ENGINEERING MATHEMATICS - I

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Class Content

- Monotone Convergence Theorem
- Infinite Series
- Sequence of Partial Sums
- Convergence and Divergence of a series
- Examples

Monotone Convergence Theorem

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Infinite Series

If $\{a_n\}$ is a sequence of real numbers then the expression $a_1 + a_2 + \cdots + a_n + \cdots$ is called an infinite series. It is usually denoted by $\sum_{n=1}^{\infty} a_n$ or more briefly, by $\sum a_n$

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Sequence of partial sums

(Partial Sums). If $\sum a_n$ is a series of positive terms, then $S_n = a_1 + a_2 + a_3 + \cdots + a_n$ is called the n^{th} partial sum of $\sum a_n$.

$$S_1 = a_1 : \ \rightarrow \text{the first partial sum}$$
$$S_2 = a_1 + a_2 : \ \rightarrow \text{the second partial sum}$$
$$\vdots$$
$$S_n = a_1 + a_2 + a_3 + \cdots + a_n : \ \rightarrow \text{the } n^{th} \text{ partial sum}$$

Thus, $\{S_n\}$ for $n \in \mathbb{N}$ is a sequence of partial sums corresponding to the

infinite series $\sum a_n$.
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Convergence and divergence of a Series

[Convergence] An infinite series $\sum_{i=1}^{\infty} a_n = a_1 + a_2 + a_3 + \cdots + a_n + \cdots$ **converges to L** if the sequence of partial sums $S_n = a_1 + a_2 + \cdots a_n$ converges to a limit L.

If $\lim_{n\to\infty} S_n = L$, a finite quantity, then the series converges.

If $\lim_{n\to\infty} S_n = +\infty$, the series diverges.

If $\lim_{n\to\infty} S_n = -\infty$, the series diverges.

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Examples

Discuss the convergence or otherwise of the series
$\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\cdots+\frac{1}{n(n+1)}+\cdots$ to ∞.
Solution.

$$a_n=\frac{1}{n(n+1)}=\frac{1}{n}-\frac{1}{(n+1)}, a_1=\frac{1}{1.2}=1-\frac{1}{2},$$
$$a_2=\frac{1}{2}-\frac{1}{3}, etc.. \cdots ,$$
$$a_n=1-\frac{1}{n+1}$$
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Examples

Discuss the convergence or divergence of $\sum_{n=0}^{\infty}(-1)^n$.

Solution:

$$S_n = 1 - 1 + 1 - 1 + \cdots \text{upto n terms.}$$
$$= \begin{cases} 0 & \text{if } n \text{ is even} \\ 1 & \text{if } n \text{ is odd} \end{cases}$$

S_{2n-1} converges to 1 but the subsequence S_{2n} converges to 0.

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Examples

Show that the series $-1-2-3-\cdots-n\cdots$ diverges to $-\infty$.

Solution:

$S_n = -1-2-3-\cdots-n = -(1+2+3+\cdots+\cdots+n)$

$S_n = -\left\{\dfrac{n(n+1)}{2}\right\}$

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Problems:

Determine a general formula for the n^{th} partial sum of the following series:

$$\sum_{n=1}^{\infty} \frac{1}{2^n} = \sum_{n=1}^{\infty} 2^{-n} = \underbrace{\underbrace{\frac{1}{2}}+\frac{1}{2^2}}+\frac{1}{2^3}+\cdots\cdots+\frac{1}{2^n}+\cdots\infty$$

$$S_1 = \frac{1}{2}$$

$$S_2 = \frac{1}{2}+\frac{1}{2^2} = \frac{3}{4}$$

$$S_3 = \frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3} = \frac{7}{8} \qquad S_4 = \frac{15}{16}$$

$$\vdots$$

$$S_n = \frac{1}{2}+\frac{1}{2^2}+\cdots\cdots\frac{1}{2^n}$$

$\{S_n\} = \left\{ \frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \frac{15}{16}, \ldots \ldots \infty \right\}$

n^{th} term $= S_n = \dfrac{2^n - 1}{2^n}$ $\quad n=1, \; n=2,$

$\lim\limits_{n\to\infty} S_n = \lim\limits_{n\to\infty} 1 - \dfrac{1}{2^n} = 1$ finite

Given sequence of Partial Sums is convergent.

$\Rightarrow$ Given series $\sum a_n$ is again convergent

2. Determine if the series represented by $\sum_{n=1}^{\infty} n$ is convergent.

$\rightarrow 1+2+3+\cdots\cdots\infty \rightarrow$ Divergent

Solution:

$S_1 = 1$

$S_2 = 1+2 = 3$

$S_3 = 1+2+3 = 6$

$\{S_n\} = \{1, 3, 6 \ldots\ldots \infty\}$

n^{th} term

$S_n = 1+2+3+\cdots+n = \frac{n(n+1)}{2}$

$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{n(n+1)}{2} = \infty$

$\Rightarrow$ Given sequence of Partial Sums Diverges

3. Determine a general Formula for the n^{th} Partial Sum of the following series.

$$\sum_{n=1}^{\infty} \frac{1+(-1)^n(2n-1)}{4}$$

Solution:

$$\sum_{n=1}^{\infty} \frac{1}{4} + \frac{1}{4}(-1)^n(2n-1) = \sum S_n$$

n^{th} partial Sum

$$S_n = \left(\frac{1}{4} - \frac{1}{4}\right) + \left(\frac{1}{4} + \frac{3}{4}\right) + \left(\frac{1}{4} - \frac{5}{4}\right) + \left(\frac{1}{4} + \frac{7}{4}\right) + \cdots$$

$$= \frac{0}{4} + \frac{4}{4} + \left(\frac{-4}{4}\right) + \left(\frac{8}{4}\right) + \left(\frac{-8}{4}\right) + \left(\frac{12}{4}\right) + \left(\frac{-12}{4}\right) + \cdots$$

THANK YOU

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