ENGINEERING MATHEMATICS - I

Ordinary Differential Equations

Dr. Karthiyayini
Department of Science and Humanities

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Unit 3 : Ordinary Differential Equations

Session : 4

Sub Topic : Exact Differential Equations

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Definition :

A differential equation of the form

$$M(x,y)dx + N(x,y)dy = 0$$

is said to be exact if its left hand member is the exact differential of some function $u(x,y)$.

That is, $du = M(x,y)dx + N(x,y)dy = 0$

Therefore, its solution is $u(x,y) = c$

Example : Consider, the differential equation,

$$ydx + xdy = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(1)$$

Note that

$$d(xy) = ydx + xdy = 0$$

Therefore, the solution of equation (1) is

$$xy = c$$

Exact Differential Equations

Theorem : The necessary and sufficient condition for the differential equation $M(x,y)dx + N(x,y)dy = 0$ to be exact is $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

Note : The solution of an exact differential equation is given by,

$$\int_{y\ constant} M dx + \int N(y) dy = c$$

where $N(y)$ = terms of N which contain y alone.

Exact Differential Equations – Problems

1. Test the differential equation for exactness & solve
$$(x^2 - 4xy - 2y^2)dx + (\boldsymbol{y}^2 - 4xy - 2\boldsymbol{x}^2)dy = 0$$

Solution : The given equation is of the form
$Mdx + Ndy = 0$, where
$M = x^2 - 4xy - 2y^2$ and $N = y^2 - 4xy - 2x^2$.

Then, $\frac{\partial M}{\partial y} = -4x - 4y$; $\quad \frac{\partial N}{\partial x} = -4x - 4y$

Since $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ the given equation is exact.

The solution is $\int Mdx + \int N(y)dy = C$

$$\int x^2 - 4xy - 2y^2 dx + \int y^2 dy = C$$

$$\boldsymbol{\frac{x^3}{3} - \frac{4x^2y}{2} - 2xy^2 + \frac{y^3}{3} = c}$$

Exact Differential Equations - Problems

2. Solve : $(x^3 - 3xy^2)dx + (y^3 - 3x^2y)dy = 0$

Solution : The given equation is of the form

$Mdx + Ndy = 0$, where

$M = x^3 - 3xy^2$ and $N = (y^3 - 3x^2y)$

Then, $\frac{\partial M}{\partial y} = -6xy$; $\quad \frac{\partial N}{\partial x} = -6xy$

Since $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ the given equation is exact.

The solution is $\int Mdx + \int N(y)dy = C$

$$\int(x^3 - 3xy^2)dx + \int y^3dy = C$$

$$\boldsymbol{x^4 - 6x^2y^2 + y^4 = 4c}$$

THANK YOU

Dr. Karthiyayini
Department of Science & Humanities

Karthiyayini.roy@pes.edu

+91 80 6618 6651