ENGINEERING MATHEMATICS - I

Partial Differentiation

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UNIT 2 : Partial Differentiation
Session : 3
Sub Topic: Problems on Partial differentiation

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Problems on Partial differentiation

1. Give $x = r\ cos\theta$, $y = r\ sin\theta$, find $(\frac{\partial x}{\partial r})_\theta$, $(\frac{\partial x}{\partial r})_y$, $(\frac{\partial r}{\partial x})_y$, $(\frac{\partial \theta}{\partial y})_x$.

Solution:

Given $x = rcos\theta$

Therefore, $\left(\frac{\partial x}{\partial r}\right)_\theta = cos\theta$

To find $\left(\frac{\partial x}{\partial r}\right)_y$, we have to express x in terms of r and y.

Since $x = rcos\theta\ and\ y = rsin\theta$, we have $x^2 + y^2 = r^2$.

Hence $x = \sqrt{r^2 - y^2}$

$$\left(\frac{\partial x}{\partial r}\right)_y = \frac{r}{\sqrt{r^2-y^2}} = \frac{r}{x} = sec\theta$$

Contd…..

To determine $\left(\frac{\partial r}{\partial x}\right)_y$, we have that $\mathrm{r} = \sqrt{x^2 + y^2}$

$$\left(\frac{\partial r}{\partial x}\right)_y = \frac{x}{\sqrt{x^2 + y^2}} = \frac{x}{r} = cos\theta$$

As $\theta = tan^{-1}\left(\frac{y}{x}\right)$,

$$\left(\frac{\partial \theta}{\partial y}\right)_x = \frac{x}{x^2 + y^2}$$

2. Verify that $\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial^2 z}{\partial x \partial y}$ for the function $z = \tan^{-1}(\frac{x}{y})$.

Solution:

LHS= $\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$

$$\frac{\partial z}{\partial x} = \frac{y}{x^2 + y^2}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) = \frac{x^2 - y^2}{(x^2+y^2)^2} \quad ------ -(1)$$

Contd…..

$$\text{RHS} = \frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial z}{\partial y} = \frac{-x}{x^2 + y^2}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = \frac{x^2 - y^2}{(x^2+y^2)^2} \quad ------ -(2)$$

(1) and (2) verifies the result.

3. For the point on the surface $x^x\, y^y\, z^z = c$, where $x = y = z$, show that $\frac{\partial^2 z}{\partial x \partial y} = -[x\log(ex)]^{-1}$.

Solution:

$$xlogx + ylogy + zlogz = logc$$

Differentiating partially with respect to ‘y’.

$$\frac{\partial z}{\partial y} = \frac{-\log(ex)}{\log(ez)}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = \frac{1}{z}\frac{\log(ex)}{\log(ez)^2}\frac{\partial z}{\partial x}$$

$$\frac{\partial z}{\partial x} = \frac{-\log(ex)}{\log(ez)}$$

Contd…

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = \frac{1}{z}\frac{\log(ex)}{\log(ez)^2}\frac{\partial z}{\partial x}$$

$$\frac{\partial z}{\partial x} = \frac{-\log(ex)}{\log(ez)}$$

Substituting this in the above expression, we get

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{1}{z}\frac{\log(ex)}{\log(ez)^2}\frac{-\log(ex)}{\log(ez)}$$

At $x = y = z$, we get

$$\frac{\partial^2 z}{\partial x \partial y} = -[x\log(ex)]^{-1}.$$

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