

## Homework 1

Many of these questions are taken from Grinstead and Snell or from Feller; some have been lightly edited.

1. Give a possible sample space  $\Omega$  for each of the following experiments.
  - (a) An election decides between two candidates  $A$  and  $B$ .
  - (b) A two-sided coin is tossed.
  - (c) A student is asked for the month and day-of-week on which her birthday falls.
  - (d) A student is chosen at random from a class of ten students.
  - (e) You receive a grade in this course.
  - (f) You choose the color of your new car's exterior (choices: red, black, silver, green) and interior (choices: black, beige).
2. In each of the following situations, define the sample space  $\Omega$ .
  - (a) A fair coin is tossed 200 times in a row.
  - (b) You count the number of people who enter a department store on a particular Sunday.
  - (c) You open up *Hamlet* and pick a word at random.
3. Let  $A$ ,  $B$ , and  $C$  be events defined on a particular sample space  $\Omega$ . Write expressions for the following combinations of events:
  - (a) All three events occur.
  - (b) At least one of the events occurs.
  - (c)  $A$  and  $B$  occur, but not  $C$ .
4. Consider a sample space  $\Omega = \{a, b, c\}$  with probabilities  $\Pr(a) = 1/2$  and  $\Pr(b) = 1/3$ .
  - (a) What is  $\Pr(c)$ ?
  - (b) How many distinct events can be defined on this space?
  - (c) Find the probabilities of each of these possible events.
5. A fair coin is tossed three times in succession. Describe in words each of the following events on sample space  $\{H, T\}^3$ .
  - (a)  $E_1 = \{HHH, HHT, HTH, HTT\}$
  - (b)  $E_2 = \{HHH, TTT\}$
  - (c)  $E_3 = \{HHT, HTH, THH\}$

What are the probabilities of each of these events?

6. Let  $A$  and  $B$  be events defined on a sample space  $\Omega$  such that  $\Pr(A \cap B) = 1/4$ ,  $\Pr(A^c) = 1/3$ , and  $\Pr(B) = 1/2$ . Here  $A^c = \Omega \setminus A$  is the event that  $A$  *doesn't* happen. What is  $\Pr(A \cup B)$ ?

7. A student must choose one of the following subjects as an elective: art, geology, or psychology. She is equally likely to choose art or psychology, and is twice as likely to choose geology. What are the respective probabilities that she picks art, or geology, or psychology?
8. Recall that a chessboard has 64 squares arranged in an  $8 \times 8$  grid. A rook is a particular chess piece that is said to *attack* anything that shares either the same row or the same column. Suppose two rooks are placed at random on a chessboard (in distinct locations). What is the chance that they are attacking each other?
9. In Morse code, each letter is formed by a succession of dashes and dots. For instance, the letter *S* is represented by three dots and the letter *O* is represented by three dashes. Suppose a child types a sequence of 9 dots/dashes at random (each position is equally likely to be a dot or a dash). What is the probability that it spells out *SOS*?
10. A die is loaded in such a way that the probability of each face turning up is proportional to the number of dots on that face (for instance, a six is three times as probable as a two). What is the probability of getting an even number in one throw?
11. Five people of different heights are lined up against a wall in random order. What is the probability that they just happen to be in increasing order of height (left-to-right)?
12. Five people get on an elevator that stops at five floors. Assuming that each person has an equal probability of going to any one floor, find the probability that they all get off at different floors.
13. A deck of ordinary cards is shuffled and a hand of 13 cards are dealt.
  - (a) What is the probability that the first and second cards are of the same suit?
  - (b) What is the probability that the cards are all of the same suit?
14. A barrel contains 90 good apples and 10 rotten apples. If ten of the apples are chosen at random, what is the probability that they are all good?
15. A computing center has 3 processors that receive  $n$  jobs, with the jobs assigned to the processors purely at random so that all of the  $3^n$  possible assignments are equally likely.
  - (a) Find the probability that one processor gets all the jobs.
  - (b) Find the probability that exactly one processor has no jobs.

Answer key 1 (a)  $\{A, B\}$ . (b)  $\{H, T\}$ . (c)  $\{1, 2, 3, \dots, 12\} \times \{\text{Sun, Mon, Tue, Wed, Thu, Fri, Sat}\}$ . (d)  $\{1, 2, 3, \dots, 10\}$ . (e)  $\{A, B, C, D, E\}$ . (f)  $\{\text{red, black, silver, green}\} \times \{\text{black, beige}\}$ . 2 (a)  $\{H, T\}$ . 300 (b)  $\{0, 1, 2, \dots\}$ . (c)  $\{\text{all words in Hamlet}\}$ . 3 (a)  $A \cup B \cap C$ . (b)  $A \cup B \cap C$ . (c)  $(A \cap B) \setminus C$ . 4 (a)  $1/6$ . (b)  $8$ . (c)  $\Pr(\emptyset) = 0$ ,  $\Pr(\{a\}) = 1/2$ ,  $\Pr(\{b\}) = 1/3$ ,  $\Pr(\{c\}) = 1/6$ ,  $\Pr(\{a, b\}) = 5/6$ ,  $\Pr(\{a, c\}) = 2/3$ ,  $\Pr(\{b, c\}) = 1/2$ ,  $\Pr(\{a, b, c\}) = 1$ . 5 (a) The first toss is heads (prob  $1/2$ ). (b) All tosses are identical (prob  $1/4$ ). (c) There are two heads (prob  $3/8$ ). 6.  $11/12$ . 7. Art or geology:  $1/4$ ; psychology:  $1/2$ . 8.  $2/9$ . 9.  $1/512$ . 10.  $4/7$ . 11.  $1/120$ . 12.  $24/625$ . 13 (a)  $4/17$  (b)  $4/(52)$ . 14.  $(90)/(100)$ . 15 (a)  $1/3^{n-1}$  (b)  $(2^n - 2)/3^{n-1}$ .