ENGINEERING MATHEMATICS - I

Ordinary Differential Equations

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Unit 3 : Ordinary Differential Equations

Session : 5

Sub Topic : Equations Reducible to Exact form

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Equations Reducible to Exact Form

Sometimes a differential equation which is not exact may become so, on multiplication by a suitable function known as the integrating factor (IF).

The integrating factor can be obtained as follows :

- **Case 1** : *If* $\frac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{N} = g(x)$ *(function of x alone) then,*

$$IF = e^{\int g(x)dx}.$$

- **Case 2** : *If* $\frac{\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}}{M} = h(y)$ *(function of y alone) then,*

$$IF = e^{\int h(y)dy}.$$

Equations Reducible to Exact Form

- **Case 3 :** In the given differential equation $Mdx + Ndy = 0$, if $M(x, y)$ *and* $N(x, y)$ is homogeneous of the same degree then,

  $\boldsymbol{IF = \frac{1}{Mx+Ny}}$, *provided that* $Mx + Ny \neq 0.$

Note : If $Mx + Ny = 0$ *then* $IF = \frac{1}{x^2}$ *or* $\frac{1}{y^2}$ *or* $\frac{1}{xy}$.

Equations Reducible to Exact Form

- **Case 4 :** If the differential equation is of the form $f_1(xy)ydx + f_2(xy)xdy = 0$, then

$$IF = \frac{1}{Mx - Ny},$$

where $M = f_1(xy)y$ & $N = f_2(xy)x$,

provided that $Mx - Ny \neq 0$.

Note : If $Mx - Ny = 0$ then $\frac{M}{N} = \frac{y}{x}$ and the given differential equation reduces to $xdy + ydx = 0$ and its solution is $xy = c$.

Equations Reducible to Exact Form - Problems

1. Solve $(1+(x+y)tany)\frac{dx}{dy}+1=0$

Solution :

$M=1, N=1+(x+y)tany$

Consider, $\frac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{M}=\frac{0-tany}{1}=-tany=g(y)$ (function of y only)

Then, $IF=e^{-\int g(y)dy}=e^{\int tanydy}=e^{\log(secy)}=secy$

Multiplying the given equation by the *IF* we have,

$M=secy\;, N=secy+(x+y)tanysecy$

Equations Reducible to Exact Form

Contd...

The solution is given by

$$\int Mdx + \int N(y)dy = C$$

$$\int secydx + \int secy + ytanysecydy = C$$

$$xsecy + \log(secy + tany) + ysecy - \log(secytany) = c$$

$$xsecy + ysecy = c$$

Equations Reducible to Exact Form - Problems

2. Solve $(2xlogx - xy)dy + 2ydx = 0$

Solution :

$M = 2y, N = 2xlogx - xy$

Consider, $\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{2-(2logx+2-y)}{2xlogx-xy} = \frac{-(2logx-y)}{x(2logx-y)} = \frac{-1}{x} = f(x)$

(function of x only)

Then, $IF = e^{\int \frac{-1}{x} dx} = e^{-logx} = \frac{1}{x}$

Multiplying the given equation by the IF we have,

$M = \frac{2y}{x}, N = 2logx - y$

Equations Reducible to Exact Form

Contd...

The solution is given by

$$\int M dx + \int N(y) dy = C$$

$$\int \frac{2y}{x} dx - \int y dy = c$$

$$2ylogx - \frac{y^2}{2} = c$$

THANK YOU

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