JOINT CFO/PHN/CIR ESTIMATION SCHEME TO COMBAT CFO AND PHN EFFECTS IN A SISO OFDM SYSTEM

by

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ABSTRACT

Orthogonal Frequency Division Multiplexing (OFDM) is fast becoming the leading 4G application for efficient and reliable communication in multipath environments. OFDM remains an attractive option due to its robustness under frequency selective fading and its ability to mitigate both inter-symbol interference (ICI) and inter-carrier interference (ICI) while delivering high data rates. However certain limitations to OFDM, such as frequency synchronization issues brought on my carrier frequency offset (CFO) and phase noise (PHN) effects can have a detrimental effect on system performance. This however can be mitigated through the use of a Joint Carrier Frequency Offset/Phase Noise/Channel Impulse Response Estimator (JCPCE) that finds optimal estimates for the CFO, PHN and CIR. This thesis aims to examine the impact of CFO and PHN on a Single-Input Single-Output OFDM system and investigates the viability of the JCPCE algorithm in mitigating these limitations to the OFDM system.

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LIST *of* SYMBOLS

Symbol	Meaning
T_s	Symbol Period (s)
B	OFDM Channel Bandwidth (Hz)
τ	Channel Delay Spread (s)
N	Number of Subcarriers
$[\ ]^*$	Conjugate Operator
$[\ ]^H$	Hermitian Operator
$[\ ]^T$	Transpose Operator
Δf	Frequency Separation (between subcarriers) (Hz)
ξ	Carrier Frequency Offset (frequency domain) (Hz)
ε	Normalized Carrier Frequency Offset (frequency domain) (Hz)
α	Standard Deviation of Innovation Noise
x, y	Vectors are displayed in bold lower case letters
X, Y	Vectors are displayed in bold upper case letters

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CHAPTER 1

Introduction

1.1 The Argument for Improvement in Wireless Systems

In a world obsessed with information and an incessant demand for precise and clear communication, the role being played by wireless technologies is of ever increasing importance. Traditional wireless systems (such as GSM, GPRS, FDMA etc.) are often band or power limited transmitting at slower data rates while being spectrally inefficient. Fourth generation (4G) communication systems allow for a fast, reliable and robust broadband communications system that is also spectrally efficient. Through evolving protocols such as Orthogonal Frequency Division Multiplexing (OFDM), Multiple-Input and Multiple-Output (MIMO) antennas, along with the Turbo principle, 4G technologies are vastly capable of exploiting the frequency selective channel property, improving spectral efficiency and improving SNR (Signal to Noise ratio) at the receiver end. It is this promise of higher data rates and increased immunity in frequency selective channels that make 4G technologies such as OFDM extremely intriguing.

1.2 Orthogonal Frequency Division Multiplexing

OFDM is a multicarrier modulation technique that has been widely adopted in a variety of high data rate communication systems such as digital subscriber lines, wireless LANs (802.11a/g/n), WiMAX [1], digital audio broadcasting (DAB), digital video broadcasting (DVB), wireless local area networks (WLAN), and more recently wireless local loop (WLL). One of the many reasons OFDM is so attractive to these high data rate applications is due its efficient and flexible management of inter-symbol interference (ISI) in highly dispersive channels. In single carrier modulation schemes data is sent serially over a multipath fading channel at a symbol period of T_s. Time dispersion, τ_d, under these circumstances can be very significant compare to T_s, resulting in ISI. A complex equaliser is then required to compensate for the channel distortion [2]. In multicarrier modulation schemes (MMS) however a high bit rate data stream is divided into several lower bit rate streams modulated on separate carriers. Hence to minimize the effects of ISI the T_s much be divided such that τ (multipath delay spread induced by the channel) is small. By definition, a high data rate system will have the property such that $\tau > T_s$, since the number of symbols sent per second is high (reducing T_s compared to τ caused by multipath delay). Hence the effects of ISI can be quite severe. In non-line of sight (NLOS) systems, such as WiMax, the delay spread can also be quite large. Furthermore, short, wireless broadband systems all suffer from severe ISI. Although the 802.16 standards utilize single carrier modulation schemes, the vast majority of, if not all, 802.16-compliant systems use the OFDM modes. Through MMS, OFDM is able to mitigate this concern by dividing a given high bit rate stream into N different orthogonal substreams. These N substreams are chosen so that the effect of channel induced delay is essentially negated, such that $\tau << NT_s$, hence effectively making these channels ISI free (ISI is completely negated through the use of cyclic prefix). However due to the stringent requirement of orthogonality between data symbols OFDM synchronization, in the form of time and frequency synchronization, becomes particularly important at the receiver end so that all symbols are maintained to be individually discernible in the frequency domain. Although time synchronization is less of a concern than in single carrier systems, the effects of frequency synchronization and oscillator phase noise can be of significant concern while demodulating an OFDM signal. Compensation techniques, such as joint carrier frequency offset, phase noise and channel estimation (JCPCE) studied in [3]-[4] aim to mitigate some of these effects. Notwithstanding, over the past decade OFDM has been particularly successful in numerous wireless applications, where its superior performance in multi-path environments and it's ability to deliver virtually ISI free transmission under high data rate demands makes it very desirable [1].

1.3 Related Work

In [3]-[4] D. D. Lin and T. J. Lim propose an optimal training-based OFDM channel impulse response (CIR) estimation algorithm that addresses the carrier frequency offset (CFO) and phase noise (PHN) problem that plagues OFDM systems. The solution involves optimizing (maximum a posteriori) a joint estimator for CIR, CFO and PHN using a systematic probabilistic method to model PHN. The papers deal with both Weiner and Gaussian phase noise and demonstrate the effect of the Joint CFO/PHN/CIR Estimator (JCPCE) for an OFDM system. It also bears mentioning that the paper does not model the effects of common and random phase noise separately. It is the aim of this thesis to confirm the results of [3]-[4] for a 64 subcarrier SISO OFDM model.

1.4 Thesis Objectives

It is the scope of this paper to simulate the workings of a SISO OFDM system and to ascertain the following objectives:

- Explore the impact of phase noise (PHN) and to determine an acceptable level that can be tolerated for a modern high data rate system.
- Explore the impact of carrier frequency offset (CFO) and to determine an acceptable level that can be tolerated for a modern high data rate system.
- Explore the workings of the JCPCE algorithm, discussed in [3]-[4], and to confirm the impact it has on mitigating CFO and PHN in SISO OFDM systems.

1.5 Thesis Outline

This thesis is outlined in the following manner:

- *Chapter 2:* introduces OFDM principles, along with an understanding of why this particular 4G MMS is best able to mitigate ISI. OFDM specifics such as the cyclic prefix, FFT, modulation and demodulation techniques are also explored. Finally OFDM is compared to the single carrier system.
- *Chapter 3:* looks at the origins of the CFO and PHN effects and demonstrates how these effects were modelled. The JCPCE algorithm is similarly explored along with an understanding of how it fits into the OFDM system and how it can be modelled.
- *Chapter 4:* looks at the various numerical results through simulation. First the effects of CFO and PHN on the SISO OFDM system are investigated, followed by the effects of the JCPCE algorithm on resolving these effects through the use of the joint optimization of a complete log-likelihood function over the unknown CIR, PHN and CFO [3].
- *Chapter 5:* summarizes the main results of this thesis and provides scope for further work.

CHAPTER 2

Orthogonal Frequency Division Multiplexing

Under traditional single carrier digital modulation schemes high data rate streams lead to the condition that the symbol period T_s is made much smaller than the multipath channel induced delay τ. Hence the effect of ISI is very severe. However under MMS and particularly with OFDM, ISI can be completely mitigated under these conditions. This chapter aims to introduce the concept of Orthogonal Frequency Division Multiplexing from a conceptual and modelling perspective. It also looks at the advantages and disadvantages of the OFDM compared to the single carrier systems.

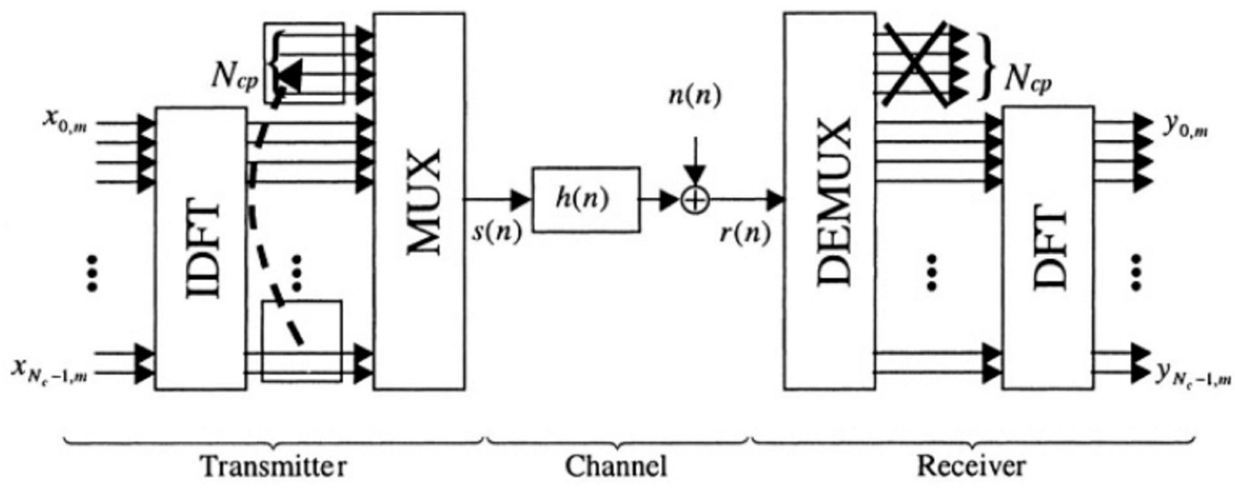


Figure i OFDM Multicarrier Modulation Scheme (MMS) represented in discrete time [2]

2.1 Multicarrier Modulation Schemes

Unlike single carrier modulation schemes, multicarrier modulation schemes (MMS) aim to overcome the limitation $\tau << T_s$ for ISI free transmission to obtain high data rates. Although this is impossible for single carrier modulation schemes it is indeed achievable through MMS. The primary aim of MMS is to divide a high data rate system into an N number of lower rate data subcarriers operating in parallel, as shown in Figure ii. These subcarriers comprise to make up the complete transmitted signal.

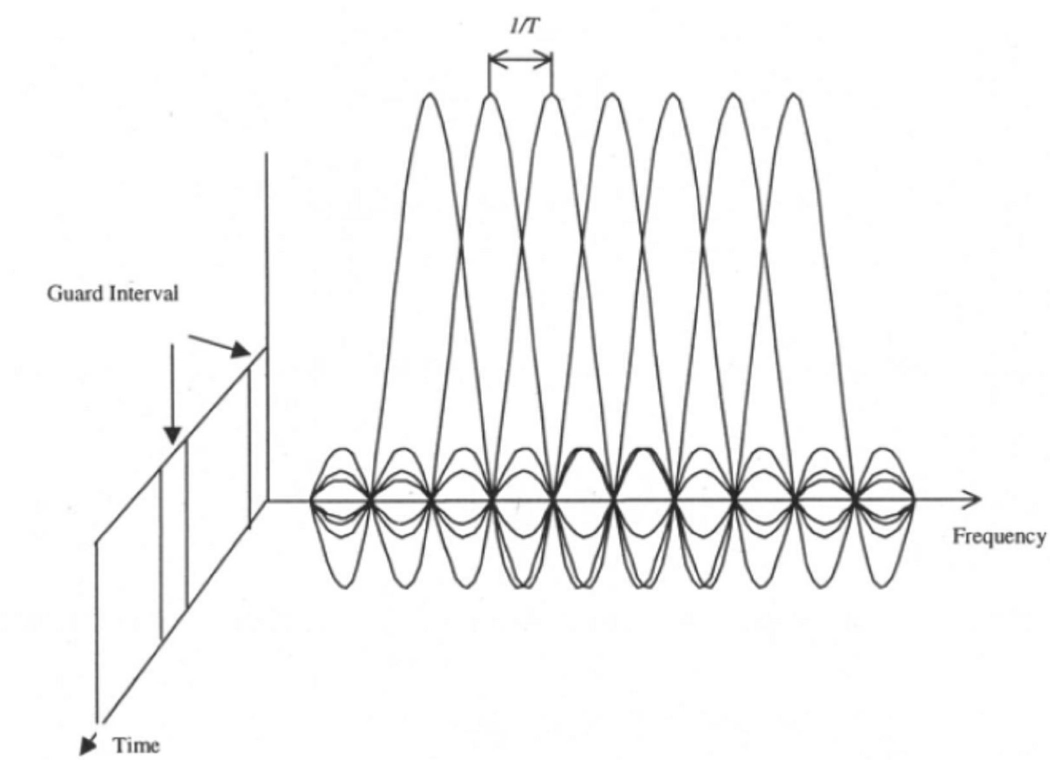


Figure ii OFDM (a type of MMS) in both frequency and time domain [5]

By dividing the symbol period, T_s, over N subcarriers MMS effectively achieves the condition for ISI free transmission such that $\tau << T$, where $T = NT_s$. This lends itself to a more intuitive and equivalent statement in the frequency domain. Each subcarrier now has a bandwidth of B/N. The number N, for the number of subcarriers, cannot be arbitrarily increased, because too long a symbol duration will make the transmission too sensitive against time incoherence of the channel that is related to the maximum Doppler frequency ν_{max}. Also each subcarrier channel must be chosen such that the bandwidth of these sub-channels is less than the coherence bandwidth of the channel [6] so that the signals can still be considered, across the N sub-channels, to be essentially perfectly correlated. Hence the condition for ISI free transmission can be stated as the following:

$$\tau << T_s \Leftrightarrow BW_{subchannel} << BW_{coherence} \tag{1.1}$$

This is an important characteristic of MMS systems because it means that individual subcarriers experience flat fading while the channel experiences frequency selective fading, shown in Figure iii below. Herein lies the key reason for the negligible ISI experienced by each channel [5].

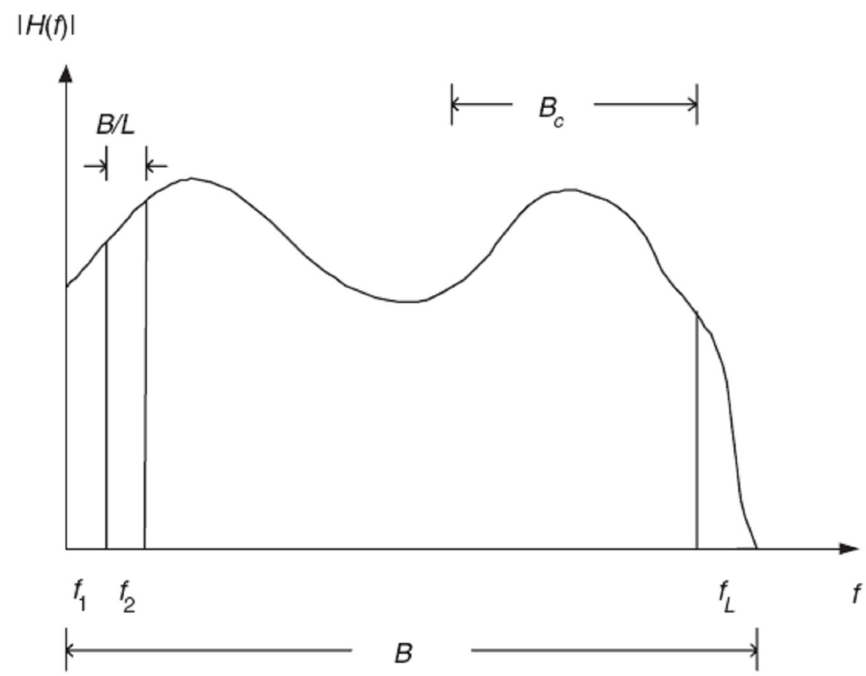


Figure iii The transmitted multicarrier signal experiences approximately flat fading on each on each sub-channel, since B/L (or N) << B_c, even though the overall channel experiences frequency-selective fading B >> B_c [1].

2.2 Principles of OFDM

Many MMS schemes use guard intervals between successive carrier symbol blocks to prevent symbols from interfering with each other (also known as inter carrier interference – ICI). This however is spectrally inefficient and increases the effective data rate of the system. In OFDM subcarriers are orthogonal to each other leading to spectral efficiency since guard intervals are not employed. Signal processing techniques such as DFT, which lends itself to the highly efficient implementation of FFT, allow for the creation of these orthogonal sub-channels.

2.2.1 Discrete Fourier Transform

In order to maintain spectral overlap each data subcarrier must be made mathematically orthogonal. Two signals are said to be orthogonal if $\theta_n(t)$ and $\theta_m(t)$ satisfy the following equation:

$$\int_a^b \theta_n(t)\theta_m^*(t)dt = k_n \delta_{nm} \tag{1.2}$$

Where δ_{nm} represents the Kronecker delta and k_n is a constant over the interval $a<t<b$. This is shown in Figure ii, wherein the sinc function is used to represent orthogonal spectral overlap in the frequency domain. In order to overcome the daunting requirement of N RF radios at both the transmitter and receiver to modulate and demodulate N orthogonal subcarriers, OFDM uses the computationally efficient method of Discrete Fourier Transform (DFT) which lends itself to the more efficient implementation of Fast Fourier Transform (FFT). The FFT and its inverse, the IFFT, help create a set of orthogonal subcarriers using just a single radio. [1].

Consider an input data stream x[n] being sent through a linear time-invariant Finite Impulse Response (FIR) channel h[n], the output of which is the linear convolution of the input data signal and the channel, yielding y[n].

$$y[n] = x[n] * h[n] \tag{1.3}$$

However let us imagine y[n] being taken through circular convolution as shown below:

$$y[n] = x[n] \otimes h[n] = h[n] \otimes x[n] \tag{1.4}$$

where,

$$x[n] \otimes h[n] = h[n] \otimes x[n] \equiv \sum_{k=0}^{L-1} h[k]x[n-k]_L \quad (1.5)$$

and the circular function $x[n]_L$ = x[n mod L] is a periodic version of x[n] with period L. In other words, each value of y[n] is the sum of the product of N terms. The corresponding DFT of the output could then be taken which yields the following result in frequency domain:

$$Y[m] = H[m]X[m] \quad (1.6)$$

where the L point DFT and IDFT are defined as follows:

$$DFT\{x[n]\} = X[m] \equiv \frac{1}{\sqrt{L}} \sum_{n=0}^{L-1} x[n]e^{-j\frac{2\pi nm}{L}} \quad (1.7)$$

$$IDFT\{X[m]\} = x[n] \equiv \frac{1}{\sqrt{L}} \sum_{n=0}^{L-1} X[m]e^{j\frac{2\pi nm}{L}} \quad (1.8)$$

Note that Equation 1.6 essentially describes an ISI free channel in the frequency domain, as each input X[m] is scaled by the channel frequency response H[m]. Note the orthogonal frequency spectra shown in Figure iv below. The receiver must sample at the peak of each symbol (located $1/T_s$) apart) to effectively negate the impact of ICI [1].

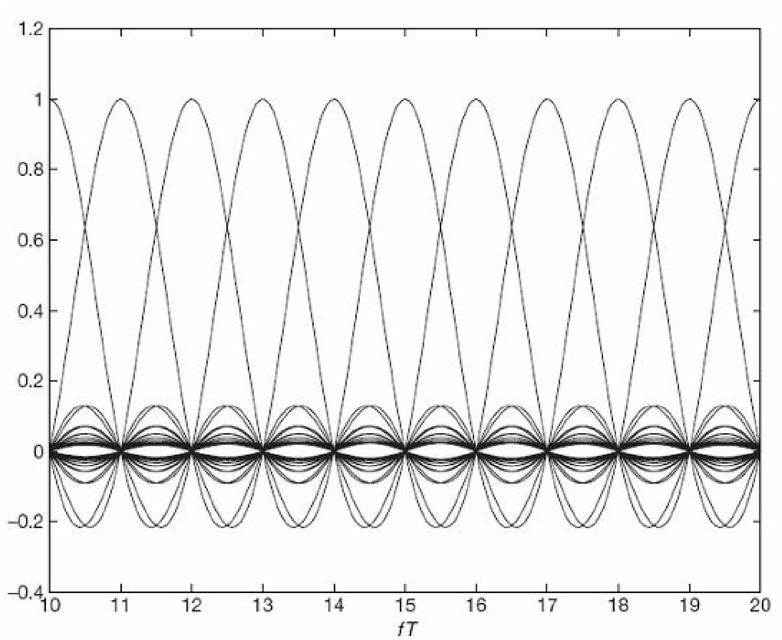


Figure iv Orthogonal frequency spectra of OFDM [5]

The circular convolution used above is faked by adding a specific prefix, the cyclic prefix, onto the transmitted signal. The presence of this cyclic prefix allows for the DFT to be taken thereby producing orthogonal subcarriers without the use of guard intervals. This has the effect of removing ICI under ideal sampling conditions.

2.2.2 Cyclic Prefix

The key to a practical OFDM system is through the use of the FFT algorithm. However in order to use FFT/IFFT to create an ISI & ICI free channel, the channel must appear to provide a circular convolution as seen in Equation 1.6. Adding a cyclic prefix to the transmitted signal to simulate a circular convolution helps accomplish this, as shown in Figure v. It is important to note that in the absence of a circular prefix the channel length of L_c would convolve with the OFDM received block of length L_r. This length is naturally increased by a factor of N as the number of subcarriers is also increased. However in this case Equation 1.3 leads to the first L_c samples of the OFDM block being distorted. The cyclic prefix however allows us to simulate a use of a circular convolution by using the last L_v samples of the OFDM block and placing it at the prefix, where $L_v > L_c$. Thus, by mimicking a circular convolution, a cyclic prefix that is at least as long as the channel duration allows the channel output y[n] to be decomposed into a simple multiplication of the order of Equation 1.6. Hence ISI and ICI effects are altogether mitigated [1].

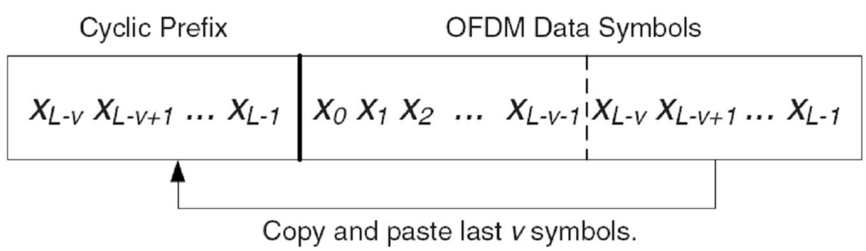


Figure v The OFDM Cyclic Prefix [1]

2.2.3 Frequency Equalization

In order for the received symbols to be estimated the complex gain (amplitude and phase) of each subcarrier must be known. This is because Equation 1.6 translated into the time domain results in the convolution of the OFDM transmitted block with the channel frequency response. Hence each subcarrier experiences amplitude and phase distortion due to the channel. This can be removed by first performing an FFT, converting back into frequency domain, and then by estimating the data symbols using a one-tap frequency domain equalizer, or FEQ:

$$\hat{X}_m = Y_m x H_m^* \tag{1.9}$$

Where H^*_m, is the complex conjugate of the channel frequency response at the subcarrier frequency. The FEQ hence corrects the phase and equalizes the amplitude before the decision device in the OFDM system [1].

2.2.4 OFDM Block Model

Presented below is the overall block model for the SISO OFDM system.

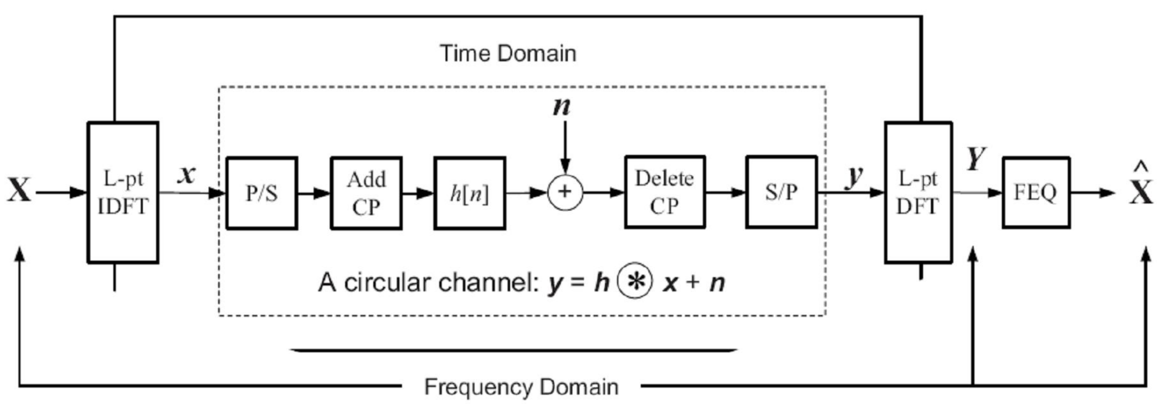


Figure vi OFDM system block in vector notation [1]

The flow of the OFDM system naturally follows from the logic of the system model in the diagram above. Note that L above refers to N to match the terminology used in this thesis. It can be described as follows [1]:

1. The first step involves dividing the input signal of bandwidth B into N narrowband subcarriers each of bandwidth B/N. Hence each subcarrier experiences only flat fading and ISI is completely mitigated as long as the length of the cyclic prefix (L_v) exceeds the length of the delay spread in the channel (L_c). The L (or N) subcarriers for a given OFDM symbol are represented by the vector **x** above. The IFFT (Equation 1.8) operation is used to modulate N independent narrowband signals from just a single wideband radio.
2. A cyclic prefix of length L_v is appended after the IFFT operation. The resulting N + L_v symbols are sent through the channel serially. This has the effect of suppressing ISI in the orthogonal subcarriers decomposed by the IFFT.
3. The cyclic prefix is then discarded at the receiver end and the N received symbols are demodulated via the FFT operation (Equation 1.7). This results in N data symbols each of the form $\widehat{X}_m = X_m H_m + N_m$ for the mth subcarrier where N_m is the noise.
4. Finally each subcarrier is equalized via FEQ, using Equation 1.9 producing an estimate of the symbol vector that is then sent to a decision device.

2.3 Advantages & Disadvantages of OFDM

Despite their flexibility and ease of use OFDM systems come with their own vices. This section discusses the differences between OFDM systems and traditional Single Carrier Systems.

2.3.1 Advantages of OFDM Compared to the Single Carrier System

Naturally the use of OFDM has many advantages over single carrier systems. The natural MMS structure of OFDM allows for each subcarrier to operate in a narrow frequency band experiencing only flat fading while the entire OFDM block may suffer from frequency selective fading. Furthermore the use of cyclic prefix (of length greater the channel delay multipath spread) helps to completely mitigate ISI and ICI to a level that cannot be matched by single carrier systems.

Furthermore due to spectral overlapping OFDM systems are also much more bandwidth conservative since the FFT/IFFT operations ensure that the subcarriers do not interfere with each other. The equalization is also much simpler compared to single carrier systems. All of this allows for OFDM systems to be more robust in multipath environments than single carrier systems.

2.3.2 Disadvantages of OFDM Compared to the Single Carrier System

A few drawbacks to OFDM systems are:

1. *Sensitivity to Phase Noise (PHN):* OFDM systems suffer significantly from phase noise created by band conversion oscillators used at the transmitter and receiver ends. These oscillators often suffer from jitter, which is essentially a random phase disturbance in the oscillator. Since an OFDM system has N subcarriers, each of which has it's own phase noise due to the oscillator, the phase noise effects in an OFDM system are significantly higher than in a single carrier system.
2. *Carrier Frequency Offset (CFO):* Carrier frequency offset is a result of frequency synchronization errors between the transmitting and receiving oscillators. Frequency offset can also be a result of the Doppler spread experienced if one end of the OFDM transmitter/receiver pair is mobile. Due to FFT/IFFT operations OFDM systems are much more bandwidth efficient than other systems due to the orthogonal spectral overlapping characteristic of its subcarrier signals. This prevents ICI and under ideal sampling conditions results in ease of detection. However the price paid for this efficiency is directly related to the CFO, which is a result of the spectral overlap rather than isolation, as is the case in other MMS systems and single carrier systems. Slight mismatches in frequency at the receiver end due to CFO will quite easily lead to ICI. Hence OFDM systems are also more sensitive to ICI.
3. *Peak-To-Average Power Ratio (PARP):* OFDM signals also experience a higher PARP than single carrier systems. The reason for this is that in the time domain, a multicarrier signal is

the sum of many narrowband signals. At some instances the sum is large while at other times it's small, meaning the peak value of the signal is substantially larger than the average value. This drawback of OFDM reduces its efficiency and increases the cost of the RF power amplifier making the system more expensive [1]. The PARP problem of OFDM systems is not addressed in this thesis.

2.4 Summary

This chapter examined the basic principles of OFDM systems, having explored its various components such as the use of Fast Fourier Transforms and cyclic prefixes. The computational advantages of an OFDM system were also explored due to the ease of implementing the FFT/IFFT algorithms. Finally the chapter looked at the various advantages and disadvantages of OFDM systems compared to single carrier systems. In particular, the problem of Carrier Frequency Offset (CFO) and Phase Noise (PHN) was examined, both of which are of considerable concern to the performance of an OFDM system.

CHAPTER 3

Mitigating Carrier Frequency Offset and Phase Noise Effects

OFDM systems have been applied to a variety of application scenarios primarily because of their robustness in frequency selective fading channels. However, as was examined earlier, OFDM systems are still highly susceptible to inter carrier interference (ICI). This is due in large part to the various synchronization problems that may result in the presence of a carrier frequency offset (CFO) or phase noise (PHN) within the OFDM transmission. This chapter looks to further examine and model the effects of CFO and PHN. It also details the use of an optimal CFO, PHN and Channel Impulse Response (CIR) joint estimator that can be used to mitigate the synchronization problems encountered in OFDM systems.

3.1 Carrier Frequency Offset (CFO)

Orthogonality between subcarriers requires that the frequency separation between subcarriers is maintained consistently in order to avoid synchronization problems. Consider a subcarrier 'm' being modulated such that:

$$\theta_m(t) = e^{j(2\pi m\Delta f)t} \tag{1.10}$$

where m = 0… N-1, and Δf is the frequency separation between subcarriers. Using equation 1.2:

$$\int_o^{NTs} \theta_m(t)\theta_n^*(t)dt = \int_o^{NTs} e^{j(2\pi m\Delta f)t} e^{-j(2\pi n\Delta f)t} dt = NT\delta_{nm} \tag{1.11}$$

where the orthogonality condition is satisfied when,

$$\Delta f = \frac{1}{NT} \tag{1.12}$$

Hence for the OFDM system to experience zero ICI the strict condition of Equation 1.12 must be met [7]. Carrier frequency offset (CFO) however violates the orthogonality of sub-carriers by distorting Δf above. Clearly the effect of such a frequency offset amounts to ICI which is similar to ISI of a single carrier signal due to timing jitters. CFO can occur due to mismatch in the frequency of operation for the oscillators at the receiver and transmitter [1]. This offset, ξ, leads to a translation in the frequency separation, Δf, between subcarriers. CFO can also be caused by the relative movement of the transmitter/ receiver block with respect to each other. Due to the presence of CFO the orthogonal condition satisfied by Equation 1.12 is no longer valid; hence the FFT/ IFFT operations that ensue in the OFDM system translate this disparity into ICI at the receiver end. Sensitivity of OFDM systems to CFO is sometimes too constraining and therefore, in practise, the zero ICI condition is partly relaxed to provide more robustness under frequency offset [7]. The problem of frequency synchronization is best illustrated by Figure vii below:

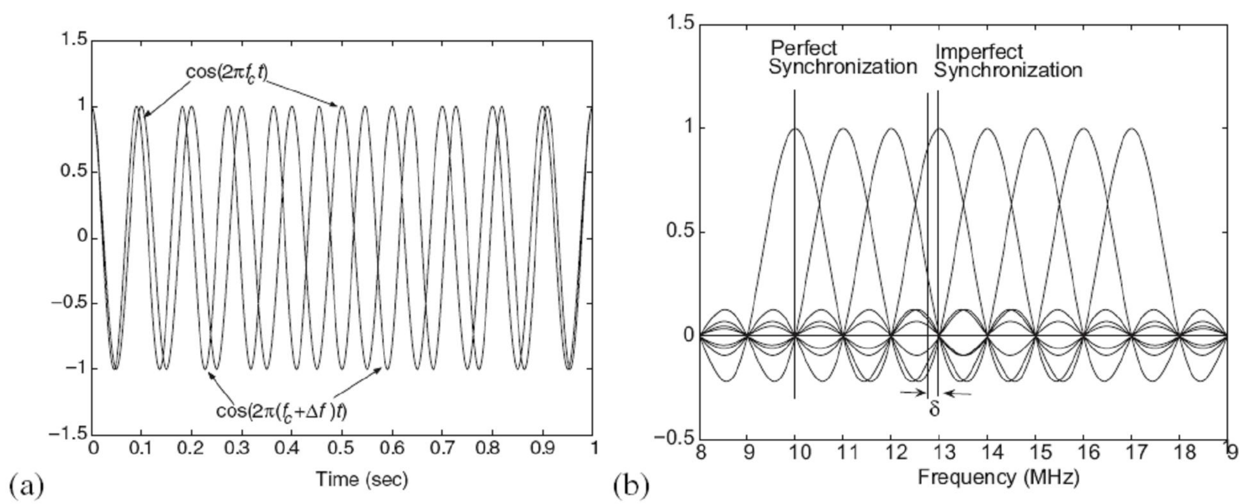


Figure vii CFO effects on an OFDM system [1]

3.1.1 *Carrier Frequency Offset Model*

The CFO model used for the purposes of simulations is derived directly from Equation 1.10. Consider a CFO, ξ, such that the frequency separation, Δf, is now: Δf + ξ. Hence equation 1.10 is now:

$$\theta_{m\xi}(t) = e^{j(2\pi m\Delta f + \xi)tT_s} = \theta_m(t)e^{j(2\pi m\xi)tT_s} \tag{1.13}$$

where $\theta_{m\xi}(t)$ is the signal with CFO and $\theta_m(t)$ is the signal without CFO. Hence the effect of frequency offset can be modelled by the equation below:

$$CFO(t) = e^{j(2\pi m\xi)tT_s} = e^{j(2\pi m\varepsilon)t} \Leftrightarrow CFO[m] = e^{j(2\pi m\xi)T_s} \tag{1.14}$$

where $\varepsilon = \xi T_s$ is the normalized CFO. In the simulations to follow the value of frequency offset is controlled by manipulating frequency offset ξ.

3.2 Phase Noise (PHN)

Timing jitter is a common phenomenon that plagues all OFDM and single carrier systems. Jitter is a time domain measure of the timing accuracy of the oscillator period. There are bound to be variations in the length of this period, which cause uncertainty at the next edge of the signal. This uncertainty is classified as time jitter. Phase noise is quite simply the frequency domain representation of this time jitter that results due to imperfections in the local oscillator [8], see Figure viii. Unlike channel impulse response (CIR), phase noise changes rapidly over the duration of an OFDM block and can hence be thought of as consisting of two components; the common phase rotation (CPR), which depends on the average PHN value over an OFDM symbol and has the same effect of all subcarriers, and random PHN, which induces ICI [4]. This paper follows the method developed in [4] and does not distinguish between these two types of phase noises.

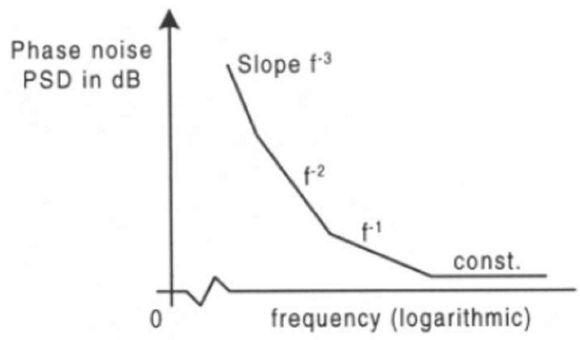


Figure viii Phase noise PSD for a typical oscillator [7]

Two different models of PHN are available. The first one models a free-running oscillator and assumes that the PHN process is nonstationary with a power that grows over time. This is known as the Wiener process. The second models an oscillator controlled by phase-locked loop (PLL) and approximates the PHN process as a zero-mean Gaussian process with finite power [4]. This thesis makes use of the Wiener model, which is explained in more detail below.

3.2.1 Wiener Phase Noise Model

The Wiener process (also known as Brownian motion) [8], is a nonstationary process with variance that grows over time. The process is modelled as the output of the voltage controller oscillator (VCO) by $\theta(t)$. The samples of $\theta(t)$ within the m^{th} OFDM symbol, θ_m, have a multivariate Gaussian prior distribution, where the samples are taken at a rate of N/T samples per second [3]. The Wiener phase noise is hence modelled by:

$$\theta(t) = \int_0^t \phi(\tau) d\tau \tag{1.15}$$

where $\phi(\tau)$ is a zero-mean stationary Gaussian process. The discrete-time samples of $\theta(t)$ form a random-walk process such that:

$$\theta_k = \theta_{k-1} + \phi_k \tag{1.16}$$

for k = 0…N-1, and $\theta_{-1} = 0$ due to perfect synchronization at the beginning of the OFDM symbol. The Gaussian-distributed PHN vector $\theta = [\theta_0, ..., \theta_{N-1}]^T$ also has a covariance matrix [4]:

$$\Phi = \alpha_\phi^2 \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & 2 & \cdots & 2 \\ \vdots & \vdots & \ddots & \\ 1 & 2 & & N \end{bmatrix} \quad (1.17)$$

In the simulations to follow α will be varied to represent different severities of phase noise. Finally, the model employed for phase noise experienced by the mth subcarrier is as follows:

$$Phn(t) = e^{jm\theta t} \Leftrightarrow Phn[m] = e^{j\theta_m} \quad (1.18)$$

3.3 Joint Carrier Frequency Offset, Phase Noise, Channel Impulse Response Estimator (JCPCE)

The Joint Carrier Frequency Offset, Phase Noise, Channel Impulse Estimator (JCPCE), discussed in [3]-[4], aims to tackle the channel estimation problem when CFO and PHN are present from a maximum likelihood stand-point [4]. Using special features of the likelihood function a joint estimation scheme is achievable that decouples the dependence of the CFO, PHN and CIR on each other. From the previous sections it is easy to see the motivation for such an algorithm that can effectively reduce CFO and PHN in an OFDM system. In the sections to follow, a received signal model for the received OFDM symbol is first developed followed by the workings of the JCPCE algorithm, which is able to perform accurate channel estimation even with CFO and PHN impairment [4].

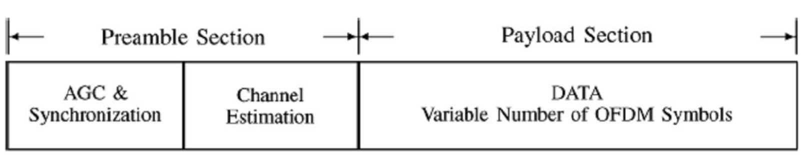


Figure ix OFDM packet structure

3.3.1 Signal Model

In order to use the JCPCE algorithm devised in [3]-[4] the received signal must first be modelled. In the model the channel impulse response (CIR) is assumed to be constant during the initial preambles (for synchronization and channel estimation) as well as the variable-length payload that follows (see Figure ix above). The received signal of an OFDM symbol, within the training period, can hence be written as [4]:

$$r_n = \frac{1}{\sqrt{N}} e^{j(\theta_n + 2\pi\varepsilon n/N)} \sum_{k=0}^{N-1} h_k d_k e^{j2\pi nk/N} + \eta_n \tag{1.19}$$

for n = 0...N-1, where h_k is the channel frequency response for subcarrier k, d_k is the transmitted data symbol and η_n is the Gaussian noise. Note the CFO and PHN term in the above equation is represented by $e^{j(\theta_n + 2\pi\varepsilon n/N)}$. The matrix form of this equation, also shown in [3]-[4], is given by:

$$r = EPF^H Hd + n \tag{1.20}$$

where $F \in C^{NxN}$ is the DFT matrix of form Equation 1.6 in each cell,

d is the data vector, n is the noise vector,

$P = diag([e^{j\theta_0},...,e^{j\theta_{N-1}}]^T)$ is the PHN matrix,

$E = diag([1, e^{j2\pi\varepsilon/N},...,e^{j2\pi l(N-1)\varepsilon/N}]^T)$ is the CFO matrix

$H = diag(h) = diag([h_0,...,h_{N-1}]^T)$ is the channel matrix

This is further simplified to give the following form for the received signal vector:

$$r = EPF^H DWg + n \tag{1.21}$$

where $D = diag(d)$ is the data vector,

$g = [g_0,...,g_{L-1}]^T$ is the CIR, where L is the channel length and is related to the channel frequency response by $h = Wg$

W is the partition of the F matrix given by $F = [W \mid V], W \in C^{NxL}, V \in C^{NxN-L}$

In this thesis we assume timing synchronization thereby assuming that the OFDM transmits only in training mode, hence the data vector, **d**, is always known. It is also worth noting that in the absence of PHN, **P**, and CFO, **E**, the CIR, **g**, can be completely, optimally estimated given the training symbol, **d**, and the received signal, **r**. Figure x shows how these system parameters come into play in the OFDM system [4].

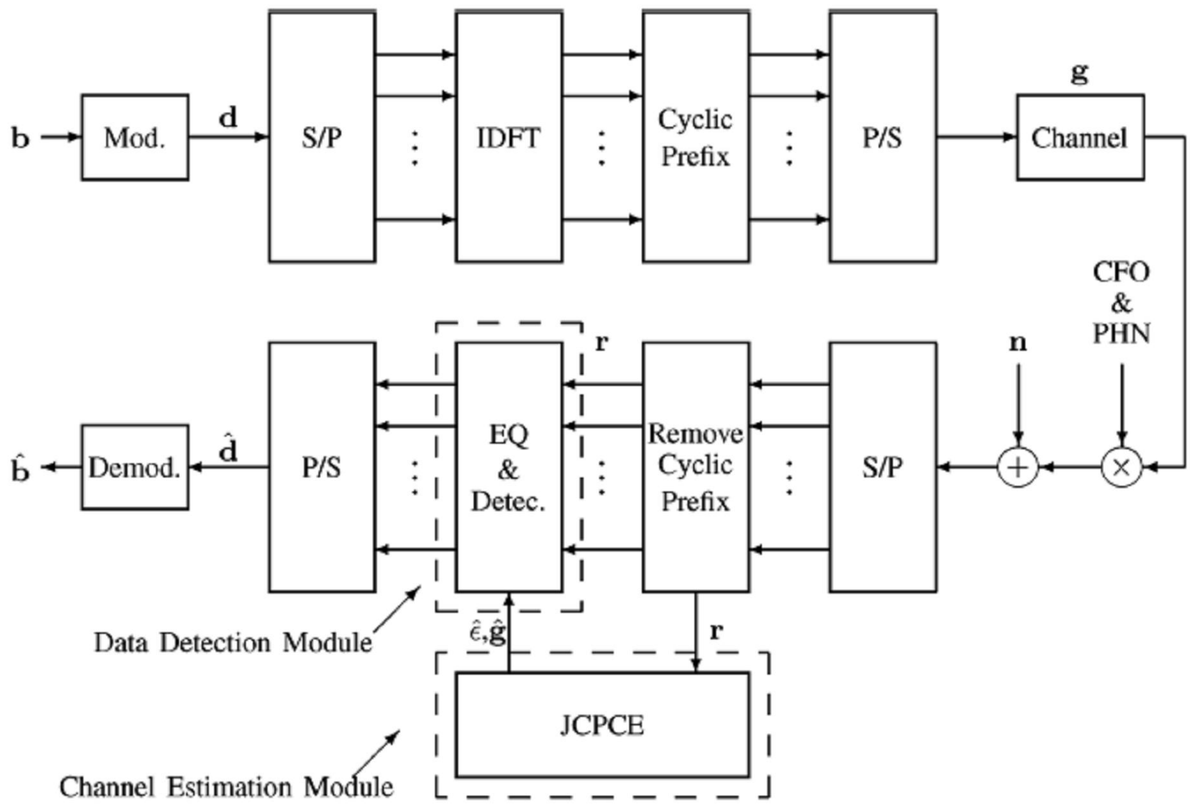


Figure x OFDM System with JCPCE [4]

3.3.2 Joint CFO/ PHN/ CIR Estimation

The complete derivation for the JCPCE algorithm can be found in [3]-[4]. This section highlights the main equations responsible for the modelling of JCPCE and provides an explanation of how they come to bear.

Equation 1.21 shows that the optimal estimates for E, P and g are coupled. However using the complete negative log-likelihood function (shown below), which is proportional to a posteriori distribution we are able to find estimates for the CIR, $\hat{g}$, normalized CFO, $\hat{\varepsilon}$ and PHN $\hat{\theta}$. No prior knowledge of $\hat{g}$, $\hat{\varepsilon}$ are assumed, however $\hat{\theta}$ is assumed known through prior statistical knowledge. The complete negative log-likelihood function is given by:

$$\begin{aligned} L(\varepsilon,\theta,g) &= -\log p(r\,|\,\varepsilon,\theta,g) - \log p(\theta) \\ &= \frac{1}{2\sigma^2}(r - EPF^H DWg)^H (r - EPF^H DWg) + \frac{1}{2}\theta^T \Phi^{-1}\theta \end{aligned} \quad (1.22)$$

where Φ is the covariance matrix for the Wiener process. Hence the objective of the JCPCE algorithm can best be stated as finding the optimal CFO, PHN and CIR estimates that satisfy Equation 1.23 below.

$$(\hat{\varepsilon},\hat{\theta},\hat{g}) = \arg\min_{\varepsilon,\theta,g} L(\varepsilon,\theta,g) \quad (1.23)$$

3.3.3 JCPCE Algorithm

The algorithm developed in [3]-[4] is shown below. The interested reader is encouraged to see [3]-[4] to analyze the complete derivation of the algorithm.

Table i Joint CFO/PHN/CIR Estimator Algorithm

JCPCE Algorithm		
Step 1: CFO	$\hat{\varepsilon} = \arg\min_{\varepsilon} 1^T ECC^H E^H 1 - 1^T \operatorname{Im}(ECC^H E^H)^T$ $x[\operatorname{Re}(ECC^H E^H) + 2\sigma^2\rho^2\Phi^{-1}]^{-1} \operatorname{Im}(ECC^H E^H)1$	(1.24)
	$\hat{E} = diag([1, e^{j2\pi\hat{\varepsilon}/N}, \ldots, e^{j2\pi l(N-1)\hat{\varepsilon}/N}]^T)$	(1.25)
Step 2: PHN	$\hat{\varepsilon} = [\operatorname{Re}(\hat{E}CC^H\hat{E}^H) + 2\sigma^2\rho^2\Phi^{-1}]^{-1} \operatorname{Im}(\hat{E}CC^H\hat{E}^H)1;$	(1.26)
	$\hat{P} = diag([e^{j\hat{\theta}_0}, \ldots, e^{j\hat{\theta}_{N-1}}]^T)$	(1.27)
Step 3: CIR	$\hat{g} = (2\rho^2)^{-1} W^H D^H F \hat{P}^H \hat{E}^H r$	(1.28)

where $D^H D = 2p^2 I$ and $C = R^H F^H DV$

3.4 Summary

This chapter provided further insight on the origins and mechanics that govern the carrier frequency offset and phase noise effects that are detrimental to OFDM systems. The Joint CFO/PHN/CIR Estimator was also explored, whereby prior knowledge of phase noise (through the evaluation of a completely likelihood function) is able to provide estimates for the PHN, CFO and CIR of an OFDM system. These estimates allow for the receiver to combat the phase noise and carrier frequency offset effects and to better model the channel effects upon equalization.

CHAPTER 4

Numerical Analysis

This chapter provides the numerical analysis and mathematical simulations [9] to illustrate the effects of the carrier frequency offset and phase noise on the SISO OFDM system. CFO and PHN effects are observed for various values and for various different numbers of subcarriers. Their combined effects are also analyzed. Finally the JCPCE algorithm is also put to the test for varying levels of PHN and CFO.

The following model details are important to note:

- The channel is modelled to experience Rayleigh fading.
- Each subcarrier is modulated using BPSK.
- The SISO OFDM system is assumed to be line-of-sight (LOS).
- A SNR loss of less than 1 dB at 10^{-2} BER is considered desirable [1].
- CFO is parameterized by using ξ, and not the normalized CFO, ε. The two are related by $\varepsilon = \xi T_s$. ξ is described as a percentage of the carrier frequency spacing Δf.
- PHN is parameterized by the standard deviation α.

The system parameters for the simulations are shown in the tables below:

Table ii System parameters for 4.1-4.3

Parameters	**Values**
OFDM Bandwidth, $B=1/T_s$	20 MHz
Number of Subcarriers, N	64
Subcarrier spacing, Δf	312.50 KHz
Symbol Length	64,000

Table iii System parameters for 4.4

Parameters	Values
OFDM Bandwidth, $B=1/T_s$	20 MHz
Number of Subcarriers, N	64
Subcarrier spacing, Δf	312.50 KHz
Symbol Length	1,280

It is also worth noting that in [3] the following parameters are used:

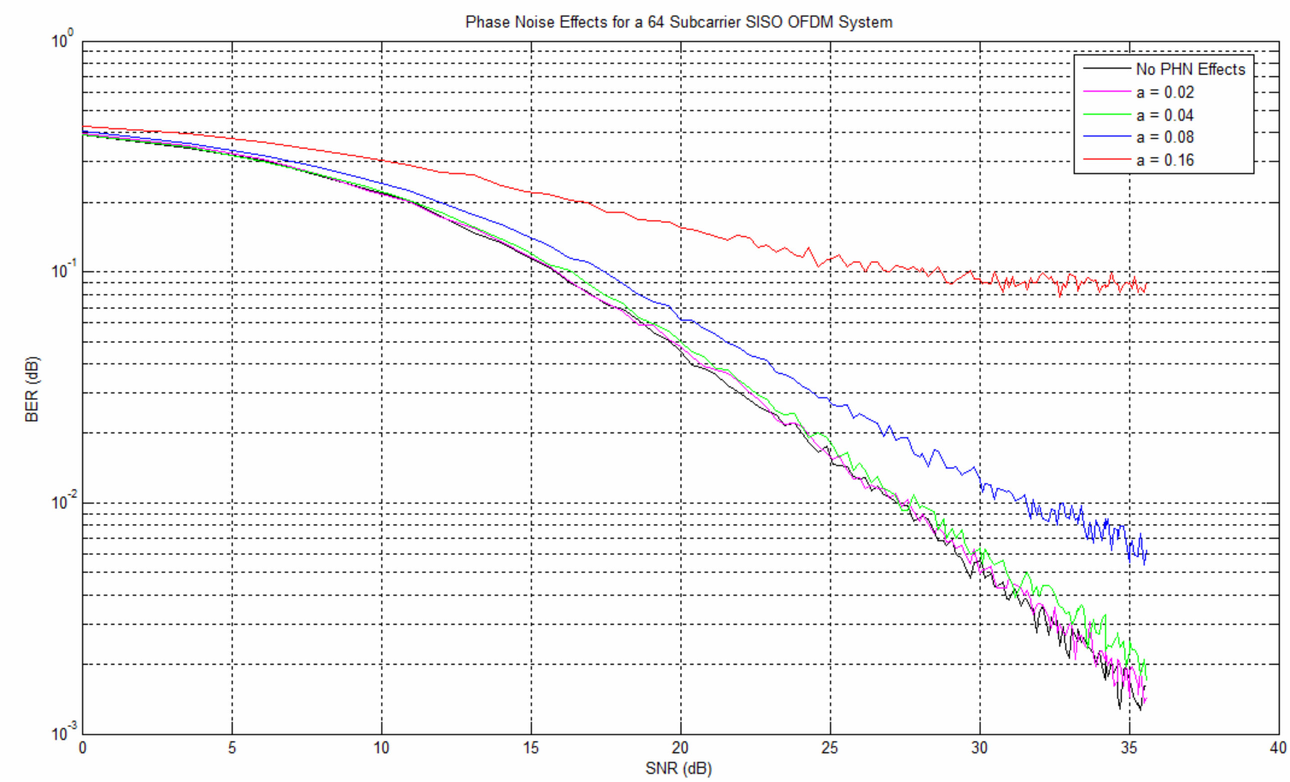
Table iv System parameters for [3]

Parameters	Values
OFDM Bandwidth, $B=1/T_s$	20 MHz
Number of Subcarriers, N	64
Subcarrier spacing, Δf	312.50 KHz
Modulation Scheme	QPSK
Wiener Phase Noise, α	0.01rad, 0.6^o

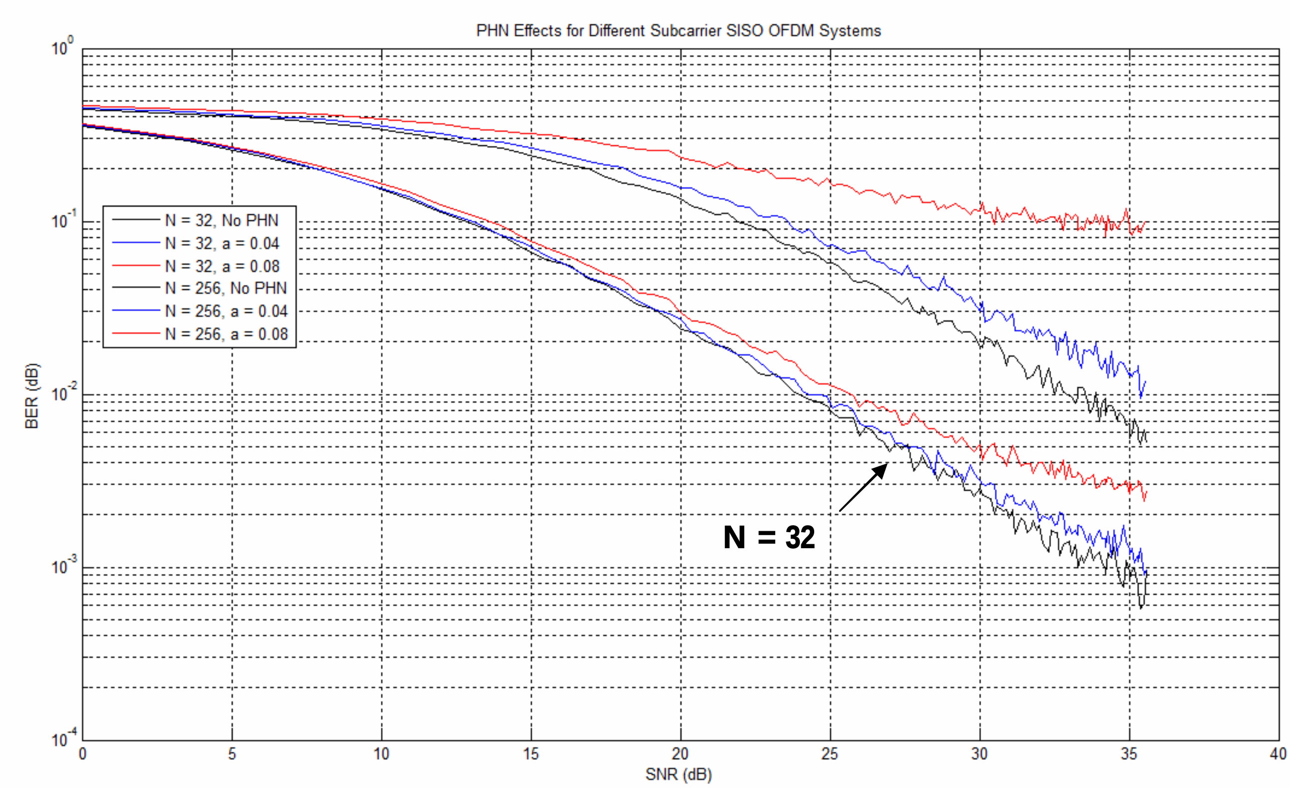
4.1 Impact of Phase Noise

The Wiener Phase Noise is modelled as a random walk process using Equation 1.18. The sections below look at the impact of phase noise on the SISO OFDM system when the Wiener PHN is varied by increasing its variance α^2, along with the impact that varying the number of subcarriers in the OFDM system has on the impact of phase noise. For all models a high symbol rate (as mentioned in Table ii) is used. All α values are in radians.

4.1.1 Impact with Varying Levels of Phase Noise



4.1.2 Impact with Varying Levels of Subcarriers



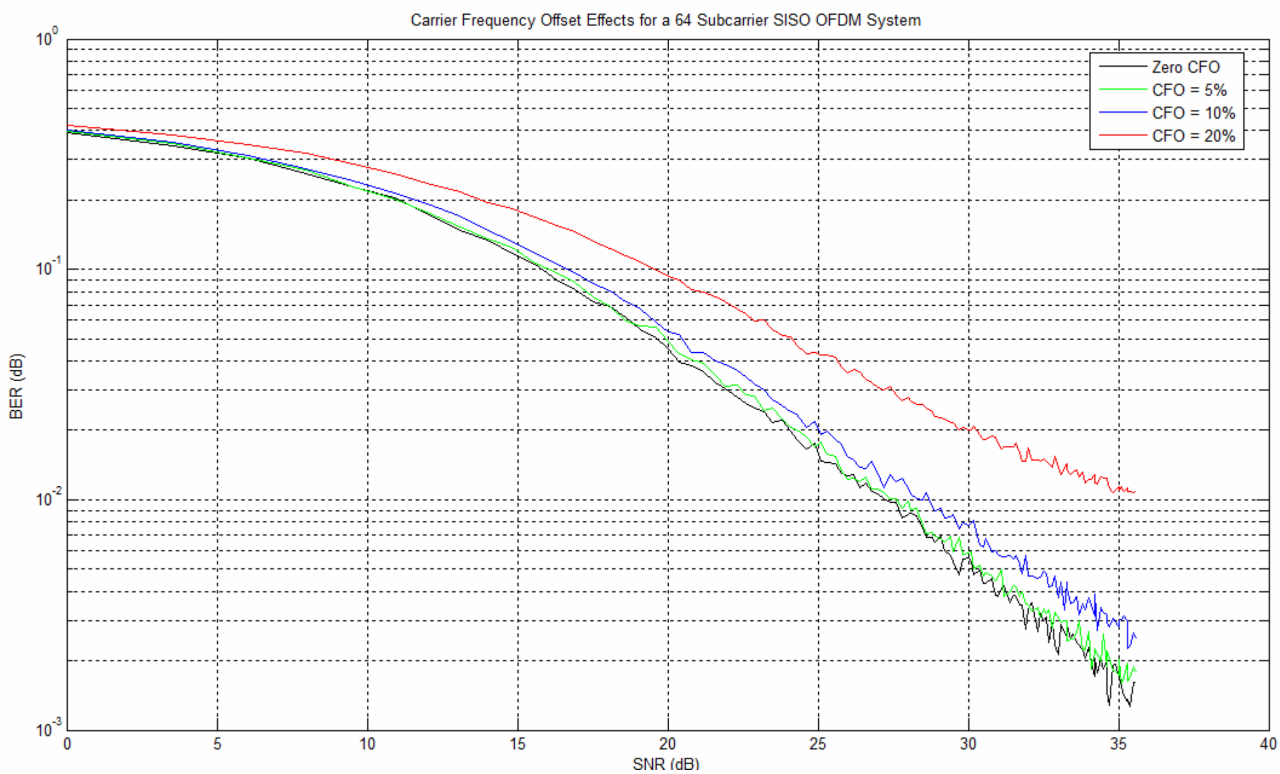
4.1.3 Summary of Results

It is quite obvious from the graphs above that an increase in the PHN results in system degradation. In particular we see that the effects of PHN distortion are more severe as the number of subcarriers increase. This is to be expected as an increase in the number of subcarriers simply means that the effective influence of phase noise has now increased. In all cases above PHN also increasingly worsens the performance of the system with increasing SNR. If a SNR of 1dB at a BER of 10^{-2} is considered acceptable [1], then it is observable that PHN corresponding to $\alpha \leq 0.04$ is desirable. This holds true for the 32 and 64 subcarrier OFDM systems but not for the 256 subcarrier system.

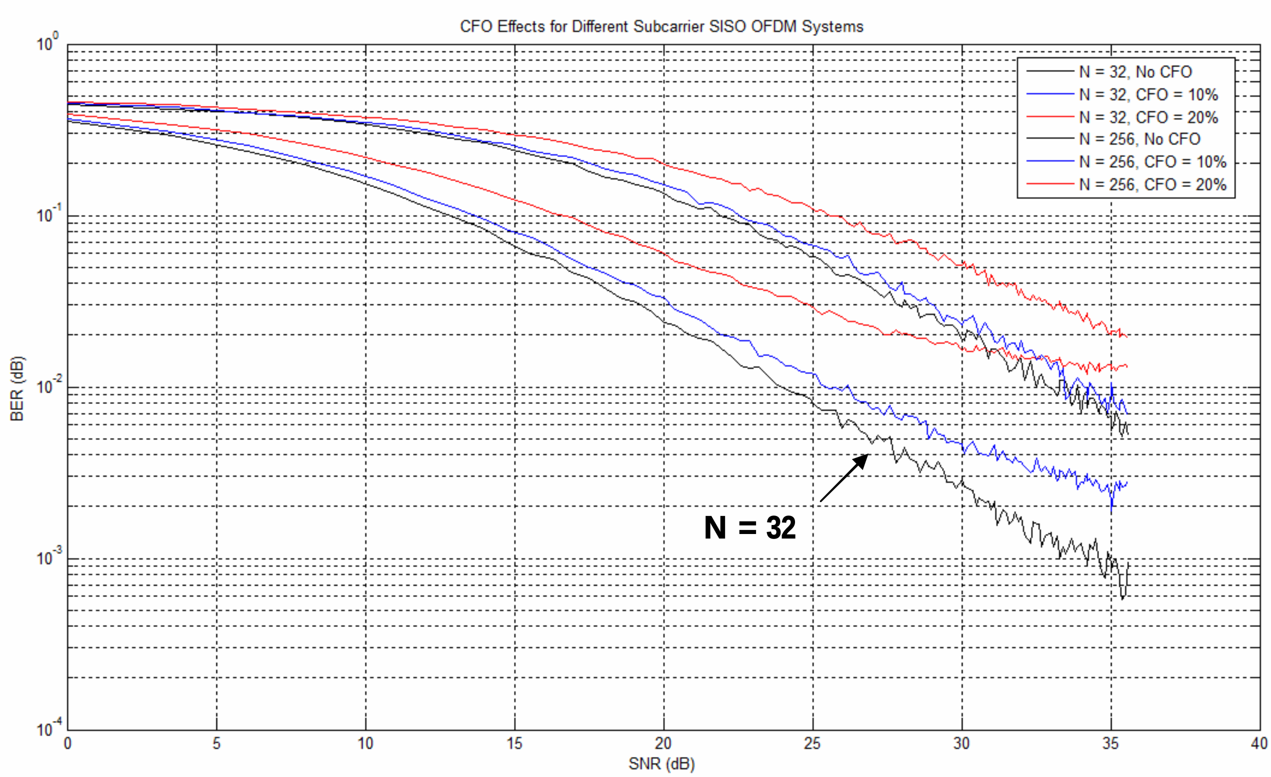
4.2 Impact of Carrier Frequency Offset

The Carrier Frequency Offset is modelled using Equation 1.14. The sections below look at the impact of carrier frequency offset on the SISO OFDM system when the ξ is varied, along with the impact that varying the number of subcarriers in the OFDM system has on the impact of frequency synchronization. It is worth mentioning again that CFO is often the result of frequency synchronization mismatches between the transmitter and receiver in the OFDM system. It can also be a result of the Doppler spread in a mobile system. In either case the impact of CFO makes the OFDM system highly sensitive to ICI and severely hampers performance. For all models a high symbol rate (as mentioned in Table ii) is used. All ξ values are expressed as a percentage of the carrier frequency separation Δf and are related to the normalized CFO, ε, by $\varepsilon = \xi T_s$. Hence for example a CFO value of 10% is translated into a frequency offset of 31.25 KHz.

4.2.1 Impact with Varying Levels of Carrier Frequency Offset



4.2.2 Impact with Varying Levels of Subcarriers

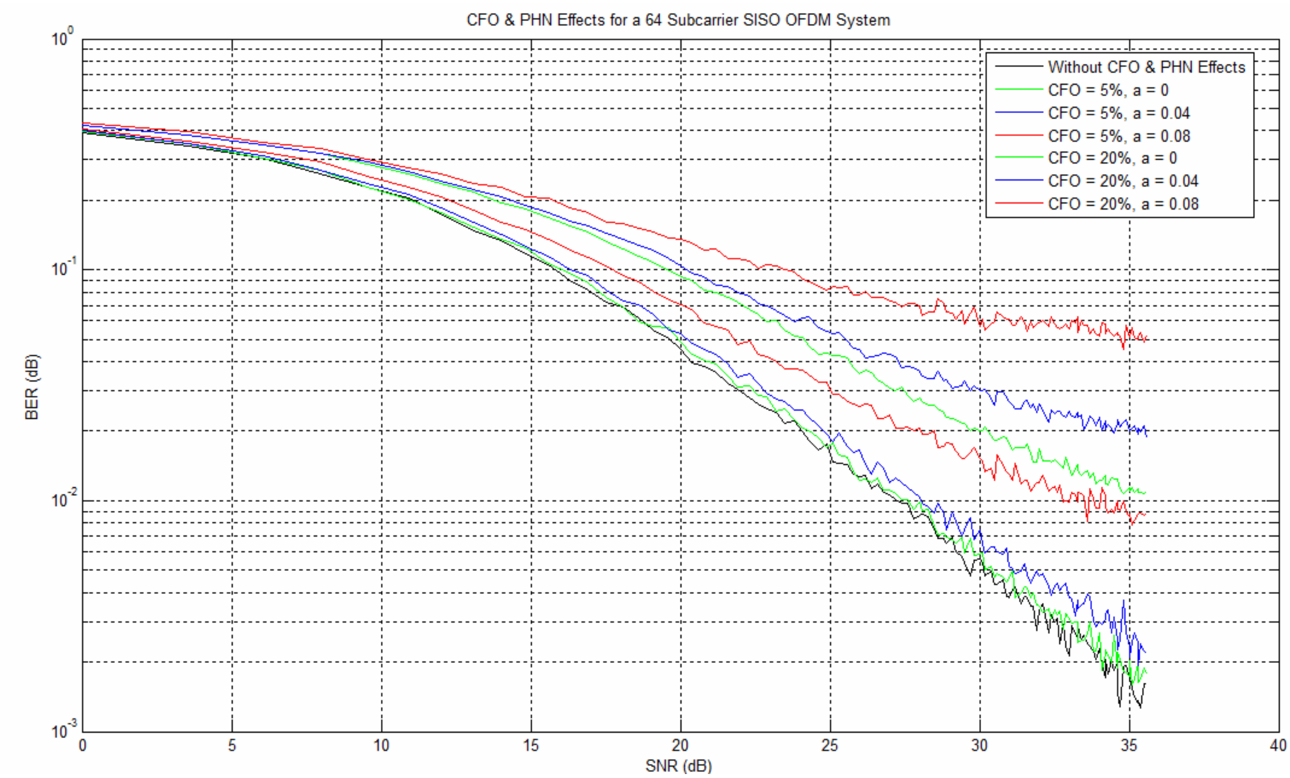


4.2.3 Summary of Results

It is apparent from the graphs above that increasing the CFO results in system degradation. Once again the impact of CFO on the system is seen to increase with increasing SNR. Furthermore it is observable that a CFO of 5% is the only possible offset value that allows 1dB SNR degradation at 10^{-2} BER. It is also interesting to note that unlike with PHN where increasing the number of subcarriers worsened the impact of PHN, the reverse effect is observe for CFO impacted systems. The impact of CFO on a 32 subcarrier OFDM system is considerably larger than that on a 256 subcarrier system. This follows naturally from the fact that the 32 subcarrier system divided the existing signal bandwidth, B into 32 subchannels, thereby increasing the frequency spacing between adjacent channels (compared to the 64 subcarrier system). In the case for 256 subcarriers, the frequency spacing is naturally reduced (by a factor of 8). Hence the impact of the frequency offset is likewise reduced by a similar factor, resulting in less degradation for high subcarrier OFDM systems.

4.3 Impact of CFO & PHN

To simulate the effects of the real world both Carrier Frequency Offset and Phase Noise effects were included in the SISO OFDM system. A CFO of 5% is first examined since it was revealed to be the optimum carrier offset acceptable in a reliable, practical system. Different levels of PHN are then compared to see the effect it has on such a system. A CFO of 20% is also examined to get a comparative sense of these impacts with different permutations of CFO and PHN. Once again the model uses a high symbol rate (as mentioned in Table ii). All CFO ξ values are expressed as a percentage of the carrier frequency separation Δf. While all phase noise α values are expressed in radians.



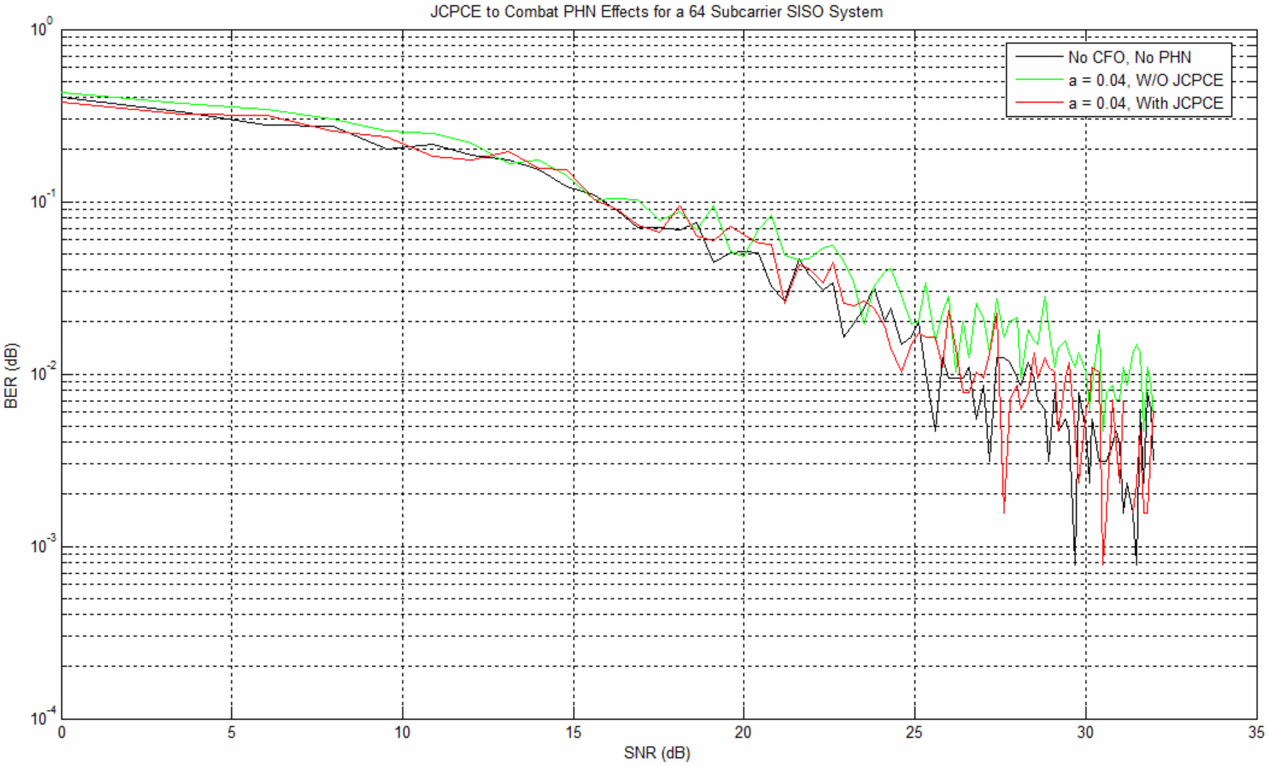
4.3.1 Summary of Results

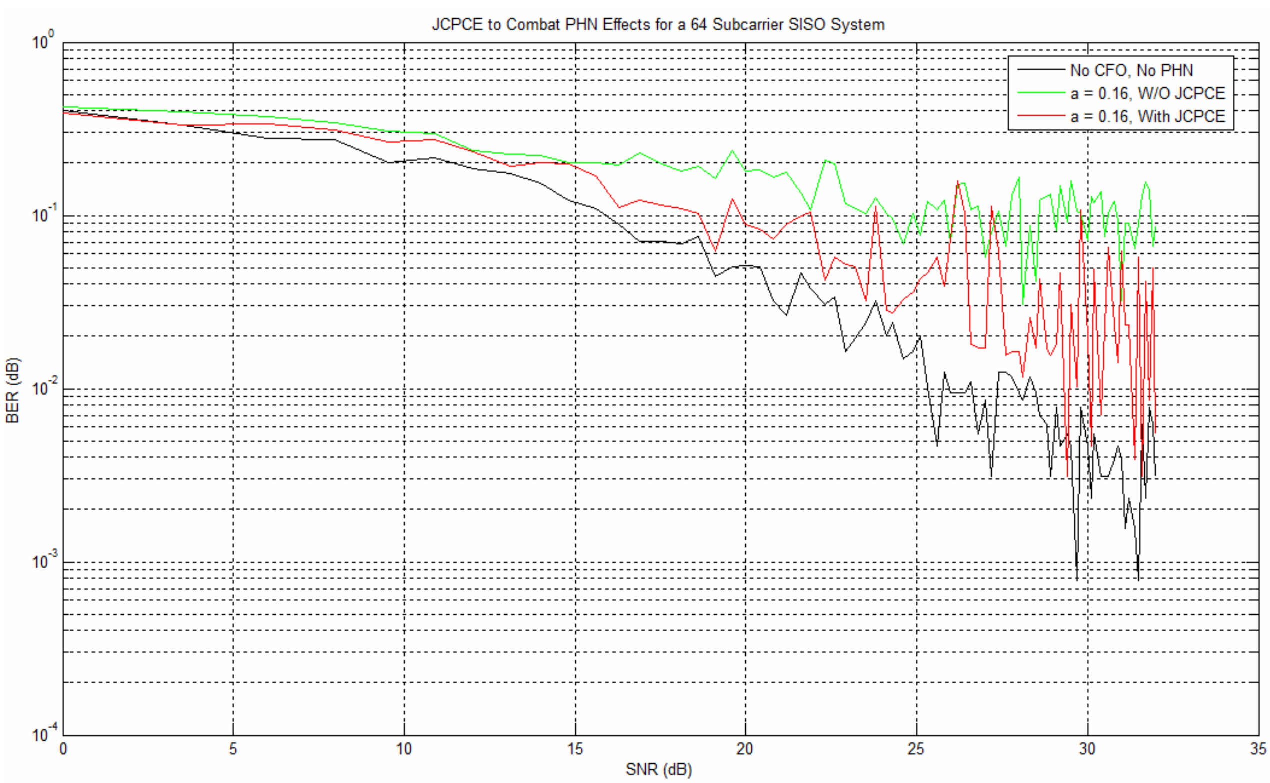
It is observable from the results above that increasing the impact of CFO has a detrimental effect on the integrity of the OFDM system. Once again we observe that a CFO value of 5% coupled with $\alpha \leq 0.04$ allows for a practically desirable system. It is also easily discernible that the CFO and PHN effects have an additive effect on decreasing the integrity of the SISO OFDM system.

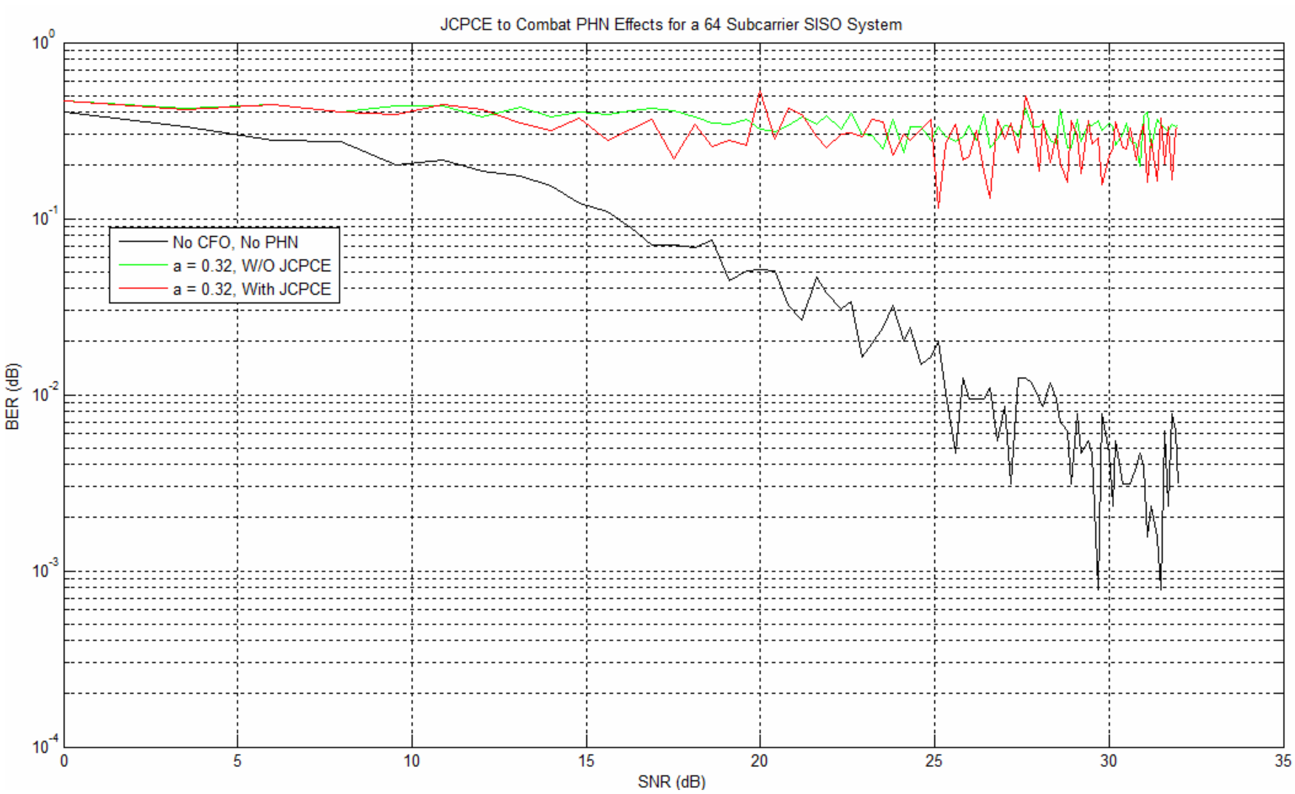
4.4 Impact of JCPCE

In this section we analyze the impact of the Joint CFO/PHN/CIR Estimation algorithm in combating phase noise and carrier frequency offset effects. The impact of the JCPCE algorithm is seen on the individual effects of PHN and CFO and also on their combined impact. A CFO system value of 10% and $\alpha = 0.04$ was chosen to simulate real world values. The system parameters are the same as those listed in Table iii.

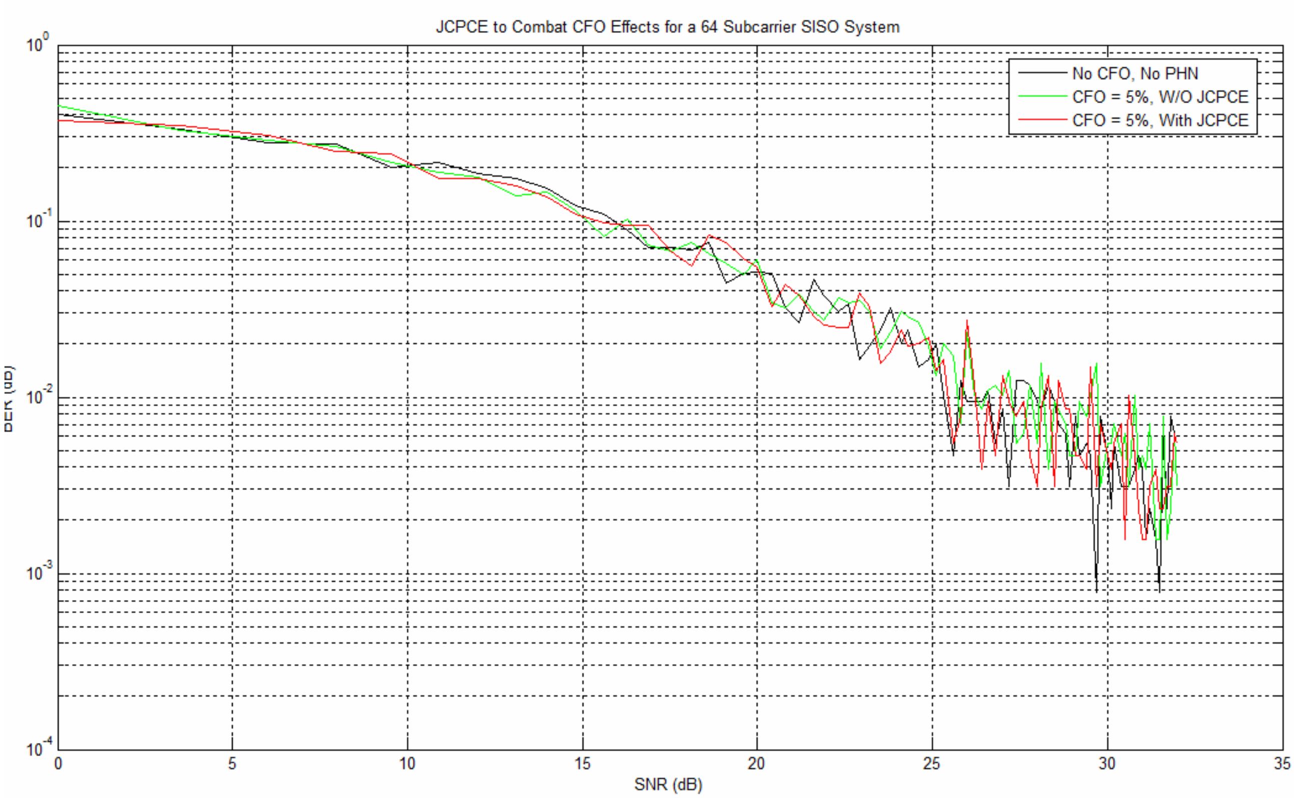
4.4.1 Impact of JCPCE on Varying Phase Noise

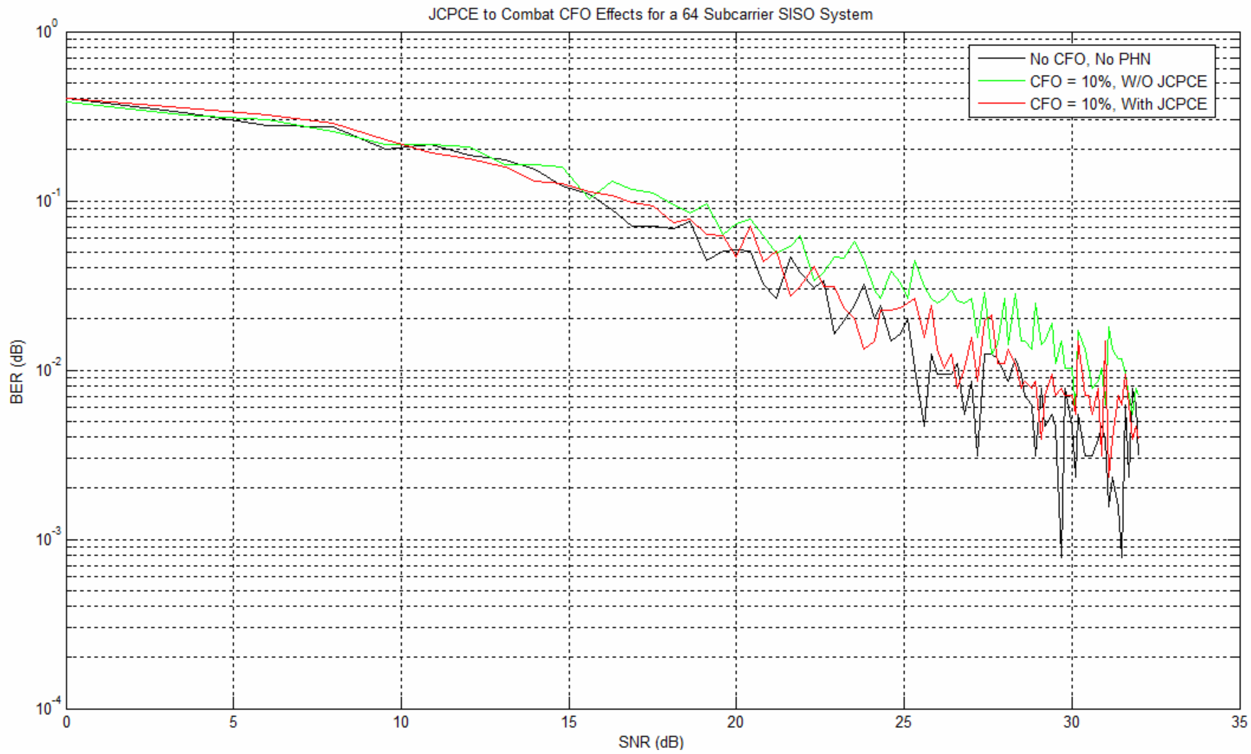


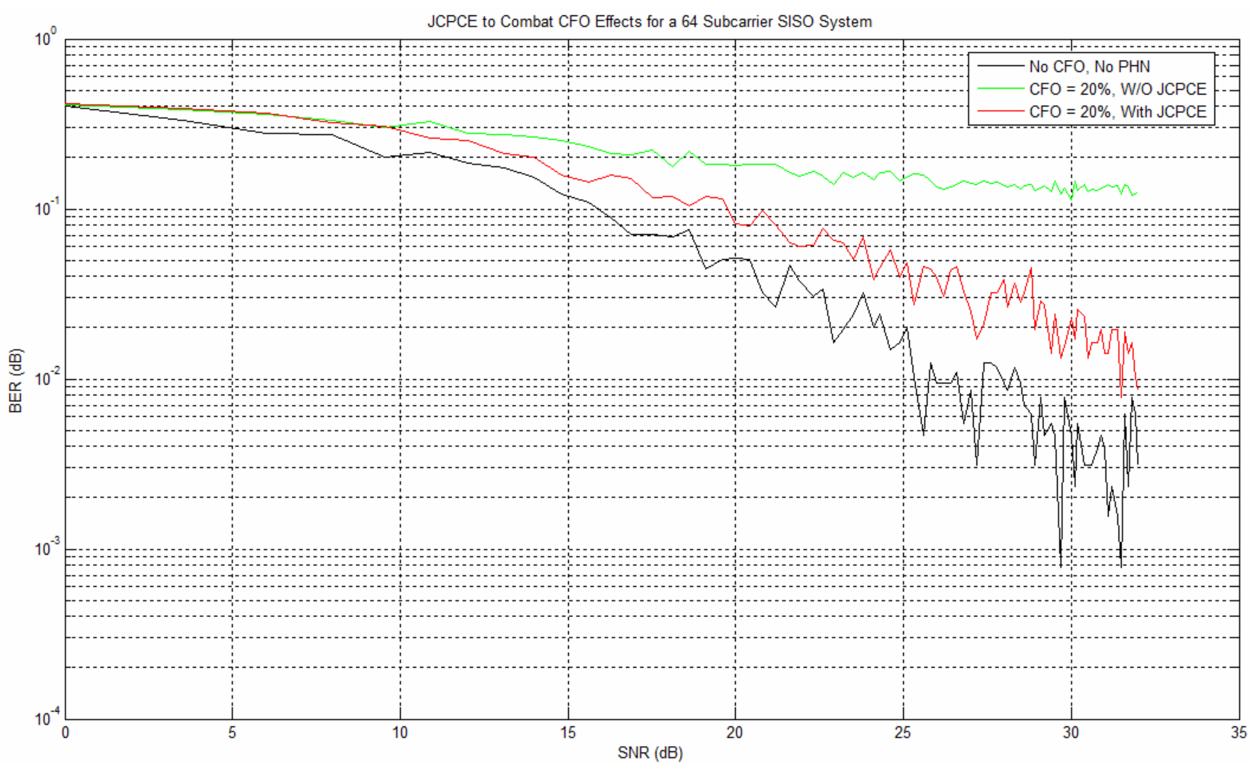




4.4.2 Impact of JCPCE on Varying Carrier Frequency Offset




JCPCE to Combat CFO Effects for a 64 Subcarrier SISO System
No CFO, No PHN
CFO = 10%, W/O JCPCE
CFO = 10%, With JCPCE
BER (dB)
SNR (dB)
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0
5
10
15
20
25
30
35


JCPCE to Combat CFO Effects for a 64 Subcarrier SISO System
No CFO, No PHN
CFO = 20%, W/O JCPCE
CFO = 20%, With JCPCE
BER (dB)
SNR (dB)
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0
5
10
15
20
25
30
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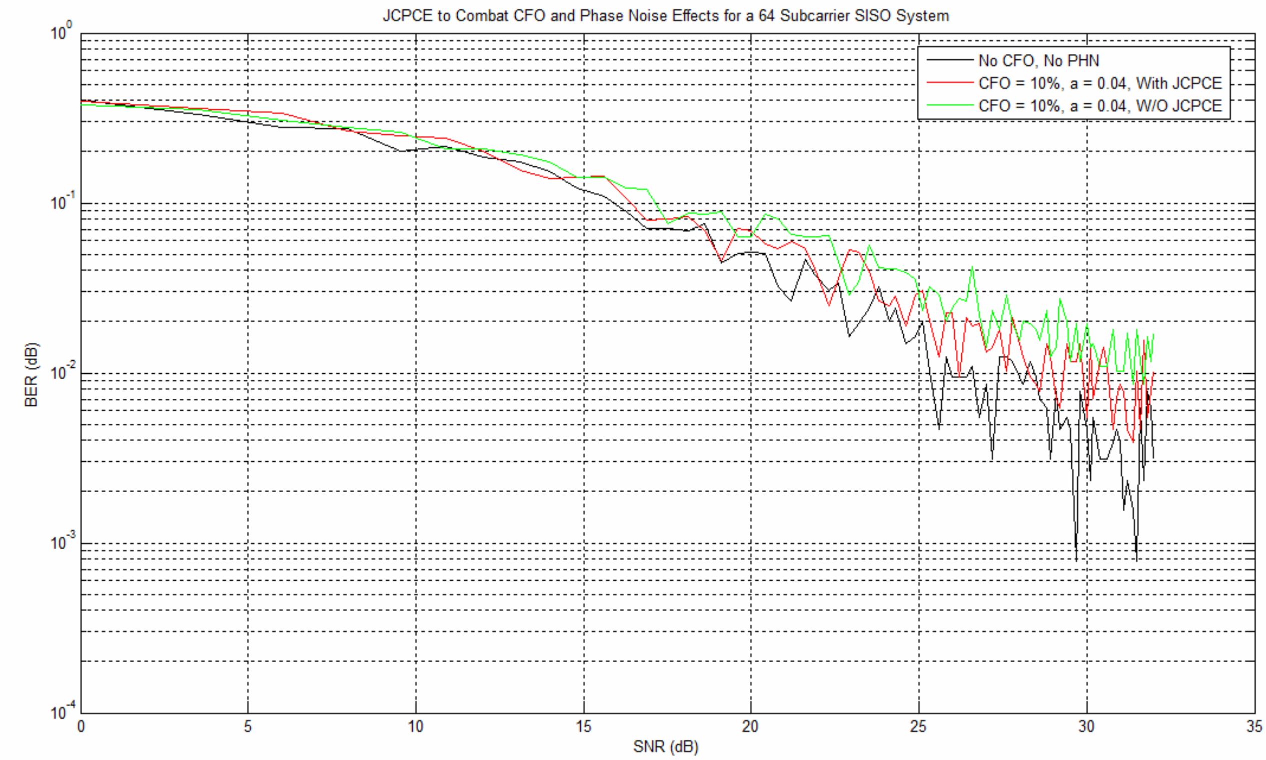
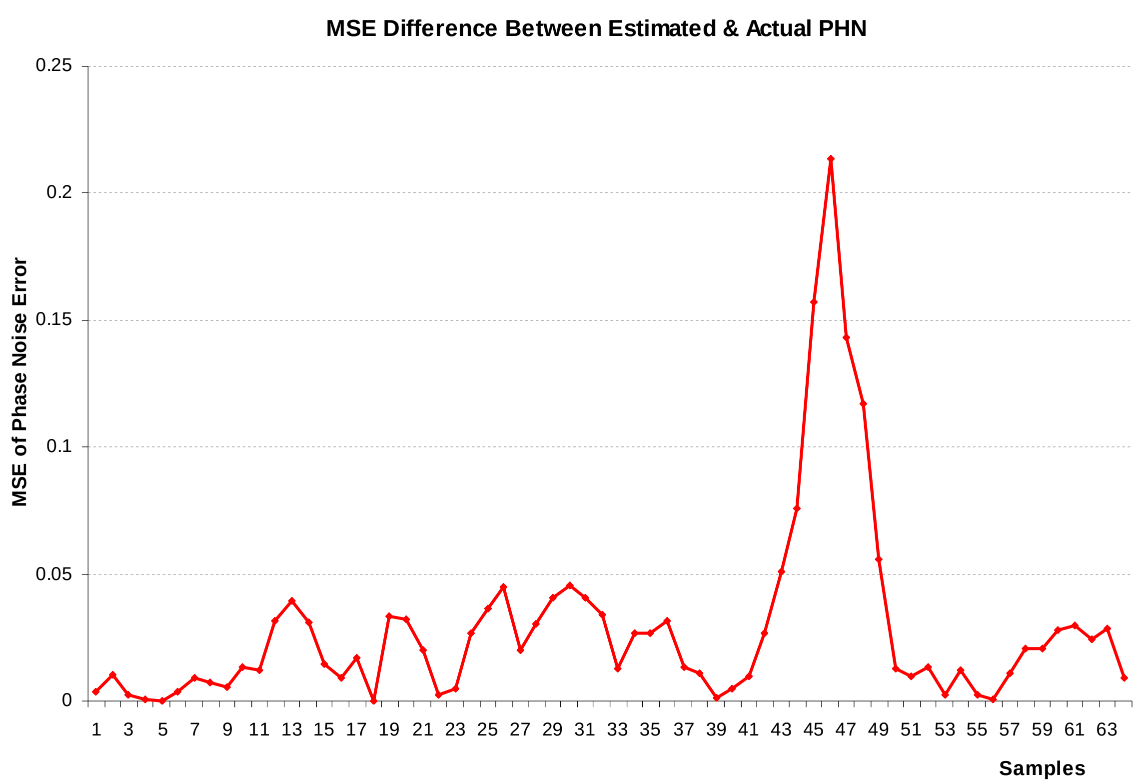
4.4.3 Impact of JCPCE on Combined CFO & PHN Effects



4.4.4 Estimation Approximations



4.4.5 Summary of Results

A number of interesting results are observable from the simulations above. However before they are examined it must be noted that unlike the simulations in sections 4.1-4.3, which used 64,000 symbols for transmission, the graphs in section 4.5 only utilized 1,280 symbols. This was largely due to a limit on computational ability since a 64,000 symbol signal processed by the JCPCE algorithm would exceed hours of simulation time. Hence a smaller transmission package was chosen. This is clearly representative in the highly variable BER rates for each SNR value. A higher symbol rate would certainly have fixed this issue. Notwithstanding, the graphs above reveal a few interesting points:

1. Firstly it is sees that for wiener phase noise effects, seen in 4.4.1, the JCPCE algorithm works best for $\alpha < 0.04$. For values higher than this the algorithm is not precise for the purposes of practical implementation, although it does reduce the effect of phase noise. The estimation for the wiener phase noise by the JCPCE is shown in 4.4.4, revealing that the average mean squared error between the actual and estimated PHN is lower than 0.05. This shows that the algorithm accurately estimates the wiener phase noise model as predicted in [3].
2. The estimation for carrier frequency offset, seen in 4.4.2, by the JCPCE algorithm appears to be more accurate than that for the phase noise. This is seen particularly when the offset is nearly 20% while the JCPCE algorithm brings the signal to within 5dB at a BER of 10^{-2}.
3. Finally the combined effects of CFO and PHN, seen in 4.4.3, for a practically realizable model of 10% CFO and $\alpha = 0.04$, shows that the JCPCE algorithm is able to successfully estimate the signal to within 1dB. This is an encouraging result for all practical applications of the OFDM system.

4.5 Summary

This chapter provided insight into the effects of PHN and CFO on the SISO OFDM model along with the impact that the Joint CFO/PHN/CIR Estimator has on mitigating these effects. It was first observed that both PHN and CFO have detrimental effects on the integrity of the system. However it was seen that increasing the number of subcarriers increases the impact of PHN while seeing a decline in the impact of CFO. The impact of JCPCE on both varying PHN and CFO was also observed and a legitimate reduction in CFO and PHN effects was observed in all cases. Lastly it was discovered that for a practical OFDM system to be realizable under the SISO OFDM model, a 10% CFO and $\alpha = 0.04$, is desirable. These parameters for PHN and CFO yield a system that only experiences 1dB drop in SNR at a BER of 10^{-2}.

CHAPTER 5

Conclusion & Future Work

5.1 Conclusion

OFDM remains a highly attractive communication system application for situations that demand high data rates such as DAB, DVB and WAN. Due to its high spectral efficiency, immunity to frequency selective fading, and ability to mitigate both ISI & ICI, OFDM remains as the primed leader amongst all 4G wireless systems. However serious limitations to OFDM such as its high sensitivity to ICI due to the effects of carrier frequency offset and the impact of phase noise hamper the viability of OFDM systems in certain environments.

The feasibility of algorithms, to mitigate carrier frequency offset and phase noise such as the Joint CFO/PHN/CIR Estimator, is a highly sought-after commodity in an ever increasing wireless marketplace. It was observed that the JCPCE algorithm (as seen in Table i) is able to effectively mitigate the effects of CFO and PHN. It was also found that for a practically desirable system the ideal parameters for CFO and PHN must be such that the carrier frequency offset is only 10% of the subcarrier frequency separation, and that α is a mere 0.04rad (or 2.3^o) for a 64 subcarrier SISO OFDM. Hence this thesis helped to verify the results found in [3]-[4] and provided a better picture of the practicalities involved in employing a SISO OFDM system in real world applications.

5.2 Future Work

Certainly future work can be conducted on issues that this thesis did not address. Some of the issues are:

1. It would be interesting to see the effectiveness of the JCPCE algorithm for a MIMO system and to understand the effects of CFO and PHN on the FSB beamforming technique used in basic MIMO systems.
2. In this thesis the transmitter and receiver were assumed to be within line of sight. It would hence be interesting to model the impact of diffraction and reflection into the OFDM model and observe the effects of PHN and CFO over different Fresnel zones and over varying distances between transmitter and receiver.
3. Certain other OFDM issues, such as other timing synchronization effects and the Peak-to-Average Power Ratio (PARP) problem could also be modelled, and their impact on the system measured.
4. Due to time limitations it was not possible to perform the JCPCE algorithm for higher symbol lengths. If this were indeed possible the graphs detailing the impact of the JCPCE algorithm would be far more accurate and reliable.
5. Finally the phase noise in this thesis was modelled as Wiener PHN. It would be interesting to see a similar analysis performed for Gaussian PHN and impact this would have on the common phase rotation phase noise.

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