ENGINEERING MATHEMATICS - I

Ordinary Differential Equations

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Unit 3 : Ordinary Differential Equations

Session : 9

Sub Topic : Non Linear Differential Equations

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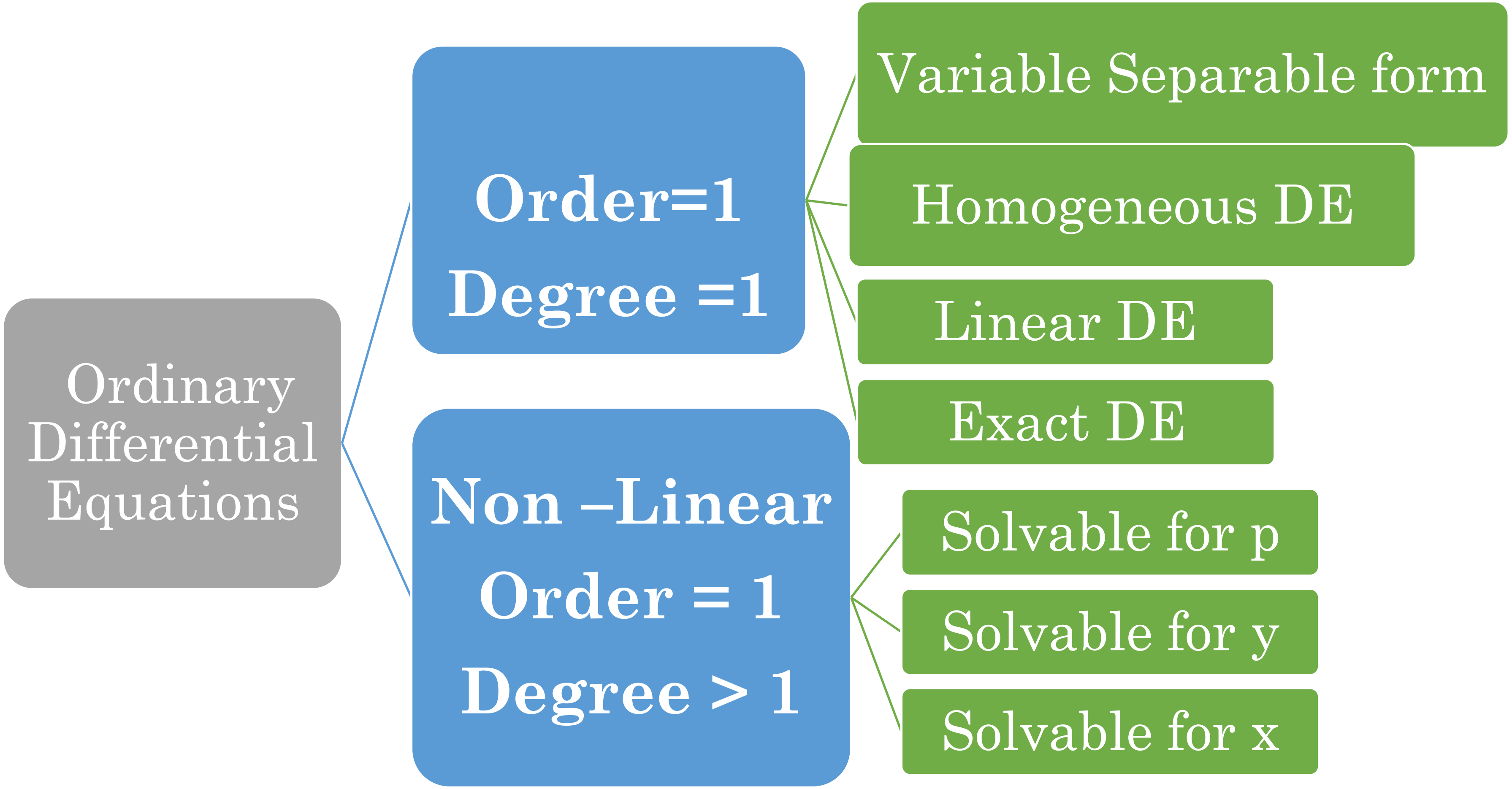
ODE- Non linear Differential Equation



ODE- Non linear Differential Equation

A differential equation of first order and higher degree is of the form $\boldsymbol{f(x, y, y') = 0}$ **or** $\boldsymbol{f(x, y, p) = 0}$............(1)

where $p = y' = \frac{dy}{dx}$.

Eq (1) is a non linear first order Differential Equation.

Example : $\boldsymbol{p^4 - (x + 2y)p^3 + (x + y + 2xy)p^2 - 2xyp = 0}$

is a *first order, 4th degree, non linear DE.*

ODE- Non linear Differential Equation

In general, a first order non linear DE of nth degree is of the form,

$$p^n + a_1 p^{n-1} + a_2 p^{n-2} + \cdots \ldots \ldots \ldots \ldots \ldots \ldots \ldots + a_{n-1} p + a_n = 0$$

where the coefficients $a_1, a_2, \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots, a_n$ *are functions x and y.*

Solutions to such non linear equations can be obtained by reducing to differential equations of first order and first degree by,

i. Solving for p   ii. Solving for y   iii. Solving for x

ODE- Non linear Differential Equation

- Equations solvable for *p*

Step 1. Given an nth degree non linear DE, express it as a nth degree polynomial in p.

Step 2. Resolve the polynomial into n linear real factors in the form

$$(\boldsymbol{p}-\boldsymbol{b}_1)(\boldsymbol{p}-\boldsymbol{b}_2)\ldots\ldots\ldots\ldots(\boldsymbol{p}-\boldsymbol{b}_n)=0,$$

where $b_1, b_2, \ldots\ldots\ldots\ldots b_n$ *are functions of x and y*

Step 3. Equate the n factors in the LHS to zero which reduces to n differential equations of first order and first degree given by,

$$\frac{dy}{dx}=\boldsymbol{b}_1(\boldsymbol{x},\boldsymbol{y}),\ \frac{dy}{dx}=\boldsymbol{b}_2(\boldsymbol{x},\boldsymbol{y}),\ \ldots\ldots\ldots\ldots\ \frac{dy}{dx}=\boldsymbol{b}_n(\boldsymbol{x},\boldsymbol{y})$$

ODE- Non linear Differential Equation

Step 4. Solve the n differential equations to obtain the solutions of the form, $\boldsymbol{f_1(x,y,c)=0,\ f_2(x,y,c)=0}$**.........**

.... $\boldsymbol{f_n(x,y,c)=0}$

Step 5. The general solution is then given by,

$\boldsymbol{f_1(x,y,c).\,f_2(x,y,c)}$**................** $\boldsymbol{f_n(x,y,c)=0}$

Note: The general or complete solution of a differential equation is that in which the number of arbitrary constants is equal to the order of the differential equation

ODE- Non linear Differential Equation - Solvable for p

1. Solve : $p^2 + 2xp - 3x^2 = 0$

Answer : Solving the equation for *p* we have,

$$p = \frac{-2x \pm \sqrt{4x^2 + 12x^2}}{2} = -x \pm 2x = x, -3x$$

- $p = x$; $p = -3x$
- $\frac{dy}{dx} = x$; $\frac{dy}{dx} = -3x$
- $dy - xdx = 0$; $dy + 3xdx = 0$
- $y - \frac{x^2}{2} = c$; $y + \frac{3x^2}{2} = c$
- $y - \frac{x^2}{2} - c = 0$; $y + \frac{3x^2}{2} - c = 0$
- *General Solution :* $\left(y - \frac{x^2}{2} - c\right)\left(y + \frac{3x^2}{2} - c\right) = 0$

2. Solve : $p^3 + 2xp^2 - y^2p^2 - 2xy^2p = 0$

Answer : Solving the equation for *p* we have,

- $\boldsymbol{p^2(p + 2x) - y^2p(p + 2x) = 0}$
- $\boldsymbol{(p^2 - y^2p)(p + 2x) = 0}$
- $\boldsymbol{p(p - y^2)(p + 2x) = 0}$
- $\boldsymbol{p = 0 \quad ; \quad p - y^2 = 0 \quad ; \quad p + 2x = 0}$

ODE- Non linear Differential Equation - Solvable for p

- $\frac{dy}{dx} = 0$; $\frac{dy}{dx} - y^2 = 0$; $\frac{dy}{dx} + 2x = 0$
- $dy = 0$; $\frac{dy}{y^2} - dx = 0$; $dy + 2xdx = 0$
- $y = c$; $-\frac{1}{y} - x = c$; $y + x^2 = c$
- $y - c = 0$; $(x + c)y + 1 = 0$; $y + x^2 - c = 0$
- *General Solution* :$(y - c)[(x + c)y + 1](y + x^2 - c) = 0$

THANK YOU

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