ENGINEERING MATHEMATICS - I

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Class Content

- More examples on comparison test

More examples on comparision test

Problem 3. Test the convergence of the series:

$$\frac{\sqrt{2}}{3\,\sqrt{4}} + \frac{\sqrt{3}}{4\,\sqrt{7}} + \frac{\sqrt{4}}{5\,\sqrt{10}} + \cdots \;\; to\ \infty$$

Solution: Here, $\sum a_n = \sum \dfrac{\sqrt{n+1}}{(n+2)\sqrt{3n+1}}$

$$\sum a_n = \sum \frac{\sqrt{n}\,\sqrt{1+\dfrac{1}{n}}}{n\sqrt{n}\left(1+\dfrac{2}{n}\right)\sqrt{3+\dfrac{1}{n}}}$$

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Choosing $\sum b_n = \frac{1}{n}$, the p-series with p $=$1, a divergent series, we get,

$lim_{n\to\infty}\frac{a_n}{b_n} = \frac{1}{\sqrt{3}} \neq 0$ and finite.

Hence by comparision test $\sum a_n$ and b_n behave alike.

This implies $\sum a_n$ is also a divergent series.

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Problem 4. Test the convergence /divergence of the series:

$$\frac{2^p}{1^q} + \frac{3^p}{2^q} + \frac{4^p}{3^q} + \cdots \ to \infty$$

Solution : Here, $\sum a_n = \sum \frac{(n+1)^p}{n^q}$

$$a_n = \sum \frac{n^p}{n^q}\left(1 + \frac{1}{n}\right)^p.$$

Choosing $\sum b_n = \sum \frac{1}{n^{q-p}}$, a p-series such that,

i) $\sum b_n$ is convergent whenever $q - p > 1$,

ii) $\sum b_n$ is divergent whenever $q - p \leq 1$,

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By comparision test, $lim_{n\to\infty}\dfrac{a_n}{b_n} = 1$,
a finite and non - zero quantity. Thus, the given series,
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Problem 5. Test the convergence /divergence of the series: $\sum \dfrac{n^2+n+1}{n^4+1}$

Solution : Here, $a_n = \dfrac{n^2+n+1}{n^4+1}$

$$a_n = \frac{n^2\left(1+\dfrac{1}{n}+\dfrac{1}{n^2}\right)}{n^4\left(1+\dfrac{1}{n^4}\right)}$$

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Hence, choosing $\sum b_n = \frac{1}{n^2}$, a p-series with $p = 2$, a convergent series since $p > 1$, we get,

$lim_{n\to\infty} \frac{a_n}{b_n} = 1$,

a finite and non zero quantity.

By comarision test, $\sum a_n$ and b_n behave alike.

Hence, the given series $\sum a_n$ is also convergent.

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Problems :

Test the following series for convergence :

1) $\sum_{n=1}^{\infty} \frac{1}{2^n + 3^n}$;

Soln : n^{th} term : $a_n = \frac{1}{2^n + 3^n} = \frac{1}{3^n\left[1 + \frac{2^n}{3^n}\right]}$;

Choose $b_n = \frac{1}{3^n}$; $\sum b_n = \sum \frac{1}{3^n}$; Geometric series with common ratio $r = \frac{1}{3} < 1$

Convergent series

$\lim_{n \to \infty} \frac{a_n}{b_n} = \lim_{n \to \infty} \frac{1}{1 + \left(\frac{2}{3}\right)^n} = \frac{1}{1+0} = 1 \neq 0$, finite quantity.

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Problems :

$\sum a_n$ and $\sum b_n$ behave alike.

$\because \sum b_n$ is convergent, $\sum a_n$, the given series is also convergent.

2) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n^{\alpha + \beta/n}}$

Solution: n^{th} term : $a_n = \frac{1}{n^{\alpha + \beta/n}} = \frac{1}{n^{\alpha} (n^{1/n})^{\beta}}$

$\because \lim_{n\to\infty} n^{1/n} = 1$

Choose $\sum b_n = \sum \frac{1}{n^{\alpha}}$ $\longrightarrow$ Convergent $\alpha > 1$

$\longrightarrow$ divergent $\alpha \leq 1$.

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{1}{(n^{1/n})^{\beta}} = 1 \neq 0$, finite quantity

$\sum a_n$ & $\sum b_n$ behave alike.

By comparision test, the given series $\sum a_n$ converges $\alpha > 1$; diverges $\alpha \le 1$.

3) Discuss the convergence of the series:

$$\sum_{n=1}^{\infty} \left(a^{1/n} - 1\right) ; \quad a \neq 1 \text{ and } a > 0.$$

Solution: n^{th} term: $a_n = \left(a^{1/n} - 1\right)$

Case 1): $a > 1$; choose $\sum b_n = \sum \frac{1}{n}$; p-series, $p = 1$, diverged.

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{a^{1/n} - 1}{(1/n)}$; $\frac{1}{n} = m$; as $n \to \infty$, $m \to 0$

$$\lim_{m\to 0} \frac{q^m - 1}{m} \quad ; \quad m = \frac{1}{n}$$

$$= \log q \neq 0 \;, \quad \because q > 1 \,.$$

By comparison test, $\sum a_n$ is divergent.

Conclusion :

for $q > 1$ & $0 < q < 1$

$\sum a_n$ is always divergent.

Case ii) : Let $0 < q < 1$

Here $a_n = q^{1/n} - 1 = q^{1/n}\left[1 - \left(\frac{1}{q}\right)^{1/n}\right] = -q^{1/n}\left[\left(\frac{1}{q}\right)^{1/n} - 1\right]$

choose $\sum b_n = \sum \frac{1}{n}$; p-series, $p = 1$, divergent.

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{-q^{1/n}\left[\left(\frac{1}{q}\right)^{1/n} - 1\right]}{\left(1/n\right)} = -1 \times \log\left(\frac{1}{q}\right) \neq 0$$

By comparison test, $\sum a_n$ is a divergent series.

THANK YOU

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