# Solved Problem 1

Chicken Palace periodically offers carryout five-piece chicken dinners at special prices. Let $Y$ be the number of dinners sold and $X$ be the price. Based on the historical observations and calculations in the following table, determine the regression equation, correlation coefficient, and coefficient of determination. How many dinners can Chicken Palace expect to sell at \$3.00 each?

| Observation | Price ($X$) | Dinners Sold ($Y$) |
|---|---|---|
| 1 | \$2.70 | 760 |
| 2 | \$3.50 | 510 |
| 3 | \$2.00 | 980 |
| 4 | \$4.20 | 250 |
| 5 | \$3.10 | 320 |
| 6 | \$4.05 | 480 |
| Total | \$19.55 | 3,300 |
| Average | \$3.258 | 550 |

### SOLUTION

We use the computer (*Regression Analysis* Solver of OM Explorer or *Regression Projector* module of POM for Windows) to calculate the best values of $a$, $b$, the correlation coefficient, and the coefficient of determination.

$$\begin{aligned} a &= 1{,}454.60 \\ b &= -277.63 \\ r &= -0.84 \\ r^2 &= 0.71 \end{aligned}$$

The regression line is

$$Y = a + bX = 1{,}454.60 - 277.63X$$

The correlation coefficient $(r = -0.84)$ shows a negative correlation between the variables. The coefficient of determination $(r^2 = 0.71)$ is not too large, which suggests that other variables (in addition to price) might appreciably affect sales.

If the regression equation is satisfactory to the manager, estimated sales at a price of \$3.00 per dinner may be calculated as follows:

$$\begin{aligned} Y &= a + bX = 1{,}454.60 - 277.63(3.00) \\ &= 621.71 \text{ or } 622 \text{ dinners} \end{aligned}$$

# Solved Problem 2

The Polish General's Pizza Parlor is a small restaurant catering to patrons with a taste for European pizza. One of its specialties is Polish Prize pizza. The manager must forecast weekly demand for these special pizzas so that he can order pizza shells weekly. Recently, demand has been as follows:

| Week | Pizzas | Week | Pizzas |
|---|---|---|---|
| June 2 | 50 | June 23 | 56 |
| June 9 | 65 | June 30 | 55 |
| June 16 | 52 | July 7 | 60 |

**a.** Forecast the demand for pizza for June 23 to July 14 by using the simple moving average method with $n = 3$. Then, repeat the forecast by using the weighted moving average method with $n = 3$ and weights of 0.50, 0.30, and 0.20, with 0.50 applying to the most recent demand.

**b.** Calculate the MAD for each method.

**SOLUTION**

**a.** The simple moving average method and the weighted moving average method give the following results:

| Current Week | Simple Moving Average Forecast for Next Week | Weighted Moving Average Forecast for Next Week |
|---|---|---|
| June 16 | $\frac{52 + 65 + 50}{3} = 55.7$ or 56 | $[(0.5 \times 52) + (0.3 \times 65) + (0.2 \times 50)] = 55.5$ or 56 |
| June 23 | $\frac{56 + 52 + 65}{3} = 55.7$ or 58 | $[(0.5 \times 56) + (0.3 \times 52) + (0.2 \times 65)] = 56.6$ or 57 |
| June 30 | $\frac{55 + 56 + 52}{3} = 54.3$ or 54 | $[(0.5 \times 55) + (0.3 \times 56) + (0.2 \times 52)] = 54.7$ or 55 |
| July 7 | $\frac{60 + 55 + 56}{3} = 57.0$ or 57 | $[(0.5 \times 60) + (0.3 \times 55) + (0.2 \times 56)] = 57.7$ or 58 |

Forecasts in each row are for the next week's demand. For example, the simple moving average and weighted moving average forecasts (both are 56 units) calculated after learning the demand on June 16 apply to June 23's demand forecast.

**b.** The mean absolute deviation is calculated as follows:

| | | SIMPLE MOVING AVERAGE | | WEIGHTED MOVING AVERAGE | |
|---|---|---|---|---|---|
| **Week** | **Actual Demand** | **Forecast for This Week** | **Absolute Errors $\lvert E_t \rvert$** | **Forecast for This Week** | **Absolute Errors $\lvert E_t \rvert$** |
| June 23 | 56 | 56 | $\lvert 56 - 56 \rvert = 0$ | 56 | $\lvert 56 - 56 \rvert = 0$ |
| June 30 | 55 | 58 | $\lvert 55 - 58 \rvert = 3$ | 57 | $\lvert 55 - 57 \rvert = 2$ |
| July 7 | 60 | 54 | $\lvert 60 - 54 \rvert = 6$ | 55 | $\lvert 60 - 55 \rvert = 5$ |
| | | | $\text{MAD} = \frac{0 + 3 + 6}{3} = 3.0$ | | $\text{MAD} = \frac{0 + 2 + 5}{3} = 2.3$ |

For this limited set of data, the weighted moving average method resulted in a slightly lower mean absolute deviation. However, final conclusions can be made only after analyzing much more data.

# Solved Problem 3

MyOMLab Video

The monthly demand for units manufactured by the Acme Rocket Company has been as follows:

| Month | Units | Month | Units |
|---|---|---|---|
| May | 100 | September | 105 |
| June | 80 | October | 110 |
| July | 110 | November | 125 |
| August | 115 | December | 120 |

**a.** Use the exponential smoothing method to forecast the number of units for June to January. The initial forecast for May was 105 units; $\alpha = 0.2$.

**b.** Calculate the absolute percentage error for each month from June through December and the MAD and MAPE of forecast error as of the end of December.

**c.** Calculate the tracking signal as of the end of December. What can you say about the performance of your forecasting method?

**SOLUTION**

**a.**

| Current Month, $t$ | Calculating Forecast for Next Month $F_{t+1} = \alpha D_t + (1 - \alpha)F_t$ | Forecast for Month $t + 1$ |
|---|---|---|
| May | $0.2(100) + 0.8(105) = 104.0$ or 104 | June |
| June | $0.2(80) + 0.8(104.0) = 99.2$ or 99 | July |
| July | $0.2(110) + 0.8(99.2) = 101.4$ or 101 | August |
| August | $0.2(115) + 0.8(101.4) = 104.1$ or 104 | September |
| September | $0.2(105) + 0.8(104.1) = 104.3$ or 104 | October |
| October | $0.2(110) + 0.8(104.3) = 105.4$ or 105 | November |
| November | $0.2(125) + 0.8(105.4) = 109.3$ or 109 | December |
| December | $0.2(120) + 0.8(109.3) = 111.4$ or 111 | January |

**b.**

| Month, $t$ | Actual Demand, $D_t$ | Forecast, $F_t$ | Error, $E_t = D_t - F_t$ | Absolute Error, $\lvert E_t \rvert$ | Absolute Percentage Error, $(\lvert E_t \rvert / D_t)(100\%)$ |
|---|---|---|---|---|---|
| June | 80 | 104 | −24 | 24 | 30.0% |
| July | 110 | 99 | 11 | 11 | 10.0 |
| August | 115 | 101 | 14 | 14 | 12.0 |
| September | 105 | 104 | 1 | 1 | 1.0 |
| October | 110 | 104 | 6 | 6 | 5.5 |
| November | 125 | 105 | 20 | 0 | 16.0 |
| December | 120 | 109 | 11 | 11 | 9.2 |
| Total | 765 | | 39 | 87 | 83.7% |

$$\text{MAD} = \frac{\Sigma|E_t|}{n} = \frac{87}{7} = 12.4 \text{ and MAPE} = \frac{(\Sigma|E_t|/D_t)(100)}{n} = \frac{83.7\%}{7} = 11.96\%$$

**c.** As of the end of December, the cumulative sum of forecast errors (CFE) is 39. Using the mean absolute deviation calculated in part (b), we calculate the tracking signal:

$$\text{Tracking signal} = \frac{\text{CFE}}{\text{MAD}} = \frac{39}{12.4} = 3.14$$

The probability that a tracking signal value of 3.14 could be generated completely by chance is small. Consequently, we should revise our approach. The long string of forecasts lower than actual demand suggests use of a trend method.

# Solved Problem 4

The Northville Post Office experiences a seasonal pattern of daily mail volume every week. The following data for two representative weeks are expressed in thousands of pieces of mail:

| Day | Week 1 | Week 2 |
|---|---|---|
| Sunday | 5 | 8 |
| Monday | 20 | 15 |
| Tuesday | 30 | 32 |
| Wednesday | 35 | 30 |
| Thursday | 49 | 45 |
| Friday | 70 | 70 |
| Saturday | 15 | 10 |
| Total | 224 | 210 |

**a.** Calculate a seasonal factor for each day of the week.

**b.** If the postmaster estimates 230,000 pieces of mail to be sorted next week, forecast the volume for each day of the week.

**SOLUTION**

**a.** Calculate the average daily mail volume for each week. Then, for each day of the week, divide the mail volume by the week's average to get the seasonal factor. Finally, for each day, add the two seasonal factors and divide by 2 to obtain the average seasonal factor to use in the forecast (see part [b]).

| | WEEK 1 | | WEEK 2 | | |
|---|---|---|---|---|---|
| **Day** | **Mail Volume** | **Seasonal Factor (1)** | **Mail Volume** | **Seasonal Factor (2)** | **Average Seasonal Factor [(1) + (2)]/2** |
| Sunday | 5 | $5/32 = 0.15625$ | 8 | $8/30 = 0.26667$ | 0.21146 |
| Monday | 20 | $20/32 = 0.62500$ | 15 | $15/30 = 0.50000$ | 0.56250 |
| Tuesday | 30 | $30/32 = 0.93750$ | 32 | $32/30 = 1.06667$ | 1.00209 |
| Wednesday | 35 | $35/32 = 1.09375$ | 30 | $30/30 = 1.00000$ | 1.04688 |
| Thursday | 49 | $49/32 = 1.53125$ | 45 | $45/30 = 1.50000$ | 1.51563 |
| Friday | 70 | $70/32 = 2.18750$ | 70 | $70/30 = 2.33333$ | 2.26042 |
| Saturday | 15 | $15/32 = 0.46875$ | 10 | $10/30 = 0.33333$ | 0.40104 |
| Total | 224 | | 210 | | |
| Average | $224/7 = 32$ | | $210/7 = 30$ | | |

**b.** The average daily mail volume is expected to be $230{,}000/7 = 32{,}857$ pieces of mail. Using the average seasonal factors calculated in part (a), we obtain the following forecasts:

| **Day** | **Calculation** | | **Forecast** |
|---|---|---|---|
| Sunday | $0.21146(32{,}857) =$ | | 6,948 |
| Monday | $0.56250(32{,}857) =$ | | 18,482 |
| Tuesday | $1.00209(32{,}857) =$ | | 32,926 |
| Wednesday | $1.04688(32{,}857) =$ | | 34,397 |
| Thursday | $1.51563(32{,}857) =$ | | 49,799 |
| Friday | $2.26042(32{,}857) =$ | | 74,271 |
| Saturday | $0.40104(32{,}857) =$ | | 13,177 |
| | | Total | 230,000 |

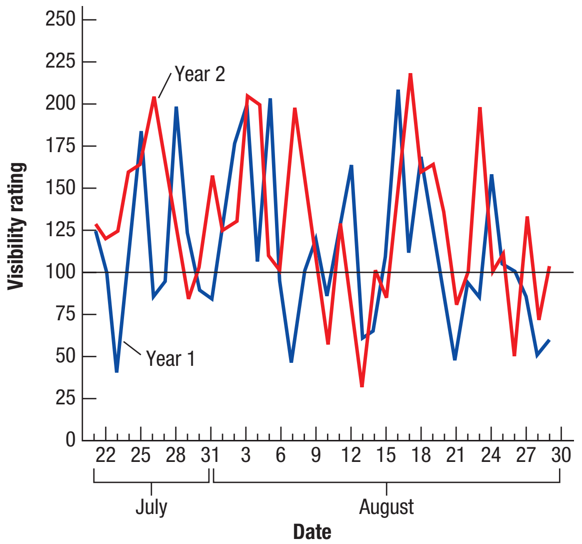
# Discussion Questions

1. Figure 8.9 shows summer air visibility measurements for Denver, Colorado. The acceptable visibility standard is 100, with readings above 100 indicating clean air and good visibility, and readings below 100 indicating temperature inversions caused by forest fires, volcanic eruptions, or collisions with comets.

   **a.** Is a trend evident in the data? Which time-series techniques might be appropriate for estimating the average of these data?

   **b.** A medical center for asthma and respiratory diseases located in Denver has great demand for its services when air quality is poor. If you were in charge of developing a short-term (say, 3-day) forecast of visibility, which causal factor(s) would you analyze? In other words, which external factors hold the potential to significantly affect visibility in the *short term*?

   **c.** Tourism, an important factor in Denver's economy, is affected by the city's image. Air quality, as measured by visibility, affects the city's image. If you were responsible for development of tourism, which causal factor(s) would you analyze to forecast visibility for the *medium term* (say, the next two summers)?




▲ **FIGURE 8.9**
Summer Air Visibility Measurements

**d.** The federal government threatens to withhold several hundred million dollars in Department of Transportation funds unless Denver meets visibility standards within 8 years. How would you proceed to generate a *long-term* judgment forecast of technologies that will be available to improve visibility in the next 10 years?

**2.** Kay and Michael Passe publish *What's Happening?*—a biweekly newspaper to publicize local events. *What's Happening?* has few subscribers; it typically is sold at checkout stands. Much of the revenue comes from advertisers of garage sales and supermarket specials. In an effort to reduce costs associated with printing too many papers or delivering them to the wrong location, Michael implemented a computerized system to collect sales data. Sales-counter scanners accurately record sales data for each location. Since the system was implemented, total sales volume has steadily declined. Selling advertising space and maintaining shelf space at supermarkets are getting more difficult.

Reduced revenue makes controlling costs all the more important. For each issue, Michael carefully makes a forecast based on sales data collected at each location. Then, he orders papers to be printed and distributed in quantities matching the forecast. Michael's forecast reflects a downward trend, which *is* present in the sales data. Now only a few papers are left over at only a few locations. Although the sales forecast accurately predicts the actual sales at most locations, *What's Happening?* is spiraling toward oblivion. Kay suspects that Michael is doing something wrong in preparing the forecast but can find no mathematical errors. Tell her what is happening.

# Problems

The OM Explorer and POM for Windows software is available to all students using the 10th edition of this textbook. Go to **http://www.pearsonglobaleditions.com/krajewski** to download these computer packages. If you purchased MyOMLab, you also have access to Active Models software and significant help in doing the following problems. Check with your instructor on how best to use these resources. In many cases, the instructor wants you to understand how to do the calculations by hand. At the least, the software provides a check on your calculations. When calculations are particularly complex and the goal is interpreting the results in making decisions, the software entirely replaces the manual calculations.

## Causal Methods: Linear Regression

**1.** Demand for oil changes at Garcia's Garage has been as follows:

| Month | Number of Oil Changes |
|---|---|
| January | 41 |
| February | 46 |
| March | 57 |
| April | 52 |
| May | 59 |
| June | 51 |
| July | 60 |
| August | 62 |

**a.** Use simple linear regression analysis to develop a forecasting model for monthly demand. In this application, the dependent variable, $Y$, is monthly demand and the independent variable, $X$, is the month. For January, let $X = 1$; for February, let $X = 2$; and so on.

**b.** Use the model to forecast demand for September, October, and November. Here, $X = 9$, 10, and 11, respectively.

**2.** At a hydrocarbon processing factory, process control involves periodic analysis of samples for a certain process quality parameter. The analytic procedure currently used is costly and time consuming. A faster and more economical alternative procedure has been proposed. However, the numbers for the quality parameter given by the alternative procedure are somewhat different from those given by the current procedure, not because of any inherent errors but because of changes in the nature of the chemical analysis.

Management believes that if the numbers from the new procedure can be used to forecast reliably the corresponding numbers from the current procedure, switching to the new procedure would be reasonable and cost effective. The following data were obtained for the quality parameter by analyzing samples using both procedures:

| Current ($Y$) | Proposed ($X$) | Current ($Y$) | Proposed ($X$) |
|---|---|---|---|
| 3.0 | 3.1 | 3.1 | 3.1 |
| 3.1 | 3.9 | 2.7 | 2.9 |
| 3.0 | 3.4 | 3.3 | 3.6 |
| 3.6 | 4.0 | 3.2 | 4.1 |
| 3.8 | 3.6 | 2.1 | 2.6 |
| 2.7 | 3.6 | 3.0 | 3.1 |
| 2.7 | 3.6 | 2.6 | 2.8 |

**a.** Use linear regression to find a relation to forecast $Y$, which is the quality parameter from the current procedure, using the values from the proposed procedure, $X$.

**b.** Is there a strong relationship between $Y$ and $X$? Explain.

**3.** Ohio Swiss Milk Products manufactures and distributes ice cream in Ohio, Kentucky, and West Virginia. The company wants to expand operations by locating another plant in northern Ohio. The size of the new plant will be a function of the expected demand for ice cream within the area served by the plant. A market survey is currently under way to determine that demand.

Ohio Swiss wants to estimate the relationship between the manufacturing cost per gallon and the number of gallons sold in a year to determine the demand for ice cream and, thus, the size of the new plant. The following data have been collected:

**a.** Develop a regression equation to forecast the cost per gallon as a function of the number of gallons produced.

| Plant | Cost per Thousand Gallons ($Y$) | Thousands of Gallons Sold ($X$) |
|---|---|---|
| 1 | $ 1,015 | 416.9 |
| 2 | 973 | 472.5 |
| 3 | 1,046 | 250.0 |
| 4 | 1,006 | 372.1 |
| 5 | 1,058 | 238.1 |
| 6 | 1,068 | 258.6 |
| 7 | 967 | 597.0 |
| 8 | 997 | 414.0 |
| 9 | 1,044 | 263.2 |
| 10 | 1,008 | 372.0 |
| Total | $10,182 | 3,654.4 |

**b.** What are the correlation coefficient and the coefficient of determination? Comment on your regression equation in light of these measures.

**c.** Suppose that the market survey indicates a demand of 325,000 gallons in the Bucyrus, Ohio area. Estimate the manufacturing cost per gallon for a plant producing 325,000 gallons per year.

**4.** A manufacturing firm has developed a skills test, the scores from which can be used to predict workers' production rating factors. Data on the test scores of various workers and their subsequent production ratings are shown.

| Worker | Test Score | Production Rating | Worker | Test Score | Production Rating |
|---|---|---|---|---|---|
| A | 53 | 45 | K | 54 | 59 |
| B | 36 | 43 | L | 73 | 77 |
| C | 88 | 89 | M | 65 | 56 |
| D | 84 | 79 | N | 29 | 28 |
| E | 86 | 84 | O | 52 | 51 |
| F | 64 | 66 | P | 22 | 27 |
| G | 45 | 49 | Q | 76 | 76 |
| H | 48 | 48 | R | 32 | 34 |
| I | 39 | 43 | S | 51 | 60 |
| J | 67 | 76 | T | 37 | 32 |

**a.** Using POM for Windows' least squares-linear regression module, develop a relationship to forecast production ratings from test scores.

**b.** If a worker's test score was 80, what would be your forecast of the worker's production rating?

**c.** Comment on the strength of the relationship between the test scores and production ratings.

**5.** The materials handling manager of a manufacturing company is trying to forecast the cost of maintenance for the company's fleet of over-the-road tractors. The manager believes that the cost of maintaining the tractors increases with their age. The following data was collected:

| Age (years) | Yearly Maintenance Cost ($) | Age (years) | Yearly Maintenance Cost ($) |
|---|---|---|---|
| 4.5 | 619 | 5.0 | 1,194 |
| 4.5 | 1,049 | 0.5 | 163 |
| 4.5 | 1,033 | 0.5 | 182 |
| 4.0 | 495 | 6.0 | 764 |
| 4.0 | 723 | 6.0 | 1,373 |
| 4.0 | 681 | 1.0 | 978 |
| 5.0 | 890 | 1.0 | 466 |
| 5.0 | 1,522 | 1.0 | 549 |
| 5.5 | 987 | | |

**a.** Use POM for Windows' least squares-linear regression module to develop a relationship to forecast the yearly maintenance cost based on the age of a tractor.

**b.** If a section has 20 three-year-old tractors, what is the forecast for the annual maintenance cost?

## Time-Series Methods

**6.** The owner of a computer store rents printers to some of her preferred customers. She is interested in arriving at a forecast of rentals so that she can order the correct quantities of supplies that go with the printers. Data for the last 10 weeks are shown here.

| Week | Rentals | Week | Rentals |
|---|---|---|---|
| 1 | 26 | 6 | 34 |
| 2 | 25 | 7 | 32 |
| 3 | 36 | 8 | 45 |
| 4 | 28 | 9 | 33 |
| 5 | 38 | 10 | 28 |

**a.** Prepare a forecast for weeks 6 through 10 by using a 5-week moving average. What is the forecast for week 11?

**b.** Calculate the mean absolute deviation as of the end of week 10.

**7.** Sales for the past 12 months at Computer Success are given here.

| Month | Sales ($) | Month | Sales ($) |
|---|---|---|---|
| January | 3,000 | July | 6,300 |
| February | 3,400 | August | 7,200 |
| March | 3,700 | September | 6,400 |
| April | 4,100 | October | 4,600 |
| May | 4,700 | November | 4,200 |
| June | 5,700 | December | 3,900 |

**a.** Use a three-month moving average to forecast the sales for the months May through December.

**b.** Use a four-month moving average to forecast the sales for the months May through December.

**c.** Compare the performance of the two methods by using the mean absolute deviation as the performance criterion. Which method would you recommend?

**d.** Compare the performance of the two methods by using the mean absolute percent error as the performance criterion. Which method would you recommend?

**e.** Compare the performance of the two methods by using the mean squared error as the performance criterion. Which method would you recommend?

**8.** Karl's Copiers sells and repairs photocopy machines. The manager needs weekly forecasts of service calls so that he can schedule service personnel. Use the actual demand in the first period for the forecast for the first week so error measurement begins in the second week. The manager uses exponential smoothing with $\alpha = 0.06$. Forecast the number of calls for week 6, which is next week.

| Week | Actual Service Calls |
|---|---|
| 1 | 24 |
| 2 | 36 |
| 3 | 36 |
| 4 | 25 |
| 5 | 30 |

**9.** Consider the sales data for Computer Success given in Problem 7.

**a.** Use a 3-month weighted moving average to forecast the sales for the months April through December. Use weights of (4/8), (3/8), and (1/8), giving more weight to more recent data.

**b.** Use exponential smoothing with $\alpha = 0.6$ to forecast the sales for the months April through December. Assume that the initial forecast for January was $3,200. Start error measurement in April.

**c.** Compare the performance of the two methods by using the mean absolute deviation as the performance criterion, with error measurement beginning in April. Which method would you recommend?

**d.** Compare the performance of the two methods by using the mean absolute percent error as the performance criterion, with error measurement beginning in April. Which method would you recommend?

**e.** Compare the performance of the two methods by using the mean squared error as the performance criterion, with error measurement beginning in April. Which method would you recommend?

**10.** A convenience store recently started to carry a new brand of soft drink. The management is interested in estimating future sales volume to determine whether it should continue to carry the new brand or replace it with another brand. The table below provides the number of cans sold per week. Use both the trend projection with regression and the exponential smoothing (let $\alpha = 0.4$ with an initial forecast for week 1 of 578) methods to forecast demand for week 13. Compare these methods by using the mean absolute deviation and mean absolute percent error performance criteria. Does your analysis suggest that sales are trending and if so, by how much?

| Week | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Sales | 578 | 624 | 648 | 737 | 678 | 614 | 756 | 675 | 759 | 692 | 672 | 749 |

**11.** Community Federal Bank in Dothan, Alabama, recently increased its fees to customers who use employees as tellers. The management is interested in whether its new fee policy has increased the number of customers now using its automatic teller machines to that point that more machines are required. The following table provides the number of automatic teller transactions by week. Use trend projection with regression to forecast usage for weeks 13–16.

| Week | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Transactions | 711 | 716 | 838 | 643 | 691 | 733 | 784 | 710 | 724 | 838 | 825 | 666 |

**12.** The number of heart surgeries performed at Heartville General Hospital has increased steadily over the past several years. The hospital's administration is seeking the best method to forecast the demand for such surgeries in year 6. The data for the past five years are shown.

| Year | Demand |
|---|---|
| 1 | 41 |
| 2 | 46 |
| 3 | 54 |
| 4 | 55 |
| 5 | 58 |

The hospital's administration is considering the following forecasting methods. Begin error measurement in year 3 so all methods are compared for the same years.

**i.** Exponential smoothing, with $\alpha = 0.6$. Let the initial forecast for year 1 be 41, the same as the actual demand.

**ii.** Exponential smoothing, with $\alpha = 0.9$. Let the initial forecast for year 1 be 41, the same as the actual demand.

**iii.** Trend projection with regression.

**iv.** Two-year moving average.

**v.** Two-year weighted moving average, using weights 0.6 and 0.4, with more recent data given more weight.

**vi.** If MAD is the performance criterion chosen by the administration, which forecasting method should it choose?

**vii.** If MSE is the performance criterion chosen by the administration, which forecasting method should it choose?

**viii.** If MAPE is the performance criterion chosen by the administration, which forecasting method should it choose?

**13.** The following data are for calculator sales in units at an Electronics store over the past 9 weeks:

| Week | Sales | Week | Sales |
|---|---|---|---|
| 1 | 44 | 6 | 53 |
| 2 | 50 | 7 | 59 |
| 3 | 46 | 8 | 53 |
| 4 | 49 | 9 | 65 |
| 5 | 55 | | |

Use trend projection with regression to forecast sales for weeks 10–13. What are the error measures (CFE, MSE, $\sigma$, MAD, and MAPE) for this forecasting procedure? How about $r^2$?

**14.** The demand for Krispee Crunchies, a favorite breakfast cereal of people born in the 1940s, is experiencing a decline. The company wants to monitor demand for this product closely as it nears the end of its life cycle. The following table shows the actual sales history for January–October. Generate forecasts for November–December, using the trend projection with regression method. Looking at the accuracy of its forecasts over the history file, as well as the other statistics provided, how confident are you in these forecasts for November–December?

| Month | Sales | Month | Sales |
|---|---|---|---|
| January | 890,000 | July | 710,000 |
| February | 800,000 | August | 730,000 |
| March | 825,000 | September | 680,000 |
| April | 840,000 | October | 670,000 |
| May | 730,000 | November | |
| June | 780,000 | December | |

**15.** Forrest and Dan make boxes of chocolates for which the demand is uncertain. Forrest says, "That's life." On the other hand, Dan believes that some demand patterns exist that could be useful for planning the purchase of sugar, chocolate, and shrimp. Forrest insists on placing a surprise chocolate-covered shrimp in some boxes so that "You never know what you'll get." Quarterly demand (in boxes of chocolates) for the last 3 years follows:

| Quarter | Year 1 | Year 2 | Year 3 |
|---|---|---|---|
| 1 | 3,000 | 3,300 | 3,502 |
| 2 | 1,700 | 2,100 | 2,448 |
| 3 | 900 | 1,500 | 1,768 |
| 4 | 4,400 | 5,100 | 5,882 |
| Total | 10,000 | 12,000 | 13,600 |

**a.** Use intuition and judgment to estimate quarterly demand for the fourth year.

**b.** If the expected sales for chocolates are 14,800 cases for year 4, use the multiplicative seasonal method to prepare a forecast for each quarter of the year. Are any of the quarterly forecasts different from what you thought you would get in part (a)?

**16.** The manager of Alaina's Garden Center must make the annual purchasing plans for rakes, gloves, and other gardening items. One of the items the company stocks is Fast-Grow, a liquid fertilizer. The sales of this item are seasonal, with peaks in the spring, summer, and fall months. Quarterly demand (in cases) for the past 2 years follows:

| Quarter | Year 1 | Year 2 |
|---|---|---|
| 1 | 45 | 67 |
| 2 | 339 | 444 |
| 3 | 299 | 329 |
| 4 | 222 | 283 |
| Total | 905 | 1,123 |

If the expected sales for Fast-Grow are 1,850 cases for year 3, use the multiplicative seasonal method to prepare a forecast for each quarter of the year.

**17.** The manager of a utility company in the Texas panhandle wants to develop quarterly forecasts of power loads for the next year. The power loads are seasonal, and the data on the quarterly loads in megawatts (MW) for the last 4 years are as follows:

| Quarter | Year 1 | Year 2 | Year 3 | Year 4 |
|---|---|---|---|---|
| 1 | 103.5 | 94.7 | 118.6 | 109.3 |
| 2 | 126.1 | 116.0 | 141.2 | 131.6 |
| 3 | 144.5 | 137.1 | 159.0 | 149.5 |
| 4 | 166.1 | 152.5 | 178.2 | 169.0 |

The manager estimates the total demand for the next year at 600 MW. Use the multiplicative seasonal method to develop the forecast for each quarter.

**18.** D Franklin Tooling, Inc., manufactures specialty tooling for firms in the paper-making industry. All of their products are engineer-to-order and so the company never knows exactly what components to purchase for a tool until a customer places an order. However, the company believes that weekly demand for a few components is fairly stable. Component 135.AG is one such item. The last 26 weeks of historical use of component 135.AG is recorded below.

| Week | Demand | Week | Demand |
|---|---|---|---|
| 1 | 137 | 14 | 131 |
| 2 | 136 | 15 | 132 |
| 3 | 143 | 16 | 124 |
| 4 | 136 | 17 | 121 |
| 5 | 141 | 18 | 127 |
| 6 | 128 | 19 | 118 |
| 7 | 149 | 20 | 120 |
| 8 | 136 | 21 | 115 |
| 9 | 134 | 22 | 106 |
| 10 | 142 | 23 | 120 |
| 11 | 125 | 24 | 113 |
| 12 | 134 | 25 | 121 |
| 13 | 118 | 26 | 119 |

Use OM Explorer's *Time Series Forecasting* Solver to evaluate the following forecasting methods. Start error measurement in the fifth week, so all methods are evaluated over the same time interval. Use the default settings for initial forecasts.

**i.** Naïve (1-Period Moving Average)

**ii.** 3-Period Moving Average

**iii.** Exponential Smoothing, with $\alpha = .28$

**iv.** Trend Projection with Regression

**v.** Which forecasting method should management use, if the performance criterion it chooses is:

- CFE?
- MSE?
- MAD?
- MAPE?

**19.** D Create an Excel spreadsheet on your own that can create combination forecasts for Problem 18. Create a combination forecast using all four techniques from Problem 18. Give each technique an equal weight. Create a second combination forecast by using the three techniques that seem best based on MAD. Give equal weight to each technique. Finally, create a third forecast by equally weighting the two best techniques. Calculate CFE, MAD, MSE, and MAPE for the combination forecast. Are these forecasts better or worse than the forecasting techniques identified in Problem 18?

**20.** D The director of a large public library must schedule employees to reshelf books and periodicals checked out of the library. The number of items checked out will determine the labor requirements. The following data reflect the number of items checked out of the library for the past 3 years:

| Month | Year 1 | Year 2 | Year 3 |
|---|---|---|---|
| January | 1,847 | 2,045 | 1,986 |
| February | 2,669 | 2,321 | 2,564 |
| March | 2,467 | 2,419 | 2,635 |
| April | 2,432 | 2,088 | 2,150 |
| May | 2,464 | 2,667 | 2,201 |
| June | 2,378 | 2,122 | 2,663 |
| July | 2,217 | 2,206 | 2,055 |
| August | 2,445 | 1,869 | 1,678 |
| September | 1,894 | 2,441 | 1,845 |
| October | 1,922 | 2,291 | 2,065 |
| November | 2,431 | 2,364 | 2,147 |
| December | 2,274 | 2,189 | 2,451 |

The director needs a time-series method for forecasting the number of items to be checked out during the next month. Find the best simple moving average forecast you can. Decide what is meant by "best" and justify your decision.

D = Difficult Problem

**21.** D Using the data in Problem 20 and the Time Series Solver of OM Explorer, find the best exponential smoothing parameter alpha that minimizes MAD. Let the forecast for period 1 be the actual data for period 1, and begin the error analysis in period 2.

**22.** D Using the data in Problem 20, find the trend projection with regression model using the Time Series Forecasting Solver of OM Explorer. Compare the performance of this method with the exponential smoothing method from Problem 21. Let the error analysis begin in period 2 (so that both exponential smoothing and trend projection are analyzed over the same time horizon).Which of these two methods would you choose if MAD is the key error measure?

**23.** D Cannister, Inc., specializes in the manufacture of plastic containers. The data on the monthly sales of 10-ounce shampoo bottles for the past 5 years are as follows:

| Year | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| January | 742 | 741 | 896 | 951 | 1,030 |
| February | 697 | 700 | 793 | 861 | 1,032 |
| March | 776 | 774 | 885 | 938 | 1,126 |
| April | 898 | 932 | 1,055 | 1,109 | 1,285 |
| May | 1,030 | 1,099 | 1,204 | 1,274 | 1,468 |
| June | 1,107 | 1,223 | 1,326 | 1,422 | 1,637 |
| July | 1,165 | 1,290 | 1,303 | 1,486 | 1,611 |
| August | 1,216 | 1,349 | 1,436 | 1,555 | 1,608 |
| September | 1,208 | 1,341 | 1,473 | 1,604 | 1,528 |
| October | 1,131 | 1,296 | 1,453 | 1,600 | 1,420 |
| November | 971 | 1,066 | 1,170 | 1,403 | 1,119 |
| December | 783 | 901 | 1,023 | 1,209 | 1,013 |

**a.** Using the multiplicative seasonal method, calculate the monthly seasonal indices.

**b.** Develop a simple linear regression equation to forecast annual sales. For this regression, the dependent variable, $Y$, is the demand in each year and the independent variable, $X$, is the index for the year (i.e.,$X = 1$ for year 1, $X = 2$ for year 2, and so on until $X = 5$ for year 5).

**c.** Forecast the annual sales for year 6 by using the regression model you developed in part (b).

**d.** Prepare the seasonal forecast for each month by using the monthly seasonal indices calculated in part (a).

**24.** D The Midwest Computer Company serves a large number of businesses in the Great Lakes region. The company sells supplies and replacements and performs service on all computers sold through seven sales offices. Many items are stocked, so close inventory control is necessary to assure customers of efficient service. Recently, business has been increasing, and management is concerned about stockouts. A forecasting method is needed to estimate requirements several months in advance so that adequate replenishment quantities can be purchased. An example of the sales growth experienced during the last 50 months is the growth in demand for item EP-37, a laser printer cartridge, shown in Table 8.3.

**a.** Develop a trend projection with regression solution using OM Explorer. Forecast demand for month 51.

**b.** A consultant to Midwest's management suggested that new office building leases would be a good leading indicator for company sales. The consultant quoted a recent university study finding that new office building leases precede office equipment and supply sales by 3 months. According to the study findings, leases in month 1 would affect sales in month 4, leases in month 2 would affect sales in month 5, and so on. Use POM for Windows' linear regression module to develop a forecasting model for sales, with leases as the independent variable. Forecast sales for month 51.

**c.** Which of the two models provides better forecasts? Explain.

**TABLE 8.3** | **EP-37 SALES AND LEASE DATA**

| Month | EP-37 Sales | Leases | Month | EP-37 Sales | Leases |
|---|---|---|---|---|---|
| 1 | 80 | 32 | 26 | 1,296 | 281 |
| 2 | 132 | 29 | 27 | 1,199 | 298 |
| 3 | 143 | 32 | 28 | 1,267 | 314 |
| 4 | 180 | 54 | 29 | 1,300 | 323 |
| 5 | 200 | 53 | 30 | 1,370 | 309 |
| 6 | 168 | 89 | 31 | 1,489 | 343 |
| 7 | 212 | 74 | 32 | 1,499 | 357 |
| 8 | 254 | 93 | 33 | 1,669 | 353 |
| 9 | 397 | 120 | 34 | 1,716 | 360 |
| 10 | 385 | 113 | 35 | 1,603 | 370 |
| 11 | 472 | 147 | 36 | 1,812 | 386 |
| 12 | 397 | 126 | 37 | 1,817 | 389 |

D = Difficult Problem

| Month | EP-37 Sales | Leases | Month | EP-37 Sales | Leases |
|---|---|---|---|---|---|
| 13 | 476 | 138 | 38 | 1,798 | 399 |
| 14 | 699 | 145 | 39 | 1,873 | 409 |
| 15 | 545 | 160 | 40 | 1,923 | 410 |
| 16 | 837 | 196 | 41 | 2,028 | 413 |
| 17 | 743 | 180 | 42 | 2,049 | 439 |
| 18 | 722 | 197 | 43 | 2,084 | 454 |
| 19 | 735 | 203 | 44 | 2,083 | 441 |
| 20 | 838 | 223 | 45 | 2,121 | 470 |
| 21 | 1,057 | 247 | 46 | 2,072 | 469 |
| 22 | 930 | 242 | 47 | 2,262 | 490 |
| 23 | 1,085 | 234 | 48 | 2,371 | 496 |
| 24 | 1,090 | 254 | 49 | 2,309 | 509 |
| 25 | 1,218 | 271 | 50 | 2,422 | 522 |

**25.** D A certain food item at P&Q Supermarkets has the demand pattern shown in the following 24-period table.

**a.** Use the Combination forecasting method of the Time Series Forecasting Solver of OM Explorer. Let error analysis begin in month 6, and include (1) a 5-period moving average (with a combination weight of 0.33), (2) an exponential smoothing model with $\alpha = 0.20$ (with a combination weight of 0.33), and (3) trend projection (with a combination weight of 0.33). What is the MAD of this model? What is the forecast for month 25?

**b.** The need to account for seasonality is apparent if you look at a graph of the trend line. There is a spike in demand in the 5th period of each 5-period cycle. Unfortunately, OM Explorer's Seasonal Forecasting Solver does not cover the case where there are 5 periods in a cycle (or seasons in a year). You must do some manual calculations. Begin by calculating the seasonal factor for each period in each of the first 4 cycles, and then calculating the average seasonal factor for each period (see Example 8.6). Now estimate the total demand for cycle 5 using OM Explorer's Trend Projection routine in the Time Series Solver. The dependent variables (see pages 309–311) are the total demands for the first 4 cycles. Now multiply the average demand estimate for the 5th cycle by the seasonal factor for the 5th period. This is your forecast for month 25. To calculate the errors (including MAD) for the multiplicative seasonal method for all cycles (except for the 5th month in the 5th cycle), calculate MAD manually. You might instead use the Error Analysis Module of POM for Windows.

**c.** How do the forecasts by the two methods compare? Which one is likely to give the better forecast, based on MAD?

| Period | Demand | Period | Demand |
|---|---|---|---|
| 1 | 33 | 13 | 37 |
| 2 | 37 | 14 | 43 |
| 3 | 31 | 15 | 56 |
| 4 | 39 | 16 | 41 |
| 5 | 54 | 17 | 36 |
| 6 | 38 | 18 | 39 |
| 7 | 42 | 19 | 41 |
| 8 | 40 | 20 | 58 |
| 9 | 41 | 21 | 42 |
| 10 | 54 | 22 | 45 |
| 11 | 43 | 23 | 41 |
| 12 | 39 | 24 | 38 |

**26.** D The data for the visibility chart in Discussion Question 1 are shown in Table 8.4. The visibility standard is set at 100. Readings below 100 indicate that air pollution has reduced visibility, and readings above 100 indicate that the air is clearer.

**a.** Use several methods to generate a visibility forecast for August 31 of the second year. Which method seems to produce the best forecast?

**b.** Use several methods to forecast the visibility index for the summer of the third year. Which method seems to produce the best forecast? Support your choice.

D = Difficult Problem

**TABLE 8.4 | VISIBILITY DATA**

| Date | Year 1 | Year 2 | Date | Year 1 | Year 2 | Date | Year 1 | Year 2 |
|---|---|---|---|---|---|---|---|---|
| July 22 | 125 | 130 | Aug 5 | 105 | 200 | Aug 19 | 170 | 160 |
| 23 | 100 | 120 | 6 | 205 | 110 | 20 | 125 | 165 |
| 24 | 40 | 125 | 7 | 90 | 100 | 21 | 85 | 135 |
| 25 | 100 | 160 | 8 | 45 | 200 | 22 | 45 | 80 |
| 26 | 185 | 165 | 9 | 100 | 160 | 23 | 95 | 100 |
| 27 | 85 | 205 | 10 | 120 | 100 | 24 | 85 | 200 |
| 28 | 95 | 165 | 11 | 85 | 55 | 25 | 160 | 100 |
| 29 | 200 | 125 | 12 | 125 | 130 | 26 | 105 | 110 |
| 30 | 125 | 85 | 13 | 165 | 75 | 27 | 100 | 50 |
| 31 | 90 | 105 | 14 | 60 | 30 | 28 | 95 | 135 |
| Aug 1 | 85 | 160 | 15 | 65 | 100 | 29 | 50 | 70 |
| 2 | 135 | 125 | 16 | 110 | 85 | 30 | 60 | 105 |
| 3 | 175 | 130 | 17 | 210 | 150 | | | |
| 4 | 200 | 205 | 18 | 110 | 220 | | | |

**27.** (D) Tom Glass forecasts electrical demand for the Flatlands Public Power District (FPPD). The FPPD wants to take its Comstock power plant out of service for maintenance when demand is expected to be low. After shutdown, performing maintenance and getting the plant back on line takes two weeks. The utility has enough other generating capacity to satisfy 1,550 megawatts (MW) of demand while Comstock is out of service. Table 8.5 shows weekly peak demands (in MW) for the past several autumns. When next in year 6 should the Comstock plant be scheduled for maintenance?

**TABLE 8.5 | WEEKLY PEAK POWER DEMANDS**

| | AUGUST | | | SEPTEMBER | | | | OCTOBER | | | | NOVEMBER | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Year** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** | **13** |
| 1 | 2,050 | 1,925 | 1,825 | 1,525 | 1,050 | 1,300 | 1,200 | 1,175 | 1,350 | 1,525 | 1,725 | 1,575 | 1,925 |
| 2 | 2,000 | 2,075 | 2,225 | 1,800 | 1,175 | 1,050 | 1,250 | 1,025 | 1,300 | 1,425 | 1,625 | 1,950 | 1,950 |
| 3 | 1,950 | 1,800 | 2,150 | 1,725 | 1,575 | 1,275 | 1,325 | 1,100 | 1,500 | 1,550 | 1,375 | 1,825 | 2,000 |
| 4 | 2,100 | 2,400 | 1,975 | 1,675 | 1,350 | 1,525 | 1,500 | 1,150 | 1,350 | 1,225 | 1,225 | 1,475 | 1,850 |
| 5 | 2,275 | 2,300 | 2,150 | 1,525 | 1,350 | 1,475 | 1,475 | 1,175 | 1,375 | 1,400 | 1,425 | 1,550 | 1,900 |

**28.** (D) A manufacturing firm seeks to develop a better forecast for an important product, and believes that there is a trend to the data. OM Explorer's *Trend Projection with Regression* Solver has been set up with the 47 demands in the history file. Note the "Load Problem 28 Data" button in the *Trend Projection with Regression* Solver that when clicked will automatically input the demand data. Otherwise, you can enter the demand data directly into the Inputs sheet.

| Yr | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Jan | 4507 | 4589 | 4084 | 4535 |
| Feb | 4400 | 4688 | 4158 | 4477 |
| Mar | 4099 | 4566 | 4174 | 4601 |
| Apr | 4064 | 4485 | 4225 | 4648 |
| May | 4002 | 4385 | 4324 | 4860 |
| Jun | 3963 | 4377 | 4220 | 4998 |
| Jul | 4037 | 4309 | 4267 | 5003 |
| Aug | 4162 | 4276 | 4187 | 4960 |
| Sep | 4312 | 4280 | 4239 | 4943 |
| Oct | 4395 | 4144 | 4352 | 5052 |
| Nov | 4540 | 4219 | 4331 | 5107 |
| Dec | 4471 | 4052 | 4371 | |

(D) = Difficult Problem

**a.** What is your forecast for December of Year 4, making period 1 as the starting period for the regression?

**b.** The actual demand for period 48 was just learned to be 5,100. Add this demand to the Inputs file and change the starting period for the regression to period 2 so that the number of periods in the regression remains unchanged. How much or little does the forecast for period 49 change from the one for period 48? The error measures? Are you surprised?

**c.** Now change the time when the regression starts to period 25 and repeat the process. What differences do you note now? What forecast will you make for period 49?

## VIDEO CASE Forecasting and Supply Chain Management at Deckers Outdoor Corporation

Deckers Outdoor Corporation's footwear products are among some of the most well-known brands in the world. From UGG sheepskin boots and Teva sport sandals to Simple shoes, Deckers flip-flops, and Tsubo footwear, Deckers is committed to building niche footwear brands into global brands with market leadership positions. Net sales for fiscal year 2007 were close to $449 million. In addition to traditional retail store outlets for Deckers' footwear styles, the company maintains an active and growing "direct to consumer" e-commerce business. Since most retail stores cannot carry every style in every color and size, the company offers the full line for each of its brands directly to consumers through the brands' individual Web sites. Online sales at its virtual store are handled by its e-commerce group. Customers who want a pair of shoes not available at the retail store can always buy from the virtual store.

Pearson

Customer orders for Decker footwear are broken down by brand and sent to headquarters in Goletta, CA. where the order is entered into the system.

Founded in 1973, the company manufactured a single line of sandals in a small factory in Southern California. The challenges of managing the raw materials and finished goods inventories were small compared to today's global sourcing and sales challenges for the company's various brands. Today, each brand has its own development team and brand managers who generate, develop, and test-market the seasonal styles that appear on the shelves of retailers such as Nordstrom, Lord & Taylor, REI, the Walking Company, and the company's own UGG brand retail stores in the United States and Japan.

At Deckers, forecasting is the starting point for inventory management, sales and operations planning, resource planning, and scheduling—in short, managing its supply chain. It carries a considerable amount of seasonal stock. Shoes with seasonal demand that are left over at the end of their season must be sold at heavily discounted prices. Its products fall into three categories: (1) carry-over items that were sold in prior years, (2) new items that look similar to past models, and (3) completely new designs that are fashionable with no past history.

Twice a year, the brand development teams work on the fall and spring product lines. They come up with new designs about one year in advance of each season. Each brand (UGG, Teva, Simple, Tsubo, and Deckers) contains numerous products. The materials for new designs are selected and tested in prototypes. Approved designs are put into the seasonal line-up. Forecasts must be made at both the product and aggregate levels months before the season begins. "Bottoms-up" forecasts for each product begin by analyzing any available history files of past demand. Judgment forecasts are also important inputs, particularly for the second and third categories of shoes that are not carry-overs. For example, Char Nicanor-Kimball is an expert in spotting trends in shoe sales and makes forecasts for the virtual store. For new designs, historical sales on similar items are used to make a best guess on demand for those items. This process is facilitated by a forecasting and inventory system on the company's Intranet. At the same time, the sales teams for each brand call on their retail accounts and secure customer orders of approved designs for the coming season. Then, the virtual store forecasts are merged with orders from the retail store orders to get the total seasonal demand forecasted by product. Next, the product forecasts are "rolled up" by category and "top down" forecasts are also made.

These forecasts then go to top management where some adjustments may be made to account for financial market conditions, consumer credit, weather, demographic factors, and customer confidence. The impact of public relations and advertising must also be considered.

Actually, forecasting continues on throughout the year on a daily and weekly basis to "get a handle" on demand. Comparing actual demand with what was forecasted for different parts of the season also helps the forecasters make better forecasts for the future and better control inventories.

Based on initial demand forecasts, the company must begin sourcing the materials needed to produce the footwear. The company makes most of its products in China and sources many of the raw materials there as well. For UGG products sheepskin sourcing occurs in Australia with top grade producers, but the rawhide tanning still takes places in China. With potential suppliers identified and assurance from internal engineering that the footwear can be successfully made, the engineering and material data are handed over to the manufacturing department to determine how best to make the footwear in mass quantities. At this point, Deckers places a seasonal "buy" with its suppliers.

The orders for each product are fed into the manufacturing schedules at the Chinese factories. All the products for a given brand are manufactured at the same factory. While Deckers agents negotiate the raw materials contracts early in the development process, the factories only place the orders for the raw materials when the company sends in the actual orders for the finished goods. No footwear is made by the factories until orders are received.

At the factories, finished goods footwear is inspected and packaged for the month-long ocean voyage from Hong Kong to ports in the United States. Deckers ships fifty containers a week from its Chinese manufacturing sources, each holding approximately 5,000 pairs of shoes. Ownership of the finished goods transfers from the factories to Deckers in Hong Kong.

When the shipping containers arrive in the United States, the footwear is transferred to Deckers' distribution centers in Southern California. Teva products are warehoused in Ventura, California; all other products are handled by the company's state-of-the-art facility in Camarillo, California. Typically, Deckers brings product into the distribution centers two to three months in advance of expected needs so that the production at the suppliers' factories and the labor activities at the distribution centers are leveled. There are definitive spikes in the demand for footwear, with Teva spiking in Quarter 1 and UGG spiking in Quarter 4. The leveling approach works to keep costs low in the supply chain. However, it also means that Deckers must maintain sizeable inventories. Most shipments from suppliers come in to the distribution centers and are stored in inventory for one to two months awaiting a customer order. By the time the footwear is stocked in the distribution center, the company knows which retail customers will be getting the various products, based on the orders booked months earlier. Then, according to delivery schedules negotiated with the customers, the company begins filling orders and shipping products to retail locations. The warehouse tracks incoming shipments, goods placed on the shelves for customers, and outgoing orders. The inventory system helps manage the customer order filling process.

Because the booked orders are a relatively large proportion of the total orders from retailers, and the number of unanticipated orders is very small, only small safety stocks are needed to service the retailers. Occasionally, the purchase order from Deckers to one of its suppliers matches the sales order from the customer. In such a case, Deckers uses a "cross-dock" system. When the shipment is received at the distribution center, it is immediately checked in and loaded on another truck for delivery to customers. Cross docking reduces the need to store vast quantities of product for long periods of time and cuts down on warehousing expenses for Deckers. The company has been successful in turning its inventory over about four times a year, which is in line with footwear industry standards.

The online sales traffic is all managed centrally. In fact, for ordering and inventory management purposes, the online side of the business is treated just like another major retail store account. As forecasted seasonal orders are generated by each brand's sales team, a manufacturing order for the online business is placed by the e-commerce sales team at the same time. However, unlike the retail outlets that take delivery of products on a regular schedule, the inventory pledged to the online business is held in the distribution center until a Web site order is received. Only then is it shipped directly to the consumer who placed the online order. If actual demand exceeds expected demand, Char Nicanor-Kimball checks if more inventory can be secured from other customer orders that have scaled back.

The forecasting and supply chain management challenges now facing Deckers are two-fold. First, the company plans to grow the brands that have enjoyed seasonal sales activity into year-round footwear options for consumers by expanding the number of products for those brands. For example, most sales for UGG footwear occur in the fall/winter season. Sales for Teva historically have been in the spring and summer. Product managers are now working to develop styles that will allow the brands to cross over the seasons. Second, the company plans to expand internationally, and will have retail outlets in Europe, China, and other Asian locations in the very near future. Company managers are well aware of the challenges and opportunities such global growth will bring, and are taking steps now to assure that the entire supply chain is prepared to forecast and handle the demand when the time comes.

### QUESTIONS

1. How much does the forecasting process at Deckers correspond with the "typical forecasting process" described at the end of this chapter?
2. Based on what you see in the video, what kinds of information technology are used to make forecasts, maintain accurate inventory records, and project future inventory levels?
3. What factors make forecasting at Deckers particularly challenging? How can forecasts be made for seasonal, fashionable products for which there is no history file? What are the costs of over-forecasting demand for such items? Under-forecasting?
4. What are the benefits of leveling aggregate demand by having a portfolio of products that create 365-day demand?
5. Deckers plans to expand internationally, thereby increasing the volume of shoes it must manage in the supply chain and the pattern of material flows. What implications does this strategy have on forecasting, order quantities, logistics, and relationships with its suppliers and customers?

## CASE Yankee Fork and Hoe Company

The Yankee Fork and Hoe Company is a leading producer of garden tools ranging from wheelbarrows, mortar pans, and hand trucks to shovels, rakes, and trowels. The tools are sold in four different product lines ranging from the top-of-the-line Hercules products, which are rugged tools for the toughest jobs, to the Garden Helper products, which are economy tools for the occasional user. The market for garden tools is extremely competitive because of the simple design of the products and the large number of competing producers. In addition, more people are using power tools, such as lawn edgers, hedge trimmers, and thatchers, reducing demand for their manual counterparts. These factors compel Yankee to maintain low prices while retaining high quality and dependable delivery.

Garden tools represent a mature industry. Unless new manual products can be developed or a sudden resurgence occurs in home gardening, the prospects for large increases in sales are not bright. Keeping ahead of the competition is a constant battle. No one knows this better than Alan Roberts, president of Yankee.

The types of tools sold today are, by and large, the same ones sold 30 years ago. The only way to generate new sales and retain old customers is to provide superior customer service and produce a product with high customer value. This approach puts pressure on the manufacturing system, which has been having difficulties lately. Recently, Roberts has been receiving calls from long-time customers, such as Sears and True Value Hardware Stores, complaining about late shipments. These customers advertise promotions for garden tools and require on-time delivery.

Roberts knows that losing customers like Sears and True Value would be disastrous. He decides to ask consultant Sharon Place to look into the matter and report to him in one week. Roberts suggests that she focus on the bow rake as a case in point because it is a high-volume product and has been a major source of customer complaints of late.

### Planning Bow Rake Production

A bow rake consists of a head with 12 teeth spaced 1 inch apart, a hardwood handle, a bow that attaches the head to the handle, and a metal ferrule that reinforces the area where the bow inserts into the handle. The bow is a metal strip that is welded to the ends of the rake head and bent in the middle to form a flat tab for insertion into the handle. The rake is about 64 inches long.

Place decides to find out how Yankee plans bow rake production. She goes straight to Phil Stanton, who gives the following account:

> Planning is informal around here. To begin, marketing determines the forecast for bow rakes by month for the next year. Then they pass it along to me. Quite frankly, the forecasts are usually inflated—must be their big egos over there. I have to be careful because we enter into long-term purchasing agreements for steel, and having it just sitting around is expensive. So I usually reduce the forecast by 10 percent or so. I use the modified forecast to generate a monthly final-assembly schedule, which determines what I need to have from the forging and woodworking areas. The system works well if the forecasts are good. But when marketing comes to me and says they are behind on customer orders, as they often do near the end of the year, it wreaks havoc with the schedules. Forging gets hit the hardest. For example, the presses that stamp the rake heads from blanks of steel can handle only 7,000 heads per day, and the bow rolling machine can do only 5,000 per day. Both operations are also required for many other products.

Because the marketing department provides crucial information to Stanton, Place decides to see the marketing manager, Ron Adams. Adams explains how he arrives at the bow rake forecasts.

> Things do not change much from year to year. Sure, sometimes we put on a sales promotion of some kind, but we try to give Phil enough warning before the demand kicks in—usually a month or so. I meet with several managers from the various sales regions to go over shipping data from last year and discuss anticipated promotions, changes in the economy, and shortages we experienced last year. Based on these meetings, I generate a monthly forecast for the next year. Even though we take a lot of time getting the forecast, it never seems to help us avoid customer problems.

### The Problem

Place ponders the comments from Stanton and Adams. She understands Stanton's concerns about costs and keeping inventory low and Adams's concern about having enough rakes on hand to make timely shipments. Both are also somewhat concerned about capacity. Yet she decides to check actual customer demand for the bow rake over the past 4 years (in Table 8.6) before making her final report to Roberts.

## QUESTIONS

1. Comment on the forecasting system being used by Yankee. Suggest changes or improvements that you believe are justified.
2. Develop your own forecast for bow rakes for each month of the next year (year 5). Justify your forecast and the method you used.

**TABLE 8.6** FOUR-YEAR DEMAND HISTORY FOR THE BOW RAKE

| | DEMAND | | | |
|---|---|---|---|---|
| **Month** | **Year 1** | **Year 2** | **Year 3** | **Year 4** |
| 1 | 55,220 | 39,875 | 32,180 | 62,377 |
| 2 | 57,350 | 64,128 | 38,600 | 66,501 |
| 3 | 15,445 | 47,653 | 25,020 | 31,404 |
| 4 | 27,776 | 43,050 | 51,300 | 36,504 |
| 5 | 21,408 | 39,359 | 31,790 | 16,888 |
| 6 | 17,118 | 10,317 | 32,100 | 18,909 |
| 7 | 18,028 | 45,194 | 59,832 | 35,500 |
| 8 | 19,883 | 46,530 | 30,740 | 51,250 |
| 9 | 15,796 | 22,105 | 47,800 | 34,443 |
| 10 | 53,665 | 41,350 | 73,890 | 68,088 |
| 11 | 83,269 | 46,024 | 60,202 | 68,175 |
| 12 | 72,991 | 41,856 | 55,200 | 61,100 |

# EXPERIENTIAL LEARNING 8.1 Forecasting a Vital Energy Statistic

The following time-series data capture the weekly average of East Coast crude oil imports in thousands of barrels per day.

| QUARTER 2 YEAR 1 | | QUARTER 3 YEAR 1 | | QUARTER 4 YEAR 1 | | QUARTER 1 YEAR 2 | |
|---|---|---|---|---|---|---|---|
| **Week** | **Data** | **Week** | **Data** | **Week** | **Data** | **Week** | **Data** |
| 1 | 1,160 | 14 | 1,116 | 27 | 1,073 | 40 | 994 |
| 2 | 779 | 15 | 1,328 | 28 | 857 | 41 | 1,307 |
| 3 | 1,134 | 16 | 1,183 | 29 | 1,197 | 42 | 997 |
| 4 | 1,275 | 17 | 1,219 | 30 | 718 | 43 | 1,082 |
| 5 | 1,355 | 18 | 1,132 | 31 | 817 | 44 | 887 |
| 6 | 1,513 | 19 | 1,094 | 32 | 946 | 45 | 1,067 |
| 7 | 1,394 | 20 | 1,040 | 33 | 725 | 46 | 890 |
| 8 | 1,097 | 21 | 1,053 | 34 | 748 | 47 | 865 |
| 9 | 1,206 | 22 | 1,232 | 35 | 1,031 | 48 | 858 |
| 10 | 1,264 | 23 | 1,073 | 36 | 1,061 | 49 | 814 |
| 11 | 1,153 | 24 | 1,329 | 37 | 1,074 | 50 | 871 |
| 12 | 1,424 | 25 | 1,096 | 38 | 941 | 51 | 1,255 |
| 13 | 1,274 | 26 | 1,125 | 39 | 994 | 52 | 980 |

Your instructor has a "holdout" sample representing the values for Week 53 and beyond. Your task is to use the POM for Windows *Time Series Forecasting* module and the history file to project this statistic into the future. If you have MyOMLab, the demand data is available in the *Experimental Exercise 8.1* Data file. It can be pasted into the Data Table of the *Time Series Forecasting* module. Otherwise, you can enter the demand data directly into the Data Table. Prior to your next class meeting:

**a.** Use the POM for Windows *Time Series Forecasting* module to locate the best naïve, moving average, weighted moving average, and trend projection with regression models that you think will most accurately forecast demand during the holdout sample. *Begin your error calculations with week 5.*

**b.** Create an Excel spreadsheet that begins with inputs of the four forecasts from the *Time Series Forecasting* module. Its purpose is to develop a combination forecast that will serve as your team's forecasts for each period. Assign a weight to each forecast model (the sum of all four forecast weights for one period should equal 1.0) and develop a "combination forecast" by multiplying each forecast by its weight. Keep the weights constant for the whole history file as you search for the best set of weights. If you do not like a particular model, give it a weight of 0. Calculate appropriate forecast error measures for your combination forecast in your Excel spreadsheet.

**c.** Create a management report that shows your period-by-period forecasts and their overall historical CFE and MAPE performance for each model and your combination forecast.

### In-Class Exercise–Part 1

**a.** Input into your Excel spreadsheet the forecasts from the POM for Windows *Time Series Forecasting* module to get the combination forecast for the first period (week 53) in the holdout sample. The combination forecast is considered your team's forecast.

**b.** Enter the actual data announced by your instructor, and have Excel compute appropriate forecast error measures for your four models and the combination forecast. Decide on any revisions of weights for the combination forecast.

**c.** Update the POM for Windows *Time Series Forecasting* module with the actual demand for the new period and get the new forecasts.

### In-Class Exercise–Part 2

**a.** Input the forecasts from the POM for Windows *Time Series Forecasting* module into your Excel spreadsheet to get the final combination forecast for the next period (week 54). At this point, you may change this period's weights on each forecasting technique going into the combination forecast. You have no contextual information, but may observe that one model has been performing particularly well in the last few periods. Your team might have different opinions, but you must reach a consensus.

**b.** Enter the actual data announced by your instructor, with Excel computing appropriate forecast error measures for your four models and the combination forecast.

**c.** Update the POM for Windows *Time Series Forecasting* module with the actual demand for the new period and get the new forecasts.

### In-Class Exercise–Parts 3 and beyond

Continue in the fashion of Parts 1 and 2 to produce forecasts as directed by your instructor. At the end of the exercise, create a second management report that shows for the holdout sample your period-by-period forecasts, their individual forecast errors and percent deviations for each model and your combination forecast. Explain your logic regarding any changes made to your combination forecast weights over the holdout period.

*Source:* This experiential exercise was prepared as an in-class exercise prepared by Dr. John Jensen, University of South Carolina, as a basis for classroom discussion. By permission of John B. Jensen.


Michael Tercha/MCT/Newscom

Associates remove returned DVDs from their mailing envelop and then inspect them at Netflix's Carol Stream, Illinois distribution facility. This facility serves subscribers in Chicago and surrounding communities.

9

# MANAGING INVENTORIES

## Inventory Management at Netflix

Netflix is a $4 billion company specializing in delivering movies and TV programs directly to the homes of customers via streaming or DVD for a subscription fee. It employs more than 2,000 employees worldwide, has more than 44 million streaming customers in over 40 countries, and has 7 million DVD customers in the United States, which is the only DVD market for Netflix. The inventory, of course, comes in the form of DVDs (also blue ray discs). Netflix holds a DVD inventory valued at $2 billion; that amounts to about 89 million discs distributed across 39 warehouses located across the country. That inventory must be carefully managed: New releases of movies must be purchased in adequate quantities while older movies cannot simply be discarded because many customers like to see the "classics." The *composition* of the inventory of discs supports one of Netflix's competitive priorities, variety. However, if it were not for the fact that Netflix designed a process to minimize the lead time in processing customer requests for a disc, the inventory of discs needed to support variety would balloon to enormous proportions.

Another of Netflix's competitive priorities is delivery speed, as evidenced by its goal of delivering a disc by "the next business day." That is, once a customer has returned a disc in his possession, the next disc in his queue will be delivered in a day. This goal was necessary to compete against the bricks-and-mortar video stores where customers could get a movie the same day they wanted it. The *size* of the inventory helps support delivery speed. The inventory of discs consists of two parts: those discs in the possession of customers, and those in stock waiting for a request. Netflix had two options for processing a customer's request for a new disc once his disc is returned to the warehouse: (1) send the next disc in the customer's queue from the stockpiled inventory of discs, or (2) process the returned discs quickly and satisfy the customer's request for the next disc in his queue from those being returned that day. Each warehouse can handle 60,000 orders per day, so a good number of requests could be satisfied with in-transit returns. Netflix opted for the second option, thereby reducing the need for a larger stockpile of discs. The process at a typical warehouse is as follows:

1. ARRIVAL: Six nights a week, unmarked trucks arrive at the post office at around 3 A.M. to pick up cartons of returned discs. The trucks deliver the discs to the Netflix warehouse.
2. INSPECTION: Employees remove the returned discs from their mailing envelopes, confirm that the disc title matches the disc sleeve, clean the disc, check for cracks or scratches, place it back into the sleeve, and file the disc in one of two bins, one for damaged discs or discs in the wrong sleeve, and one for acceptable discs. Each employee does this process about 650 times per hour. After 65 minutes of inspection, a bell rings and the employees stand up for exercise drills.
3. INVENTORY: The discs are scanned into the inventory by a machine that reads 30,000 bar codes per hour. As soon as the barcode on the disc sleeve is scanned, the last renter receives an email confirming that the disc has been received and what disc that renter will receive next.
4. SORTING: Discs are scanned a second time to see if anyone has ordered them online. Ordered discs are sorted by ZIP code in preparation for shipment. Others are shelved for future use.
5. STUFFING: Outgoing discs are stuffed into new mailing envelopes and sent through a label machine that scans their bar codes and prints shipping addresses. Around 5 P.M., warehouse trucks are loaded with the cartons of discs and returned to the post office.

While it is clear that streaming will eventually overtake the business of shipping DVDs directly to the homes of customers, what lessons can we take away from Netflix's DVD process? First, properly managed inventories can support the competitive priorities of variety and delivery speed. Second, process design can help to reduce the need for excessive inventory investment. These are lessons that can be applied to most any business.

*Sources*: Christopher Borrelli, "How Netflix Gets Your Movies to Your Mailbox So Fast," Chicago Tribune (August 4, 2009); Tracy V. Wilson and Stephanie Crawford, "How Netflix Works," How Stuff Works, **http://electronics.howstufworks.com**; Rick Newman, "How Netflix (and Blockbuster) Killed Blockbuster," US News, **http://moneyusnews.com** (September 23, 2010); Netflix Annual Report 2013.

## LEARNING GOALS *After reading this chapter, you should be able to:*

1. Identify the advantages, disadvantages, and costs of holding inventory.
2. Define the different types of inventory and the roles they play in supply chains.
3. Explain the tactics for reducing inventories in supply chains.
4. Use ABC analysis to determine the items deserving most attention and tightest inventory control.
5. Calculate the economic order quantity and apply it to various situations.
6. Determine the order quantity and reorder point for a continuous review inventory control system.
7. Determine the review interval and target inventory level for a periodic review inventory control system.

Inventory management, the planning and controlling of inventories to meet the competitive priorities of the organization, is an important concern for managers in all types of businesses. Effective inventory management is essential for realizing the full potential of any supply chain. The challenge is not to pare inventories to the bone to reduce costs or to have plenty around to satisfy all demands, but to have the right amount to achieve the competitive priorities of the business most efficiently. That is the strategy Netflix applied to their inventory of discs. This type of efficiency can only happen if the right amount of inventory is flowing through the supply chain—through suppliers, the firm, warehouses or distribution centers, and customers. Much of inventory management involves *lot sizing*, which is the determination of how frequently and in what quantity to order inventory. We make ample reference to the term **lot size**, which is the quantity of an inventory item management either buys from a supplier or manufactures using internal processes. In this chapter, we focus on the decision-making aspects of inventory management.

**inventory management**
The planning and controlling of inventories to meet the competitive priorities of the organization.

**lot size**
The quantity of an inventory item management either buys from a supplier or manufactures using internal processes.

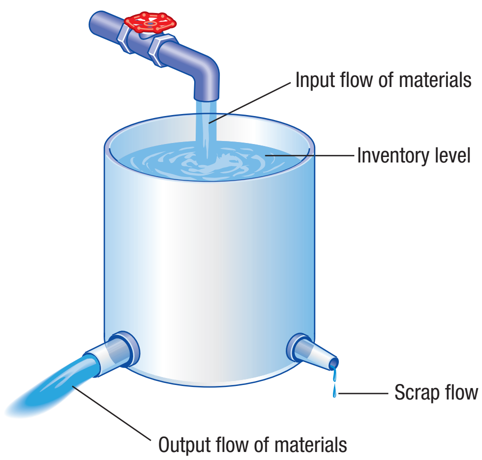
Inventories are important to all types of organizations, their employees, and their supply chains. Inventories profoundly affect everyday operations because they must be counted, paid for, used in operations, used to satisfy customers, and managed. Inventories require an investment of funds, as does the purchase of a new machine. Monies invested in inventory are not available for investment in other things; thus, they represent a drain on the cash flows of an organization. Nonetheless, companies realize that the availability of products is a key selling point in many markets and downright critical in many more.

So, is inventory a boon or a bane? Certainly, too much inventory on hand reduces profitability, and too little inventory on hand creates shortages in the supply chain and ultimately damages customer confidence. Inventory management, therefore, involves trade-offs. Let us discover how companies can effectively manage inventories across the organization.

## Inventory Trade-Offs

The value of inventory management becomes apparent when the complexity of the supply chain is recognized. The performance of numerous suppliers determines the inward flow of materials and services to a firm. The performance of the firm determines the outward flow of services or products to the next stage of the supply chain. The flow of materials, however, determines inventory levels. **Inventory** is a stock of materials used to satisfy customer demand or to support the production of services or goods. Figure 9.1 shows how inventories are created at one node in a supply chain through the analogy of a water tank. The flow of water into the tank raises the water level. The inward flow of water represents input materials, such as steel, component parts, office supplies, or a finished product. The water level represents the amount of inventory held at a plant, service facility, warehouse, or retail outlet. The flow of water from the tank lowers the water level in the tank. The outward flow of water represents the demand for materials in inventory, such as customer orders for a Huffy bicycle or service requirements for supplies such as soap, food, or furnishings. The rate of the outward flow also reflects the ability of the firm to match the demand for services or products. Another possible outward flow is that of scrap, which also lowers the level of useable inventory. Together, the difference between input flow rate and the output flow rate determines the level of inventory. Inventories rise when more material flows into the tank than flows out; they fall when more material flows out than flows in. Figure 9.1 also shows clearly why firms utilize Six Sigma and total quality management (TQM) to reduce defective materials: The larger the scrap flows, the larger the input flow of materials required for a given level of output.

**inventory**
A stock of materials used to satisfy customer demand or to support the production of services or goods.



▲ FIGURE 9.1
Creation of Inventory

A fundamental question in supply chain management is how much inventory to have. The answer to this question involves a trade-off between the advantages and disadvantages of holding inventory. Depending on the situation, the pressures for having small inventories may or may not exceed the pressures for having large inventories.

## Pressures for Small Inventories

An inventory manager's job is to balance the advantages and disadvantages of both small and large inventories and find a happy medium between the two levels. The primary reason for keeping inventories small is that inventory represents a temporary monetary investment. As such, the firm incurs an opportunity cost, which we call the cost of capital, arising from the money tied up in inventory that could be used for other purposes. The **inventory holding cost** (or *carrying cost*) is the sum of the cost of capital plus the variable costs of keeping items on hand, such as storage and handling costs and taxes, insurance, and shrinkage costs. When these components change with inventory levels, so does the holding cost.

**inventory holding cost**

The sum of the cost of capital and the variable costs of keeping items on hand, such as storage and handling, taxes, insurance, and shrinkage.

Companies usually state an item's holding cost per period of time as a percent of its value. The annual cost to maintain one unit in inventory typically ranges from 15 to 35 percent of its value. Suppose that a firm's holding cost is 20 percent. If the average value of total inventory is 20 percent of sales, the average annual cost to hold inventory is 4 percent [0.20(0.20)] of total sales. This cost is sizable in terms of gross profit margins, which often are less than 10 percent. Thus, the components of holding cost create pressures for small inventories.

Using Operations to Create Value

PROCESS MANAGEMENT
Process Strategy and Analysis
Managing Quality
Planning Capacity
Managing Process Constraints
Designing Lean Systems
Managing Effective Projects

CUSTOMER DEMAND MANAGEMENT
Forecasting Demand
→ Managing Inventories
Planning and Scheduling Operations
Efficient Resource Planning

SUPPLY CHAIN MANAGEMENT
Designing Effective Supply Chains
Supply Chains and Logistics
Integrating the Supply Chain
Managing Supply Chain Sustainability

**Cost of Capital** The cost of capital is the opportunity cost of investing in an asset relative to the expected return on assets of similar risk. Inventory is an asset; consequently, we should use a cost measure that adequately reflects the firm's approach to financing assets. Most firms use the *weighted average cost of capital (WACC)*, which is the average of the required return on a firm's stock equity and the interest rate on its debt, weighted by the proportion of equity and debt in its portfolio. The cost of capital usually is the largest component of holding cost, as high as 15 percent of inventory value, depending on the particular capitalization portfolio of the firm. Firms typically update the WACC on an annual basis because it is used to make many financial decisions.

**Storage and Handling Costs** Inventory takes up space and must be moved into and out of storage. Storage and handling costs may be incurred when a firm rents space on either a long- or short-term basis. An inventory holding cost is incurred when a firm could use storage space productively in some other way.

**Taxes, Insurance, and Shrinkage** More taxes are paid if end-of-year inventories are high, and the cost of insuring the inventories increases, too. Shrinkage takes three forms. The first, *pilferage*, or theft of inventory by customers or employees, is a significant percentage of sales for some businesses. The second form of shrinkage, called *obsolescence*, occurs when inventory cannot be used or sold at full value, owing to model changes, engineering modifications, or unexpectedly low demand. Obsolescence is a big expense in the retail clothing industry. Drastic discounts on seasonal clothing frequently must be offered on many of these products at the end of a season. Finally, *deterioration* through physical spoilage or damage due to rough or excessive material handling results in lost value. Food and beverages, for example, lose value and might even have to be discarded when their shelf life is reached. When the rate of deterioration is high, building large inventories may be unwise.

## Pressures for Large Inventories

Given the costs of holding inventory, why not eliminate it altogether? Let us look briefly at the pressures related to maintaining large inventories.

**Customer Service** Creating inventory can speed delivery and improve the firm's on-time delivery of goods. High inventory levels reduce the potential for stockouts and backorders, which are key concerns of wholesalers and retailers. A *stockout* is an order that cannot be satisfied, resulting in loss of the sale. A *backorder* is a customer order that cannot be filled when promised or demanded but is filled later. Customers do not like waiting for backorders to be filled. Many of them will take their business elsewhere. Sometimes, customers are given discounts for the inconvenience of waiting.

**Ordering Cost** Each time a firm places a new order, it incurs an **ordering cost**, or the cost of preparing a purchase order for a supplier or a production order for manufacturing. For the same item, the ordering cost is the same, regardless of the order size. The purchasing agent must take the time to decide how much to order and, perhaps, select a supplier and negotiate terms. Time also is spent on paperwork, follow-up, and receiving the item(s). In the case of a production order for a manufactured item, a blueprint and routing instructions often must accompany the order. However, the Internet streamlines the order process and reduces the costs of placing orders.

**ordering cost**

The cost of preparing a purchase order for a supplier or a production order for manufacturing.

**Setup Cost** The cost involved in changing over a machine or workspace to produce a different item is the **setup cost**. It includes labor and time to make the changeover, cleaning, and sometimes new tools or equipment. Scrap or rework costs are also higher at the start of the production run. Setup cost also is independent of order size, which creates pressure to make or order a large supply of the items and hold them in inventory rather than order smaller batches.

**setup cost**

The cost involved in changing over a machine or workspace to produce a different item.

**Labor and Equipment Utilization** By creating more inventory, management can increase workforce productivity and facility utilization in three ways. First, placing larger, less frequent production orders reduces the number of unproductive setups, which add no value to a service or product. Second, holding inventory reduces the chance of the costly rescheduling of production orders because the components needed to make the product are not in inventory. Third, building inventories improves resource utilization by stabilizing the output rate when demand is cyclical or seasonal. The firm uses inventory built during slack periods to handle extra demand in peak seasons. This approach minimizes the need for extra shifts, hiring, layoffs, overtime, and additional equipment.

**Transportation Cost** Sometimes, outbound transportation cost can be reduced by increasing inventory levels. Having inventory on hand allows more full-carload shipments to be made and minimizes the need to expedite shipments by more expensive modes of transportation. Inbound transportation costs can also be reduced by creating more inventory. Sometimes, several items are ordered from the same supplier. Placing these orders at the same time will increase inventories because some items will be ordered before they are actually needed; nonetheless, it may lead to rate discounts, thereby decreasing the costs of transportation and raw materials.

**Payments to Suppliers** A firm often can reduce total payments to suppliers if it can tolerate higher inventory levels. Suppose that a firm learns that a key supplier is about to increase its prices. In this case, it might be cheaper for the firm to order a larger quantity than usual—in effect delaying the price increase—even though inventory will increase temporarily. A firm can also take advantage of quantity discounts this way. A **quantity discount**, whereby the price per unit drops when the order is sufficiently large, is an incentive to order larger quantities. Supplement C, "Special Inventory Models," shows how to determine order quantities in such a situation.

**quantity discount**

A drop in the price per unit when an order is sufficiently large.

## MANAGERIAL PRACTICE 9.1 Inventory Management at Walmart

In the market for shaver blade replacements? A printer? First-aid supplies? Dog food? Hair spray? If so, you expect that the store you shop at will have what you want. However, making sure that the shelves are stocked with tens of thousands of products is no simple matter for inventory managers at Walmart, which has 10,700 Walmart stores and Sam's Club locations in 27 countries, employs more than 2.2 million associates, serves 245 million customers per week worldwide, and uses 100,000 suppliers. You can imagine in an operation this large that some things can get lost. Linda Dillman, then CIO at Walmart, recounts the story of the missing hair spray at one of the stores. The shelf needed to be restocked with a specific hair spray; however, it took three days to find the case in the backroom. Most customers will not swap hair sprays, so Walmart lost three days of sales on that product.

Knowing what is in stock, in what quantity, and where it is being held is critical to effective inventory management. Without accurate inventory information, companies can make major mistakes by ordering too much, not enough, or shipping products to the wrong location. Companies can have large inventories and still have stockouts of product because they have too much inventory of some products and not enough of others. Walmart, a $466 billion company with inventories in excess of $44 billion, is certainly aware of the potential benefits from improved inventory management and is constantly experimenting with ways to reduce inventory investment. The economics of retailing are circular: You order products, they are delivered, and you have 30 days to pay for it. The faster you sell the merchandise, the less it costs to finance. Ideally you want to sell it before the 30 days are up; the vendors would actually be financing your inventory. That is why reducing the amount of

Paul J. Milette/Palm Beach Post/ZUMA Press/Newscom

An employee stacks a shipment of pet supplies at a Walmart distribution center in Fort Pierce, Florida. The 1.2-million-square-foot facility serves 45 Walmart stores on the east coast of Florida.

inventory on hand is so important to a company that handles a lot of inventory. Knowing when to replenish inventory stocks and how much to order each time is critical when dealing with so much inventory investment. The application of technology is also important, such as using radio frequency identification (RFID) to track inventory shipments and stock levels at stores and warehouses throughout the supply chain (see Chapter 14, Integrating the Supply Chain). Of course, having a lot of technology will not help if there are not enough employees available to move the products from the storeroom to the shelves, as Walmart has found out. Cutting the hours of employees can cause stockouts even if the product is available in the stockroom. Nonetheless, one handheld RFID reader could have found the missing case of hair spray in a few minutes.

*Source:* Laurie Sullivan, "Walmart's Way," **http://Informationweek.com** (September 27, 2004), pp. 36–50; Bill Saporito, "The Trouble Lurking on Walmart's Empty Shelves," **http://business.time.com**, (April 9, 2013); Todd Traub, "Walmart Used Technology to Become Supply Chain Leader," *Arkansas Business* (July 2, 2012); and Walmart 2013 Annual Report.

# Types of Inventory

Inventories can be classified in several ways. In this section we discuss accounting inventories and operational inventories.

**raw materials (RM)**
The inventories needed for the production of services or goods.

**work-in-process (WIP)**
Items, such as components or assemblies, needed to produce a final product in manufacturing or service operations.

**finished goods (FG)**
The items in manufacturing plants, warehouses, and retail outlets that are sold to the firm's customers.

**independent demand items**
Items for which demand is influenced by market conditions and is not related to the inventory decisions for any other item held in stock or produced.

## Accounting Inventories

Inventory exists in three aggregate categories that are useful for accounting purposes. **Raw materials (RM)** are the inventories needed for the production of services or goods. They are considered to be inputs to the transformation processes of the firm. **Work-in-process (WIP)** consists of items, such as components or assemblies, needed to produce a final product in manufacturing. WIP is also present in some service operations, such as repair shops, restaurants, check-processing centers, and package delivery services. **Finished goods (FG)** in manufacturing plants, warehouses, and retail outlets are the items sold to the firm's customers. The finished goods of one firm may actually be the raw materials for another.

Figure 9.2 shows how inventory can be held in different forms and at various stocking points. In this example, raw materials—the finished goods of the supplier—are held both by the supplier and the manufacturer. Raw materials at the plant pass through one or more processes, which transform them into various levels of WIP inventory. Final processing of this inventory yields finished goods inventory. Finished goods can be held at the plant, the distribution center (which may be a warehouse owned by the manufacturer or the retailer), and retail locations.

An important distinction regarding the three categories of inventories is the nature of the demand they experience. For example, take finished goods, which are **independent demand items**—that is, items for which demand is influenced by market conditions and is not related to the inventory decisions for any other item held in stock or produced. Retailers, such as JCPenney and Dillards, deal with finished goods. Examples of independent demand items include:

- Wholesale and retail merchandise
- Service support inventory, such as stamps and mailing labels for post offices, office supplies for law firms, and laboratory supplies for research universities
- Product and replacement-part distribution inventories
- Maintenance, repair, and operating (MRO) supplies—that is, items that do not become part of the final service or product, such as employee uniforms, fuel, paint, and machine repair parts

Managing an independent demand inventory can be tricky because demand is influenced by external factors. For example, the owner of a bookstore may not be sure how many copies of the latest best-seller novel customers will purchase during the coming month. As a result, the manager may decide to stock extra copies as a safeguard. Independent demand, such as the demand for various book titles, must be *forecasted* using the techniques we discussed in Chapter 8, "Forecasting Demand." There is, however, a whole different type of demand for certain items that must be considered.

Marcin Balcerzak/Shutterstock.com

Raw materials, work-in-progress, and finished goods inventories can all be stocked in the same facility. Modern warehouses allow for efficient inventory access.

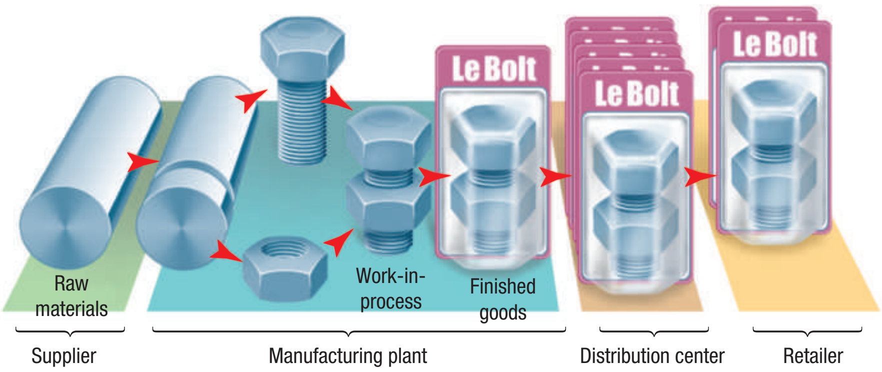



**FIGURE 9.2** Inventory of Successive Stocking Points

**Dependent demand items**, consisting of raw materials and work-in-process inventories, are those items whose required quantity varies with the production plans for other items held in the firm's inventory. These items are required as components or inputs to a service or product. Dependent demand should be calculated, not forecasted, and exhibits a pattern very different from that of independent demand (see Chapter 11, "Efficient Resource Planning").

**dependent demand items**
Items whose required quantity varies with the production plans for other items held in the firm's inventory.

## Operational Inventories

Inventories can also be classified by how they are created. In this context, inventory takes four forms: (1) cycle, (2) safety stock, (3) anticipation, and (4) pipeline. They cannot be identified physically; that is, an inventory manager cannot look at a pile of widgets and identify which ones are cycle inventory and which ones are safety stock inventory. However, conceptually, each of the four types comes into being in an entirely different way. Once you understand these differences, you can prescribe different ways to reduce inventory.

**Cycle Inventory** The portion of total inventory that varies directly with lot size is called **cycle inventory**. Determining how frequently to order, and in what quantity, is called **lot sizing**. Two principles apply.

**cycle inventory**
The portion of total inventory that varies directly with lot size.

**lot sizing**
The determination of how frequently and in what quantity to order inventory.

1. The lot size, $Q$, varies directly with the elapsed time (or cycle) between orders. If a lot is ordered every 5 weeks, the average lot size must equal 5 weeks' demand.
2. The longer the time between orders for a given item, the greater the cycle inventory must be.

At the beginning of the interval, the cycle inventory is at its maximum, or $Q$. At the end of the interval, just before a new lot arrives, cycle inventory drops to its minimum, or 0. The average cycle inventory is the average of these two extremes:

$$\text{Average cycle inventory} = \frac{Q + 0}{2} = \frac{Q}{2}$$

This formula is exact only when the demand rate is constant and uniform. However, it does provide a reasonably good estimate even when demand rates are not constant. Factors other than the demand rate (e.g., scrap losses) also may cause estimating errors when this simple formula is used.

**Safety Stock Inventory** To avoid customer service problems and the hidden costs of unavailable components, companies hold safety stock. **Safety stock inventory** is surplus inventory that protects against uncertainties in demand, lead time, and supply changes. Safety stocks are desirable when suppliers fail to deliver either the desired quantity on the specified date or items of acceptable quality, or when manufactured items require significant amounts of scrap or rework. Safety stock inventory ensures that operations are not disrupted when such problems occur, allowing subsequent operations to continue.

**safety stock inventory**
Surplus inventory that a company holds to protect against uncertainties in demand, lead time, and supply changes.

To create safety stock, a firm places an order for delivery earlier than when the item is typically needed.[1] The replenishment order therefore arrives ahead of time, giving a cushion against uncertainty. For example, suppose that the average lead time from a supplier is 3 weeks, but a firm orders 5 weeks in advance just to be safe. This policy creates a safety stock equal to a 2 weeks' supply $(5 - 3)$.

**Anticipation Inventory** Inventory used to absorb uneven rates of demand or supply, which businesses often face, is referred to as **anticipation inventory**. Predictable, seasonal demand patterns lend themselves to the use of anticipation inventory. Uneven demand can motivate a manufacturer to stockpile anticipation inventory during periods of low demand so that output levels do not have to be increased much when demand peaks. Anticipation inventory also can help when suppliers are threatened with a strike or have severe capacity limitations.

**anticipation inventory**
Inventory used to absorb uneven rates of demand or supply.

[1]When orders are placed at fixed intervals, a second way to create safety stock is used. Each new order placed is larger than the quantity typically needed through the next delivery date.



Pipeline inventories result from moving items and materials from one location to another. Because trains offer an economical way to transport large quantities of goods, they are a favorite choice to reduce the costs of pipeline inventories.

**pipeline inventory**

Inventory that is created when an order for an item is issued but not yet received.

**Pipeline Inventory** Inventory that is created when an order for an item is issued but not yet received is called **pipeline inventory**. This form of inventory exists because the firm must commit to enough inventory (on-hand plus in-transit) to cover the lead time for the order. Longer lead times or higher demands per week create more pipeline inventory. As such, the average pipeline inventory between two stocking points can be measured as the average demand during lead time, $\overline{D}_L$, which is the average demand for the item per period $(\overline{d})$ multiplied by the number of periods in the item's lead time $(L)$ to move between the two points, or

$$\text{Pipeline inventory} = \overline{D}_L = \overline{d}L$$

The equation assumes that both $\overline{d}$ and $L$ are constants and that $L$ is not affected by the order or lot size, $Q$. Changing an item's lot size does not directly affect the average level of the pipeline inventory. Nonetheless, the lot size can *indirectly* affect pipeline inventory if it is related to the lead time. In such a case, pipeline inventory will change depending on the relationship of $L$ to $Q$. Example 9.1 shows how this can happen.

## EXAMPLE 9.1 Estimating Inventory Levels

**MyOMLab**

Tutor 9.1 in MyOMLab provides a new example to practice the estimation of inventory levels.

A plant makes monthly shipments of electric drills to a wholesaler in average lot sizes of 280 drills. The wholesaler's average demand is 70 drills a week, and the lead time from the plant is 3 weeks. The wholesaler must pay for the inventory from the moment the plant makes a shipment. If the wholesaler is willing to increase its purchase quantity to 350 units, the plant will give priority to the wholesaler and guarantee a lead time of only 2 weeks. What is the effect on the wholesaler's cycle and pipeline inventories?

### SOLUTION

The wholesaler's current cycle and pipeline inventories are

$$\text{Cycle inventory} = \frac{Q}{2} = \frac{280}{2} = 140 \text{ drills}$$

$$\text{Pipeline inventory} = \overline{D}_L = \overline{d}L = (70 \text{ drills/week})(3 \text{ weeks}) = 210 \text{ drills}$$

Figure 9.3 shows the cycle and pipeline inventories if the wholesaler accepts the new proposal.

**FIGURE 9.3 ▶** Estimating Inventory Levels Using Tutor 9.1

1. Enter the average lot size, average demand during a period, and the number of periods of lead time:

| | |
|---|---|
| Average lot size | 350 |
| Average demand | 70 |
| Lead time | 2 |

2. To compute cycle inventory, simply divide average lot size by 2. To compute pipeline inventory, multiply average demand by lead time:

| | |
|---|---|
| Cycle inventory | 175 |
| Pipeline inventory | 140 |

### DECISION POINT

The effect of the new proposal on cycle inventories is to increase them by 35 units, or 25 percent. The reduction in pipeline inventories, however, is 70 units, or 33 percent. The proposal would reduce the total investment in cycle and pipeline inventories. Also, it is advantageous to have shorter lead times because the wholesaler only has to commit to purchases 2 weeks in advance, rather than 3 weeks.

# Inventory Reduction Tactics

Managers are always eager to find cost-effective ways to reduce inventory in supply chains. In this section we discuss the basic tactics (which we call *levers*) for reducing cycle, safety stock, anticipation, and pipeline inventories in supply chains. A primary lever is one that must be activated if inventory is to be reduced. A secondary lever reduces the penalty cost of applying the primary lever and the need for having inventory in the first place.

## Cycle Inventory

The primary lever to reduce cycle inventory is simply to reduce the lot sizes of items moving in the supply chain. However, making such reductions in $Q$ without making any other changes can be devastating. For example, setup costs or ordering costs can skyrocket. If these changes occur, two secondary levers can be used:

1. Streamline the methods for placing orders and making setups to reduce ordering and setup costs and allow $Q$ to be reduced. This may involve redesigning the infrastructure for information flows or improving manufacturing processes.
2. Increase repeatability to eliminate the need for changeovers. **Repeatability** is the degree to which the same work can be done again. Repeatability can be increased through high product demand; the use of specialization; the devotion of resources exclusively to a product; the use of the same part in many different products; the use of *flexible automation*; the use of the *one-worker, multiple-machines* concept; or through *group technology*. Increased repeatability may justify new setup methods, reduce transportation costs, and allow quantity discounts from suppliers.

**repeatability**

The degree to which the same work can be done again.

## Safety Stock Inventory

The primary lever to reduce safety stock inventory is to place orders closer to the time when they must be received. However, this approach can lead to unacceptable customer service unless demand, supply, and delivery uncertainties can be minimized. Four secondary levers can be used in this case:

1. Improve demand forecasts so that fewer surprises come from customers. Design the mechanisms to increase collaboration with customers to get advanced warnings for changes in demand levels.
2. Cut the lead times of purchased or produced items to reduce demand uncertainty. For example, local suppliers with short lead times could be selected whenever possible.
3. Reduce supply uncertainties. Suppliers are likely to be more reliable if production plans are shared with them. Put in place the mechanisms to increase collaboration with suppliers. Surprises from unexpected scrap or rework can be reduced by improving manufacturing processes. Preventive maintenance can minimize unexpected downtime caused by equipment failure.
4. Rely more on equipment and labor buffers, such as capacity cushions and cross-trained workers. These buffers are important to businesses in the service sector because they generally cannot inventory their services.

## Anticipation Inventory

The primary lever to reduce anticipation inventory is simply to match demand rate with production rate. Secondary levers can be used to even out customer demand in one of the following ways:

1. Add new products with different demand cycles so that a peak in the demand for one product compensates for the seasonal low for another.
2. Provide off-season promotional campaigns.
3. Offer seasonal pricing plans.

## Pipeline Inventory

An operations manager has direct control over lead times but not demand rates. Because pipeline inventory is a function of demand during the lead time, the primary lever is to reduce the lead time. Two secondary levers can help managers cut lead times:

1. Find more responsive suppliers and select new carriers for shipments between stocking locations or improve materials handling within the plant. Improving the information system could overcome information delays between a distribution center and retailer.
2. Change $Q$ in those cases where the lead time depends on the lot size.

Inventories in supply chains are managed with the help of inventory control systems. These systems manage the levels of cycle, safety stock, anticipation, and pipeline inventories in a firm. Regardless of whether an item experiences independent or dependent demand, three important questions must be answered: What degree of control should we impose on an item? How much should we order? and When should we place the order? An approach called ABC analysis, which we address in the next section, helps with the first question. Inventory control systems respond to the last two questions. In selecting an inventory control system for a particular application, the nature of the demands imposed on the inventory items is crucial. In this chapter, we focus on inventory control systems for independent demand items, which is the type of demand the bookstore owner, other retailers, service providers, and distributors face. Even though demand from any one customer is difficult to predict, low demand from some customers for a particular item often is offset by high demand from others. Thus, total demand for any independent demand item may follow a relatively smooth pattern, with some random fluctuations. For items facing dependent demands, such as raw materials and work-in-process inventories, material requirements planning (MRP) systems are useful. We devote Chapter 11, "Efficient Resource Planning" to this important inventory control system.

In the remainder of this chapter we first address the question of what degree of control to impose on an item, and then answer the question of how much to order. In the last two sections we discuss and compare two inventory control systems: (1) the continuous review system, called a *Q* system, and (2) the periodic review system, called a *P* system.

**stock-keeping unit (SKU)**

An individual item or product that has an identifying code and is held in inventory somewhere along the supply chain.

**ABC analysis**

The process of dividing SKUs into three classes, according to their dollar usage, so that managers can focus on items that have the highest dollar value.

**cycle counting**

An inventory control method, whereby storeroom personnel physically count a small percentage of the total number of items each day, correcting errors that they find.

## ABC Analysis

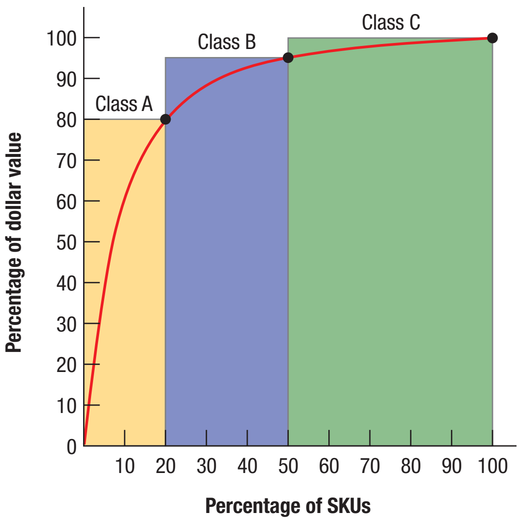
Thousands of items, often referred to as stock-keeping units, are held in inventory by a typical organization, but only a small percentage of them deserve management's closest attention and tightest control. A **stock-keeping unit (SKU)** is an individual item or product that has an identifying code and is held in inventory somewhere along the supply chain. **ABC analysis** is the process of dividing SKUs into three classes according to their dollar usage so that managers can focus on items that have the highest dollar value. This method is the equivalent of creating a *Pareto chart* except that it is applied to inventory rather than to process errors. As Figure 9.4 shows, class A items typically represent only about 20 percent of the SKUs but account for 80 percent of the dollar usage. Class B items account for another 30 percent of the SKUs but only 15 percent of the dollar usage. Finally, 50 percent of the SKUs fall in class C, representing a mere 5 percent of the dollar usage. The goal of ABC analysis is to identify the class A SKUs so management can closely control their inventory levels.

The analysis begins by multiplying the annual demand rate for an SKU by the dollar value (cost) of one unit of that SKU to determine its dollar usage. After ranking the SKUs on the basis of dollar usage and creating the Pareto chart, the analyst looks for "natural" changes in slope. The dividing lines in Figure 9.4 between classes are inexact. Class A SKUs could be somewhat higher or lower than 20 percent of all SKUs but normally account for the bulk of the dollar usage.

Class A SKUs are reviewed frequently to reduce the average lot size and to ensure timely deliveries from suppliers. It is important to maintain high inventory turnover for these items. By contrast, class B SKUs require an intermediate level of control. Here, less frequent monitoring of suppliers coupled with adequate safety stocks can provide cost-effective coverage of demands. For class C SKUs, much looser control is appropriate. While a stockout of a class C SKU can be as crucial as for a class A SKU, the inventory holding cost of class C SKUs tends to be low. These features suggest that higher inventory levels can be tolerated and that more safety stock and larger lot sizes may suffice for class C SKUs. See Solved Problem 2 for a detailed example of ABC analysis.

Creating ABC inventory classifications is useless unless inventory records are accurate. Technology can help; many companies are tracking inventory wherever it exists in the supply chain. Chips imbedded in product packaging contain information on the product and send signals that can be accessed by sensitive receivers and transmitted to a central location for processing. There are other, less sophisticated approaches of achieving accuracy that can be used. One way is to assign responsibility to specific employees for issuing and receiving materials and accurately reporting each transaction. Another method is to secure inventory behind locked doors or gates to prevent unauthorized or unreported withdrawals. This method also guards against accidentally storing newly received inventory in the wrong locations, where it can be lost for months. **Cycle counting** can also be used, whereby storeroom personnel physically count a small percentage of the total number of SKUs each day, correcting errors that they find. Class A SKUs are counted most frequently. A final method is to make logic error checks on each transaction reported and fully investigate any discrepancies. The discrepancies can include (1) actual

▼ **FIGURE 9.4**
Typical Chart Using ABC Analysis



receipts when no receipts are scheduled, (2) disbursements that exceed the current on-hand inventory balance, and (3) receipts with an inaccurate (nonexistent) SKU number.

Now that we have identified the inventory items deserving of most attention, we turn to the decision of how much to order.

# Economic Order Quantity

Supply chain managers face conflicting pressures to keep inventories low enough to avoid excess inventory holding costs but high enough to reduce ordering and setup costs. *Inventory holding cost* is the sum of the cost of capital and the variable costs of keeping items on hand, such as storage and handling, taxes, insurance, and shrinkage. *Ordering cost* is the cost of preparing a purchase order for a supplier or a production order for the shop, while *setup cost* is the cost of changing over a machine to produce a different item. In this section, we will address the *cycle inventory*, which is that portion of total inventory that varies directly with lot size. A good starting point for balancing these conflicting pressures and determining the best cycle-inventory level for an item is finding the **economic order quantity (EOQ)**, which is the lot size that minimizes total annual cycle-inventory holding and ordering costs. The approach to determining the EOQ is based on the following assumptions:

**economic order quantity (EOQ)** The lot size that minimizes total annual inventory holding and ordering costs.

1. The demand rate for the item is constant (for example, always 10 units per day) and known with certainty.
2. No constraints are placed (such as truck capacity or materials handling limitations) on the size of each lot.
3. The only two relevant costs are the inventory holding cost and the fixed cost per lot for ordering or setup.
4. Decisions for one item can be made independently of decisions for other items. In other words, no advantage is gained in combining several orders going to the same supplier.
5. The lead time is constant (e.g., always 14 days) and known with certainty. The amount received is exactly what was ordered and it arrives all at once rather than piecemeal.

The economic order quantity will be optimal when all five assumptions are satisfied. In reality, few situations are so simple. Nonetheless, the EOQ is often a reasonable approximation of the appropriate lot size, even when several of the assumptions do not quite apply. Here are some guidelines on when to use or modify the EOQ.

- **Do not use the EOQ**
  - If you use the "make-to-order" strategy and your customer specifies that the entire order be delivered in one shipment
  - If the order size is constrained by capacity limitations such as the size of the firm's ovens, amount of testing equipment, or number of delivery trucks
- **Modify the EOQ**
  - If significant quantity discounts are given for ordering larger lots
  - If replenishment of the inventory is not instantaneous, which can happen if the items must be used or sold as soon as they are finished without waiting until the entire lot has been completed (see Supplement C, "Special Inventory Models," for several useful modifications to the EOQ)
- **Use the EOQ**
  - If you follow a "make-to-stock" strategy and the item has relatively stable demand
  - If your carrying costs per unit and setup or ordering costs are known and relatively stable

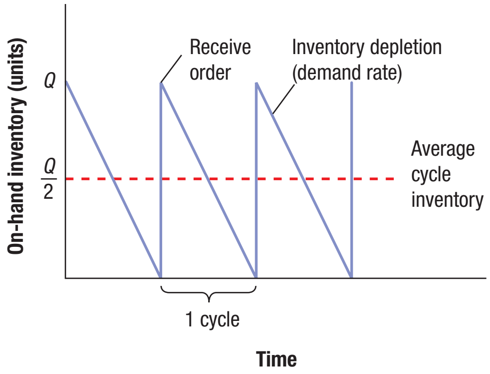
The EOQ was never intended to be an optimizing tool. Nonetheless, if you need to determine a reasonable lot size, it can be helpful in many situations.

## Calculating the EOQ

We begin by formulating the total cost for any lot size $Q$ for a given SKU. Next, we derive the EOQ, which is the $Q$ that minimizes total annual cycle-inventory cost. Finally, we describe how to convert the EOQ into a companion measure, the elapsed time between orders.

When the EOQ assumptions are satisfied, cycle inventory behaves as shown in Figure 9.5. A cycle begins with $Q$ units held in inventory, which happens when a new order is received. During the cycle, on-hand inventory is used at a constant rate and, because demand is known with certainty and the lead time is a constant, a new lot can be ordered so that inventory falls to 0 precisely when the new lot is received. Because inventory varies uniformly between $Q$ and 0, the average cycle inventory equals half the lot size, $Q$.

**FIGURE 9.5 ▶**
Cycle-Inventory Levels




The annual holding cost for this amount of inventory, which increases linearly with $Q$, as Figure 9.6(a) shows, is

$$\text{Annual holding cost} = (\text{Average cycle inventory})(\text{Unit holding cost})$$

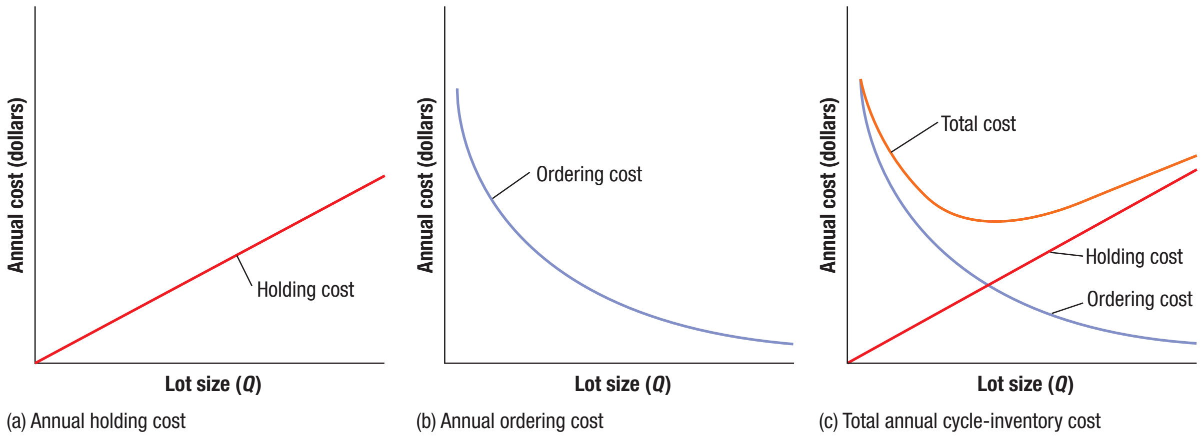
The annual ordering cost is

$$\text{Annual ordering cost} = (\text{Number of orders/Year})(\text{Ordering or setup cost})$$

The average number of orders per year equals annual demand divided by $Q$ For example, if 1,200 units must be ordered each year and the average lot size is 100 units, then 12 orders will be placed during the year. The annual ordering or setup cost decreases nonlinearly as $Q$ increases, as shown in Figure 9.6(b), because fewer orders are placed.

MyOMLab Animation

**FIGURE 9.6 ▶**
Graphs of Annual Holding, Ordering, and Total Costs




The total annual cycle-inventory cost,[2] as graphed in Figure 9.6(c), is the sum of the two cost components:

Total cost = Annual holding cost + Annual ordering or setup cost[3]

$$C = \frac{Q}{2}(H) + \frac{D}{Q}(S)$$

[2] Expressing the total cost on an annual basis usually is convenient (although not necessary). Any time horizon can be selected as long as $D$ and $H$ cover the same time period. If the total cost is calculated on a monthly basis, $D$ must be monthly demand and $H$ must be the cost of holding a unit for 1 month.

[3] The number of orders actually placed in any year is always a whole number, although the formula allows for the use of fractional values. However, rounding is not needed because what is being calculated is an average of multiple years. Such averages often are nonintegers.

where

$C$ = total annual cycle-inventory cost
$Q$ = lot size, in units
$H$ = cost of holding one unit in inventory for a year, often expressed as a percentage of the item's value
$D$ = annual demand, in units per year
$S$ = cost of ordering or setting up one lot, in dollars per lot

## EXAMPLE 9.2 The Cost of a Lot-Sizing Policy

A museum of natural history opened a gift shop two years ago. Managing inventories has become a problem. Low inventory turnover is squeezing profit margins and causing cash-flow problems.

One of the top-selling SKUs in the container group at the museum's gift shop is a bird feeder. Sales are 18 units per week, and the supplier charges \$60 per unit. The cost of placing an order with the supplier is \$45. Annual holding cost is 25 percent of a feeder's value, and the museum operates 52 weeks per year. Management chose a 390-unit lot size so that new orders could be placed less frequently. What is the annual cycle-inventory cost of the current policy of using a 390-unit lot size? Would a lot size of 468 be better?

**MyOMLab**

Tutor 9.2 in MyOMLab provides a new example of the application of ABC analysis.

### SOLUTION

We begin by computing the annual demand and holding cost as

$$D = (18 \text{ units/week})(52 \text{ weeks/year}) = 936 \text{ units}$$

$$H = 0.25(\$60/\text{unit}) = \$15$$

The total annual cycle-inventory cost for the current policy is

$$C = \frac{Q}{2}(H) + \frac{D}{Q}(S)$$

$$= \frac{390}{2}(\$15) + \frac{936}{390}(\$45) = \$2{,}925 + \$108 = \$3{,}033$$

The total annual cycle-inventory cost for the alternative lot size is

$$C = \frac{468}{2}(\$15) + \frac{936}{468}(\$45) = \$3{,}510 + \$90 = \$3{,}600$$

### DECISION POINT

The lot size of 468 units, which is a half-year supply, would be a more expensive option than the current policy. The savings in ordering costs are more than offset by the increase in holding costs. Management should use the total annual cycle-inventory cost function to explore other lot-size alternatives.

Figure 9.7 displays the impact of using several $Q$ values for the bird feeder in Example 9.2. Eight different lot sizes were evaluated in addition to the current one. Both holding and ordering costs were plotted, but their sum—the total annual cycle-inventory cost curve—is the important feature. The graph shows that the best lot size, or EOQ, is the lowest point on the total annual cost curve, or between 50 and 100 units. Obviously, reducing the current lot-size policy $Q = 390$ can result in significant savings.

A more efficient approach is to use the EOQ formula:

$$\text{EOQ} = \sqrt{\frac{2DS}{H}}$$

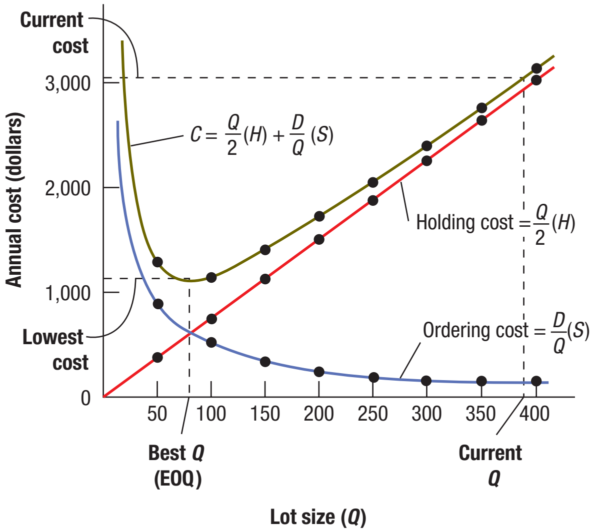



**FIGURE 9.7**
Total Annual Cycle-Inventory Cost Function for the Bird Feeder

We use calculus to obtain the EOQ formula from the total annual cycle-inventory cost function. We take the first derivative of the total annual cycle-inventory cost function with respect to $Q$, set it equal to 0, and solve for $Q$. As Figure 9.7 indicates, the EOQ is the order quantity for which annual holding cost equals annual ordering cost. Using this insight, we can also obtain the EOQ formula by equating the formulas for annual ordering cost and annual holding cost and solving for $Q$. The graph in Figure 9.7 also reveals that when the annual holding cost for any $Q$ exceeds the annual ordering cost, as with the 390-unit order, we can immediately conclude that $Q$ is too high. A lower $Q$ reduces holding cost and increases ordering cost, bringing them into balance. Similarly, if the annual ordering cost exceeds the annual holding cost, $Q$ should be increased.

**time between orders (TBO)**

The average elapsed time between receiving (or placing) replenishment orders of $Q$ units for a particular lot size.

Sometimes, inventory policies are based on the time between replenishment orders, rather than on the number of units in the lot size. The **time between orders (TBO)** for a particular lot size is the average elapsed time between receiving (or placing) replenishment orders of $Q$ units. Expressed as a fraction of a year, the TBO is simply $Q$ divided by annual demand. When we use the EOQ and express time in terms of months, the TBO is

$$\text{TBO}_{\text{EOQ}} = \frac{\text{EOQ}}{D}(12 \text{ months/year})$$

In Example 9.3, we show how to calculate TBO for years, months, weeks, and days.

## EXAMPLE 9.3 Finding the EOQ, Total Cost, and TBO

**MyOMLab** Tutor 9.3 in MyOMLab provides a new example to practice the application of the EOQ model.

**MyOMLab** Active Model 9.1 in MyOMLab provides additional insight on the EOQ model and its uses.

For the bird feeder in Example 9.2, calculate the EOQ and its total annual cycle-inventory cost. How frequently will orders be placed if the EOQ is used?

### SOLUTION

Using the formulas for EOQ and annual cost, we get

$$\text{EOQ} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(936)(45)}{15}} = 74.94, \text{ or } 75 \text{ units}$$

Figure 9.8 shows that the total annual cost is much less than the $3,033 cost of the current policy of placing 390-unit orders.

**FIGURE 9.8** Total Annual Cycle-Inventory Costs Based on EOQ Using Tutor 9.3

| **Parameters** | | | |
|---|---|---|---|
| Current Lot Size (Q) | 390 | Economic Order Quantity | 75 |
| Demand (D) | 936 | | |
| Order Cost (S) | $45 | | |
| Unit Holding Cost (H) | $15 | | |
| **Annual Costs** | | **Annual Costs based on EOQ** | |
| Orders per Year | 2.4 | Orders per Year | 12.48 |
| Annual Ordering Cost | $108.00 | Annual Ordering Cost | $561.60 |
| Annual Holding Cost | $2,925.00 | Annual Holding Cost | $562.50 |
| Annual Inventory Cost | $3,033.00 | Annual Inventory Cost | $1,124.10 |

When the EOQ is used, the TBO can be expressed in various ways for the same time period.

$$\text{TBO}_{\text{EOQ}} = \frac{\text{EOQ}}{D} = \frac{75}{936} = 0.080 \text{ year}$$

$$\text{TBO}_{\text{EOQ}} = \frac{\text{EOQ}}{D}(12 \text{ months/year}) = \frac{75}{936}(12) = 0.96 \text{ month}$$

$$\text{TBO}_{\text{EOQ}} = \frac{\text{EOQ}}{D}(52 \text{ weeks/year}) = \frac{75}{936}(52) = 4.17 \text{ weeks}$$

$$\text{TBO}_{\text{EOQ}} = \frac{\text{EOQ}}{D}(365 \text{ days/year}) = \frac{75}{936}(365) = 29.25 \text{ days}$$

### DECISION POINT

Using the EOQ, about 12 orders per year will be required. Using the current policy of 390 units per order, an average of 2.4 orders will be needed each year (every 5 months). The current policy saves on ordering costs but incurs a much higher cost for carrying the cycle inventory. Although it is easy to see which option is best on the basis of total ordering and holding costs, other factors may affect the final decision. For example, if the supplier would reduce the price per unit for large orders, it may be better to order the larger quantity.

## Managerial Insights from the EOQ

Subjecting the EOQ formula to *sensitivity analysis* can yield valuable insights into the management of inventories. Sensitivity analysis is a technique for systematically changing crucial parameters to determine the effects of a change. Table 9.1 shows the effects on the EOQ when we substitute different values into the numerator or denominator of the formula.

**TABLE 9.1** | **SENSITIVITY ANALYSIS OF THE EOQ**

| Parameter | EOQ | Parameter Change | EOQ Change | Comments |
|---|---|---|---|---|
| **Demand** | $\sqrt{\frac{2DS}{H}}$ | ↑ | ↑ | Increase in lot size is in proportion to the square root of *D*. |
| **Order/Setup Costs** | $\sqrt{\frac{2DS}{H}}$ | ↓ | ↓ | Weeks of supply decreases and inventory turnover increases because the lot size decreases. |
| **Holding Costs** | $\sqrt{\frac{2DS}{H}}$ | ↓ | ↑ | Larger lots are justified when holding costs decrease. |

**continuous review ($Q$) system**
A system designed to track the remaining inventory of a SKU each time a withdrawal is made to determine whether it is time to reorder.

**reorder point (ROP) system**
See continuous review ($Q$) system.

As Table 9.1 shows, the EOQ provides support for some of the intuition you may have about inventory management. However, the effect of ordering or setup cost changes on inventories is especially important for *lean systems*. This relationship explains why manufacturers are so concerned about reducing setup time and costs; it makes small lot production economic. Actually, lean systems provide an environment conducive to the use of the EOQ. For example, yearly, monthly, daily, or hourly demand rates are known with reasonable certainty in lean systems, and the rate of demand is relatively uniform. Lean systems (see Chapter 6, "Designing Lean Systems") may have few process constraints if the firm practices *constraint management* (see Chapter 5, "Managing Process Constraints"). In addition, lean systems strive for constant delivery lead times and dependable delivery quantities from suppliers, both of which are assumptions of the EOQ. Consequently, the EOQ as a lot sizing tool is quite compatible with the principles of lean systems.

We now turn to a discussion of the two most common independent demand inventory control systems: the continuous review ($Q$) system and the periodic review ($P$) system.

Aurelia Ventura/La Opinion/Newscom

Retailers typically face independent demands for the products on their shelves. Thousands of customers may shop at a large store, each looking for a different selection of products. The products must be restocked from a distribution center in the region. Here shoppers look for bargains at a JCPenney store in the Glendale Galleria in California.

# Continuous Review System

A **continuous review ($Q$) system**, sometimes called a **reorder point (ROP) system** or *fixed order-quantity system*, tracks the remaining inventory of a SKU each time a withdrawal is made to determine whether it is time to reorder. In practice, these reviews are done frequently (e.g., daily) and often continuously (after each withdrawal). The advent of computers and electronic cash registers linked to inventory records has made continuous reviews easy. At each review, a decision is made about a SKU's inventory position. If it is judged to be too low, the system triggers a new order. The **inventory position (IP)** measures the SKU's ability to satisfy future demand. It includes **scheduled receipts (SR)**, which are orders that have been placed but have not yet been received, plus on-hand inventory (OH) minus backorders (BO). Sometimes, scheduled receipts are called **open orders**. More specifically,

$$\text{Inventory position} = \text{On-hand inventory} + \text{Scheduled receipts} - \text{Backorders}$$

$$\text{IP} = \text{OH} + \text{SR} - \text{BO}$$

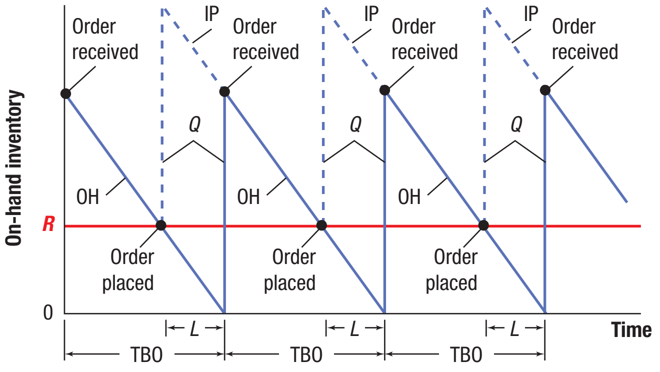
When the inventory position reaches a predetermined minimum level, called the **reorder point ($R$)**, a fixed quantity $Q$ of the SKU is ordered. In a continuous review system, although the order quantity $Q$ is

**inventory position (IP)**
The measurement of a SKU's ability to satisfy future demand.

**scheduled receipts (SR)**
Orders that have been placed but have not yet been received.

**open orders**
See scheduled receipts (SR).

**reorder point ($R$)**
The predetermined minimum level that an inventory position must reach before a fixed quantity $Q$ of the SKU is ordered.




▲ **FIGURE 9.9**
*Q* System When Demand and Lead Time Are Constant and Certain

fixed, the time between orders can vary. Hence, *Q* can be based on the EOQ, a price break quantity (the minimum lot size that qualifies for a quantity discount), a container size (such as a truckload), or some other quantity selected by management.

## Selecting the Reorder Point When Demand and Lead Time Are Constant

To demonstrate the concept of a reorder point, suppose that the demand for feeders at the museum gift shop in Example 9.3 is always 18 per week, the lead time is a constant 2 weeks, and the supplier always ships the exact number ordered on time. With both demand and lead time constant, the museum's buyer can wait until the inventory position drops to 36 units, or (18 units/week) (2 weeks), to place a new order. Thus, in this case, the reorder point, *R*, equals the *total demand during lead time*, with no added allowance for safety stock.

Figure 9.9 shows how the system operates when demand and lead time are constant. The downward-sloping line represents the on-hand inventory, which is being depleted at a constant rate. When it reaches reorder point *R* (the horizontal line), a new order for *Q* units is placed. The on-hand inventory continues to drop throughout lead time *L* until the order is received. At that time, which marks the end of the lead time, on-hand inventory jumps by *Q* units. A new order arrives just when inventory drops to 0. The TBO is the same for each cycle.

The inventory position, IP, shown in Figure 9.9 corresponds to the on-hand inventory, except during the lead time. Just after a new order is placed, at the start of the lead time, IP increases by *Q*, as shown by the dashed line. The IP exceeds OH by this same margin throughout the lead time.[4] At the end of the lead time, when the scheduled receipts convert to on-hand inventory, IP = OH once again. The key point here is to compare IP, not OH, with *R* in deciding whether to reorder. A common error is to ignore scheduled receipts or backorders.

**EXAMPLE 9.4** **Placing a New Order When Demand and Lead Time Are Constant**

Demand for chicken soup at a supermarket is always 25 cases a day and the lead time is always 4 days. The shelves were just restocked with chicken soup, leaving an on-hand inventory of only 10 cases. No backorders currently exist, but there is one open order in the pipeline for 200 cases. What is the inventory position? Should a new order be placed?

**SOLUTION**

$$R = \text{Total demand during lead time} = (25)(4) = 100 \text{ cases}$$

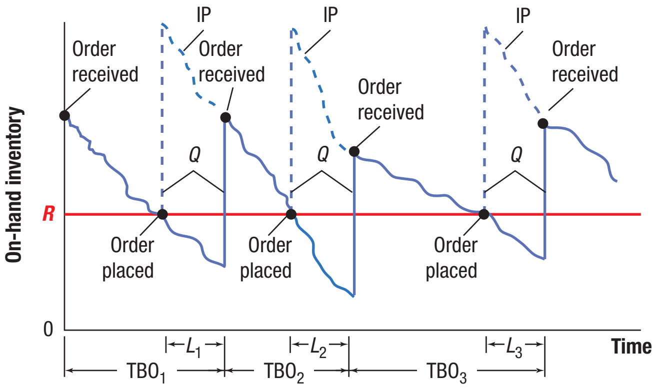
$$\begin{aligned} \text{IP} &= \text{OH} + \text{SR} - \text{BO} \\ &= 10 + 200 - 0 = 210 \text{ cases} \end{aligned}$$

**DECISION POINT**

Because IP exceeds *R* (210 versus 100), do not reorder. Inventory is almost depleted, but a new order need not be placed because the scheduled receipt is in the pipeline.

MyOMLab Animation

▼ **FIGURE 9.10**
*Q* System When Demand Is Uncertain




## Selecting the Reorder Point When Demand Is Variable and Lead Time Is Constant

In reality demand is not always predictable. Figure 9.10 shows how the *Q* system operates when demand is variable and lead time is constant. The wavy downward-sloping line indicates that demand varies from day to day. Its slope is steeper in the second cycle, which means that the demand rate is higher during this time period. The changing demand rate means that the time between orders changes, so $\text{TBO}_1 \neq \text{TBO}_2 \neq \text{TBO}_3$. Example 9.5 shows the mechanics of the continuous review system when demand is variable and the lead time is constant.

[4]A possible exception is the situation when more than one scheduled receipt is open at the same time because of long lead times or larger than average demands during the lead time. Such is the case in Example 9.5.

## EXAMPLE 9.5 Placing a New Order When Demand is Variable and Lead Time is Constant

A distribution center (DC) in Wisconsin stocks Sony plasma TV sets. The center receives its inventory from a mega warehouse in Kansas with a lead time ($L$) of 5 days. The DC uses a reorder point ($R$) of 300 sets and a fixed order quantity ($Q$) of 250 sets. The current on-hand inventory (OH) at the end of Day 1 is 400 sets, there are no scheduled receipts (SR), and there are no backorders (BO). Assume that all demands and receipts occur at the end of the day. The inventory position is compared to the reorder point after demands and receipts are accounted for. If necessary, an order is placed and the inventory position is updated. Given the demand schedule in the table below, determine when to order using a ($Q$) system.

### SOLUTION

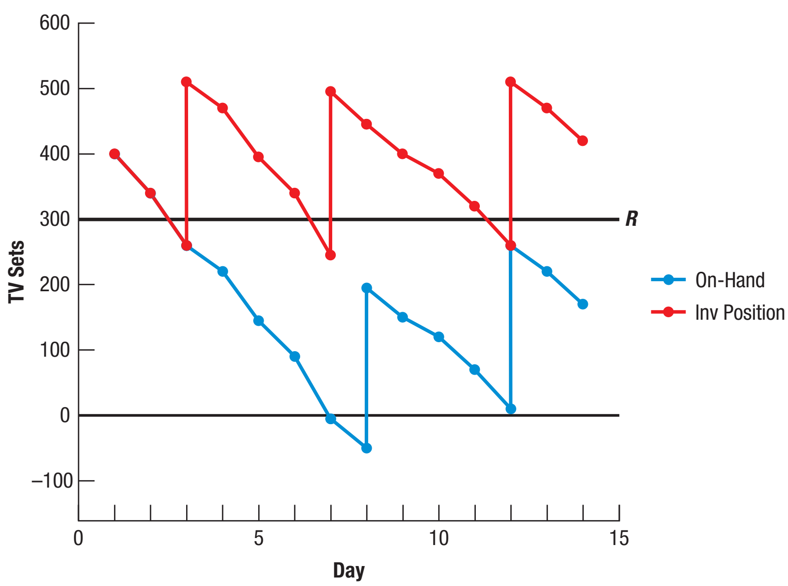
We use the following equation:

Inventory position ($IP$) $=$ OH $+$ SR $-$ BO

| Day | Demand | OH | SR | BO | IP | *Q* |
|---|---|---|---|---|---|---|
| 1 | 50 | 400 | | | 400 + 0 = 400 | |
| 2 | 60 | 340 | | | 340 + 0 = 340 | |
| 3 | 80 | 260 | **250** after ordering | | 260 < *R* before ordering<br>260 + **250** = 510 after ordering | **250** (due Day 8) |
| 4 | 40 | 220 | **250** | | 220 + **250** = 470 | |
| 5 | 75 | 145 | **250** | | 145 + **250** = 395 | |
| 6 | 55 | 90 | **250** | | 90 + **250** = 340 | |
| 7 | 95 | 0 | **250** + **250** = 500 after ordering | 5 | 0 + **250** − 5 = 245 < *R* before ordering<br>245 + **250** = 495 after ordering | **250** (due Day 12) |
| 8 | 50 | 0 + **250** − 50 − 5 = 195 | **250** | | 195 + **250** = 445 | |
| 9 | 45 | 195 − 45 = 150 | **250** | | 150 + **250** = 400 | |
| 10 | 30 | 120 | **250** | | 120 + **250** = 370 | |
| 11 | 50 | 70 | **250** | | 70 + **250** = 320 | |
| 12 | 60 | 70 − 60 + **250** = 260 | **250** after ordering | | 260 < *R* before ordering<br>260 + **250** = 510 after ordering | **250** (due Day 17) |
| 13 | 40 | 260 − 40 = 220 | **250** | | 220 + **250** = 470 | |
| 14 | 50 | 170 | **250** | | 170 + **250** = 420 | |

### DECISION POINT

The figure to the right shows the relationship between the on-hand quantity of TV sets and the inventory position. The IP at the DC drops below the reorder point of 300 sets for the first time on Day 3, triggering an order for 250 sets. On Day 7, demand exceeded the supply of TVs, generating a backorder of 5 sets. Notice that the calculation for IP accounts for the backorder as well as the fact that there are two scheduled receipts on the books once the new order is placed. This situation occurred because the reorder point was breached one day before the open order for 250 sets was received. On Day 8, the shipment of 250 sets arrives and the backorders are satisfied. Note that the on-hand inventory satisfies the demand for that day, as well as the backorders, from the shipment of 250 sets, leaving only 195 sets for inventory. The demands at the DC are fairly volatile and can cause the reorder point to be breached quite dramatically at times. This often happens with continuous review systems



when customers place orders in large quantities, rather than one unit at a time. The customers of the DC could be large retailers who purchase large volumes of TV sets for sales promotions. Another possible reason is that the DC in this example performs all inventory transactions at the end of the day; even if shipments to customers were only one unit at a time, they were treated as one large shipment for purposes of inventory control. With today's technology and the use of product bar codes, the DC could continuously monitor inventory and replenishment orders would be placed just as the reorder point was reached.

As shown in Example 9.5, because of uncertainty, demands during the lead time are unpredictable and backorders or stockouts can occur. That is why managers add safety stock to hedge against lost sales. Consequently $R$ is higher in Figure 9.10 than in Figure 9.9. It also explains why the on-hand inventory usually does not drop to 0 by the time a replenishment order arrives for well-designed continuous review systems. The greater the safety stock and thus the higher reorder point $R$, the less likely a stockout. In general

$$\text{Reorder point} = \text{Average demand during lead time} + \text{Safety stock}$$
$$= \bar{d}L + \text{safety stock}$$

where

$$\bar{d} = \text{average demand per week or day or month}$$
$$L = \text{constant lead time in weeks or days or months}$$

Because the average demand during lead time is variable, the real decision to be made when selecting $R$ concerns the safety stock level. Deciding on a small or large safety stock is a trade-off between customer service and inventory holding costs. Cost minimization models can be used to find the best safety stock, but they require estimates of stockout and backorder costs, which are usually difficult to make with any precision because it is hard to estimate the effect of lost sales, lost customer confidence, future loyalty of customers, and market share because the customer went to a competitor. The usual approach for determining $R$ is for management—based on judgment—to set a reasonable service-level policy for the inventory and then determine the safety stock level that satisfies this policy. There are three steps to arrive at a reorder point:

1. Choose an appropriate service-level policy.
2. Determine the distribution of demand during lead time.
3. Determine the safety stock and reorder point levels.

**service level**
The desired probability of not running out of stock in any one ordering cycle, which begins at the time an order is placed and ends when it arrives in stock.

**cycle-service level**
See service level.

**protection interval**
The period over which safety stock must protect the user from running out of stock.

**Step 1: Service Level Policy** Select a **service level**, or **cycle-service level** (the desired probability of not running out of stock in any one ordering cycle), which begins at the time an order is placed and ends when it arrives in stock. The intent is to provide coverage over the **protection interval**, or the period over which safety stock must protect the user from running out of stock. For the $Q$ system, the lead time is the protection interval. For example, in a bookstore the manager may select a 90 percent cycle-service level for a book. In other words, the probability is 90 percent that demand will not exceed the supply during the lead time. The probability of running short *during the protection interval*, creating a stockout or backorder, is only 10 percent $(100 - 90)$ in our example. This stockout risk, which occurs only during the lead time in the $Q$ system, is greater than the overall risk of a stockout because the risk is nonexistent outside the ordering cycle.

**Step 2: Distribution of Demand during Lead Time** Determine the distribution of demand during lead time, which requires the specification of its mean and standard deviation. To translate a cycle-service level policy into a specific safety stock level, we must know how demand during the lead time is distributed. If demand and lead times vary little around their averages, the safety stock can be small. Conversely, if they vary greatly from one order cycle to the next, the safety stock must be large. Variability is measured by the distribution of demand during lead time. Sometimes, average demand during the lead time and the standard deviation of demand during the lead time are not directly available and must be calculated by combining information on the demand rate with information on the lead time. Suppose that lead time is constant and demand is variable, but records on demand are not collected for a time interval that is exactly the same as the lead time. The same inventory control system may be used to manage thousands of different SKUs, each with a different lead time. For example, if demand is reported *weekly*, these records can be used directly to compute the average and the standard deviation of demand during the lead time if the lead time is exactly 1 week. However, if the lead time is 3 weeks, the computation is more difficult.

We can determine the demand during the lead time distribution by making some reasonable assumptions. Suppose that the average demand, $\bar{d}$, is known along with the standard deviation of demand, $\sigma_d$, over some time interval such as days or weeks. Also, suppose that the probability distributions of demand for each time interval are identical and independent of each other. For example, if the time interval is a week, the probability distributions of demand are assumed to be the same each week (identical $\bar{d}$ and $\sigma_d$), and the total demand in 1 week does not affect the total demand in another week. Let $L$

be the constant lead time, expressed in the same time units as the demand. Under these assumptions, average demand during the lead time will be the sum of the averages for each of the $L$ identical and independent distributions of demand, or $\bar{d} + \bar{d} + \bar{d} + \ldots = \bar{d}L$. In addition, the variance of the distribution of demand during lead time will be the sum of the variances of the $L$ identical and independent distributions of demand, or

$$\sigma_d^2 + \sigma_d^2 + \sigma_d^2 + \ldots = \sigma_d^2 L$$

Finally, the standard deviation of the distribution of demand during lead time is

$$\sigma_{dLT} = \sqrt{\sigma_d^2 L} = \sigma_d \sqrt{L}$$

Figure 9.11 shows how the demand distribution of the lead time is developed from the individual distributions of weekly demands, where $\bar{d} = 75$, $\sigma_d = 15$, and $L = 3$. In this example, average demand during the lead time is $(75)(3) = 225$ units and $\sigma_{dLT} = 15\sqrt{3} = 25.98$.

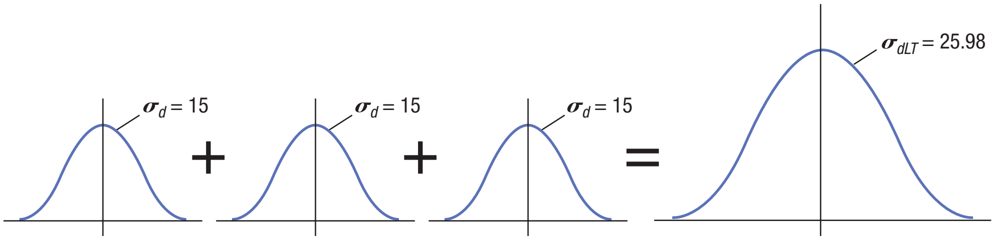



**FIGURE 9.11**
Development of Distribution of Demand during Lead Time

**Step 3: Safety Stock and Reorder Point** When selecting the safety stock, the inventory planner often assumes that demand during the lead time is normally distributed, as shown in Figure 9.12.

The average demand during the lead time is the centerline of the graph, with 50 percent of the area under the curve to the left and 50 percent to the right. Thus, if a cycle-service level of 50 percent were chosen, the reorder point $R$ would be the quantity represented by this centerline. Because $R$ equals the average demand during the lead time plus the safety stock, the safety stock is 0 when $R$ equals this average demand. Demand is less than average 50 percent of the time and, thus, having no safety stock will be sufficient only 50 percent of the time.

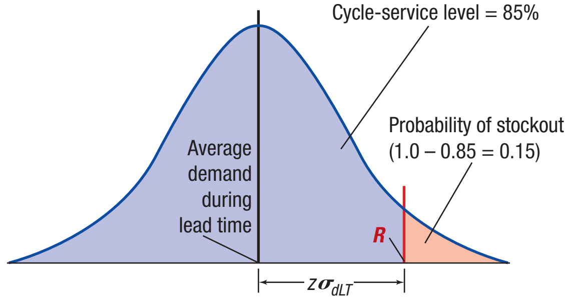



**FIGURE 9.12**
Finding Safety Stock with Normal Probability Distribution for an 85 Percent Cycle-Service Level

To provide a service level above 50 percent, the reorder point must be higher than the average demand during the lead time. As Figure 9.12 shows, that requires moving the reorder point to the right of the centerline so that more than 50 percent of the area under the curve is to the left of $R$. An 85 percent cycle-service level is achieved in Figure 9.12 with 85 percent of the area under the curve to the left of $R$ (in **blue**) and only 15 percent to the right (in pink). We compute the safety stock as follows:

$$\text{Safety stock} = z\sigma_{dLT}$$

where

$z$ = the number of standard deviations needed to achieve the cycle-service level

$\sigma_{dLT}$ = standard deviation of demand during the lead time

The reorder point becomes

$$R = \bar{d}L + \text{safety stock}$$

The higher the value of $z$, the higher the safety stock and the cycle-service level should be. If $z = 0$, there is no safety stock, and stockouts will occur during 50 percent of the order cycles. For a cycle-service level of 85 percent, $z = 1.04$. Example 9.6 shows how to use the Normal Distribution appendix to find the appropriate $z$ value, safety stock, and reorder point.

## EXAMPLE 9.6 Reorder Point for Variable Demand and Constant Lead Time

Let us return to the bird feeder in Example 9.3. The EOQ is 75 units. Suppose that the average demand is 18 units per week with a standard deviation of 5 units. The lead time is constant at 2 weeks. Determine the safety stock and reorder point if management wants a 90 percent cycle-service level.

MyOMLab

Tutor 9.4 in MyOMLab provides a new example to determine the safety stock and the reorder point for a $Q$ system.

**SOLUTION**

In this case, $\sigma_d = 5$, $\bar{d} = 8$ units, and $L = 2$ weeks, so $\sigma_{dLT} = \sigma_d\sqrt{L} = 5\sqrt{2} = 7.07$. Consult the body of the table in the Normal Distribution appendix for 0.9000, which corresponds to a 90 percent cycle-service level. The closest number is 0.8997, which corresponds to 1.2 in the row heading and 0.08 in the column heading. Adding these values gives a $z$ value of 1.28. With this information, we calculate the safety stock and reorder point as follows:

$$\text{Safety stock} = z\sigma_{dLT} = 1.28(7.07) = 9.05, \text{ or } 9 \text{ units}$$
$$\text{Reorder point} = \bar{d}L + \text{Safety stock}$$
$$= 2(18) + 9 = 45 \text{ units}$$

**DECISION POINT**

The $Q$ system for the bird feeder operates as follows: Whenever the inventory position reaches 45 units, order the EOQ of 75 units. Various order quantities and safety stock levels can be used in a $Q$ system. For example, management could specify a different order quantity (because of shipping constraints) or a different safety stock (because of storage limitations).

## Selecting the Reorder Point When Both Demand and Lead Time Are Variable

In practice, it is often the case that both the demand and the lead time are variable. Unfortunately, the equations for the safety stock and reorder point become more complicated. In the model below we make two simplifying assumptions. First, the demand distribution and the lead time distribution are measured in the same time units. For example, both demand and lead time are measured in weeks. Second, demand and lead time are *independent*. That is, demand per week is not affected by the length of the lead time.

$$\text{Safety stock} = z\sigma_{dLT}$$
$$R = (\text{Average weekly demand} \times \text{Average lead time in weeks}) + \text{Safety stock}$$
$$= \bar{d}\bar{L} + \text{Safety stock}$$

where

$$\bar{d} = \text{Average weekly or daily or monthly demand}$$
$$\bar{L} = \text{Average weekly or daily or monthly lead time}$$
$$\sigma_d = \text{Standard deviation of weekly or daily or monthly demand}$$
$$\sigma_{LT} = \text{Standard deviation of the lead time, and}$$
$$\sigma_{dLT} = \sqrt{\bar{L}\sigma_d^2 + \bar{d}^2\sigma_{LT}^2}$$

Now that we have determined the mean and standard deviation of the distribution of demand during lead time under these more complicated conditions, we can select the reorder point as we did before for the case where the lead time was constant.

### EXAMPLE 9.7 Reorder Point for Variable Demand and Variable Lead Time

The Office Supply Shop estimates that the average demand for a popular ball-point pen is 12,000 pens per week with a standard deviation of 3,000 pens. The current inventory policy calls for replenishment orders of 156,000 pens. The average lead time from the distributor is 5 weeks, with a standard deviation of 2 weeks. If management wants a 95 percent cycle-service level, what should the reorder point be?

**SOLUTION**

We have $\bar{d} = 12{,}000$ pens, $\sigma_d = 3{,}000$ pens, $\bar{L} = 5$ weeks, and $\sigma_{LT} = 2$ weeks.

$$\sigma_{dLT} = \sqrt{\bar{L}\sigma_d^2 + \bar{d}^2\sigma_{LT}^2} = \sqrt{(5)(3{,}000)^2 + (12{,}000)^2(2)^2} = 24{,}919.87 \text{ pens}$$

Consult the body of the Normal Distribution appendix for 0.9500, which corresponds to a 95 percent cycle-service level. That value falls exactly in the middle of the tabular values of 0.9495 (for a $z$ value of 1.64) and 0.9505 (for a $z$ value of 1.65). Consequently, we will use the more conservative value of 1.65. We calculate the safety stock and reorder point as follows:

$$\text{Safety stock} = z\sigma_{dLT} = (1.65)(24{,}919.87) = 41{,}117.79, \text{ or } 41{,}118 \text{ pens}$$
$$\text{Reorder point} = \bar{d}\bar{L} + \text{Safety stock} = (12{,}000)(5) + 41{,}118 = 101{,}118 \text{ pens}$$

**DECISION POINT**

Whenever the stock of ball-point pens drops to 101,118, management should place another replenishment order of 156,000 pens to the distributor.

Sometimes, the theoretical distributions for demand and lead time are not known. In those cases, we can use simulation to find the distribution of demand during lead time using discrete distributions for demand and lead times. Simulation can also be used to estimate the performance of an inventory system. More discussion, and an example, can be found in MyOMLab.

MyOMLab

## Systems Based on the *Q* System

Two systems based on the *Q* system are the two-bin system and the base-stock system.

**Two-Bin System** The concept of a *Q* system can be incorporated in a **visual system**, that is, a system that allows employees to place orders when inventory visibly reaches a certain marker. Visual systems are easy to administer because records are not kept on the current inventory position. The historical usage rate can simply be reconstructed from past purchase orders. Visual systems are intended for use with low-value SKUs that have a steady demand, such as nuts and bolts or office supplies. Overstocking is common, but the extra inventory holding cost is minimal because the items have relatively little value.

**visual system**

A system that allows employees to place orders when inventory visibly reaches a certain marker.

A visual system version of the *Q* system is the **two-bin system** in which a SKU's inventory is stored at two different locations. Inventory is first withdrawn from one bin. If the first bin is empty, the second bin provides backup to cover demand until a replenishment order arrives. An empty first bin signals the need to place a new order. Premade order forms placed near the bins let workers send one to purchasing or even directly to the supplier. When the new order arrives, the second bin is restored to its normal level and the rest is put in the first bin. The two-bin system operates like a *Q* system, with the normal level in the second bin being the reorder point *R*. The system also may be implemented with just one bin by marking the bin at the reorder point level.

**two-bin system**

A visual system version of the *Q* system in which a SKU's inventory is stored at two different locations.

**Base-Stock System** In its simplest form, the **base-stock system** issues a replenishment order, *Q*, each time a withdrawal is made, for the same amount as the withdrawal. This one-for-one replacement policy maintains the inventory position at a base-stock level equal to expected demand during the lead time plus safety stock. The base-stock level, therefore, is equivalent to the reorder point in a *Q* system. However, order quantities now vary to keep the inventory position at *R* at all times. Because this position is the lowest IP possible that will maintain a specified service level, the base-stock system may be used to minimize cycle inventory. More orders are placed, but each order is smaller. This system is appropriate for expensive items, such as replacement engines for jet airplanes. No more inventory is held than the maximum demand expected until a replacement order can be received.

**base-stock system**

An inventory control system that issues a replenishment order, *Q*, each time a withdrawal is made, for the same amount of the withdrawal.

## Calculating Total *Q* System Costs

Total costs for the continuous review ($Q$) system is the sum of three cost components:

$$\text{Total cost} = \text{Annual cycle inventory holding cost} + \text{annual ordering cost} \\ + \text{annual safety stock holding cost}$$

$$C = \frac{Q}{2}(H) + \frac{D}{Q}(S) + (H)(\text{Safety stock})$$

The annual cycle-inventory holding cost and annual ordering cost are the same equations we used for computing the total annual cycle-inventory cost in Example 9.2. The annual cost of holding the safety stock is computed under the assumption that the safety stock is on hand at all times. Referring to Figure 9.10 in each order cycle, we will sometimes experience a demand greater than the average demand during lead time, and sometimes we will experience less. On average over the year, we can assume the safety stock will be on hand. See Solved Problems 4 and 6 at the end of this chapter for an example of calculating the total costs for a *Q* system.

## Advantages of the *Q* System

The primary advantages of *Q* systems are the following:

1. The review frequency of each SKU may be individualized. Tailoring the review frequency to the SKU can reduce total ordering and holding costs.
2. Fixed lot sizes, if large enough, can result in quantity discounts. The firm's physical limitations, such as its truckload capacities, materials handling methods, and shelf space might also necessitate a fixed lot size.
3. The system requires low levels of safety stock for the amount of uncertainty in demands during the lead time.

## Periodic Review System

**periodic review (*P*) system**

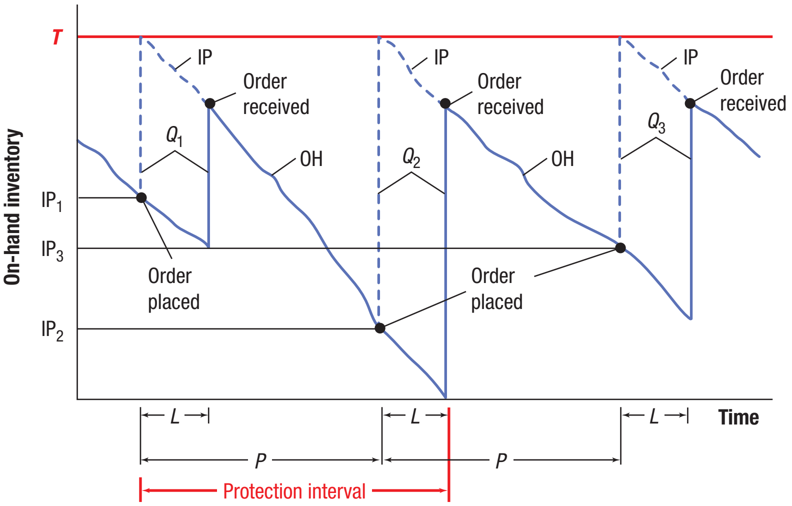
A system in which an item's inventory position is reviewed periodically rather than continuously.

An alternative inventory control system is the **periodic review (*P*) system**, sometimes called a *fixed interval reorder system* or *periodic reorder system*, in which an item's inventory position is reviewed periodically rather than continuously. Such a system can simplify delivery scheduling because it establishes a routine. A new order is always placed at the end of each review, and the time between orders (TBO) is fixed at *P*. Demand is a random variable, so total demand between reviews varies. In a *P* system, the lot size, *Q*, may change from one order to the next, but the time between orders is fixed. An example of a periodic review system is that of a soft-drink supplier making weekly rounds of grocery stores. Each week, the supplier reviews the store's inventory of soft drinks and restocks the store with enough items to meet demand and safety stock requirements until the next week.

Under a *P* system, four of the original EOQ assumptions are maintained: (1) no constraints are placed on the size of the lot, (2) the relevant costs are holding and ordering costs, (3) decisions for one SKU are independent of decisions for other SKUs, and (4) lead times are certain and supply is known. However, demand uncertainty is again allowed for. Figure 9.13 shows the periodic review system under these assumptions. The downward-sloping line again represents on-hand inventory. When the predetermined time, *P*, has elapsed since the last review, an order is placed to bring the inventory position, represented by the dashed line, up to the target inventory level, *T*. The lot size for the first review is $Q_1$, or the difference between inventory position $IP_1$ and *T*. As with the continuous review system, IP and OH differ only during the lead time. When the order arrives at the end of the lead time, OH and IP again are identical. Figure 9.13 shows that lot sizes vary from one order cycle to the next. Because the inventory position is lower at the second review, a greater quantity is needed to achieve an inventory level of *T*.

MyOMLab Animation

**▼ FIGURE 9.13**
*P* System When Demand Is Uncertain




### EXAMPLE 9.8 Determining How Much to Order in a *P* System

Return to the distribution center (DC) in Example 9.5. Suppose that management wants to use a periodic review system for the Sony TV sets. The first review of the inventory is scheduled for the end of Day 2. Assume that all demands and receipts occur at the end of the day. On the scheduled review day, inventory replenishment orders are placed after the demands and receipts have been accounted for. The lead time is 5 days, and management has set $T = 620$ and $P = 6$ days. Given the demand schedule in the table below, determine how much to order ($Q$) using a *P* system.

**SOLUTION**

We use the following equations:

$$\text{Inventory Position (IP)} = \text{OH} + \text{SR} - \text{BO}$$

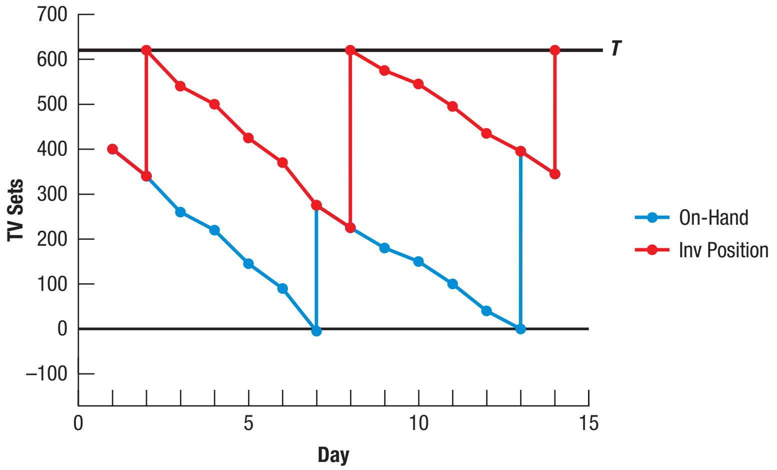
$$\text{Order Quantity } (Q) = T - \text{IP}$$

| Day | Demand | OH | SR | BO | IP | *Q* |
|---|---|---|---|---|---|---|
| 1 | 50 | 400 | | | 400 | |
| 2 | 60 | 340 | **280** after ordering | | 340 before ordering<br>340 + **280** = 620 after ordering | 620 − 340 = **280**<br>(due Day 7) |
| 3 | 80 | 260 | **280** | | 260 + **280** = 540 | |
| 4 | 40 | 220 | **280** | | 220 + **280** = 500 | |
| 5 | 75 | 145 | **280** | | 145 + **280** = 425 | |
| 6 | 55 | 90 | **280** | | 90 + **280** = 370 | |
| 7 | 95 | 90 + **280** − 95 = 275 | | | 275 + 0 = 275 | |
| 8 | 50 | 225 | **395** after ordering | | 225 + 0 = 225 before ordering<br>225 + **395** = 620 after ordering | 620 − 225 = **395**<br>(due Day 13) |

| Day | Demand | OH | SR | BO | IP | *Q* |
|---|---|---|---|---|---|---|
| 9 | 45 | 180 | **395** | | 180 + **395** = 575 | |
| 10 | 30 | 150 | **395** | | 150 + **395** = 545 | |
| 11 | 50 | 100 | **395** | | 100 + **395** = 495 | |
| 12 | 60 | 40 | **395** | | 40 + **395** = 435 | |
| 13 | 40 | 40 + **395** − 40 = 395 | | | 395 + 0 = **395** | |
| 14 | 50 | 345 | **275** after ordering | | 345 + 0 = 345 before ordering<br>345 + **275** = 620 after ordering | 620 − 345 = **275**<br>(due Day 19) |

**DECISION POINT**

The figure to the right shows the relationship between on-hand inventory and the inventory position. The DC did not experience any backorders because on Day 7 the replenishment order arrived in the nick of time. Notice that the order quantities vary in size while the time between orders remains a constant. Compare the operation of the *P* system in this example to the *Q* system in Example 9.5. The *Q* system requires constant monitoring to determine when the order point is reached. However, the average daily inventory is only 188 sets, compared to 226 sets for the *P* system. Granted, the *Q* system experienced backorders because of some unexpectedly large orders. Nonetheless, it is a general rule that to gain the benefits of periodic ordering, the *P* system requires more inventory for the same level of protection against stockouts or backorders. We will see why this is the case as we develop the parameters for the *P* system.

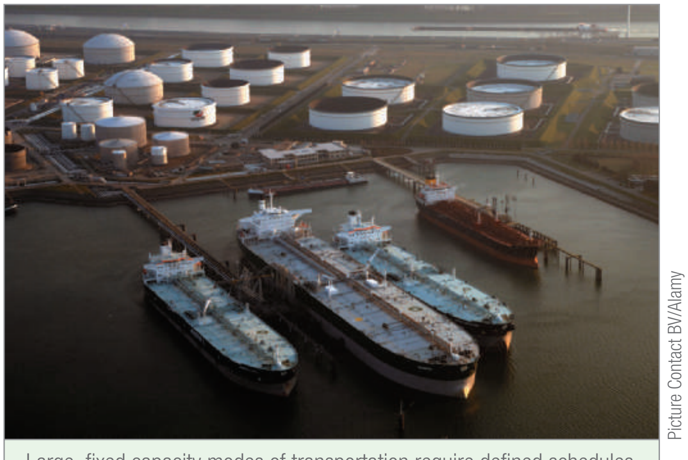



## Selecting the Time between Reviews

To run a *P* system, managers must make two decisions: the length of time between reviews, *P*, and the target inventory level, *T*. Let us first consider the time between reviews, *P*. It can be any convenient interval, such as each Friday or every other Friday. Another option is to base *P* on the cost trade-offs of the EOQ. In other words, *P* can be set equal to the average time between orders for the economic order quantity, or $\text{TBO}_{\text{EOQ}}$. Because demand is variable, some orders will be larger than the EOQ and some will be smaller. However, over an extended period of time, the average lot size should be close to the EOQ. If other models are used to determine the lot size (e.g., those described in Supplement C, "Special Inventory Models"), we divide the lot size chosen by the annual demand, *D*, and use this ratio as *P*. It will be expressed as the fraction of a year between orders, which can be converted into months, weeks, or days as needed.

## Selecting the Target Inventory Level When Demand Is Variable and Lead Time Is Constant

Now, let us calculate the target inventory level, *T*, when demand is variable but the lead time is constant. Figure 9.13 reveals that an order must be large enough to make the inventory position, IP, last beyond the next review, which is *P* time periods away. The checker must wait *P* periods to revise, correct, and reestablish the inventory position. Then, a new order is placed, but it does not arrive until after the lead time, *L*. Therefore, as Figure 9.13 shows, a protection interval of $P + L$ periods is needed. A fundamental difference between the *Q* and *P* systems is the length of time needed for stockout protection. A *Q* system needs stockout protection only during the lead time because orders can be placed as soon as they are needed and will be



Picture Contact BV/Alamy

Large, fixed capacity modes of transportation require defined schedules of operation. Such a situation supports the use of periodic review systems. Here ocean vessels await loads of petro-chemicals at the Vopak terminal in the Port of Rotterdam.

received $L$ periods later. A $P$ system, however, needs stockout protection for the longer $P + L$ protection interval because orders are placed only at fixed intervals, and the inventory is not checked until the next designated review time.

As with the $Q$ system, we need to develop the appropriate distribution of demand during the protection interval to specify the system fully. In a $P$ system, we must develop the distribution of demand for $P + L$ time periods. The target inventory level $T$ must equal the expected demand during the protection interval of $P + L$ periods, plus enough safety stock to protect against demand uncertainty over this same protection interval. We assume that lead time is constant and that demand in one period is independent of demand in the next period. Thus, the average demand during the protection interval is $\bar{d}(P + L)$, or

$$T = \bar{d}(P + L) + \text{Safety stock for the protection interval}$$

We compute safety stock for a $P$ system much as we did for the $Q$ system. However, the safety stock must cover demand uncertainty for a longer period of time. When using a normal probability distribution, we multiply the desired standard deviations to implement the cycle-service level, $z$, by the standard deviation of demand during the protection interval, $\sigma_{P+L}$. The value of $z$ is the same as for a $Q$ system with the same cycle-service level. Thus,

$$\text{Safety stock} = z\sigma_{P+L}$$

Based on our earlier logic for calculating $\sigma_{dLT}$ we know that the standard deviation of the distribution of demand during the protection interval is

$$\sigma_{P+L} = \sigma_d\sqrt{P + L}$$

Because a $P$ system requires safety stock to cover demand uncertainty over a longer time period than a $Q$ system, a $P$ system requires more safety stock; that is, $\sigma_{P+L}$ exceeds $\sigma_{dLT}$. Hence, to gain the convenience of a $P$ system requires that overall inventory levels be somewhat higher than those for a $Q$ system.

**EXAMPLE 9.9**

## Calculating *P* and *T*

**MyOMLab**

Tutor 9.5 in MyOMLab provides a new example to determine the review interval and the target inventory for a *P* system.

Again, let us return to the bird feeder example. Recall that demand for the bird feeder is normally distributed with a mean of 18 units per week and a standard deviation in weekly demand of 5 units. The lead time is 2 weeks, and the business operates 52 weeks per year. The $Q$ system developed in Example 9.5 called for an EOQ of 75 units and a safety stock of 9 units for a cycle-service level of 90 percent. What is the equivalent $P$ system? Answers are to be rounded to the nearest integer.

### SOLUTION

We first define $D$ and then $P$. Here, $P$ is the time between reviews, expressed in weeks because the data are expressed as demand *per week*:

$$D = (18\text{ units/week})(52\text{ weeks/year}) = 936\text{ units}$$

$$P = \frac{\text{EOQ}}{D}(52) = \frac{75}{936}(52) = 4.2,\text{ or }4\text{ weeks}$$

With $\bar{d} = 18$ units per week, an alternative approach is to calculate $P$ by dividing the EOQ by $\bar{d}$ to get $75/18 = 4.2$, or 4 weeks. Either way, we would review the bird feeder inventory every 4 weeks. We now find the standard deviation of demand over the protection interval $(P + L = 6)$:

$$\sigma_{P+L} = \sigma_d\sqrt{P + L} = 5\sqrt{6} = 12.25\text{ units}$$

Before calculating $T$, we also need a $z$ value. For a 90 percent cycle-service level, $z = 1.28$ (see the Normal Distribution appendix). The safety stock becomes

$$\text{Safety stock} = z\sigma_{P+L} = 1.28(12.25) = 15.68,\text{ or }16\text{ units}$$

We now solve for $T$:

$$\begin{aligned} T &= \text{Average demand during the protection interval} + \text{Safety stock} \\ &= \bar{d}(P + L) + \text{Safety stock} \\ &= (18\text{ units/week})(6\text{ weeks}) + 16\text{ units} = 124\text{ units} \end{aligned}$$

**DECISION POINT**

Every 4 weeks we would order the number of units needed to bring inventory position IP (counting the new order) up to the target inventory level of 124 units. The *P* system requires 16 units in safety stock, while the *Q* system only needs 9 units. If cost were the only criterion, the *Q* system would be the choice for the bird feeder. As we discuss later, other factors may sway the decision in favor of the *P* system.

## Selecting the Target Inventory Level When Demand and Lead Time Are Variable

A useful approach for finding *P* and *T* in practice is simulation. Given discrete probability distributions for demand and lead time, simulation can be used to estimate the demand during the protection interval distribution. The "*Demand During the Protection Interval Simulator*" in OM Explorer can be used to determine the distribution. Once determined, the distribution can be used to select a value for *T*, given a desired cycle-service level. More discussion, and an example, can be found in MyOMLab.

MyOMLab

## Systems Based on the *P* System

Two systems based on the *P* system are the single-bin system and the optional replenishment system.

**Single-Bin System** The concept of a *P* system can be translated into a simple visual system of inventory control. In the **single-bin system**, a maximum level is marked on the storage shelf or bin, and the inventory is brought up to the mark periodically—say, once a week. The single bin may be, for example, a gasoline storage tank at a service station or a storage bin for small parts at a manufacturing plant.

**single-bin system**

A system of inventory control in which a maximum level is marked on the storage shelf or bin, and the inventor is brought up to the mark periodically.

**Optional Replenishment System** Sometimes called the optional review, min–max, or $(s, S)$ system, the **optional replenishment system** is much like the *P* system. It is used to review the inventory position at fixed time intervals and, if the position has dropped to (or below) a predetermined level, to place a variable-sized order to cover expected needs. The new order is large enough to bring the inventory position up to a target inventory, similar to *T* for the *P* system. However, orders are not placed after a review unless the inventory position has dropped to the predetermined minimum level. The minimum level acts as the reorder point *R* does in a *Q* system. If the target is 100 and the minimum level is 60, the minimum order size is 40 (or $100 - 60$). Because continuous reviews need not be made, this system is particularly attractive when both review and ordering costs are high.

**optional replenishment system**

A system used to review the inventory position at fixed time intervals and, if the position has dropped to (or below) a predetermined level, to place a variable-sized order to cover expected needs.

## Calculating Total *P* System Costs

The total costs for the *P* system are the sum of the same three cost elements for the *Q* system. The differences are in the calculation of the order quantity and the safety stock. As shown in Figure 9.13, the average order quantity will be the average consumption of inventory during the *P* periods between orders. Consequently, $Q = \bar{d}P$. Total costs for the *P* system are

$$C = \frac{\bar{d}P}{2}(H) + \frac{D}{\bar{d}P}(S) + (H)(\text{Safety stock})$$

See Solved Problem 5 at the end of this chapter for an example of calculating total *P* system costs.

## Advantages of the *P* System

The primary advantages of *P* systems are the following:

1. The system is convenient because replenishments are made at fixed intervals. Fixed replenishment intervals allow for standardized pickup and delivery times. In contrast, individual items are ordered on their own best intervals with the *Q* system, which can differ widely.
2. Orders for multiple items from the same supplier can be combined into a single purchase order. This approach reduces ordering and transportation costs and can result in a price break from the supplier.
3. The inventory position, IP, needs to be known only when a review is made (not continuously, as in a *Q* system). However, this advantage is moot for firms using computerized record-keeping systems, in which a transaction is reported upon each receipt or withdrawal. When inventory records are always current, the system is called a **perpetual inventory system**.

**perpetual inventory system**

A system of inventory control in which the inventory records are always current.

Both the *Q* system and the *P* system have their advantages. Indeed, the advantages for one system become the disadvantages for the other. In conclusion, the choice between *Q* and *P* systems is not clear cut. Which system is better depends on the relative importance of its advantages in various situations.

## LEARNING GOALS IN REVIEW

| Learning Goal | Guidelines for Review | MyOMLab Resources |
|---|---|---|
| 1 Identify the advantages, disadvantages, and costs of holding inventory. | We cover these important aspects of inventories in the section "Inventory Trade-offs," pp. 339–342. Focus on the pressures for small or large inventories and Figure 9.1. | **Video**: Inventory and Textbooks |
| 2 Define the different types of inventory and the roles they play in supply chains. | The section "Types of Inventory," pp. 342–344, explains each type of inventory and provides an example in Figure 9.2. Example 9.1 and Solved Problem 1 show how to estimate inventory levels. Be sure to understand the distinction between independent and dependent inventories. | **OM Explorer Tutor:** 9.1: Estimating Inventory Levels |
| 3 Explain the tactics for reducing inventories in supply chains. | See the section "Inventory Reduction Tactics," pp. 345–346, for important approaches to managing inventory levels. The main tools for eliminating unneeded inventories are inventory control systems. | |
| 4 Use ABC analysis to identify the items deserving most attention and tightest inventory control. | The section "ABC Analysis," pp. 346–347, shows a simple approach to categorizing inventory items for ease of management oversight. Figure 9.4 has an example. Solved Problem 2 demonstrates the calculations. | **OM Explorer Tutor:** 9.2: ABC Analysis<br>**POM for Windows**: ABC Analysis |
| 5 Calculate the economic order quantity and apply it to various situations. | See the section "Economic Order Quantity," pp. 347–351, for a complete discussion of the EOQ model. Focus on Figures 9.5, 9.6, and 9.7 to see how the EOQ model affects inventory levels under the standard assumptions and how the EOQ provides the lowest cost solution. Review Examples 9.2 and 9.3 and Solved Problem 3 for help in calculating the total costs of various lot-size choices. Table 9.1 reveals important managerial insights from the EOQ. See also the Active Model Exercise on p. 373. | **Active Model:** 9.1: Economic Order Quantity<br>**OM Explorer Tutor:** 9.3: Finding EOQ and Total Cost<br>**POM for Windows:** Economic Order Quantity (EOQ) Model<br>**Tutor Exercise:** 9.1: Finding EOQ; Safety Stock; *R, P, T* at Bison College Bookstore |
| 6 Determine the order quantity and reorder point for a continuous review inventory control system. | The section "Continuous Review System," pp. 351–357, builds the essence of the *Q* system from basic principles to more realistic assumptions. Be sure to understand Figures 9.10 and 9.12. Examples 9.4, 9.5, and 9.6 and Solved Problems 4 and 6 show how to determine the parameters *Q* and *B* under various assumptions. | **OM Explorer Solvers:** Inventory Systems Designer; Demand During Protection Interval Simulator; *Q* System Simulator<br>**OM Explorer Tutor:** 9.4: Finding the Safety Stock and *R*<br>**Tutor Exercise:** 9.1: Finding EOQ; Safety Stock; *R,P,T* at Bison College Bookstore<br>**Tutorial on Inventory Management Systems:** Using Simulation to Develop Inventory Management Systems<br>**Advanced Problems**: Office Supply Shop Simulation; Floral Shop Simulation Simquick Simulation Exercise |
| 7 Determine the review interval and target inventory level for a periodic review inventory control system. | We summarize the key concepts in the section "Periodic Review System," pp. 358–361. Figure 9.13 shows how a *P* system operates while Examples 9.8 and 9.9 and Solved Problem 5 demonstrate how to calculate the parameters *P* and *T*. | **OM Explorer Solver:** Inventory Systems Designer; Demand During Protection Interval Simulator<br>**OM Explorer Tutor:** 9.5: Calculating *P* and *T*<br>**Tutor Exercise:** 9.1: Finding EOQ; Safety Stock; *R,P,T* at Bison College Bookstore<br>**Tutorial on Inventory Management Systems**: Using Simulation to Develop Inventory Management Systems<br>**Advanced Problem**: Grocery Store Simulation; Simquick Simulation Exercise |

# Key Equations

## Types of Inventory

1. Average cycle inventory: $\frac{Q}{2}$

2. Pipeline inventory: $\overline{D}_L = \overline{d}L$

## Economic Order Quantity

**3.** Total annual cycle-inventory cost = Annual holding cost + Annual ordering or setup cost:

$$C = \frac{Q}{2}(H) + \frac{D}{Q}(S)$$

**4.** Economic order quantity:

$$\text{EOQ} = \sqrt{\frac{2DS}{H}}$$

**5.** Time between orders, expressed in weeks:

$$\text{TBO}_{\text{EOQ}} = \frac{\text{EOQ}}{D}(52 \text{ weeks/year})$$

## Continuous Review System

**6.** Inventory position = On-hand inventory + Scheduled receipts − Backorders:

$$\text{IP} = \text{OH} + \text{SR} - \text{BO}$$

**7.** Continuous review system:

Protection interval = Lead time $(L)$

Standard deviation of demand during the lead time (constant $L$) $= \sigma_{dLT} = \sigma_d\sqrt{L}$

Standard deviation of demand during the lead time (variable $L$) $= \sigma_{dLT} = \sqrt{\bar{L}\sigma_d^2 + \bar{d}^2\sigma_{LT}^2}$

Safety stock $= z\sigma_{dLT}$

Reorder point $R$ for constant lead time $= \bar{d}L$ + Safety stock

Reorder point $R$ for variable lead time $= \bar{d}\bar{L}$ + Safety stock

Order quantity = EOQ

Replenishment rule: Order EOQ units when IP $\leq R$

Total $Q$ system cost: $C = \frac{Q}{2}(H) + \frac{D}{Q}(S) + (H)(\text{Safety stock})$

## Periodic Review System

**8.** Periodic review system:

Review interval = Time between orders = $P$

Protection interval = Time between orders + Lead time = $P + L$

Standard deviation of demand during the protection interval $\sigma_{P+L} = \sigma_d\sqrt{P + L}$

Safety stock $= z\sigma_{P+L}$

Target inventory level $(T)$ = Average demand during the protection interval + Safety stock

$= \bar{d}(P + L)$ + Safety stock

Order quantity Target inventory level − Inventory position = $T - \text{IP}$

Replenishment rule: Every $P$ time periods, order $T - \text{IP}$ units

Total $P$ system cost: $C = \frac{\bar{d}P}{2}(H) + \frac{D}{\bar{d}P}(S) + (H)(\text{Safety stock})$

## Key Terms

ABC analysis 346
anticipation inventory 343
base-stock system 357
continuous review ($Q$) system 351
cycle counting 346
cycle inventory 343
cycle-service level 354
dependent demand items 343
economic order quantity (EOQ) 347
finished goods (FG) 342
independent demand items 342
inventory 339
inventory holding cost 340
inventory management 339
inventory position (IP) 351
lot size 339
lot sizing 343
open orders 351
optional replenishment system 361
ordering cost 340
periodic review ($P$) system 358
perpetual inventory system 361
pipeline inventory 344
protection interval 354
quantity discount 341
raw materials (RM) 342
reorder point ($R$) 351
reorder point (ROP) system 351
repeatability 345
safety stock inventory 343
scheduled receipts (SR) 351
service level 354
setup cost 341
single-bin system 361
stock-keeping unit (SKU) 346
time between orders (TBO) 350
two-bin system 357
visual system 357
work-in-process (WIP) 342

## Solved Problem 1

A distribution center experiences an average weekly demand of 50 units for one of its items. The product is valued at $650 per unit. Inbound shipments from the factory warehouse average 350 units. Average lead time (including ordering delays and transit time) is 2 weeks. The distribution center operates 52 weeks per year; it carries a 1-week supply of inventory as safety stock and no anticipation inventory. What is the value of the average aggregate inventory being held by the distribution center?

### SOLUTION

| Type of Inventory | Calculation of Aggregate Average Inventory | |
|---|---|---|
| Cycle | $\frac{Q}{2} = \frac{350}{2}$ | = 175 units |
| Safety stock | 1-week supply | = 50 units |
| Anticipation | None | |
| Pipeline | $\bar{d}L$ = (50 units/week) (2 weeks) | = 100 units |
| | *Average aggregate inventory* | = 325 units |
| | *Value of aggregate inventory* | = $650(325)<br>= $211,250 |

## Solved Problem 2

**MyOMLab**

Tutor 9.2 in MyOMLab provides a new example of the application of ABC analysis.

Booker's Book Bindery divides SKUs into three classes according to their dollar usage. Calculate the usage values of the following SKUs and determine which is most likely to be classified as class A.

### SOLUTION

The annual dollar usage for each SKU is determined by multiplying the annual usage quantity by the value per unit. As shown in Figure 9.14, the SKUs are then sorted by annual dollar usage, in declining order. Finally, A–B and B–C class lines are drawn roughly, according to the guidelines presented in the text. Here, class A includes only one SKU (signatures), which represents only 1/7, or 14 percent, of the SKUs but accounts for 83 percent of annual dollar usage. Class B includes the next two SKUs, which taken together represent 28 percent of the SKUs and account for 13 percent of annual dollar usage. The final four SKUs, class C, represent over half the number of SKUs but only 4 percent of total annual dollar usage.

| SKU Number | Description | Quantity Used per Year | Unit Value ($) |
|---|---|---|---|
| 1 | Boxes | 500 | 3.00 |
| 2 | Cardboard (square feet) | 18,000 | 0.02 |
| 3 | Cover stock | 10,000 | 0.75 |
| 4 | Glue (gallons) | 75 | 40.00 |
| 5 | Inside covers | 20,000 | 0.05 |
| 6 | Reinforcing tape (meters) | 3,000 | 0.15 |
| 7 | Signatures | 150,000 | 0.45 |

| SKU Number | Description | Quantity Used per Year | | Unit Value ($) | | Annual Dollar Usage ($) |
|---|---|---|---|---|---|---|
| 1 | Boxes | 500 | × | 3.00 | = | 1,500 |
| 2 | Cardboard (square feet) | 18,000 | × | 0.02 | = | 360 |
| 3 | Cover stock | 10,000 | × | 0.75 | = | 7,500 |
| 4 | Glue (gallons) | 75 | × | 40.00 | = | 3,000 |
| 5 | Inside covers | 20,000 | × | 0.05 | = | 1,000 |
| 6 | Reinforcing tape (meters) | 3,000 | × | 0.15 | = | 450 |
| 7 | Signatures | 150,000 | × | 0.45 | = | 67,500 |
| | | | | | Total | 81,310 |

| SKU # | Description | Qty Used/Year | Value | Dollar Usage | Pct of Total | Cumulative % of Dollar Value | Cumulative % of SKU | Class |
|---|---|---|---|---|---|---|---|---|
| 7 | Signatures | 150,000 | $0.45 | $67,500 | 83.0% | 83.0% | 14.3% | A |
| 3 | Cover stock | 10,000 | $0.75 | $7,500 | 9.2% | 92.2% | 28.6% | B |
| 4 | Glue | 75 | $40.00 | $3,000 | 3.7% | 95.9% | 42.9% | B |
| 1 | Boxes | 500 | $3.00 | $1,500 | 1.8% | 97.8% | 57.1% | C |
| 5 | Inside covers | 20,000 | $0.05 | $1,000 | 1.2% | 99.0% | 71.4% | C |
| 6 | Reinforcing tape | 3,000 | $0.15 | $450 | 0.6% | 99.6% | 85.7% | C |
| 2 | Cardboard | 18,000 | $0.02 | $360 | 0.4% | 100.0% | 100.0% | C |
| Total | | | | $81,310 | | | | |

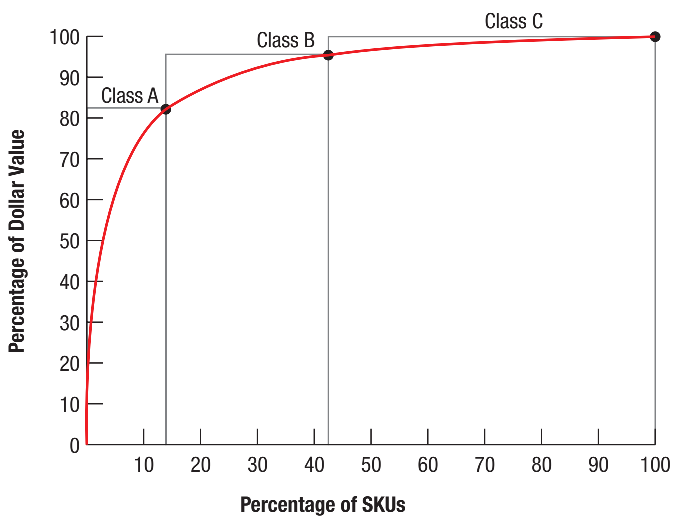



**FIGURE 9.14**
Annual Dollar Usage for Class A, B, and C SKUs Using Tutor 9.2

## Solved Problem 3

Nelson's Hardware Store stocks a 19.2 volt cordless drill that is a popular seller. Annual demand is 5,000 units, the ordering cost is $15, and the inventory holding cost is $4/unit/year.

**a.** What is the economic order quantity?

**b.** What is the total annual cost for this inventory item?

**SOLUTION**

**a.** The order quantity is

$$\text{EOQ} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(5{,}000)(\$15)}{\$4}} = \sqrt{37{,}500}$$

$$= 193.65, \text{ or } 194 \text{ drills}$$

**b.** The total annual cost is

$$C = \frac{Q}{2}(H) + \frac{D}{Q}(S) = \frac{194}{2}(\$4) + \frac{5{,}000}{194}(\$15) = \$774.60$$

## Solved Problem 4

MyOMLab Video

A regional distributor purchases discontinued appliances from various suppliers and then sells them on demand to retailers in the region. The distributor operates 5 days per week, 52 weeks per year. Only when it is open for business can orders be received. The following data are estimated for a counter-top mixer:

Average daily demand $(\bar{d}) = 100$ mixers

Standard deviation of daily demand $(\sigma_d) = 30$ mixers

Lead time $(L) = 3$ days

Holding cost $(H) = \$9.40/\text{unit}/\text{year}$

Ordering cost $(S) = \$35/\text{order}$

Cycle-service level $= 92$ percent

The distributor uses a continuous review $Q$ system.

**a.** What order quantity $Q$, and reorder point, $R$, should be used?

**b.** What is the total annual cost of the system?

**c.** If on-hand inventory is 40 units, one open order for 440 mixers is pending, and no backorders exist, should a new order be placed?

**SOLUTION**

**a.** Annual demand is

$$D = (5 \text{ days/week})(52 \text{ weeks/year})(100 \text{ mixers/day}) = 26{,}000 \text{ mixers/year}$$

The order quantity is

$$\text{EOQ} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(26{,}000)(\$35)}{\$9.40}} = \sqrt{193{,}167} = 440.02, \text{ or } 440 \text{ mixers}$$

The standard deviation of the distribution of demand during lead time is

$$\sigma_{dLT} = \sigma_d\sqrt{L} = 30\sqrt{3} = 51.96$$

A 92 percent cycle-service level corresponds to $z = 1.41$ (see the Normal Distribution appendix). Therefore,

$$\text{Safety stock} = z\sigma_{dLT} = 1.41(51.96 \text{ mixers}) = 73.26, \text{ or } 73 \text{ mixers}$$

$$\text{Average demand during the lead time} = \bar{d}L = 100(3) = 300 \text{ mixers}$$

$$\text{Reorder point } R = \text{Average demand during the lead time} + \text{Safety stock}$$

$$= 300 \text{ mixers} + 73 \text{ mixers} = 373 \text{ mixers}$$

With a continuous review system, $Q = 440$ and $R = 373$.

**b.** The total annual cost for the $Q$ systems is

$$C = \frac{Q}{2}(H) + \frac{D}{Q}(S) + (H)(\text{Safety stock})$$

$$C = \frac{440}{2}(\$9.40) + \frac{26{,}000}{440}(35) + (\$9.40)(73) = \$4{,}822.38$$

**c.** Inventory position $=$ On-hand inventory $+$ Scheduled receipts $-$ Backorders

$$\text{IP} = \text{OH} + \text{SR} - \text{BO} = 40 + 440 - 0 = 480 \text{ mixers}$$

Because IP (480) exceeds $R$ (373), do not place a new order.

# Solved Problem 5

Suppose that a periodic review $(P)$ system is used at the distributor in Solved Problem 4, but otherwise the data are the same.

**a.** Calculate the $P$ (in workdays, rounded to the nearest day) that gives approximately the same number of orders per year as the EOQ.

**b.** What is the target inventory level, $T$? Compare the $P$ system to the $Q$ system in Solved Problem 4.

**c.** What is the total annual cost of the $P$ system?

**d.** It is time to review the item. On-hand inventory is 40 mixers; receipt of 440 mixers is scheduled, and no backorders exist. How much should be reordered?

## SOLUTION

**a.** The time between orders is

$$P = \frac{\text{EOQ}}{D}(260 \text{ days/year}) = \frac{440}{26{,}000}(260) = 4.4, \text{ or } 4 \text{ days}$$

**b.** Figure 9.15 shows that $T = 812$ and safety stock $= (1.41)(79.37) = 111.91$, or about 112 mixers. The corresponding $Q$ system for the counter-top mixer requires less safety stock.

**c.** The total annual cost of the $P$ system is

$$C = \frac{\bar{d}P}{2}(H) + \frac{D}{\bar{d}P}(S) + (H)(\text{Safety stock})$$

$$C = \frac{(100)(4)}{2}(\$9.40) + \frac{26{,}000}{(100)(4)}(\$35) + (\$9.40)(1.41)(79.37)$$

$$= \$5{,}207.80$$

**d.** Inventory position is the amount on hand plus scheduled receipts minus backorders, or

$$\text{IP} = \text{OH} + \text{SR} - \text{BO} = 40 + 440 - 0 = 480 \text{ mixers}$$

The order quantity is the target inventory level minus the inventory position, or

$$Q = T - \text{IP} = 812 \text{ mixers} - 480 \text{ mixers} = 332 \text{ mixers}$$

An order for 332 mixers should be placed.

| Continuous Review (Q) System | |
|---|---|
| z | 1.41 |
| Safety Stock | 73 |
| Reorder Point | 373 |
| Annual Cost | $4,822.38 |

| Periodic Review (P) System | | |
|---|---|---|
| Time Between Reviews (P) | 4.00 | Days |
| | ☑ Enter manually | |
| Standard Deviation of Demand During Protection Interval | 79.37 | |
| Safety Stock | 112 | |
| Average Demand During Protection Interval | 700 | |
| Target Inventory Level (T) | 812 | |
| Annual Cost | $5,207.80 | |

**FIGURE 9.15**
*OM Explorer* Solver for Inventory Systems

## Solved Problem 6

Grey Wolf Lodge is a popular 500-room hotel in the North Woods. Managers need to keep close tabs on all room service items, including a special pine-scented bar soap. The daily demand for the soap is 275 bars, with a standard deviation of 30 bars. Ordering cost is \$10 and the inventory holding cost is \$0.30/bar/year. The lead time from the supplier is 5 days, with a standard deviation of 1 day. The lodge is open 365 days a year.

**a.** What is the economic order quantity for the bar of soap?

**b.** What should the reorder point be for the bar of soap if management wants to have a 99 percent cycle-service level?

**c.** What is the total annual cost for the bar of soap, assuming a $Q$ system will be used?

### SOLUTION

**a.** We have $D = (275)(365) = 100{,}375$ bars of soap; $S = \$10$; and $H = \$0.30$. The EOQ for the bar of soap is

$$\text{EOQ} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(100{,}375)(\$10)}{\$0.30}} = \sqrt{6{,}691{,}666.7}$$
$$= 2{,}586.83, \text{ or } 2{,}587 \text{ bars}$$

**b.** We have $\bar{d} = 275$ bars/day, $\sigma_d = 30$ bars, $\bar{L} = 5$ days, and $\sigma_{LT} = 1$ day.

$$\sigma_{dLT} = \sqrt{\bar{L}\sigma_d^2 + \bar{d}^2\sigma_{LT}^2} = \sqrt{(5)(30)^2 + (275)^2(1)^2} = 283.06 \text{ bars}$$

Consult the body of the Normal Distribution appendix for 0.9900, which corresponds to a 99 percent cycle-service level. The closest value is 0.9901, which corresponds to a $z$ value of 2.33. We calculate the safety stock and reorder point as follows:

$$\text{Safety stock} = z\sigma_{dLT} = (2.33)(283.06) = 659.53, \text{ or } 660 \text{ bars}$$
$$\text{Reorder point} = \bar{d}\bar{L} + \text{Safety stock} = (275)(5) + 600 = 2{,}035 \text{ bars}$$

**c.** The total annual cost for the $Q$ system is

$$C = \frac{Q}{2}(H) + \frac{D}{Q}(S) + (H)(\text{Safety stock})$$
$$= \frac{2{,}587}{2}(\$0.30) + \frac{100{,}375}{2{,}587}(\$10) + (\$0.30)(660) = \$974.05$$

## Discussion Questions

**1.** What is the relationship between inventory and the nine competitive priorities we discussed in Chapter 1, "Using Operations to Create Value"? Suppose that two competing manufacturers, Company H and Company L, are similar except that Company H has much higher investments in raw materials, work-in-process, and finished goods inventory than Company L. In which of the nine competitive priorities will Company H have an advantage?

**2.** Suppose that a large discount retailer with a lot of purchasing power in a supply chain requires that all suppliers incorporate a new information system that will reduce the cost of placing orders between the retailer and its suppliers as well as between the suppliers and their suppliers. Suppose also that order quantities and lead times are related; the smaller the order quantity the shorter the lead time from suppliers. Assume that all members of the supply chain use a continuous review system and EOQ order quantities. Explain the implications of the new information system for the supply chain in general and the inventory systems of the supply chain members in particular.

**3.** Will organizations ever get to the point where they will no longer need inventories? Why or why not?

## Problems

The OM Explorer and POM for Windows software is available to all students using the 11th edition of this textbook. Go to **http://www.pearsonglobaleditions.com/krajewski** to download these computer packages. If you purchased MyOMLab, you also have access to Active Models software and significant help in doing the following problems. Check with your instructor on how best to use these resources. In many cases, the instructor wants you to understand how to do the calculations by hand. At the least, the software provides a check on your calculations. When calculations are particularly complex and the goal is interpreting the results in making decisions, the software replaces entirely the manual calculations.

## Types of Inventory

1. A part is produced in lots of 1,400 units. It is assembled from 2 components worth $70 total. The value added in production (for labor and variable overhead) is $60 per unit, bringing total costs per completed unit to $130. The average lead time for the part is 8 weeks and annual demand is 3,600 units, based on 50 business weeks per year.
   a. How many units of the part are held, on average, in cycle inventory? What is the dollar value of this inventory?
   b. How many units of the part are held, on average, in pipeline inventory? What is the dollar value of this inventory? (*Hint:* Assume that the typical part in pipeline inventory is 50 percent completed. Thus, half the labor and variable overhead cost has been added, bringing the unit cost to $100, or $70 + $60/2).
2. Prince Electronics, a manufacturer of consumer electronic goods, has five distribution centers in different regions of the country. For one of its products, a high-speed modem priced at $370 per unit, the average weekly demand at *each* distribution center is 70 units. Average shipment size to each distribution center is 450 units, and average lead time for delivery is 3 weeks. Each distribution center carries 3 weeks' supply as safety stock but holds no anticipation inventory.
   a. On average, how many dollars of pipeline inventory will be in transit to each distribution center?
   b. How much total inventory (cycle, safety, and pipeline) does Prince hold for all five distribution centers?
3. Terminator, Inc. manufactures a motorcycle part in lots of 350 units. The raw materials cost for the part is $160, and the value added in manufacturing 1 unit from its components is $300, for a total cost per completed unit of $460. The lead time to make the part is 2 weeks, and the annual demand is 4,000 units. Assume 50 working weeks per year.
   a. How many units of the part are held, on average, as cycle inventory? What is its value?
   b. How many units of the part are held, on average, as pipeline inventory? What is its value?

## Inventory Reduction Tactics

4. Ruby-Star Incorporated is considering two different vendors for one of its top-selling products which has an average weekly demand of 50 units and is valued at $75 per unit. Inbound shipments from vendor 1 will average 350 units with an average lead time (including ordering delays and transit time) of 2 weeks. Inbound shipments from vendor 2 will average 500 units with an average lead time of 1 week. Ruby-Star operates 52 weeks per year; it carries a 2-week supply of inventory as safety stock and no anticipation inventory.
   a. What would be the average aggregate inventory value of this product if Ruby-Star used vendor 1 exclusively?
   b. What would be the average aggregate inventory value of this product if Ruby-Star used vendor 2 exclusively?
   c. How would your analysis change if average weekly demand increased to 100 units per week?
5. Haley Photocopying purchases paper from an out-of-state vendor. Average weekly demand for paper is 150 cartons per week for which Haley pays $15 per carton. Inbound shipments from the vendor average 1000 cartons with an average lead time of 3 weeks. Haley operates 52 weeks per year; it carries a 4-week supply of inventory as safety stock and no anticipation inventory. The vendor has recently announced that they will be building a facility near Haley Photocopying that will reduce lead time to one week. Further, they will be able to reduce shipments to 200 cartons. Haley believes that they will be able to reduce safety stock to a 1-week supply. What impact will these changes make to Haley's average inventory level and its average aggregate inventory value?

## ABC Analysis

6. Oakwood Hospital is considering using ABC analysis to classify laboratory SKUs into three categories: those that will be delivered daily from their supplier (Class A items), those that will be controlled using a continuous review system (B items), and those that will be held in a two-bin system (C items).

   The following table shows the annual dollar usage for a sample of eight SKUs. Rank the SKUs, and assign them to their appropriate category.

| SKU Code | Dollar Value | Annual Usage |
|---|---|---|
| 1 | $4.50 | $3,150 |
| 2 | $1.00 | $30,000 |
| 3 | $0.55 | $61 |
| 4 | $1.50 | $225 |
| 5 | $0.20 | $13,000 |
| 6 | $0.90 | $315 |
| 7 | $0.02 | $2,300 |
| 8 | $0.01 | $11 |

7. Southern Markets, Inc. is considering the use of ABC analysis to focus on the most critical SKUs in its inventory. Currently, there are approximately 20,000 different SKUs with a total dollar usage of $10,000,000 per year.
   a. What would you expect to be the number of SKUs and the total annual dollar usage for A items, B items, and C items at Southern Markets, Inc.?

**b.** The following table provides a random sample of the unit values and annual demands of eight SKUs. Categorize these SKUs as A, B, and C items.

| SKU Code | Unit Value | Demand (Units) |
|---|---|---|
| A104 | $0.85 | 370 |
| D205 | $0.03 | 3,500 |
| X104 | $4.35 | 60 |
| U404 | $2.20 | 2,700 |
| L205 | $4.70 | 25 |
| S104 | $2.40 | 30 |
| X205 | $0.23 | 270 |
| L104 | $0.36 | 1,040 |

**8.** New Wave Shelving's Inventory Manager would like to start using an ABC inventory classification system. The following table shows the annual inventory usage of all the 19 component items that the company holds. Assign them to their appropriate category.

| SKU # | Description | Quantity Used Per Year | Dollar Value Per Unit |
|---|---|---|---|
| a-1 | Steel panel | 500 | $ 25.00 |
| a-2 | Steel bumper | 750 | $ 135.00 |
| a-3 | Steel clamp | 3500 | $ 5.00 |
| a-4 | Steel brace | 200 | $ 20.00 |
| b-1 | Copper coil | 1250 | $ 260.00 |
| b-2 | Copper panel | 1250 | $ 50.00 |
| b-3 | Copper brace 1 | 250 | $ 75.00 |
| b-4 | Copper brace 2 | 150 | $ 125.00 |
| c-1 | Rubber bumper | 8500 | $ 0.75 |
| c-2 | Rubber foot | 6500 | $ 0.75 |
| c-3 | Rubber seal 1 | 1500 | $ 1.00 |
| c-4 | Rubber seal 2 | 3500 | $ 1.00 |
| c-5 | Rubber seal 3 | 1200 | $ 2.25 |
| d-1 | Plastic fastener kit | 1500 | $ 3.50 |
| d-2 | Plastic handle | 2000 | $ 0.75 |
| d-3 | Plastic panel | 1000 | $ 6.50 |
| d-4 | Plastic bumper | 2000 | $ 1.25 |
| d-5 | Plastic coil | 450 | $ 6.00 |
| d-6 | Plastic foot | 6000 | $ 0.25 |

## Economic Order Quantity

**9.** Yellow Press, Inc., buys paper in 1,500-pound rolls for printing. Annual demand is 2,500 rolls. The cost per roll is $800, and the annual holding cost is 15 percent of the cost. Each order costs $50 to process.

**a.** How many rolls should Yellow Press, Inc., order at a time?

**b.** What is the time between orders?

**10.** Babble, Inc., buys 400 blank cassette tapes per month for use in producing foreign language courseware. The ordering cost is $12.50. Holding cost is $0.12 per cassette per year.

**a.** How many tapes should Babble, Inc., order at a time?

**b.** What is the time between orders?

**11.** At Dot Com, a large retailer of popular books, demand is constant at 20,400 books per year. The cost of placing an order to replenish stock is $35, and the annual cost of holding is $6 per book. Stock is received 5 working days after an order has been placed. No backordering is allowed. Assume 250 working days a year.

**a.** What is Dot Com's optimal order quantity?

**b.** What is the optimal number of orders per year?

**c.** What is the optimal interval (in working days) between orders?

**d.** What is demand during the lead time?

**e.** What is the reorder point?

**f.** What is the inventory position immediately after an order has been placed?

**12.** Leaky Pipe, a local retailer of plumbing supplies, faces demand for one of its SKUs at a constant rate of 30,000 units per year. It costs Leaky Pipe $10 to process an order to replenish stock and $1 per unit per year to carry the item in stock. Stock is received 4 working days after an order is placed. No backordering is allowed. Assume 300 working days a year.

**a.** What is Leaky Pipe's optimal order quantity?

**b.** What is the optimal number of orders per year?

**c.** What is the optimal interval (in working days) between orders?

**d.** What is the demand during the lead time?

**e.** What is the reorder point?

**f.** What is the inventory position immediately after an order has been placed?

## Continuous Review System

**13.** Sam's Cat Hotel operates 52 weeks per year, 7 days per week, and uses a continuous review inventory system. It purchases kitty litter for $10.75 per bag. The following information is available about these bags.

Demand = 95 bags/week

Order cost = $58/order

Annual holding cost = 25 percent of cost

Desired cycle-service level = 90 percent

Lead time = 4 weeks (28 working days)

Standard deviation of *weekly* demand = 16 bags

Current on-hand inventory is 315 bags, with no open orders or backorders.

**a.** What is the EOQ? What would be the average time between orders (in weeks)?

**b.** What should $R$ be?

**c.** An inventory withdrawal of 10 bags was just made. Is it time to reorder?

**d.** The store currently uses a lot size of 490 bags (i.e., $Q = 490$). What is the annual holding cost of this policy? Annual ordering cost? Without calculating the EOQ, how can you conclude from these two calculations that the current lot size is too large?

**e.** What would be the annual cost saved by shifting from the 490-bag lot size to the EOQ?

**14.** Consider again the kitty litter ordering policy for Sam's Cat Hotel in Problem 13.

**a.** Suppose that the weekly demand forecast of 95 bags is incorrect and actual demand averages only 65 bags per week. How much higher will total costs be, owing to the distorted EOQ caused by this forecast error?

**b.** Suppose that actual demand is 65 bags but that ordering costs are cut to only $10 by using the Internet to automate order placing. However, the buyer does not tell anyone, and the EOQ is not adjusted to reflect this reduction in $S$. How much higher will total costs be, compared to what they could be if the EOQ were adjusted?

**15.** In a $Q$ system, the demand rate for strawberry ice cream is normally distributed, with an average of 300 pints *per week*. The lead time is 9 weeks. The standard deviation of *weekly* demand is 15 pints.

**a.** What is the standard deviation of demand during the 9-week lead time?

**b.** What is the average demand during the 9-week lead time?

**c.** What reorder point results in a cycle-service level of 99 percent?

**16.** Petromax Enterprises uses a continuous review inventory control system for one of its SKUs. The following information is available on the item. The firm operates 50 weeks in a year.

Demand = 50,000 units/year

Ordering cost = $35/order

Holding cost = $2/unit/year

Average lead time = 3 weeks

Standard deviation of weekly demand = 125 units

**a.** What is the economic order quantity for this item?

**b.** If Petromax wants to provide a 90 percent cycle-service level, what should be the safety stock and the reorder point?

**17.** In a continuous review inventory system, the lead time for door knobs is 5 weeks. The standard deviation of demand during the lead time is 85 units. The desired cycle-service level is 99 percent. The supplier of door knobs streamlined its operations and now quotes a one-week lead time. How much can safety stock be reduced without reducing the 99 percent cycle-service level?

**18.** In a two-bin inventory system, the demand for three-inch lag bolts during the 2-week lead time is normally distributed, with an average of 53 units per week. The standard deviation of weekly demand is 5 units.

**a.** What is the probability of demand exceeding the reorder point when the normal level in the second bin is set at 130 units?

**b.** What is the probability of demand exceeding the 130 units in the second bin if it takes 3 weeks to receive a replenishment order?

**19.** You are in charge of inventory control of a highly successful product retailed by your firm. Weekly demand for this item varies, with an average of 200 units and a standard deviation of 16 units. It is purchased from a wholesaler at a cost of $12.50 per unit. You are using a continuous review system to control this inventory. The supply lead time is 4 weeks. Placing an order costs $50, and the inventory carrying rate per year is 20 percent of the item's cost. Your firm operates 5 days per week, 50 weeks per year.

**a.** What is the optimal ordering quantity for this item?

**b.** How many units of the item should be maintained as safety stock for 99 percent protection against stockouts during an order cycle?

**c.** If supply lead time can be reduced to 2 weeks, what is the percent reduction in the number of units maintained as safety stock for the same 99 percent stockout protection?

**d.** If through appropriate sales promotions, the demand variability is reduced so that the standard deviation of weekly demand is 8 units instead of 16, what is the percent reduction (compared to that in part [b]) in the number of units maintained as safety stock for the same 99 percent stockout protection?

**20.** Your firm uses a continuous review system and operates 52 weeks per year. One of the SKUs has the following characteristics.

Demand ($D$) = 20,000 units/year

Ordering cost ($S$) = $40/order

Holding cost ($H$) = $2/unit/year

Lead time ($L$) = 2 weeks

Cycle-service level = 95 percent

Demand is normally distributed, with a standard deviation of *weekly* demand of 100 units.

Current on-hand inventory is 1,040 units, with no scheduled receipts and no backorders.

**a.** Calculate the item's EOQ. What is the average time, in weeks, between orders?

**b.** Find the safety stock and reorder point that provide a 95 percent cycle-service level.

**c.** For these policies, what are the annual costs of (i) holding the cycle inventory and (ii) placing orders?

**d.** A withdrawal of 15 units just occurred. Is it time to reorder? If so, how much should be ordered?

**21.** A company begins a review of ordering policies for its continuous review system by checking the current policies for a sample of SKUs. Following are the characteristics of one item.

Demand $(D) = 64$ units/week (Assume 52 weeks per year)

Ordering or setup cost $(S) = \$50$/order

Holding cost $(H) = \$13$/unit/year

Lead time $(L) = 2$ weeks

Standard deviation of *weekly* demand $= 12$ units

Cycle-service level $= 88$ percent

**a.** What is the EOQ for this item?

**b.** What is the desired safety stock?

**c.** What is the reorder point?

**d.** What are the cost implications if the current policy for this item is $Q = 200$ and $R = 180$?

**22.** Osprey Sports stocks everything that a musky fisherman could want in the Great North Woods. A particular musky lure has been very popular with local fishermen as well as those who buy lures on the Internet from Osprey Sports. The cost to place orders with the supplier is \$30/order; the demand averages 4 lures per day, with a standard deviation of 1 lure; and the inventory holding cost is \$1.00/lure/year. The lead time from the supplier is 10 days, with a standard deviation of 3 days. It is important to maintain a 97 percent cycle-service level to properly balance service with inventory holding costs. Osprey Sports is open 350 days a year to allow the owners the opportunity to fish for muskies during the prime season. The owners want to use a continuous review inventory system for this item.

**a.** What order quantity should be used?

**b.** What reorder point should be used?

**c.** What is the total annual cost for this inventory system?

**23.** The Farmer's Wife is a country store specializing in knick-knacks suitable for a farm-house décor. One item experiencing a considerable buying frenzy is a miniature Holstein cow. Average weekly demand is 30 cows, with a standard deviation of 5 cows. The cost to place a replenishment order is \$15 and the holding cost is \$0.75/cow/year. The supplier, however, is in China. The lead time for new orders is 8 weeks, with a standard deviation of 2 weeks. The Farmer's Wife, which is open only 50 weeks a year, wants to develop a continuous review inventory system for this item with a cycle-service level of 90 percent.

**a.** Specify the continuous review system for the cows. Explain how it would work in practice.

**b.** What is the total annual cost for the system you developed?

**24.** (D) Muscle Bound is a chain of fitness stores located in many large shopping centers. Recently, an internal memo from the CEO to all operations personnel complained about the budget overruns at Muscle Bound's central warehouse. In particular, she said that inventories were too high and that the budget will be cut dramatically and proportionately equal for all items in stock. Consequently, warehouse management set up a pilot study to see what effect the budget cuts would have on customer service. They chose 5-pound barbells, which are a high volume SKU and consume considerable warehouse space. Daily demand for the barbells is 1,000 units, with a standard deviation of 150 units. Ordering costs are \$40 per order. Holding costs are \$2/unit/year. The supplier is located in the Philippines; consequently, the lead time is 35 days with a standard deviation of 5 days. Muscle Bound stores operate 313 days a year (no Sundays).

Suppose that the barbells are allocated a budget of \$16,000 for total annual costs. If Muscle Bound uses a continuous review system for the barbells and cannot change the ordering costs and holding costs or the distributions of demand or lead time, what is the best cycle-service level management can expect from their system?

*It may be helpful to review MyOMLab Supplement E, "Simulation," before working Problem 25.*

**25.** (D) The Georgia Lighting Center stocks more than 3,000 lighting fixtures, including chandeliers, swags, wall lamps, and track lights. The store sells at retail, operates 6 days per week, and advertises itself as the "brightest spot in town." One expensive fixture is selling at an average rate of 5 units per day. The reorder policy is $Q = 40$ and $R = 15$. A new order is placed on the day the reorder point is reached. The lead time is 3 business days. For example, an order placed on Monday will be delivered on Thursday. Simulate the performance of this $Q$ system for the next 3 weeks (18 work days). Any stockouts result in lost sales (rather than backorders). The beginning inventory is 19 units, and no receipts are scheduled. Table 9.2 simulates the first week of operation. Extend Table 9.2 to simulate operations for the next 2 weeks if demand for the next 12 business days is 7, 4, 2, 7, 3, 6, 10, 0, 5, 10, 4, and 7.

**a.** What is the average daily ending inventory over the 18 days? How many stockouts occurred?

**b.** Using the same beginning inventory and daily demand data, simulate the inventory performance of the same item assuming a $Q = 30$, $R = 20$ system is used. Calculate the average inventory level and number of stockouts and compare with part (a).

**TABLE 9.2** FIRST WEEK OF OPERATION

| Workday | Beginning Inventory | Orders Received | Daily Demand | Ending Inventory | Inventory Position | Order Quantity |
|---|---|---|---|---|---|---|
| 1. Monday | 19 | — | 5 | 14 | 14 | 40 |
| 2. Tuesday | 14 | — | 3 | 11 | 51 | — |
| 3. Wednesday | 11 | — | 4 | 7 | 47 | — |
| 4. Thursday | 7 | 40 | 1 | 46 | 46 | — |
| 5. Friday | 46 | — | 10 | 36 | 36 | — |
| 6. Saturday | 36 | — | 9 | 27 | 27 | — |

(D) = Difficult Problem

## Periodic Review System

**26.** Nationwide Auto Parts uses a periodic review inventory control system for one of its stock items. The review interval is 6 weeks, and the lead time for receiving the materials ordered from its wholesaler is 3 weeks. Weekly demand is normally distributed, with a mean of 100 units and a standard deviation of 20 units.

**a.** What is the average and the standard deviation of demand during the protection interval?

**b.** What should be the target inventory level if the firm desires 97.5 percent stockout protection?

**c.** If 350 units were in stock at the time of a periodic review, how many units should be ordered?

**27.** In a $P$ system, the lead time for a box of weed-killer is 2 weeks and the review period is 1 week. Demand during the protection interval averages 218 boxes, with a standard deviation of 40 boxes.

**a.** What is the cycle-service level when the target inventory is set at 300 boxes?

**b.** In the fall season, demand for weed-killer decreases but also becomes more highly variable. Assume that during the fall season, demand during the protection interval is expected to decrease to 180 boxes, but with a standard deviation of 50 boxes. What would be the cycle-service level if management keeps the target inventory level set at 300 boxes?

**28.** Suppose that Sam's Cat Hotel in Problem 10 uses a $P$ system instead of a $Q$ system. The average daily demand is $\bar{d} = 90/6 = 15$ bags and the standard deviation of *daily*

demand is $\sigma_d = \dfrac{\sigma_{week}}{\sqrt{6}} = (15/\sqrt{6}) = 6.124$ bags.

**a.** What $P$ (in working days) and $T$ should be used to approximate the cost trade-offs of the EOQ?

**b.** How much more safety stock is needed than with a $Q$ system?

**c.** It is time for the periodic review. How much kitty litter should be ordered?

**29.** Your firm uses a periodic review system for all SKUs classified, using ABC analysis, as B or C items. Further, it uses a continuous review system for all SKUs classified as A items. The demand for a specific SKU, currently classified as an A item, has been dropping. You have been asked to evaluate the impact of moving the item from continuous review to periodic review. Assume your firm operates 52 weeks per year; the item's current characteristics are:

Demand $(D) = 15{,}080$ units/year

Ordering cost $(S) = \$125.00$/order

Holding cost $(H) = \$3.00$/unit/year

Lead time $(L) = 5$ weeks

Cycle-service level $= 95$ percent

Demand is normally distributed, with a standard deviation of weekly demand of 64 units.

**a.** Calculate the item's EOQ.

**b.** Use the EOQ to define the parameters of an appropriate continuous review and periodic review system for this item.

**c.** Which system requires more safety stock and by how much?

**30.** Using the same information as in Problem 21, develop the best policies for a periodic review system.

**a.** What value of $P$ gives the same approximate number of orders per year as the EOQ? Round to the nearest week.

**b.** What safety stock and target inventory level provide an 88 percent cycle-service level?

**31.** Wood County Hospital consumes 1,000 boxes of bandages per week. The price of bandages is \$35 per box, and the hospital operates 52 weeks per year. The cost of processing an order is \$15, and the cost of holding one box for a year is 15 percent of the value of the material.

**a.** The hospital orders bandages in lot sizes of 900 boxes. What *extra* cost does the hospital incur, which it could save by using the EOQ method?

**b.** Demand is normally distributed, with a standard deviation of weekly demand of 100 boxes. The lead time is 2 weeks. What safety stock is necessary if the hospital uses a continuous review system and a 97 percent cycle-service level is desired? What should be the reorder point?

**c.** If the hospital uses a periodic review system, with $P = 2$ weeks, what should be the target inventory level, $T$?

**32.** A golf specialty wholesaler operates 50 weeks per year. Management is trying to determine an inventory policy for its 1-irons, which have the following characteristics:

Demand $(D) = 2{,}000$ units/year

Demand is normally distributed

Standard deviation of *weekly* demand $= 3$ units

Ordering cost $= \$40$/order

Annual holding cost $(H) = \$5$/units

Desired cycle-service level $= 90$ percent

Lead time $(L) = 4$ weeks

**a.** If the company uses a periodic review system, what should $P$ and $T$ be? Round $P$ to the nearest week.

**b.** If the company uses a continuous review system, what should $R$ be?

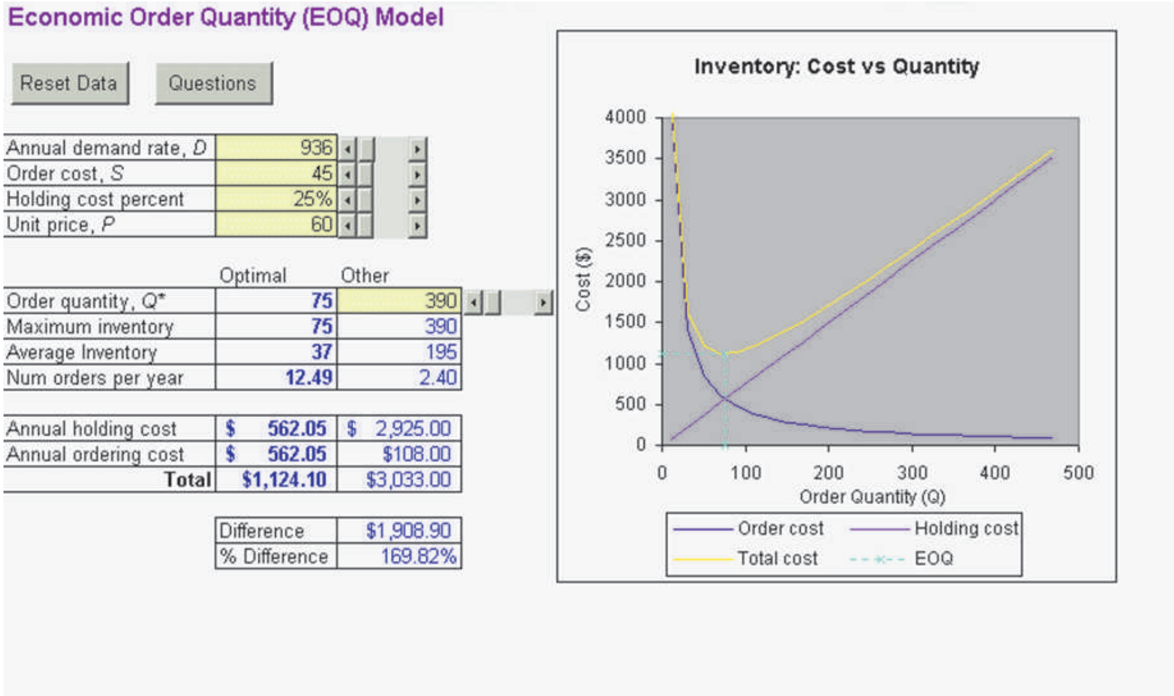
## Active Model Exercise

Active Model 9.1, "Economic Order Quantity," appears in MyOMLab. It allows you to evaluate the sensitivity of the EOQ and associated costs to changes in the demand and cost parameters.

### QUESTIONS

1. What is the EOQ and what is the lowest total cost?
2. What is the annual cost of holding inventory at the EOQ and the annual cost of ordering inventory at the EOQ?

3. From the graph, what can you conclude about the relationship between the lowest total cost and the costs of ordering and holding inventory?
4. How much does the total cost increase if the store manager orders twice as many bird feeders as the EOQ? How much does the total cost increase if the store manager orders half as many bird feeders as the EOQ?
5. What happens to the EOQ and the total cost when demand is doubled? What happens to the EOQ and the total cost when unit price is doubled?
6. Scroll through the lower order cost values and describe the changes to the graph. What happens to the EOQ?
7. Comment on the sensitivity of the EOQ model to errors in demand or cost estimates.




# VIDEO CASE Inventory Management at Crayola

Managing inventory at Crayola is a fine balancing act. With the back-to-school period driving 42% of company demand for crayons, markers, paints, modeling compounds and other products, production starts in February so enough finished goods are in the 800,000 square foot warehouse in time to supply 3,600 Walmarts, 1,400 Targets, and thousands of other retailers in the United States for the fall school supply rush.


Pearson

Crayola must supply customers with nearly 1,500 products, which requires an average inventory investment of $110 million. Finished goods inventory, shown here, must be stored in advance of seasonal demand peaks, such as the back-to-school period, which accounts for 42 percent of annual demand.

This means demand forecasts for raw materials in the master production schedule must be developed months before any of the finished products move to those retail customers. Lead times range from 60 days for domestic raw materials sources to upwards of 90 days for finished goods from suppliers outside the United States. As production ramps up for the back-to-school season well before the first day of classes, Crayola plans inventory levels for the entire year so that production remains reasonably steady. While the back-to-school season represents the lion's share of annual sales, holiday sales account for 35% of revenues, and the rest comes from spring sales. Crayola has over 1,500 SKUs, with close to 225 top sellers, so accurate forecasts are essential.

Historical sales patterns as well as orders generated by its U.S. sales divisions located in Easton, Pennsylvania (headquarters), Bentonville, Arkansas (near Walmart's headquarters), and Minneapolis (near Target's headquarters) help managers attain the accuracy needed. Marketing co-branding for the latest movies and comic books plays a role in creating the forecast for new SKUs and bundles, which must be coordinated to hit retailers the same time the movies and comics debut or the company risks missing the market and ending up with inventory that can't easily be sold.

Crayola's inventory holding costs run about 25%, and its average inventory value is $110 million. The company must assure there is warehouse space for finished goods as well as raw materials used in production. Pigments, clays, and packaging materials are moved from the warehouse and positioned close to the production lines, using a Kanban system to pull raw materials inventory as needed. Rail tanker cars from Louisiana and Pennsylvania carrying paraffin wax are delivered twice a week for crayon production. Since the rail cars feed directly into production, any disruption in delivery has the potential for shutting down production. Bad weather is a particular risk in this part of the company's supply chain since it can prevent the transport of goods during hurricanes or snowstorms.

Crayola attempts to source as many raw materials from domestic sources as possible. Cartons, clay, ink, labels and corrugated boxes come from the mid-Atlantic region of the United States, while those plastic components Crayola does not manufacture on site, such as nibs for markers, are sourced from Asia and can take up to 120 days to ship through the Panama Canal to the Port of Newark. Materials used in kits and bundles come from Korea, China, Vietnam and Brazil, and face similar shipping logistics.

When considering work-in-process inventories at Crayola, paints, markers, modeling clays, and many of the crayons coming off the production line are boxed into trays for use downstream in creating kits and bundles. These items are considered work-in-process items, even though the individual units are finished goods (i.e., a crayon or marker is completely manufactured once it comes off the line). The same is true for marker barrels, paint pots and other plastics. Specialized equipment is used to make these items which feed downstream production.

Recently, Crayola's leadership expected that actual demand for its popular Marker Maker© toy product might come in higher than the original forecast. As a countermeasure, Crayola established duplicate capacities in China and the U.S. to meet the aggregate potential demand. In China, the company produced the original forecast and delivered to customers as planned. However, when the actual demand was 26% over the original forecast, Crayola could meet the surge in demand because it had positioned the long lead time ink bottles in its Pennsylvania plants and was able to mold the plastic parts using marker components from its core marker product. By utilizing existing machine capacity in its plants, reducing the lead time of ink bottles by making them in Pennsylvania, and by duplicating tooling, Crayola was able to ensure that its customers and consumers were satisfied during the holiday season.

### QUESTIONS

1. Consider the pressures for small vs. large inventories. Which situation does Crayola seem to fit, and why?
2. Explain how both independent and dependent demand items are present at Crayola.
3. The Marker Maker© product recently experienced an unexpected surge in demand and the supply chain's agility was credited with helping to meet the crisis. We have discussed four ways to classify operational inventories by how they are created. Regarding the ways managers can use these inventories to satisfy demand, explain how Crayola can achieve the flexibility to adjust to unexpected demand surges.

## EXPERIENTIAL LEARNING Swift Electronic Supply, Inc.

It was a typical fall afternoon in Southern California, with thousands of tourists headed to the beaches to have fun. About 40 miles away, however, Steven Holland, the CEO of Swift Electronic Supply, Inc., faced a severe problem with Swift's inventory management.

An Intel veteran, Steven Holland worked in the electronic components distribution industry for more than 20 years. Seven years ago, he founded Swift Electronic Supply, Inc., an electronic distributor. After several successful years, the company is now troubled with eroding profit margins. Recent economic downturns further worsened the situation. Factors such as the growth of B2B e-commerce, the globalization of markets, the increased popularity of value-added services, and ongoing consolidations among electronic distributors affect the future of Swift.

To reverse these influences, Holland talked to a prestigious local university. After consultation, Holland found the most effective way to increase profitability is to cut inventory costs. As a starting point, he studied in detail a representative product, dynamic random access memory (DRAM), as the basis for his plan.

### Industry and Company Preview

Owing to a boom in the telecommunications industry and the information technology revolution, electronics distributors experienced double-digit annual growth over the last decade. To cut the cost of direct purchasing forces, large component manufacturers such as Intel, Cisco, and Texas Instruments decided to outsource their procurement so that they could focus on product development and manufacturing. Therefore, independent electronic distributors like Swift started offering procurement services to these companies.

Swift serves component manufacturers in California and Arizona. Working as the intermediary between its customers and overseas original equipment manufacturers (OEMs), Swift's business model is quite simple. Forecasting customer demand, Swift places orders to a number of OEMs, stocks those products, breaks the quantities down, and delivers the products to its end customers.

Recently, due to more intense competition and declines in demand, Swift offered more flexible delivery schedules and was willing to accommodate small order quantities. However, customers can always shift to Swift's competitors should Swift not fulfill their orders. Steven Holland was in a dilemma: The intangible costs of losing customers can be enormous; however, maintaining high levels of inventory can also be costly.

### Dram

Holland turned his attention to DRAM as a representative product. Previously, the company ordered a large amount every time it felt it was necessary. Holland's assistant developed a table (Table 9.3) that has 2 months of demand history. From Holland's experience, the demand for DRAM is relatively stable in the company's product line and it had no sales seasonality. The sales staff agrees that conditions in the current year will not be different from those of past years, and historical demand will be a good indicator of what to expect in the future.

The primary manufacturers of DRAM are those in Southeast Asia. Currently, Swift can purchase one unit of 128M DRAM for $10. After negotiation with a reputable supplier, Holland managed to sign a long-term agreement, which kept the price at $10 and allowed Swift to place orders at any time. The supplier also supplies other items in Swift's inventory. In addition, it takes the supplier of the DRAM 2 days to deliver the goods to Swift's warehouse using air carriers.

When Swift does not have enough inventory to fill a customer's order, the sales are lost; that is, Swift is not able to backorder the shortage because its customers fill their requirements through competitors. The customers will accept partial shipments, however.

It costs Swift $200 to place an order with the suppliers. This amount covers the corresponding internal ordering costs and the costs of delivering the products to the company. Holland estimates that the cost of lost sales amounts to $2 per unit of DRAM. This rough estimate includes the loss of profits, as well as the intangible damage to customer goodwill.

To simplify its inventory management system, Swift has a policy of maintaining a cycle-service level of 95 percent. The holding cost per day per unit is estimated to be 0.5 percent of the cost of goods, regardless of the product. Inventory holding costs are calculated on the basis of the ending inventory each day. The current balance is 1,700 units of DRAM in stock.

The daily purchasing routine is as follows. Orders are placed at the *beginning* of the day, before Swift is open for customer business. The orders arrive at the beginning of the day, 2 days later, and can be used for sales that day. For example, an order placed at the beginning of day 1 will arrive at Swift before Swift is open for business on day 3. The actual daily demand is always recorded at the *end* of the day, after Swift has closed for customer business. All cost computations are done at the end of the day after the total demand has been recorded.

**TABLE 9.3** | **HISTORICAL DEMAND DATA FOR THE DRAM (UNITS)**

| Day | Demand | Day | Demand | Day | Demand |
|---|---|---|---|---|---|
| 1 | 869 | 21 | 663 | 41 | 959 |
| 2 | 902 | 22 | 1,146 | 42 | 703 |
| 3 | 1,109 | 23 | 1,016 | 43 | 823 |
| 4 | 947 | 24 | 1,166 | 44 | 862 |
| 5 | 968 | 25 | 829 | 45 | 966 |
| 6 | 917 | 26 | 723 | 46 | 1,042 |
| 7 | 1,069 | 27 | 749 | 47 | 889 |
| 8 | 1,086 | 28 | 766 | 48 | 1,002 |
| 9 | 1,066 | 29 | 996 | 49 | 763 |
| 10 | 929 | 30 | 1,122 | 50 | 932 |
| 11 | 1,022 | 31 | 962 | 51 | 1,052 |
| 12 | 959 | 32 | 829 | 52 | 1,062 |
| 13 | 756 | 33 | 862 | 53 | 989 |
| 14 | 882 | 34 | 793 | 54 | 1,029 |
| 15 | 829 | 35 | 1,039 | 55 | 823 |
| 16 | 726 | 36 | 1,009 | 56 | 942 |
| 17 | 666 | 37 | 979 | 57 | 986 |
| 18 | 879 | 38 | 976 | 58 | 736 |
| 19 | 1,086 | 39 | 856 | 59 | 1,009 |
| 20 | 992 | 40 | 1,036 | 60 | 852 |

### Simulation

Holland believes that simulation is a useful approach to assess various inventory control alternatives. The historical data from Table 9.3 could be used to develop attractive inventory policies. The table was developed to record various costs and evaluate different alternatives. An example showing some recent DRAM inventory decisions is shown in Table 9.4.

1. Design a new inventory system for Swift Electronic Supply, Inc., using the data provided.
2. Provide the rationale for your system, which should include the decision rules you would follow to determine how much to order and when.
3. Simulate the use of your inventory system and record the costs. Develop a table such as Table 9.4 to record your results. Your instructor will provide actual demands on a day-to-day basis during the simulation.

**TABLE 9.4 | EXAMPLE SIMULATION**

| Day | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Beginning inventory position | 1,700 | 831 | 1,500 | 391 | 3,000 | 3,232 | 2,315 | | | |
| Number ordered | 1,500 | | 3,000 | 1,200 | | | 1,900 | | | |
| Daily demand | 869 | 902 | 1,109 | 947 | 968 | 917 | 1,069 | | | |
| Day-ending inventory | 831 | –71 | 391 | –556 | 2,032 | 2,315 | 1,246 | | | |
| Ordering costs ($200 per order) | 200 | | 200 | 200 | | | 200 | | | |
| Holding costs ($0.05 per piece per day) | 41.55 | 0.00 | 19.55 | 0.00 | 101.60 | 115.75 | 62.30 | | | |
| Shortage costs ($2 per piece) | 0 | 142 | 0 | 1,112 | 0 | 0 | 0 | | | |
| Total cost for day | 241.55 | 142.00 | 219.55 | 1,312.00 | 101.60 | 115.75 | 262.30 | | | |
| Cumulative cost from last day | 0.00 | 241.55 | 383.55 | 603.10 | 1,915.10 | 2,016.70 | 2,132.45 | | | |
| Cumulative costs to date | 241.55 | 383.55 | 603.10 | 1,915.10 | 2,016.70 | 2,132.45 | 2,394.75 | | | |

## CASE Parts Emporium

Parts Emporium, Inc., is a wholesale distributor of automobile parts formed by two disenchanted auto mechanics, Dan Block and Ed Spriggs. Originally located in Block's garage, the firm showed slow but steady growth for 7 years before it relocated to an old, abandoned meat-packing warehouse on Chicago's South Side. With increased space for inventory storage, the company was able to begin offering an expanded line of auto parts. This increased selection, combined with the trend toward longer car ownership, led to an explosive growth of the business. Fifteen years later, Parts Emporium was the largest independent distributor of auto parts in the north central region.

Recently, Parts Emporium relocated to a sparkling new office and warehouse complex off Interstate 55 in suburban Chicago. The warehouse space alone occupied more than 100,000 square feet. Although only a handful of new products have been added since the warehouse was constructed, its utilization increased from 65 percent to more than 90 percent of capacity. During this same period, however, sales growth stagnated. These conditions motivated Block and Spriggs to hire the first manager from outside the company in the firm's history.

It is June 6, Sue McCaskey's first day in the newly created position of materials manager for Parts Emporium. A recent graduate of a prominent business school, McCaskey is eagerly awaiting her first real-world problem. At approximately 8:30 A.M., it arrives in the form of status reports on inventory and orders shipped. At the top of an extensive computer printout is a handwritten note from Joe Donnell, the purchasing manager: "Attached you will find the inventory and customer service performance data. Rest assured that the individual inventory levels are accurate because we took a complete physical inventory count at the end of last week. Unfortunately, we do not keep compiled records in some of the areas as you requested. However, you are welcome to do so yourself. Welcome aboard!"

A little upset that aggregate information is not available, McCaskey decides to randomly select a small sample of approximately 100 items and compile inventory and customer service characteristics to get a feel for the "total picture." The results of this experiment reveal to her why Parts Emporium decided to create the position she now fills. It seems that the inventory is in all the wrong places. Although an *average* of approximately 60 days of inventory is on hand, the firm's customer service is inadequate. Parts Emporium tries to backorder the customer orders not immediately filled from stock, but some 10 percent of demand is being lost to competing distributorships. Because stockouts are costly, relative to inventory holding costs, McCaskey believes that a cycle-service level of at least 95 percent should be achieved.

McCaskey knows that although her influence to initiate changes will be limited, she must produce positive results immediately. Thus, she decides to concentrate on two products from the extensive product line: the EG151 exhaust gasket and the DB032 drive belt. If she can demonstrate significant gains from proper inventory management for just two products, perhaps Block and Spriggs will give her the backing needed to change the total inventory management system.

The EG151 exhaust gasket is purchased from an overseas supplier, Haipei, Inc. Actual demand for the first 21 weeks of this year is shown in the following table:

| Week | Actual Demand | Week | Actual Demand |
|---|---|---|---|
| 1 | 104 | 12 | 97 |
| 2 | 103 | 13 | 99 |
| 3 | 107 | 14 | 102 |
| 4 | 105 | 15 | 99 |
| 5 | 102 | 16 | 103 |
| 6 | 102 | 17 | 101 |
| 7 | 101 | 18 | 101 |
| 8 | 104 | 19 | 104 |
| 9 | 100 | 20 | 108 |
| 10 | 100 | 21 | 97 |
| 11 | 103 | | |

A quick review of past orders, shown in another document, indicates that a lot size of 150 units is being used and that the lead time from Haipei is fairly constant at 2 weeks. Currently, at the end of week 21, no inventory is on hand, 11 units are backordered, and the company is awaiting a scheduled receipt of 150 units.

The DB032 drive belt is purchased from the Bendox Corporation of Grand Rapids, Michigan. Actual demand so far this year is shown in the following table:

| Week | Actual Demand | Week | Actual Demand |
|---|---|---|---|
| 11 | 18 | 17 | 50 |
| 12 | 33 | 18 | 53 |
| 13 | 53 | 19 | 54 |
| 14 | 54 | 20 | 49 |
| 15 | 51 | 21 | 52 |
| 16 | 53 | | |

Because this product is new, data are available only since its introduction in week 11. Currently, 324 units are on hand, with no backorders and no scheduled receipts. A lot size of 1,000 units is being used, with the lead time fairly constant at 3 weeks.

The wholesale prices that Parts Emporium charges its customers are \$12.99 for the EG151 exhaust gasket and \$8.89 for the DB032 drive belt. Because no quantity discounts are offered on these two highly profitable items, gross margins based on current purchasing practices are 32 percent of the wholesale price for the exhaust gasket and 48 percent of the wholesale price for the drive belt.

Parts Emporium estimates its cost to hold inventory at 21 percent of its inventory investment. This percentage recognizes the opportunity cost of tying money up in inventory and the variable costs of taxes, insurance, and shrinkage. The annual report notes other warehousing expenditures for utilities and maintenance and debt service on the 100,000-square-foot warehouse, which was built for \$1.5 million. However, McCaskey reasons that these warehousing costs can be ignored because they will not change for the range of inventory policies that she is considering.

Out-of-pocket costs for Parts Emporium to place an order with suppliers are estimated to be \$20 per order for exhaust gaskets and \$10 per order for drive belts. On the outbound side, the company can charge a delivery fee. Although most customers pick up their parts at Parts Emporium, some orders are delivered to customers. To provide this service, Parts Emporium contracts with a local company for a flat fee of \$21.40 per order, which is added to the customer's bill. McCaskey is unsure whether to increase the ordering costs for Parts Emporium to include delivery charges.

## QUESTIONS

1. Put yourself in Sue McCaskey's position and prepare a detailed report to Dan Block and Ed Spriggs on managing the inventory of the EG151 exhaust gasket and the DB032 drive belt. Be sure to present a proper inventory system and recognize all relevant costs.
2. By how much do your recommendations for these two items reduce annual cycle inventory, stockout, and ordering costs?

SUPPLEMENT

# C

# SPECIAL INVENTORY MODELS

Many real world problems require relaxation of certain assumptions on which the economic order quantity (EOQ) model is based. This supplement addresses three realistic situations that require going beyond the simple EOQ formulation.

1. Noninstantaneous Replenishment. Particularly in situations in which manufacturers use a continuous process to make a primary material, such as a liquid, gas, or powder, production is not instantaneous. Thus, inventory is replenished gradually, rather than in lots.
2. Quantity Discounts. Three annual costs are (1) the inventory holding cost, (2) the fixed cost for ordering and setup, and (3) the cost of materials. For service providers and for manufacturers alike, the unit cost of purchased materials sometimes depends on the order quantity.
3. One-Period Decisions. Retailers and manufacturers of fashion goods often face a situation in which demand is uncertain and occurs during just one period or season.

This supplement assumes you have read Chapter 9, "Managing Inventories," and Supplement A, "Decision Making Models."

## LEARNING GOALS *After reading this supplement, you should be able to:*

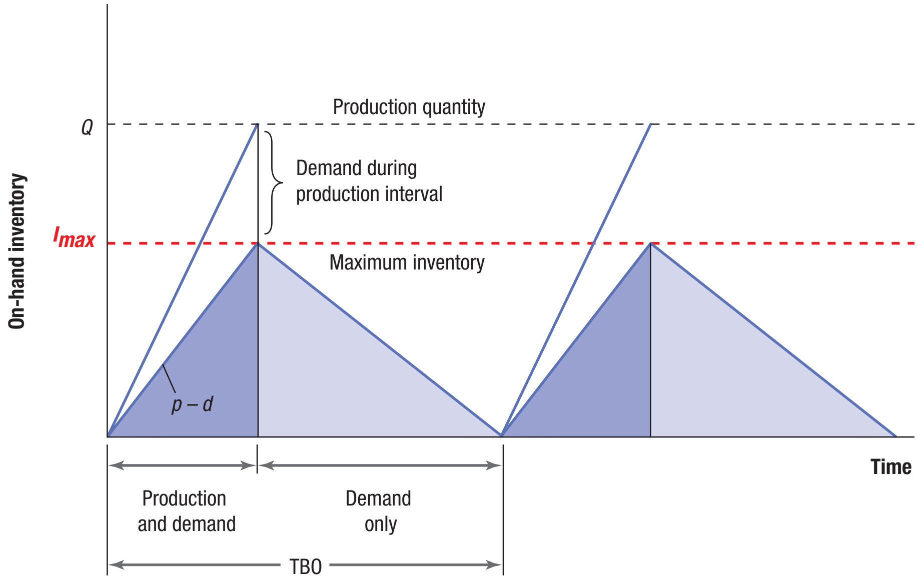
1. Calculate the optimal lot size when replenishment is not instantaneous.
2. Determine the optimal order quantity when materials are subject to quantity discounts.
3. Calculate the order quantity that maximizes the expected profits for a one-period inventory decision.

## Noninstantaneous Replenishment

If an item is being produced internally rather than purchased, finished units may be used or sold as soon as they are completed, without waiting until a full lot is completed. For example, a restaurant that bakes its own dinner rolls begins to use some of the rolls from the first pan even before the baker finishes a five-pan batch. The inventory of rolls never reaches the full five-pan level, the way it would if the rolls all arrived at once on a truck sent by a supplier.

Figure C.1 depicts the usual case, in which the production rate, $p$, *exceeds* the demand rate, $d$. If demand and production were equal, manufacturing would be continuous with no buildup of cycle inventory. If the production rate is lower than the demand rate, sales opportunities are being missed on an ongoing basis. We assume that $p > d$ in this supplement.

**FIGURE C.1 ▶**
Lot Sizing with Noninstantaneous Replenishment





Ulrich Mueller/Shutterstock.com

This chemical plant stores its products in stainless steel silos. The production of each product is scheduled to start when its silo is nearly empty.

Cycle inventory accumulates faster than demand occurs; that is, a buildup of $p - d$ units occurs per time period. For example, if the production rate is 100 units per day and the demand is 5 units per day, the buildup is 95 (or $100 - 5$) units each day. This buildup continues until the lot size, $Q$, has been produced, after which the inventory depletes at a rate of 5 units per day. Just as the inventory reaches 0, the next production interval begins. To be consistent, both $p$ and $d$ must be expressed in units of the same time period, such as units per day or units per week. Here, we assume that they are expressed in units per day.

The $p - d$ buildup continues for $Q/p$ days because $Q$ is the lot size and $p$ units are produced each day. In our example, if the lot size is 300 units, the production interval is 3 days (300/100). For the given rate of buildup over the production interval, the maximum cycle inventory, $I_{\max}$, is

$$I_{\max} = \frac{Q}{p}(p - d) = Q\left(\frac{p - d}{p}\right)$$

Cycle inventory is no longer $Q/2$, as it was with the basic EOQ method; instead, it is $I_{\max}/2$. Setting up the total annual cost equation for this production situation, where $D$ is annual demand, and as before $d$ is daily demand, we get

$$\text{Total annual cost} = \text{Annual holding cost} + \text{Annual ordering or setup cost}$$

$$C = \frac{I_{\max}}{2}(H) + \frac{D}{Q}(S) = \frac{Q}{2}\left(\frac{p - d}{p}\right)(H) + \frac{D}{Q}(S)$$

**economic production lot size (ELS)**

The optimal lot size in a situation in which replenishment is not instantaneous.

Based on this cost function, the optimal lot size, often called the **economic production lot size (ELS)**, is

$$\text{ELS} = \sqrt{\frac{2DS}{H}}\sqrt{\frac{p}{p - d}}$$

Because the second term is a ratio greater than 1, the ELS results in a larger lot size than the EOQ.

## EXAMPLE C.1 Finding the Economic Production Lot Size

A plant manager of a chemical plant must determine the lot size for a particular chemical that has a steady demand of 30 barrels per day. The production rate is 190 barrels per day, annual demand is 10,500 barrels, setup cost is $200, annual holding cost is $0.21 per barrel, and the plant operates 350 days per year.

**a.** Determine the economic production lot size (ELS).

**b.** Determine the total annual setup and inventory holding cost for this item.

**c.** Determine the time between orders (TBO), or cycle length, for the ELS.

**d.** Determine the production time per lot.

What are the advantages of reducing the setup time by 10 percent?

MyOMLab

Tutor C.1 in MyOMLab provides a new example to determine the ELS.

MyOMLab

Active Model C.1 in MyOMLab provides additional insight on the ELS model and its uses.

### SOLUTION

**a.** Solving first for the ELS, we get

$$\text{ELS} = \sqrt{\frac{2DS}{H}}\sqrt{\frac{p}{p-d}} = \sqrt{\frac{2(10{,}500)(\$200)}{\$0.21}}\sqrt{\frac{190}{190-30}}$$

$$= 4{,}873.4 \text{ barrels}$$

**b.** The total annual cost with the ELS is

$$C = \frac{Q}{2}\left(\frac{p-d}{p}\right)(H) + \frac{D}{Q}(S)$$

$$= \frac{4{,}873.4}{2}\left(\frac{190-30}{190}\right)(\$0.21) + \frac{10{,}500}{4{,}873.4}(\$200)$$

$$= \$430.91 + \$430.91 = \$861.82$$

**c.** Applying the TBO formula to the ELS, we get

$$\text{TBO}_{\text{ELS}} = \frac{\text{ELS}}{D}(350 \text{ days/year}) = \frac{4{,}873.4}{10{,}500}(350)$$

$$= 162.4, \quad \text{or} \quad 162 \text{ days}$$

**d.** The production time during each cycle is the lot size divided by the production rate:

$$\frac{\text{ELS}}{p} = \frac{4{,}873.4}{190} = 25.6, \quad \text{or} \quad 26 \text{ days}$$

### DECISION POINT

As OM Explorer shows in Figure C.2, the net effect of reducing the setup cost by 10 percent is to reduce the lot size, the time between orders, and the production cycle time. Consequently, total annual costs are also reduced. This adds flexibility to the manufacturing process because items can be made more quickly with less expense. Management must decide whether the added cost of improving the setup process is worth the added flexibility and inventory cost reductions.

| | |
|---|---|
| Period Used in Calculations | Day |
| Demand per Day | 30 |
| Production Rate/Day | 190 |
| Annual Demand | 10,500 |
| Setup Cost | $180 |
| Annual Holding Cost ($) | $0.21 |
| Operating Days per Year | 350 |
| Economic Lot Size (ELS) | 4,623 |
| Annual Total Cost | $817.60 |
| Time Between Orders (days) | 154.1 |
| Production Time | 24.3 |

Enter Holding Cost Manually    Holding Cost As % of Value

**FIGURE C.2**
*OM Explorer* Solver for the Economic Production Lot Size Showing the Effect of a 10 Percent Reduction in Setup Cost

Corbis Premium RF/Alamy

Many hospitals join cooperatives (or co-ops) to gain the clout needed to garner price discounts from suppliers. Here a hospital pharmacist checks inventory records of supplies in preparation for placing an order.

## Quantity Discounts

Quantity discounts, which are price incentives to purchase large quantities, create pressure to maintain a large inventory. For example, a supplier may offer a price of \$4.00 per unit for orders between 1 and 99 units, a price of \$3.50 per unit for orders between 100 and 199 units, and a price of \$3.00 per unit for orders of 200 or more units. The item's price is no longer fixed, as assumed in the EOQ derivation; instead, if the order quantity is increased enough, the price is discounted. Hence, a new approach is needed to find the best lot size—one that balances the advantages of lower prices for purchased materials and fewer orders (which are benefits of large order quantities) against the disadvantage of the increased cost of holding more inventory.

The total annual cost now includes not only the holding cost, $(Q/2)(H)$, and the ordering cost, $(D/Q)(S)$, but also the cost of purchased materials. For any per-unit price level, $P$, the total cost is

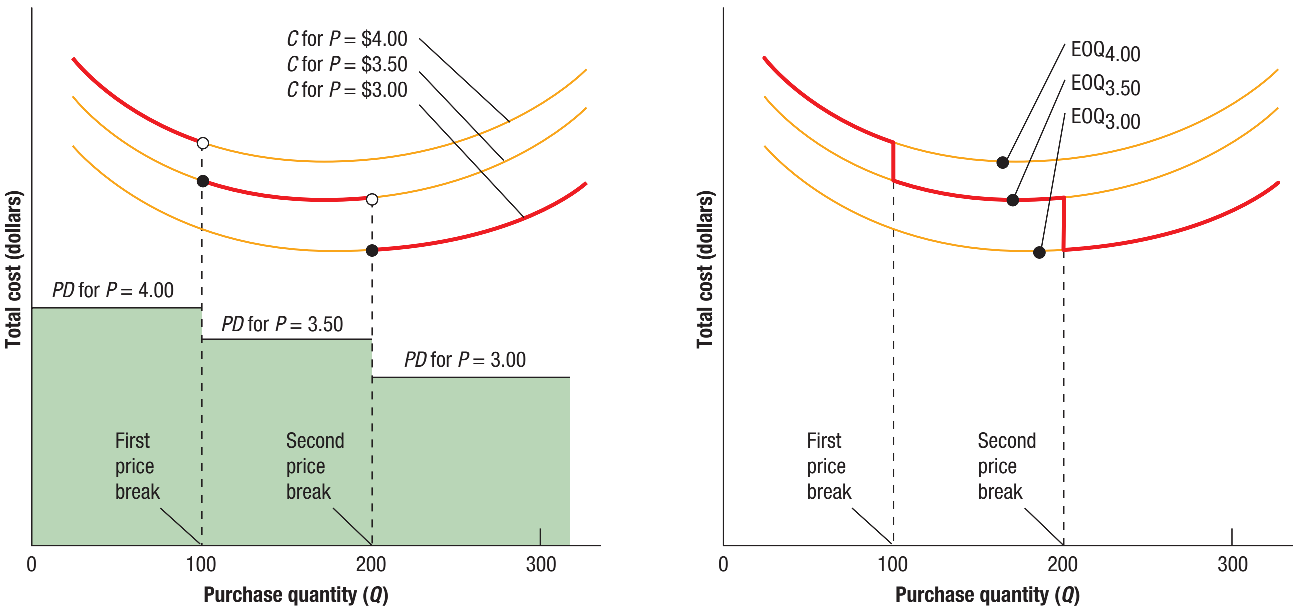
$$\text{Total annual cost} = \text{Annual holding cost} + \text{Annual ordering or setup cost} + \text{Annual cost of materials}$$

$$C = \frac{Q}{2}(H) + \frac{D}{Q}(S) + PD$$

The unit holding cost, $H$, usually is expressed as a percent of the unit price because the more valuable the item held in inventory, the higher is the holding cost. Thus, the lower the unit price, $P$, the lower is $H$. Conversely, the higher $P$ is, the higher is $H$.

The total cost equation yields U-shaped total cost curves. Adding the annual cost of materials to the total cost equation raises each total cost curve by a fixed amount, as shown in Figure C.3(a). The three cost curves illustrate each of the price levels. The top curve applies when no discounts are received; the lower curves reflect the discounted price levels. No single curve is relevant to all purchase quantities. The relevant, or feasible, total cost begins with the top curve, then drops down, curve by curve, at the price breaks. A price break is the minimum quantity needed to get a discount. In Figure C.3(a), two price breaks occur at $Q = 100$ and $Q = 200$. The result is a total cost curve, with steps at the price breaks.

**FIGURE C.3**
Total Cost Curves with Quantity Discounts




(a) Total cost curves with purchased materials added

(b) EOQs and price break quantities

Figure C.3(b) also shows three additional points—the minimum point on each curve—obtained with the EOQ formula at each price level. These EOQs do not necessarily produce the best lot size for two reasons.

1. The EOQ at a particular price level may not be feasible. The lot size may not lie in the range corresponding to its per-unit price. Figure C.3(b) illustrates two instances of an infeasible EOQ. First, the minimum point for the \$3.00 curve appears to be fewer than 200 units. However, the supplier's quantity discount schedule does not allow purchases of that small a quantity at the \$3.00 unit price. Similarly, the EOQ for the \$4.00 price level is greater than the first price break, so the price charged would be only \$3.50.
2. The EOQ at a particular price level may be feasible but may not be the best lot size. The feasible EOQ may have a higher cost than is achieved by the EOQ or price break quantity on a lower price curve. In Figure C.3(b), for example, the 200-unit price break quantity for the \$3.00 price level has a lower total cost than the feasible EOQ for the \$3.50 price level. A feasible EOQ is always better than any feasible point on cost curves with higher price levels, but not necessarily those with lower levels. Thus, the only time we can immediately conclude, without comparing total costs, that a feasible EOQ is the best order quantity is when it is on the curve for the lowest price level. This conclusion is not possible in Figure C.3(b) because the only feasible EOQ is at the middle price level, $P =$ \$3.50.

We must, therefore, pay attention only to feasible price–quantity combinations, shown as solid lines in Figure C.3(b), as we search for the best lot size. The following two-step procedure may be used to find the best lot size.

*Step 1.* Beginning with the lowest price, calculate the EOQ for each price level until a feasible EOQ is found. It is feasible if it lies in the range corresponding to its price. Each subsequent EOQ is smaller than the previous one because $P$, and thus $H$, gets larger and because the larger $H$ is in the denominator of the EOQ formula.

*Step 2.* If the first feasible EOQ found is for the lowest price level, this quantity is the best lot size. Otherwise, calculate the total cost for the first feasible EOQ and for the larger price break quantity at each lower price level. The quantity with the lowest total cost is optimal.

## EXAMPLE C.2 Finding Q with Quantity Discounts at St. LeRoy Hospital

A supplier for St. LeRoy Hospital has introduced quantity discounts to encourage larger order quantities of a special catheter. The price schedule is

| Order Quantity | Price per Unit |
|---|---|
| 0 to 299 | \$60.00 |
| 300 to 499 | \$58.80 |
| 500 or more | \$57.00 |

**MyOMLab** Tutor C.2 in MyOMLab provides a new example for choosing the best order quantity when discounts are available.

**MyOMLab** Active Model C.2 in MyOMLab provides additional insight on the quantity discount model and its uses.

The hospital estimates that its annual demand for this item is 936 units, its ordering cost is \$45.00 per order, and its annual holding cost is 25 percent of the catheter's unit price. What quantity of this catheter should the hospital order to minimize total costs? Suppose the price for quantities between 300 and 499 is reduced to \$58.00. Should the order quantity change?

### SOLUTION

**Step 1.** Find the first feasible EOQ, starting with the lowest price level:

$$\text{EOQ}_{57.00} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(936)(\$45.00)}{0.25(\$57.00)}} = 77 \text{ units}$$

A 77-unit order actually costs \$60.00 per unit, instead of the \$57.00 per unit used in the EOQ calculation, so this EOQ is infeasible. Now, try the \$58.80 level:

$$\text{EOQ}_{58.80} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(936)(\$45.00)}{0.25(\$58.80)}} = 76 \text{ units}$$

This quantity also is infeasible because a 76-unit order is too small to qualify for the \$58.80 price. Try the highest price level:

$$EOQ_{60.00} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(936)(45.00)}{0.25(60.00)}} = 75 \text{ units}$$

This quantity is feasible because it lies in the range corresponding to its price, $P = \$60.00$.

**Step 2.** The first feasible EOQ of 75 does not correspond to the lowest price level. Hence, we must compare its total cost with the price break quantities (300 and 500 units) at the lower price levels (\$58.80 and \$57.00):

$$C = \frac{Q}{2}(H) + \frac{D}{Q}(S) + PD$$

$$C_{75} = \frac{75}{2}[(0.25)(\$60.00)] + \frac{936}{75}(\$45.00) + \$60.00(936) = \$57{,}284$$

$$C_{300} = \frac{300}{2}[(0.25)(\$58.80)] + \frac{936}{300}(\$45.00) + \$58.80(936) = \$57{,}382$$

$$C_{500} = \frac{500}{2}[(0.25)(\$57.00)] + \frac{936}{500}(\$45.00) + \$57.00(936) = \$56{,}999$$

The best purchase quantity is 500 units, which qualifies for the deepest discount.

### DECISION POINT

If the price per unit for the range of 300 to 499 units is reduced to \$58.00, the best decision is to order 300 catheters, as shown by OM Explorer in Figure C.4. This result shows that the decision is sensitive to the price schedule. A reduction of slightly more than 1 percent is enough to make the difference in this example. In general, however, it is not always the case that you should order more than the economic order quantity when given price discounts. When discounts are small, holding cost $H$ is large, and demand $D$ is small; small lot sizes are better even though price discounts are foregone.

**FIGURE C.4 ▶** *OM Explorer* Solver for Quantity Discounts Showing the Best Order Quantity

More | Fewer

| Min. Amount Req'd for Price Point | Lot Sizes | Price/Unit |
|---|---|---|
| | 0–299 | \$60.00 |
| 300 | 300–499 | \$58.00 |
| 500 | 500 or more | \$57.00 |

| | |
|---|---|
| Annual Demand | 936 |
| Order Cost | \$45 |
| Holding Cost (% or price) | 25% |
| Best Order Quantity | 300 |

| | Price Point | EOQ or Req'd Order for Price Point | Inventory Cost | Order Cost | Purchase Cost | Total Cost | |
|---|---|---|---|---|---|---|---|
| | \$60.00 | 75 | \$562.50 | \$561.60 | \$56,160 | \$57,284 | |
| >> | \$58.00 | 300 | \$2,175 | \$140.40 | \$54,288 | \$56,603 | << |
| | \$57.00 | 500 | \$3,563 | \$84.24 | \$53,352 | \$56,999 | |

# One-Period Decisions

One of the dilemmas facing many retailers is how to handle seasonal goods, such as winter coats. Often, they cannot be sold at full markup the next year because of changes in styles. Furthermore, the lead time can be longer than the selling season, allowing no second chance to rush through another order to cover unexpectedly high demand. A similar problem exists for manufacturers of other fashion goods.

This type of situation is often called the *newsboy problem*. If the newspaper seller does not buy enough newspapers to resell on the street corner, sales opportunities are lost. If the seller buys too many newspapers, the overage cannot be sold because nobody wants yesterday's newspaper.

The following process is a straightforward way to analyze such problems and decide on the best order quantity.

1. List the different levels of demand that are possible, along with the estimated probability of each.

2. Develop a *payoff* table that shows the profit for each purchase quantity, $Q$, at each assumed demand level, $D$. Each row in the table represents a different order quantity, and each column represents a different demand level. The payoff for a given quantity–demand combination depends on whether all units are sold at the regular profit margin during the regular season, which results in two possible cases.

   **a.** If demand is high enough $(Q \leq D)$, then all units are sold at the full profit margin, $p$, during the regular season,

$$\text{Payoff} = (\text{Profit per unit})(\text{Purchase quantity}) = pQ$$

   **b.** If the purchase quantity exceeds the eventual demand $(Q > D)$, only $D$ units are sold at the full profit margin, and the remaining units purchased must be disposed of at a loss, $l$, after the season. In this case,

$$\begin{aligned}\text{Payoff} &= \begin{pmatrix}\text{Profit per unit sold}\\ \text{during season}\end{pmatrix}(\text{Demand}) - \begin{pmatrix}\text{Loss per}\\ \text{unit}\end{pmatrix}\begin{pmatrix}\text{Amount disposed of}\\ \text{after season}\end{pmatrix}\\ &= pD - l(Q - D)\end{aligned}$$

3. Calculate the expected payoff for each $Q$ (or row in the payoff table) by using the *expected value* decision rule. For a specific $Q$, first multiply each payoff in the row by the demand probability associated with the payoff, and then add these products.

4. Choose the order quantity $Q$ with the highest expected payoff.

Using this decision process for all such items over many selling seasons will maximize profits. However, it is not foolproof, and it can result in an occasional bad outcome.

## EXAMPLE C.3 Finding *Q* for One-Period Inventory Decisions

One of many items sold at a museum of natural history is a Christmas ornament carved from wood. The gift shop makes a \$10 profit per unit sold during the season, but it takes a \$5 loss per unit after the season is over. The following discrete probability distribution for the season's demand has been identified:

| Demand | 10 | 20 | 30 | 40 | 50 |
|---|---|---|---|---|---|
| Demand Probability | 0.2 | 0.3 | 0.3 | 0.1 | 0.1 |

How many ornaments should the museum's buyer order?

MyOMLab

Tutor C.3 in MyOMLab provides a new example to practice the one-period inventory decision.

MyOMLab

Active Model C.3 in MyOMLab provides additional insight on the one-period inventory decision model and its uses.

### SOLUTION

Each demand level is a candidate for best order quantity, so the payoff table should have five rows. For the first row, where $Q = 10$, demand is at least as great as the purchase quantity. Thus, all five payoffs in this row are

$$\text{Payoff} = pQ = (\$10)(10) = \$100$$

This formula can be used in other rows but only for those quantity–demand combinations where all units are sold during the season. These combinations lie in the upper-right portion of the payoff table, where $Q \leq D$. For example, the payoff when $Q = 40$ and $D = 50$ is

$$\text{Payoff} = pQ = (\$10)(40) = \$400$$

The payoffs in the lower-left portion of the table represent quantity–demand combinations where some units must be disposed of after the season $(Q > D)$. For this case, the payoff must be calculated with the second formula. For example, when $Q = 40$ and $D = 30$,

$$\text{Payoff} = pD - l(Q - D) = (\$10)(30) - (\$5)(40 - 30) = \$250$$

OM Explorer or POM for Windows can be used to analyze this problem. Using OM Explorer, we obtain the payoff table in Figure C.5.

▼ **FIGURE C.5**
*OM Explorer* Solver for One-Period Inventory Decisions Showing the Payoff Table

| | | |
|---|---|---|
| **Profit** | \$10.00 | (if sold during preferred period) |
| **Loss** | \$5.00 | (if sold after preferred period) |

Enter the possible demands along with the probability of each occurring. Use the buttons to increase or decrease the number of allowable demand forecasts. NOTE: Be sure to enter demand forecasts and probabilities in all tinted cells, and be sure probabilities add up to 1.

< >

| | | | | | |
|---|---|---|---|---|---|
| **Demand** | 10 | 20 | 30 | 40 | 50 |
| **Profitability** | 0.2 | 0.3 | 0.3 | 0.1 | 0.1 |

**Payoff Table**

| | | **Demand** | | | | |
|---|---|---|---|---|---|---|
| | | 10 | 20 | 30 | 40 | 50 |
| **Quantity** | 10 | 100 | 100 | 100 | 100 | 100 |
| | 20 | 50 | 200 | 200 | 200 | 200 |
| | 30 | 0 | 150 | 300 | 300 | 300 |
| | 40 | −50 | 100 | 250 | 400 | 400 |
| | 50 | −100 | 50 | 200 | 350 | 500 |

Now we calculate the expected payoff for each $Q$ by multiplying the payoff for each demand quantity by the probability of that demand and then adding the results. For example, for $Q = 30$,

$$\text{Payoff} = 0.2(\$0) + 0.3(\$150) + 0.3(\$300) + 0.1(\$300) + 0.1(\$300) = \$195$$

Using OM Explorer, Figure C.6 shows the expected payoffs.

**DECISION POINT**

Because $Q = 30$ has the highest payoff at \$195, it is the best order quantity. Management can use OM Explorer or POM for Windows to do sensitivity analysis on the demands and their probabilities to see how confident they are with that decision.

**FIGURE C.6 ▶**
*OM Explorer* Solver Showing the Expected Payoffs for One-Period Inventory Decisions

**Weighted Payoffs**

| Order Quantity | Expected Payoff |
|---|---|
| 10 | 100 |
| 20 | 170 |
| 30 | 195 |
| 40 | 175 |
| 50 | 140 |

| | |
|---|---|
| Greatest Expected Payoff | 195 |
| Associated with Order Quantity | 30 |

The need for one-time inventory decisions also can arise in manufacturing plants when (1) customized items are made (or purchased) to a single order, and (2) scrap quantities are high. A customized item produced for a single order is never intentionally held in stock because the demand for it is too unpredictable. In fact, it may never be ordered again so the manufacturer would like to make just the amount requested by the customer—no more, no less. The manufacturer also would like to satisfy an order in just one run to avoid an extra setup and a delay in delivering goods ordered. These two goals may conflict if the likelihood of some units being scrapped is high. Suppose that a customer places an order for 20 units. If the manager orders 20 units from the shop or from the supplier, one or two units may have to be scrapped. This shortage will force the manager to place a second (or even third) order to replace the defective units. Replacement can be costly if setup time is high and can also delay shipment to the customer. To avoid such problems, the manager could order more than 20 units the first time. If some units are left over, the customer might be willing to buy the extras or the manager might find an internal use for them. For example, some manufacturing companies set up a special account for obsolete materials. These materials can be "bought" by departments within the company at less than their normal cost, as an incentive to use them.

## LEARNING GOALS IN REVIEW

| Learning Goal | Guidelines | MyOMLab Resources |
|---|---|---|
| 1 Calculate the optimal lot size when replenishment is not instantaneous. | See the section "Noninstantaneous Replenishment," pp. 379–381. Study Example C.1 and Solved Problem 1 for help on determining the ELS. | **Active Model:** C.1: Economic Production Lot Size<br>**OM Explorer Solver:** Economic Production Lot size<br>**OM Explorer Tutor:** C.1: Economic Production Lot Size<br>**POM for Windows:** Economic Production Lot Size |
| 2 Determine the optimal order quantity when materials are subject to quantity discounts. | See the section "Quantity Discounts," pp. 382–384. Study Example C.2 and Solved Problem 2 for a step-by-step approach to determine the best order quantity. | **Active Model:** C.2: Quantity Discounts<br>**OM Explorer Solver:** Quantity Discounts<br>**OM Explorer Tutor:** C.2: Finding Q with Quantity Discounts<br>**POM for Windows:** Quantity Discount Model |
| 3 Calculate the order quantity that maximizes the expected profits for a one-period inventory decision. | See the section "One-Period Decisions," pp. 384–386. Be sure to understand Example C.3 and Solved Problem 3. | **Active Model:** C.3: One-Time Inventory Decisions<br>**OM Explorer Solver:** One-Period Inventory Decisions<br>**OM Explorer Tutor:** C.3: One-Period Inventory Decisions<br>**POM for Windows:** Decision Tables |

# Key Equations

## Noninstantaneous Replenishment

**1.** Maximum cycle inventory: $I_{\max} = Q\left(\frac{p-d}{p}\right)$

**2.** Total annual cost = Annual holding cost + Annual ordering or setup cost

$$C = \frac{Q}{2}\left(\frac{p-d}{p}\right)(H) + \frac{D}{Q}(S)$$

**3.** Economic production lot size: $\text{ELS} = \sqrt{\frac{2DS}{H}}\sqrt{\frac{p}{p-d}}$

**4.** Time between orders, expressed in years: $\text{TBO}_{\text{ELS}} = \frac{\text{ELS}}{D}$

## Quantity Discounts

**5.** Total annual cost = Annual holding cost + Annual ordering or setup cost + Annual cost of material

$$C = \frac{Q}{2}(H) + \frac{D}{Q}(S) + PD$$

## One-period Decisions

**6.** Payoff matrix: Payoff $= \begin{cases} pQ & \text{if } Q \le D \\ pD - l(Q-D) & \text{if } Q > D \end{cases}$

# Key Term

economic production lot size (ELS) 380

# Solved Problem 1

Peachy Keen, Inc., makes mohair sweaters, blouses with Peter Pan collars, pedal pushers, poodle skirts, and other popular clothing styles of the 1950s. The average demand for mohair sweaters is 100 per week. Peachy's production facility has the capacity to sew 400 sweaters per week. Setup cost is \$351. The value of finished goods inventory is \$40 per sweater. The annual per-unit inventory holding cost is 20 percent of the item's value.

**a.** What is the economic production lot size (ELS)?

**b.** What is the average time between orders (TBO)?

**c.** What is the total of the annual holding cost and setup cost?

**SOLUTION**

**a.** The production lot size that minimizes total cost is

$$\text{ELS} = \sqrt{\frac{2DS}{H}}\sqrt{\frac{p}{p-d}} = \sqrt{\frac{2(100 \times 52)(\$351)}{0.20(\$40)}}\sqrt{\frac{400}{(400-100)}}$$

$$= \sqrt{456{,}300}\sqrt{\frac{4}{3}} = 780 \text{ sweaters}$$

**b.** The average time between orders is

$$\text{TBO}_{\text{ELS}} = \frac{\text{ELS}}{D} = \frac{780}{5{,}200} = 0.15 \text{ year}$$

Converting to weeks, we get

$$\text{TBO}_{\text{ELS}} = (0.15 \text{ year})(52 \text{ weeks/year}) = 7.8 \text{ weeks}$$

**c.** The minimum total of setup and holding costs is

$$C = \frac{Q}{2}\left(\frac{p-d}{p}\right)(H) + \frac{D}{Q}(S) = \frac{780}{2}\left(\frac{400-100}{400}\right)(0.20 \times \$40) + \frac{5{,}200}{780}(\$351)$$

$$= \$2{,}340/\text{year} + \$2{,}340/\text{year} = \$4{,}680/\text{year}$$

# Solved Problem 2

MyOMLab Video

A hospital buys disposable surgical packages from Pfisher, Inc. Pfisher's price schedule is $50.25 per package on orders of 1 to 199 packages and $49.00 per package on orders of 200 or more packages. Ordering cost is $64 per order, and annual holding cost is 20 percent of the per-unit purchase price. Annual demand is 490 packages. What is the best purchase quantity?

**SOLUTION**

We first calculate the EOQ at the *lowest* price:

$$EOQ_{49.00} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(490)(\$64.00)}{0.20(\$49.00)}} = \sqrt{6{,}400} = 80 \text{ packages}$$

This solution is infeasible because, according to the price schedule, we cannot purchase 80 packages at a price of $49.00 each. Therefore, we calculate the EOQ at the next lowest price ($50.25):

$$EOQ_{50.25} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(490)(\$64.00)}{0.20(\$50.25)}} = \sqrt{6{,}241} = 79 \text{ packages}$$

This EOQ is feasible, but $50.25 per package is not the lowest price. Hence, we have to determine whether total costs can be reduced by purchasing 200 units and thereby obtaining a quantity discount.

$$C = \frac{Q}{2}(H) + \frac{D}{Q}(S) + PD$$

$$C_{79} = \frac{79}{2}(0.20 \times \$50.25) + \frac{490}{79}(\$64.00) + \$50.25(490)$$

$$= \$396.98/\text{year} + \$396.68/\text{year} + \$24{,}622.50/\text{year} = 25{,}416.44/\text{year}$$

$$C_{200} = \frac{200}{2}(0.20 \times \$49.00) + \frac{490}{200}(\$64.00) + \$49.00(490)$$

$$= \$980.00/\text{year} + \$156.80/\text{year} + \$24{,}010.00/\text{year} = \$25{,}146.80/\text{year}$$

Purchasing 200 units per order will save $269.64/year, compared to buying 79 units at a time.

# Solved Problem 3

Swell Productions is sponsoring an outdoor conclave for owners of collectible and classic Fords. The concession stand in the T-Bird area will sell clothing such as T-shirts and official Thunderbird racing jerseys. Jerseys are purchased from Columbia Products for $40 each and are sold during the event for $75 each. If any jerseys are left over, they can be returned to Columbia for a refund of $30 each. Jersey sales depend on the weather, attendance, and other variables. The following table shows the probability of various sales quantities. How many jerseys should Swell Productions order from Columbia for this one-time event?

| Sales Quantity | Probability | Quantity Sales | Probability |
|---|---|---|---|
| 100 | 0.05 | 400 | 0.34 |
| 200 | 0.11 | 500 | 0.11 |
| 300 | 0.34 | 600 | 0.05 |

**SOLUTION**

Table C.1 is the payoff table that describes this one-period inventory decision. The upper-right portion of the table shows the payoffs when the demand, $D$, is greater than or equal to the order quantity, $Q$. The payoff is equal to the per-unit profit (the difference between price and cost) multiplied by the order quantity. For example, when the order quantity is 100 and the demand is 200.

**TABLE C.1** | PAYOFFS

| | DEMAND, *D* | | | | | | |
|---|---|---|---|---|---|---|---|
| *Q* | 100 | 200 | 300 | 400 | 500 | 600 | Expected Payoff |
| 100 | $3,500 | $3,500 | $ 3,500 | $ 3,500 | $ 3,500 | $ 3,500 | $ 3,500 |
| 200 | $2,500 | $7,000 | $ 7,000 | $ 7,000 | $ 7,000 | $ 7,000 | $ 6,775 |
| 300 | $1,500 | $6,000 | $10,500 | $10,500 | $10,500 | $10,500 | $ 9,555 |
| 400 | $ 500 | $5,000 | $ 9,500 | $14,000 | $14,000 | $14,000 | $10,805 |
| 500 | ($ 500) | $4,000 | $ 8,500 | $13,000 | $17,500 | $17,500 | $10,525 |
| 600 | ($1,500) | $3,000 | $ 7,500 | $12,000 | $16,500 | $21,000 | $ 9,750 |

$$\text{Payoff} = (p - c)Q = (\$75 - \$40)100 = \$3{,}500$$

The lower-left portion of Table C.1 shows the payoffs when the order quantity exceeds the demand. Here the payoff is the profit from sales, $pD$, minus the loss associated with returning overstock, $l(Q - D)$, where $l$ is the difference between the cost and the amount refunded for each jersey returned and $Q - D$ is the number of jerseys returned. For example, when the order quantity is 500 and the demand is 200,

$$\textit{Payoff} = pD - l(Q - D) = (\$75 - \$40)200 - (\$40 - \$30)(500 - 200) = \$4{,}000$$

The highest expected payoff occurs when 400 jerseys are ordered:

$$\begin{aligned}\text{Expected payoff}_{400} &= (\$500 \times 0.05) + (\$5{,}000 \times 0.11) + (\$9{,}500 \times 0.34) \\ &\quad + (\$14{,}000 \times 0.34) + (\$14{,}000 \times 0.11) + (\$14{,}000 \times 0.05) \\ &= \$10{,}805\end{aligned}$$

# Problems

The OM Explorer and POM for Windows software is available to all students using the 11th edition of this textbook. Go to **http://www.pearsonglobaleditions.com/krajewski** to download these computer packages. If you purchased MyOMLab, you also have access to Active Models software and significant help in doing the following problems. Check with your instructor on how best to use these resources. In many cases, the instructor wants you to understand how to do the calculations by hand. At the least, the software provides a check on your calculations. When calculations are particularly complex and the goal is interpreting the results in making decisions, the software entirely replaces the manual calculations. The software also can be a valuable resource well after your course is completed.

## Noninstantaneous Replenishment

**1.** Bold Vision, Inc., makes laser printer and photocopier toner cartridges. The demand rate is 625 EP cartridges per week. The production rate is 1,736 EP cartridges per week, and the setup cost is $100. The value of inventory is $130 per unit, and the holding cost is 20 percent of the inventory value. Bold Vision operates 52 weeks a year. What is the economic production lot size?

**2.** Sharpe Cutter is a small company that produces specialty knives for paper cutting machinery. The annual demand for a particular type of knife is 100,000 units. The demand is uniform over the 250 working days in a year. Sharpe Cutter produces this type of knife in lots and, on average, can produce 450 knives a day. The cost to set up a production lot is $300, and the annual holding cost is $1.20 per knife.

- **a.** Determine the economic production lot size (ELS).
- **b.** Determine the total annual setup and inventory holding cost for this item.
- **c.** Determine the TBO, or cycle length, for the ELS.
- **d.** Determine the production time per lot.

**3.** Suds's Bottling Company does bottling, labeling, and distribution work for several local microbreweries. The demand rate for Wortman's beer is 600 cases (24 bottles each) per week. Suds's bottling production rate is 2,400 cases per week, and the setup cost is $800. The value of inventory is $12.50 per case, and the annual holding cost is 30 percent of the inventory value. Suds's facilities operate 52 weeks each year. What is the economic production lot size?

**4.** One-Eyed Toad Pottery makes custom planters for up-scale clients. The average demand for planters is 20 per week. One-Eyed Toad's production facility has the capacity to make 25 planters per week. Setup cost is $1500. The value of finished goods inventory is $250 per planter. The annual per-unit inventory holding cost is 35 percent of the item's value.

- **a.** What is the economic production lot size (ELS)?
- **b.** What is the average time between orders (TBO)?
- **c.** What is the total of the annual holding cost and setup cost?

## Quantity Discounts

**5.** The Bucks Grande exhibition baseball team plays 50 weeks each year and uses an average of 350 baseballs per week. The team orders baseballs from Coopers-Town, Inc., a ball manufacturer noted for six-sigma-level consistency and high product quality. The cost to order baseballs is $100 per order and the annual holding cost per ball is 38 percent of the purchase price. Coopers-Town's price structure is:

| Order Quantity | Price per Unit |
|---|---|
| 1–999 | $7.50 |
| 1,000–4999 | $7.25 |
| 5,000 or more | $6.50 |

**a.** How many baseballs should the team buy per order?

**b.** What is the total annual cost associated with the best order quantity?

**c.** Coopers-Town, Inc., discovers that, owing to special manufacturing processes required for the Buck's baseballs, it has underestimated the setup time required on a capacity-constrained piece of machinery. Coopers-Town adds another category to the price structure to provide an incentive for larger orders and thereby hopes to reduce the number of setups required. If the Bucks buy 15,000 baseballs or more, the price will drop to $6.25 each. Should the Bucks revise their order quantity?

**6.** To boost sales, Pfisher (refer to Solved Problem 2) announces a new price structure for disposable surgical packages. Although the price break no longer is available at 200 units, Pfisher now offers an even greater discount if larger quantities are purchased. On orders of 1 to 499 packages, the price is $50.25 per package. For orders of 500 or more, the price per unit is $47.80. Ordering costs, annual holding costs, and annual demand remain at $64 per order, 20 percent of the per-unit cost, and 490 packages per year, respectively. What is the new lot size?

**7.** The University Bookstore at a prestigious private university buys mechanical pencils from a wholesaler. The wholesaler offers discounts for large orders according to the following price schedule:

| Order Quantity | Price per Unit |
|---|---|
| 0 to 200 | $4.00 |
| 201 to 2,000 | $3.50 |
| 2,001 or more | $3.25 |

The bookstore expects an annual demand of 2,500 units. It costs $10 to place an order, and the annual cost of holding a unit in stock is 30 percent of the unit's price. Determine the best order quantity.

**8.** Mac-in-the-Box, Inc., sells computer equipment by mail and telephone order. Mac sells 1,200 flat-bed scanners per year. Ordering cost is $300, and annual holding cost is 16 percent of the item's price. The scanner manufacturer offers the following price structure to Mac-in-the-Box:

| Order Quantity | Price per Unit |
|---|---|
| 0 to 11 | $520 |
| 12 to 143 | $500 |
| 144 or more | $400 |

What order quantity minimizes total annual costs?

**9.** As inventory manager, you must decide on the order quantity for an item that has an annual demand of 2,000 units. Placing an order costs you $20 each time. Your annual holding cost, expressed as a percentage of average inventory value, is 20 percent. Your supplier has provided the following price schedule:

| Minimum Order Quantity | Price per Unit |
|---|---|
| 1 | $2.50 |
| 200 | $2.40 |
| 300 | $2.25 |
| 1,000 | $2.00 |

What ordering policy do you recommend?

**10.** Bold Vision, Inc. (from Problem 1), must purchase toner from a local supplier. The company does not wish to carry raw material inventory and therefore only purchases enough toner to satisfy the demand of each individual batch of cartridges. Each toner cartridge requires one pound of toner. The raw material supplier offers Bold Vision a purchase discount of $2.00 per pound if the company orders at least 2,000 pounds at a time. Should Bold Vision accept this offer and alter its toner purchase quantity?

## One-Period Decisions

**11.** Downtown Health Clinic needs to order influenza vaccines for the next flu season. The Clinic charges its patients $15.00 per vaccination and each dose of vaccine costs the clinic $4.00 to purchase. The Center for Disease Control has a long standing policy of buying back unused vaccines for $1.00 per dose. The Clinic estimates the following probability distribution for the season's demand:

| Demand | Probability |
|---|---|
| 2,000 | 0.05 |
| 3,000 | 0.20 |
| 4,000 | 0.25 |
| 5,000 | 0.40 |
| 6,000 | 0.10 |

**a.** How many vaccines should the Clinic order to maximize its expected profit?

**b.** The Clinic is trying to determine if they should participate in a new Federal program in which the cost of each dose is reduced to $2.00. However, to participate in the program, they can charge no more than $10.00 per vaccine. On strictly a profit maximizing basis, should the Clinic agree to participate?

**12.** Dorothy's pastries are freshly baked and sold at several specialty shops throughout Perth. When they are a day old, they must be sold at reduced prices. Daily demand is distributed as follows:

| Demand | Probability |
|---|---|
| 50 | 0.25 |
| 150 | 0.50 |
| 200 | 0.25 |

Each pastry sells for $1.00 and costs $0.60 to make. Each one not sold at the end of the day can be sold the next day for $0.30 as day-old merchandise. How many pastries should be baked each day?

**13.** The Aggies will host Tech in this year's homecoming football game. Based on advance ticket sales, the athletic department has forecast hot dog sales as shown in the following table. The school buys premium hot dogs for $1.50 and sells them during the game at $3.00 each. Hot dogs left over after the game will be sold for $0.50 each to the Aggie student cafeteria, where they will be used in making hotdog casserole.

| Sales Quantity | Probability |
|---|---|
| 2,000 | 0.10 |
| 3,000 | 0.30 |
| 4,000 | 0.30 |
| 5,000 | 0.20 |
| 6,000 | 0.10 |

Use a payoff matrix to determine the number of hot dogs to buy for the game.

**14.** The Lake Sharkey BBQ Pit serves slow cooked beef brisket by the pound. Based on historical sales during the Labor Day weekend, management has forecasted brisket sales in pounds as shown in the following table. Lake Sharkey spends $14 to produce each pound of brisket for which it charges $23 per pound. Any unsold brisket at the end of the weekend is ground into chili which sells for $12 per pound. How many pounds of brisket should The Lake Sharkey BBQ Pit prepare for sale this Labor Day?

| Demand in pounds | Probability |
|---|---|
| 500 | 0.10 |
| 1000 | 0.40 |
| 1500 | 0.30 |
| 2000 | 0.15 |
| 2500 | 0.05 |

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Ian Simpson/Alamy

# 10

# PLANNING AND SCHEDULING OPERATIONS

In 1997 Cooper Tires purchased Avon Rubber PLC of Melksham, Wiltshire, in the United Kingdom. The Cooper facility in Melksham is a major employer in the region. Avon, heavily involved with Formula One racing since 1982, had been the sole supplier of tires for the British Formula Three Championship and, from 2009, its tires were re-branded "Cooper" as Cooper became the championship's sole sponsor. For its own part, Cooper became the official tire of the A1 Grand Prix for the initial season and was under contract to produce slick and treaded rain tires until 2008. It also became the official tire for two other championships: Champ Car Atlantic Championship and USF2000 National Championship. Cooper Tires remains a major source of racing tires worldwide.

## Cooper Tire and Rubber Company

The Cooper Tire and Rubber Company is a $3.4 billion company with 13,000 employees worldwide. The company is the fourth-largest tire manufacturer in North America and the 11th largest globally. Rather than participating in both the original equipment and replacement tire markets as do its major competitors, Bridgestone, Goodyear Tire and Rubber, and Michelin, it focuses on producing and selling replacement tires for the passenger car, light truck, motorcycle, race car, commercial, and off-road vehicle markets worldwide. It has nine manufacturing facilities located in North America, Europe, and China and 40 distribution centers worldwide. Tires are distributed through independent dealers, regional retailers and wholesalers, and national retailers.

Cooper has three key strategic imperatives to guide operations: (1) Develop a competitive cost structure and improve profitability, (2) drive top-line profitable growth, and (3) build organizational capabilities. Indeed, while everything we have to offer in this text comes to bear in supporting these imperatives, let us see what Cooper has done regarding operations planning and scheduling, the topic of this chapter. We examine several press releases to gain insight into the nature of operations planning and scheduling at a large manufacturer.

**October 1, 2008.** Cooper Tire and Rubber Company announced it continues to adjust production schedules at its U.S. facilities primarily due to raw material shortages and soft demand in the North American market. This adjustment was largely due to the hurricanes, which hit the Gulf Coast. Raw materials were allocated to plants with critical customer demands and labor schedules were flexed at all plants while the raw materials shortages are allocated. By flexing work schedules Cooper attempted to avoid layoffs at any of the plants. The estimated impact of the production adjustments was $9 to $11 million.

**October 5, 2009.** Cooper Tire and Rubber Company announced plans to increase production capacity at its Texarkana, Arkansas, facility by changing to 24/7 operations and to add approximately 200 additional employees to meet a growing demand for its products. The shift in production took several months in 2010.

**March 7, 2013.** Cooper Tire and Rubber Company temporarily idled production at its Findlay, Ohio, plant due to high tire inventories and Cooper's implementation of a new software system. Cooper built the extra tires in 2012's fourth-quarter to offset any lost production in 2013. However, company officials said the expiration of tariffs on imported Chinese-made tires in the fall of 2012 resulted in higher-than-normal tire inventories, resulting in fewer orders. In its annual report, Cooper said that the tariff expiration was expected to affect sales and production in the first and possibly second quarter of 2013.

**September 16, 2013.** JDA Software Group and Cooper Tire and Rubber Company announced the implementation of JDA Production Scheduling—Discrete at its manufacturing facilities. With the size proliferation in original equipment tires, including the phasing out of 12-, 13-, and 14-inch tires and the increase in bigger tires and better designs, Cooper realized that the changes in designs and sizes would make it increasingly more difficult to meet the needs of its customers. Before the software change, the company operated with a manual, weekly planning cycle. The schedulers reviewed the demand data SKU by SKU to create a curing schedule for each plant. The curing process is a capacity-limited resource that controls the flow of tires in a plant. The scheduling process was time consuming and the data were not up to date by the time the schedule was completed. With the new software, the scheduling lead time was reduced by nine days; all of the plant constraints were automatically recognized, and inventory management was improved.

Cooper Tire's example shows us how employee hires, changes to workforce schedules, temporary use of undertime by idling a plant, and facility scheduling can be used to achieve its strategic initiatives.

*Sources:* **http://coopertire.com/About-Us**, 2014; **http://coopertire.com/News/Corporate-News-Releases**, (October 5, 2009); **http://www.answers.com/topic/cooper-tire-rubber-company**, 2014; **http://www.bloomberg.com**, (October 1, 2008); **http://www.toledoblade.com**, (March 7, 2013); JDA Software Group, "Case Study: Optimizing the Planning Schedule," (September 16, 2013).

## LEARNING GOALS *After reading this chapter, you should be able to:*

1. Explain the rationale behind the levels in the operations planning and scheduling process.
2. Describe the supply options used in sales and operations planning.
3. Compare the chase planning strategy to the level planning strategy for developing sales and operations plans.
4. Use spreadsheets for sales and operations planning.
5. Develop workforce and workstation schedules.

Managing supply chains effectively requires more than just good demand forecasts or knowing how much to order and when. Demand is the first half of the equation, and the other half is supply. As Cooper Tires in the chapter opener has shown, the firm must develop plans to supply the resources needed to meet the forecasted demand. These resources include the workforce, materials, inventories, dollars, and equipment capacity.

**Operations planning and scheduling** is the process of making sure that demand and supply plans are in balance, from the aggregate level down to the short-term scheduling level. Operations planning and scheduling lies at the core of supply chain integration, around which plans are made up and down the supply chain, from supplier deliveries to customer due dates and services. Why is it so important? First, it requires managerial inputs from all of the firm's functions. Marketing provides inputs on demand and accounting provides important cost data and a firm's financial condition. Second, each function is affected by the plan. A plan that calls for expanding the workforce has a direct impact on the hiring and training requirements for the human resources function. As the plan is implemented, it creates revenue and cost streams that finance must deal with as it manages the firm's cash flows. Third, each department and group in a firm has its own workforce. Managers of these departments must make choices on hiring, overtime, and vacations. Finally, whether the business is an airline, hotel, computer manufacturer, or university, schedules are a part of everyday life. Schedules involve an enormous amount of detail and affect every process in a firm. For example, service, product, and employee schedules determine specific cash flow requirements, trigger the firm's billing process, and initiate requirements for the employee training process. Firms use the scheduling process to lower their costs and improve their responsiveness, affecting operations up and down the supply chain worldwide. Table 10.1 defines several types of plans related to operations planning and scheduling.


Using Operations to Create Value

**PROCESS MANAGEMENT**

Process Strategy and Analysis
Managing Quality
Planning Capacity
Managing Process Constraints
Designing Lean Systems
Managing Effective Projects

**CUSTOMER DEMAND MANAGEMENT**

Forecasting Demand
Managing Inventories
→ **Planning and Scheduling Operations**
Efficient Resource Planning

**SUPPLY CHAIN MANAGEMENT**

Designing Effective Supply Chains
Supply Chains and Logistics
Integrating the Supply Chain
Managing Supply Chain Sustainability


**TABLE 10.1 | TYPES OF PLANS WITH OPERATIONS PLANNING AND SCHEDULING**

| Key Term | Definition |
|---|---|
| **Sales and operations plan (S&OP)** | A plan of future aggregate resource levels so that supply is in balance with demand. It states a company's or department's production rates, workforce levels, and inventory holdings that are consistent with demand forecasts and capacity constraints. The S&OP is time-phased, meaning that it is projected for several time periods (such as months or quarters) into the future. |
| **Aggregate plan** | Another term for the sales and operations plan. |
| **Production plan** | A sales and operations plan for a *manufacturing firm* that centers on production rates and inventory holdings. |
| **Staffing plan** | A sales and operations plan for a *service firm*, which centers on staffing and on other human resource-related factors. |
| **Resource plan** | An intermediate step in the planning process that lies between S&OP and scheduling. It determines requirements for materials and other resources on a more detailed level than the S&OP. It is covered in the next chapter. |
| **Schedule** | A detailed plan that allocates resources over shorter time horizons to accomplish specific tasks. |

In this chapter, we begin by discussing the three levels of operations planning and scheduling: (1) sales and operations planning (S&OP), (2) resource planning, and (3) scheduling. We explain the purpose of aggregation in sales and operations planning and the various information inputs required for its development. We examine how S&OP relates with other plans and functional areas within the firm and describe the supply options and planning strategies for effective S&OP. We show how spreadsheets can help find good solutions. Then, we conclude with scheduling, including performance measures and some basic techniques for creating schedules. MyOMLab Supplement J, "Operations Scheduling," provides additional help with scheduling problems.

MyOMLab

**operations planning and scheduling**

The process of balancing supply with demand, from the aggregate level down to the short-term scheduling level.

## Levels in Operations Planning and Scheduling

Managers develop plans for their operations covering varying time spans, from the long term to the short term. These plans form a hierarchy: the long-term plans form an umbrella under which short-term plans exist. Sales and operations plans exist at Level 1 and represent the long-term operations plans. These plans form the basis for major outlays for materials and resources and consequently cannot be very specific regarding products or services. Resource plans exist at Level 2 and are more detailed than the sales and operations plans and cover a shorter term. The most detailed plans are the schedules in Level 3, which cover very short time horizons and relate to specific products and resources. Level 2 plans must be consistent with Level 1 plans, and Level 3 plans must be consistent with Level 2 plans.

## Level 1: Sales and Operations Planning

In this section, we explain why companies begin with plans that take a macro, or big-picture, view of their business. We also describe how these plans relate to their other plans and how the long-term plans ultimately are translated into detailed schedules ready for immediate action.

**Aggregation** The sales and operations plan is useful because it focuses on a general course of action, consistent with the company's strategic goals and objectives, without getting bogged down in details. We must first aggregate, and then use the targets and resources from the plan to create effective, coordinated schedules. A company's managers must determine whether they can satisfy budgetary goals without having to schedule each of the company's thousands of products and employees individually. While schedules with such detail are the goal, the operations planning and scheduling process begins at the aggregate level.

In general, companies perform aggregation along three dimensions: (1) services or products, (2) workforce, and (3) time.

**product family**

A group of services or products that have similar demand requirements and common process, labor, and materials requirements.

1. **Services or products** A group of customers, services, or products that have similar demand requirements and common process, workforce, and materials requirements is called a **product family**. Sometimes, product families relate to market groupings or to specific processes. A firm can aggregate its services or products into a set of relatively broad families, avoiding too much detail at this stage of the planning process. For instance, a manufacturer of bicycles that produces 12 different models of bikes might divide them into two groups, mountain bikes and road bikes, for the purpose of preparing the sales and operations plan. Common and relevant measurements should be used.
2. **Workforce** A company can aggregate its workforce in various ways as well, depending on its flexibility. For example, if workers at the bicycle manufacturer are trained to work on either mountain bikes or road bikes, for planning purposes management can consider its workforce to be a single aggregate group, even though the skills of individual workers may differ.
3. **Time** The planning horizon covered by a sales and operations plan typically is one year, although it can differ in various situations. To avoid the expense and disruptive effect of frequent changes in output rates and the workforce, adjustments usually are made monthly or quarterly. In other words, the company looks at time in the aggregate—months, quarters, or seasons—rather than in weeks, days, or hours.

**business plan**

A projected statement of income, costs, and profits.

**annual plan (or financial plan)**

A plan for financial assessment used by a nonprofit service organization.

**Information Inputs** Just as it is needed to manage the demand side, consensus is needed among the firm's departments when decisions for the supply side are made. Information inputs are sought to create a sales and operations plan that works for all. Figure 10.1 lists inputs from each functional area. They must be accounted for to make sure that the plan is a good one and also doable. Such coordination helps synchronize the flow of services, materials, and information through the supply chain to best balance supply with customer demand.


Jim West/Alamy

This plant is manufacturing bikes for Shinola, which makes three bike models along with other consumer goods such as watches and leather goods. The bikes would represent a product family for sales and operations planning purposes.

**Related Plans** A financial assessment of the organization's near future—that is, for 1 or 2 years ahead—is called either a business plan (in for-profit firms) or an annual plan (in nonprofit service organizations). A **business plan** is a projected statement of income, costs, and profits. It usually is accompanied by budgets, a projected (pro forma) balance sheet, and a projected cash flow statement showing sources and allocations of funds. The business plan unifies the plans and expectations of a firm's operations, finance, sales, and marketing managers. In particular, it reflects plans for market penetration, new product introduction, and capital investment. Manufacturing firms and for-profit service organizations, such as a retail store, a firm of attorneys, or a hospital, prepare such plans. A nonprofit service organization, such as the United Way or a municipal government, prepares a different type of plan for financial assessment, called an **annual plan** or **financial plan**.

Figure 10.2 illustrates the relationships among the business or annual plan, constraint management, forecasting, operations strategy, sales and

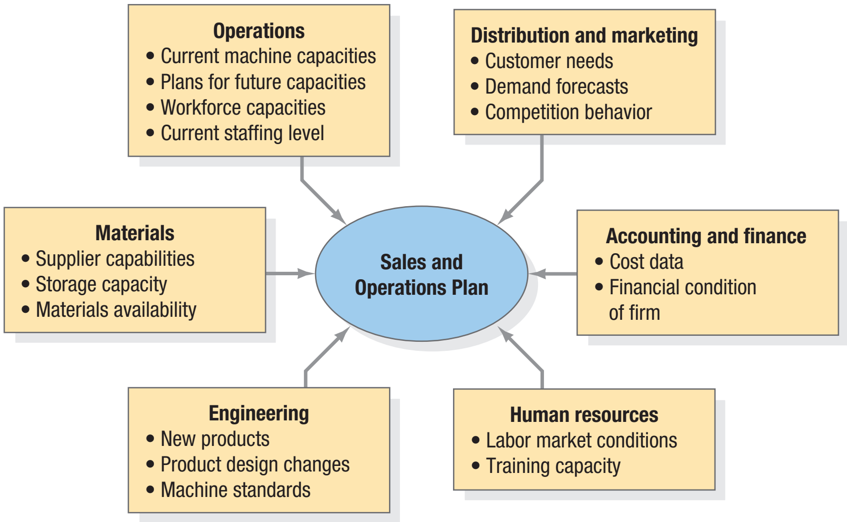



**FIGURE 10.1** Managerial Inputs from Functional Areas to Sales and Operations Plans

operations plan, and the detailed plans and schedules derived from it. For *service providers* in the supply chain, top management sets the organization's direction and objectives in the business plan (in a for-profit organization) or annual plan (in a not-for-profit organization). This plan then provides the framework for developing the sales and operations plan, which typically focuses on staffing and other human resource–related factors at a more aggregate level. It presents the number and types of employees needed to meet the objectives of the business or annual plan. For *manufacturing firms* in the supply chain, top management sets the company's strategic objectives for at least the next year in the business plan. It provides the overall framework, along with inputs coming from operations strategy, forecasting, and capacity constraint management. The sales and operations plan specifies product family production rates, inventory levels, and workforce levels. Regardless of whether the firm is a service provider or a manufacturer, the sales and operations plan sets the stage for the two levels to follow.

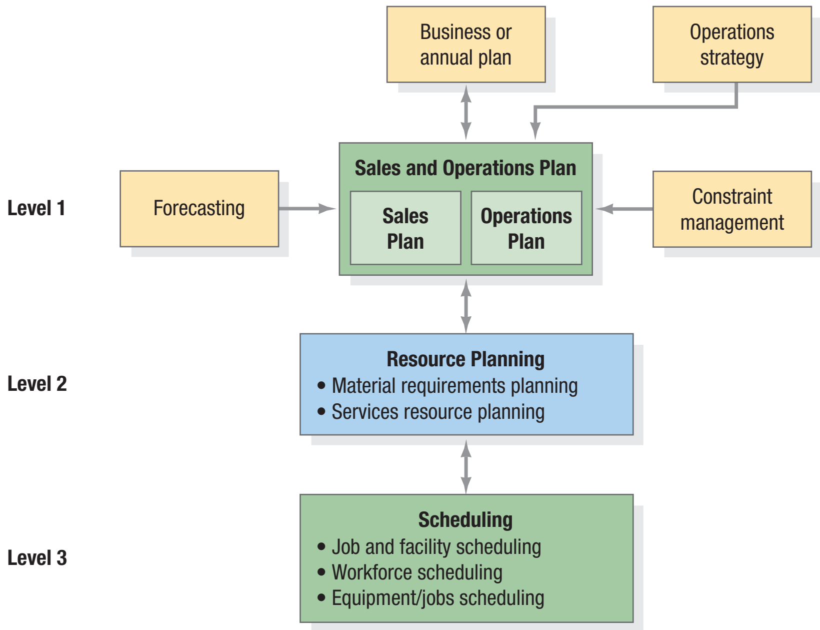



**FIGURE 10.2** The Relationship of Sales and Operations Plans and Schedules to Other Plans

MyOMLab Animation

## Level 2: Resource Planning

The next planning level is *resource planning*, which is a process that takes sales and operations plans; processes time standards, routings, and other information on how services or products are produced; and then plans the timing of capacity and material requirements. It decomposes the aggregate quantities of product families, workforce, and time to arrive at the material and resource requirements implied in the sales and operations plan over a shorter time horizon. For a manufacturing firm, the resource plan gets specific as to individual products within each product family, purchased materials, and resources on a detailed level. A major input is the *master production schedule*, which specifies the timing and size of production quantities for each product in the product families. The *material requirements planning* process then derives plans for components, purchased materials, and workstations. For a service firm, the resource plan may specify the daily or weekly facility capacity requirements for service facilities or labor over the next several months. In essence, the resource planning activity provides due dates for the supply of materials, components, products, and other resources such as labor, space, vehicles, and dollars. This activity sets up Level 3, Scheduling. Because of its importance, we devote Chapter 11, "Efficient Resource Planning," to this topic.

### Level 3: Scheduling

Scheduling takes the resource plan and translates it into specific operational tasks on a detailed basis. Facility schedules can be developed by assigning activities to facilities so as to utilize them efficiently. For example, surgeries for specific patients can be assigned to operating rooms so as to meet the needs of the patients while adhering to the capacity constraints of the operating rooms. Another important schedule is the *workforce schedule*, which details the specific work schedule for each category of employee. For example, a sales and operations plan might allocate 10 police officers for the day shift in a particular district; the resource plan may determine the police protection requirements for a typical week, and the workforce schedule might assign five of them to work Monday through Friday and the other five to work Wednesday through Sunday to meet the varying daily needs for police protection in that district. Finally, given the material requirements plan for a group of jobs in a manufacturing plant, the specific sequence of those jobs can be scheduled on a bottleneck machine. We will address scheduling problems later in this chapter. Thus, the sales and operations plan plays a key role in translating the strategies of the business plan into an operational plan for the manufacturing process.

As the arrows in Figure 10.2 indicate, information flows in two directions: from the top down (broad to detailed) and from the bottom up (detailed to broad). If a sales and operations plan cannot be developed to satisfy the objectives of the business or annual plan with the existing resources, the business or annual plan might need some adjustment. Similarly, if a feasible capacity requirements or material requirements plan cannot be developed, the sales and operations plan might need some adjustment. The planning process is dynamic, with periodic plan revisions or adjustments based on two-way information flows, typically on a monthly basis.

## S&OP Supply Options

Developing sales and operations plans means making decisions. In this section, we concentrate on the supply options that ultimately are combined to develop a sales and operations plan. Given demand forecasts, operations managers must develop a plan to meet the demand. There are six options that can be used singly or in combination to arrive at a plan.


Photo by Michael Springer/Bloomberg via Getty Images

An employee stocks a Whirlpool air conditioner at a Lowe's store in Westborough, Massachusetts. The demand for window units is highly seasonal and also depends on variations in the weather. Typically, Whirlpool begins production of room air conditioners in the fall and holds them as inventory until they are shipped in the spring. Building anticipation inventory in the slack season allows the company to even out production rates over much of the year and still satisfy demand in the peak periods (spring and summer) when retailers are placing most of their orders.

1. **Anticipation Inventory** *Anticipation inventory* can be used to absorb uneven rates of demand or supply. For example, a plant facing seasonal demand can stock anticipation inventory during light demand periods and use it during heavy demand periods. Manufacturers of air conditioners, such as Whirlpool, can experience 90 percent of their annual demand during just three months of a year. Extra, or anticipation inventory, also can help when supply, rather than demand, is uneven. For example, a company can stock up on a certain purchased item if the company's suppliers expect severe capacity limitations. Despite its advantages, anticipation inventory can be costly to hold, particularly if stocked in its finished state. Moreover, when services or products are customized, anticipation inventory is not usually an option. Service providers in the supply chain generally cannot use anticipation inventory because services cannot be stocked.

2. **Workforce Adjustment** Management can adjust workforce levels by hiring or laying off employees. The use of this alternative can be attractive if the workforce is largely unskilled or semiskilled and the labor pool is large. These conditions are more likely found in some countries than in others. However, for a particular company, the size of the qualified labor pool may limit the number of new employees that can be hired at any one time. Also, new employees must be trained, and the capacity of the training facilities

themselves might limit the number of new hires at any one time. In some industries, laying off employees is difficult or unusual for contractual reasons (unions); in other industries, such as tourism and agriculture, seasonal layoffs and hirings are the norm.

3. **Workforce Utilization** An alternative to a workforce adjustment is a change in workforce utilization involving overtime and undertime. **Overtime** means that employees work longer than the regular workday or workweek and receive additional pay for the extra hours. It can be used to satisfy output requirements that cannot be completed on regular time. Overtime is expensive (typically 150 percent of the regular-time pay rate), and workers often do not want to work a lot of overtime for an extended period of time. Excessive overtime also can result in declining quality and productivity. On the other hand, it helps avoid the costly fringe benefits (such as health insurance, dental care, Social Security, retirement funds, paid vacations, and holidays) that come with hiring a new full-time employee. **Undertime** means that employees do not have enough work for the regular-time workday or workweek. For example, they cannot be fully utilized for eight hours per day or for five days per week. Undertime occurs when labor capacity exceeds demand requirements (net of anticipation inventory), and this excess capacity cannot or should not be used productively to build up inventory or to satisfy customer orders earlier than the delivery dates already promised.

   Undertime can either be paid or unpaid. An example of *paid undertime* is when employees are kept on the payroll rather than being laid off. In this scenario, employees work a full day and receive their full salary but are not as busy because of the light workload. Some companies use paid undertime (though they do not call it that) during slack periods, particularly with highly skilled, hard-to-replace employees or when there are obstacles to laying off workers. The disadvantages of paid undertime include the cost of paying for work not performed and lowered productivity.

**overtime**
The time that employees work that is longer than the regular workday or workweek for which they receive additional pay.

**undertime**
The situation that occurs when employees do not have enough work for the regular-time workday or workweek.

4. **Part-Time Workers** Another option apart from undertime is to hire part-time workers, who are paid only for the hours and days worked. Perhaps they only work during the peak times of the day or peak days of the week. Sometimes, part-time arrangements provide predictable work schedules, but in other cases workers are not called in if the workload is light. Such arrangements are more common in low-skill positions or when the supply of workers seeking such an arrangement is sufficient. Part-time workers typically do not receive fringe benefits.
5. **Subcontractors** Subcontractors can be used to overcome short-term capacity shortages, such as during peaks of the season or business cycle. Subcontractors can supply services, make components and subassemblies, or even assemble an entire product.
6. **Vacation Schedules** A manufacturer can shut down during an annual lull in sales, leaving a skeleton crew to cover operations and perform maintenance. Hospital employees might be encouraged to take all or part of their allowed vacation time during slack periods. The use of this alternative depends on whether the employer can mandate the vacation schedules of its employees. In any case, employees may be strongly discouraged from taking vacations during peak periods or encouraged to take vacations during slack periods.

# S&OP Strategies

Here we focus on supply options that define output rates and workforce levels. Two basic strategies are useful starting points in searching for the best plan.

## Chase Strategy

The **chase strategy** involves hiring and laying off employees to match the demand forecast over the planning horizon. Varying the workforce's regular-time capacity to equate supply to demand requires no inventory investment, overtime, or undertime. The drawbacks are the expense of continually adjusting workforce levels, the potential alienation of the workforce, and the loss of productivity and quality because of constant changes in the workforce.

**chase strategy**
A strategy that involves hiring and laying off employees to match the demand forecast.

## Level Strategy

The **level strategy** involves keeping the workforce constant (except possibly at the beginning of the planning horizon). It can vary its utilization to match the demand forecast via overtime, undertime (paid or unpaid), and vacation planning (i.e., paid vacations when demand is low). A constant workforce can be sized at many levels: Managers can choose to maintain a large workforce so as to minimize the planned use of overtime during peak periods (which, unfortunately, also maximizes the need for undertime during slack periods). Alternately, they can choose to maintain a smaller workforce and rely heavily on overtime during the peak periods (which places a strain on the workforce and endangers quality).

**level strategy**
A strategy that keeps the workforce constant, but varies its utilization via overtime, undertime, and vacation planning to match the demand forecast.


Charles Buchanan/Daily Free Press/Associated Press

The greeting card business is highly seasonal, which poses problems for the producers of those cards. Hallmark strives to keep a level strategy to maintain some security for their workforce. Here a shopper is selecting a Valentine's Day card at a Hallmark store in Kinston, NC.

These two "pure" strategies used alone usually do not produce the best sales and operations plan. It might not be best to keep the workforce exactly level or to vary it to exactly match forecasted demand on a period-by-period basis. The best strategy, therefore, usually is a **mixed strategy** that considers the full range of supply options. The chase strategy is limited to just hiring and laying off employees. The level strategy is limited to overtime, undertime, and vacation schedules. The mixed strategy opens things up to all options, including anticipation inventory, part-time workers, subcontractors, backorders, and stockouts.

## Constraints and Costs

An acceptable sales and operations plan must recognize relevant constraints or costs. Constraints can be either physical limitations or related to managerial policies. Examples of physical constraints might be machine capacities that limit maximum output or inadequate inventory storage space. Policy constraints might include limitations on the number of backorders or the use of subcontractors or overtime, as well as the minimum inventory levels needed to achieve desired safety stocks. Ethical issues may also be involved, such as excessive layoffs or required overtime.

**mixed strategy**

A strategy that considers the full range of supply options.

Typically, many plans can contain a number of constraints. Table 10.2 lists the costs that the planner considers when preparing sales and operations plans.

## Sales and Operations Planning as a Process

Sales and operations planning is a decision-making process, involving both planners and management. It is dynamic and continuing, as aspects of the plan are updated periodically when new information becomes available and new opportunities emerge. It is a cross-functional process that seeks a set of plans that all of a firm's functions can support. For each product family, decisions are made based on cost trade-offs, recent history, recommendations by planners and middle management, and the executive team's judgment.

Figure 10.3 shows a typical plan for a manufacturer. The plan is for one of the manufacturer's make-to-stock product families expressed in aggregate units. This simple format shows the interplay between demand and supply. The history on the left for January through March shows how forecasts are tracking actual sales and how well actual production conforms to the plan. The inventory projections are of particular interest to finance because they significantly affect the manufacturer's cash requirements. The last two columns on the top right show how current fiscal year sales projections match up with the current business plan.

This plan is projected out for 18 months, beginning with April. The forecast, operations, and inventory sections for the first 6 months are shown on a month-by-month basis. They then are shown on a

**TABLE 10.2 | TYPES OF COSTS WITH SALES AND OPERATIONS PLANNING**

| Cost | Definition |
|---|---|
| **Regular time** | Regular-time wages paid to employees plus contributions to benefits, such as health insurance, dental care, Social Security, retirement funds, and pay for vacations, holidays, and certain other types of absences. |
| **Overtime** | Wages paid for work beyond the normal workweek, typically 150 percent of regular-time wages (sometimes up to 200 percent for Sundays and holidays), exclusive of fringe benefits. Overtime can help avoid the extra cost of fringe benefits that come with hiring another full-time employee. |
| **Hiring and layoff** | Costs of advertising jobs, interviews, training programs for new employees, scrap caused by the inexperience of new employees, loss of productivity, and initial paperwork. Layoff costs include the costs of exit interviews, severance pay, retaining and retraining remaining workers and managers, and lost productivity. |
| **Inventory holding** | Costs that vary with the level of inventory investment: the costs of capital tied up in inventory, variable storage and warehousing costs, pilferage and obsolescence costs, insurance costs, and taxes. |
| **Backorder and stockout** | Additional costs to expedite past-due orders, the costs of lost sales, and the potential cost of losing a customer to a competitor (sometimes called loss of goodwill). |

**Artic Air Company—April Sales and Operations Plan**

*Family:* Medium window units (make-to-stock) *Unit of measure:* 100 units

| | HISTORY | | | | | | | | | 3rd | 4th | Mos | Fiscal Year | Business |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **SALES** | J | F | M | A* | M | J | J | A | S | 3 Mos** | 3 Mos | 13–18 | Projection ($000) | Plan ($000) |
| New forecast | 45 | 55 | 60 | 70 | 85 | 95 | 130 | 110 | 70 | 150 | 176 | 275 | $8,700 | $8,560 |
| Actual sales | 52 | 40 | 63 | | | | | | | | | | | |
| Diff for month | 7 | –15 | 3 | | | | | | | | | | | |
| Cum | | –8 | –5 | | | | | | | | | | | |
| **OPERATIONS** | | | | | | | | | | | | | | |
| New Plan | 75 | 75 | 75 | 75 | 75 | 85 | 85 | 85 | 75 | 177 | 225 | | | |
| Actual | 75 | 78 | 76 | | | | | | | | | | | |
| Diff for month | 0 | 3 | 1 | | | | | | | | | | | |
| Cum | | 3 | 4 | | | | | | | | | | | |
| **INVENTORY** | | | | | | | | | | | | | | |
| Plan | 85 | 105 | 120 | 125 | 115 | 105 | 60 | 35 | 40 | 198 | 321 | | | |
| Actual | 92 | 130 | 143 | | | | | | | | | | | |

**DEMAND ISSUES AND ASSUMPTIONS**

1. New product design to be launched in January of next year.

**SUPPLY ISSUES**

1. Vacations primarily in November and December.
2. Overtime in June–August.

* April is the first month of the planning horizon for this current plan. When next month's plan is developed, its first month in the planning horizon will be May, and the most recent month of the history will be April (with January no longer shown in the history).

** This column provides the sales, operations, and inventory totals for October through December. For example, the forecast of 150 units translates into an average of 50 units per month (or 150/3 = 50).

**FIGURE 10.3**
Sales and Operations Plan for Make-to-Stock Product Family

quarterly basis for the second 6 months. Finally, the totals for the last 6 months in the time horizon are given in just one column. This display gives more precision to the short term and yet gives coverage well into the future—all with a limited number of columns.

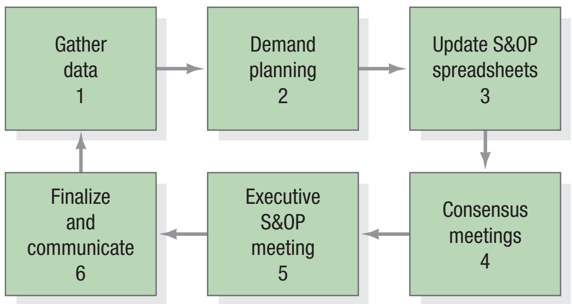
The medium window product family is a make-to-stock product that experiences highly seasonal demand. The operations plan is to build up anticipation inventory in the slack season of January through April, schedule vacations as much as possible in November and December, and use overtime in the peak season of June, July, and August. For example, the Operations plan increases monthly production from 75 to 85 for June through August, returns to 75 for September, and then drops to an average of only 59 (or 177/3) for October through December. Sales and operations plans use different formats depending on the production and inventory strategy. For an assemble-to-order strategy, the inventory does not consist of finished goods. Instead, it is inventory of standardized components and subassemblies built for the finishing and assembly operations. For a make-to-order strategy, the inventory section in the plan of Figure 10.3 is replaced by a section showing the planned and actual order backlog quantities.

Sales and operations plans for service providers are quite different. For one thing, their plan does not contain an inventory section, but focuses instead on the demand and supply of human resources. Forecasts are typically expressed in terms of employees required, with separate rows for regular time, overtime, vacations, part-time workers, and so on. Different departments or worker classifications replace product families.

The S&OP process itself, typically done on a monthly basis, consists of six basic steps. They are much like the forecasting process steps we discussed in Chapter 8, "Forecasting Demand."

***Step 1.*** Begin to "roll forward" the plan for the new planning horizon. Start preliminary work right after the month's end. Update files with actual sales, production, inventory, costs, and constraints.

***Step 2.*** Participate in the forecasting and demand planning process to create the authorized demand forecasts. For service providers, the forecasts are staff requirements for each workforce group. For example, a director of nursing in a hospital can develop a workload index for a nursing staff and translate a projection of the month-to-month patient load into an equivalent total amount of nursing care time—and thus the number of nurses—required for each month of the year.




***Step 3.*** Update the sales and operations plans for each family, recognizing relevant constraints and costs including availability of materials from suppliers, training facilities capable of handling only so many new hires at a time, machine capacities, or limited storage space. Policy constraints might include limitations on the number of backorders, or the use of subcontractors or overtime, as well as the minimum inventory levels needed to achieve desired safety stocks. Typically, many plans can satisfy a specific set of constraints.

Cultura Creative/Alamy

FUJIFILM Imaging Colorants makes inks and dyes, primarily for inkjet printer cartridges, and operates an effective S&OP process. It must coordinate between a U.S. finishing plant and a UK bulk manufacturing plant. Managers from all functions teleconference at the U.S. site with seven other UK managers. At this Partnership meeting (step 4 in the S&OP process), they review the demand, production, and inventory plans, as well as the projected working capital plan.

The planner searches for a plan that best balances costs, customer service, and workforce stability, which may necessitate revising the plan several times.

***Step 4.*** Have one or more consensus meetings with the stakeholders on how best to balance supply with demand. Participants could include the supply chain manager, plant manager, controller, purchasing manager, production control manager, or logistics manager. The goal is one set of recommendations to present at the firm's executive sales and operations planning (S&OP) meeting. Where agreement cannot be reached, prepare scenarios of alternative plans. Also prepare an updated financial view of the total business by rolling up the plans for all product families into a spreadsheet expressed in total dollars.

***Step 5.*** Present recommendations by product family at the executive S&OP meeting, which typically includes the firm's president and the vice presidents of functional areas. The plan is reviewed relative to the business plan, new product issues, special projects, and other relevant factors. The executives may ask for final changes to the plan, such as to balance conflicting objectives better. Acceptance of this authorized plan does not necessarily mean that everyone is in total agreement, but it does imply that everyone will work to achieve the plan.

***Step 6.*** Update the plans to reflect the outcome of the executive S&OP meeting, and communicate them to the important stakeholders for implementation. Important recipients include those who do resource planning, covered in the next chapter.

# Spreadsheets for Sales and Operations Planning

The sales and operations plan in Figure 10.3 does not show much on the supply options used in the operations plan or their cost implications. Here we discuss using spreadsheets that do just that. Supplement D, "Linear Programming Models," describes using the transportation method for production planning. Both spreadsheets and linear programming could be used on the side as a planner develops prospective plans in step 3 of the S&OP process.

Various spreadsheets can be used, including ones that you develop on your own. Here we work with the *Sales and Operations Planning with Spreadsheets* Solver in OM Explorer.

## Spreadsheets for a Manufacturer

Figure 10.4 shows a plan for a manufacturer, which uses all supply options except overtime. The top part of the spreadsheet shows the *input values* that consist of the forecasted demand requirements and the supply option choices period by period. Vary these "levers" as you search for better plans.

The next part of the spreadsheet (in green) shows the *derived values* that follow from the input values. The first row of derived values is called *utilized time*, which is that portion of the workforce's regular time that is paid for and productively used. In any period, the utilized time equals the workforce level minus undertime and vacation time. For example, in period 1 the utilized time is 94 (or $120 - 6 - 20$). Given the utilized time of the workforce, the *inventory* can be calculated by subtracting the forecast from the utilized time, adding last periods ending inventory, and adding subcontracting time and backorders. In period 1, assuming last period's ending inventory is zero, the inventory is 70 (or $94 - 24 + 0 + 0 + 0$). The *hires* and *layoffs* rows can be derived from the workforce levels. In this example, the workforce is increased for period 2 from its initial size of 120 employees to 158, which means that 38 employees are hired. Because the workforce size remains constant at 158 throughout the rest of the planning horizon, no other hirings or layoffs happen. When additional alternatives, such as vacations, inventory, and backorders are all possible, the overtime and undertime cannot be derived just from information on forecasted demand and workforce levels. Thus, undertime and overtime are shown as input values (rather than derived values) in the spreadsheet, and the user must be careful to specify consistent input values.

The final part of the spreadsheet, the *calculated values* of the plan, shows the plan's cost consequences. Along with qualitative considerations, the cost of each plan determines whether the plan is

| | 1 | 2 | 3 | 4 | 5 | 6 | Total |
|---|---|---|---|---|---|---|---|
| **Inputs** | | | | | | | |
| Forecasted demand | 24 | 142 | 220 | 180 | 136 | 168 | 870 |
| Workforce level | 120 | 158 | 158 | 158 | 158 | 158 | 910 |
| Undertime | 6 | 0 | 0 | 0 | 0 | 0 | 6 |
| Overtime | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Vacation time | 20 | 6 | 0 | 0 | 4 | 10 | 40 |
| Subcontracting time | 0 | 0 | 0 | 0 | 0 | 6 | 6 |
| Backorders | 0 | 0 | 0 | 4 | 0 | 0 | 4 |
| **Derived** | | | | | | | |
| Utilized time | 94 | 152 | 158 | 158 | 154 | 148 | 864 |
| Inventory | 70 | 80 | 18 | 0 | 14 | 0 | 182 |
| Hires | 0 | 38 | 0 | 0 | 0 | 0 | 38 |
| Layoffs | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **Calculated** | | | | | | | |
| Utilized time cost | $376,000 | $608,000 | $632,000 | $632,000 | $616,000 | $592,000 | $3,456,000 |
| Undertime cost | $24,000 | $0 | $0 | $0 | $0 | $0 | $24,000 |
| Overtime cost | $0 | $0 | $0 | $0 | $0 | $0 | $0 |
| Vacation time cost | $80,000 | $24,000 | $0 | $0 | $16,000 | $40,000 | $160,000 |
| Inventory cost | $2,800 | $3,200 | $720 | $0 | $560 | $0 | $7,280 |
| Backorders cost | $0 | $0 | $0 | $4,000 | $0 | $0 | $4,000 |
| Hiring cost | $0 | $91,200 | $0 | $0 | $0 | $0 | $91,200 |
| Layoff cost | $0 | $0 | $0 | $0 | $0 | $0 | $0 |
| Subcontracting cost | $0 | $0 | $0 | $0 | $0 | $43,200 | $43,200 |
| **Total cost** | $482,800 | 726,400 | 632,720 | 636,000 | 632,560 | 675,200 | $3,785,680 |

**FIGURE 10.4** Manufacturer's Plan Using a Spreadsheet and Mixed Strategy

satisfactory or whether a revised plan should be considered. When seeking clues about how to improve a plan already evaluated, we identify its highest cost elements. Revisions that would reduce these specific costs might produce a new plan with lower overall costs. Spreadsheet programs make analyzing these plans easy, and they present a whole new set of possibilities for developing sound sales and operations plans.

The plan in Figure 10.4 definitely is for a manufacturer because it uses inventory to advantage, particularly in the first two periods. It is a mixed strategy, and not just because it uses anticipation inventory, backorders, and subcontracting. The workforce level changes in period 2, but it does not exactly match the forecasted demand as with a chase strategy. It has some elements of the level strategy, because undertime and vacation time are part of the plan, but it does not rely exclusively on these supply options.

Care must be taken to recognize differences in how inputs are measured. The workforce level might be expressed as the number of employees, but the forecasted demand and inventory are expressed as units of the product. The OM Explorer spreadsheets require a common unit of measure, so we must translate some of the data prior to entering the input values. Perhaps the easiest approach is to express the forecasted demand and supply options as *employee-period equivalents*. If demand forecasts are given as units of product, we can convert them to employee-period equivalents by dividing them by the productivity of a worker. For example, if the demand is for 1,500 units of product and the average employee produces 100 units in one period, the demand requirement is 15 employee-period equivalents.

## Spreadsheeets for a Service Provider

The same spreadsheets can be used by service providers, except anticipation inventory is not an option. You can unprotect the sheet and then hide the rows that are not relevant. It is useful not to hide the inventory row until the end, however, because positive or negative values signal an inconsistency in your plan. Whereas Figure 10.4 shows a good plan found after several revisions, here we illustrate with Example 10.1 how to find a good plan for a service provider beginning with the chase and level (ignoring vacations) strategies. These plans can provide insights that lead to even better mixed strategy plans.

### EXAMPLE 10.1 Using the Chase and Level Strategies as Starting Points

The manager of a large distribution center must determine how many part-time stockpickers to maintain on the payroll. She wants to develop a staffing plan that minimizes total costs, and wants to begin with the chase strategy and level strategy. For the level strategy, she wants to first try the workforce level that meets demand with the minimum use of undertime and not consider vacation scheduling.

First, the manager divides the next year into six time periods, each one 2 months long. Each part-time employee can work a maximum of 20 hours per week on regular time, but the actual number can be less. Instead of paying undertime, each worker's day is shortened during slack periods. Once on the payroll, each worker is used each day, but they may work only a few hours. Overtime can be used during peak periods.

The distribution center's forecasted demand is shown as the number of part-time employees required for each time period at the maximum regular time of 20 hours per week. For example, in period 3, an estimated 18 part-time employees working 20 hours per week on regular time will be needed.

| | 1 | 2 | 3 | 4 | 5 | 6 | Total |
|---|---|---|---|---|---|---|---|
| Forecasted demand* | 6 | 12 | 18 | 15 | 13 | 14 | 78 |

*Number of part-time employees

Currently, 10 part-time clerks are employed. They have not been subtracted from the forecasted demand shown. Constraints and cost information are as follows:

**a.** The size of training facilities limits the number of new hires in any period to no more than 10.

**b.** No backorders are permitted; demand must be met each period.

**c.** Overtime cannot exceed 20 percent of the regular-time capacity (that is, 4 hours) in any period. Therefore, the most that any part-time employee can work is $1.20(20) = 24$ hours per week.

**d.** The following costs can be assigned:

| | |
|---|---|
| Regular-time wage rate | $2,000 per time period at 20 hours per week |
| Overtime wages | 150 percent of the regular-time rate |
| Hires | $1,000 per person |
| Layoffs | $500 per person |

AP Photos

Framed by thousands of ski poles, a part-time worker sorts and inventories new products in the receiving department of REI's distribution center in Sumner, Washington. REI employs a high percentage of part-time workers, many of whom are college students. They tend to be young people who participate in outdoor sports and are familiar with the equipment that REI sells.

MyOMLab

Tutor 10.1 in MyOMLab provides a new example for planning using the chase strategy with hiring and layoffs.

**SOLUTION**

**a.** Chase Strategy

This strategy simply involves adjusting the workforce as needed to meet demand, as shown in Figure 10.5. Rows in the spreadsheet that do not apply (such as inventory and vacations) are hidden. The workforce level row is identical to the forecasted demand row. A large number of hirings and layoffs begin with laying off four part-time employees immediately because the current staff is 10 and the staff level required in period 1 is only six. However, many employees, such as college students, prefer part-time work. The total cost is $173,500, and most of the cost increase comes from frequent hiring and layoffs, which add $17,500 to the cost of utilized regular-time costs.

| | 1 | 2 | 3 | 4 | 5 | 6 | Total |
|---|---|---|---|---|---|---|---|
| **Inputs** | | | | | | | |
| Forecasted demand | 6 | 12 | 18 | 15 | 13 | 14 | 78 |
| Workforce level | 6 | 12 | 18 | 15 | 13 | 14 | 78 |
| Undertime | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Overtime | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **Derived** | | | | | | | |
| Utilized time | 6 | 12 | 18 | 15 | 13 | 14 | 78 |
| Hires | 0 | 6 | 6 | 0 | 0 | 1 | 13 |
| Layoffs | 4 | 0 | 0 | 3 | 2 | 0 | 9 |
| **Calculated** | | | | | | | |
| Utilized time cost | $12,000 | $24,000 | $36,000 | $30,000 | $26,000 | $28,000 | $156,000 |
| Undertime cost | $0 | $0 | $0 | $0 | $0 | $0 | $0 |
| Hiring cost | $0 | $6,000 | $6,000 | $0 | $0 | $1,000 | $13,000 |
| Layoff cost | $2,000 | $0 | $0 | $1,500 | $1,000 | $0 | $4,500 |
| **Total cost** | $14,000 | 30,000 | 42,000 | 31,500 | 27,000 | 29,000 | $173,500 |

◀ **FIGURE 10.5** Spreadsheet for Chase Strategy

**b.** Level Strategy

To minimize undertime, the maximum use of overtime possible must occur in the peak period. For this particular level strategy (other workforce options are possible), the most overtime that the manager can use is 20 percent of the regular-time capacity, *w*, so

$$1.20w = 18 \text{ employees required in peak period (period 3)}$$

$$w = \frac{18}{1.20} = 15 \text{ employees}$$

A 15-employee staff size minimizes the amount of undertime for this level strategy. Because the staff already includes 10 part-time employees, the manager should immediately hire five more. The complete plan is shown in Figure 10.6. The total cost is $164,000, which seems reasonable because the minimum conceivable cost is only $156,000 $(78 \text{ employee-periods} \times \$2{,}000/\text{employee-period})$. This cost could be achieved only if the manager found a way to cover the forecasted demand for all 78-employee periods with regular time. The plan seems reasonable primarily because it involves the use of large amounts of undertime (15-employee periods), which in this example are unpaid.

**MyOMLab**
Tutor 10.2 in MyOMLab provides a new example for planning using the level strategy with overtime and undertime.

| | 1 | 2 | 3 | 4 | 5 | 6 | Total |
|---|---|---|---|---|---|---|---|
| **Inputs** | | | | | | | |
| Forecasted demand | 6 | 12 | 18 | 15 | 13 | 14 | 78 |
| Workforce level | 15 | 15 | 15 | 15 | 15 | 15 | 90 |
| Undertime | 9 | 3 | 0 | 0 | 2 | 1 | 15 |
| Overtime | 0 | 0 | 3 | 0 | 0 | 0 | 3 |
| **Derived** | | | | | | | |
| Utilized time | 6 | 12 | 15 | 15 | 13 | 14 | 75 |
| Hires | 5 | 0 | 0 | 0 | 0 | 0 | 5 |
| Layoffs | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **Calculated** | | | | | | | |
| Utilized time cost | $12,000 | $24,000 | $30,000 | $30,000 | $26,000 | $28,000 | $150,000 |
| Undertime cost | $0 | $0 | $0 | $0 | $0 | $0 | $0 |
| Overtime cost | $0 | $0 | $9,000 | $0 | $0 | $0 | $9,000 |
| Hiring cost | $5,000 | $0 | $0 | $0 | $0 | $0 | $5,000 |
| Layoff cost | $0 | $0 | $0 | $0 | $0 | $0 | $0 |
| **Total cost** | $17,000 | 24,000 | 39,000 | 30,000 | 26,000 | 28,000 | $164,000 |

◀ **FIGURE 10.6** Spreadsheet for Level Strategy

### DECISION POINT

The manager, now having a point of reference with which to compare other plans, decided to evaluate some other plans before making a final choice, beginning with the chase strategy. The only way to reduce costs is somehow to reduce the premium in period 3 for three overtime employee periods $(3 \text{ employee-periods} \times \$3{,}000/\text{employee-period})$ or to reduce the hiring cost of five employees $(5 \text{ hires} \times \$1{,}000/\text{person})$. Nonetheless, better solutions may be possible. For example, undertime can be reduced by delaying the hiring until period 2 because the current workforce is sufficient until then. This delay would decrease the amount of unpaid undertime, which is a qualitative improvement. See Active Model 10.1 for additional insights.

**MyOMLab**
Active Model 10.1 in MyOMLab shows the impact of changing the workforce level, the cost structure, and overtime capacity.

# Scheduling

Scheduling is the last step in Figure 10.2. It takes the operations and scheduling process from planning to execution, and is where the "rubber meets the road." This important aspect of supply chain management is itself a process. It requires gathering data from sources such as demand forecasts or specific customer orders, resource availability from the sales and operations plan, due dates for resource or material requirements from resource planning activities, and specific constraints to be reckoned with from employees and customers. It then involves generating a schedule for the supply of resources or materials to meet the needs determined in resource planning. Here we cover job and facility scheduling, workforce scheduling, job sequencing at a workstation, and software support.

## Job and Facility Scheduling

Schedules can be displayed in various ways. For different jobs or activities, schedules can simply list the job due dates, show in a table their start and finish times, or show in a graph their start and finish times. The *Gantt chart* uses the third approach. Figure 7.4 in Chapter 7, "Managing Effective Projects," demonstrates how a "picture can be worth a thousand words" in managing projects. Associates not familiar with scheduling techniques can still grasp the essence of the plan by just looking at such a chart. This tool can be used to monitor the progress of work and to view the load on workstations or other facilities. The chart takes two basic forms: (1) The job or activity progress chart, which can be used to monitor and revise schedules, and (2) the workstation chart, which can be used to schedule the capacity of facilities.

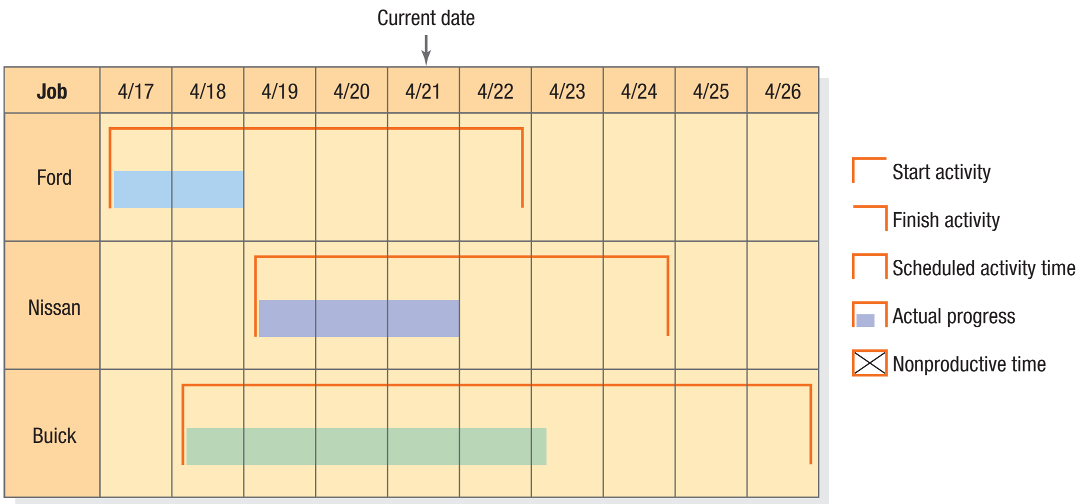
Gennadiy Poznyakov/Fotolia

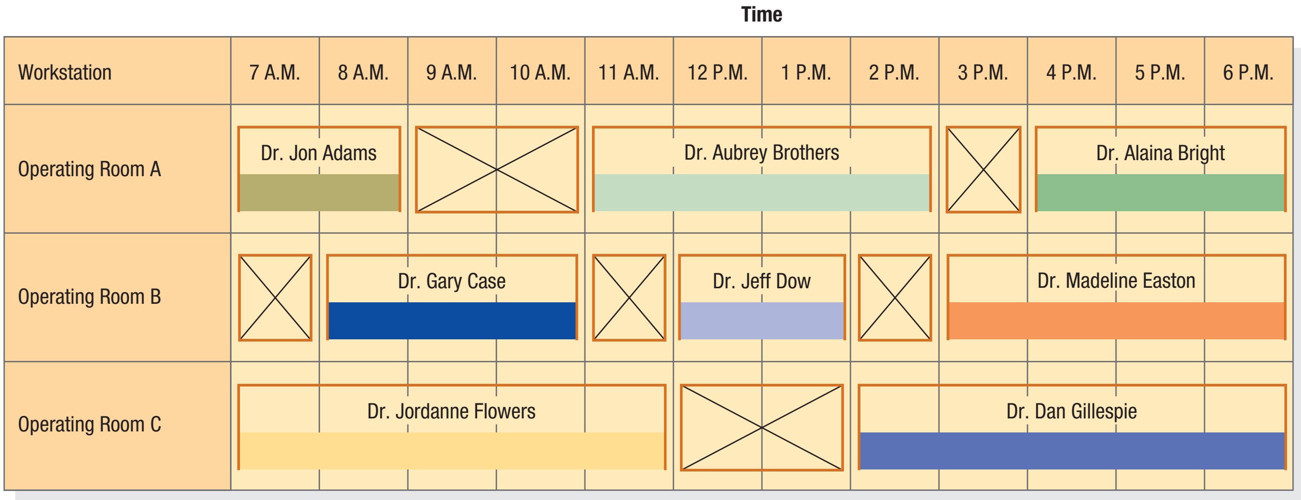
Operating room represents a fixed capacity that must be scheduled carefully to avoid unused capacity. Any time not used by surgeons is time lost forever.

**Gantt Progress Chart** The *Gantt progress chart* graphically displays the current status of each job or activity relative to its scheduled completion date. For example, suppose that an automobile parts manufacturer has three jobs under way, one each for Ford, Nissan, and Buick. The actual status of these orders is shown by the colored bars in Figure 10.7; the red lines indicate the desired schedule for the start and finish of each job. For the current date, April 21, this Gantt chart shows that the Ford order is behind schedule because operations has completed only the work scheduled through April 18. The Nissan order is exactly on schedule, and the Buick order is ahead of schedule.

**Gantt Workstation Chart** Figure 10.8 shows a *Gantt workstation chart* of the operating rooms at a hospital for a particular day. Using the same notation as in Figure 10.7, the chart shows the load on the operating rooms and the nonproductive time. The time slots assigned to each doctor include the time needed to clean the room prior to the next surgery. The chart can be used to identify time slots for unscheduled emergency surgeries. It can also be used to accommodate requests to change the time of surgeries.

**FIGURE 10.7** ▶
Gantt Progress Chart for an Auto Parts Company






▲ **FIGURE 10.8**
Gantt Workstation Chart for Operating Rooms at a Hospital

For example, Dr. Flowers may be able to change the start of her surgery to 2 P.M. by swapping time slots with Dr. Gillespie in operating room C or by asking Dr. Brothers to start her surgery one hour earlier in operating room A and asking Dr. Bright to schedule her surgery for the morning in operating room C. In any event, the hospital administrator would have to get involved in rescheduling the surgeries.

## Workforce Scheduling

Another way to manage capacity is **workforce scheduling**, which is a type of scheduling that determines when employees work. Of particular interest are situations when not all employees work the same five days a week, and same eight hours per day. The schedule specifies the on-duty and off-duty periods for each employee over a certain time period, as in assigning postal clerks, nurses, pilots, attendants, or police officers to specific workdays and shifts. This approach is used when customers demand quick response and total demand can be forecasted with reasonable accuracy. In these instances, capacity is adjusted to meet the expected loads on the service system. Managerial Practice 10.1 shows how Air New Zealand derives its flight and crew schedules from higher-level plans.

**workforce scheduling**
A type of scheduling that determines when employees work.

### MANAGERIAL PRACTICE 10.1 Scheduling at Air New Zealand

How important is scheduling to an airline company? Certainly, customer satisfaction regarding on-time schedule performance is critical in a highly competitive industry such as air transportation. In addition, airlines lose a lot of money when expensive equipment, such as an aircraft, is idle. Flight and crew scheduling, however, is a complex process. For example, Air New Zealand is a group of five airlines with a combined fleet of 105 aircraft, with another 21 more on order. The average utilization is 8:28 hours per day. It has undergone an $800 million upgrade to its long-haul service, refitting its Boeing 747 fleet and adding eight new Boeing 777-200 aircraft for flights to North America. It directly serves 50 ports—26 domestic and 24 international within 15 countries. It carries 13.4 million passengers annually, and its network incorporates flight times ranging from 15 minutes to 13 hours.

Operations planning and scheduling at the aggregate level begins with a market plan that identifies the new and existing flight segments that are needed to remain competitive. This general plan is further refined to a three-year plan and then is put into an annual budget in which flight segments have specific departure and arrival times. Twenty-six weeks prior to the day of operation, the timetable for all the flights Air New Zealand will fly is created. This is followed by the assignment of aircraft types to flights and the development of aircraft routes. Six weeks prior to the day of operation schedulers determine sequences of flights that can be flown in a feasible way at minimum cost. The two types of crews—pilots and attendants—each come with its own set of constraints. Pilots, for example, cannot be scheduled for

Tupungato/Shutterstock

Operations planning and scheduling at an airline like Air New Zealand goes through several stages to match supply with demand, from aggregate plans to short-term schedules. Even after finalizing flights and crew roster schedules, severe weather conditions or mechanical failures can cause last-minute changes. Air New Zealand's long-term competitive strength depends on how well it performs this process.

more than 35 hours in a 7-day week and no more than 100 hours in a 28-day cycle. They also must have a 36-hour break every 7 days and 30 days off in an 84-day cycle. Each pilot's tour of duty begins and ends at a crew base and consists of an alternating sequence of duty periods and rest periods, with duty periods including one or more flights. Finally, four weeks prior to the day of operation, actual rosters for an individual crew member are developed. The scheduler must ensure that each flight has a qualified crew and that each crew member has a feasible tour of duty over the roster period. From the crew's point of view, it is also important to satisfy as many crew requests and preferences as possible.

*Source:* Rasmussen, Matias Sevel, Lusby, Richard M., and Ryan, David M., "A Subsequence Generation Approach for the Airline Crew Pairing Problem," **http://www.agifors.org/award/submissions2011/MatiasRasmussen**; "Service Scheduling at Air New Zealand," *Operations Management 10e Video Library* (Upper Saddle River, NJ: Prentice Hall, 2010); **http://www.airnewzealand.com** (May, 2014). See video in MyOMLab.

As the Air New Zealand example shows, workforce schedules translate the staffing plan, which has been decomposed into specific, time-based staff requirements, into schedules of work for each employee. Determining the workdays for each employee in itself does not make the staffing plan operational. Daily workforce requirements, stated in aggregate terms in the staffing plan and decomposed in the resource requirements plan, must be satisfied. The workforce capacity available each day must meet or exceed daily workforce requirements. If it does not, the scheduler must try to rearrange days off until the requirements are met. If no such schedule can be found, management might have to change the staffing plan and hire more employees, authorize overtime hours, or allow for larger backlogs.

**Constraints** The technical constraints imposed on the workforce schedule are the resources provided by the staffing plan and the requirements placed on the operating system. However, other constraints, including legal and behavioral considerations, also can be imposed. For example, Air New Zealand is required to have at least a minimum number of flight attendants on duty at all times. Similarly, a minimum number of fire and safety personnel must be on duty at a fire station at all times. Such constraints limit management's flexibility in developing workforce schedules.

The constraints imposed by the psychological needs of workers complicate scheduling even more. Some of these constraints are written into labor agreements. For example, an employer might agree to give employees a certain number of consecutive days off per week or to limit employees' consecutive workdays to a certain maximum. Other provisions might govern the allocation of vacations, days off for holidays, or rotating shift assignments. In addition, the preferences of the employees themselves need to be considered.

**rotating schedule**

A schedule that rotates employees through a series of workdays or hours.

**fixed schedule**

A schedule that calls for each employee to work the same days and hours each week.

One way that managers deal with certain undesirable aspects of scheduling is to use a **rotating schedule**, which rotates employees through a series of workdays or hours. Thus, over a period of time, each person has the same opportunity to have weekends and holidays off and to work days, as well as evenings and nights. A rotating schedule gives each employee the next employee's schedule the following week. In contrast, a **fixed schedule** calls for each employee to work the same days and hours each week.

**Developing a Workforce Schedule** Suppose that we are interested in developing a workforce schedule for a company that operates seven days a week and provides each employee with two consecutive days off. In this section, we demonstrate a method that recognizes this constraint. The objective is to identify the two consecutive days off for each employee that will minimize the amount of total slack capacity, thereby maximizing the utilization of the workforce. The work schedule for each employee, then, is the five days that remain after the two days off have been determined. The procedure involves the following steps.

***Step 1.*** From the schedule of net requirements for the week, derived from the resource plan in Level 2, find all the pairs of consecutive days, excluding the day (or days) with the maximum daily requirement. Select the unique pair that has the lowest total requirements for the two days. In some unusual situations, all pairs may contain a day with the maximum requirements. If so, select the pair with the lowest total requirements. Suppose that the numbers of employees required are

| | | |
|---|---|---|
| Monday: 8 | Thursday: 12 | Saturday: 4 |
| Tuesday: 9 | Friday: 7 | Sunday: 2 |
| Wednesday: 2 | | |

The maximum daily requirement is 12 employees, on Thursday. The consecutive pair with the lowest total requirements is Saturday and Sunday, with $4 + 2 = 6$.

***Step 2.*** If a tie occurs, choose one of the tied pairs, consistent with the provisions written into the labor agreement, if any. Alternatively, the tie could be broken by asking the employee being scheduled to make the choice. As a last resort, the tie could be broken arbitrarily. For example, preference could be given to Saturday–Sunday pairs.

***Step 3.*** Assign the employee the selected pair of days off. Subtract the requirements satisfied by the employee from the net requirements for each day the employee is to work. In this example, the employee is assigned Saturday and Sunday off. After requirements are subtracted, Monday's requirement is 7, Tuesday's is 8, Wednesday's is 1, Thursday's is 11, and Friday's is 6. Saturday's and Sunday's requirements do not change because no employee is yet scheduled to work those days.

***Step 4.*** Repeat steps 1 through 3 until all the requirements have been satisfied or a certain number of employees have been scheduled.

This method reduces the amount of slack capacity assigned to days with low requirements and forces the days with high requirements to be scheduled first. It also recognizes some of the behavioral and contractual aspects of workforce scheduling in the tie-breaking rules.

## EXAMPLE 10.2 Developing a Workforce Schedule

The Amalgamated Parcel Service is open seven days a week. The schedule of requirements is

| Day | M | T | W | Th | F | S | Su |
|---|---|---|---|---|---|---|---|
| Required number of employees | 6 | 4 | 8 | 9 | 10 | 3 | 2 |

**MyOMLab**

Tutor 10.3 in MyOMLab provides a new example to practice workforce scheduling.

The manager needs a workforce schedule that provides two consecutive days off and minimizes the amount of total slack capacity. To break ties in the selection of off days, the scheduler gives preference to Saturday and Sunday if it is one of the tied pairs. If not, she selects one of the tied pairs arbitrarily.

### SOLUTION

Friday contains the maximum requirements, and the pair S–Su has the lowest total requirements. Therefore, Employee 1 is scheduled to work Monday through Friday.

Note that Friday still has the maximum requirements and that the requirements for the S–Su pair are carried forward because these are Employee 1's days off. These updated requirements are the ones the scheduler uses for the next employee.

The day-off assignments for the employees are shown in the following table.

**SCHEDULING DAYS OFF**

| M | T | W | Th | F | S | Su | Employee | Comments |
|---|---|---|---|---|---|---|---|---|
| 6 | 4 | 8 | 9 | 10 | 3 | 2 | **1** | The S–Su pair has the lowest total requirements. Assign Employee **1** to a Monday through Friday schedule and update the requirements. |
| 5 | 3 | 7 | 8 | 9 | 3 | 2 | **2** | The S–Su pair has the lowest total requirements. Assign Employee **2** to a Monday through Friday schedule and update the requirements. |
| 4 | 2 | 6 | 7 | 8 | 3 | 2 | **3** | The S–Su pair has the lowest total requirements. Assign Employee **3** to a Monday through Friday schedule and update the requirements. |
| 3 | 1 | 5 | 6 | 7 | 3 | 2 | **4** | The M–T pair has the lowest total requirements. Assign Employee **4** to a Wednesday through Sunday schedule and update the requirements. |
| 3 | 1 | 4 | 5 | 6 | 2 | 1 | **5** | The S–Su pair has the lowest total requirements. Assign Employee **5** to a Monday through Friday schedule and update the requirements. |
| 2 | 0 | 3 | 4 | 5 | 2 | 1 | **6** | The M–T pair has the lowest total requirements. Assign Employee **6** to a Wednesday through Sunday schedule and update the requirements. |
| 2 | 0 | 2 | 3 | 4 | 1 | 0 | **7** | The S–Su pair has the lowest total requirements. Assign Employee **7** to a Monday through Friday schedule and update the requirements. |
| 1 | 0 | 1 | 2 | 3 | 1 | 0 | **8** | Four pairs have the minimum requirement and the lowest total: S–Su, Su–M, M–T, and T–W. Choose the S–Su pair according to the tie-breaking rule. Assign Employee **8** to a Monday through Friday schedule and update the requirements. |
| 0 | 0 | 0 | 1 | 2 | 1 | 0 | **9** | Arbitrarily choose the Su–M pair to break ties because the S–Su pair does not have the lowest total requirements. Assign Employee **9** to a Tuesday through Saturday schedule and update the requirements. |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | **10** | Choose the S–Su pair according to the tie-breaking rule. Assign Employee **10** to a Monday through Friday schedule. |

In this example, Friday always has the maximum requirements and should be avoided as a day off. The final schedule for the employees is shown in the following table.

**FINAL SCHEDULE**

| Employee | M | T | W | Th | F | S | Su | Total |
|---|---|---|---|---|---|---|---|---|
| 1 | X | X | X | X | X | off | off | |
| 2 | X | X | X | X | X | off | off | |
| 3 | X | X | X | X | X | off | off | |
| 4 | off | off | X | X | X | X | X | |
| 5 | X | X | X | X | X | off | off | |
| 6 | off | off | X | X | X | X | X | |
| 7 | X | X | X | X | X | off | off | |
| 8 | X | X | X | X | X | off | off | |
| 9 | off | X | X | X | X | X | off | |
| 10 | X | X | X | X | X | off | off | |
| Capacity, C | 7 | 8 | 10 | 10 | 10 | 3 | 2 | 50 |
| Requirements, R | 6 | 4 | 8 | 9 | 10 | 3 | 2 | 42 |
| Slack, C – R | 1 | 4 | 2 | 1 | 0 | 0 | 0 | 8 |

**DECISION POINT**

With its substantial amount of slack capacity, the schedule is not unique. Employee 9, for example, could have Sunday and Monday, Monday and Tuesday, or Tuesday and Wednesday off without causing a capacity shortage. Indeed, the company might be able to get by with one fewer employee because of the total of eight slack days of capacity. However, all 10 employees are needed on Fridays. If the manager were willing to get by with only nine employees on Fridays or if someone could work one day of overtime on a rotating basis, he would not need Employee 10. As indicated in the table, the net requirement left for Employee 10 to satisfy amounts to only one day, Friday. Thus, Employee 10 can be used to fill in for vacationing or sick employees.

## Sequencing Jobs at a Workstation

**sequencing**

Determining the order in which jobs or customers are processed in the waiting line at a workstation.

**priority sequencing rule**

A rule that specifies the job or customer processing sequence when several jobs are waiting in line at a workstation.

**first-come, first-served (FCFS)**

A priority sequencing rule that specifies that the job or customer arriving at the workstation first has the highest priority.

**earliest due date (EDD)**

A priority sequencing rule that specifies that the job or customer with the earliest due date is the next job to be processed.

Another aspect of scheduling is sequencing work at workstations. **Sequencing** determines the order in which jobs are processed in the waiting line at a workstation. In this regard, the term "job" refers to either production orders or human customers. When combined with the expected processing times, the sequence allows you to estimate the start and finish times of each job and use a workstation Gantt chart to display the schedule.

**Priority Sequencing Rules** One way to determine what job to process next is with the help of a **priority sequencing rule**. The following two priority sequencing rules are commonly used in practice.

- *First-Come, First-Served.* The job arriving at the workstation first has the highest priority under a **first-come, first-served (FCFS)** rule. This rule is "fair" in that each job is treated equally, with no one stepping ahead of others already in line. It is commonly used at service facilities and is the rule that was assumed in Supplement B, "Waiting Lines Models."
- *Earliest Due Date.* The job with the **earliest due date (EDD)** is the next one to be processed. The *due date* specifies when work on a job should be finished. Due dates are commonly used by manufacturers and suppliers in the supply chain. For example, a product cannot be assembled until all of its purchased and produced components are available. If these components were not already in inventory, they must be ordered prior to when the product assembly can begin. Their due date is the start date for assembling the product to be assembled. This simple relationship is fundamental to coordinating with suppliers and with the manufacturer's own shops in the supply chain. It is also the key to **expediting**, which is the process of completing a job sooner than would otherwise be done. Expediting can be done by revising the due date, moving the job to the front of the waiting line, making a special appeal by phone or e-mail to the supplier, adding extra capacity, or even putting a red tag on the job that says the job is urgent.

Neither rule guarantees finding an optimal solution. Different sequences found by trial and error can produce better schedules. In fact, there are multiple performance measures for judging a schedule. A schedule that does well on one measure may do poorly on another.

**Performance Measures** The quality of a schedule can be judged in various ways. Two commonly used performance measures are flow time and past due.

- *Flow Time.* The amount of time a job spends in the service or manufacturing system is called **flow time**. It is the sum of the waiting time for servers or machines; the process time, including setups; the time spent moving between operations; and delays resulting from machine breakdowns, unavailability of facilitating goods or components, and the like. Flow time is sometimes referred to as *throughput time* or *time spent in the system, including service*. For a set of jobs to be processed at a single workstation, a job's flow time is

$$\text{Flow time} = \text{Finish time} + \text{Time since job arrived at workstation}$$

When using this equation, we assume for convenience that the first job scheduled starts at time zero (0). At time 0, all the jobs were available for processing at the workstation.

- *Past Due.* The measure **past due** can be expressed as the amount of time by which a job missed its due date (also referred to as **tardiness**) or as the percentage of total jobs processed over some period of time that missed their due dates. Minimizing these past due measures supports the competitive priorities of cost (penalties for missing due dates), quality (perceptions of poor service), and time (on-time delivery).

**expediting**

The process of completing a job or finishing with a customer sooner than would otherwise be done.

**flow time**

The amount of time a job spends in the service or manufacturing system.

**past due**

The amount of time by which a job missed its due date.

**tardiness**

See past due.

## EXAMPLE 10.3 Using the FCFS and EDD Priority Sequencing Rules

Currently a consulting company has five jobs in its backlog. The time since the order was placed, processing time, and promised due dates are given in the following table. Determine the schedule by using the FCFS rule, and calculate the average days past due and flow time. How can the schedule be improved, if average flow time is the most critical?

| Customer | Time Since Order Arrived (days ago) | Processing Time (days) | Due Date (days from now) |
|---|---|---|---|
| A | 15 | 25 | 29 |
| B | 12 | 16 | 27 |
| C | 5 | 14 | 68 |
| D | 10 | 10 | 48 |
| E | 0 | 12 | 80 |

### SOLUTION

**a.** The FCFS rule states that Customer A should be the first one in the sequence, because that order arrived earliest—15 days ago. Customer E's order arrived today, so it is processed last. The sequence is shown in the following table, along with the days past due and flow times.

| Customer Sequence | Start Time (days) | | Processing Time (days) | | Finish Time (days) | Due Date | Days Past Due | Days Ago Since Order Arrived | Flow Time (days) |
|---|---|---|---|---|---|---|---|---|---|
| A | 0 | + | 25 | = | 25 | 29 | **0** | 15 | **40** |
| B | 25 | + | 16 | = | 41 | 27 | **14** | 12 | **53** |
| D | 41 | + | 10 | = | 51 | 48 | **3** | 10 | **61** |
| C | 51 | + | 14 | = | 65 | 68 | **0** | 5 | **70** |
| E | 65 | + | 12 | = | 77 | 80 | **0** | 0 | **77** |

The *finish time* for a job is its start time plus the processing time. Its finish time becomes the start time for the next job in the sequence, assuming that the next job is available for immediate processing. The days past due for a job is zero (0) if its due date is equal to or exceeds the finish time. Otherwise it equals the shortfall. The flow time for each job equals its finish time plus the number of days ago since the order first arrived at the workstation. For example, Customer C's flow time is its scheduled finish time of 65 days plus the 5 days since the order arrived, or 70 days. The days past due and average flow time performance measures for the FCFS schedule are

$$\text{Average days past due} = \frac{0 + 14 + 3 + 0 + 0}{5} = 3.4 \text{ days}$$

$$\text{Average flow time} = \frac{40 + 53 + 61 + 70 + 77}{5} = 60.2 \text{ days}$$

MyOMLab

**b.** The average flow time can be reduced. One possibility is the sequence shown in the following table, which uses the Shortest Processing Time (SPT) rule, which is one of several rules developed more fully in MyOMLab Supplement J, "Operations Scheduling." (For still another possibility, see Solved Problem 3, which applies the EDD rule.)

| Customer Sequence | Start Time (days) | | Processing Time (days) | | Finish Time (days) | Due Date | Days Past Due | Days Ago Since Order Arrived | Flow Time (days) |
|---|---|---|---|---|---|---|---|---|---|
| D | 0 | + | 10 | = | 10 | 48 | **0** | 10 | **20** |
| E | 10 | + | 12 | = | 22 | 80 | **0** | 0 | **22** |
| C | 22 | + | 14 | = | 36 | 68 | **0** | 5 | **41** |
| B | 36 | + | 16 | = | 52 | 27 | **25** | 12 | **64** |
| A | 52 | + | 25 | = | 77 | 29 | **48** | 15 | **92** |

$$\text{Average days past due} = \frac{0 + 0 + 0 + 25 + 48}{5} = 14.6 \text{ days}$$

$$\text{Average flow time} = \frac{20 + 22 + 41 + 64 + 92}{5} = 47.8 \text{ days}$$

This schedule reduces the average flow time from 60.2 to 47.8 days—a 21 percent improvement. However, the past due times for jobs A and B have increased.

**DECISION POINT**

Management decided to use a modified version of the second schedule, adding overtime when Customer B is processed. Further, Customer A agreed to extend its due date to 77 days, because in this case the advanced warning allowed it to reschedule its own operations with little problem.

Nataliya Hora/Shutterstock

Scheduling an automobile assembly line is a challenging task, and requires the help of sophisticated software. Even with robots doing much of the work, parts and components must be timed to arrive at the precise time the frame is available for assembly. Here robotic arms assemble a Skoda in a Czech factory.

## Software Support

Computerized scheduling systems are available to cope with the complexity of workforce scheduling, such as the myriad constraints and concerns at Air New Zealand. In some types of firms, such as telephone companies, mail-order catalog houses, or emergency hotline agencies, employees must be on duty 24 hours a day, 7 days a week.

Sometimes a portion of the staff is part time, which allows management a great deal of flexibility but adds considerable complexity to the scheduling requirements. The flexibility comes from the opportunity to match anticipated loads closely through the use of overlapping shifts or odd shift lengths; the complexity comes from the need to evaluate the numerous possible alternatives. Management also must consider the timing of lunch breaks and rest periods, the number and starting times of shift schedules, and the days off for each employee. The programs select the schedule that minimizes the sum of expected costs of over- and understaffing.

Software is also available for sequencing jobs at workstations. They help firms design and manage the linkages between customers and suppliers in the supply chain. True integration requires the manipulation of large amounts of complex data in real time because the customer order work flow must be synchronized with the required material, manufacturing, and distribution activity. Coupled with the Internet and improved data storage and manipulation methods, such computer software has given rise to **advanced planning and scheduling (APS) systems**, which seek to optimize resources across the supply chain and align daily operations with strategic goals. A firm's ability to change its schedules quickly and still keep the goods and services flowing smoothly through the supply chain provides a competitive edge.

**advanced planning and scheduling (APS) systems**
Computer software systems that seek to optimize resources across the supply chain and align daily operations with strategic goals.

## LEARNING GOALS IN REVIEW

| Learning Goal | Guidelines for Review | MyOMLab Resources |
|---|---|---|
| 1 Explain the rationale behind the levels in the operations planning and scheduling process. | The section "Levels in Operations Planning and Scheduling," pp. 395–398, shows the various levels in a hierarchy of plans and how they relate to each other. There are two key figures in this section: Figure 10.1 shows the information inputs to the sales and operations plan, which is at the top of the hierarchy, and Figure 10.2 shows the levels of plans. | **Video:** Sales and Operations Planning at Starwood |
| 2 Describe the supply options used in sales and operations planning. | See the section "S&OP Supply Options," pp. 398–399, for the six ways managers can satisfy demands with the sales and operations plan. | |
| 3 Compare the chase planning strategy to the level planning strategy for developing sales and operations plans. | "S&OP Strategies," pp. 399–402, explains the chase and level strategies, the related constraints and costs, and the process managers use to develop a sales and operations plan. Figure 10.3 shows what a sales and operations plan looks like. | **Active Model Exercise:** 10.1: Level Strategy<br>**OM Explorer Tutors:** 10.1: Chase Strategy; 10.2: Level Strategy<br>**Tutor Exercise:** 10.1: Results of Different Scenarios with a Level Strategy |
| 4 Use spreadsheets for sales and operations planning. | See "Spreadsheets for Sales and Operations Planning," pp. 402–405, for a detailed discussion of how to use spreadsheets in S&OP. Example 10.1, pp. 403–405, demonstrates the procedure for doing S&OP for a service facility. See also Solved Problem 1, pp. 414–415. | **OM Explorer Solver:** Sales and Operations Planning with Spreadsheets<br>**OM Explorer Tutor:** 10.4: Staffing Strategies with Spreadsheets |
| 5 Develop workforce and workstation schedules. | The section "Scheduling," pp. 406–413, shows how to use Gantt charts, create workforce schedules, and sequence jobs at a workstation. Additional help can be found in Example 10.2, pp. 409–410, and Solved Problem 2, pp. 415–416, for workforce schedules, and Example 10.3, pp. 411–412, and Solved Problem 3, pp. 416–417, for sequencing jobs. | **Video:** Air New Zealand: Service Scheduling<br>**OM Explorer Solvers:** Workforce Scheduler; Single-Workstation Scheduler<br>**OM Explorer Tutor:** 10.3: Developing a Workforce Schedule<br>**Tutor Exercise:** 10.2: Staffing for the Newest MBA Class<br>**POM for Windows:** Scheduling<br>**Case:** Food King |

## Key Terms

advanced planning and scheduling (APS) systems 413
aggregate plan 395
annual plan (or financial plan) 396
backorder and stockout cost 400
business plan 396
chase strategy 399
earliest due date (EDD) 410
expediting 411
first-come, first-served (FCFS) 410
fixed schedule 408
flow time 411
hiring and layoff cost 400
inventory holding cost 400
level strategy 399
mixed strategy 400
operations planning and scheduling 395
overtime 399
overtime cost 400
past due 411
priority sequencing rule 410
product family 396
production plan 395
regular time cost 400
resource plan 395
rotating schedule 408
sales and operations plan (S&OP) 395
schedule 395
sequencing 410
staffing plan 400
tardiness 411
undertime 399
workforce scheduling 407

# Solved Problem 1

MyOMLab

Tutor 10.4 in MyOMLab provides another example for practicing sales and operations planning using a variety of strategies.

MyOMLab Video

The Cranston Telephone Company employs workers who lay telephone cables and perform various other construction tasks. The company prides itself on good service and strives to complete all service orders within the planning period in which they are received.

Each worker puts in 600 hours of regular time per planning period and can work as many as an additional 100 hours of overtime. The operations department has estimated the following staff requirements for such services over the next four planning periods:

| Planning Period | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Demand (hours) | 21,000 | 18,000 | 30,000 | 12,000 |

Cranston pays regular-time wages of $6,000 per employee per period for any time worked up to 600 hours (including undertime). The overtime pay rate is $15 per hour over 600 hours. Hiring, training, and outfitting a new employee costs $8,000. Layoff costs are $2,000 per employee. Currently, 40 employees work for Cranston in this capacity. No delays in service, or backorders, are allowed. Use the spreadsheet approach to answer the following questions:

**a.** Prepare a chase strategy using only hiring and layoffs. What are the total numbers of employees hired and laid off?

**b.** Develop a staffing plan that uses the level strategy, relaying only on overtime and undertime. Maximize the use of overtime during the peak period so as to minimize the workforce level and amount of undertime.

**c.** Propose an effective mixed-strategy plan.

**d.** Compare the total costs of the three plans.

### SOLUTION

**a.** The chase strategy workforce level is calculated by dividing the demand for each period by 600 hours, or the amount or regular-time work for one employee during one period. This strategy calls for a total of 20 workers to be hired and 40 to be laid off during the four-period plan. Figure 10.9 shows the "chase strategy" solution that OM Explorer's *Sales and Operations Planning with Spreadsheets* Solver produces. We simply hide any unneeded columns and rows in this general-purpose solver.

**FIGURE 10.9 ▶** Spreadsheet for Chase Strategy

| | 1 | 2 | 3 | 4 | Total |
|---|---|---|---|---|---|
| **Inputs** | | | | | |
| Forecasted demand | 35 | 30 | 50 | 20 | 135 |
| Workforce level | 35 | 30 | 50 | 20 | 135 |
| Undertime | 0 | 0 | 0 | 0 | 0 |
| Overtime | 0 | 0 | 0 | 0 | 0 |
| **Derived** | | | | | |
| Utilized time | 35 | 30 | 50 | 20 | 135 |
| Hires | 0 | 0 | 20 | 0 | 20 |
| Layoffs | 5 | 5 | 0 | 30 | 40 |
| **Calculated** | | | | | |
| Utilized time cost | $210,000 | $180,000 | $300,000 | $120,000 | $810,000 |
| Undertime cost | $0 | $0 | $0 | $0 | $0 |
| Overtime cost | $0 | $0 | $0 | $0 | $0 |
| Hiring cost | $0 | $0 | $160,000 | $0 | $160,000 |
| Layoff cost | $10,000 | $10,000 | $0 | $60,000 | $80,000 |
| **Total cost** | $220,000 | 190,000 | 460,000 | 180,000 | $1,050,000 |

**b.** The peak demand is 30,000 hours in period 3. As each employee can work 700 hours per period (600 on regular time and 100 on overtime), the workforce level of the level strategy that minimizes undertime is $30{,}000/700 = 42.86$, or 43 employees. This strategy calls for three employees to be hired in the first quarter and for none to be laid off. To convert the demand requirements into employee-period equivalents, divide the demand in hours by 600. For example, the demand of 21,000 hours in period 1 translates into 35 employee-period equivalents (21,000/600) and demand in period 3 translates into 50 employee-period equivalents (30,000/600). Figure 10.10 shows OM Explorer's spreadsheet for this level strategy that minimizes undertime.

| | 1 | 2 | 3 | 4 | Total |
|---|---|---|---|---|---|
| **Inputs** | | | | | |
| Forecasted demand | 35 | 30 | 50 | 20 | 135 |
| Workforce level | 43 | 43 | 43 | 43 | 172 |
| Undertime | 8 | 13 | 0 | 23 | 44 |
| Overtime | 0 | 0 | 7 | 0 | 7 |
| **Derived** | | | | | |
| Utilized time | 35 | 30 | 43 | 20 | 128 |
| Hires | 3 | 0 | 0 | 0 | 3 |
| Layoffs | 0 | 0 | 0 | 0 | 0 |
| **Calculated** | | | | | |
| Utilized time cost | $210,000 | $180,000 | $258,000 | $120,000 | $768,000 |
| Undertime cost | $48,000 | $78,000 | $0 | $138,000 | $264,000 |
| Overtime cost | $0 | $0 | $63,000 | $0 | $63,000 |
| Hiring cost | $24,000 | $0 | $0 | $0 | $24,000 |
| Layoff cost | $0 | $0 | $0 | $0 | $0 |
| **Total cost** | $282,000 | 258,000 | 321,000 | 258,000 | $1,119,000 |

**FIGURE 10.10** Spreadsheet for Level Strategy

**c.** The mixed-strategy plan that we propose uses a combination of hires, layoffs, and overtime to reduce total costs. The workforce is reduced by 5 at the beginning of the first period, increased by 8 in the third period, and reduced by 13 in the fourth period. Figure 10.11 shows the results.

| | 1 | 2 | 3 | 4 | Total |
|---|---|---|---|---|---|
| **Inputs** | | | | | |
| Forecasted demand | 35 | 30 | 50 | 20 | 135 |
| Workforce level | 35 | 35 | 43 | 30 | 143 |
| Undertime | 0 | 5 | 0 | 10 | 15 |
| Overtime | 0 | 0 | 7 | 0 | 7 |
| **Derived** | | | | | |
| Utilized time | 35 | 30 | 43 | 20 | 128 |
| Hires | 0 | 0 | 8 | 0 | 8 |
| Layoffs | 5 | 0 | 0 | 13 | 18 |
| **Calculated** | | | | | |
| Utilized time cost | $210,000 | $180,000 | $258,000 | $120,000 | $768,000 |
| Undertime cost | $0 | $30,000 | $0 | $60,000 | $90,000 |
| Overtime cost | $0 | $0 | $63,000 | $0 | $63,000 |
| Hiring cost | $0 | $0 | $64,000 | $0 | $64,000 |
| Layoff cost | $10,000 | $0 | $0 | $26,000 | $36,000 |
| **Total cost** | $220.000 | 210,000 | 385,000 | 206,000 | $1,021,000 |

**FIGURE 10.11** Spreadsheet for Mixed Strategy

**d.** The total cost of the chase strategy is $1,050,000. The level strategy results in a total cost of $1,119,000. The mixed-strategy plan was developed by trial and error and results in a total cost of $1,021,000. Further improvements are possible.

# Solved Problem 2

The Food Bin grocery store operates 24 hours per day, 7 days per week. Fred Bulger, the store manager, has been analyzing the efficiency and productivity of store operations recently. Bulger decided to observe the need for checkout clerks on the first shift for a one-month period. At the end of the month, he calculated the average number of checkout registers that should be open during the first shift each day. His results showed peak needs on Saturdays and Sundays.

| Day | M | T | W | Th | F | S | Su |
|---|---|---|---|---|---|---|---|
| Number of Clerks Required | 3 | 4 | 5 | 5 | 4 | 7 | 8 |

Bulger now has to come up with a workforce schedule that guarantees each checkout clerk two consecutive days off, but still covers all requirements.

**a.** Develop a workforce schedule that covers all requirements while giving two consecutive days off to each clerk. How many clerks are needed? Assume that the clerks have no preference regarding which days they have off.

**b.** Plans can be made to use the clerks for other duties if slack or idle time resulting from this schedule can be determined. How much idle time will result from this schedule, and on what days?

**SOLUTION**

**a.** We use the method demonstrated in Example 10.2 to determine the number of clerks needed. The minimum number of clerks is eight.

| | DAY | | | | | | |
|---|---|---|---|---|---|---|---|
| | M | T | W | Th | F | S | Su |
| Requirements | 3 | 4 | 5 | 5 | 4 | 7 | 8* |
| Clerk 1 | off | off | X | X | X | X | X |
| Requirements | 3 | 4 | 4 | 4 | 3 | 6 | 7* |
| Clerk 2 | off | off | X | X | X | X | X |
| Requirements | 3 | 4 | 3 | 3 | 2 | 5 | 6* |
| Clerk 3 | X | X | X | off | off | X | X |
| Requirements | 2 | 3 | 2 | 3 | 2 | 4 | 5* |
| Clerk 4 | X | X | X | off | off | X | X |
| Requirements | 1 | 2 | 1 | 3 | 2 | 3 | 4* |
| Clerk 5 | X | off | off | X | X | X | X |
| Requirements | 0 | 2 | 1 | 2 | 1 | 2 | 3* |
| Clerk 6 | off | off | X | X | X | X | X |
| Requirements | 0 | 2* | 0 | 1 | 0 | 1 | 2* |
| Clerk 7 | X | X | off | off | X | X | X |
| Requirements | 0 | 1* | 0 | 1* | 0 | 0 | 1* |
| Clerk 8 | X | X | X | X | off | off | X |
| Requirements | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

*Maximum requirements

**b.** Based on the results in part (a), the number of clerks on duty minus the requirements is the number of idle clerks available for other duties:

| | M | T | W | Th | F | S | Su |
|---|---|---|---|---|---|---|---|
| Number on duty | 5 | 4 | 6 | 5 | 5 | 7 | 8 |
| Requirements | 3 | 4 | 5 | 5 | 4 | 7 | 8 |
| Idle clerks | 2 | 0 | 1 | 0 | 1 | 0 | 0 |

The slack in this schedule would indicate to Bulger the number of employees he might ask to work part time (fewer than 5 days per week). For example, Clerk 7 might work Tuesday, Saturday, and Sunday, and Clerk 8 might work Tuesday, Thursday, and Sunday. That would eliminate slack from the schedule.

# Solved Problem 3

Revisit Example 10.3, where the consulting company has five jobs in its backlog. Create a schedule using the EDD rule, calculating the average days past due and flow time. In this case, does EDD outperform the FCFS rule?

**SOLUTION**

| Customer Sequence | Start Time (days) | | Processing Time (days) | | Finish Time (days) | Due Date | Days Past Due | Days Ago Since Order Arrived | Flow Time (days) |
|---|---|---|---|---|---|---|---|---|---|
| B | 0 | + | 16 | = | 16 | 27 | **0** | 12 | **28** |
| A | 16 | + | 25 | = | 41 | 29 | **12** | 15 | **56** |
| D | 41 | + | 10 | = | 51 | 48 | **3** | 10 | **61** |
| C | 51 | + | 14 | = | 65 | 68 | **0** | 5 | **70** |
| E | 65 | + | 12 | = | 77 | 80 | **0** | 0 | **77** |

The days past due and average flow time performance measures for the EDD schedule are

$$\text{Average days past due} = \frac{0 + 12 + 3 + 0 + 0}{5} = 3.0 \text{ days}$$

$$\text{Average flow time} = \frac{28 + 56 + 61 + 70 + 77}{5} = 58.4 \text{ days}$$

By both measures, EDD outperforms the FCFS (3.0 versus 3.4 days past due and 58.4 versus 60.2 days flow time). However, the solution found in part (b) of Example 10.3 still has the best average flow time of only 47.8 days.

# Discussion Questions

1. List the types of costs incurred when employees are laid off. What costs are difficult to estimate in monetary terms? Suppose that a firm is facing a downturn in business, each employee has skills valued at $40,000 per year, and it costs $100,000 to lay off an employee. If business is expected to improve in one year, are layoffs financially justified? What is the "payback" period for the layoff decision?

2. In your community, some employers maintain stable workforces at all costs, and others furlough and recall workers seemingly at the drop of a hat. What are the differences in markets, management, products, financial position, skills, costs, and competition that could explain these two extremes in personnel policy?

3. Consider Managerial Practice 10.1 and the manner in which Air New Zealand schedules its flight crews. Relate each step in the process to the levels in the operations planning and scheduling process shown in Figure 10.2.

4. Explain why management should be concerned about priority systems in service and manufacturing organizations.

# Problems

The OM Explorer and POM for Windows software is available to all students using the 11th edition of this textbook. Go to **http://www.pearsonglobaleditions.com/krajewski** to download these computer packages. If you purchased MyOMLab, you also have access to Active Models software and significant help in doing the following problems. Check with your instructor on how best to use these resources. In many cases, the instructor wants you to understand how to do the calculations by hand. At the least, the software provides a check on your calculations. When calculations are particularly complex and the goal is interpreting the results in making decision, the software entirely replaces the manual calculations.

## S&OP Strategies

1. The Barberton Municipal Division of Road Maintenance is charged with road repair in the city of Barberton and the surrounding area. Cindy Kramer, road maintenance director, must submit a staffing plan for the next year based on a set schedule for repairs and on the city budget. Kramer estimates that the labor hours required for the next four quarters are 6,500, 13,000, 20,000, and 9,000, respectively. Each of the 11 workers on the workforce can contribute 500 hours per quarter. Payroll costs are $6,000 in wages per worker for regular time worked up to 500 hours, with an overtime pay rate of $20 for each overtime hour. Overtime is limited to 20 percent of the regular-time capacity in any quarter. Although unused overtime capacity has no cost, unused regular time is paid at $12 per hour. The cost of hiring a worker is $3,100, and the cost of laying off a worker is $1,300. Subcontracting is not permitted.

   **a.** Find a level staffing plan that relies just on overtime and the minimum amount of undertime possible. Overtime can be used to its limits in any quarter. What is the total cost of the plan and how many undertime hours does it call for?

   **b.** Use a chase strategy that varies the workforce level without using overtime or undertime. What is the total cost of this plan?

   **c.** Propose a plan of your own. Compare your plan with those in part (a) and part (b) and discuss its comparative merits.

2. Bob Carlton's golf camp estimates the following staff requirements for its services over the next 2 years.

| Quarter | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Demand (hours) | 4,200 | 6,400 | 3,000 | 4,800 |
| Quarter | 5 | 6 | 7 | 8 |
| Demand (hours) | 4,400 | 6,240 | 3,600 | 4,800 |

Each certified instructor puts in 480 hours per quarter regular time and can work an additional 120 hours overtime. Regular-time wages and benefits cost Carlton $7,200 per employee per quarter for regular time worked up to 480 hours, with an overtime cost of $20 per hour. Unused regular time for certified instructors is paid at $15 per hour. There is no cost for unused overtime capacity. The cost of hiring, training, and certifying a new employee is $10,000. Layoff costs are $4,000 per employee. Currently, eight employees work in this capacity.

a. Find a staffing plan using the level strategy that allows for no delay in service. It should rely only on overtime and the minimum amount of undertime necessary. What is the total cost of this plan?

b. Use a chase strategy that varies the workforce level without using overtime or undertime. What is the total cost of this plan?

c. Propose a better plan and calculate its total cost.

3. Continuing Problem 2, now assume that Carlton is permitted to employ some uncertified, part-time instructors, provided they represent no more than 15 percent of the total workforce hours (regular, overtime, and part-time) in any quarter. Each part-time instructor can work up to 240 hours per quarter, with no overtime or undertime cost. Labor costs for part-time instructors are $12 per hour. Hiring and training costs are $2,000 per uncertified instructor, and there are no layoff costs.

a. Propose a low-cost, mixed-strategy plan and calculate its total cost.

b. What are the primary advantages and disadvantages of having a workforce consisting of both regular and temporary employees?

4. The Donald Fertilizer Company produces industrial chemical fertilizers. The projected manufacturing requirements (in thousands of gallons) for the next four quarters are 90, 60, 90, and 140, respectively. A level workforce is desired, relying only on anticipation inventory as a supply option. Stockouts and backorders are to be avoided, as are overtime and undertime.

a. Determine the quarterly production rate required to meet total demand for the year, and minimize the anticipation inventory that would be left over at the end of the year. Beginning inventory is 0.

b. Specify the anticipation inventory that will be produced.

c. Suppose that the requirements for the next four quarters are revised to 60, 90, 140, and 90, respectively. If total demand is the same, what level of production rate is needed now, using the same strategy as part (a)?

5. Management at the Kerby Corporation has determined the following aggregated demand schedule (in units):

| Month | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Demand | 500 | 800 | 1,000 | 1,400 |
| Month | 5 | 6 | 7 | 8 |
| Demand | 2,000 | 3,000 | 2,700 | 1,500 |
| Month | 9 | 10 | 11 | 12 |
| Demand | 1,400 | 1,500 | 2,000 | 1,200 |

An employee can produce an average of 10 units per month. Each worker on the payroll costs $2,000 in regular-time wages per month. Undertime is paid at the same rate as regular time. In accordance with the labor contract in force, Kerby Corporation does not work overtime or use subcontracting. Kerby can hire and train a new employee for $2,000 and lay off one for $500. Inventory costs $32 per unit on hand at the end of each month. At present, 140 employees are on the payroll and anticipation inventory is zero.

a. Prepare a production plan that only uses a level workforce and anticipation inventory as its supply options. Minimize the inventory left over at the end of the year. Layoffs, undertime, vacations, subcontracting, backorders, and stockouts are not options. The plan may call for a one-time adjustment of the workforce before month 1 begins.

b. Prepare a production plan using a chase strategy, relying only on hiring and layoffs.

c. Prepare a mixed-strategy production plan that uses only a level workforce and anticipation inventory through month 7 (an adjustment of the workforce may be made before month 1 begins) then switches to a chase strategy for months 8–12.

d. Contrast these three plans on the basis of annual costs.

6. Gretchen's Kitchen is a fast-food restaurant located in an ideal spot near the local high school. Gretchen Lowe must prepare an annual staffing plan. The only menu items are hamburgers, chili, soft drinks, shakes, and French fries. A sample of 1,000 customers taken at random revealed that they purchased 2,100 hamburgers, 200 pints of chili, 1,000 soft drinks and shakes, and 1,000 bags of French fries. Thus, for purposes of estimating staffing requirements, Lowe assumes that each customer purchases 2.1 hamburgers, 0.2 pint of chili, 1 soft drink or shake, and 1 bag of French fries. Each hamburger requires 4 minutes of labor, a pint of chili requires 3 minutes, and a soft drink or shake and a bag of fries each take 2 minutes of labor.

The restaurant currently has 10 part-time employees who work 80 hours a month on staggered shifts. Wages are $400 per month for regular time and $7.50 per hour for overtime. Hiring and training costs are $250 per new employee, and layoff costs are $50 per employee.

Lowe realizes that building up seasonal inventories of hamburgers (or any of the products) would not be wise because of shelf-life considerations. Also, any demand not satisfied is a lost sale and must be avoided. Three strategies come to mind.

- Use a level strategy relying on overtime and undertime, with up to 20 percent of regular-time capacity on overtime.
- Maintain a base of 10 employees, hiring and laying off as needed to avoid any overtime.
- Utilize a chase strategy, hiring and laying off employees as demand changes to avoid overtime.

When performing her calculations, Lowe always rounds to the next highest integer for the number of employees. She also follows a policy of not using an employee more than 80 hours per month, except when overtime is needed. The projected demand by month (number of customers) for next year is as follows:

| | | | |
|---|---|---|---|
| Jan. | 3,200 | July | 4,800 |
| Feb. | 2,600 | Aug. | 4,200 |
| Mar. | 3,300 | Sept. | 3,800 |
| Apr. | 3,900 | Oct. | 3,600 |
| May | 3,600 | Nov. | 3,500 |
| June | 4,200 | Dec. | 3,000 |

**a.** Develop the schedule of service requirements (hours per month) for the next year.

**b.** Which strategy is most effective?

**c.** Suppose that an arrangement with the high school enables the manager to identify good prospective employees without having to advertise in the local newspaper. This source reduces the hiring cost to $50, which is mainly the cost of charred hamburgers during training. If cost is her only concern, will this method of hiring change Gretchen Lowe's strategy? Considering other objectives that may be appropriate, do you think she should change strategies?

**7.** A manager faces peak (weekly) demand for one of her operations, but is not sure how long the peak will last. She can either use overtime from the current workforce, or hire/lay off and just pay regular-time wages. Regular-time pay is $550 per week, overtime is $825 per week, the hiring cost is $2,000, and the layoff cost is $3,000. Assuming that people are available seeking such a short-term arrangement, how many weeks must the surge in demand last to justify a temporary hire? *Hint:* Use break-even analysis (see Supplement A, "Decision Making Models"). Let $w$ be the number of weeks of the high demand (rather than using $Q$ for the break-even quantity). What is the fixed cost for the regular-time option? Overtime option?

## Spreadsheets for Sales and Operations Planning

**8.** Tax Prep Advisers, Inc., has forecasted the following staffing requirements for tax preparation associates over the next 12 months. Management would like three alternative staffing plans to be developed.

| **Month** | **1** | **2** | **3** | **4** |
|---|---|---|---|---|
| **Demand** | 5 | 8 | 10 | 13 |
| **Month** | **5** | **6** | **7** | **8** |
| **Demand** | 18 | 20 | 20 | 14 |
| **Month** | **9** | **10** | **11** | **12** |
| **Demand** | 12 | 8 | 2 | 1 |

The company currently has 10 associates. No more than 10 new hires can be accommodated in any month because of limited training facilities. No backorders are allowed, and overtime cannot exceed 25 percent of regular-time capacity on any month. There is no cost for unused overtime capacity. Regular-time wages are $1,500 per month, and overtime wages are 150 percent of regular-time wages. Undertime is paid at the same rate as regular time. The hiring cost is $2,500 per person, and the layoff cost is $2.000 per person.

**a.** Prepare a staffing plan utilizing a level workforce strategy, minimizing undertime. The plan may call for a one-time adjustment of the workforce before month 1.

**b.** Using a chase strategy, prepare a plan that is consistent with the constraint on hiring and minimizes use of overtime.

**c.** Prepare a mixed strategy in which the workforce level is slowly increased by two employees per month through month 5 and is then decreased by two employees per month starting in month 6 and continuing through month 12. Does this plan violate the hiring or overtime constraints set the company?

**d.** Contrast these three plans on the basis of annual costs.

**9.** Climate Control, Inc., makes expedition-quality rain gear for outdoor enthusiasts. Management prepared a forecast of sales (in suits) for next year and now must prepare a production plan. The company has traditionally maintained a level workforce strategy. All nine workers are treated like family and have been employed by the company for a number of years. Each employee can produce 2,000 suits per month. At present, finished goods inventory holds 24,000 suits. The demand forecast follows:

| **Month** | **1** | **2** | **3** | **4** |
|---|---|---|---|---|
| **Demand** | 25,000 | 16,000 | 15,000 | 19,000 |
| **Month** | **5** | **6** | **7** | **8** |
| **Demand** | 32,000 | 29,000 | 27,000 | 22,000 |
| **Month** | **9** | **10** | **11** | **12** |
| **Demand** | 14,000 | 15,000 | 20,000 | 6,000 |

Use the *Sales and Operations Planning with Spreadsheets* Solver in OM Explorer or develop your own spreadsheet models to address the following questions.

**a.** Management is willing to authorize overtime in periods for which regular production and current levels of anticipation inventory do not satisfy demand. However, overtime must be strictly limited to no more than 20 percent of regular-time capacity. Management wants to avoid stockouts and backorders and is not willing to accept a plan that calls for shortages. Is it feasible to hold the workforce constant, assuming that overtime is only used in periods for which shortages would occur?

**b.** Assume that management is not willing to authorize any overtime. Instead, management is willing to negotiate with customers so that backorders may be used as a supply option. However, management is not willing to carry more than 5,000 suits from one month to the next in backorder. Is it feasible to hold the workforce constant, assuming that a maximum backorder of 5,000 suits may be maintained from month to month?

**c.** Assume management is willing to authorize the use of overtime over the next four months to build additional anticipation inventory. However, overtime must be strictly limited to no more than 20 percent of regular-time capacity. Management wants to avoid stockouts and backorders and is not willing to accept a plan that calls for shortages. Is it feasible to hold the workforce constant, assuming that overtime is only used in months 1–4? If not, in which months would additional overtime be required?

**10.** The Kool King Company has followed a policy of no layoffs for most of the manufacturer's life, even though the demand for its air conditioners is highly seasonal. Management wants to evaluate the cost-effectiveness of this policy. Competitive pressures are increasing, and ways need to be found to reduce costs. The following demand (expressed in employee–month equivalents) has been forecast for next year:

| | | | | | |
|---|---|---|---|---|---|
| Jan. | 70 | May | 130 | Sept. | 110 |
| Feb. | 90 | June | 170 | Oct. | 60 |
| Mar. | 100 | July | 170 | Nov. | 20 |
| Apr. | 100 | Aug. | 150 | Dec. | 40 |

Additional planning data follow, with costs, inventory, and backorders expressed in employee–month equivalents:

| | | | |
|---|---|---|---|
| Regular-time production cost | $1,500 | Hire cost | $500/ person |
| Overtime production cost | 150% of regular-time production cost | Layoff cost | $2,000/ person |
| Subcontracting cost | $2,500 | Current backorders | 10 |
| Inventory holding cost | $100 | Current inventory | 0 |
| Backorder cost | $1,000 | Desired ending inventory | 0 |
| Maximum overtime | 20% of regular-time capacity | Current employment | 130 employees |

Hiring costs are lower than layoff costs because the facility is located near a Technical Training School. Undertime is paid at the rate equivalent to regular-time production. Each employee who has been with the company at least one year also received 0.5 months of paid vacation. All 130 employees currently employed qualify for vacations next year, assuming that they remain on the workforce. Answer the following questions using *Sales and Operations Planning with Spreadsheets* Solver in OM Explorer, or an Excel spreadsheet that you developed on your own.

**a.** Develop an S&OP with the level strategy using overtime, undertime, and vacations as the only supply options. Use the maximum amount of overtime so as to minimize undertime. What is the total cost of this plan, and what are its advantages and disadvantages?

**b.** Develop an S&OP with the chase strategy. Part of your decision will be when and how many vacation periods to grant. What is the total cost of this plan, and what are its advantages and disadvantages?

**c.** Develop an S&OP with a lower cost than found with either the level or chase strategy, being open to the full range of supply options (including anticipation inventory). Subcontractors can supply up to 50 employee-month equivalents. What is the total cost of this plan, and what are its advantages and disadvantages?

**11.** Jane Dapna, the operations manager of Classico Inc., is using the *Sales and Operations Planning with Spreadsheets* Solver in OM Explorer to develop a five month production plan. Her initial plan is to maintain a level workforce and use overtime and backorders as shown below. Use the provided inputs to calculate the derived inputs: Utilized time, Inventory, Hires and Layoffs, and the associated costs. (*Hint:* Don't forget to express the forecasted demand and reactive alternatives as employee-period equivalents.)

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Inputs | | | | | | | |
| | | | | | | | |
| Starting Workforce | | 20 | | Cost to Hire One Worker | | | $ 10,000.00 |
| Wages per Worker per Period | | $ 4,000.00 | | Cost to Lay Off One Worker | | | $ 20,000.00 |
| Overtime Pay Percentage | | 150% | | Initial Inventory Level | | | 10 |
| Subcontracting Cost per Period | | 0 | | Initial Backorders | | | 0 |
| | | | | Inventory Cost | | | $ 100.00 |
| | | | | Backorder Cost | | | $ 250.00 |
| | | | | | | | |
| **Period** | **January** | **February** | **March** | **April** | **May** | | **Total** |
| **Inputs** | | | | | | | |
| Forecasted demand | 20 | 30 | 40 | 30 | 20 | | 140 |
| Workforce level | 20 | 20 | 20 | 20 | 20 | | 100 |
| Undertime | 0 | 0 | 0 | 0 | 0 | | 0 |
| Overtime | 0 | 0 | 10 | 20 | 0 | | 30 |
| Vacation time | 0 | 0 | 0 | 0 | 0 | | 0 |
| Subcontracting time | 0 | 0 | 0 | 0 | 0 | | 0 |
| Backorders | 0 | 0 | 10 | 0 | 0 | | 10 |
| **Derived** | | | | | | | |
| Utilized time | | | | | | | 100 |
| Inventory | | | | | | | 10 |
| Hires | | | | | | | 0 |
| Layoffs | | | | | | | 0 |
| **Calculated** | | | | | | | |
| Utilized time cost | | | | | | | $ 400,000.00 |
| Undertime cost | | | | | | | $ - |
| Overtime cost | | | | | | | $ 180,000.00 |
| Vacation time cost | | | | | | | $ - |
| Inventory cost | | | | | | | $ 1,000.00 |
| Backorders cost | | | | | | | $ 2,500.00 |
| Hiring cost | | | | | | | $ - |
| Layoff cost | | | | | | | $ - |
| Subcontracting cost | | | | | | | $ - |
| | | | | | | | |
| Total cost | | | | | | | $ 583,500.00 |
| | | | | | | | |
| Reminder: Express the forecasted demand and reactive alternatives as employee-period equivalents. | | | | | | | |

**12.** Gemini Inc. is using the *Sales and Operations Planning with Spreadsheets* Solver in OM Explorer to develop a five month production plan. Use the provided Derived outputs and Calculated costs to specify the model's inputs. (*Hint:* Don't forget that the forecasted demand and reactive alternatives are expressed as employee-period equivalents.)

| Inputs | | | | | |
|---|---|---|---|---|---|
| Starting Workforce | 20 | | Cost to Hire One Worker | | $ 20,000.00 |
| Wages per Worker per Period | $ 2,500.00 | | Cost to Lay Off One Worker | | $ 5,000.00 |
| Overtime Pay Percentage | 150% | | Initial Inventory Level | | 0 |
| Subcontracting Cost per Period | 0 | | Initial Backorders | | 10 |
| | | | Inventory Cost | | $ 100.00 |
| | | | Backorder Cost | | $ 250.00 |

| Period | January | February | March | April | May | | Total |
|---|---|---|---|---|---|---|---|
| **Inputs** | | | | | | | |
| Forecasted demand | | | | | | | 165 |
| Workforce level | | | | | | | 130 |
| Undertime | | | | | | | 0 |
| Overtime | | | | | | | 35 |
| Vacation time | | | | | | | 0 |
| Subcontracting time | | | | | | | 0 |
| Backorders | | | | | | | 10 |
| **Derived** | | | | | | | |
| Utilized time | 20 | 20 | 30 | 30 | 30 | | 130 |
| Inventory | 0 | 10 | 15 | 15 | 0 | | 40 |
| Hires | 0 | 0 | 10 | 0 | 0 | | 10 |
| Layoffs | 0 | 0 | 0 | 0 | 0 | | 0 |
| **Calculated** | | | | | | | |
| Utilized time cost | $ 50,000.00 | $ 50,000.00 | $ 75,000.00 | $ 75,000.00 | $ 75,000.00 | | $ 325,000.00 |
| Undertime cost | $ - | $ - | $ - | $ - | $ - | | $ - |
| Overtime cost | $ - | $ - | $ 56,250.00 | $ 37,500.00 | $ 37,500.00 | | $ 131,250.00 |
| Vacation time cost | $ - | $ - | $ - | $ - | $ - | | $ - |
| Inventory cost | $ - | $ 1,000.00 | $ 1,500.00 | $ 1,500.00 | $ - | | $ 4,000.00 |
| Backorders cost | $ - | $ - | $ - | $ - | $ 2,500.00 | | $ 2,500.00 |
| Hiring cost | $ - | $ - | $ 20,000.00 | $ - | $ - | | $ 20,000.00 |
| Layoff cost | $ - | $ - | $ - | $ - | $ - | | $ - |
| Subcontracting cost | $ - | $ - | $ - | $ - | $ - | | $ - |
| Total cost | $ 50,000.00 | $ 51,000.00 | $ 152,750.00 | $ 114,000.00 | $ 115,000.00 | | $ 482,750.00 |

Reminder: Express the forecasted demand and reactive alternatives as employee-period equivalents.

## Scheduling

**13.** Gerald Glynn manages the Michaels Distribution Center. After careful examination of his database information, he has determined the daily requirements for part-time loading dock personnel. The distribution center operates 7 days a week, and the daily part-time staffing requirements are

| Day | M | T | W | Th | F | S | Su |
|---|---|---|---|---|---|---|---|
| **Requirements** | 6 | 3 | 5 | 3 | 7 | 2 | 3 |

Find the minimum number of workers Glynn must hire. Prepare a workforce schedule for these individuals so that each will have two consecutive days off per week and all staffing requirements will be satisfied. Give preference to the S–Su pair in case of a tie.

**14.** Cara Ryder manages a ski school in a large resort and is trying to develop a schedule for instructors. The instructors receive little salary and work just enough to earn room and board. They receive free skiing and spend most of their free time tackling the resort's notorious double black-diamond slopes. Hence, the instructors work only 4 days a week. One of the lesson packages offered at the resort is a 4-day beginner package. Ryder likes to keep the same instructor with a group over the 4-day period, so she schedules the instructors for 4 consecutive days and then 3 days off. Ryder uses years of experience with demand forecasts provided by management to formulate her instructor requirements for the upcoming month.

| Day | M | T | W | Th | F | S | Su |
|---|---|---|---|---|---|---|---|
| Requirements | 7 | 5 | 4 | 5 | 6 | 9 | 8 |

**a.** Determine how many instructors Ryder needs to employ. Give preference to Saturday and Sunday off. (*Hint:* Look for the group of 3 days with the lowest requirements.)

**b.** Specify the work schedule for each employee. How much slack does your schedule generate for each day?

**15.** The mayor of Cambridge, Colorado, wanting to be environmentally progressive, decides to implement a recycling plan. All residents of the city will receive a special three-part bin to separate their glass, plastic, and aluminum, and the city will be responsible for picking up the materials. A young city and regional planning graduate, Michael Duffy, has been hired to manage the recycling program. After carefully studying the city's population density, Duffy decides that the following numbers of recycling collectors will be needed:

| Day | M | T | W | Th | F | S | Su |
|---|---|---|---|---|---|---|---|
| Requirements | 12 | 7 | 9 | 9 | 5 | 3 | 6 |

The requirements are based on the populations of the various housing developments and subdivisions in the city and surrounding communities. To motivate residents of some areas to have their pickups scheduled on weekends, a special tax break will be given.

**a.** Find the minimum number of recycling collectors required if each employee works 5 days a week and has two consecutive days off. Give preference to the S–Su pair when that pair is involved in a tie.

**b.** Specify the work schedule for each employee. How much slack does your schedule generate for each day?

**c.** Suppose that Duffy can smooth the requirements further through greater tax incentives. The requirements then will be eight collectors on Monday and seven on the other days of the week. How many collectors will be needed now? Does smoothing of requirements have capital investment implications? If so, what are they?

**16.** Little 6, Inc., an accounting firm, forecasts the following weekly workload during the tax season:

| | DAY | | | | | | |
|---|---|---|---|---|---|---|---|
| | M | T | W | Th | F | S | Su |
| Personal Tax Returns | 24 | 14 | 18 | 18 | 10 | 28 | 16 |
| Corporate Tax Returns | 16 | 10 | 12 | 15 | 24 | 12 | 4 |

Corporate tax returns each require 4 hours of an accountant's time, and personal returns each require 90 minutes. During tax season, each accountant can work up to 10 hours per day. However, error rates increase to unacceptable levels when accountants work more than 5 consecutive days per week.

*Hint:* Read Supplement D before doing this problem. Let $x_i =$ number for each working schedule, e.g., $x_1 =$ number for Tuesday through Saturday.

**a.** Create an effective and efficient work schedule by formulating the problem as a linear program and solve using POM for Windows.

**b.** Assume that management has decided to offer a pay differential to those accountants who are scheduled to work on a weekend day. Normally, accountants earn $1,200 per week, but management will pay a bonus of $100 for Saturday work and $150 for Sunday work. What schedule will cover all demand as well as minimize payroll cost?

**c.** Assume that Little 6 has three part-time employees available to work Friday, Saturday, and Sunday at a rate of $800. Could these employees be cost effectively utilized?

**17.** Return to Problem 13 and the workforce schedule for part-time loading dock workers. Suppose that each part-time worker can work only 3 days, but the days must be consecutive. Formulate and solve this workforce scheduling problem as a linear program and solve it using POM for Windows. Your objective is to minimize total slack capacity. What is the minimum number of loaders needed now, and what are their schedules?

*Hint:* Read Supplement D before doing this problem. Let $x_i =$ number of workers for each 3-day schedule, for instance, $x_1 =$ number of workers for Tuesday through Thursday.

**18.** The Hickory Company manufactures wooden desks. Management schedules overtime every weekend to reduce the backlog on the most popular models. The automatic routing machine is used to cut certain types of edges on the desktops. The following orders need to be scheduled for the routing machine:

| Order | Time Since Order Arrived (hours ago) | Estimated Machine Time (hours) | Due Date (hours from now) |
|---|---|---|---|
| 1 | 6 | 10 | 12 |
| 2 | 5 | 3 | 8 |
| 3 | 3 | 15 | 18 |
| 4 | 1 | 9 | 20 |
| 5 | 0 | 7 | 21 |

The due dates reflect the need for the order to be at its next operation.

**a.** Develop separate schedules by using the FCFS and EDD rules. Compare the schedules on the basis of average flow time and average past due hours.

**b.** Comment on the performance of the two rules relative to these measures.

**19.** Currently a company that designs Web sites has five customers in its backlog. The day when the order arrived, processing time, and promised due dates are given in the following table. The customers are listed in the order of when they arrived. They are ready to be scheduled today, which is the start of day 190.

| Customer | Time Since Order Arrived (days ago) | Processing Time (days) | Due Date (days from now) |
|---|---|---|---|
| A | 10 | 20 | 26 |
| B | 8 | 12 | 50 |
| C | 6 | 28 | 66 |
| D | 3 | 24 | 58 |
| E | 2 | 32 | 100 |

**a.** Develop separate schedules by using the FCFS and EDD rules. Compare the schedules on the basis of average flow time and average days past due.

**b.** Comment on the performance of the two rules relative to these measures. Which one gives the best schedule? Why?

**20.** The Mowry Machine Shop still has five jobs to be processed as of 8 A.M. today (day 23) at its bottleneck operation. The day when the order arrived, processing time, and promised due dates are given in the following table. The jobs are listed in the order of arrival.

| Job | Time Since Order Arrived (days ago) | Processing Time (days) | Due Date (days from now) |
|---|---|---|---|
| A | 11 | 10 | 22 |
| B | 10 | 8 | 13 |
| C | 8 | 4 | 19 |
| D | 6 | 4 | 16 |
| E | 1 | 3 | 30 |

**a.** Develop separate schedules by using the FCFS and EDD rules. Compare the schedules on the basis of average flow time and average days past due.

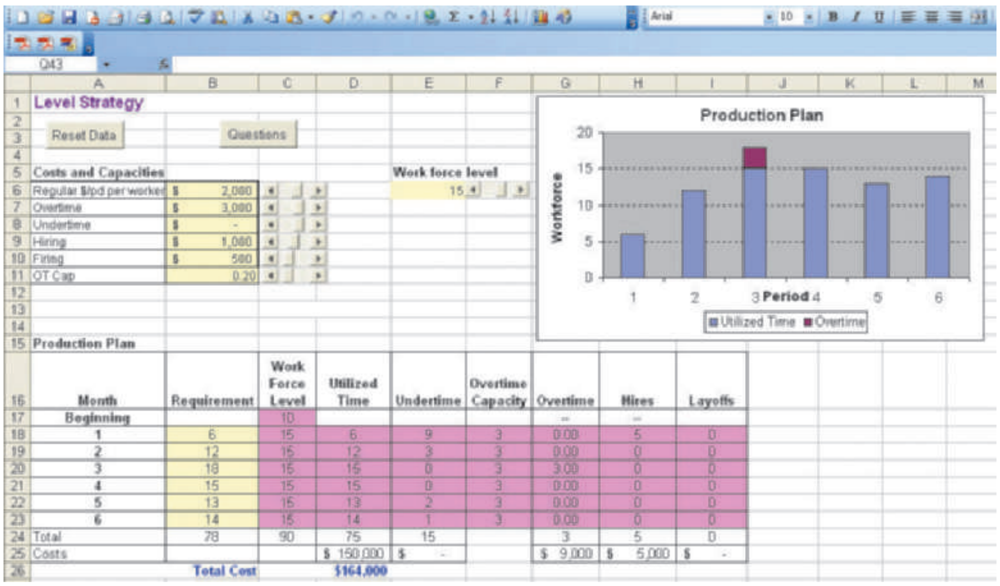
**b.** Which rule gives the best schedule, in your judgment? Why?

# Active Model Exercise

Active Model 10.1, "Level Strategy," appears in MyOMLab. It allows you to evaluate the effects of modifying the size of a constant workforce.

### QUESTIONS

1. If we use the same number of workers in each period, what happens as the number of workers increases from 15?
2. If we use the same number of workers in each period, what happens as the number of workers decreases from 15?
3. Suppose the hiring cost is $1,100. What happens as the number of workers increases?
4. Suppose the overtime cost is $3,300. What happens as the number of workers increases?
5. Suppose the undertime cost is the same as the regular-time cost (i.e., paid undertime). What is the best number of workers to have in each month and still meet the demand?
6. If the overtime capacity increases to 30 percent, what is the minimum number of workers that meets the demand in every month?



# VIDEO CASE Sales and Operations Planning at Starwood

Business travel often means staying overnight in a hotel. Upon arrival, you may be greeted by a doorman or valet to assist you with your luggage. Front desk staff awaits your check-in. Behind the scenes, housekeeping, maintenance, and culinary staff prepare for your stay. Making a reservation gives the hotel notice of your plan to stay, but even before your trip is ever conceived, the hotel is staffed and ready. How? Through a process called *sales and operations planning*.

Sales and operations planning is a process every organization performs to some degree. Called a staffing plan (or service resource plan if more detailed) in service organizations, the plan must strike the right level of customer service while maintaining workforce stability and cost control so as to achieve the organization's profit expectations. So where do companies begin? Let us take a look at Starwood Hotels and Resorts to see how it is done.

Starwood operates in more than 750 locations around the globe. At the highest levels, Starwood engages in sales and operations planning on an annual basis, with adjustments made as needed each month by region and by property. Budgeted revenues and other projections come from headquarters; the regions and individual properties then break down the forecasts to meet their expected occupancies. Typically, the director of human resources determines the staffing mix needed across divisions such as food and beverage service, rooms (including housekeeping, spa, and guest services), engineering, Six Sigma (see Chapter 3, "Managing Quality"), revenue management, and accounting.

At the property level, general managers and their staff must provide input into next year's plan while implementing and monitoring activity in the current year. For most properties, payroll is close to 40 percent of budgeted revenues and represents the largest single expense the hotel incurs. It is also the most controllable expense. Many of Starwood's hotels and most resorts experience patterns of seasonality that affect demand for rooms and services. This seasonality, in turn, significantly affects the organization's staffing plan.

To determine the staffing levels, the company uses a proprietary software program that models occupancy demand based on historical data. The key drivers of staffing are occupied rooms and restaurant meals, called "covers." Starwood knows on a *per room* and *per cover* basis how many staff are required to function properly. When occupancy and covers are entered into the software program, the output models a recommended staffing level for each division. This recommendation is then reviewed by division managers and adjusted as needed to be sure staffing is in line with budgeted financial plans. Job fairs to recruit nonmanagement staff are held several times a year so a qualified candidate pool of both part-time and full-time staff is ready when needed. Most hotels maintain a pool of part-time workers who can contract or expand the hours worked if required by property guest levels. Vacations for management are scheduled for the low season. Overtime will be worked as needed, but is less desirable than scheduling the appropriate level of staff in each division.

The program also takes into account both the complexity and positioning of the property within Starwood. For example, a 400-room city hotel that is essentially a high-rise building is not as complex as a 400-room sprawling resort with golf, spa, convention, and other services not offered by the city hotel. Positioning also is important. A five-star resort hotel's customer service expectations are much greater than a three-star airport hotel location and requires much higher ratios of staff to guests. Finally, if the hotel is a brand new property, historical data from similar properties is used to model staffing for the first year or two of operation.



A software program that forecasts occupancy based on historical data helps Starwood maintain proper staffing levels at its hotels. Managers know on a per-room, and "per-cover," basis how many hotel employees should be scheduled so that customers get good service.

Starwood attempts to modify demand and smooth out the peaks and valleys of its demand patterns. Many of the company's hotels experience three seasons: high, mid (called "shoulder"), and low season. Starwood, like its competitors, offers special rates, family packages, and weekend specials to attract different segments of the market during slower business periods. Staff is cross-trained to work in multiple areas, such as front reception and the concierge desk, so additional staff does not have to be added across seasons. Employees may also be temporarily redeployed among Starwood's properties to help out during peak periods. For example, when occupancy is forecast to be high in one region of the country, staff from areas entering their low season will be assigned to cover the demand.

## QUESTIONS

1. At what points in the planning process would you expect accounting/finance, marketing, information systems, and operations to play a role? What inputs should these areas provide, and why?
2. Does Starwood employ a chase, level, or mixed strategy? Why is this approach the best choice for the company?
3. How would staffing for the opening of a new hotel or resort differ from that of an existing property? What data might Starwood rely upon to make sure the new property is not over- or understaffed in its first year of operation?

# CASE Memorial Hospital

Memorial Hospital is a 265-bed regional health care facility located in the mountains of western North Carolina. The mission of the hospital is to provide quality health care to the people of Ashe County and the six surrounding counties. To accomplish this mission, Memorial Hospital's CEO has outlined three objectives: (1) maximize customer service to increase customer satisfaction, (2) minimize costs to remain competitive, and (3) minimize fluctuations in workforce levels to help stabilize area employment.

The hospital's operations are segmented into eight major wards for the purposes of planning and scheduling the nursing staff. These wards are listed in Table 10.3, along with the number of beds, targeted patient-to-nurse ratios, and average patient census for each ward. The overall demand for hospital services remained relatively constant over the past few years even though the population of the seven counties served increased. This stable demand can be attributed to increased competition from other hospitals in the area and the rise in alternative health care delivery systems, such as health maintenance organizations (HMOs). However, demand for Memorial Hospital's services does vary considerably by type of ward and time of year. Table 10.4 provides a historical monthly breakdown of the average daily patient census per ward.

The director of nursing for Memorial Hospital is Darlene Fry. Each fall she confronts one of the most challenging aspects of her job: planning the nurse-staffing levels for the next calendar year. Although the average demand for nurses has remained relatively stable over the past couple of years, the staffing plan usually changes because of changing work policies, changing pay structures, and temporary nurse availability and cost. With fall quickly approaching, Fry is collecting information to plan next year's staffing levels.

The nurses at Memorial Hospital work a regular schedule of four 10-hour days per week. The average regular-time pay across all nursing grades is $12.00 per hour. Overtime may be scheduled when necessary. However, because of the intensity of the demands placed on nurses, only a limited amount of overtime is permitted per week. Nurses may be scheduled for as many as 12 hours per day, for a maximum of 5 days per week. Overtime is compensated at a rate of $18.00 per hour. In periods of extremely high demand, temporary part-time nurses may be hired for a limited period of time. Temporary nurses are paid $15.00 per hour. Memorial Hospital's policy limits the proportion of temporary nurses to 15 percent of the total nursing staff.

Finding, hiring, and retaining qualified nurses is an ongoing problem for hospitals. One reason is that various forms of private practice lure many nurses away from hospitals with higher pay and greater flexibility. This situation has caused Memorial to guarantee its full-time staff nurses pay for a minimum of 30 hours per week, regardless of the demand placed on nursing services. In addition, each nurse receives 4 weeks of paid vacation each year. However, vacation scheduling may be somewhat restricted by the projected demand for nurses during particular times of the year.

**TABLE 10.3 | WARD CAPACITY DATA**

| Ward | Number of Beds | Patients per Nurse | Patient Census* |
|---|---|---|---|
| Intensive Care | 20 | 2 | 10 |
| Cardiac | 25 | 4 | 15 |
| Maternity | 30 | 4 | 10 |
| Pediatric | 40 | 4 | 22 |
| Surgery | 5 | † | † |
| Post-Op | 15 | 5 | 8 (T–F daily equivalent)‡ |
| Emergency | 10 | 3 | 5 (daily equivalent)‡ |
| General | 120 | 8 | 98 |

*Yearly average per day

†The hospital employs 20 surgical nurses. Routine surgery is scheduled on Tuesdays and Fridays; five surgeries can be scheduled per day per operating room (bed) on these days. Emergency surgery is scheduled as needed.

‡Daily equivalents are used to schedule nurses because patients flow through these wards in relatively short periods of time. A daily equivalent of 5 indicates that throughout a typical day, an average of five patients are treated in the ward.

At present, the hospital employs 130 nurses, including 20 surgical nurses. The other 110 nurses are assigned to the remaining seven major areas of the hospital. The personnel department informed Fry that the average cost to the hospital for hiring a new full-time nurse is $400 and for laying off or firing a nurse is $150. Although layoffs are an option, Fry is aware of the hospital's objective of maintaining a level workforce.

After looking over the information that she collected, Darlene Fry wants to consider staffing changes in all areas except the surgery ward, which is already correctly staffed.

## QUESTIONS

1. Explain the alternatives available to Darlene Fry as she develops a nurse-staffing plan for Memorial Hospital. How does each alternative plan meet the objective stated by the CEO?
2. Based on the data presented, develop a nurse-staffing plan for Memorial Hospital. Explain your rationale for this plan.

**TABLE 10.4** AVERAGE DAILY PATIENT CENSUS PER MONTH

| | MONTH | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Ward** | **J** | **F** | **M** | **A** | **M** | **J** | **J** | **A** | **S** | **O** | **N** | **D** |
| Intensive Care | 13 | 10 | 8 | 7 | 7 | 6 | 11 | 13 | 9 | 10 | 12 | 14 |
| Cardiac | 18 | 16 | 15 | 13 | 14 | 12 | 13 | 12 | 13 | 15 | 18 | 20 |
| Maternity | 8 | 8 | 12 | 13 | 10 | 8 | 13 | 13 | 14 | 10 | 8 | 7 |
| Pediatric | 22 | 23 | 24 | 24 | 25 | 21 | 22 | 20 | 18 | 20 | 21 | 19 |
| Surgery* | 20 | 18 | 18 | 17 | 16 | 16 | 22 | 21 | 17 | 18 | 20 | 22 |
| Post-Op† | 10 | 8 | 7 | 7 | 6 | 6 | 10 | 10 | 7 | 8 | 9 | 10 |
| Emergency† | 6 | 4 | 4 | 7 | 8 | 5 | 5 | 4 | 4 | 3 | 4 | 6 |
| General | 110 | 108 | 100 | 98 | 95 | 90 | 88 | 92 | 98 | 102 | 107 | 94 |

*Source:* This case was prepared by Dr. Brooke Saladin, Wake Forest University, North Carolina, as a basis for classroom discussion. 

*Average surgeries per day on Tuesday and Thursday.

†Daily equivalents

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SUPPLEMENT

# D

# LINEAR PROGRAMMING MODELS

In many business situations, resources are limited and demand for them is great. For example, a limited number of vehicles may have to be scheduled to make multiple trips to customers, or a staffing plan may have to be developed to cover expected variable demand with the fewest employees. In this supplement, we describe a technique called **linear programming**, which is useful for allocating scarce resources among competing demands. The resources may be time, money, or materials, and the limitations are known as constraints. Linear programming can help managers find the best allocation solution and provide information about the value of additional resources.

**linear programming**

A technique that is useful for allocating scarce resources among competing demands.

**LEARNING GOALS** *After reading this supplement, you should be able to:*

1. Define the seven characteristics of all linear programming models.
2. Formulate a linear programming model.
3. Perform a graphic analysis and derive a solution for a two-variable linear programming model.
4. Use a computer routine to solve a linear programming problem.
5. Apply the transportation method to sales and operations planning (S&OP) problems.

## Characteristics of Linear Programming Models

Before we can demonstrate how to solve problems in operations and supply chain management with linear programming, we must first explain seven characteristics of all linear programming models: (1) objective function, (2) decision variables, (3) constraints, (4) feasible region, (5) parameters, (6) linearity, and (7) nonnegativity.

**Objective Function.** Linear programming is an *optimization* process. A single **objective function** states mathematically what is being maximized (e.g., profit or present value) or minimized (e.g., cost or scrap). The objective function provides the scorecard on which the attractiveness of different solutions is judged.

**objective function**

An expression in linear programming models that states mathematically what is being maximized or minimized.

**Decision Variables.** Solving a linear programming model yields the optimal values for **decision variables**, which represent choices that the decision maker can control. For example, a decision variable could be the number of units of a product to make next month or the number of units of inventory to hold next month. Linear programming is based on the assumption that decision variables are *continuous*; they can be fractional quantities and need not be whole numbers. Often, this assumption is realistic, as when the decision variable is expressed in dollars, hours, or some other continuous measure. Even when the

**decision variables**

Variables that represent the choices the decision maker can control.

decision variables represent nondivisible units, such as workers, tables, or trucks, we sometimes can simply round the linear programming solution up or down to get a reasonable solution that does not violate any constraints, or we can use a more advanced technique, called *integer programming*.

**Constraints.** Limitations that restrict the permissible choices for the decision variables are called *constraints*. Each limitation can be expressed mathematically in one of three ways: a less-than-or-equal-to ($\leq$), an equal-to ($=$), or a greater-than-or-equal-to ($\geq$) constraint. A $\leq$ constraint puts an upper limit on some function of decision variables and most often is used with maximization problems. For example, a $\leq$ constraint may specify the maximum number of customers who can be served or the capacity limit of a machine. An $=$ constraint means that the function must equal some value. For example, 100 (not 99 or 101) units of one product must be made. An $=$ constraint often is used for certain mandatory relationships, such as the fact that ending inventory always equals beginning inventory plus production minus sales. A $\geq$ constraint puts a lower limit on some function of decision variables. For example, a $\geq$ constraint may specify that production of a product must exceed or equal demand.

**feasible region**

A region that represents all permissible combinations of the decision variables in a linear programming model.

**Feasible Region.** Every linear programming problem must have one or more constraints. Taken together, the constraints define a **feasible region**, which represents all permissible combinations of the decision variables. In some unusual situations, the problem is so tightly constrained that there is only one possible solution—or perhaps none. However, in the usual case, the feasibility region contains infinitely many possible solutions, assuming that the feasible combinations of the decision variables can be fractional values. The goal of the decision maker is to find the best possible solution.

**parameter**

A value that the decision maker cannot control and that does not change when the solution is implemented.

**certainty**

The word that is used to describe that a fact is known without doubt.

**Parameters.** The objective function and constraints are functions of decision variables and parameters. A **parameter**, also known as a *coefficient* or *given constant*, is a value that the decision maker cannot control and that does not change when the solution is implemented. Each parameter is assumed to be known with **certainty**. For example, a computer programmer may know that running a software program will take 30 minutes—no more, no less.

**linearity**

A characteristic of linear programming models that implies proportionality and additivity—there can be no products or powers of decision variables.

**Linearity.** The objective function and constraint equations are assumed to be linear. **Linearity** implies proportionality and additivity—there can be no products (e.g., $10x_1x_2$) or powers (e.g., $x_1^3$) of decision variables. Suppose that the profit gained by producing two types of products (represented by decision variables $x_1$ and $x_2$) is $2x_1 + 3x_2$. Proportionality implies that one unit of $x_1$ contributes \$2 to profits and two units contribute \$4, regardless of how much of $x_1$ is produced. Similarly, each unit of $x_2$ contributes \$3, whether it is the first or the tenth unit produced. Additivity means that the total objective function value equals the profits from $x_1$ plus the profits from $x_2$.

**nonnegativity**

An assumption that the decision variables must be positive or zero.

**Nonnegativity.** Finally, we make an assumption of **nonnegativity**, which means that the decision variables must be positive or zero. A firm that makes spaghetti sauce, for example, cannot produce a negative number of jars. To be formally correct, a linear programming formulation should show a $\geq 0$ constraint for each decision variable.

Although the assumptions of linearity, certainty, and continuous variables are restrictive, linear programming can help managers analyze many complex resource allocation problems. The process of building the model forces managers to identify the important decision variables and constraints, which is a useful step in its own right. Identifying the nature and scope of the problem represents a major step toward solving it. In a later section, we show how sensitivity analysis can help the manager deal with uncertainties in the parameters and answer "what-if" questions.

# Formulating a Linear Programming Model

**product-mix problem**

A one-period type of planning problem, the solution of which yields optimal output quantities (or product mix) of a group of services or products subject to resource capacity and market demand constraints.

Linear programming applications begin with the formulation of a *model* of the problem with the general characteristics just described. We illustrate the modeling process here with the **product-mix problem**, which is a one-period type of planning problem, the solution of which yields optimal output quantities (or product mix) of a group of services or products subject to resource capacity and market demand constraints. This problem was first introduced in Chapter 5, "Managing Process Constraints," and now we take it up more formally. Formulating a model to represent each unique problem, using the following three-step sequence, is the most creative and perhaps the most difficult part of linear programming.

**Step 1.** *Define the Decision Variables.* What must be decided? Define each decision variable specifically, remembering that the definitions used in the objective function must be equally useful in the constraints. The definitions should be as specific as possible. Consider the following two alternative definitions:

$$x_1 = \text{product 1}$$
$$x_1 = \text{number of units of product 1 to be produced and sold next month}$$

The second definition is much more specific than the first, making the remaining steps easier.

**Step 2.** *Write Out the Objective Function.* What is to be maximized or minimized? If it is next month's profits, write out an objective function that makes next month's profits a linear function of the decision variables. Identify parameters to go with each decision variable. For example, if each unit of $x_1$ sold yields a profit of \$7, the total profit from product $x_1 = 7x_1$. If a variable has no impact on the objective function, its objective function coefficient is 0. The objective function often is set equal to $Z$, and the goal is to maximize or minimize $Z$.

**Step 3.** *Write Out the Constraints.* What limits the values of the decision variables? Identify the constraints and the parameters for each decision variable in them. As with the objective function, the parameter for a variable that has no impact in a constraint is 0. To be formally correct, also write out the nonnegativity constraints.

As a consistency check, make sure that the same unit of measure is being used on both sides of each constraint and in the objective function. For example, suppose that the right-hand side of a constraint is hours of capacity per month. Then, if a decision variable on the left-hand side of the constraint measures the number of units produced per month, the dimensions of the parameter that is multiplied by the decision variable must be hours per unit because

$$\left(\frac{\text{Hours}}{\cancel{\text{Unit}}}\right)\left(\frac{\cancel{\text{Units}}}{\text{Month}}\right) = \left(\frac{\text{Hours}}{\text{Month}}\right)$$

Of course, you can also skip around from one step to another, depending on the part of the problem that has your attention. If you cannot get past step 1, try a new set of definitions for the decision variables. Often the problem can be modeled correctly in more than one way.

## EXAMPLE D.1 Formulating a Linear Programming Model

The Stratton Company produces two basic types of plastic pipe. Three resources are crucial to the output of pipe: extrusion hours, packaging hours, and a special additive to the plastic raw material. The following data represent next week's situation. All data are expressed in units of 100 feet of pipe.

| | PRODUCT | | |
|---|---|---|---|
| **Resource** | **Type 1** | **Type 2** | **Resource Availability** |
| Extrusion | 4 hr | 6 hr | 48 hr |
| Packaging | 2 hr | 2 hr | 18 hr |
| Additive mix | 2 lb | 1 lb | 16 lb |

The contribution to profits and overhead per 100 feet of pipe is \$34 for type 1 and \$40 for type 2. Formulate a linear programming model to determine how much of each type of pipe should be produced to maximize contribution to profits and to overhead, assuming that everything produced can be sold.

### SOLUTION

**Step 1.** To define the decision variables that determine product mix, we let

$x_1$ = amount of type 1 pipe to be produced and sold next week, measured in 100-foot increments (e.g., $x_1 = 2$ means 200 feet of type 1 pipe)

and

$x_2$ = amount of type 2 pipe to be produced and sold next week, measured in 100-foot increments

**Step 2.** Next, we define the objective function. The goal is to maximize the total contribution that the two products make to profits and overhead. Each unit of $x_1$ yields \$34, and each unit of $x_2$ yields \$40. For specific values of $x_1$ and $x_2$, we find the total profit by multiplying the number of units of each product produced by the profit per unit and adding them. Thus, our objective function becomes

$$\text{Maximize: } \$34x_1 + \$40x_2 = Z$$

**Step 3.** The final step is to formulate the constraints. Each unit of $x_1$ and $x_2$ produced consumes some of the critical resources. In the extrusion department, a unit of $x_1$ requires 4 hours and a unit of $x_2$ requires

6 hours. The total must not exceed the 48 hours of capacity available, so we use the $\leq$ sign. Thus, the first constraint is

$$4x_1 + 6x_2 \leq 48$$

Similarly, we can formulate constraints for packaging and raw materials:

$$2x_1 + 2x_2 \leq 18 \text{ (packaging)}$$
$$2x_1 + x_2 \leq 16 \text{ (additive mix)}$$

These three constraints restrict our choice of values for the decision variable because the values we choose for $x_1$ and $x_2$ must satisfy all of the constraints. Negative values for $x_1$ and $x_2$ do not make sense, so we add nonnegativity restrictions to the model:

$$x_1 \geq 0 \text{ and } x_2 \geq 0 \text{ (nonnegativity restrictions)}$$

We can now state the entire model, made complete with the definitions of variables.

$$\begin{aligned} \text{Maximize: } & \$34x_1 + \$40x_2 = Z \\ \text{Subject to: } & 4x_1 + 6x_2 \leq 48 \\ & 2x_1 + 2x_2 \leq 18 \\ & 2x_1 + x_2 \leq 16 \\ & x_1 \geq 0 \text{ and } x_2 \geq 0 \end{aligned}$$

where

$x_1$ = amount of type 1 pipe to be produced and sold next week, measured in 100-foot increments
$x_2$ = amount of type 2 pipe to be produced and sold next week, measured in 100-foot increments

## Graphic Analysis

With the model formulated, we now seek the optimal solution. In practice, linear programming problems are solved with a computer. However, insight into the meaning of the computer output—and linear programming concepts in general—can be gained by analyzing a simple two-variable problem with the **graphic method of linear programming**. Hence, we begin with the graphic method, even though it is not a practical technique for solving problems that have three or more decision variables. The five basic steps are (1) *plot the constraints*, (2) *identify the feasible region*, (3) *plot an objective function line*, (4) *find the visual solution*, and (5) *find the algebraic solution*.

**graphic method of linear programming**

A type of graphic analysis that involves the following five steps: plotting the constraints, identifying the feasible region, plotting an objective function line, finding a visual solution, and finding the algebraic solution.

### Plot the Constraints

We begin by plotting the constraint equations, disregarding the inequality portion of the constraints ($<$ or $>$). Making each constraint an equality ($=$) transforms it into the equation for a straight line. The line can be drawn as soon as we identify two points on it. Any two points reasonably spread out may be chosen; the easiest ones to find are the *axis intercepts*, where the line intersects each axis. To find the $x_1$ axis intercept, set $x_2$ equal to 0 and solve the equation for $x_1$. For the Stratton Company in Example D.1, the equation of the line for the extrusion process is

$$4x_1 + 6x_2 = 48$$

For the $x_1$ axis intercept, $x_2 = 0$, so

$$\begin{aligned} 4x_1 + 6(0) &= 48 \\ x_1 &= 12 \end{aligned}$$

To find the $x_2$ axis intercept, set $x_1 = 0$ and solve for $x_2$:

$$\begin{aligned} 4(0) + 6x_2 &= 48 \\ x_2 &= 8 \end{aligned}$$

We connect points (0, 8) and (12, 0) with a straight line, as shown in Figure D.1.

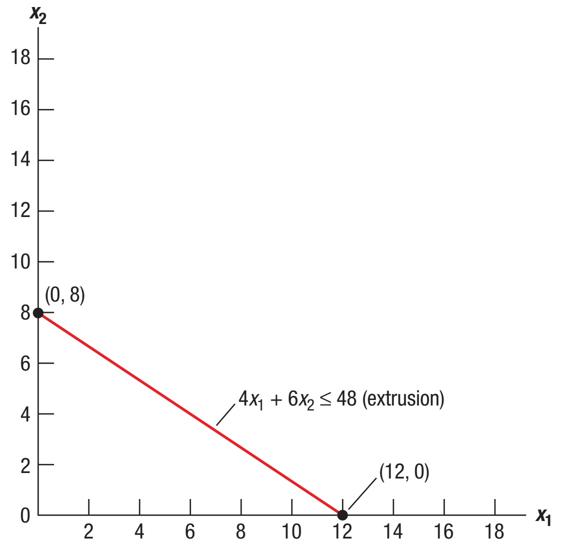



**FIGURE D.1**
Graph of the Extrusion Constraint

## EXAMPLE D.2 Plotting the Constraints

For the Stratton Company problem, plot the other constraints: one constraint for packaging and one constraint for the additive mix.

**MyOMLab**
Active Model D.1 in MyOMLab offers many insights on graphic analysis and sensitivity analysis. Use it when studying Examples D.2 through D.4.

**MyOMLab**
Tutor D.1 in MyOMLab provides a new practice example for plotting the constraints.

### SOLUTION

The equation for the packaging process's line is $2x_1 + 2x_2 = 18$. To find the $x_1$ intercept, set $x_2 = 0$:

$$2x_1 + 2(0) = 18$$
$$x_1 = 9$$

To find the $x_2$ axis intercept, set $x_1 = 0$:

$$2(0) + 2x_2 = 18$$
$$x_2 = 9$$

The equation for the additive mix's line is $2x_1 + x_2 = 16$. To find the $x_1$ intercept, set $x_2 = 0$:

$$2x_1 + 0 = 16$$
$$x_1 = 8$$

To find the $x_2$ axis intercept, set $x_1 = 0$:

$$2(0) + x_2 = 16$$
$$x_2 = 16$$

With a straight line, we connect points (0, 9) and (9, 0) for the packaging constraint and points (0, 16) and (8, 0) for the additive mix constraint. Figure D.2 shows the graph with all three constraints plotted.

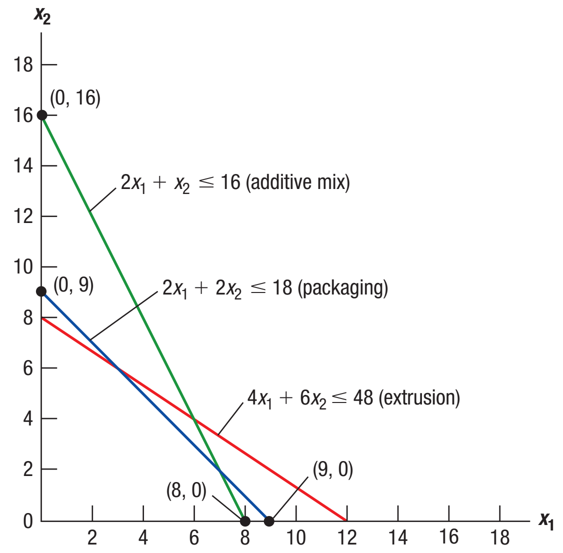



**FIGURE D.2**
Graph of the Three Constraints

## Identify the Feasible Region

The feasible region is the area on the graph that contains the solutions that satisfy all the constraints simultaneously, including the nonnegativity restrictions. To find the feasible region, first locate the feasible points for each constraint and then the area that satisfies all constraints. Generally, the following three rules identify the feasible points for a given constraint:

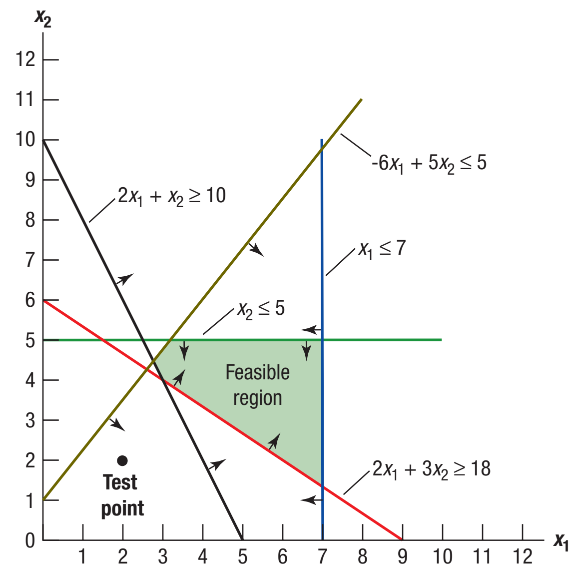
1. For the $=$ constraint, only the points on the line are feasible solutions.
2. For the $\leq$ constraint, the points on the line and the points below or to the left of the line are feasible solutions.
3. For the $\geq$ constraint, the points on the line and the points above or to the right of the line are feasible solutions.

Exceptions to these rules occur when one or more of the parameters on the left-hand side of a constraint are negative. In such cases, we draw the constraint line and test a point on one side of it. If the point does not satisfy the constraint, it is in the infeasible part of the graph. Suppose that a linear programming model has the following five constraints plus the two nonnegativity constraints:

$$
\begin{aligned}
2x_1 + x_2 &\geq 10 \\
2x_1 + 3x_2 &\geq 18 \\
x_1 &\leq 7 \\
x_2 &\leq 5 \\
-6x_1 + 5x_2 &\leq 5 \\
x_1, x_2 &\geq 0
\end{aligned}
$$

The feasible region is the shaded portion of Figure D.3. The arrows shown on each constraint identify which side of each line is feasible. The rules work for all but the fifth constraint, which has a negative parameter, $-6$, for $x_1$. We arbitrarily select (2, 2) as the test point, which Figure D.3 shows is below the line and to the right. At this point, we find $-6(2) + 5(2) = -2$. Because $-2$ does not exceed 5, the portion of the figure containing (2, 2) is feasible, at least for this fifth constraint.

**FIGURE D.3** ▶
Identifying the Feasible Region




**EXAMPLE D.3** **Identifying the Feasible Region**

Identify the feasible region for the Stratton Company problem.

**SOLUTION**

Because the problem contains only $\leq$ constraints, and the parameters on the left-hand side of each constraint are not negative, the feasible portions are to the left of and below each constraint. The feasible region, shaded in Figure D.4, satisfies all three constraints simultaneously.

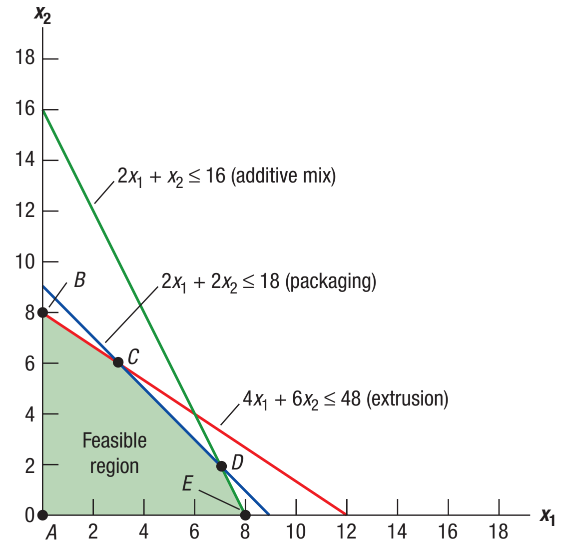



**FIGURE D.4**
Identifying the Feasible Region

## Plot the Objective Function Line

Now we want to find the solution that optimizes the objective function. Even though all the points in the feasible region represent possible solutions, we can limit our search to the corner points. A **corner point** lies at the intersection of two (or possibly more) constraint lines on the boundary of the feasible region. No interior points in the feasible region need be considered because at least one corner point is better than any interior point. Similarly, other points on the boundary of the feasible region can be ignored because a corner point is at least as good as any of them.

**corner point**
A point that lies at the intersection of two (or possibly more) constraint lines on the boundary of the feasible region.

In Figure D.4, the five corner points are marked *A*, *B*, *C*, *D*, and *E*. Point *A* is the origin (0, 0) and can be ignored because any other feasible point is a better solution. We could try each of the other corner points in the objective function and select the one that maximizes *Z*. For example, corner point *B* lies at (0, 8). If we substitute these values into the objective function, the resulting *Z* value is 320:

$$
\begin{aligned}
34x_1 \quad + 40x_2 \quad &= Z \\
34(0) + 40(8) &= 320
\end{aligned}
$$

However, we may not be able to read accurately the values of $x_1$ and $x_2$ for some of the points (e.g., *C* or *D*) on the graph. Algebraically solving two linear equations for each corner point also is inefficient when there are many constraints and, thus, many corner points.

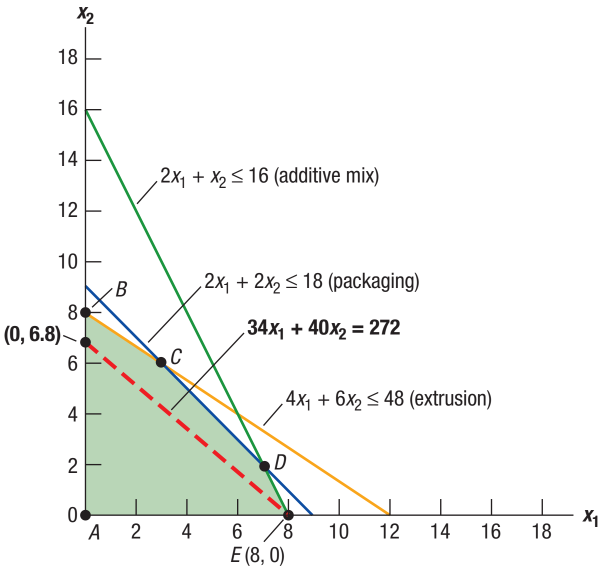
The best approach is to plot the objective function on the graph of the feasible region for some arbitrary *Z* values. From these objective function lines, we can spot the best solution visually. If the objective function is profits, each line is called an *iso-profit line* and every point on that line will yield the same profit. If *Z* measures cost, the line is called an *iso-cost line* and every point on it represents the same cost. We can simplify the search by plotting the first line in the feasible region—somewhere near the optimal solution, we hope. For the Stratton Company example, let us pass a line through point *E* (8, 0). This point is a corner point. It might even be the optimal solution because it is far from the origin. To draw the line, we first identify its *Z* value as $34(8) + 40(0) = 272$. Therefore, the equation for the objective function line passing through *E* is

$$34x_1 + 40x_2 = 272$$

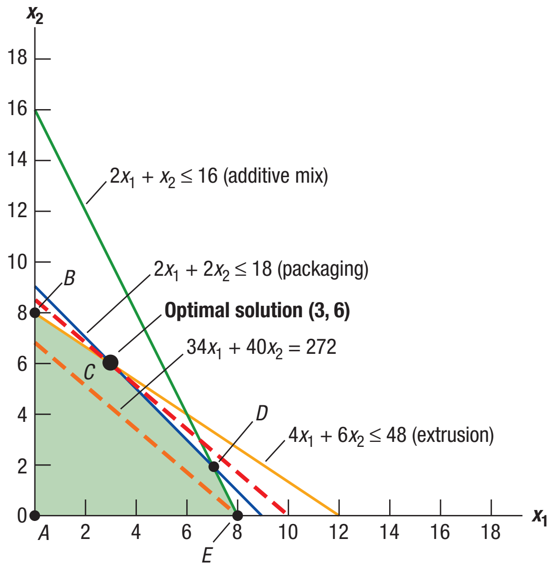
Every point on the line defined by this equation has an objective function *Z* value of 272. To draw the line, we need to identify a second point on it and then connect the two points. Let us use the $x_2$ intercept, where $x_1 = 0$:

$$
\begin{aligned}
34(0) + 40x_2 &= 272 \\
x_2 &= 6.8
\end{aligned}
$$

Figure D.5 shows the iso-profit line that connects points (8, 0) and (0, 6.8). A series of other dashed lines could be drawn parallel to this first line. Each would have its own *Z* value. Lines above the first line we drew would have higher *Z* values. Lines below it would have lower *Z* values.




▲ **FIGURE D.5**
Passing an Iso-Profit Line Through (8, 0)




▲ **FIGURE D.6**
Drawing the Second Iso-Profit Line

## Find the Visual Solution

We now eliminate corner points $A$ and $E$ from consideration as the optimal solution because better points lie above and to the right of the $Z = 272$ iso-profit line. Our goal is to maximize profits, so the best solution is a point on the iso-profit line *farthest* from the origin but still touching the feasible region. (For minimization problems, it is a point in the feasible region on the iso-cost line *closest* to the origin.)[1] To identify which of the remaining corner points is optimal ($B$, $C$, or $D$), we draw, parallel to the first line, one or more iso-profit lines that give better $Z$ values (higher for maximization and lower for minimization). The line that just touches the feasible region identifies the optimal solution. For the Stratton Company problem, Figure D.6 shows the second iso-profit line. The optimal solution is the last point touching the feasible region: point $C$. It appears to be in the vicinity of (3, 6), but the visual solution is not exact.

A linear programming problem can have more than one optimal solution. This situation occurs when the objective function is parallel to one of the faces of the feasible region. Such would be the case if our objective function in the Stratton Company problem were $\$38x_1 + \$38x_2$. Points (3, 6) and (7, 2) would be optimal, as would any other point on the line connecting these two corner points. In such a case, management probably would base a final decision on nonquantifiable factors. It is important to understand, however, that we need to consider only the corner points of the feasible region when optimizing an objective function.

## Find the Algebraic Solution

To find an exact solution, we must use algebra. We begin by identifying the pair of constraints that define the corner point at their intersection. We then list the constraints as equations and solve them simultaneously to find the coordinates $x_1$, $x_2$ of the corner point. Simultaneous equations can be solved several ways. For small problems, the easiest way is as follows:

***Step 1.*** Develop an equation with just one unknown. Start by multiplying both sides of one equation by a constant so that the coefficient for one of the two decision variables is *identical* in both equations. Then, subtract one equation from the other and solve the resulting equation for its single unknown variable.

***Step 2.*** Insert this decision variable's value into either one of the original constraints and solve for the other decision variable.

[1]The statements "farthest from the origin" or "closest to the origin" would no longer be true if there are negative coefficients in the objective function.

**EXAMPLE D.4** **Finding the Optimal Solution Algebraically**

Find the optimal solution algebraically for the Stratton Company problem. What is the value of $Z$ when the decision variables have optimal values?

MyOMLab

Tutor D.2 in MyOMLab provides a new practice example for finding the optimal solution.

**SOLUTION**

**Step 1.** Figure D.6 showed that the optimal corner point lies at the intersection of the extrusion and packaging constraints. Listing the constraints as equalities, we have

$$4x_1 + 6x_2 = 48 \text{ (extrusion)}$$
$$2x_1 + 2x_2 = 18 \text{ (packaging)}$$

We multiply each term in the packaging constraint by 2. The packaging constraint now is $4x_1 + 4x_2 = 36$. Next, we subtract the packaging constraint from the extrusion constraint. The result will be an equation from which $x_1$ has dropped out. (Alternatively, we could multiply the second equation by 3 so that $x_2$ drops out after the subtraction.) Thus,

$$\begin{aligned} 4x_1 + 6x_2 &= 48 \\ -(4x_1 + 4x_2 &= 36) \\ \hline 2x_2 &= 12 \\ x_2 &= 6 \end{aligned}$$

**Step 2.** Substituting the value of $x_2$ into the extrusion equation, we get

$$\begin{aligned} 4x_1 + 6(6) &= 48 \\ 4x_1 &= 12 \\ x_1 &= 3 \end{aligned}$$

Thus, the optimal point is (3, 6). This solution gives a total profit of $34(3) + 40(6) = \$342$.

**DECISION POINT**

Management at the Stratton Company decided to produce 300 feet of type 1 pipe and 600 feet of type 2 pipe for the next week for a total profit of $342.

## Slack and Surplus Variables

Figure D.6 shows that the optimal product mix will exhaust all the extrusion and packaging resources because at the optimal corner point (3, 6) the two constraints are equalities. Substituting the values of $x_1$ and $x_2$ into these constraints shows that the left-hand sides equal the right-hand sides:

$$4(3) + 6(6) = 48 \text{ (extrusion)}$$
$$2(3) + 2(6) = 18 \text{ (packaging)}$$

A constraint (such as the one for extrusion) that helps form the optimal corner point is called a **binding constraint** because it limits the ability to improve the objective function. If a binding constraint is relaxed, or made less restrictive, a better solution is possible. Relaxing a constraint means increasing the right-hand-side parameter for a $\leq$ constraint or decreasing it for a $\geq$ constraint. No improvement is possible from relaxing a constraint that is not binding, such as the additive mix constraint in Figure D.6. If the right-hand side was increased from 16 to 17 and the problem was solved again, the optimal solution would not change. In other words, there is already more additive mix than needed.

**binding constraint**

A constraint that helps form the optimal corner point; it limits the ability to improve the objective function.

For nonbinding inequality constraints, knowing how much the left and right sides differ is helpful. Such information tells us how close the constraint is to becoming binding. For a $\leq$ constraint, the amount by which the left-hand side falls short of the right-hand side is called **slack**. For a $\geq$ constraint, the amount by which the left-hand side exceeds the right-hand side is called **surplus**. To find the slack for a $\leq$ constraint algebraically, we *add* a slack variable to the constraint and convert it to an equality. Then, we substitute in the values of the decision variables and solve for the slack. For example, the additive mix constraint in Figure D.6, $2x_1 + x_2 \leq 16$, can be rewritten by adding slack variable $s_1$:

**slack**

The amount by which the left-hand side of a linear programming constraint falls short of the right-hand side.

**surplus**

The amount by which the left-hand side of a linear programming constraint exceeds the right-hand side.

$$2x_1 + x_2 + s_1 = 16$$

We then find the slack at the optimal solution (3, 6):

$$\begin{aligned} 2(3) + 6 + s_1 &= 16 \\ s_1 &= 4 \end{aligned}$$

The procedure is much the same to find the surplus for a $\geq$ constraint, except that we *subtract* a surplus variable from the left-hand side. Suppose that $x_1 + x_2 \geq 6$ was another constraint in the Stratton Company problem, representing a lower bound on the number of units produced. We would then rewrite the constraint by subtracting a surplus variable $s_2$:

$$x_1 + x_2 - s_2 = 6$$

MyOMLab
Tutor D.3 in MyOMLab provides another practice example for finding slack.

The surplus at the optimal solution (3, 6) would be

$$\begin{aligned} 3 + 6 - s_2 &= 6 \\ s_2 &= 3 \end{aligned}$$

## Sensitivity Analysis

Rarely are the parameters in the objective function and constraints known with certainty. Often, they are just estimates of actual values. For example, the available packaging and extrusion hours for the Stratton Company are estimates that do not reflect the uncertainties associated with absenteeism or personnel transfers, and the required hours per unit to package and extrude may be time estimates that essentially are averages. Likewise, profit contributions used for the objective function coefficients do not reflect uncertainties in selling prices and such variable costs as wages, raw materials, and shipping.

MyOMLab

Despite such uncertainties, initial estimates are needed to solve the problem. Accounting, marketing, and time-standard information systems (see MyOMLab Supplement H, "Measuring Output Rates") often provide these initial estimates. After solving the problem using these estimated values, the analyst can determine how much the optimal values of the decision variables and the objective function value $Z$ would be affected if certain parameters had different values. This type of postsolution analysis for answering "what-if" questions is called *sensitivity analysis*.

One way of conducting sensitivity analysis for linear programming problems is the brute-force approach of changing one or more parameter values and resolving the entire problem. This approach may be acceptable for small problems, but it is inefficient if the problem involves many parameters. For example, brute-force sensitivity analysis using 3 separate values for each of 20 objective function coefficients requires $3^{20}$, or 3,486,784,401, separate solutions! Fortunately, efficient methods are available for getting sensitivity information without resolving the entire problem, and they are routinely used in most linear programming computer software packages. Table D.1 describes the four basic types of sensitivity analysis information provided by linear programming.

**TABLE D.1 | SENSITIVITY ANALYSIS INFORMATION PROVIDED BY LINEAR PROGRAMMING**

| Key Term | Definition |
|---|---|
| **Reduced cost** | How much the objective function coefficient of a decision variable must improve (increase for maximization or decrease for minimization) before the optimal solution changes and the decision variable "enters" the solution with some positive number |
| **Shadow price** | The marginal improvement in $Z$ (increase for maximization and decrease for minimization) caused by relaxing the constraint by one unit |
| **Range of optimality** | The interval (lower and upper bounds) of an objective function coefficient over which the optimal values of the decision variables remain unchanged |
| **Range of feasibility** | The interval (lower and upper bounds) over which the right-hand-side parameter of a constraint can vary while its shadow price remains valid |

# Computer Analysis

Most real-world linear programming problems are solved on a computer, so we concentrate here on understanding the use of linear programming and the logic on which it is based. The solution procedure in computer codes is some form of the **simplex method**, which is an iterative algebraic procedure for solving linear programming problems.

**simplex method**
An iterative algebraic procedure for solving linear programming problems.

## Simplex Method

The graphic analysis gives insight into the logic of the simplex method, beginning with the focus on corner points. If there is any feasible solution to a problem, at least one corner point will always be the optimum, even when multiple optimal solutions are available. Thus, the simplex method starts with an initial corner point and then systematically evaluates other corner points in such a way that the objective function improves (or, at worst, stays the same) at each iteration. In the Stratton Company problem, an improvement would be an increase in profits. When no more improvements are possible, the optimal solution has been found.[2] The simplex method also helps generate the sensitivity analysis information that we developed graphically.

Each corner point has no more than $m$ variables that are greater than 0, where $m$ is the number of constraints (not counting the nonnegativity constraints). The $m$ variables include slack and surplus variables, not just the original decision variables. Because of this property, we can find a corner point by simultaneously solving $m$ constraints, where all but $m$ variables are set equal to 0. For example, point $B$ in Figure D.6 has three nonzero variables: $x_2$, the slack variable for packaging, and the slack variable for the additive mix. Their values can be found by simultaneously solving the three constraints, with $x_1$ and the slack variable for extrusion equal to 0. After finding this corner point, the simplex method applies information similar to the reduced costs to decide which new corner point to find next that gives an even better $Z$ value. It continues in this way until no better corner point is possible. The final corner point evaluated is the optimal one.

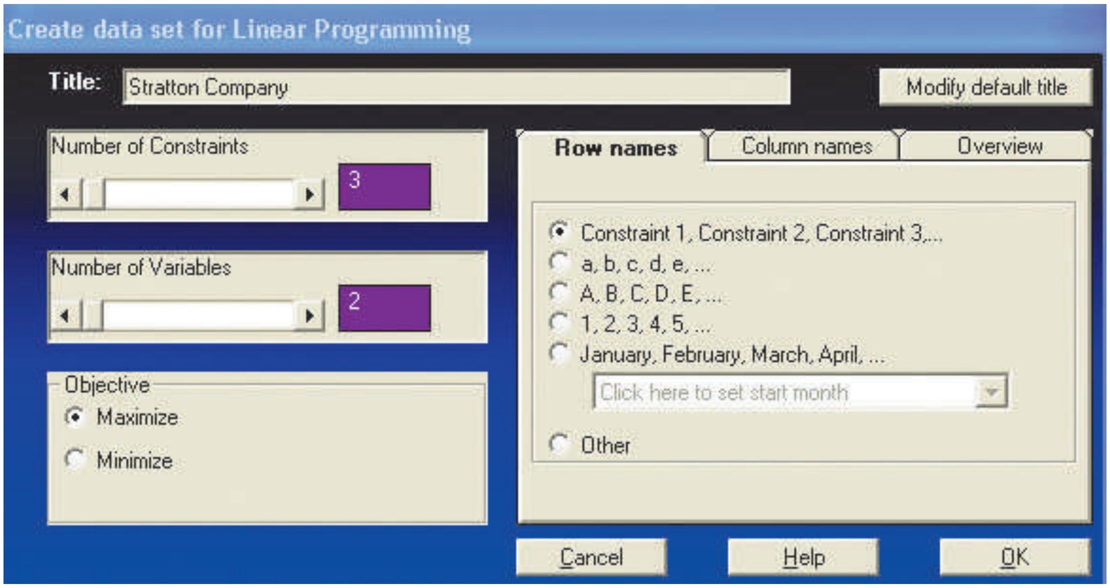
## Computer Output

Computer programs dramatically reduce the amount of time required to solve linear programming problems. The capabilities and displays of software packages are not uniform. For example, POM for Windows in MyOMLab can handle small- to medium-sized linear programming problems. Inputs are made easily and nonnegativity constraints need not be entered. Microsoft's *Excel* Solver offers a second option for similar problem sizes. More advanced software for larger problems is available from multiple sources.

MyOMLab

Here we show output from POM for Windows when applied to the Stratton Company. Figure D.7 shows the two *data entry* screens. The first screen allows you to enter the problem's name, specify the

**FIGURE D.7**
Data Entry Screens




Objective
Maximize
Minimize

Instruction
Enter the name for this constraint. Almost any character is permissible.

Stratton Company

| | X1 | X2 | | RHS | Equation form |
|---|---|---|---|---|---|
| Maximize | 34 | 40 | | | Max 34X1 + 40X2 |
| Extrusion | 4 | 6 | <= | 48 | 4X1 + 6X2 <= 48 |
| Packaging | 2 | 2 | <= | 18 | 2X1 + 2X2 <= 18 |
| Additive | 2 | 1 | <= | 16 | 2X1 + X2 <= 16 |

[2]For more information on how to perform the simplex method manually, see Render, Barry, Ralph M. Stair, and Michael E. Hanna. *Quantitative Analysis for Management*, 12th ed. or any other current textbook on management science.

number of constraints and decision variables, and choose between maximization and minimization. After making these inputs and clicking the "OK" button, the data table screen is shown. Enter the parameters, give names to each constraint and decision variable (as desired), and specify the type of relationship ($\leq, =, \geq$) for each constraint. The second screen in Figure D.7 shows the completed data table. The user may customize labels for the decision variables, right-hand-side values, and constraints. Here, the first decision variable is labeled as "X1," the right-hand-side values as "RHS," and the extrusion constraint as "Extrusion." Slack and surplus variables will be added automatically as needed. When all of the inputs are made, click the green arrow labeled "Solve" in the upper-right corner.

Figure D.8 displays the solution with the *Results* screen. All output confirms our earlier calculations and the graphic analysis. Of particular interest is the bottom row that gives the optimal values of the decision variables ($x_1 = 3$ and $x_2 = 6$), and also the optimal value of the objective function ($342). The shadow prices for each constraint are given in the last column.

**▼ FIGURE D.8**
Results Screen

| Stratton Company Solution | | | | | |
|---|---|---|---|---|---|
| | X1 | X2 | | RHS | Shadow Price |
| Maximize | 34 | 40 | | | |
| Constraint 1 | 4 | 6 | <= | 48 | 3 |
| Constraint 2 | 2 | 2 | <= | 18 | 11 |
| Constraint 3 | 2 | 1 | <= | 16 | 0 |
| Solution-> | 3 | 6 | | 342 | |

Click on the Window icon and select the second option in the drop down menu to switch to the *Ranging* screen, as shown in Figure D.9. The top half deals with the decision variables. Of particular interest are the reduced costs and the lower and upper bounds. Two tips on interpreting the *reduced cost* information are as follows:

**▼ FIGURE D.9**
Ranging Screen

| Variable | Value | Reduced Cost | Original Val | Lower Bound | Upper Bound |
|---|---|---|---|---|---|
| X1 | 3 | 0 | 34 | 26.6667 | 40 |
| X2 | 6 | 0 | 40 | 34 | 51 |
| Constraint | Shadow Price | Slack/Surplus | Original Val | Lower Bound | Upper Bound |
| Constraint 1 | 3 | 0 | 48 | 40 | 54 |
| Constraint 2 | 11 | 0 | 18 | 16 | 20 |
| Constraint 3 | 0 | 4 | 16 | 12 | Infinity |

1. It is relevant only for a decision variable that is 0 in the optimal solution. If the decision variable is greater than 0, ignore the reduced cost number. Thus, for the Stratton Company problem the reduced cost numbers provide no new insight because they are always 0 when decision variables have positive values in the optimal solution. Look instead at the lower and upper bounds on the objective function coefficients.
2. It tells how much the objective function coefficient of a decision variable that is 0 in the optimal solution must *improve* (increase for maximization problems or decrease for minimization problems) before the optimal solution would change. At that point, the decision variable associated with the coefficient enters the optimal solution at some positive level. To learn the new solution, apply POM for Windows again with a coefficient improved by slightly more than the reduced cost number.

The top half of this screen also gives the *range of optimality*, or the lower and upper bound over which the objective function coefficients can range without affecting the optimal values of the decision variables. Note that the objective function coefficient for $x_1$, which currently has a value of $34, has a range of optimality from $26.6667 to $40. While the objection function's $Z$ value would change with coefficient changes over this range, the optimal values of the decision variables remain the same.

The bottom half of Figure D.9 deals with the constraints, including the slack or surplus variables and the original right-hand-side values. Of particular interest are the shadow prices. Two tips on interpreting a *shadow price* follow:

1. The number is relevant only for a binding constraint, where the slack or surplus variable is 0 in the optimal solution. For a nonbinding constraint, the shadow price is 0.
2. The sign of the shadow price can be a positive or negative. The sign depends on whether the objective function is being maximized or minimized, and whether it is a $\leq$ constraint or $\geq$ constraint. If you simply ignore the sign, interpret the shadow price as the benefit of increasing the right-hand side by one unit of resource for a $\leq$ constraint and reducing it by one unit of resource for a $\geq$ constraint. The benefit is a reduction in the objective function value for minimization problems, and an increase for maximization problems. The shadow price can also be interpreted as the marginal loss (or penalty) in $Z$ caused by making the constraint more restrictive by one unit of resource.

Thus, the Stratton Company problem has 4 pounds of the additive mix slack, so the shadow price is $0. Packaging, on the other hand, is a binding constraint because it has no slack. The shadow price of one more packaging hour is $11. Example D.5 shows how shadow prices can be used for decision making.

Finally, Figure D.9 reports the lower and upper bounds for the *range of feasibility*, over which the right-hand-side parameters can range without changing the shadow prices. For example, the $11 shadow price for packaging is valid over the range from 16 to 20 hours.

The number of variables in the optimal solution (counting the decision variables, slack variables, and surplus variables) that are greater than 0 never exceeds the number of constraints. Such is the case for the Stratton Company problem, with its three constraints (not counting the implicit nonnegativity constraints) and three nonzero variables in the optimal solution ($x_1$,$x_2$, and the additive mix slack variable). On some rare occasions, the number of nonzero variables in the optimal solution can be less than the number of constraints—a condition called **degeneracy**. When degeneracy occurs, the sensitivity analysis information is suspect. If you want more "what-if" information, simply run your software package again using the new parameter values that you want to investigate.

**degeneracy**

A condition that occurs when the number of nonzero variables in the optimal solution is less than the number of constraints.

### EXAMPLE D.5 Using Shadow Prices for Decision Making

The Stratton Company needs answers to three important questions: (1) Would increasing capacities in the extrusion or packaging area pay if it cost an extra $8 per hour over and above the normal costs already reflected in the objective function coefficients? (2) Would increasing packaging capacity pay if it cost an additional $6 per hour? (3) Would buying more raw materials pay?

#### SOLUTION

Expanding extrusion capacity would cost a premium of $8 per hour, but the shadow price for that capacity is only $3 per hour. However, expanding packaging hours would cost only $6 per hour more than the price already reflected in the objective function, and the shadow price is $11 per hour. Finally, buying more raw materials would not pay because a surplus of 4 pounds already exists; the shadow price is $0 for that resource.

#### DECISION POINT

Management decided to increase its packaging capacity but did not decide to expand extrusion capacity or buy more raw materials.

## The Transportation Method

A special case of linear programming is the **transportation problem**, which can be represented as a standard table, sometimes called a *tableau*. Figure D.10 is an example, where the rows are supply sources and the columns are demands. Both the supplies and demands can be broken down into several periods into the future. Typically, the rows of the table are linear constraints that impose capacity limitations, and the columns are linear constraints that require certain demand levels to be met. Each cell in the tableau is a decision variable, and a per-unit cost is shown in the upper-right corner of each cell. Figure D.10 implies 52 decision variables (13 rows $\times$ 4 columns) and 17 constraints (13 rows $+$ 4 columns).

**transportation problem**

A special case of linear programming that has linear constraints for capacity limitations and demand requirements.

Transportation problems can be formulated as a conventional linear programming problem and solved as usual. The **transportation method** simplifies data input and is a more efficient solution technique, but it does not provide sensitivity analysis as in Figure D.9, nor can you add any constraints on the decision variables beyond capacity and demand. Here we show how production-planning problems can

**transportation method**

A more efficient solution technique than the simplex method for solving transportation problems.

**FIGURE D.10 ▶**
Example of Transportation Tableau

| Source of Supply | | Time Period | | | | Capacity |
|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | |
| Period | Initial Inventory | | | | | |
| 1 | Type 1 | | | | | |
| | Type 2 | | | | | |
| | Type 3 | | | | | |
| 2 | Type 1 | | | | | |
| | Type 2 | | | | | |
| | Type 3 | | | | | |
| 3 | Type 1 | | | | | |
| | Type 2 | | | | | |
| | Type 3 | | | | | |
| 4 | Type 1 | | | | | |
| | Type 2 | | | | | |
| | Type 3 | | | | | |
| Demand | | | | | | |

be formulated as transportation problems. Chapter 13, "Supply Chains and Logistics," shows an entirely different application of the transportation method: how to solve location problems. We focus on the setup and interpretation of the problem, leaving the rest of the solution process to a computer software package.

## Transportation Method for Sales and Operations Planning

Making sure that demand and supply are in balance is central to sales and operations planning (S&OP), so it is no surprise that the transportation method can be applied to it. The *transportation method for sales and operations planning* is particularly helpful in determining anticipation inventories. Thus, it relates more to manufacturers' production plans than to service providers' staffing plans. In fact, the workforce levels for each period are inputs to the transportation method rather than outputs from it. Different workforce adjustment plans should be evaluated. Thus, several transportation method solutions may be obtained before a final plan is selected.

Using the transportation method for production planning is based on the assumption that a demand forecast is available for each period, along with a possible workforce adjustment plan. Capacity limits on overtime and the use of subcontractors also are needed for each period. Another assumption is that all costs are linearly related to the amount of goods produced; that is, a change in the amount of goods produced creates a proportionate change in costs.

To develop a sales and operations plan for a manufacturer, we do the following:

1. Obtain the demand forecasts for each period to be covered by the sales and operations plan and identify the initial inventory level currently available that can be used to meet future demand.
2. Select a candidate workforce adjustment plan, using a chase strategy, level strategy, or a mixed strategy. Specify the capacity limits of each production alternative (regular time, overtime, and subcontracting) for each period covered by the plan.
3. Estimate the cost of holding inventory and the cost of possible production alternatives (regular-time production, overtime production, and subcontracting). Translate all costs to a common unit of measure, such as a unit of product. For example, regular-time wages, overtime wages, inventory holding costs, and so on could be expressed as dollars per unit. Identify the cost of undertime, if idle regular-time capacity is paid.

4. Input the information gathered in steps 1–3 into a computer routine that solves the transportation problem. After getting the solution, calculate the anticipation inventory levels and identify high-cost elements of the plan.
5. Repeat the process with other plans for regular-time, overtime, and subcontracting capacities until you find the solution that best balances cost and qualitative considerations. Even though this process involves trial and error, the transportation method yields the best mix of regular time, overtime, and subcontracting for each supply plan.

Example D.6 demonstrates this approach using the *Transportation Method (Production Planning)* module in the POM for Windows package.

## EXAMPLE D.6 Preparing a Production Plan with the Transportation Method

The Tru-Rainbow Company produces a variety of paint products for both commercial and private use. The demand for paint is highly seasonal, peaking in the third quarter. Initial inventory is 250,000 gallons, and ending inventory should be 300,000 gallons.

Tru-Rainbow's manufacturing manager wants to determine the best production plan using the following demand requirements and capacity plan. Demands and capacities here are expressed in thousands of gallons (rather than employee-period equivalents). The manager knows that the regular-time cost is $1.00 per unit, overtime cost is $1.50 per unit, subcontracting cost is $1.90 per unit, and inventory holding cost is $0.30 per unit per quarter. Undertime is paid and the cost is $0.50 per unit. It is less than the regular-time cost because only labor costs are involved, not materials and variable overhead going into paint production.

| | Demand | Regular-time Capacity | Overtime Capacity | Subcontractor Capacity |
|---|---|---|---|---|
| **Quarter 1** | 300 | 450 | 90 | 200 |
| **Quarter 2** | 850 | 450 | 90 | 200 |
| **Quarter 3** | 1,500 | 750 | 150 | 200 |
| **Quarter 4** | 350 | 450 | 90 | 200 |
| **Totals** | 3,000 | 2,100 | 420 | 800 |

The following constraints apply:

**a.** The maximum allowable overtime in any quarter is 20 percent of the regular-time capacity in that quarter.

**b.** The subcontractor can supply a maximum of 200,000 gallons in any quarter. Production can be subcontracted in one period and the excess held in inventory for a future period to avoid a stockout.

**c.** No planned backorders or stockouts are permitted.

### SOLUTION

Figure D.11 shows the POM for Windows screen of data inputs. Figure D.12 shows the POM for Windows screen that displays the optimal solution for this particular workforce adjustment plan. It looks much like the table shown previously, but with one exception. The demand for quarter 4 is shown to be 650,000 gallons rather than the demand forecast of only 350,000. The larger number reflects the desire of the manager to have an ending inventory in quarter 4 of 300,000 gallons. Some points to note in Figures D.11 and D.12 include the following:

1. There is a row in Figure D.12 for each supply alternative (instead of the "source of supply" in Figure D.10) on a quarter-by-quarter basis. The last column in each row indicates the maximum amount that can be used

▼ **FIGURE D.11**
POM for Windows Screens for Tru-Rainbow Company

| Period | Demand | Regular tm Capacity | Overtime Capacity | Subcontract Capacity | | Unit costs | Value |
|---|---|---|---|---|---|---|---|
| Quarter 1 | 300 | 450 | 90 | 200 | | Regular time | 1 |
| Quarter 2 | 850 | 450 | 90 | 200 | | Overtime | 1.5 |
| Quarter 3 | 1500 | 750 | 150 | 200 | | Subcontracting | 1.9 |
| Quarter 4 | 650 | 450 | 90 | 200 | | Holding cost | .3 |
| | | | | | | Lost sales cost | Not allowed |
| | | | | | | Idle RT Capacity | .5 |
| | | | | | | | |
| | | | | | | Initial Inventory | 250 |
| | | | | | | | |

| Optimal cost = $4,010 | Quarter 1 | Quarter 2 | Quarter 3 | Quarter 4 | Excess Capacity | Capacity |
|---|---|---|---|---|---|---|
| Init Inventory | 230 | | 20 | | | 250 |
| Quarter 1 RegTime | 50 | 400 | | | | 450 |
| Quarter 1 Overtime | | | 90 | | | 90 |
| Quarter 1 Subcontract | 20 | | | | 180 | 200 |
| Quarter 2 RegTime | | 450 | | | | 450 |
| Quarter 2 Overtime | | | 90 | | | 90 |
| Quarter 2 Subcontract | | | 200 | | | 200 |
| Quarter 3 RegTime | | | 750 | | | 750 |
| Quarter 3 Overtime | | | 150 | | | 150 |
| Quarter 3 Subcontract | | | 200 | | | 200 |
| Quarter 4 RegTime | | | | 450 | | 450 |
| Quarter 4 Overtime | | | | 90 | | 90 |
| Quarter 4 Subcontract | | | | 110 | 90 | 200 |
| Demand | 300 | 850 | 1500 | 650 | 270 | |

▲ **FIGURE D.12**
Solution Screen for Prospective Tru-Rainbow Company Production Plan

to meet demand. The first row is the initial inventory available, and the rows that follow are for regular-time, overtime, and subcontracting production in each of the four quarters. The initial inventory can be used to satisfy demand in any of the four quarters. The second row (regular-time production in period 1) can also be used to satisfy demand in any of the four periods the plan will cover, and so on. The numbers in the last column give the maximum capacity made available for the supply alternatives. For example, the regular-time capacity for quarter 3 increases from the usual 450,000 gallons to 750,000 gallons, to help with the peak demand forecasted to be 1,500,000 gallons.

2. A column indicates each future quarter of demand and the last row gives its demand forecast. The demand for the fourth quarter is shown to be 650 units, because it includes the desired amount of ending inventory. The Excess Capacity column shows the cost of unused capacity, and the number in the last row (270 units) is the amount by which total capacity exceeds total demand. The unit costs for unused capacity are 0, except for the $0.50 unit cost of unused regular-time capacity.
3. The numbers in the other cells (excluding the cells in the last row or last column) show the cost of producing a unit in one period and, in some cases, carrying the unit in inventory for sale in a future period. These numbers correspond to the costs in the upper-right corners of the cells in Figure D.10. For example, the cost per unit of regular-time production in quarter 1 is $1.00 per gallon if it is used to meet the demand in quarter 1. This cost is found in row 2 and column 1 of the tableau. However, if it is produced to meet demand in quarter 2, the cost increases to $1.30 (or $1.00 + $0.30) because we must hold the unit in inventory for one quarter. Satisfying a unit of demand in quarter 3 by producing in quarter 1 on regular time and carrying the unit for two quarters costs $1.60, or [$1.00 + (2 × $0.30)], and so on. A similar approach is used for the costs of overtime and subcontracting.
4. The cells in the bottom left portion of the tableau with a cost of $9,999 are associated with backorders (or producing in a period to satisfy demand in a period after it was needed). Here we disallow backorders by making the backorder cost an arbitrarily large number, in this case $9,999 per unit. If backorder costs are so large, the transportation method will try to avoid backorders because it seeks a solution that minimizes total cost. If that is not possible, we increase the staffing plan and the overtime and subcontracting capacities.
5. The least expensive alternatives are those in which the output is produced and sold in the same period. For example, the cost for quarter 2 overtime production is only $1.50 per gallon if it is designated to meet demand in quarter 2 (row 6, column 2). The cost increases to $2.10 if it is designated for quarter 4 demand. However, we may not always be able to avoid alternatives that create inventory because of capacity restrictions.
6. Finally, the per-unit holding cost for the beginning inventory in period 1 is 0 because it is a function of previous production-planning decisions.

The first row in Figure D.12 shows that 230 units of the initial inventory are used to help satisfy the demand in quarter 1. The remaining 20 units in the first row are earmarked for helping supply the demand in quarter 3. The sum of the allocations across row 1 for the four quarters $(230 + 0 + 20 + 0)$ does not exceed the maximum capacity of 250, given in the right column. With the transportation method, this result must occur with each row. Any shortfalls are unused capacity, given in the "Excess Capacity" column. In this case, the undertime cost of \$0.50 per unit was sufficiently large that no regular-time capacity went unused.

Similarly, the sum of the allocations down each column must equal the total demand for the quarter. For example, the demand for quarter 1 is supplied from 230 units of the initial inventory, 50 units of quarter 1 regular-time production, and 20 units of quarter 1 subcontracting production. Summed together, they equal the forecasted demand of 300 units.

To further interpret the solution, we can convert Figure D.12 into the following table. For example, the total regular-time production in quarter 1 is 450,000 gallons (50,000 gallons to help meet demand in quarter 1 and 400,000 gallons to help satisfy demand in quarter 2).

The anticipation inventory held at the end of each quarter is obtained in the last column. For any quarter, it is the quarter's beginning inventory plus total supply (regular-time and overtime production, plus subcontracting) minus demand. For example, for quarter 1 the beginning inventory (250,000) plus the total from production and subcontracting (560,000) minus quarter 1 demand (300,000) results in an ending inventory of 510,000, which also is the beginning inventory for quarter 2.

| Quarter | Regular-time Production | Overtime Production | Subcontracting | Total Supply | Anticipation Inventory |
|---|---|---|---|---|---|
| 1 | 450 | 90 | 20 | 560 | $250 + 560 - 300 = 510$ |
| 2 | 450 | 90 | 200 | 740 | $510 + 740 - 850 = 400$ |
| 3 | 750 | 150 | 200 | 1,100 | $400 + 1{,}100 - 1{,}500 = 0$ |
| 4 | 450 | 90 | 110 | 650 | $0 + 650 - 350 = 300$ |
| Totals | 2,100 | 420 | 530 | 3,050 | |

*Note:* Anticipation inventory is the amount at the end of each quarter, where Beginning inventory + Total production − Actual Demand = Ending inventory.

The breakdown of costs can be found by multiplying the allocation in each cell of Figure D.12 by the cost per unit in that cell in the last column of Figure D.11 . Computing the cost column by column (it can also be done on a row-by-row basis) yields a total cost of \$4,010,000, or $\$4{,}010 \times 1{,}000$.

| Cost Calculations by Column | | |
|---|---|---|
| *Quarter 1* | $230(\$0) + 50(\$1.00) + 20(\$1.90)$ | $= \$\ 88$ |
| *Quarter 2* | $400(\$1.30) + 450(\$1.00)$ | $= 970$ |
| *Quarter 3* | $20(\$0.60) + 90(\$2.10) + 90(\$1.80) + 200(\$2.20) + 750(\$1.00) + 150(\$1.50) + 200(\$1.90)$ | $= 2{,}158$ |
| *Quarter 4* | $450(\$1.00) + 90(\$1.50) + 110(\$1.90)$ | $= 794$ |
| | | Total $= \$4{,}010$ |

## DECISION POINT

This plan requires too much overtime and subcontracting, and the anticipation inventory cost is substantial. The manager decided to search for a better capacity plan—with increases in the workforce to boost regular-time production capacity—that could lower production costs, perhaps even low enough to offset the added capacity costs.

Many problems in operations and supply chain management, and in other functional areas, lend themselves to linear programming and the transportation method. In addition to the examples already introduced in this supplement, applications also exist in process management, constraint management, shipping assignments, inventory control, and shift scheduling. The review problems at the end of this supplement and at the end of previous chapters illustrate many of these types of problems. Once the decision maker knows how to formulate a problem generally, he or she can then adapt it to the situation at hand.

## LEARNING GOALS IN REVIEW

| Learning Goal | Guidelines for Review | MyOMLab Resources |
|---|---|---|
| 1 Define the seven characteristics of all linear programming models. | See the section "Characteristics of Linear Programming Models," pp. 429–430. | |
| 2 Formulate a linear programming model. | Review the section "Formulating a Linear Programming Model," pp. 430–432, and pay particular attention to Example D.1. | |
| 3 Perform a graphic analysis and derive a solution for a two-variable linear programming model. | See the section "Graphic Analysis," pp. 432–438, where we show how to plot the constraints, identify the feasible region, plot the objective function, and find the solution. Do not overlook the sections on slack and surplus variables and sensitivity analysis because they will set the foundation for the use of computers. Review Solved Problem 1. | **Active Model Exercise:** D.1: LP Graph<br>**POM for Windows:** Linear Programming<br>**OM Explorer Tutors:** D.1: Plotting the Constraints; D.2: Finding the Optimal Solution; D.3: Finding Slack at the Optimal Solution; D.4: Graphic and Algebraic Solution |
| 4 Use a computer routine to solve a linear programming problem. | The section "Computer Analysis," pp. 438–441, goes into detail as to how computers can be used to solve linear programming problems. Computer output from POM for Windows is displayed and interpreted on pp. 439–441. Key information relates to the optimal values of the decision variables, objective function value, slack variables, and surplus variables. The shadow prices, reduced costs, lower bounds, and upper bounds can be valuable information for sensitivity analysis. | **POM for Windows:** Linear Programming |
| 5 Apply the transportation method to Sales and Operations Planning (S&OP) problems. | The section "The Transportation Method," pp. 441–445, gives a step-by-step description on how to set up the problem as a transportation problem and then how to interpret the POM for Windows output. Pay particular attention to Figures D.11 and D.12. Example D.6 and Solved Problem 2 are also helpful. | **POM for Windows:** Transportation Method (Production Planning) |

## Key Terms

binding constraint 437
certainty 430
corner point 435
decision variables 429
degeneracy 441
feasible region 430
graphic method of linear programming 432
linearity 430
linear programming 429
nonnegativity 430
objective function 429
parameter 430
product-mix problem 430
range of feasibility 438
range of optimality 438
reduced cost 438
shadow price 438
simplex method 438
slack 437
surplus 437
transportation method 441
transportation problem 441

## Solved Problem 1

MyOMLab
Tutor D.4 in MyOMLab provides a practice example for finding the graphic and algebraic solution.

MyOMLab Video

O'Connel Airlines is considering air service from its hub of operations in Cicely, Alaska, to Rome, Wisconsin, and Seattle, Washington. O'Connel has one gate at the Cicely Airport, which operates 12 hours per day. Each flight requires 1 hour of gate time. Each flight to Rome consumes 15 hours of pilot crew time and is expected to produce a profit of $2,500. Serving Seattle uses 10 hours of pilot crew time per flight and will result in a profit of $2,000 per flight. Pilot crew labor is limited to 150 hours per day. The market for service to Rome is limited to nine flights per day.

**a.** Use the graphic method of linear programming to maximize profits for O'Connel Airlines.

**b.** Identify positive slack and surplus variables, if any.

**SOLUTION**

**a.** The objective function is to maximize profits, $Z$:

$$\text{Maximize: } \$2{,}500x_1 + \$2{,}000x_2 = Z$$

where

$$x_1 = \text{number of flights per day to Rome, Wisconsin}$$
$$x_2 = \text{number of flights per day to Seattle, Washington}$$

The constraints are

$$x_1 + x_2 \leq 12 \text{ (gate capacity)}$$
$$15x_1 + 10x_2 \leq 150 \text{ (labor)}$$
$$x_1 \leq 9 \text{ (market)}$$
$$x_1 \geq 0 \text{ and } x_2 \geq 0$$

A careful drawing of iso-profit lines parallel to the one shown in Figure D.13 will indicate that point *D* is the optimal solution. It is at the intersection of the labor and gate capacity constraints. Solving algebraically, we get

$$\begin{aligned} 15x_1 + 10x_2 &= 150 \text{ (labor)} \\ -10x_1 - 10x_2 &= -120 \text{ (gate} \times -10) \\ 5x_1 + 0x_2 &= 30 \\ x_1 &= 6 \\ 6 + x_2 &= 12 \text{ (gate)} \\ x_2 &= 6 \end{aligned}$$

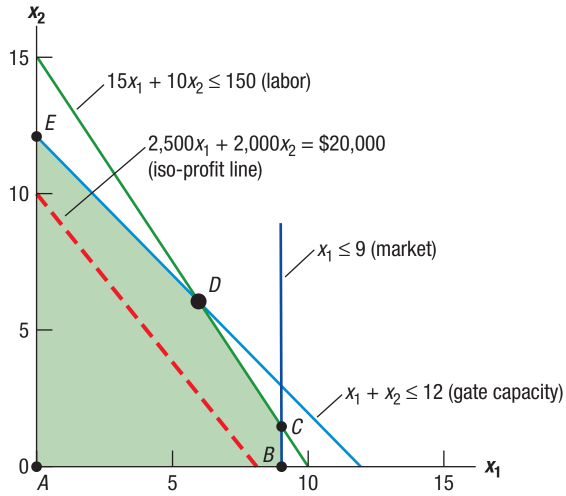
The maximum profit results from making six flights to Rome and six flights to Seattle:

$$\$2{,}500(6) + \$2{,}000(6) = \$27{,}000$$

**b.** The market constraint has three units of slack, so the demand for flights to Rome is not fully met:

$$\begin{aligned} x_1 &\leq 9 \\ x_1 + s_3 &= 9 \\ 6 + s_3 &= 9 \\ s_3 &= 3 \end{aligned}$$

**▼ FIGURE D.13**
Graphic Solution for O'Connel Airlines




# Solved Problem 2

The Arctic Air Company produces residential air conditioners. The manufacturing manager wants to develop a sales and operations plan for the next year based on the following demand and capacity data (in hundreds of product units):

| | Demand | Regular-time Capacity | Overtime Capacity | Subcontractor Capacity |
|---|---|---|---|---|
| **Jan–Feb (1)** | 50 | 65 | 13 | 10 |
| **Mar–Apr (2)** | 60 | 65 | 13 | 10 |
| **May–Jun (3)** | 90 | 65 | 13 | 10 |
| **Jul–Aug (4)** | 120 | 80 | 16 | 10 |
| **Sep–Oct (5)** | 70 | 80 | 16 | 10 |
| **Nov–Dec (6)** | 40 | 65 | 13 | 10 |
| **Totals** | 430 | 420 | 84 | 60 |

Undertime is unpaid, and no cost is associated with unused overtime or subcontractor capacity. Producing one air conditioning unit on regular time costs $1,000, including $300 for labor. Producing a unit on overtime costs $1,150. A subcontractor can produce a unit to Arctic Air specifications for $1,250. Holding an air conditioner in stock costs $60 for each 2-month period, and 200 air conditioners are currently in stock. The plan calls for 400 units to be in stock at the end of period 6. No backorders are allowed. Use the transportation method to develop a plan that minimizes costs.

## SOLUTION

The following tables identify the optimal production and inventory plans. Figure D.14 shows the tableau that corresponds to this solution. An arbitrarily large cost ($99,999 per period) was used for backorders, which effectively ruled them out. Again, all production quantities are in hundreds of units. Note that demand in period 6 is 4,400. That amount is the period 6 demand plus the desired ending inventory of 400. The anticipation inventory is measured as the amount at the end of each period. Cost calculations

**FIGURE D.14** ▶
Tableau for Optimal Production and Inventory Plans

| Period | Alternatives | Time Period 1 | 2 | 3 | 4 | 5 | 6 | Unused Capacity | Total Capacity |
|---|---|---|---|---|---|---|---|---|---|
| Period | Initial Inventory | 0 | 60 | 120 | 180<br>**2** | 240 | 300 | **0** | 2 |
| 1 | Regular time | 1,000<br>**50** | 1,060<br>**15** | 1,120 | 1,180 | 1,240 | 1,300 | **0** | 65 |
| | Overtime | 1,150 | 1,210 | 1,270 | 1,330 | 1,390 | 1,450 | **13** | 13 |
| | Subcontract | 1,250 | 1,310 | 1,370 | 1,430 | 1,490 | 1,550 | **10** | 10 |
| 2 | Regular time | 99,999 | 1,000<br>**41** | 1,060<br>**12** | 1,120<br>**12** | 1,180 | 1,240 | **0** | 65 |
| | Overtime | 99,999 | 1,150<br>**4** | 1,210 | 1,270 | 1,330 | 1,390 | **9** | 13 |
| | Subcontract | 99,999 | 1,250 | 1,310 | 1,370 | 1,430 | 1,490 | **10** | 10 |
| 3 | Regular time | 99,999 | 99,999 | 1,000<br>**65** | 1,060 | 1,120 | 1,180 | **0** | 65 |
| | Overtime | 99,999 | 99,999 | 1,150<br>**13** | 1,210 | 1,270 | 1,330 | **0** | 13 |
| | Subcontract | 99,999 | 99,999 | 1,250 | 1,310 | 1,370 | 1,430 | **10** | 10 |
| 4 | Regular time | 99,999 | 99,999 | 99,999 | 1,000<br>**80** | 1,060 | 1,120 | **0** | 80 |
| | Overtime | 99,999 | 99,999 | 99,999 | 1,150<br>**16** | 1,210 | 1,270 | **0** | 16 |
| | Subcontract | 99,999 | 99,999 | 99,999 | 1,250<br>**10** | 1,310 | 1,370 | **0** | 10 |
| 5 | Regular time | 99,999 | 99,999 | 99,999 | 99,999 | 1,000<br>**70** | 1,060 | **10** | 80 |
| | Overtime | 99,999 | 99,999 | 99,999 | 99,999 | 1,150 | 1,210 | **16** | 16 |
| | Subcontract | 99,999 | 99,999 | 99,999 | 99,999 | 1,250 | 1,310 | **10** | 10 |
| 6 | Regular time | 99,999 | 99,999 | 99,999 | 99,999 | 99,999 | 1,000<br>**44** | **21** | 65 |
| | Overtime | 99,999 | 99,999 | 99,999 | 99,999 | 99,999 | 1,150 | **13** | 13 |
| | Subcontract | 99,999 | 99,999 | 99,999 | 99,999 | 99,999 | 1,250 | **10** | 10 |
| | Demand | 50 | 60 | 90 | 120 | 70 | 44 | 132 | 566 |

are based on the assumption that workers are not paid for undertime or are productively put to work elsewhere in the organization whenever they are not needed for this work.

One initially puzzling aspect of this solution is that it allocates the initial inventory of 200 units to meet demand in period 4 rather than in period 1. The explanation is that multiple optimal solutions exist and this solution is only one of them. However, all solutions result in the same production and anticipation inventory plans derived as follows:

**PRODUCTION PLAN**

| Period | Regular-time Production | Overtime Production | Subcontracting | Total |
|---|---|---|---|---|
| 1 | 6,500 | — | — | 6,500 |
| 2 | 6,500 | 400 | — | 6,900 |
| 3 | 6,500 | 1,300 | — | 7,800 |
| 4 | 8,000 | 1,600 | 1,000 | 10,600 |
| 5 | 7,000 | — | — | 7,000 |
| 6 | 4,400 | — | — | 4,400 |

**ANTICIPATION INVENTORY**

| Period | Beginning Inventory Plus Total Production Minus Demand | Anticipation (Ending) Inventory |
|---|---|---|
| 1 | 200 + 6,500 − 5,000 | 1,700 |
| 2 | 1,700 + 6,900 − 6,000 | 2,600 |
| 3 | 2,600 + 7,800 − 9,000 | 1,400 |
| 4 | 1,400 + 10,600 − 12,000 | 0 |
| 5 | 0 + 7,000 − 7,000 | 0 |
| 6 | 0 + 4,400 − 4,000 | 400 |

# Discussion Questions

**1.** A particular linear programming maximization problem has the following less-than-or-equal-to constraints: (1) raw materials, (2) labor hours, and (3) storage space. The optimal solution occurs at the intersection of the raw materials and labor hours constraints, so those constraints are binding. Management is considering whether to authorize overtime. What useful information could the linear programming solution provide to management in making this decision? Suppose a warehouse becomes available for rent at bargain rates. What would management need to know to decide whether to rent the warehouse? How could the linear programming model be helpful?

**2.** Linear programming and the transportation method promise optimal solutions. However, wise managers sometimes, after seeing the optimal results such as in Figures D.8 or D.12, might decide to implement different plans. How do you explain such decision making?

# Problems

The OM Explorer and POM for Windows software is available to all students using the 10th edition of this textbook. Go to **http://www.pearsonglobaleditions.com/krajewski** to download these computer packages. Check with your instructor on how best to use it. For linear programming problems with more than two variables, and for transportation problems, use POM for Windows to solve your model formulations. The emphasis in these cases is on modeling problems and interpreting computer output.

## Formulating a Linear Programming Model

**1.** Happy Dog Inc. produces three types of dog food. Puppy Blend is produced for dogs that are less than a year old, Adult Blend for dogs between 1 and 8 years old, and Geriatric Blend for dogs older than 8 years. Each blend, sold in 5 pound bags, has a unique recipe that requires, among other ingredients, exact quantities of certain raw materials.

| | Chicken | Fish Meal | Soy Flour | Demand (in 5 lb. bags) |
|---|---|---|---|---|
| Puppy Blend | 2.5 lbs. | 1.0 lbs. | 0.5 lbs. | 2,000 |
| Adult Blend | 1.5 lbs. | 2.0 lbs. | 0.5 lbs. | 8,000 |
| Geriatric Blend | 1.0 lbs. | 2.0 lbs. | 1.0 lbs. | 1,000 |
| Availability of raw material | 10,000 lbs. | 20,000 lbs. | 5,000 lbs. | |

**a.** Formulate a linear programming model that produces as many bags of dog food as possible without exceeding the demand or the available supply of raw material.

**b.** Reformulate the linear programming model if the company is now interested in maximizing their profit (price − raw material cost) from dog food production. Assume that Puppy Blend sells for $9.50 per bag, Adult Blend sells for $8.50 per bag, and Geriatric Blend sells for $9.00 per bag. Further, Chicken costs $2.50 per pound Fish Meal costs $1.25 per pound, and Soy Flour costs $2.00 per pound. How does this new information change your linear programming model?

**2.** Amazing Dairy produces yogurt, sour cream, kefir, and cottage cheese in 10 pound containers for the food service industry. Each product is made in 10 pound batches and requires similar processing through three machines. Details on Amazing Dairy's product line, including the processing time in minutes at each machine, follows.

| | Yogurt | Sour Cream | Kefir | Cottage Cheese |
|---|---|---|---|---|
| Price per pound | $1.50 | $2.00 | $3.50 | $1.25 |
| Weekly demand | 200 lbs. | 350 lbs. | 50 lbs. | 150 lbs. |
| Processing Time at Machine 1 | 15 minutes per 10 lb. batch | 10 minutes per 10 lb. batch | 15 minutes per 10 lb. batch | 5 minutes per 10 lb. batch |
| Processing Time at Machine 2 | 25 minutes per 10 lb. batch | 10 minutes per 10 lb. batch | 15 minutes per 10 lb. batch | 5 minutes per 10 lb. batch |
| Processing Time at Machine 3 | 5 minutes per 10 lb. batch | 15 minutes per 10 lb. batch | 30 minutes per 10 lb. batch | 20 minutes per 10 lb. batch |

Amazing Dairy has 2,400 minutes of processing time available for each machine each week. Develop a linear program that maximizes total revenue.

3. The town of Lexington purchases gasoline from three vendors to fuel its fleet of municipal vehicles. Each vendor delivers gasoline in 6,000 gallon quantities. Since vendors must deliver the fuel, a transportation fee (based on the distance the vendor must travel) is added to each delivery. The number of deliveries required by each location, the cost of supplying a truckload of gasoline to each location from each vendor, and the number of truckloads each vendor can deliver are provided below:

| | Police Station | Fire Station | Bus Depot | Public Works Garage | Deliveries (truckloads) available |
|---|---|---|---|---|---|
| Vendor A | $500 | $525 | $550 | $600 | 20 |
| Vendor B | $350 | $425 | $450 | $575 | 14 |
| Vendor C | $400 | $375 | $625 | $475 | 10 |
| Deliveries (truckloads) required | 12 | 2 | 18 | 6 | |

Assuming that the price per gallon of fuel is the same for each vendor, develop a linear programming model that minimizes the total transportation fee that Lexington must pay for supplying all of its facilities' fuel needs.

4. JPMorgan Chase has a scheduling problem. Operators work 8-hour shifts and can begin work at midnight, 4 A.M., 8 A.M., noon, 4 P.M., or 8 P.M. Operators are needed to satisfy the following demand pattern. Formulate a linear programming model to cover the demand requirements with the minimum number of operators.

| Time Period | Operators Needed |
|---|---|
| Midnight to 4 A.M. | 4 |
| 4 A.M. to 8 A.M. | 6 |
| 8 A.M. to noon | 90 |
| Noon to 4 P.M. | 85 |
| 4 P.M. to 8 P.M. | 55 |
| 8 P.M. to 12 midnight | 20 |

## Graphic Analysis

5. The Sports Shoe Company is a manufacturer of basketball and football shoes. The manager of marketing must decide the best way to spend advertising resources. Each football team sponsored requires 120 pairs of shoes. Each basketball team requires 32 pairs of shoes. Football coaches receive $300,000 for shoe sponsorship, and basketball coaches receive $1,000,000. The manager's promotional budget is $30,000,000. The company has a limited supply (4 liters, or 4,000 cubic centimeters) of flubber, a rare and costly compound used in promotional athletic shoes. Each pair of basketball shoes requires 3 cc of flubber, and each pair of football shoes requires 1 cc. The manager wants to sponsor as many basketball and football teams as resources will allow.

 a. Create a set of linear equations to describe the objective function and the constraints.

 b. Use graphic analysis to find the visual solution.

 c. What is the maximum number of each type of team that the company can sponsor?

6. A business student at Nowledge College must complete a total of 65 courses to graduate. The number of business courses must be greater than or equal to 23. The number of nonbusiness courses must be greater than or equal to 20. The average business course requires a textbook costing $60 and 120 hours of study. Nonbusiness courses require a textbook costing $24 and 200 hours of study. The student has $3,000 to spend on books.

 a. Create a set of linear equations to describe the objective function and the constraints.

 b. Use graphic analysis to find the visual solution.

 c. What combination of business and nonbusiness courses minimizes total hours of study?

 d. Identify the slack or surplus variables.

7. In Problem 6, suppose that the objective is to minimize the cost of books and that the student's total study time is limited to 12,600 hours.

 a. Use graphic analysis to determine the combination of courses that minimizes the total cost of books.

 b. Identify the slack or surplus variables.

8. Mile-High Microbrewery makes a light beer and a dark beer. Mile-High has a limited supply of barley, limited bottling capacity, and a limited market for light beer. Profits are $0.20 per bottle of light beer and $0.50 per bottle of dark beer.

 a. The following table shows resource availability of products at the Mile-High Microbrewery. Use the graphic method of linear programming to maximize profits. How many bottles of each product should be produced per month?

| | PRODUCT | | |
|---|---|---|---|
| Resource | Light Beer ($x_1$) | Dark Beer ($x_2$) | Resource Availability (per month) |
| Barley | 0.1 gram | 0.6 gram | 2,000 grams |
| Bottling | 1 bottle | 1 bottle | 6,000 bottles |
| Market | 1 bottle | — | 4,000 bottles |

 b. Identify any constraints with slack or surplus.

9. The plant manager of a plastic pipe manufacturer has the opportunity to use two different routings for a particular type of plastic pipe. Routing 1 uses extruder A, and routing 2 uses extruder B. Both routings require the same melting process. The following table shows the time requirements and capacities of these processes:

| | TIME REQUIREMENTS (HR/100 FT) | | |
|---|---|---|---|
| Process | Routing 1 | Routing 2 | Capacity (hr) |
| Melting | 1 | 1 | 45 |
| Extruder A | 3 | 0 | 90 |
| Extruder B | 0 | 1 | 160 |

Each 100 feet of pipe processed on routing 1 uses 5 pounds of raw material, whereas each 100 feet of pipe processed on routing 2 used only 4 pounds. This difference results from differing scrap rates of the extruding machines. Consequently, the profit per 100 feet of pipe processed on routing 1 is $60 and on routing 2 is $80. A total of 200 pounds of raw material is available.

**a.** Create a set of linear equations to describe the objective function and the constraints.

**b.** Use graphic analysis to find the visual solution.

**c.** What is the maximum profit?

**10.** A manufacturer of textile dyes can use two different processing routings for a particular type of dye. Routing 1 uses drying press A, and routing 2 uses drying press B. Both routings require the same mixing vat to blend chemicals for the dye before drying. The following table shows the time requirements and capacities of these processes:

| | TIME REQUIREMENTS (HR/KG) | | |
|---|---|---|---|
| Process | Routing 1 | Routing 2 | Capacity (hr) |
| Mixing | 2 | 2 | 54 |
| Dryer A | 6 | 0 | 120 |
| Dryer B | 0 | 8 | 180 |

Each kilogram of dye processed on routing 1 uses 20 liters of chemicals, whereas each kilogram of dye processed on routing 2 uses only 15 liters. The difference results from differing yield rates of the drying presses. Consequently, the profit per kilogram processed on routing 1 is $50 and on routing 2 is $65. A total of 450 liters of input chemicals is available.

**a.** Write the constraints and objective function to maximize profits.

**b.** Use the graphic method of linear programming to find the optimal solution.

**c.** Identify any constraints with slack or surplus.

## Computer Analysis

**11.** The Trim-Look Company makes several lines of skirts, dresses, and sport coats. Recently, a consultant suggested that the company reevaluate its South Islander line and allocate its resources to products that would maximize contribution to profits and to overhead. Each product requires the same polyester fabric and must pass through the cutting and sewing departments. The following data were collected for the study:

| | PROCESSING TIME (HR) | | |
|---|---|---|---|
| Product | Cutting | Sewing | Material (yd) |
| Skirt | 1 | 1 | 1 |
| Dress | 3 | 4 | 1 |
| Sport coat | 4 | 6 | 4 |

The cutting department has 100 hours of capacity, sewing has 180 hours of capacity, and 60 yards of material are available. Each skirt contributes $5 to profits and overhead; each dress, $17; and each sport coat, $30.

**a.** Specify the objective function and constraints for this problem.

**b.** Use a computer package such as POM for Windows to solve the problem.

**12.** Consider Problem 11 further.

**a.** How much would you be willing to pay for an extra hour of cutting time? For an extra hour of sewing time? For an extra yard of material? Explain your response to each question.

**b.** Determine the range of right-hand-side values over which the shadow price would be valid for the cutting constraint and for the material constraint.

**13.** Polly Astaire makes fine clothing for big and tall men. A few years ago Astaire entered the sportswear market with the Sunset line of shorts, pants, and shirts. Management wants to make the amount of each product that will maximize profits. Each type of clothing is routed through two departments, A and B. The relevant data for each product are as follows:

| | PROCESSING TIME (HR) | | |
|---|---|---|---|
| Product | Department A | Department B | Material (yd) |
| Shirts | 2 | 1 | 2 |
| Shorts | 2 | 3 | 1 |
| Pants | 3 | 4 | 4 |

Department A has 120 hours of capacity, department B has 160 hours of capacity, and 90 yards of material are available. Each shirt contributes $10 to profits and overhead; each pair of shorts, $10; and each pair of pants, $23.

**a.** Specify the objective function and constraints for this problem.

**b.** Use a computer package such as POM for Windows to solve the problem.

**c.** How much should Astaire be willing to pay for an extra hour of department A capacity? How much for an extra hour of department B capacity? For what range of right-hand values are these shadow prices valid?

**14.** The Butterfield Company makes a variety of knives. Each knife is processed on four machines. The processing times required are as follows. Machine capacities (in hours) are 1,500 for machine 1; 1,400 for machine 2; 1,600 for machine 3; and 1,500 for machine 4.

| | PROCESSING TIME (HR) | | | |
|---|---|---|---|---|
| Knife | Machine 1 | Machine 2 | Machine 3 | Machine 4 |
| A | 0.05 | 0.10 | 0.15 | 0.05 |
| B | 0.15 | 0.10 | 0.05 | 0.05 |
| C | 0.20 | 0.05 | 0.10 | 0.20 |
| D | 0.15 | 0.10 | 0.10 | 0.10 |
| E | 0.05 | 0.10 | 0.10 | 0.05 |

Each product contains a different amount of two basic raw materials. Raw material 1 costs \$0.50 per ounce, and raw material 2 costs \$1.50 per ounce. There are 75,000 ounces of raw material 1 and 100,000 ounces of raw material 2 available.

| | REQUIREMENTS (OZ/UNIT) | | |
|---|---|---|---|
| Knife | Raw Material 1 | Raw Material 2 | Selling Price (\$/unit) |
| A | 4 | 2 | 15.00 |
| B | 6 | 8 | 25.50 |
| C | 1 | 3 | 14.00 |
| D | 2 | 5 | 19.50 |
| E | 6 | 10 | 27.00 |

**a.** If the objective is to maximize profit, specify the objective function and constraints for the problem. Assume that labor costs are negligible.

**b.** Solve the problem with a computer package such as POM for Windows.

**15.** The Nutmeg Corporation produces three different products, each in a 1-pound can: Almond-Lovers Mix, Walnut-Lovers Mix, and the Thrifty Mix. Three types of nuts are used in Nutmeg's products: almonds, walnuts, and peanuts. Nutmeg currently has 350 pounds of almonds, 150 pounds of walnuts, and 1000 pounds of peanuts. Each of Nutmeg's products must contain a certain percentage of each type of nut, as shown in the following table. The table also shows the revenue per can as well as the cost per pound to purchase nuts.

| | PERCENTAGE REQUIREMENTS PER CAN | | | |
|---|---|---|---|---|
| | Almonds | Walnuts | Peanuts | Revenue per can |
| Almond-Lovers Mix | 80% | 20% | 0% | \$8.00 |
| Walnut-Lovers Mix | 20% | 80% | 0% | \$10.00 |
| Thrifty Mix | 10% | 10% | 80% | \$4.50 |
| Cost per pound | \$4.50 | \$6.00 | \$3.00 | |

**a.** Given Nutmeg's current stock of nuts, how many cans of each product should be produced to maximize revenue?

**b.** Does the solution you developed in part a change if Nutmeg is interested in maximizing contribution margin (defined as revenue per unit − raw material cost)?

**c.** If 50 additional pounds of walnuts became available, how would your contribution-margin maximizing solution from part (b) change?

**16.** A problem often of concern to managers in processing industries is blending. The Nutmeg Corporation, from Problem 15, is considering a new product it intends to sell to active and health-concerned adults. This new product will be a 4 ounce package of nuts that conforms to specific heath requirements. First, the 4 ounce package can contain no more than 720 calories. It must deliver at least 20 grams of protein. Finally, the package must provide at least 15 percent of the adult daily requirement (ADR) of calcium and 20 percent of the ADR of iron. Nutmeg would like to use only almonds, walnuts, and peanuts in this new product. The following table provides nutritional data on each of these ingredients as well as their cost to Nutmeg.

| Ingredients | Calories per ounce | Grams of protein per ounce | Percent ADR of calcium per ounce | Percent ADR of iron per ounce | Cost per ounce |
|---|---|---|---|---|---|
| Almonds | 180 | 6 | 8% | 6% | \$0.28 |
| Walnuts | 190 | 4 | 2% | 6% | \$0.38 |
| Peanuts | 170 | 7 | 0% | 4% | \$0.12 |

**a.** Use linear programming to find the cost minimizing number of ounces of each ingredient that Nutmeg should use in each 4 ounce package. What is the per package cost of raw materials?

**b.** The marketing department at Nutmeg insists that each package should contain at least $\frac{1}{2}$ ounce of almonds, at least $\frac{1}{2}$ ounce of walnuts, and no more than 1 ounce of peanuts. Does the solution developed for part (a) satisfy these new constraints? If not, use linear programming to find a solution that includes these marketing requirements. What is the new cost of raw materials?

**17.** A small fabrication firm makes three basic types of components for use by other companies. Each component is processed on three machines. The processing times follow. Total capacities (in hours) are 1,600 for machine 1; 1,400 for machine 2; and 1,500 for machine 3.

| | PROCESSING TIME (HR) | | |
|---|---|---|---|
| Component | Machine 1 | Machine 2 | Machine 3 |
| A | 0.25 | 0.10 | 0.05 |
| B | 0.20 | 0.15 | 0.10 |
| C | 0.10 | 0.05 | 0.15 |

Each component contains a different amount of two basic raw materials. Raw material 1 costs \$0.20 per ounce, and raw material 2 costs \$0.35 per ounce. At present, 200,000 ounces of raw material 1 and 85,000 ounces of raw material 2 are available.

| | REQUIREMENT (OZ/UNIT) | | |
|---|---|---|---|
| Component | Raw Material 1 | Raw Material 2 | Selling Price (\$/unit) |
| A | 32 | 12 | 40 |
| B | 26 | 16 | 28 |
| C | 19 | 9 | 24 |

**a.** Assume that the company must make at least 1,200 units of component B, that labor costs are negligible, and that the objective is to maximize profits. Specify the objective function and constraints for the problem.

**b.** Use a computer package such as POM for Windows to solve the problem.

**18.** The following is a linear programming model for analyzing the product mix of Maxine's Hat Company, which produces three hat styles:

$$\text{Maximize: } \$7x_1 + \$8x_2 + \$6x_3 = Z$$

$$\begin{aligned}\text{Subject to: } & 2x_1 + 4x_2 + 2x_3 \le 120 \text{ (machine A time)}\\ & 5x_1 + 3x_2 + 2x_3 \le 400 \text{ (machine B time)}\\ & 2x_1 + 2x_2 + 4x_3 \le 110 \text{ (machine C time)}\\ & x_1 \ge 0, x_2 \ge 0, \text{ and } x_3 \ge 0\end{aligned}$$

The POM for Windows printout in Figure D.15 shows the optimal solution to the problem.

Consider each of the following statements independently, and state whether it is true or false. Explain each answer.

**a.** If the price of hat 3 were increased to \$11.50, it would be part of the optimal product mix. *Hint:* Hat 3 is represented by $x_3$ and its optimal value currently is 0, which means that it is not to be produced (and not part of the optimal product mix).

**b.** The capacity of machine B can be reduced to 280 hours without affecting profits.

**c.** If machine C had a capacity of 115 hours, the production output would remain unchanged.

(a) Results Screen

| | | | | | | |
|---|---|---|---|---|---|---|
| Maximize | 7 | 8 | 6 | | | |
| machine A time | 2 | 4 | 2 | <= | 120 | .5 |
| machine B time | 5 | 3 | 2 | <= | 400 | 0 |
| machine C time | 2 | 2 | 4 | <= | 110 | 3 |
| Solution-> | 50 | 5 | 0 | | 390 | |

(b) Ranging Screen

| Variable | Value | Reduced Cost | Original Val | Lower Bound | Upper Bound |
|---|---|---|---|---|---|
| X1 | 50 | 0 | 7 | 4.6667 | 8 |
| X2 | 5 | 0 | 8 | 7 | 14 |
| X3 | 0 | 7 | 6 | -Infinity | 13 |
| Constraint | Shadow Price | Slack/Surplus | Original Val | Lower Bound | Upper Bound |
| machine A time | .5 | 0 | 120 | 110 | 220 |
| machine B time | 0 | 135 | 400 | 265 | Infinity |
| machine C time | 3 | 0 | 110 | 60 | 120 |

▲ **FIGURE D.15**
Solution Screens for Maxine's Hat Company

**19.** The Washington Chemical Company produces chemicals and solvents for the glue industry. The production process is divided into several "focused factories," each producing a specific set of products. The time has come to prepare the production plan for one of the focused factories. This particular factory produces five products, which must pass through both the reactor and the separator. Each product also requires a certain combination of raw materials. Production data are shown in Table D.2.

**TABLE D.2 | PRODUCTION DATA FOR WASHINGTON CHEMICAL**

| | PRODUCT | | | | | |
|---|---|---|---|---|---|---|
| **Resource** | **1** | **2** | **3** | **4** | **5** | **Total Resources Available** |
| Reactor (hr/lb) | 0.05 | 0.10 | 0.80 | 0.57 | 0.15 | 7,500 hr* |
| Separator (hr/lb) | 0.20 | 0.02 | 0.20 | 0.09 | 0.30 | 7,500 hr* |
| Raw material 1 (lb) | 0.20 | 0.50 | 0.10 | 0.40 | 0.18 | 10,000 lb |
| Raw material 2 (lb) | — | 0.70 | — | 0.50 | — | 6,000 lb |
| Raw material 3 (lb) | 0.10 | 0.20 | 0.40 | — | — | 7,000 lb |
| Profit contribution (\$/lb) | 4.00 | 7.00 | 3.50 | 4.00 | 5.70 | |

*The total time available has been adjusted to account for setups. The five products have a prescribed sequence owing to the cost of changeovers between products. The company has a 35-day cycle (or 10 changeovers per year per product). Consequently, the time for these changeovers has been deducted from the total time available for these machines.

The Washington Chemical Company has a long-term contract with a major glue manufacturer that requires annual production of 3,000 pounds of both products 3 and 4. More of these products could be produced because demand currently exceeds production capacity.

**a.** Determine the annual production quantity of each product that maximizes contribution to profits. Assume the company can sell all it can produce.

**b.** Specify the lot size for each product.

## The Transportation Method

**20.** The Warwick Manufacturing Company produces shovels for industrial and home use. Sales of the shovels are seasonal, and Warwick's customers refuse to stockpile them during slack periods. In other words, the customers want to minimize inventory, insist on shipments according to their schedules, and will not accept backorders.

Warwick employs manual, unskilled laborers who require only basic training. Producing 1,000 shovels costs $3,500 on regular time and $3,700 on overtime. These amounts include materials, which account for more than 85 percent of the cost. Overtime is limited to production of 15,000 shovels per quarter. In addition, subcontractors can be hired at $4,200 per thousand shovels, but Warwick's labor contract restricts this type of production to 5,000 shovels per quarter.

The current level of inventory is 30,000 shovels, and management wants to end the year at that level. Holding 1,000 shovels in inventory costs $280 per quarter. The latest annual demand forecast is as follows:

| Quarter | Demand |
|---|---|
| 1 | 70,000 |
| 2 | 150,000 |
| 3 | 320,000 |
| 4 | 100,000 |
| Totals | 640,000 |

Use the Transportation Method (Production Planning) module in POM for Windows to determine the *best* regular-time capacity plan. Assume the following:

- The firm has 30 workers now, and management wants to have the same number in quarter 4.
- Each worker can produce 4,000 shovels per quarter.
- Hiring a worker costs $1,000, and laying off a worker costs $600.

**21.** The management of Warwick Manufacturing Company is willing to give price breaks to its customers as an incentive to purchase shovels in advance of the traditional seasons. Warwick's sales and marketing staff estimates that the demand for shovels resulting from the price breaks would be as follows:

| Quarter | Demand | Original Demand |
|---|---|---|
| 1 | 120,000 | 70,000 |
| 2 | 180,000 | 150,000 |
| 3 | 180,000 | 320,000 |
| 4 | 160,000 | 100,000 |
| Totals | 640,000 | 640,000 |

Use the Transportation Method (Production Planning) module in POM for Windows to calculate the optimal production plan (including the workforce staffing plan) under the new demand schedule. Compare it to the optimal production plan under the original demand schedule. Evaluate the potential effects of demand management.

**22.** The Bull Grin Company produces a feed supplement for animal foods produced by a number of other companies. Producing 1,000 pounds of supplement costs $810 on regular time and $900 on overtime. These amounts include materials, which account for more than 80 percent of the cost. The plant can produce 400,000 pounds of supplement per quarter using regular time, but overtime is limited to the production of 40,000 pounds per quarter. The current level of inventory is 40,000 pounds, and management wants to end the year at that level. Holding 1,000 pounds of feed supplement in inventory costs $110 per quarter. Assume hiring and layoff costs are negligible. The latest annual demand forecast follows:

| Quarter | Demand (in Pounds) |
|---|---|
| 1 | 100,000 |
| 2 | 410,000 |
| 3 | 770,000 |
| 4 | 440,000 |

**a.** Formulate this production-planning problem as a linear program after defining all decision variables.

**b.** Solve your formulation using a computer package such as POM for Windows.

**c.** Assume that subcontractors can be hired at $1,100 per thousand pounds to produce as much supplement as Bull Grin requires. Does this change the cost minimizing solution found in part (b)?

**d.** If Bull Grin realizes that the current level of inventory is actually 0 pounds, are the resources assumed in part (c) adequate to satisfy all demand and still end the year with 40,000 pounds in ending inventory? If so, how much will the cost of Bull Grin's production plan increase?

**23.** Supertronics, Inc., would like to know how the firm's profitability is altered by product mix. Currently, product mix is determined by giving priority to the product with the highest per-unit contribution margin (defined as the difference between price and material cost). Details on the Supertronics product line, including processing time at each workstation, follow:

| | PRODUCT | | | |
|---|---|---|---|---|
| | Alpha | Beta | Delta | Gamma |
| Price | $350.00 | $320.00 | $400.00 | $500.00 |
| Material Cost | $50.00 | $40.00 | $125.00 | $150.00 |
| Weekly Demand in Units | 100 | 60 | 50 | 80 |
| Processing Time at Machine 1 in Minutes | 20 | 0 | 40 | 10 |
| Processing Time at Machine 2 in Minutes | 25 | 20 | 0 | 50 |
| Processing Time at Machine 3 in Minutes | 0 | 20 | 60 | 30 |

**a.** Assume that Supertronics has 5,500 minutes of capacity available at each workstation each week. Develop a linear program to define the production mix that maximizes contribution margin.

**b.** Solve your formulation using a computer package such as POM for Windows.

**c.** Given your solution for part (b), which machine is the bottleneck?

**d.** How would your formulation and solution in part (b) change if 50 units of each product were already committed to customers and thereby had to be produced?

**24.** Revisit Problem 22 on the Bull Grin Company. Some cost and demand parameters have changed. Producing 1,000 pounds of supplement now costs $830 on regular time and $910 on overtime. No additional cost is incurred for unused regular-time, overtime, or subcontractor capacity. Overtime is limited to production of a total of 20,000 pounds per quarter. In addition, subcontractors can be hired at $1,000 per thousand pounds, but only 30,000 pounds per quarter can be produced this way.

The current level of inventory is 40,000 pounds, and management wants to end the year at that level. Holding 1,000 pounds of feed supplement in inventory per quarter costs $100. The latest annual forecast is shown in Table D.3.

**TABLE D.3 | FORECASTS AND CAPACITIES**

| | PERIOD | | | | |
|---|---|---|---|---|---|
| | Quarter 1 | Quarter 2 | Quarter 3 | Quarter 4 | Total |
| Demand (pounds) | 130,000 | 400,000 | 800,000 | 470,000 | 1,800,000 |
| Capacities (pounds) | | | | | |
| Regular time | 390,000 | 400,000 | 460,000 | 380,000 | 1,630,000 |
| Overtime | 20,000 | 20,000 | 20,000 | 20,000 | 80,000 |
| Subcontract | 30,000 | 30,000 | 30,000 | 30,000 | 30,000 |

Use the transportation method of production planning in POM for Windows to find the optimal production plan and calculate its cost, or use the spreadsheet approach to find a good production plan and calculate its cost.

**25.** The Cut Rite Company is a major producer of industrial lawn mowers. The cost to Cut Rite for hiring a semiskilled worker for its assembly plant is $3,000, and the cost for laying off one is $2,000. The plant averages an output of 36,000 mowers per quarter with its current workforce of 720 employees. Regular-time capacity is directly proportional to the number of employees. Overtime is limited to a maximum of 3,000 mowers per quarter, and subcontracting is limited to 1,000 mowers per quarter. The costs to produce one mower are $2,430 on regular time (including materials), $2,700 on overtime, and $3,300 via subcontracting. Unused regular-time capacity costs $270 per mower. No additional cost is incurred for unused overtime or subcontractor capacity. The current level of inventory is 4,000 mowers, and management wants to end the year at that level. Customers do not tolerate backorders, and holding a mower in inventory per quarter costs $300. The demand for mowers this coming year is as follows:

| Quarter | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Demand | 10,000 | 41,000 | 77,000 | 44,000 |

Two workforce plans have been proposed, and management is uncertain as to which one to use. The following table shows the number of employees per quarter under each plan:

| Quarter | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Plan 1 | 720 | 780 | 920 | 720 |
| Plan 2 | 860 | 860 | 860 | 860 |

**a.** Which plan would you recommend to management? Explain, supporting your recommendation with an analysis using the transportation method of production planning.

**b.** If management used creative pricing to get customers to buy mowers in nontraditional time periods, the following demand schedule would result:

| Quarter | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Demand | 20,000 | 54,000 | 54,000 | 44,000 |

Which workforce plan would you recommend now?

**26.** The Holloway Calendar Company produces a variety of printed calendars for both commercial and private use. The demand for calendars is highly seasonal, peaking in the third quarter. Current inventory is 165,000 calendars, and ending inventory should be 200,000 calendars.

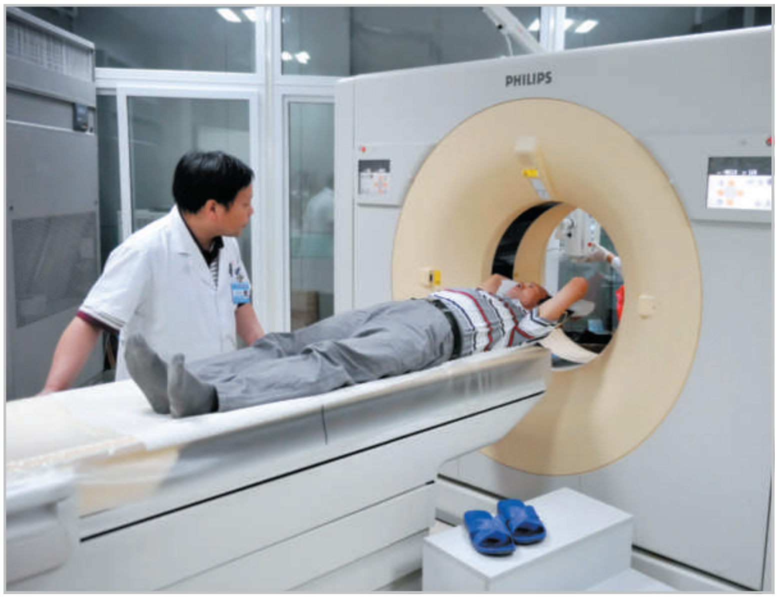
Ann Ritter, Holloway's manufacturing manager, wants to determine the best production plan for the demand requirements and capacity plan shown in the following table. (Here, demand and capacities are expressed as thousands of calendars rather than as employee-period equivalents.) Ritter knows that the regular-time cost is \$0.50 per unit, overtime cost is \$0.75 per unit, subcontracting cost is \$0.90 per unit, and inventory holding cost is \$0.10 per calendar per quarter. Unused regular-time capacity is not paid.

| | QUARTER | | | | |
|---|---|---|---|---|---|
| | **1** | **2** | **3** | **4** | **Total** |
| **Demand** | 250 | 515 | 1,200 | 325 | 2,290 |
| **Capacities** | | | | | |
| Regular time | 300 | 300 | 600 | 300 | 1,500 |
| Overtime | 75 | 75 | 150 | 75 | 375 |
| Subcontracting | 150 | 150 | 150 | 150 | 600 |

**a.** Recommend a production plan to Ritter, using the Transportation Method (Production Planning) module of POM for Windows. (Do not allow any stockouts or backorders to occur.)

**b.** Interpret and explain your recommendation.

**c.** Calculate the total cost of your recommended production plan.




Imaginechina/Corbis

A doctor operates a Philips CT scanner to examine a patient at a hospital in Beijing, China. Royal Philips Electronics NV is planning to expand its global healthcare business in China's rural areas, a market which is seen as a new driver of growth for the Dutch multinational group.

# 11

# EFFICIENT RESOURCE PLANNING

## Philips

Philips, based in Amsterdam, Netherlands, is a diversified Dutch technology conglomerate whose healthcare, consumer lifestyle, and lighting divisions employ over 122,000 employees in more than 60 countries. It is one of the largest electronics company in the world and a leader in energy efficient lighting solutions; cardiac, oral, and home healthcare devices; flat screen television sets; and personal grooming products such as electric shavers. Managing its 124 manufacturing plants in 26 different countries presents management with a complex set of decisions surrounding what needs to be made and when, and which components should be ordered in what quantities to support the production schedules that can satisfy customer demand. Complexity of sourcing and planning for all these material and labor resources increases rapidly due to the large variety of products manufactured on a regular basis by Philips.

The Medical Systems division in the United States within the healthcare segment of Philips is located in Andover, Massachusetts. Shanghai APEX Electronics, a small manufacturing company in China, designs, develops, and produces ultrasonic transducers for Philips. Component requirements from Philips plants for suppliers such as APEX are generated through a resource planning process embedded within an enterprise-wide system. These requirements are communicated to APEX and other suppliers through a standardized supply chain management application called Nocturne. In turn, Philips expected and required APEX to install Nocturne and also acquire a material requirements planning (MRP) computer-based system of its own so that it could more readily meet the

Using Operations to Create Value

**PROCESS MANAGEMENT**

Process Strategy and Analysis
Managing Quality
Planning Capacity
Managing Process Constraints
Designing Lean Systems
Managing Effective Projects

**CUSTOMER DEMAND MANAGEMENT**

Forecasting Demand
Managing Inventories
Planning and Scheduling Operations
→ **Efficient Resource Planning**

**SUPPLY CHAIN MANAGEMENT**

Designing Effective Supply Chains
Supply Chains and Logistics
Integrating the Supply Chain
Managing Supply Chain Sustainability

on-time components commitments requested by Philips. While APEX did not have the resources or the need to acquire large-scale enterprise systems similar to the ones implemented by Philips, it did procure and implement an integrated manufacturing system for small manufacturers called E-Z-MRP from Beach Access Software in Del Mar, California. This MRP system at APEX could use Nocturne to interface downstream with Philips and upstream with suppliers of APEX to communicate its raw materials requirements.

Integrating material requirements planning and all the associated resources up and down the supply chain coordinates product flows, avoids costly delays or shortages, minimizes inventory, and makes every firm within the chain more competitive and profitable. Adopting such sophisticated materials planning and co-ordination practices across its manufacturing plants and supply chains has allowed Philips to keep pace with technological changes and maintain its leadership position in a rapidly changing industry.

*Sources:* "Combination of Small MRP System and Large Supply Chain Software Spells Success for Shanghai APEX Electronics," **http://www.e-z-mrp.com/combination-small-mrp-system-large-supply-chain-software-spells-success-shanghai-apex-electronics/**; **http://www.usa.philips.com/about/company/index.page** (August 14, 2014); **http://en.wikipedia.org/wiki/Philips** (August 14, 2014).

## LEARNING GOALS *After reading this chapter, you should be able to:*

1. Explain how the concept of dependent demand in material requirements planning is fundamental to resource planning.
2. Describe a master production schedule (MPS) and compute available-to-promise quantities.
3. Apply the logic of an MRP explosion to identify production and purchase orders needed for dependent demand items.
4. Explain how enterprise resource planning (ERP) systems can foster better resource planning.
5. Apply resource planning principles to the provision of services and distribution inventories.

**Philips** demonstrates that companies can gain a competitive edge by focusing their attention on resource planning even for their upstream suppliers to ensure that all of the resources they need to produce finished services or products are available at the right time. If they are not, a firm risks losing business. For a manufacturer, this task can mean keeping track of thousands of subassemblies, components, and raw materials as well as key equipment capacities. For a service provider, this task can mean keeping track of numerous supplies and carefully scheduling the time and capacity requirements of different employees and types of equipment.

**resource planning**

A process that takes sales and operations plans; processes information in the way of time standards, routings, and other information on how services or products are produced; and then plans the input requirements.

**Resource planning** lies at the heart of any organization, cutting across all of its different functional areas. It takes sales and operations plans; processes information in the way of time standards, routings, and other information on how services or products are produced; and then plans the input requirements. It also can create reports for managers of the firm's major functional areas, such as human resources, purchasing, sales and marketing, and finance and accounting. In essence, resource planning is a process in and of itself that can be analyzed relative to the firm's competitive priorities.

We begin this chapter by first examining material requirements planning (MRP), which is a specific approach to resource planning for manufacturers that includes the master production scheduling of finished products and determines the timing of all orders for components and raw materials in support of that schedule. Subsequently we describe enterprise resource planning (ERP) systems, which have become a valuable tool for, among other things, resource planning. The concluding section of the chapter illustrates how service providers can use the principles of MRP in managing their supplies, human resources, equipment, and financial resources.

# Material Requirements Planning

Understanding resource planning begins with the concept of *dependent demand*, which sets it apart from the techniques covered in Chapter 9, "Managing Inventories." **Material requirements planning (MRP)** is a computerized information system developed specifically to help manufacturers manage dependent demand inventory and schedule replenishment orders. The key inputs of an MRP system are a master production schedule, a bill of materials database, and an inventory record database, as shown in Figure 11.1. Using this information, the MRP system identifies the actions planners must take to stay on schedule, such as releasing new production orders, adjusting order quantities, and expediting late orders.

An MRP system translates the master production schedule and other sources of demand, such as independent demand for replacement parts and maintenance items, into the requirements for all subassemblies, components, and raw materials needed to produce the required parent items. This process is called an **MRP explosion** because it converts the requirements of various final products into a material requirements plan that specifies the replenishment schedules of all the subassemblies, components, and raw materials needed to produce final products.

We first explore the nature of dependent demand and how it differs from independent demand, followed by a discussion of the key inputs to the MRP system shown in Figure 11.1.

**material requirements planning (MRP)**

A computerized information system developed specifically to help manufacturers manage dependent demand inventory and schedule replenishment orders.

**MRP explosion**

A process that converts the requirements of various final products into a material requirements plan that specifies the replenishment schedules of all the subassemblies, components, and raw materials needed to produce final products.

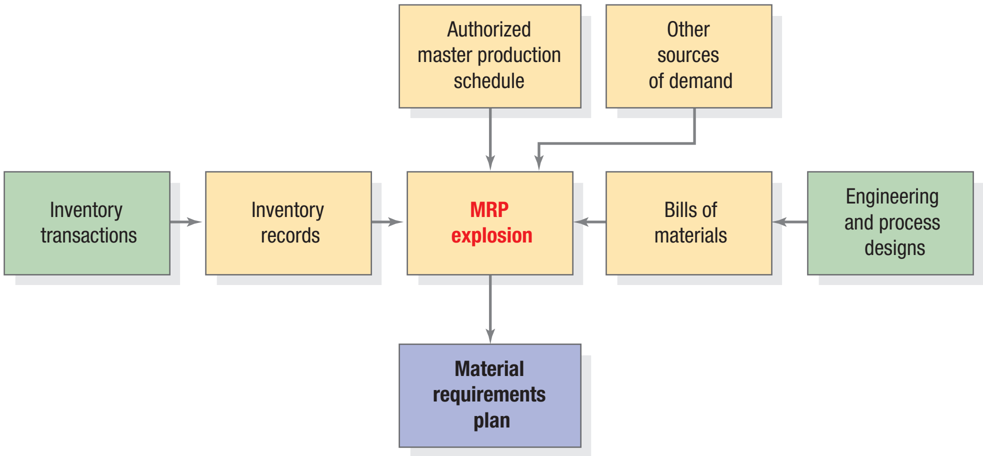



**FIGURE 11.1**
Material Requirements Plan Inputs

## Dependent Demand

For years, many companies tried to manage production and their dependent demand inventories using independent demand systems similar to those discussed in Chapter 9, "Managing Inventories," but the outcome was seldom satisfactory because dependent demand is fundamentally different from independent demand. To illustrate the concept of dependent demand, let us consider a Huffy bicycle produced for retail outlets. Demand for a final product, such as a bicycle, is called *independent demand* because it is influenced only by market conditions. In contrast, the demand for spokes going into the bicycle "depends" on the production planned for its wheels. Huffy must forecast this demand using techniques such as those discussed in Chapter 8, "Forecasting Demand." However, Huffy also keeps many other items in inventory—handlebars, pedals, frames, and wheel rims—used to make completed bicycles. Each of these items has a **dependent demand** because the quantity required varies with the production plans for other items held in the firm's inventory—finished bikes, in this case. For example, the demand for frames, pedals, and wheel rims is *dependent* on the production of completed bicycles. Operations can calculate the demand for dependent demand items once the bicycle production levels are laid out in the sales and operations plan. For example, every bicycle needs two wheel rims, so 1,000 completed bicycles need $1{,}000(2) = 2{,}000$ rims. Forecasting techniques are not needed for the rims.

The bicycle, or any other product that is manufactured from one or more components, is called a **parent**. The wheel rim is an example of a **component**—an item that goes through one or more operations to be transformed into or become part of one or more parents. A wheel rim, for example, will have several different parents if the rim is used to make more than one style of bicycle. This parent–component relationship can cause erratic dependent demand patterns for components. Suppose that every time inventory falls to 500 units (a reorder point), an order for 1,000 more bicycles is placed, as shown in Figure 11.2(a). The assembly supervisor then authorizes the withdrawal of 2,000 rims from inventory, along with other components for the finished product. The demand for the rim is shown in Figure 11.2(b). So, even though customer demand for the finished bicycle is continuous and reasonably uniform, the production demand for wheel rims is "lumpy"; that is, it occurs sporadically, usually in relatively large quantities. Thus, the production decisions for the assembly of bicycles, which account

**dependent demand**

The demand for an item that occurs because the quantity required varies with the production plans for other items held in the firm's inventory.

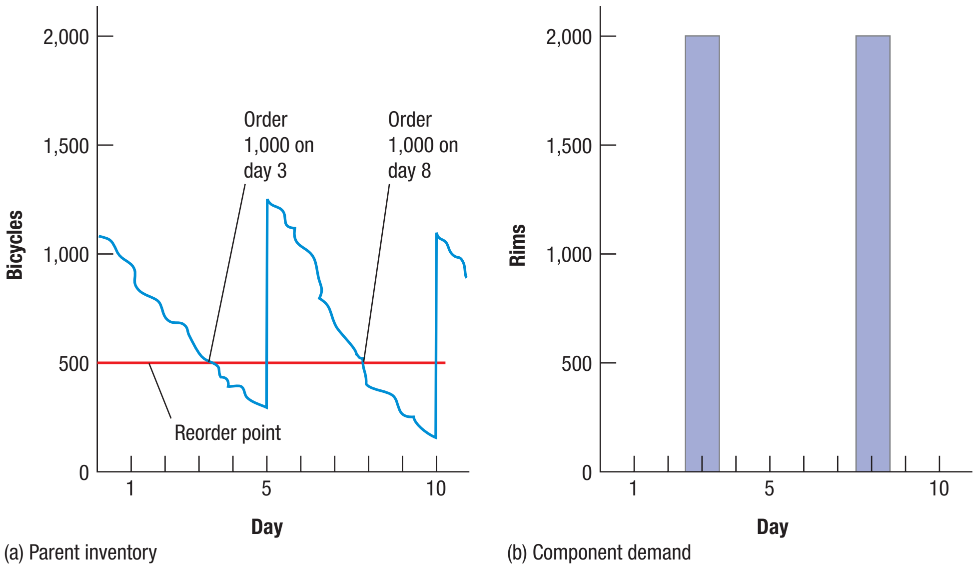
**parent**

Any product that is manufactured from one or more components.

**component**

An item that goes through one or more operations to be transformed into or become part of one or more parents.

**FIGURE 11.2** ▶
Lumpy Dependent Demand Resulting from Continuous Independent Demand




for the costs of assembling the bicycles and the projected assembly capacities at the time the decisions are made, determine the demand for wheel rims.

Managing dependent demand inventories is complicated because some components may be subject to both dependent and independent demand. For example, the shop floor needs 2,000 wheel rims for the new bicycles, but the company also sells replacement rims for old bicycles directly to retail outlets. This practice places an independent demand on the inventory of wheel rims. Materials requirements planning can be used in complex situations involving components that may have independent demand as well as dependent demand inventories.

## Master Production Scheduling

**master production schedule (MPS)**
A part of the material requirements plan that details how many end items will be produced within specified periods of time.

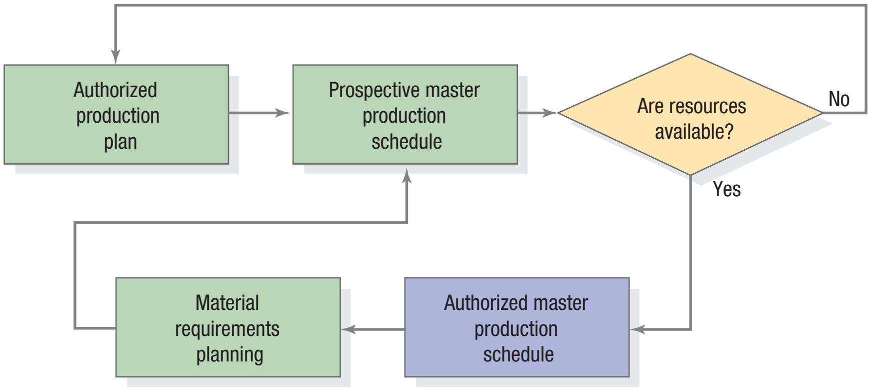
The first input into a material requirements plan is the **master production schedule (MPS)**, which details how many end items will be produced within specified periods of time. It breaks the sales and operations plan into specific product schedules. Figure 11.3 shows how a sales and operations plan at a furniture manufacturing firm for a family of chairs breaks down into the weekly MPS for each specific chair type (the time period can be hours, days, weeks, or months). The chair example demonstrates the following aspects of master scheduling:

1. The sums of the quantities in the MPS must equal those in the sales and operations plan. This consistency between the plans is desirable because of the economic analysis done to arrive at the sales and operations plan.
2. The production quantities must be allocated efficiently over time. The specific mix of chair types—the number of each type as a percent of the total family's quantity—is based on historic demand and on marketing and promotional considerations. The planner must select lot sizes for each chair type, taking into consideration economic factors such as production setup costs and inventory carrying costs.
3. Capacity limitations and bottlenecks, such as machine or labor capacity, storage space, or working capital, may determine the timing and size of MPS quantities. The planner must acknowledge these limitations by recognizing that some chair styles require more resources than others and setting the timing and size of the production quantities accordingly.

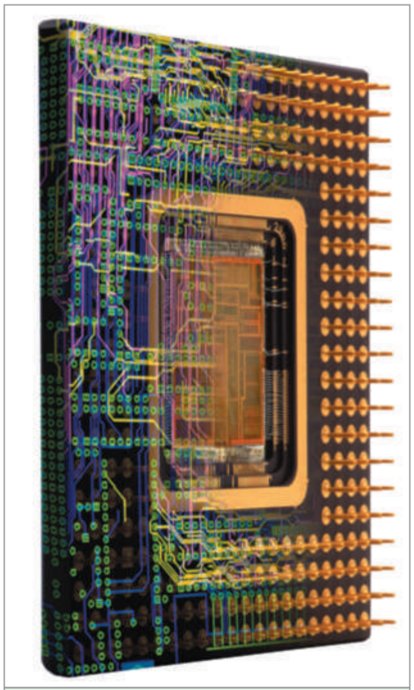
**FIGURE 11.3** ▶
MPS for a Family of Chairs

| | April | | | | May | | | |
|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Ladder-back chair | 150 | | | | | 150 | | |
| Kitchen chair | | | | 120 | | | 120 | |
| Desk chair | | 200 | 200 | | 200 | | | 200 |
| Sales and operations plan for chair family | 670 | | | | 670 | | | |

Figure 11.4 shows the MPS process. Operations must first create a prospective MPS to test whether it meets the schedule with the resources (e.g., machine capacities, workforce, overtime, and subcontractors) provided for in the sales and operations plan. Operations then revises the MPS until a schedule that satisfies all of the resource limitations is developed or until it is determined that no feasible schedule can be developed. In the latter event, the production plan must be revised to adjust production requirements or increase authorized resources. Once the firm's managers have accepted a feasible prospective MPS, operations uses the authorized MPS as an input to material requirements planning. Operations can then determine specific schedules for component production and assembly. Actual performance data such as inventory levels and shortages are inputs to preparing the prospective MPS for the next period, and so the master production scheduling process is repeated from one period to the next.




▲ **FIGURE 11.4**
Master Production Scheduling Process


WidStock/Alamy

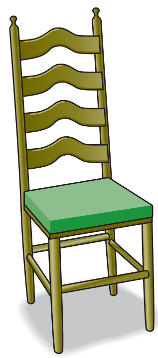
Cypress Semiconductor, a California-based company that manufactures logic devices, USB controllers, general-purpose programmable clocks, memories, and wireless connectivity solutions for consumer and automotive markets, uses commercial software solutions to manage the complexity of its master production scheduling processes.

## Developing a Master Production Schedule

The process of developing an MPS includes (1) calculating the projected on-hand inventory and (2) determining the timing and size of the production quantities of specific products. We use the ladder-back chair, shown in Figure 11.5, to illustrate the process. For simplicity, we assume that the firm does not utilize safety stocks for end items, even though many firms do. In addition, we use weeks as our planning periods, even though hours, days, or months could be used.

***Step 1.*** *Calculate Projected On-Hand Inventories.* The first step is to calculate the projected on-hand inventory, which is an estimate of the amount of inventory available each week after demand has been satisfied:

$$\begin{pmatrix}\text{Projected on-hand}\\ \text{inventory at end}\\ \text{of this week}\end{pmatrix} = \begin{pmatrix}\text{On-hand}\\ \text{inventory at}\\ \text{end of last week}\end{pmatrix} + \begin{pmatrix}\text{MPS quantity}\\ \text{due at start}\\ \text{of this week}\end{pmatrix} - \begin{pmatrix}\text{Projected}\\ \text{requirements}\\ \text{this week}\end{pmatrix}$$



▲ **FIGURE 11.5**
Ladder-Back Chair

In some weeks, no MPS quantity for a product may be needed because sufficient inventory already exists. For the projected requirements for this week, the scheduler uses whichever is larger—the forecast or the customer orders booked—recognizing that the forecast is subject to error. If actual booked orders exceed the forecast, the projection will be more accurate if the scheduler uses the booked orders because booked orders are a known quantity. Conversely, if the forecast exceeds booked orders for a week, the forecast will provide a better estimate of the requirements needed for that week because some orders may yet come in.

The manufacturer of the ladder-back chair produces the chair to stock and needs to develop an MPS for it. Marketing has forecasted a demand of 30 chairs for the first week of April, but actual customer orders booked are for 38 chairs. The current on-hand inventory is 55 chairs. No MPS quantity is due in week 1. Figure 11.6 shows an MPS record with these quantities listed. Because actual orders for week 1 are greater than the forecast, the scheduler uses that figure for actual orders to calculate the projected inventory balance at the end of week 1:

$$\text{Inventory} = \begin{pmatrix}\text{55 chairs}\\ \text{currently}\\ \text{in stock}\end{pmatrix} + \begin{pmatrix}\text{MPS quantity}\\ \text{(0 for week 1)}\end{pmatrix} - \begin{pmatrix}\text{38 chairs already}\\ \text{promised for}\\ \text{delivery in week 1}\end{pmatrix} = \text{17 chairs}$$

In week 2, the forecasted quantity exceeds actual orders booked, so the projected on-hand inventory for the end of week 2 is $17 + 0 - 30 = -13$. The shortage signals the need for more chairs to be produced and available for week 2.

| **Item:** Ladder-back chair | April | |
|---|---|---|
| **Quantity on Hand:** 55 | 1 | 2 |
| Forecast | 30 | 30 |
| Customer orders (booked) | 38 | 27 |
| Projected on-hand inventory | 17 | –13 |
| MPS quantity | 0 | 0 |
| MPS start | | |

**Explanation:**
Forecast is less than booked orders in week 1; projected on-hand inventory balance = 55 + 0 – 38 = 17.

**Explanation:**
Forecast exceeds booked orders in week 2; projected on-hand inventory balance = 17 + 0 – 30 = –13. The shortage signals a need to schedule an MPS quantity for completion in week 2.

▲ **FIGURE 11.6**
Master Production Schedule for Weeks 1 and 2

***Step 2.*** *Determine the Timing and Size of MPS Quantities.* The goal of determining the timing and size of MPS quantities is to maintain a nonnegative projected on-hand inventory balance. As shortages in inventory are detected, MPS quantities should be scheduled to cover them. The first MPS quantity should be scheduled for the week when the projected on-hand inventory reflects a shortage, such as week 2 in Figure 11.6.[1] The scheduler adds the MPS quantity to the projected on-hand inventory and searches for the next period when a shortage occurs. This shortage signals a need for a second MPS quantity, and so on.

Figure 11.7 shows an MPS for the ladder-back chair for the next 8 weeks. The order policy requires production lot sizes of 150 units. A shortage of 13 chairs in week 2 will occur unless the scheduler provides for an MPS quantity for that period. Our convention is to show blanks instead of zeroes in all rows, which improves readability and is often used in practice. The only exception is in the projected on-hand inventory row, where a number is always shown, even if it is a 0 or negative number.

Once the MPS quantity is scheduled, the updated projected inventory balance for week 2 is

$$\text{Inventory} = \begin{pmatrix}\text{17 chairs in}\\ \text{inventory at the}\\ \text{end of week 1}\end{pmatrix} + \begin{pmatrix}\text{MPS quantity}\\ \text{of 150 chairs}\end{pmatrix} - \begin{pmatrix}\text{Forecast of}\\ \text{30 chairs}\end{pmatrix} = 137 \text{ chairs}$$

▼ **FIGURE 11.7**
Master Production Schedule for Weeks 1–8

**Item:** Ladder-back chair — **Order Policy:** 150 units; **Lead Time:** 1 week

| | April | | | | May | | | |
|---|---|---|---|---|---|---|---|---|
| **Quantity on Hand:** 55 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Forecast | 30 | 30 | 30 | 30 | 35 | 35 | 35 | 35 |
| Customer orders (booked) | 38 | 27 | 24 | 8 | | | | |
| Projected on-hand inventory | 17 | 137 | 107 | 77 | 42 | 7 | 122 | 87 |
| MPS quantity | | 150 | | | | | 150 | |
| MPS start | 150 | | | | | 150 | | |

**Explanation:**
The time needed to assemble 150 chairs is 1 week. The assembly department must start assembling chairs in week 1 to have them ready by week 2.

**Explanation:**
On-hand inventory balance = 17 + 150 – 30 = 137. The MPS quantity is needed to avoid a shortage of 30 – 17 = 13 chairs in week 2.

The scheduler proceeds column by column through the MPS record until it reaches the end, filling in the MPS quantities as needed to avoid shortages. The 137 units will satisfy forecasted demands until week 7, when the inventory shortage in the absence of an MPS quantity is $7 + 0 - 35 = -28$. This shortage signals the need for another MPS quantity of 150 units. The updated inventory balance is $7 + 150 - 35 = 122$ chairs for week 7.

The last row in Figure 11.7 indicates the periods in which production of the MPS quantities must *begin* so that they will be available when indicated in the MPS quantity row. In the upper-right portion of the MPS record, a lead time of 1 week is indicated for the ladder-back chair; that is, 1 week is needed to assemble 150 ladder-back chairs, assuming that items B, C, D, and E are available. For each MPS quantity, the scheduler works backward through the lead time to determine when the assembly department must start producing chairs. Consequently, a lot of 150 units must be started in week 1 and another in week 6.

## Available-to-Promise Quantities

In addition to providing manufacturing with the timing and size of production quantities, the MPS provides marketing with information

[1]In some cases, new orders will be planned before a shortage is encountered. Two such instances occur when safety stocks and anticipation inventories are built up.

useful for negotiating delivery dates with customers. The quantity of end items that marketing can promise to deliver on specified dates is called **available-to-promise (ATP) inventory**. It is the difference between the customer orders already booked and the quantity that operations is planning to produce. As new customer orders are accepted, the ATP inventory is reduced to reflect the commitment of the firm to ship those quantities, but the actual inventory stays unchanged until the order is removed from inventory and shipped to the customer. An available-to-promise inventory is associated with each MPS quantity because the MPS quantity specifies the timing and size of new stock that can be earmarked to meet future bookings.

Figure 11.8 shows an MPS record with an additional row for the ATP quantities. The ATP in week 2 is the MPS quantity minus booked customer orders until the next MPS quantity, or $150 - (27 + 24 + 8 + 0 + 0) = 91$ units. The ATP indicates to marketing that, of the 150 units scheduled for completion in week 2, 91 units are uncommitted, and total new orders up to that quantity can be promised for delivery as early as week 2. In week 7, the ATP is 150 units because there are no booked orders in week 7 and beyond.

The procedure for calculating ATP information is slightly different for the first (current) week of the schedule than for other weeks because it accounts for the inventory currently in stock. The ATP inventory for the first week equals *current on-hand inventory* plus the MPS quantity for the first week, minus the cumulative total of booked orders up to (but not including) the week in which the next MPS quantity arrives. So, in Figure 11.8, the ATP for the first week is $55 + 0 - 38 = 17$. This information indicates to the sales department that it can promise as many as 17 units this week, 91 more units sometime in weeks 2 through 6, and 150 more units in week 7 or 8. If customer order requests exceed ATP quantities in those time periods, the MPS must be changed before the customer orders can be booked or the customers must be given a later delivery date—when the next MPS quantity arrives. See Solved Problem 2 at the end of this chapter for an example of decision making using the ATP quantities.

**Item:** Ladder-back chair

**Order Policy:** 150 units
**Lead Time:** 1 week

| Quantity on Hand: 55 | April | | | | May | | | |
|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Forecast | 30 | 30 | 30 | 30 | 35 | 35 | 35 | 35 |
| Customer orders (booked) | 38 | 27 | 24 | 8 | | | | |
| Projected on-hand inventory | 17 | 137 | 107 | 77 | 42 | 7 | 122 | 87 |
| MPS quantity | | 150 | | | | | 150 | |
| MPS start | 150 | | | | | 150 | | |
| Available-to-promise (ATP) inventory | 17 | 91 | | | | | 150 | |

**Explanation:**
The total of customer orders booked until the next MPS receipt is 38 units. The ATP = 55 (on-hand) + 0 (MPS quantity) – 38 = 17.

**Explanation:**
The total of customer orders booked until the next MPS receipt is 27 + 24 + 8 = 59 units. The ATP = 150 (MPS quantity) – 59 = 91 units.

▲ **FIGURE 11.8**
MPS Record with an ATP Row

**available-to-promise (ATP) inventory**
The quantity of end items that marketing can promise to deliver on specified dates.

**MyOMLab**
Tutor 11.1 in MyOMLab provides a new example to practice completing a master production schedule.

Inventory planners do not create master production plans manually, although they thoroughly understand the logic built into them. Figure 11.9 is typical of the computer support available. It was created with the *Master Production Scheduling* Solver in OM Explorer, and confirms the output shown in Figure 11.8.

## Freezing the MPS

The MPS is the basis of all end item, subassembly, component, and materials schedules. For this reason, changes to the MPS can be costly, particularly if they are made to MPS quantities soon to be completed. Increases in an MPS quantity can result in material shortages, delayed shipments to customers, and excessive expediting costs. Decreases in MPS quantities can result in unused materials or components (at least until another need for them arises) and valuable capacity being used to create products not needed. Similar costs occur when forecasted need dates for MPS quantities are changed. For these reasons, many firms, particularly those with a make-to-stock strategy and a focus on low-cost operations, *freeze*, or disallow changes to, the near-term portion of the MPS.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Lot Size | 150 | | | | | | | | |
| Lead Time | 1 | | | | | | | | |
| Quantity on Hand | 55 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Forecast | | 30 | 30 | 30 | 30 | 35 | 35 | 35 | 35 |
| Customer Orders (Booked) | | 38 | 27 | 24 | 8 | | | | |
| Projected On-Hand Inventory | | 17 | 137 | 107 | 77 | 42 | 7 | 122 | 87 |
| MPS Quantity | | | 150 | | | | | 150 | |
| MPS Start | | 150 | | | | | 150 | | |
| Available-to-Promise Inv (ATP) | | 17 | 91 | | | | | 150 | |

▲ **FIGURE 11.9**
Master Production Scheduling Solver Output Using *OM Explorer*

## Reconciling the MPS with Sales and Operations Plans

Because the MPS is based on both forecasts as well as actual orders received, it can differ from the sales and operations plan when summed across different periods in a month. It is important for the MPS and the S&OP plans to be consistent because the MPS specifies actual outputs while the S&OP determines the aggregate resources needed to produce those outputs. For instance, in Figure 11.3, if the sum total MPS quantities of the three models of chairs in the month of April was 725 instead of 670, then either the management must revise the sales and operations plan upwards by authorizing additional resources to match supply with demand, or reduce the quantities of MPS in the month of April to match the sales and operations plan. MPSs drive plant and supplier activity, so they must be synchronized with actual customer demands and the sales and operations plans to ensure that the firm's planning decisions are actually being implemented on an ongoing basis.

**bill of materials (BOM)**

A record of all the components of an item, the parent–component relationships, and the usage quantities derived from engineering and process designs.

**usage quantity**

The number of units of a component that are needed to make one unit of its immediate parent.

**end item**

The final product sold to a customer.

# MRP Explosion

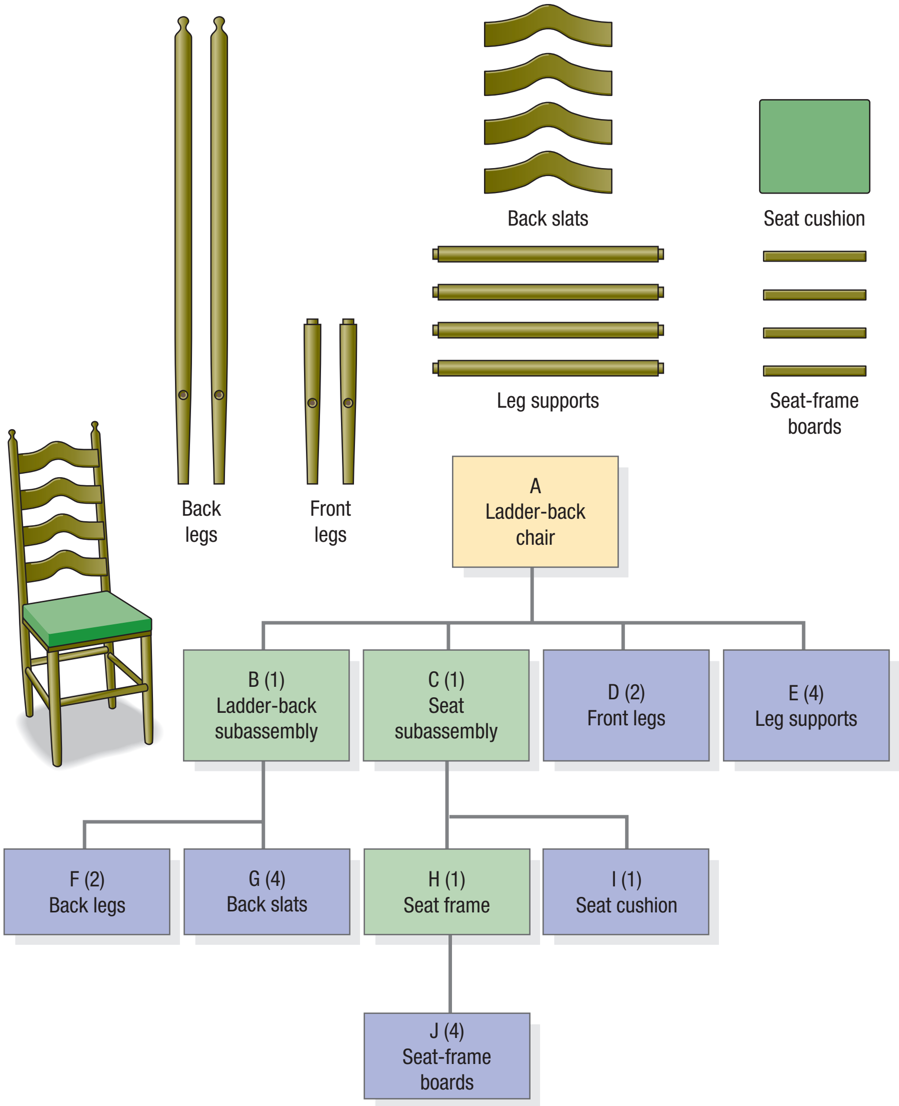
Having understood how an authorized MPS and other sources of independent demand create a time-phased statement of what is needed on a weekly basis, we next turn to exploring other inputs to the MRP system shown in Figure 11.1. These include the bills of materials and the information contained in inventory records, which in conjunction with the planning factors come together within the MRP logic to create the MRP explosion. The outputs of this explosion process and the reports that form the resultant material requirements plan are also described, along with the environments in which MRP works best.

MyOMLab Animation

## Bill of Materials

The replenishment schedule for a component is determined from the production schedules of its parents. Hence, the system needs accurate information on parent–component relationships. A **bill of materials (BOM)** is a record of all the components of an item, the parent–component relationships, and the usage quantities derived from engineering and process designs. In Figure 11.10, the BOM of our ladder-back chair from our discussion of the MPS shows that the chair is made from a ladder-back subassembly, a seat subassembly, front legs, and leg supports. In turn, the ladder-back subassembly is made from back legs and back slats, and the seat subassembly is made from a seat frame and a seat cushion. Finally, the seat frame is made from seat-frame boards. For convenience, we refer to these items by the letters shown in Figure 11.10.

▼ FIGURE 11.10
BOM for a Ladder-Back Chair




All items except item A are components because they are needed to make a parent. Items A, B, C, and H are parents because they all have at least one component. The BOM also specifies the **usage quantity**, or the number of units of a component that are needed to make one unit of its immediate parent. Figure 11.10 shows usage quantities for each parent–component relationship in parentheses. Note that one chair (item A) is made from one ladder-back subassembly (item B), one seat subassembly (item C), two front legs (item D), and four leg supports (item E). In addition, item B is made from two back legs (item F) and four back slats (item G). Item C needs one seat frame (item H) and one seat cushion (item I). Finally, item H needs four seat-frame boards (item J).

Four terms frequently used to describe inventory items are *end items*, *intermediate items*, *subassemblies*, and *purchased items*. An **end item** typically is the final product sold to the customer; it is a parent but not a

| **iPad total product costs $270** | **iPad 2 total product costs $267** |
|---|---|
| LCD display – 9.7-inch multi-touch screen **$59** | LCD display – 9.7-inch multi-touch screen **$50** |
| Camera – None **$0** | Camera – Front & Rear **$4.50** |
| Memory – Samsung **$47** | Memory – Samsung/Toshiba **$30** |
| Apple Processor – Apple A4 **$17** | Apple Processor – Apple A5 **$25** |
| Radio Components Infineon/Broadcom **$26** | Radio Components Infineon/Broadcom or Qualcomm/Broadcom **$25** |
| Sensors – STMicroelectronics **$1.00** | Sensors – STMicroelectronics **$2.50** |
| Battery **$23** | Battery **$20 – $25** |

D. Hurst/Alamy and PSL Images/Alamy

Bills of Materials for Apple's iPad and iPad2.

component. Item A in Figure 11.10, the completed ladder-back chair, is an end item. Accounting statements classify inventory of end items as either work-in-process (WIP), if work remains to be done, or finished goods. An **intermediate item**, such as item B, C, or H, has at least one parent and at least one component. Some products have several levels of intermediate items; the parent of one intermediate item can also be an intermediate item. Inventory of intermediate items—whether completed or still on the shop floor—is classified as WIP. A **subassembly** is an intermediate item that is assembled (as opposed to being transformed by other means) from more than one component. Items B and C are subassemblies. A **purchased item** has no components in the buyer's inventory because it comes from a supplier, but it has one or more parents. Examples are items D, E, F, G, I, and J in Figure 11.10. Inventory of purchased items is treated as raw materials in accounting statements.

**intermediate item**

An item that has at least one parent and at least one component.

**subassembly**

An intermediate item that is *assembled* (as opposed to being transformed by other means) from more than one component.

**purchased item**

An item that has one or more parents but no components because it comes from a supplier.

**part commonality**

The degree to which a component has more than one immediate parent.

A component may have more than one parent. **Part commonality**, sometimes called *standardization of parts* or *modularity*, is the degree to which a component has more than one immediate parent. As a result of commonality, the same item may appear in several places in the BOM for a product, or it may appear in the BOM for several different products. For example, the seat subassembly in Figure 11.10 is a component of the ladder-back chair and of a kitchen chair that is part of the same family of products. The usage quantity specified in the BOM relates to a specific parent–component relationship. The usage quantity for any component can therefore change, depending on the parent item. Part commonality, or using the same part in many parents, increases its volume and repeatability, which provides several process advantages and helps minimize inventory costs.

When the popular iPAD and iPAD 2 products from Apple were released a few years back, careful dissections of their BOMs showed that the two different generation products differed in their enclosure and battery, but were otherwise remarkably similar in their components and design. The same suppliers were used for many components, and costs were very comparable for new revisions of chips found in the previous iPad and iPhones. Standardization of designs and components across different products and generations allowed Apple to be very competitive and profitable.

## Inventory Record

Inventory records are a third major input to MRP, and inventory transactions are the basic building blocks of up-to-date records (see Figure 11.1). These transactions include releasing new orders, receiving scheduled receipts, adjusting due dates for scheduled receipts, withdrawing inventory, canceling orders, correcting inventory errors, rejecting shipments, and verifying scrap losses and stock returns. Recording the transactions accurately is essential if the firm's on-hand inventory balances are to be correct and its MRP system is to operate effectively.

**inventory record**

A record that shows an item's lot-size policy, lead time, and various time-phased data.

The **inventory record** divides the future into time periods called *time buckets*. In our discussion, we use weekly time buckets for consistency with our MPS example, although other time periods could as easily be used. The inventory record shows an item's lot-size policy, lead time, and various time-phased data. The purpose of the inventory record is to keep track of inventory levels and component replenishment needs. The time-phased information contained in the inventory record consists of (1) *gross requirements*, (2) *scheduled receipts*, (3) *projected on-hand inventory*, (4) *planned receipts*, and (5) *planned order releases*.

We illustrate the discussion of inventory records with the seat subassembly, item C that was shown in Figure 11.10. Suppose that it is used in two products: a ladder-back chair and a kitchen chair.

**gross requirements**

The total demand derived from *all* parent production plans.

**Gross Requirements** The **gross requirements** are the total demand derived from *all* parent production plans. They also include demand not otherwise accounted for, such as demand for replacement parts for units already sold. Figure 11.11 shows an inventory record for item C, the seat subassembly. Item C is produced in lots of 230 units and has a lead time of 2 weeks. The inventory record also shows item C's gross requirements for the next 8 weeks, which come from the MPS for the ladder-back and kitchen chairs (see Figure 11.3). The MPS start quantities for each parent are added to arrive at each week's gross requirements. The seat subassembly's gross requirements exhibit lumpy demand: Operations will withdraw seat subassemblies from inventory in only 4 of the 8 weeks.

The MRP system works with release dates to schedule production and delivery for components and subassemblies. Its program logic anticipates the removal of all materials required by a parent's production order from inventory at the beginning of the parent item's lead time—when the scheduler first releases the order to the shop.

**Item:** C
**Description:** Seat subassembly
**Lot Size:** 230 units
**Lead Time:** 2 weeks

| | | Week | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Gross requirements | | 150 | | | 120 | | 150 | 120 | |
| Scheduled receipts | | 230 | | | | | | | |
| Projected on-hand inventory | 37 | 117 | 117 | 117 | –3 | –3 | –153 | –273 | –273 |
| Planned receipts | | | | | | | | | |
| Planned order releases | | | | | | | | | |

**Explanation:**
Gross requirements are the total demand for the two chairs. Projected on-hand inventory in week 1 is 37 + 230 – 150 = 117 units.

▲ **FIGURE 11.11**
MRP Record for the Seat Subassembly

**Scheduled Receipts** Recall that *scheduled receipts* (sometimes called open orders) are orders that have been placed but not yet completed. For a purchased item, the scheduled receipt could be in one of several stages: being processed by a supplier, being transported to the purchaser, or being inspected by the purchaser's receiving department. If the firm is making the item in-house, the order could be on the shop floor being processed, waiting for components, waiting for a machine to become available, or waiting to be moved to its next operation. According to Figure 11.11, one 230-unit order of item C is due in week 1. Given the 2-week lead time, the inventory planner probably released the order 2 weeks ago. Scheduled receipts due in beyond the item's lead time are unusual, caused by events such as a last-minute change in the MPS.

**projected on-hand inventory**

An estimate of the amount of inventory available each week after gross requirements have been satisfied.

**Projected On-Hand Inventory** The **projected on-hand inventory** is an estimate of the amount of inventory available each week after gross requirements have been satisfied. The beginning inventory, shown as the first entry (37) in Figure 11.11, indicates the on-hand inventory available at the time the record was computed. As with scheduled receipts, entries are made for each actual withdrawal and receipt to update the MRP database. Then, when the MRP system produces the revised record, the correct inventory will appear.

Other entries in the row show inventory expected in future weeks. Projected on-hand inventory is calculated as

$$\begin{pmatrix}\text{Projected on-hand}\\ \text{inventory balance}\\ \text{at end of week } t\end{pmatrix} = \begin{pmatrix}\text{Inventory on}\\ \text{hand at end of}\\ \text{week } t-1\end{pmatrix} + \begin{pmatrix}\text{Scheduled}\\ \text{or planned}\\ \text{receipts in}\\ \text{week } t\end{pmatrix} - \begin{pmatrix}\text{Gross}\\ \text{requirements}\\ \text{in week } t\end{pmatrix}$$

**planned receipts**

Orders that are not yet released to the shop or the supplier.

The projected on-hand calculation includes the consideration of **planned receipts**, which are orders not yet released to the shop or the supplier. Planned receipts should not be confused with scheduled receipts. Planned receipts are still at the planning stage and can still change from one week to the next, whereas scheduled receipts are actual orders that are being acted upon by the shop or supplier. In

Figure 11.11, the planned receipts are all zero. The on-hand inventory calculations for each week are as follows:

| | |
|---|---|
| Week 1: | $37 + 230 - 150 = 117$ |
| Weeks 2 and 3: | $117 + 0 - 0 = 117$ |
| Week 4: | $117 + 0 - 120 = -3$ |
| Week 5: | $-3 + 0 - 0 = -3$ |
| Week 6: | $-3 + 0 - 150 = -153$ |
| Week 7: | $-153 + 0 - 120 = -273$ |
| Week 8: | $-273 + 0 - 0 = -273$ |

In week 4, the balance drops to –3 units, which indicates that a shortage of 3 units will occur unless more seat subassemblies are built. This condition signals the need for a planned receipt to arrive in week 4. In addition, unless more stock is received, the shortage will grow to 273 units in weeks 7 and 8.[2]

**Planned Receipts** Planning for the receipt of new orders will keep the projected on-hand balance from dropping below zero. The planned receipt row is developed as follows:

**planned order release**

An indication of when an order for a specified quantity of an item is to be issued.

1. Weekly on-hand inventory is projected until a shortage appears. Completion of the initial planned receipt is scheduled for the week in which the shortage is projected. The addition of the newly planned receipt should increase the projected on-hand balance so that it equals or exceeds zero. It will exceed zero when the lot size exceeds requirements in the week it is planned to arrive.
2. The projection of on-hand inventory continues until the next shortage occurs. This shortage signals the need for the second planned receipt.

This process is repeated until the end of the planning horizon by proceeding column by column through the MRP record—filling in planned receipts as needed and completing the projected on-hand inventory row. Figure 11.12 shows the planned receipts for the seat subassembly. In week 4, the projected on-hand inventory will drop below zero, so a planned receipt of 230 units is scheduled for week 4. The updated inventory on-hand balance is 117 inventory at end of week 3) + 230 (planned receipts) − 120 (gross requirements) = 227 units. The projected on-hand inventory remains at 227 for week 5 because no scheduled receipts or gross requirements are anticipated. In week 6, the projected on-hand inventory is 227(inventory at end of week 5) − 150 (gross requirements) = 77 units. This quantity is greater than zero, so no new planned receipt is needed. In week 7, however, a shortage will occur unless more seat subassemblies are received. With a planned receipt in week 7, the updated inventory balance is 77(inventory at end of week 6) + 230 (planned receipts) − 120 (gross requirements) = 187 units.

**Item:** C **Description:** Seat subassembly **Lot Size:** 230 units **Lead Time:** 2 weeks

| | | Week | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Gross requirements | | 150 | | | 120 | | 150 | 120 | |
| Scheduled receipts | | 230 | | | | | | | |
| Projected on-hand inventory | 37 | 117 | 117 | 117 | 227 | 227 | 77 | 187 | 187 |
| Planned receipts | | | | | 230 | | | 230 | |
| Planned order releases | | | 230 | | | 230 | | | |

**Explanation:**
Without a planned receipt in week 4, a shortage of 3 units will occur: 117 – 120 = –3 units. Adding the planned receipt brings the balance to 117 + 230 – 120 = 227 units. Offsetting for a 2-week lead time puts the corresponding planned order release back to week 2.

**Explanation:**
The first planned receipt lasts until week 7, when projected inventory would drop to 77 – 120 = –43 units. Adding the second planned receipt brings the balance to 77 + 230 – 120 = 187 units. The corresponding planned order release is for week 5 (or week 7 minus 2 weeks).

▲ **FIGURE 11.12**
Completed Inventory Record for the Seat Subassembly

**Planned Order Releases** A **planned order release** indicates when an order for a specified quantity of an item is to be issued. We must place the planned order release quantity in the proper time bucket. To do so, we must assume that all inventory flows—scheduled receipts,

[2] There is an exception to the rule of scheduling a planned receipt whenever the projected inventory otherwise becomes negative. When a scheduled receipt is coming in *after* the inventory becomes negative, the first recourse is to expedite the scheduled receipt (giving it an earlier due date), rather than scheduling a new planned receipt.


Justin Kase z03z/Alamy

Winnebago regularly introduces new models of motorhomes, many of which have received the Readers' Choice award from MotorHome Magazine. With a constant change in product variety, Winnebago uses a homegrown MRP system running on an IBM mainframe that can be easily modified to structure its Bill of Materials (BOM) and support production schedules of the new vehicles.

planned receipts, and gross requirements—occur at the same point of time in a time period. Some firms assume that all flows occur at the beginning of a time period; other firms assume that they occur at the end of a time period or at the middle of the time period. Regardless of when the flows are assumed to occur, we find the release date by subtracting the lead time from the receipt date. For example, the release date for the first planned order release in Figure 11.12 is 4 (planned receipt date) − 2 (lead time) = 2 (planned order release date). Figure 11.12 shows the planned order releases for the seat subassembly. If all goes according to the plan, we will release an order for 230 seat assemblies next week (in week 2). This order release sets off a series of updates to the inventory record. First, the planned order release for the order is removed. Next, the planned receipt for 230 units in week 4 is also removed. Finally, a new scheduled receipt for 230 units will appear in the scheduled receipt row for week 4.

## Planning Factors

The planning factors in a MRP inventory record play an important role in the overall performance of the MRP system. By manipulating these factors, managers can fine-tune inventory operations. In this section, we discuss planning lead time, lot-sizing rules, and safety stock.

**Planning Lead Time** Planning lead time is an estimate of the time between placing an order for an item and receiving the item in inventory. Accuracy is important in planning lead time. If an item arrives in inventory sooner than needed, inventory holding costs increase. If an item arrives too late, stockouts, excessive expediting costs, or both may occur.

For purchased items, the planning lead time is the time allowed for receiving a shipment from the supplier after the order has been sent, including the normal time to place the order. Often, the purchasing contract stipulates the delivery date. For items manufactured in-house, a rough-cut estimate of the planning lead time can be obtained by keeping track of the actual lead times for recent orders and computing an average. A more extensive estimating process consists of breaking down each of the following factors:

- Setup time
- Processing time
- Materials handling time between operations
- Waiting time

Each of these times must be estimated for every operation along the item's route. Estimating setup, processing, and materials handling times can be relatively easy, but estimating the waiting time for materials handling equipment or for a workstation to perform a particular operation can be more difficult. In a facility that uses a make-to-order strategy, such as a machine shop, the load on the shop varies considerably over time, causing actual waiting times for a particular order to fluctuate widely. Therefore, being able to accurately estimate the waiting time is especially important when it comes to estimating the planning lead time. However, in a facility that uses a make-to-stock strategy, such as an assembly plant, product routings are more standard and waiting time is more predictable; hence, waiting time generally is a less-troublesome part of planning lead times.

**Lot-Sizing Rules** A lot-sizing rule determines the timing and size of order quantities. A lot-sizing rule must be assigned to each item before planned receipts and planned order releases can be computed. The choice of lot-sizing rules is important because they determine the number of setups required and the inventory holding costs for each item. We present three lot-sizing rules: (1) fixed order quantity, (2) periodic order quantity, and (3) lot-for-lot.

**fixed order quantity (FOQ)**

A rule that maintains the same order quantity each time an order is issued.

**Fixed Order Quantity** The **fixed order quantity (FOQ)** rule maintains the same order quantity each time an order is issued.[3] For example, the lot size might be the size dictated by equipment capacity

[3]The *kanban* system essentially uses a FOQ rule, except that the order quantity is very small.

limits, such as when a full lot must be loaded into a furnace at one time. For purchased items, the FOQ could be determined by the quantity discount level, truckload capacity, or minimum purchase quantity. Alternatively, the lot size could be determined by the economic order quantity (EOQ) formula (see Chapter 9, "Managing Inventories"). Figure 11.12 illustrated the FOQ rule. However, if an item's gross requirement within a week is particularly large, the FOQ might be insufficient to avoid a shortage. In such unusual cases, the inventory planner must increase the lot size beyond the FOQ, typically to a size large enough to avoid a shortage. Another option is to make the order quantity an integer multiple of the FOQ. This option is appropriate when capacity constraints limit production to FOQ sizes (at most).

**Periodic Order Quantity** The **periodic order quantity (POQ)** rule allows a different order quantity for each order issued but issues the order for predetermined time intervals, such as every two weeks. The order quantity equals the amount of the item needed during the predetermined time between orders and must be large enough to prevent shortages. Specifically, the POQ is

**periodic order quantity (POQ)**

A rule that allows a different order quantity for each order issued but issues the order for predetermined time intervals.

$$\begin{pmatrix}\text{POQ lot size}\\ \text{to arrive in}\\ \text{week } t\end{pmatrix} = \begin{pmatrix}\text{Total gross requirements}\\ \text{for } P \text{ weeks, including}\\ \text{week } t\end{pmatrix} - \begin{pmatrix}\text{Projected on-hand}\\ \text{inventory balance at}\\ \text{end of week } t-1\end{pmatrix}$$

This amount exactly covers $P$ weeks' worth of gross requirements. That is, the projected on-hand inventory should equal zero at the end of the $P$th week.

Suppose that we want to switch from the FOQ rule used in Figure 11.12 to the POQ rule. Figure 11.13 was created with the *Single-Item MRP* Solver in OM Explorer. It shows the application of the POQ rule, with $P = 3$ weeks, to the seat subassembly inventory. The first order is required in week 4 because it is the first week that projected inventory balance will fall below zero. The first order using $P = 3$ weeks is

MyOMLab

Tutor 11.2 in MyOMLab provides a new example to practice lot-sizing decisions using FOQ, POQ, and L4L rules.

$$(\text{POQ lot size}) = \begin{pmatrix}\text{Gross requirements}\\ \text{for weeks}\\ \text{4, 5, and 6}\end{pmatrix} - \begin{pmatrix}\text{Inventory at}\\ \text{end of week 3}\end{pmatrix}$$

$$= (120 + 0 + 150) - 117 = 153 \text{ units}$$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Periods | 8 | | | | | | | | |
| Item | Seat Assembly | | Period (P) for POQ | | 3 | Lot Size (FOQ) | | | |
| Description | | | | | | Lead Time | | | 2 |
| POQ Rule | | | | | | | | | |
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Gross requirements | | 150 | | | 120 | | 150 | 120 | |
| Scheduled receipts | | 230 | | | | | | | |
| Projected on-hand inventory | 37 | 117 | 117 | 117 | 150 | 150 | | | |
| Planned receipts | | | | | 153 | | | 120 | |
| Planned order releases | | | 153 | | | 120 | | | |

**FIGURE 11.13**

Single-Item MRP Solver Output in OM Explorer using the POQ ($P = 3$) Rule for the Seat Subassembly

The second order must arrive in week 7 with a lot size of $(120 + 0) - 0 = 120$ units. This second order reflects only two weeks' worth of gross requirements—to the end of the planning horizon.

The POQ rule does not mean that the planner must issue a new order every $P$ weeks. Rather, when an order is planned, its lot size must be enough to cover $P$ successive weeks. One way to select a $P$ value is to divide the average lot size desired, such as the EOQ or some other applicable lot size, by the average weekly demand. That is, express the target lot size as the desired weeks of supply ($P$) and round to the nearest integer.

**Lot for Lot** A special case of the POQ rule is the **lot-for-lot (L4L) rule**, under which the lot size ordered covers the gross requirements of a single week. Thus, $P = 1$, and the goal is to minimize inventory levels. This rule ensures that the planned order is just large enough to prevent a shortage in the single week it covers. The L4L lot size is

**lot-for-lot (L4L) rule**

A rule under which the lot size ordered covers the gross requirements of a single week.

$$\begin{pmatrix}\text{L4L lot size}\\ \text{to arrive in}\\ \text{week } t\end{pmatrix} = \begin{pmatrix}\text{Gross requirements}\\ \text{for week } t\end{pmatrix} - \begin{pmatrix}\text{Projected on-hand}\\ \text{inventory balance at}\\ \text{end of week } t-1\end{pmatrix}$$

The projected on-hand inventory combined with the new order will equal zero at the end of week $t$. Following the first planned order, an additional planned order will be used to match each subsequent gross requirement.

| **Item:** Ladder-back chair | | **Order Policy:** L4L<br>**Lead Time:** 2 weeks | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Week | | | | | | | |
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Gross requirements | | 150 | | | 120 | | 150 | 120 | |
| Scheduled receipts | | 230 | | | | | | | |
| Projected on-hand inventory | 37 | 117 | 117 | 117 | | | | | |
| Planned receipts | | | | | 3 | | 150 | 120 | |
| Planned order releases | | | 3 | | 150 | 120 | | | |

▲ **FIGURE 11.14**
The L4L Rule for the Seat Subassembly

This time we want to switch from the FOQ rule to the L4L rule. Figure 11.14 shows the application of the L4L rule to the seat subassembly inventory. As before, the first order is needed in week 4:

$$(\text{L4L lot size}) = \begin{pmatrix}\text{Gross requirements}\\ \text{in week 4}\end{pmatrix} - \begin{pmatrix}\text{Inventory balance}\\ \text{at end of week 3}\end{pmatrix}$$
$$= 120 - 117 = 3$$

The stockroom must receive additional orders in weeks 6 and 7 to satisfy each of the subsequent gross requirements. The planned receipt for week 6 is 150 and for week 7 is 120.

**Comparing Lot-Sizing Rules** Choosing a lot-sizing rule can have important implications for inventory management. Lot-sizing rules affect inventory costs and setup and ordering costs. The FOQ, POQ, and L4L rules differ from one another in one or both respects. In our example, each rule took effect in week 4, when the first order was placed. Let us compare the projected on-hand inventory averaged over weeks 4 through 8 of the planning horizon. The data are shown in Figures 11.12, 11.13, and 11.14, respectively.

$$\text{FOQ: } \frac{227 + 227 + 77 + 187 + 187}{5} = 181 \text{ units}$$

$$\text{POQ: } \frac{150 + 150 + 0 + 0 + 0}{5} = 60 \text{ units}$$

$$\text{L4L: } \frac{0 + 0 + 0 + 0 + 0}{5} = 0 \text{ units}$$

The performance of the L4L rule with respect to average inventory levels comes at the expense of an additional planned order and its accompanying setup time and cost. We can draw three conclusions from this comparison:

1. The FOQ rule generates a high level of average inventory because it creates inventory *remnants*. A remnant is inventory carried into a week, but it is too small to prevent a shortage. Remnants occur because the FOQ does not match requirements exactly. For example, according to Figure 11.12, the stockroom must receive a planned order in week 7, even though 77 units are on hand at the beginning of that week. The remnant is the 77 units that the stockroom will carry for 3 weeks, beginning with receipt of the first planned order in week 4. Although they increase average inventory levels, inventory remnants introduce stability into the production process by buffering unexpected scrap losses, capacity bottlenecks, inaccurate inventory records, or unstable gross requirements.
2. The POQ rule reduces the amount of average on-hand inventory because it does a better job of matching order quantity to requirements. It adjusts lot sizes as requirements increase or decrease. Figure 11.13 shows that in week 7, when the POQ rule has fully taken effect, the projected on-hand inventory is zero—no remnants.
3. The L4L rule minimizes inventory investment, but it also maximizes the number of orders placed. This rule is most applicable to expensive items or items with small ordering or setup costs. It is the only rule that can be used for a low-volume item made to order. It can also approximate the small-lot inventory levels of a lean system.

By avoiding remnants, both the POQ and the L4L rule may introduce instability by tying the lot-sizing decision so closely to requirements. If any requirement changes, so must the lot size, which can disrupt component schedules. Last-minute increases in parent orders may be hindered by missing components.

**Safety Stock** An important managerial decision is the quantity of safety stock to carry. It is more complex for dependent demand items than for independent demand items. Safety stock for dependent demand items with lumpy demand (gross requirements) is helpful only when future gross requirements, the timing or size of scheduled receipts, and the amount of scrap that will be produced are uncertain. As these uncertainties are resolved, safety stock should be reduced and ultimately eliminated. The usual policy is to use safety stock for end items and purchased items to protect against fluctuating customer orders and unreliable suppliers of components but to avoid using it as much as possible for intermediate items. Safety stocks can be incorporated in the MRP logic by using the following rule: Schedule a planned receipt whenever the projected on-hand inventory balance drops below the desired safety stock level (rather than zero, as before). The objective is to keep a minimum level of planned inventories equal to the safety stock quantity. Figure 11.15 shows what happens when the safety stock requirement has just been