



ENGINEERING PHYSICS

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ENGINEERING PHYSICS

Unit I : Review of concepts leading to Quantum Mechanics



Week #1 Class #4

- Review of Electric and magnetic fields
- EM Wave equation
- Energy transported by EM Waves
- *Max Planck's Black Body Radiation equation*

Class #4

- *Black body radiation*
- *Cavity Oscillators*
- *Classical estimation of energy density*
- *Max Planck's estimation of energy density*

➤ *Suggested Reading*

1. *Concepts of Modern Physics,
Arthur Beiser, Chapter 2*
2. *Learning material prepared by the Department of Physics*

➤ *Reference Videos*

1. *DrPhysicsA/Blackbody radiation and UV catastrophe*

Gustav Robert Kirchhoff

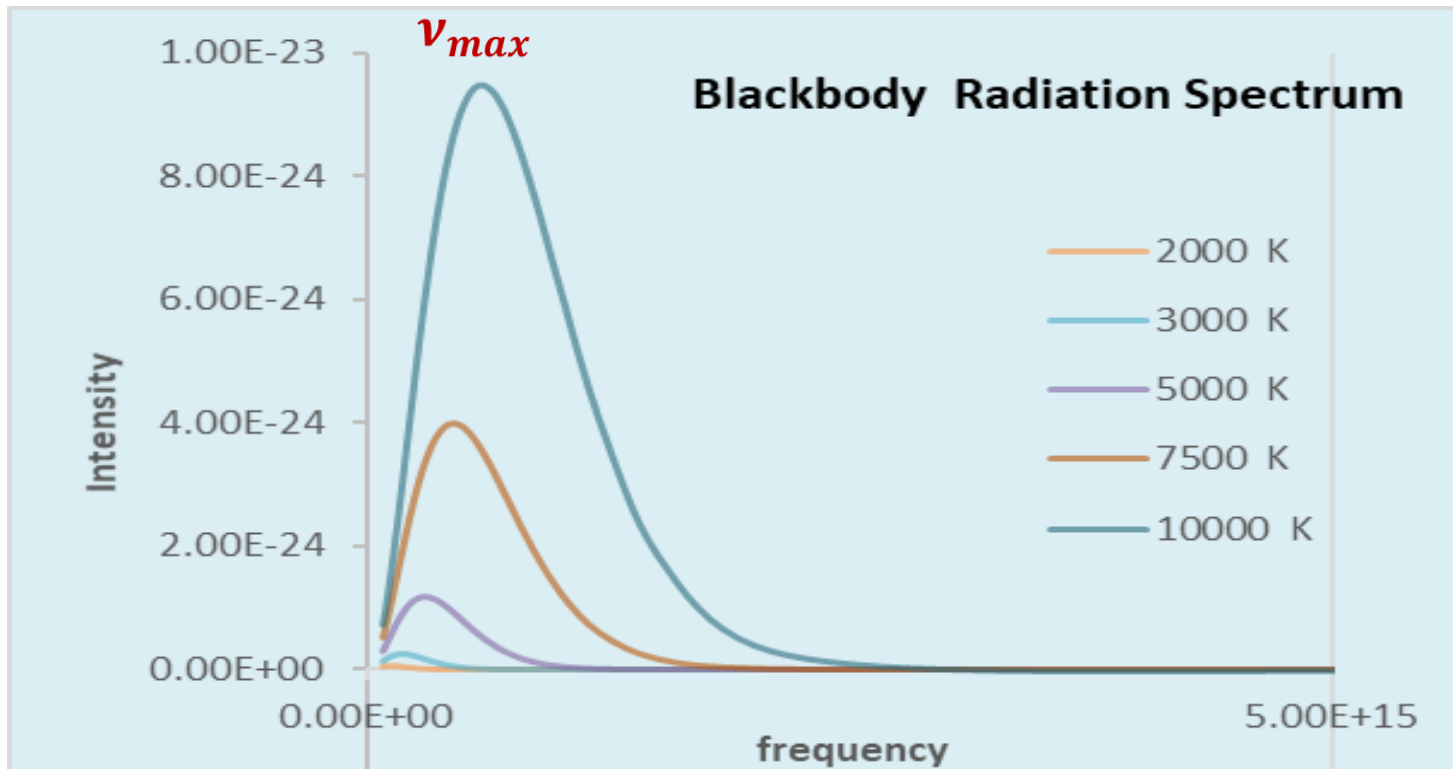
- *Interaction of radiation with materials*

Absorption and emission

- *Materials emit radiation of all wavelengths around the visible range when heated*
- *Black body – (not necessarily black)*
 - *absorbs all radiations falling on it*
 - *Emits all wavelengths (frequencies) as it absorbed*
 - *Emissivity is unity*

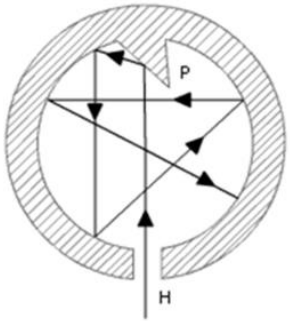
Blackbody Radiation Spectrum

- *Intensity of emitted radiations distributed about ν_{max}*
- ν_{max} *temperature dependent*
- ν_{max} *shifts to higher values at higher temperatures*



Blackbody radiators

- *Black body model*
 - *Radiation entering the cavity is trapped*
 - *When heated can emit radiation of all wavelengths*
 - *Intensity of radiations for different wavelengths*
-



**Cavity as
blackbody**

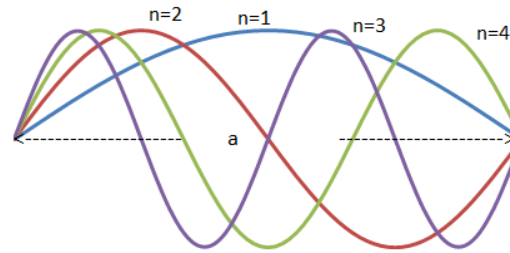
Cavity Oscillators

- Cubical cavity of side a
 - Many harmonic oscillators on the surface of the cavity
 - Standing waves satisfying $a = n \frac{\lambda_n}{2}$
 - Frequency emitted $\nu_n = n \frac{c}{2a}$
-



a

Standing waves
in cavity



Allowed states in the
cavity

Cavity Oscillators

- *3D cavity*
- *Frequency emitted* $\nu_r = r \frac{c}{2a}$
- $r = \sqrt{n_x^2 + n_y^2 + n_z^2}$

Number of standing waves in cavity

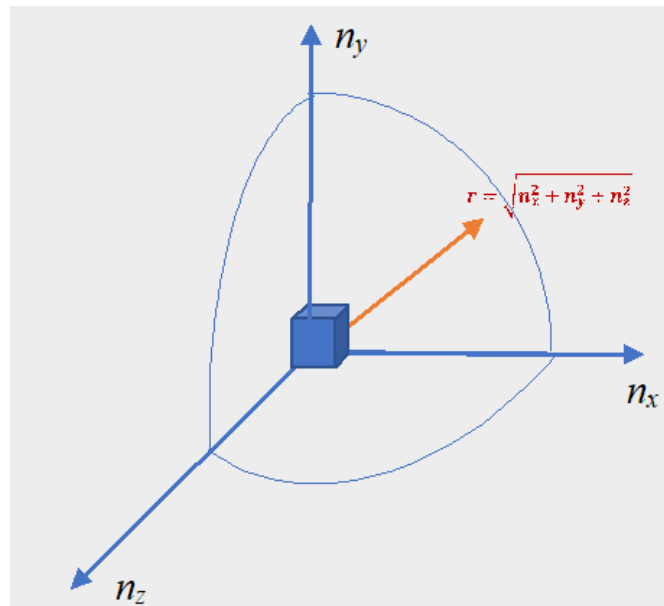


Image courtesy **Hyperphysics**

Changes in r by dr can give count of oscillators with frequencies $d\nu_r$

$$d\nu_r = dr \frac{c}{2a}$$

The number of oscillators with frequencies between ν_r and $\nu_r + d\nu_r$ can be estimated as the volume of a shell of radius r and thickness dr

$$= \frac{1}{8} (4\pi r^2 \cdot dr) = \frac{1}{2} (\pi r^2 \cdot dr)$$

Considering 2 polarization states for the radiation the number of oscillators

$$= \pi r^2 \cdot dr$$

The number of oscillators with frequencies between ν_r and $\nu_r + d\nu_r$ can be estimated as the volume of a shell of radius r and thickness dr

$$= \frac{8\pi a^3}{c^3} \nu^2 d\nu$$

where a^3 is the volume of the cavity

The number of oscillators per unit volume with frequencies between ν and $\nu + d\nu$

$$dN = \frac{8\pi}{c^3} \nu^2 d\nu$$

Average energy of Cavity Oscillators

The probability function or the partition function of oscillators with energy E in thermal equilibrium

$$P(E) = \frac{e^{-E/kT}}{kT}.$$

Average energy of these oscillators using the Boltzmann distribution function

$$\langle E \rangle = \frac{\int E * P(E) dE}{\int P(E) dE} = kT$$

Rayleigh Jean's classical distribution of the energy density of radiation

$$\rho(\nu) d\nu = \langle E \rangle * dN = \frac{8\pi}{c^3} \nu^2 d\nu k_B T$$

Max Planck's analysis of Cavity Oscillators

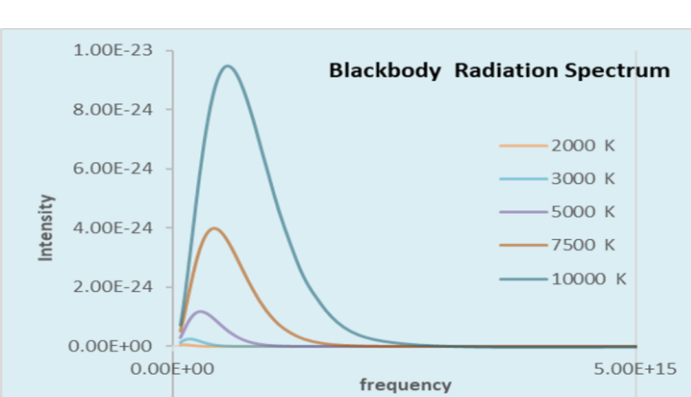
- *Harmonic oscillators have frequency which is an integer multiple of a fundamental frequency*
- *Energy states are discrete and not continuous*
- *Average energy of oscillators $\langle E \rangle \Rightarrow kT$ as $\nu \Rightarrow 0$*
- *Average energy of oscillators $\langle E \rangle \Rightarrow 0$ as $\nu \Rightarrow \infty$*
- *Average energy has to be evaluated by a summation and not an integration*

The average energy of these oscillators using the Boltzmann distribution function

$$\langle E \rangle = \frac{\sum E * P(E)}{\sum P(E)}$$
$$= \frac{h\nu}{\exp^{h\nu/kT} - 1}$$

The energy density of radiation

$$\rho(\nu)d\nu = \langle E \rangle * dN$$
$$= \frac{8\pi}{c^3} \nu^2 d\nu \frac{h\nu}{\exp^{(h\nu/kT)} - 1}$$
$$= \frac{8\pi h \nu^3}{c^3} \cdot \frac{1}{\exp^{(h\nu/kT)} - 1} d\nu$$



Max Planck's analysis of Cavity Oscillators



Max Planck could fit the experimental results with a value of

$h = 6.57 \times 10^{-34} \text{ Js} \dots$

very close to more accurate estimations of $h = 6.626 \times 10^{-34} \text{ Js}$

*Quantization of energy of radiation introduced by Planck
even before Einstein's photon hypothesis!*

The foundation stone of the era of quantum physics

The black body radiation concepts which are correct ...

- 1. Every body which is black is a blackbody**
- 2. Rayleigh and Jeans could explain the radiation curves for the higher wavelengths and not the lower wave lengths**
- 3. The radiation density is dependent of the volume of the cavity under consideration**
- 4. Classically the average energy of the oscillators cannot be found**
- 5. Max Planck suggested that the average energy of oscillators have to evaluated using a summation of energies and probabilities**



THANK YOU

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