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ENGINEERING PHYSICS

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Class #11

- One dimensional Schrodinger's time dependent wave equation
- Time dependent and position dependent wave functions
- Schrodinger's time independent wave equation

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➢ *Suggested Reading*

1. *Concepts of Modern Physics, Arthur Beiser, Chapter 5*
2. *Learning Material Unit II prepared by the Department of Physics*

➢ *Reference Videos*

1. *Video lectures : MIT 8.04 Quantum Physics I*
2. *Engineering Physics Class #10*

Schrodinger's wave equation

- *Schrodinger's formalism of a wave equation*
- *The total energy of a system* $E = KE + V$
- *This equation remains invariant when multiplied by* $\psi(x,t)$

$$E\psi(x,t) = KE\psi(x,t) + V\psi(x,t)$$

- *The terms in the equations can be rewritten in terms of operators*

$$\hat{E}\psi(x,t) = \widehat{KE}\psi(x,t) + V\psi(x,t)$$

$$\left\{i\hbar\frac{\partial}{\partial t}\right\}\psi(x,t) = \left\{-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\right\}\psi(x,t) + V\psi(x,t)$$

Schrodinger's wave equation

$$i\hbar\frac{\partial\psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} + V\psi(x,t)$$

$$\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} + i\hbar\frac{\partial\psi(x,t)}{\partial t} - V\psi(x,t) = 0$$

This is the Schrodinger's time dependent wave equation (non-relativistic)

The solution of this differential equation yields the wave function and its time evolution

Schrodinger's wave equation

In three dimensions the Schrodinger's time dependent wave equation

$$\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\psi(r,t)+i\hbar\frac{\partial\psi(r,t)}{\partial t}-V\psi(r,t)=0$$

$$\frac{\hbar^2}{2m}\nabla^2\psi(r,t)+i\hbar\frac{\partial\psi(r,t)}{\partial t}-V\psi(r,t)=0$$

Schrodinger's time independent wave equation

- *Steady state systems – observables are time invariant*

- *The wave function* $\psi = Ae^{\frac{i}{\hbar}(px-Et)}$ *can be expressed as*

$$\psi(x,t) = Ae^{\frac{i}{\hbar}(px)}e^{-\frac{i}{\hbar}Et} = \psi(x)\varphi(t)$$

where $\psi(x) = Ae^{\frac{i}{\hbar}(px)}$ *is the space dependent component*

and $\varphi(t) = e^{-\frac{i}{\hbar}Et}$ *is the time dependent component*

Schrodinger's time independent wave equation

- *Substituting for $\psi(x,t)$ in the time dependent Schrodinger's equation*

$$\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2}.\varphi(t)+i\hbar\frac{\partial\varphi(t)}{\partial t}\psi(x)-V\psi(x).\varphi(t)=0$$

- *the time derivative of* $\varphi(t)$ *yields* $-\frac{i}{\hbar}E.\varphi(t)$

- *Substituting back into the above equation*

$$\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2}.\varphi(t)+E.\varphi(t).\psi(x)-V\psi(x).\varphi(t)=0$$

Schrodinger's time independent wave equation

- *Rewriting the equation*

$$\left[\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + E.\psi(x) - V\psi(x)\right].\varphi(t) = 0$$

- *the product of two functions is zero implies that either of the terms is zero*

- $\varphi(t)$ *cannot be zero*

- *Hence*

$$\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + (E - V)\psi(x) = 0$$

which is the Schrodinger's time independent wave equation

Schrodinger's time independent wave equation

- *Writing the equation in the standard form of a differential equation*

$$\frac{\partial^2\psi(x)}{\partial x^2}+\frac{2m}{\hbar^2}(E-V)\psi(x)=0$$

- *The solution of this equation gives the wave function of a steady state system*
- *The region in which the particle is moving can be defined by the potential function*
- *The nature of the solution is very much dependent on the potential function*

Schrodinger's time independent wave equation

- *In the case when (E-V) is positive and constant*

$$\frac{\partial^2\psi(x)}{\partial x^2} + k^2\psi(x) = 0$$

- *where* $k^2 = \frac{2m}{\hbar^2}(E - V)$ *or* $k = \sqrt{\frac{2m}{\hbar^2}(E - V)}$
- *This is the familiar equation for a simple harmonic motion*
- *The solution describes a wave motion*

Framework for solving Schrodinger's wave equation

1. *Problem statement –*
   a) *The particle and its energy*
   b) *The potential energy of the particle*
   c) *The range in which the particle is can be found*
2. *Write the Schrodinger's wave equation relevant to the problem*
3. *Obtain solution of the SWE - $\psi(x)$*
4. *Verify whether $\psi(x)$ is an acceptable function*
   a) *$\psi(x)$ and it's derivatives are finite, continuous and single valued*
   b) *$\psi(x)$ is normalized*

ENGINEERING PHYSICS

Class #11 …….. Quiz….

The concepts which are true of Schrodinger's wave equations …

1. Standard wave equations involve second order derivative in space and time
2. Schrodinger's wave equation is a standard wave equation
3. The operator for the kinetic energy is $\left\{-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\right\}$
4. At t=0, $\varphi(t)$ is zero
5. The time independent wave equation can be used to evaluate steady state systems
6. The wave function is always an imaginary function

THANK YOU

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