ENGINEERING MATHEMATICS - I

Partial Differentiation

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UNIT 2 : Partial Differentiation

Session : 5

Sub Topic : Implicit Functions, Problems

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Implicit Function

Consider an implicit function $x^3y + 3xy^2 = 3$

Then,

$x^3\frac{dy}{dx} + 3yx^2 + 3y^2 + 6xy\frac{dy}{dx}$=0,

$\frac{dy}{dx} = -\frac{3y(x+y)}{x(x^2+6y)}$

Similarly, if $z = f(x_1, x_2, x_3, \ldots\ldots, x_n)$

total differential of z is defined by

$$dz = f_{x_1}\,dx_1 + f_{x_2}\,dx_2 + f_{x_3}\,dx_3 + \cdots + f_{x_n}\,dx_n$$

Implicit Function

If $z = f(x,y)$ is a function of two independent variables, then

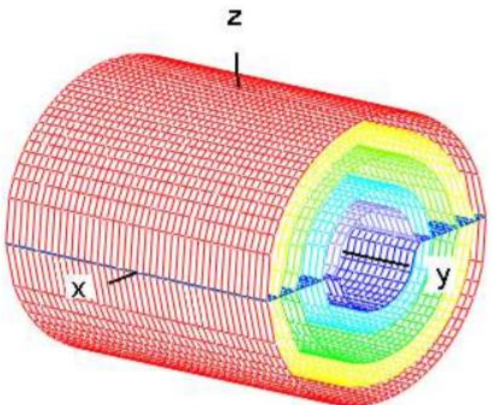
$$dz = f_x(x,y)dx + f_y(x,y)dy$$

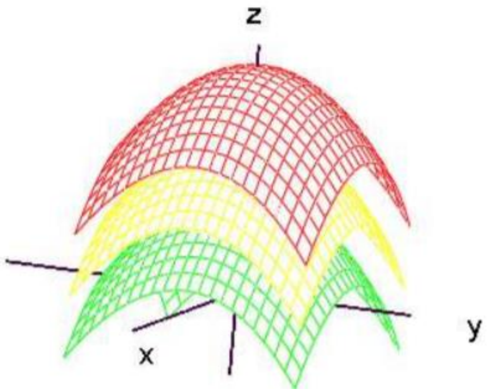
If z represents a level surface, i.e. if $z = f(x,y) = c$, then

$$dz = 0, \Rightarrow \frac{dy}{dx} = -\frac{f_y(x,y)}{f_x(x,y)}$$

The total derivative is also called the differential.

The differential is used in integration by substitution and in approximation. It is also useful to calculate the Errors in approximation.





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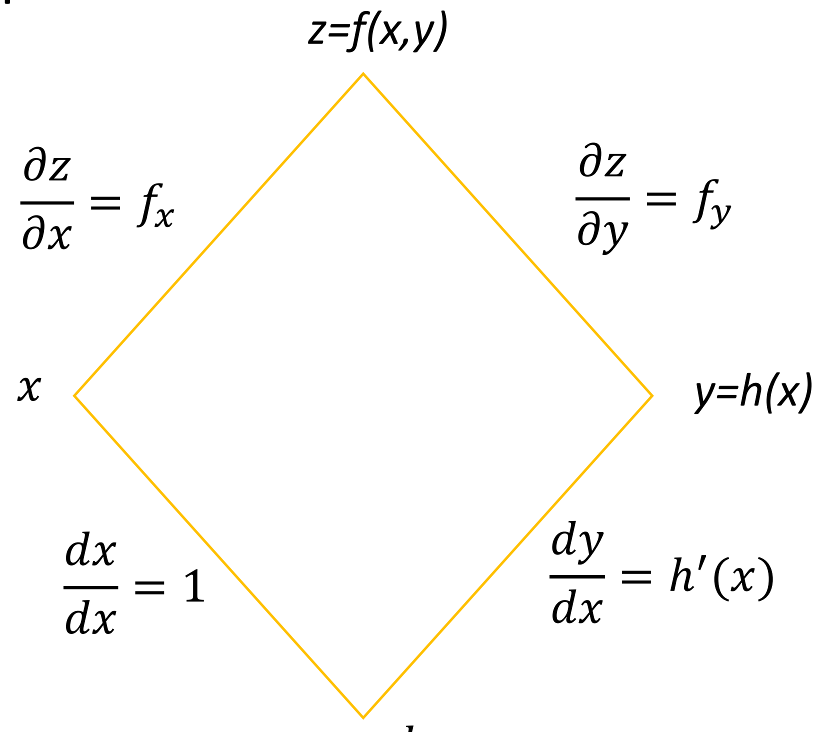
Branch diagram for differentiating $z = f(x, y)$ w.r.t x.

Setting $\frac{dz}{dx} = 0$ leads to a simple computation formula for implicit differentiation



$$f_x . 1 + f_y \frac{dy}{dx} = 0$$

Implicit function of three variables

Let $f(x,y,z)=0$ be the equation of an implicit function of three variables x,y,z. Suppose y and z are functions of x, then f is a function of one independent variable x and y , z are intermediate variables.

Keeping z constant, differentiating w.r.t. x, we get

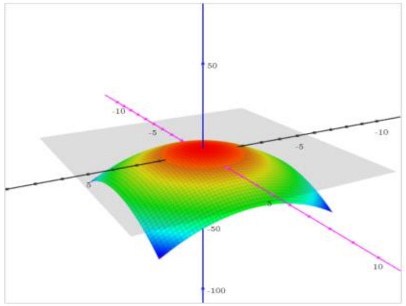
$$\frac{\partial f}{\partial x}+\frac{\partial f}{\partial y}\frac{\partial y}{\partial x}=0$$

Solving, $\frac{\partial y}{\partial x}=-\frac{f_x}{f_y}$ provided $f_y\neq 0$.

Similarly, differentiating w.r.t. x, holding y constant

$$\frac{\partial f}{\partial x}+\frac{\partial f}{\partial z}\frac{\partial z}{\partial x}=0,$$

Solving, $\frac{\partial z}{\partial x}=-\frac{f_x}{f_z}$, if $f_z\neq 0$.



https://www.intmath.com/vectors/3d-grapher.php

Implicit Functions -Problems

1. For the curve $xe^{y}+ye^{x}=0$, find the equation of the tangent line at the origin.

Solution:

$xe^{y}+ye^{x}=0$………………(1)

Differentiating w.r.t x,

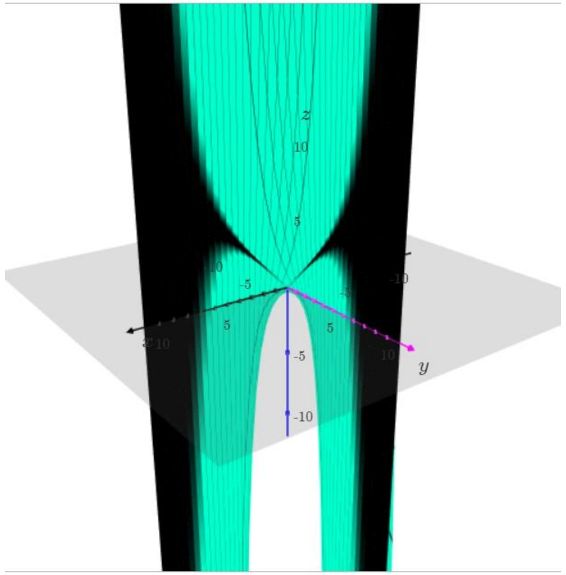
$$\frac{dy}{dx} = -\frac{ye^{x} + e^{y}}{xe^{y} + e^{x}}$$

Slope of the tangent at origin

$$\frac{dy}{dx}\, at\ (0,0) = -1$$

Equation of the tangent line at(0,0),

$$y + x = 0$$



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2. Find $\frac{dy}{dx}$ if $(cosx)^y = (siny)^x$

Solution:

$$Let\ f(x, y) = (cosx)^y - (siny)^x = 0$$

$$\frac{dy}{dx} = -\frac{f_x}{f_y}$$

$$= [\frac{y(cosx)^{y-1}(-sinx) - (siny)^x\ log(siny)}{(cosx)^y\ log(cosx) - x(siny)^{x-1}(cosy)}]$$

$$= \frac{siny(ysinx + cosxlog(siny)}{cosx(sinylog(cosx) - xcosy)}$$

3. Find $\frac{\partial y}{\partial x}$ and $\frac{\partial z}{\partial x}$ at (0,1,2) at $z^3 + xy - y^2z = 6$.

Solution:

$f = z^3 + xy - y^2z - 6 = 0$

Keeping z as constant

$\frac{\partial y}{\partial x} = \frac{-\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}} = \frac{-y}{x-2yz}$

$\frac{\partial y}{\partial x}$ at (0,1,2)=$\frac{1}{4}$

Contd.....

Keeping y as constant

$$\frac{\partial z}{\partial x} = \frac{-\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial z}}$$

$$= \frac{-y}{3z^2 - y^2}$$

$\frac{\partial z}{\partial x}$ at (0,1,2)$= \frac{-1}{11}$

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