(1) Let D = car is defective, and F_i = car is from factory i.

a. $$P(D) = P(D|F_1)P(F_1) + P(D|F_2)P(F_2) + P(D|F_3)P(F_3) = (0.05)(0.25) + (0.04)(0.35) + (0.02)(0.40)$$

b. $$P(F_1|D) = \frac{P(D|F_1)P(F_1)}{P(D)} = \frac{(0.05)(0.25)}{0.0345}$$

(2) Let M = male, F = female, and C = colorblind. Then,

$$P(M|C) = \frac{P(C|M)P(M)}{P(C)} = \frac{P(C|M)P(M)}{P(C|M)P(M) + P(C|F)P(F)} = \frac{(0.05)(0.5)}{(0.05)(0.5) + (0.01)(0.5)}$$

(3) Let p = test is positive.

a. $P(p) = P(p|d_1)P(d_1) + P(p|d_2)P(d_2) + P(p|d_3)P(d_3) = (0.8)(1/3) + (0.6)(1/3) + (0.4)(1/3)$

b. For each of the three diseases: $P(d_i|p) = \dfrac{P(p|d_i)P(d_i)}{P(d_i)}$

(4) Let A = two-headed coin and B = six heads in a row.

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} = \frac{(1)(1/65)}{(1/65)(1) + (64/65)(1/2)^6}$$

(5) Let A = tenth toss is heads, and B = total number of heads is k.

$$P(A|B) = \frac{P(A,B)}{P(B)} = \frac{\binom{n-1}{k-1}/2^n}{\binom{n}{k}/2^n} = \frac{n}{k}$$

(6) Let T = tiger and N = test is negative.

$$P(T|N) = \frac{P(N|T)P(T)}{P(N)} = \frac{(1/6)(1/3)}{(1/6)(1/3) + (2/3)(2/3)}$$

(7) Let B = bear and S = scratch.

$$P(B|S) = \frac{P(S|B)P(B)}{P(S)} = \frac{(3/5)(1/4)}{(1/10)(3/4) + (3/5)(1/4)}$$

(8) Item (1) is independent since the outcomes of each individual toss do not affect one another. Item (2) is also independent, but it may be more difficult to see why. We compute $P(A) = 1/2$, and $P(D) = 2/2^3$. We also compute $P(A, D) = P(\{(HHH)\}) = 1/2^3$. Thus, we see that $P(A)P(D) = P(A, D) = 1/8$.

(9) Item (2) is independent as the drawing of the suit and face of a single card do not affect one another. We can verify this with a bit of math: $P(A) = 1/4$ and $P(B) = 4/52$. The chance of drawing the ten of hearts is simply $P(A, B) = 1/52$. Thus, $P(A)P(B) = P(A, B) = 1/52$. We also verify that item (4) is independent. Again, $P(A) = 1/4$ and $P(B) = 4/52$. Now, $P(A, B)$ is the number of two-carded hands that contains a heart in the first card, and a ten in the second. The sample space is 52×51 since order matters. The final probability is therefore $P(A, B) = \frac{(12)(4)+(1)(3)}{(52)(51)} = 1/52$. Again, $P(A)P(B) = P(A, B)$.

(10) Let S = accepted to UCSD and L = accepted to UCLA.

a. $P(S|L) = \dfrac{P(S, L)}{P(L)} = \dfrac{0.2}{0.5}$

b. No, because $P(S|L) \neq P(S)$.

(11) Let $F_i = i$ engines fail.

$$\begin{aligned} P(\text{Land Safely}) &= 1 - P(\text{Land Unsafely}) \\ &= 1 - [P(F_3) + P(F_4)] = 1 - [(4)(0.99)(0.01)^3 + (0.01)^4] \end{aligned}$$

(12)

a. Probability of jth ball is $(1/n)$, and the probability of kth ball is $(1/n)$. So the total probability is $(1/n^2)$.

b. To get an upper-bound for $Pr(E_i)$, we can simply take the union of F_{jk} for all possible choices of two balls j and k. Thus, the upperbound we get is $\binom{m}{2}\frac{1}{n^2} \leq \frac{m^2}{2n^2}$.

c. To get an upper-bound this time, we can take the union of $Pr(E_i)$ for all possible choices of i. This results in $n\frac{m^2}{2n^2} = \frac{m^2}{2n}$.

d. Solve for m in the equation $\frac{m^2}{2n} \leq \frac{1}{2}$. The resulting answer is $m \leq n^{1/2}$.