ENGINEERING MATHEMATICS - I

Ordinary Differential Equations

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Unit 3 : Ordinary Differential Equations

Session : 2

Sub Topic : Introduction

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Contents

- Classification of Differential Equations
- Order & Degree
- Methods of solving First order Differential Equations
- Recapitulation of Linear Differential Equations

Methodology

Formulation of differential equation
(using the given physical situation)

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Solve the differential equation using suitable methods

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Physical interpretation of the solution

Differential Equations

- Ordinary Differential Equations
- Partial Differential Equations

Definitions :

- *Ordinary Differential Equation* : A differential equation in which all the differential coefficients or differentials have reference to a *single independent variable* is called an ordinary differential equation.

- *Partial Differential Equation* : A differential equation in which there are *two or more independent variables* and the partial differential coefficients are with respect to any one or more of them is called a partial differential equations.

Recall !!

- *Order* : The order of a differential equation is the order of the highest derivative in it.

- *Degree* : The degree of a differential equation is the exponent of the highest derivative.(*when the derivatives are cleared off radicals and fractions*).

Type, Order, Degree & Linearity

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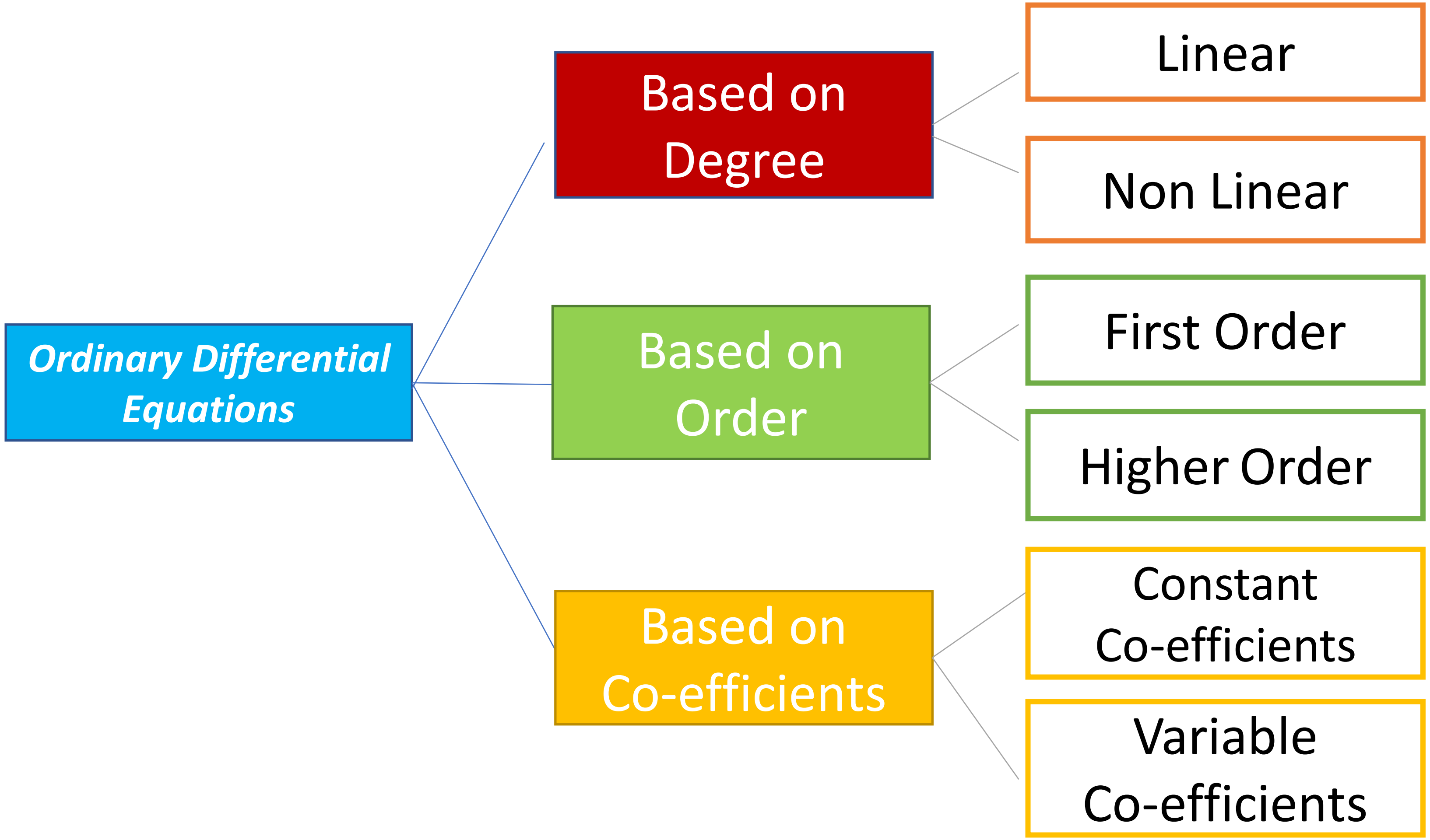
Sl No.	Differential Equation	Ordinary/ Partial	Order	Degree	Linear/ Non Linear
1	$\frac{dy}{dx} = y$	Ordinary	1	1	Linear
2	$\frac{d^2y}{dx^2} + 4y = 0$	Ordinary	2	1	Linear
3	$x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} + x = 0$	Ordinary	1	2	Non Linear
4	$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2} = c\frac{d^2y}{dx^2}$	Ordinary	2	2	Non Linear
5	$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2u$	Partial	1	1	Linear


Ordinary Differential Equations
Based on Degree
Linear
Non Linear
Based on Order
First Order
Higher Order
Based on Co-efficients
Constant Co-efficients
Variable Co-efficients

Ordinary Differential Equation (First Order, First degree)

- Equations in Variable Separable Form
- Homogeneous Equations
- Linear Equations
- Exact Equations

Standard form of a linear differential Equation and its solution

❖ Any differential equation of the form

$$\frac{dy}{dx} + Py = Q$$

where ***P and Q are functions of x only*** is called a **linear equation in *y*.**

Integrating Factor = $e^{\int P dx}$

Solution : $y(IF) = \int Q.(IF)dx + c$

Recapitulation of Linear Differential Equations

Standard form of a linear differential Equation and its solution

- Any differential equation of the form

$$\frac{dx}{dy} + Px = Q$$

where ***P and Q are functions of y only*** is called a **linear equation in *x*.**

Integrating Factor = $e^{\int P dy}$

Solution : $x(IF) = \int Q.(IF)dy + c$

Example 1

Consider, $\frac{dy}{dx} + ycotx = cosx$

This is a linear differential equation of the form,

$$\frac{dy}{dx} + Py = Q \text{ (Linear in y)}$$

where $P = cotx$ and $Q = cosx$

Integrating Factor = $\mathrm{IF} = \mathrm{e}^{\int \mathrm{P}dx} = \mathrm{e}^{\int \mathrm{cotx}dx} = \sin x.$

General Solution : $y(IF) = \int Q.(IF)dx + c$

$$\Longrightarrow y \sin x = \int \cos x \sin x \, dx + \mathrm{c}$$

$\Longrightarrow y = \frac{1}{2} sinx + c \; cosec \; x$ is the required solution.

Example 2

Consider, $(x+y+1)\frac{dy}{dx} = 1$

The given equation may be rewritten as , $\frac{dx}{dy} - x = y + 1$

This is a linear differential equation of the form,

$$\boldsymbol{\frac{dx}{dy} + Px = Q} \textbf{ (Linear in x)}$$

where $P = -1\,, Q = y + 1$

$$Integrating\ factor : IF = e^{\int Pdy} = \mathrm{e}^{\int -1\,\mathrm{d}y} = e^{-y}$$

Example 2

General Solution : $\boldsymbol{x(IF) = \int Q.(IF)dy + c}$

$$\Longrightarrow x\, e^{-y} = \int (\mathrm{y}+1)e^{-y}\mathrm{dy} + \mathrm{c}$$

$$\Longrightarrow x\, e^{-y} = -(\mathrm{y}+1)e^{-y} - e^{-y} + c$$

Simplifying further, $(x+y+2) = ce^{y}$ is the required solution.

THANK YOU

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