and point out specific instances of each strategy. Offer an opinion about which player is the better strategist.

11. (**Optional**, requires appendix) In the genetic test example in the appendix to this chapter, suppose the test comes out negative (Y is observed). What is the probability that the person tested does not have the genetic defect (B exists)? Calculate this probability by applying Bayes' theorem, and then check your answer by doing an enumeration of the 10,000 members of the population.
12. (**Optional**, requires appendix) Return to the example of the 2017 Citrus in Section 4.B. The two types of Citrus—the reliable orange and the hapless lemon—are outwardly indistinguishable to a buyer. In the example, if the fraction f of oranges in the Citrus population is less than 0.65, the seller of an orange will not be willing to part with the car for the maximum price buyers are willing to pay, so the market for oranges will collapse.

    But what if a seller has a costly way to signal her car's type? Although oranges and lemons are in nearly every respect identical, the defining difference between the two is that lemons break down much more frequently. Knowing this, owners of oranges make the following proposal. On a buyer's request, the seller will in one day take a 500-mile round-trip drive in the car. (Assume this trip will be verifiable via odometer readings and a time-stamped receipt from a gas station 250 miles away.) For the sellers of both types of Citrus, the cost of the trip in fuel and time is \$0.50 per mile (that is, \$250 for the 500-mile trip). However, with probability q, a lemon attempting the journey will break down. If a car breaks down, the cost is \$2 per mile of the total length of the attempted road trip (that is, \$1,000). Additionally, breaking down will be a sure sign that the car is a lemon, so a Citrus that does so will sell for only \$6,000.

Assume that the fraction of oranges in the Citrus population, f, is 0.6. Also, assume that the probability of a lemon breaking down, q, is 0.5.

1. Use Bayes' theorem to determine f_{updated}, the fraction of Citruses that have successfully completed a 500-mile road trip that are oranges. Assume that all Citrus owners attempt the trip. Is f_{updated} greater than or less than f? Explain why.
2. Use f_{updated} to determine the price, p_{updated}, that buyers are willing to pay for a Citrus that has successfully completed the 500-mile road trip.
3. Will an owner of an orange be willing to make the road trip and sell her car for p_{updated}? Why or why not?
4. What is the expected payoff of attempting the road trip to the seller of a lemon?
5. Would you describe the outcome of this market as pooling, separating, or semiseparating? Explain.

UNSOLVED EXERCISES

1. Consider again the case of the 2017 Citrus. Almost all cars depreciate over time, and so it is with the Citrus. With every month that passes, all sellers of Citruses—regardless of type—are willing to accept \$100 less than they were the month before. Also, with every passing month, buyers are maximally willing to pay \$400 less for an orange than they were the previous month and \$200 less for a lemon. Assume that the example in the text takes place in month 0. Eighty percent of the Citruses are oranges, and this proportion never changes.
    1. Fill out three versions of the following table for month 1, month 2, and month 3:

	Willingness to accept of sellers	**Willingness to pay of buyers**
Orange		
Lemon		

---

    2. Graph the price that sellers of oranges will be willing to accept over the next 12 months. On the same figure, graph the price that buyers will be willing to pay for a Citrus of unknown type (given that the proportion of oranges is 0.8). (Hint: Make the vertical axis range from 10,000 to 14,000.)
    3. Is there a market for oranges in month 3? Why or why not?
    4. In what month does the market for oranges collapse?
    5. If owners of lemons experienced no depreciation (that is, they were never willing to accept anything less than \$3,000), would this affect the timing of the collapse of the market for oranges? Why or why not?

In what month would the market for oranges collapse in this case?

6. If buyers experienced no depreciation in the price a lemon (that is, they were always willing to pay up to $6,000 for a lemon), would this affect the timing of the collapse of the market for oranges? Why or why not? In what month would the market for oranges collapse in this case?

2. An economy has two types of jobs, Good and Bad, and two types of workers, Qualified and Unqualified. The population consists of 60% Qualified and 40% Unqualified workers. In a Bad job, either type of worker produces 10 units of output. In a Good job, a Qualified worker produces 100 units, and an Unqualified worker produces 0. There is enough demand for workers that for each type of job, companies must pay what they expect the appointee to produce.

   Companies must hire each worker without observing his type and pay him before knowing his actual output. But Qualified workers can signal their qualification by getting educated. For a Qualified worker, the cost of getting educated to level n is $n^2/2$, whereas for an Unqualified worker, it is n^2. These costs are measured in the same units as output, and n must be an integer.

   1. What is the minimum level of n that will achieve separation?
   2. Now suppose the signal is made unavailable. Which kinds of jobs will be filled by which kinds of workers, and at what wages? Who will gain and who will lose from this change?

3. You are the Dean of the Faculty at St. Anford University. You hire assistant professors for a probationary period of seven years, after which they come up for tenure and are either promoted and gain a job for life or turned down, in which case they must find another job elsewhere.

Your assistant professors come in two types, Good and Brilliant. Any types worse than Good have already been weeded out in the hiring process, but you cannot directly distinguish between Good and Brilliant types. Each individual assistant professor knows whether he or she is Brilliant or merely Good. You would like to tenure only the Brilliant types.

The payoff from a tenured career at St. Anford is \$2 million; think of this as the expected value today of salaries, consulting fees, and book royalties, plus the monetary equivalent of the pride and joy that the faculty member and his or her family would get from being tenured at St. Anford. Anyone denied tenure at St. Anford will get a faculty position at Boondocks College, and the value today of that career is \$0.5 million.

Your faculty can do research and publish the findings. But each publication requires effort and time and causes strain on the family; all these are costly to the faculty member. The monetary equivalent of this cost is \$30,000 per publication for a Brilliant assistant professor and \$60,000 per publication for a Good one. You can set a minimum number N of publications that an assistant professor must produce in order to achieve tenure.

1. Without doing any math, describe, as completely as you can, what would happen in a separating equilibrium to this game.
2. There are two potential types of pooling outcomes to this game. Without doing any math, describe what they would look like, as completely as you can.
3. Now please go ahead and do some math. What is the set of possible N that will accomplish your goal of screening the Brilliant professors out from the merely Good ones?

4. Return to the Tudor–Fordor problem from Section 6.C, when Tudor's low per-unit production cost is 10. Let z be the probability that Tudor actually has a low per-unit cost.
   1. Rewrite the table in Figure 9.8 in terms of z.
   2. How many pure-strategy equilibria are there when z = 0? What type of equilibrium (separating, pooling, or semiseparating) occurs when z = 0? Explain.
   3. How many pure-strategy equilibria are there when z = 1? What type of equilibrium (separating, pooling, or semiseparating) occurs when z = 1? Explain.
   4. What is the lowest value of z such that there is a pooling equilibrium?
   5. Explain intuitively why the pooling equilibrium cannot occur when the value of z is too low.
5. Return to the situation in Exercise S5, where Tudor's low per-unit production cost is 6.
   1. Write the normal form of this game in terms of z, the probability that Tudor has a low per-unit cost.
   2. What is the equilibrium when z = 0.1? Is it separating, pooling, or semiseparating?
   3. Repeat part (b) for z = 0.2.
   4. Repeat part (b) for z = 0.3.
   5. Compare your answers in parts (b), (c), and (d) of this problem with part (d) of Exercise U4. When Tudor's low cost is 6 instead of 10, can pooling equilibria be achieved at lower values of z? Or are higher values of z required for pooling equilibria to occur? Explain intuitively why this is the case.
6. Corporate lawsuits may sometimes be signaling games. Here is one example: In 2003, AT&T filed suit against eBay, alleging that its Billpoint and PayPal electronic payment systems infringed on AT&T's 1994 patent on "mediation of transactions by a communications system."

   Let's consider this situation from the point in time when the suit was filed. In response to this suit, as in most patent-infringement suits, eBay can offer to settle with AT&T without going to court. If AT&T accepts eBay’ s settlement offer, there will be no trial. If AT&T rejects eBay’ s settlement offer, the outcome will be determined by the court.

The amount of damages claimed by AT&T is not publicly available. Let’ s assume that AT&T is suing for $300 million. In addition, let’ s assume that if the case goes to trial, the two parties will incur court costs (for lawyers and consultants) of $10 million each.

Because eBay is actually in the business of processing electronic payments, we might think that eBay knows more than AT&T does about its probability of winning the case. For simplicity, let’ s assume that eBay knows for sure whether it will be found innocent (i) or guilty (g) of patent infringement. From AT&T’ s point of view, there is a 25% chance that eBay is guilty (g) and a 75% chance that eBay is innocent (i).

Let’ s also suppose that eBay has two possible actions: a generous settlement offer (G) of $200 million or a stingy settlement offer (S) of $20 million. If eBay offers a generous settlement, assume that AT&T will accept, thus avoiding a costly trial. If eBay offers a stingy settlement, then AT&T must decide whether to accept (A) and avoid a trial or reject and take the case to court (C). In the trial, if eBay is found guilty, it must pay AT&T $300 million in addition to paying all the court costs. If eBay is found innocent, it will pay AT&T nothing, and AT&T will pay all the court costs.

1. Show the game in extensive form. (Be careful to label information sets correctly.)
2. Which of the two players has an incentive to bluff (that is, to give a false signal) in this game? What would bluffing consist of? Explain your reasoning.

3. Draw the payoff matrix for this game. Find all Nash equilibria. What are the expected payoffs to each player in equilibrium?

7. For the Stripped-Down Poker game that Felix and Oscar play in Exercise S7, what does the mix of Kings and Queens have to be for the game to be fair? That is, what fraction of Kings will make the expected payoff of the game \$0 for both players?
8. Bored with Stripped-Down Poker, Felix and Oscar now make the game more interesting by adding a third card type: Jack. Four Jacks are added to the deck of four Kings and four Queens. All rules remain the same as before, except for what happens when Felix Bets and Oscar Calls. When Felix Bets and Oscar Calls, Felix wins the pot if he has a King, they tie and each gets his money back if Felix is holding a Queen, and Oscar wins the pot if the card is a Jack.
   1. Show the game in extensive form. (Be careful to label information sets correctly.)
   2. How many pure strategies does Felix have in this game? Explain your reasoning.
   3. How many pure strategies does Oscar have in this game? Explain your reasoning.
   4. Represent this game in strategic form. This should be a matrix of *expected* payoffs for each player, given a pair of strategies.
   5. Find the unique pure-strategy Nash equilibrium of this game.
   6. Would you call this equilibrium a pooling equilibrium, a separating equilibrium, or a semiseparating equilibrium?
   7. In equilibrium, what is the expected payoff to Felix of playing this game? Is it a fair game?
9. Consider Michael Spence's job-market signaling model with the following specifications.[40] There are two types

of workers, 1 and 2. The productivities of the two types, as functions of their level of education E, are

$W_1(E) = E$ and $W_2(E) = 1.5E$.

The costs of education for the two types, as functions of the level of education, are

$C_1(E) = E^2/2$ and $C_2(E) = E^2/3$.

Each worker's payoff equals his or her income minus the cost of education. Companies that seek to hire these workers are perfectly competitive in the labor market.

1. If types are public information (observable and verifiable), find expressions for the levels of education, incomes, and utilities of the two types of workers.

   Now suppose each worker's type is his or her private information.

2. Suppose employers offer two kinds of employment packages: anyone educated to level x_1 will get salary y_1, and anyone educated to level x_2 will get salary y_2, where (x_1, y_1) and (x_2, y_2) are the education and salary levels for type 1 and type 2 that you found in part (a). Verify that type 2 workers do not prefer to get educated to level x_1 to get salary y_1, but type 1's do prefer to get educated to level x_2 to get salary y_2. That is, verify that separation by self-selection cannot achieve the outcome that one would obtain under public information about types.
3. If we leave the education-wage pair for type 1 as in part (a), what is the range of education–wage pairs for type 2 that can achieve separation?

4. Of the possible separating education-wage packages, which one do you expect to prevail? Give a verbal, but not a formal, explanation for your answer.
5. Who gains or loses from the information asymmetry? How much?

10. Consider the question raised in the following quotation:

    Mr. Robinson pretty much concludes that business schools are a sifting device—M.B.A. degrees are union cards for yuppies. But perhaps the most important fact about the Stanford business school is that all meaningful sifting occurs before the first class begins. No messy weeding is done within the walls. “They don’t want you to flunk. They want you to become a rich alum who’ll give a lot of money to the school.” But one wonders: If corporations are abdicating to the Stanford admissions office the responsibility for selecting young managers, why don’t they simply replace their personnel departments with Stanford admissions officers, and eliminate the spurious education? Does the very act of throwing away a lot of money and two years of one’s life demonstrate a commitment to business that employers find appealing? (From Michael Lewis, review of *Snapshots from Hell: The Making of an MBA*, by Peter Robinson, *New York Times*, May 8, 1994, Book Review section.)

    What answer to Lewis’s question can you give, based on our analysis of strategies in situations of asymmetric information?

11. (**Optional**, requires appendix) An auditor for the IRS is reviewing Wanda’s latest tax return (see Exercise S8), on which she reports having had a bad year. Assume that Wanda is playing this game according to her equilibrium strategy, and that the auditor knows this.
    1. Using Bayes’ theorem, find the probability that Wanda had a good year given that she reports having

had a bad year.

2. Explain why the answer in part (a) is more or less than the baseline probability of having a good year, 0.6.

12. (**Optional**, requires appendix) Return to Exercise S12. Assume, reasonably, that the probability of a lemon's breaking down increases over the length of the road trip. Specifically, let $q = m/(m + 500)$, where m is the number of miles in the round trip.
    1. Find the minimum integer number of miles, m, necessary to avoid the collapse of the market for oranges. That is, what is the smallest m such that the seller of an orange is willing to sell her car at the market price for a Citrus that has successfully completed the road trip? (Hint: Remember to calculate f_{updated} and p_{updated}.)
    2. What is the minimum integer number of miles, m, necessary to achieve complete separation between functioning markets for oranges and lemons? That is, what is the smallest m such that the owner of a lemon will never decide to attempt the road trip?

Endnotes

- See Michael Spence, “Job Market Signaling,” *The Quarterly Journal of Economics*, vol. 87, no. 3 (August 1973), pp. 355–374. Return to reference 40

■ Appendix: Inferring Probabilities from Observing Consequences

When players have different amounts of information in a game, they will try to use some device to ascertain their opponents' private information. As we saw in Section 3 of this chapter, it is sometimes possible for direct communication to yield a cheap talk equilibrium. But more often, players will need to determine one another's information by observing one another's actions. Seeing those actions (or their observed consequences), players can draw inferences about the underlying information possessed by others. For instance, if another player might have "good" or "bad" information, one can use their actions to form an updated belief about the probability that their information is "good" or "bad." Such belief updating requires some relatively sophisticated manipulation of the rules of probability, and we examine this process in detail here. The rules given in the appendix to Chapter 7 for manipulating and calculating the probability of events prove useful in our calculations of payoffs when individual players are differently informed.

The best way to understand this idea is by giving an example. Suppose 1% of the population has a genetic defect that can cause a disease. A test that can identify this genetic defect has 99% accuracy: When the defect is present, the test will fail to detect it 1% of the time, and the test will also falsely find a defect when none is present 1% of the time. We are interested in determining the probability that a person with a positive test result really has the defect. That is, we cannot directly observe the person's genetic defect (underlying condition), but we can observe the results of the