ENGINEERING PHYSICS

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Class #13

- Matter wave incident on a step potential with E>V
- Solutions of the SWE
- Interpretation of the wave functions
- Probabilities of reflection and transmission

Suggested Reading

1. *Concepts of Modern Physics, Arthur Beiser, Chapter 5*
2. *Learning Material prepared by the Department of Physics*

Reference Videos

1. *Video lectures : MIT 8.04 Quantum Physics I*
2. *Engineering Physics Class #12*

Problem statement

- *A particle of mass* m *with energy* E *moving from a region of zero potential for* $x < 0$ *into a region of constant potential* V_o *for* $x > 0$
- *Two possible cases*
  - $E > V_o$
  - $E < V_o$
- *The problem can be split as a two region problem for each of the two energy situations*

  Region I $x < 0 \rightarrow V = 0$

  Region II $x > 0 \rightarrow V = V_o$

Case I: Particle in a constant potential - $E > V_o$

Region I $x < 0$ $V = 0$

- *The general Schrodinger's wave equation*

$$\frac{\partial^2 \psi_I(x)}{\partial x^2} + \frac{2m}{\hbar^2}(E - V)\psi_I(x) = 0$$

- $V = 0$ *implies a free particle and the SWE reduces to*

$$\frac{\partial^2 \psi_I(x)}{\partial x^2} + \frac{2m}{\hbar^2}E\psi_I(x) = 0$$

$$\frac{\partial^2 \psi_I(x)}{\partial x^2} + k_I^2\psi_I(x) = 0$$

Where $k_I = \sqrt{\frac{2mE}{\hbar^2}}$ *and* $E = \frac{\hbar^2 k_I^2}{2m}$

Case I: Particle in a constant potential - $E > V_o$

Region I

- *The general solution for the wave function*

$$\psi_I(x) = Ae^{ik_I x} + Be^{-ik_I x}$$

$\rightarrow$ *forward moving incident wave*

$\leftarrow$ *backward moving reflected wave*

Case I: Particle in a constant potential - $E > V_o$

Region II $x > 0$ $V = V_o < E\ (E - V_o)$

- *The general Schrodinger's wave equation*

$$\frac{\partial^2 \psi_{II}(x)}{\partial x^2} + \frac{2m}{\hbar^2}(E - V)\psi_{II}(x) = 0$$

- *the SWE reduces to*

$$\frac{\partial^2 \psi_{II}(x)}{\partial x^2} + \frac{2m}{\hbar^2}(E - V_o)\psi_{II}(x) = 0$$

$$\frac{\partial^2 \psi_{II}(x)}{\partial x^2} + k_{II}^2 \psi_{II}(x) = 0$$

Where $k_{II} = \sqrt{\frac{2m(E-V_o)}{\hbar^2}}$ *and* $E = \frac{\hbar^2 k_{II}^2}{2m} + V_o$

Case I: Particle in a constant potential - $E > V_o$

Region II

- *The general solution for the wave function*

$$\psi_{II}(x) = Ce^{-ik_{II}x} + De^{ik_{II}x}$$

Beyond x=0 there is no disruption in the potential and the wave continues to move only in the forward direction only

The transmitted wave $\psi_{II}(x) = De^{ik_{II}x}$

Particle moving into a region of constant potential - $E > V_o$

- *The wave functions $\psi_I(x)$ and $\psi_{II}(x)$ and their derivatives wrt to x, $d\psi_I(x)$ and $d\psi_{II}(x)$ have to be continuous at the boundary $x = 0$*

$$\psi_I(0) = \psi_{II}(0) \quad \rightarrow \quad A + B = D \qquad ....(1)$$

$$d\psi_I(0) = ik_I(A - B) = d\psi_{II}(0) = ik_{II}D$$

$$A - B = \frac{k_{II}}{k_I}D \qquad .....(2)$$

1+2 yields $D = 2A\left(\frac{k_I}{k_I+k_{II}}\right)$

1-2 yields $2B = D\left(\frac{k_I-k_{II}}{k_I}\right) = 2A\left(\frac{k_I}{k_I+k_{II}}\right)\left(\frac{k_I-k_{II}}{k_I}\right)$

$$B = A\left(\frac{k_I - k_{II}}{k_I + k_{II}}\right)$$

Particle moving into a region of constant potential - $E > V_o$

- $B \neq 0$ implies a small probability amplitude for reflection
- Define the flux of the wave function as $\psi^*\psi \times velocity$
- The flux of incident waves

$$\left(A^* e^{-ik_I x}\right)\left(A e^{ik_I x}\right) = A^* A \times v_I$$

- The flux of reflected waves

$$\left(B^* e^{ik_I x}\right)\left(B e^{-ik_I x}\right) = B^* B \times v_I$$

- The probability of reflection or the reflection co-efficient
- $R = \dfrac{flux\ of\ reflected\ waves}{flux\ of\ incident\ waves} = \dfrac{B^* B\ v_I}{A^* A\ v_I} = \left(\dfrac{k_I - k_{II}}{k_I + k_{II}}\right)^2 > 0$

Particle moving into a region of constant potential - $E > V_o$

- *The flux of transmitted waves*
- $\left(D^* e^{-ik_{II}x}\right)\left(De^{ik_{II}x}\right) = D^* D \times v_{II}$
- *The probability of transmission over the step or the transmission co-efficient*

$$T = \frac{transmitted\ flux}{incident\ flux} = \frac{D^* D\ v_{II}}{A^* A\ v_I} = \frac{4k_I\ k_{II}}{(k_I + k_{II})^2}$$

Verify R+T=1

The total incident flux= reflected flux + transmitted flux

Summarizing Case I

- *Region I*

Region II

- $\psi_I(x) = Ae^{ik_I x} + Be^{-ik_I x}$

$\psi_{II}(x) = De^{ik_{II} x}$

- $k_I = \sqrt{\frac{2mE}{\hbar^2}}$

$k_{II} = \sqrt{\frac{2m(E-V_o)}{\hbar^2}}$

- $E = \frac{\hbar^2 k_I^2}{2m} = KE$

$E = \frac{\hbar^2 k_{II}^2}{2m} + V_o$

- $P_I = \hbar k_I$

$P_{II} = \hbar k_{II}$

- $\lambda_I = \frac{h}{\sqrt{2mE}}$

$\lambda_{II} = \frac{h}{\sqrt{2m(E-V_o)}}$

ENGINEERING PHYSICS

Class #13 …….. Quiz….

The concepts that are true …..

1. If the energy of the particle is greater than the potential step, then then there is no probability of reflection.
2. The de Broglie wavelength of the particle in the region of constant potential is greater than that of the incident particle
3. The propagation constant of the particle in the region of potential is less than that of the particle in the zero potential region.
4. The transmission probability of the particle over the region of constant potential is always less than 1.
5. The flux of transmitted waves = flux of reflected waves + flux of incident waves

THANK YOU

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