CSE 103: Probability and statistics **Fall 2010**

Homework 2

Many of these questions are taken from Grinstead and Snell or from Feller; some have been lightly edited.

1. Four women check their hats at a concert, but when each woman returns after the performance, she gets a hat chosen randomly from those remaining. What is the probability that each woman gets her own hat back?

2. Assume that whenever a child is born, it is equally likely to be a girl or boy, independent of any earlier children. What is the probability that a randomly-chosen family with six children has exactly three girls and three boys?

3. In a true-false exam consisting of ten questions, a student answers every question by randomly guessing.

   (a) What is the probability that he gets all the questions right?

   (b) What is the probability that he gets at least 80% of the questions right?

4. In a particular state, license plates consist of three distinct digits and three distinct letters in any order, for instance: $A34ZB7$. Suppose you pick a license plate at random.

   (a) How large is the sample space?

   (b) What is the probability that all the digits come *before* the letters?

   (c) What is the probability that the digits are in increasing order *and* the letters are in increasing order (but they can be interleaved)?

5. Snow White asks three of the seven dwarfs, chosen at random, to accompany her on a trip.

   (a) What is the probability that Dopey is in this group?

   (b) What is the probability that both Dopey and Sneezy are in the group?

   (c) What is the probability that neither Dopey nor Sneezy are in the group?

6. How long does a sequence of random and independent decimal digits have to be in order for the probability of the digit 7 appearing to be at least 0.9?

7. What is the probability that in a group of n people, at least two of them have their birthdays in the same *month*? Assume that a birthday is equally likely to be in any month, and that $n \leq 12$. Give your answer in terms of n.

8. A student must choose exactly two of the following three electives: art, French, or mathematics. The probability that he chooses art is 5/8, the probability he chooses French is 5/8, and the probability that he chooses both art and French is 1/4.

   (a) What is the probability that he chooses mathematics?

   (b) What is the probability that he chooses either art or French?

9. For a bill to come before the president of the United States, it must be passed by both the House of Representatives and the Senate. Assume that, of the bills presented to the two bodies, 60% pass the House, 80% pass the Senate, and 90% pass at least one of the two. Calculate the probability that the next bill presented to the two groups will come before the president.

10. In a fierce battle, not less than 70% of the soldiers lost one eye, not less than 75% lost one ear, not less than 80% lost one hand, and not less than 85% lost one leg. What is the minimal possible percentage of those who simultaneously lost one ear, one eye, one hand, and one leg?

11. A coin is tossed three times. What is the probability that there are exactly two heads, given that:

    (a) the first outcome is a head?

    (b) the first outcome is a tail?

    (c) the first two outcomes are both heads?

    (d) the first two outcomes are both tails?

    (e) the first outcome is a head and the third outcome is a tail?

12. A card is drawn at random from a standard deck. What is the probability that:

    (a) it is a heart, given that it is red?

    (b) it is higher than a ten, given that it is a heart (interpret J, Q, K, A as having numeric value $11, 12, 13, 14$)?

    (c) it is a jack, given that it is higher than a 10?

13. If $\Pr(B^c) = 1/4$ and $\Pr(A|B) = 1/2$, what is $\Pr(A \cap B)$?

14. A die is rolled twice. What is the probability that the sum of the two rolls is > 7, given that:

    (a) the first roll is a 4?

    (b) the first roll is a 1?

    (c) the first roll is > 3?

    (d) the first roll is < 5?

15. From a deck of five cards numbered 2, 4, 6, 8, and 10, respectively, a card is drawn at random and replaced. This is done three times. What is the probability that the card numbered 2 was drawn exactly two times, given that the sum of the numbers on the three draws is 12?

Answer key 1. 1/24. 2. 5/16. 3 (a) 1/1024 (b) 7/128. 4 (a) $\binom{10}{3}\binom{26}{3}6!$ (b) 1/20 (c) 1/36. 5 (a) 3/7 (b) 1/7 (c) 2/7. 6. 22. 7. $12!/((12-n)!12^n)$. 8 (a) 3/4 (b) 1. 9. 1/2. 10. 10%. 11 (a) 1/2 (b) 1/4 (c) 1/2 (d) 0 (e) 1/2. 12 (a) 1/2 (b) 4/13 (c) 1/4. 13. 3/8. 14 (a) 1/2 (b) 0 (c) 2/3 (d) 1/4. 15. 3/10.