ENGINEERING MATHEMATICS - I

Problems on Euler's Theorem

Dr. Anitha
Science and Humanities

ENGINEERING MATHEMATICS - I

UNIT 2 : Partial Differentiation

Session : 7

Sub Topic : Problems on Euler's Theorem

Dr. Anitha

Department of Science and Humanities

Problems on Euler's Theorem

1. Verify Euler's theorem for $cos^{-1}(\frac{x+y}{\sqrt{x}+\sqrt{y}})$

Solution:

Z=cosu=$\frac{x+y}{\sqrt{x}+\sqrt{y}}$ = $= x^{\frac{1}{2}} \frac{(1+\frac{y}{x})}{1+\frac{\sqrt{x}}{\sqrt{y}}} = \sqrt{x}\, f(\frac{y}{x})$

Z is a homogeneous function of degree ½

Euler's theorem is given by $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = \frac{1}{2}z$

$$x\frac{\partial z}{\partial x} = \frac{x\sqrt{x} + 2x\sqrt{y} - y\sqrt{x}}{2(\sqrt{x}+\sqrt{y})^2}$$

$$y\frac{\partial z}{\partial y} = \frac{2y\sqrt{x} + y\sqrt{y} - x\sqrt{y}}{2(\sqrt{x}+\sqrt{y})^2}$$

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = \frac{1}{2}z$$

Problems on Euler's Theorem

2. If $u = sin^{-1}\left(\frac{x+2y+3z}{\sqrt{x^8+y^8+z^8}}\right)$, show that $xu_x + yu_y + zu_z + 3tanu = 0$.

Solution: $Sinu = \left(\frac{x+2y+3z}{\sqrt{x^8+y^8+z^8}}\right) = x^{-3}g(\frac{y}{x}, \frac{z}{x})$

$Sinu$ is a homogeneous function of degree -3.

By Euler's theorem,

$$x\frac{\partial(sinu)}{\partial x} + y\frac{\partial(sinu)}{\partial y} + z\frac{\partial(sinu)}{\partial z} = -3sinu$$

$$xcosu\frac{\partial u}{\partial x} + ycosu\frac{\partial u}{\partial y} + zcosu\frac{\partial u}{\partial z} = -3\text{sinu}$$

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = -3tanu$$

Dr. Anitha

Department of Science and Humanities

nanitha@pes.edu

Extn 730