



ENGINEERING PHYSICS

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ENGINEERING PHYSICS

Unit II : Quantum Mechanics of simple systems



Class #12

- Free particle solution
- Particles field interactions – a classical experiment
- Potential Step

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Unit II : Quantum Mechanics of simple systems



➤ *Suggested Reading*

1. *Concepts of Modern Physics, Arthur Beiser, Chapter 5*
2. *Learning Material prepared by the Department of Physics*

➤ *Reference Videos*

1. *Video lectures : MIT 8.04 Quantum Physics I*
2. *Engineering Physics Class #11*

Problem statement - Free particle

- *A particle of mass m and energy E moving freely in space*
- *A particle is free if no force acts on the particle*
- *$F = 0 \Rightarrow -\frac{dV}{dx} = 0$*
- *This implies $V = 0$ or $V = \text{constant}$*
- *the simpler case of $V = 0$*
- *The general Schrodinger's wave equation*

$$\frac{\partial^2 \psi(x)}{\partial x^2} + \frac{2m}{\hbar^2} (E - V) \psi(x) = 0$$

Free particle solution

With $V = 0$ the Schrodinger's wave equation reduces to

$$\frac{\partial^2 \psi(x)}{\partial x^2} + \frac{2mE}{\hbar^2} \psi(x) = 0$$

$$\frac{\partial^2 \psi(x)}{\partial x^2} + k^2 \psi(x) = 0$$

- *Where $k^2 = \frac{2mE}{\hbar^2}$ or $k = \sqrt{\frac{2mE}{\hbar^2}}$*
- *The general solution of this equation is of the form*

$$\psi(x) = Ae^{ikx} + Be^{-ikx}$$

- *The two parts represent two possible state of motion*

$Ae^{ikx} \Rightarrow$ represents a particle moving in increasing x direction (+ve x)

$Be^{-ikx} \Rightarrow$ represents a particle moving in decreasing x direction (-ve x)

where k is the propagation constant of the wave

And the energy of the wave is $E = \frac{\hbar^2 k^2}{2m}$

- *No quantum effects since all k values are allowed and hence all energy states are allowed*
- *The free particle solution is the classical limit of quantum mechanics*

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Particle interacting with fields

A system of a split tube of a highly conducting metal with a very narrow separation

The first part is at zero potential and the other held at a constant potential V

A charged particle with energy E moving along the axis of the tube

The particle continues to move in the first tube without any change in energy

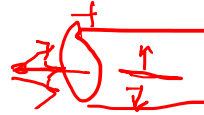
Particle with
energy E



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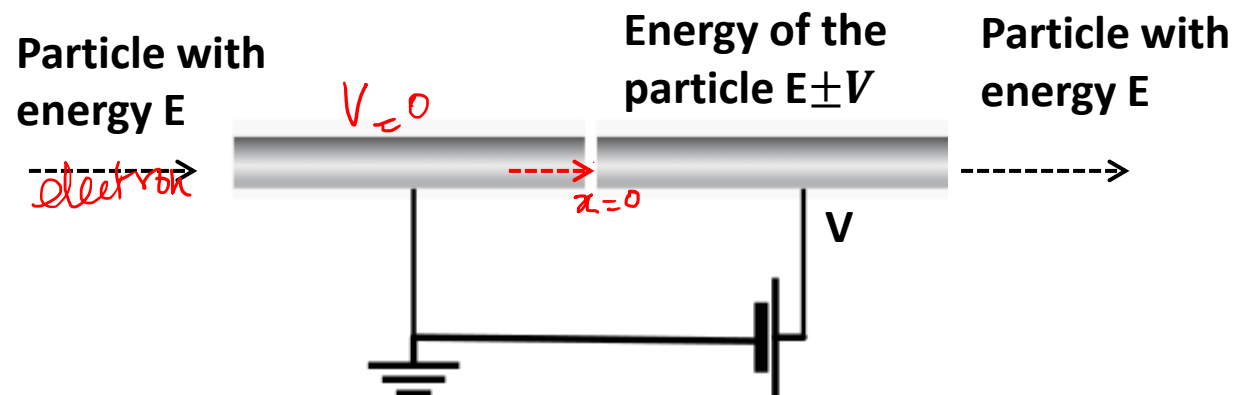
Particle interacting with fields

At the entrance of the second tube the particle feels an acceleration / deceleration



Once within the second tube the particle continues to move with energy $E \pm V$

The particle exiting the second tube feels a deceleration / acceleration and exits the tube with the initial energy



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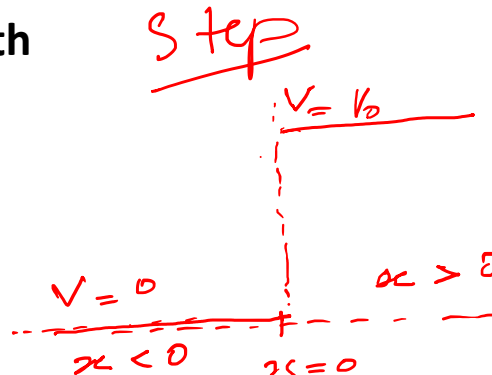
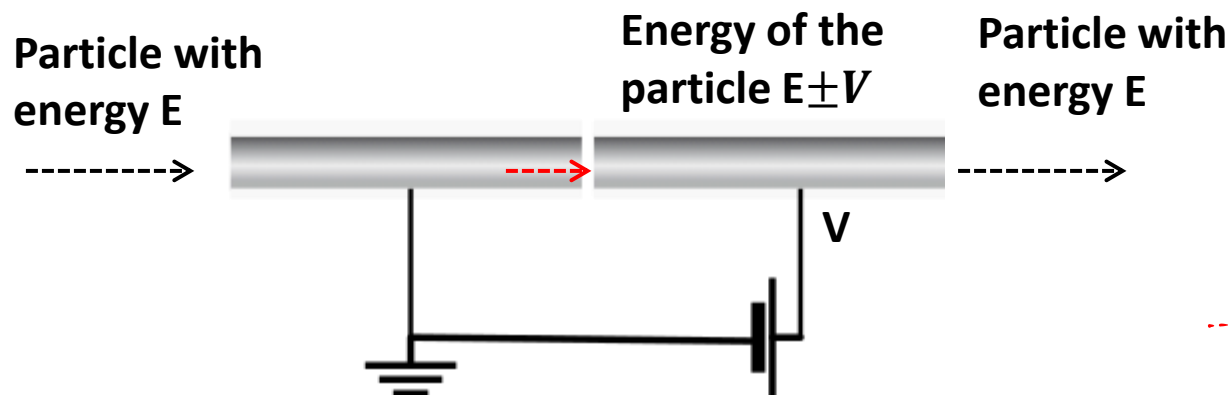
Particle interacting with fields

Classically if $E < V$ the particle is repelled if the charge on the particle is of the same sign of the potential

Particle can interact with fields –

exchange energy with the field within the framework of the uncertainty principle

The plot of the potential as a function of distance with $x=0$ at the point of the separation of the tube is a step function



The concepts which are true of topics discussed ...

- 1. A free particle always move in a zero potential field**
- 2. The two part wave function describe all possible paths of the particle**
- 3. The energy of the particle is quantized**
- 4. The propagation constant is not quantized for the free particle solution**
- 5. The free particle solution is the classical Newton's law of motion.**



THANK YOU

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