ENGINEERING MATHEMATICS - I

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Class Content

- Geometric Series
- Conditions for convergence
- Conditions for convergence
- Examples on Geometric series

Geometric Series, conditions for convergence and divergence

If the series $\sum_{n=1}^{\infty} a_n$ converges, then the limit of $a_n \to 0$.

The test says that if the terms a_i do not go to zero, *then there is* **no way** *for the series of partial sums to converge. The series Does NOT converge.*

TEST

(The Geometric Series)

The geometric series $\mathbf{1 + x + x^2 + x^3 + \cdots}$ to∞

$$i)\ \textit{converges if } -1 < x < 1 \ \textit{i.e}\ |x| < 1$$

$$ii)\ \textit{diverges if } x \geq 1$$

$$iii)\ \textit{oscillates if } x = -1$$

$$iv)\ \textit{oscillates if } x < -1$$

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Geometric Series

Proof : Let $S_n = 1 + x + x^2 + x^3 + \cdots + x^n$.

Case 1) $|x| < 1$

$$Here, S_n = \frac{1 - x^n}{1 - x} = \frac{1}{1 - x} - \frac{x^n}{1 - x}, \text{so that,}$$

$$\lim_{n \to \infty} S_n = \frac{1}{1 - x},$$

a finite quantity. Geometric series is convergent.

Case 2 : i) When $x > 1$

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Case 2) ii) When $x = 1$

$$S_n = 1 + 1 + 1 + \cdots + 1 = n$$
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Therefore the series is divergent.

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Case 3 :

i) When $x = -1$, the series becomes,
$1 - 1 + 1 - 1 + 1 \cdots to \infty$
$S_n = -1 + 1 - 1 + \cdots$ to n terms, $= 1 or \;\; 0$
according as n is odd or even.
Since the subsequences $S_{2n-1} \to 1$ and $S_{2n} \to 0$,
the sequence of partial sums S_n oscillates finitely.

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Case 3) ii) When $x < -1$

$x < -1 \rightarrow -x > 1$,

Let $r = -x$, then $r > 1$,

$\therefore r^n \rightarrow \infty$ as $n \rightarrow \infty$

$S_n = 1 + x + x^2 + x^3 + \cdots \text{to} n \text{terms}$

$= \dfrac{1 - x^n}{1 - x} = \dfrac{1 - (-r)^n}{1 + r} = \dfrac{1 - r^n}{1 + r} \text{or} \dfrac{1 + r^n}{1 + r}$

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$\therefore S_{2n} \rightarrow -\infty$ and $S_{2n} \rightarrow +\infty$

This implies that the sequence $\{S_n\}$ oscillates infinitely .

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Examples on Geometric Series

1. Determine if the series $\sum_{n=1}^{\infty} \frac{5^n}{7^n}$ converges or diverges:

Solution: The given series is a geometric series.

Here, the common ration $r = \frac{5}{7} < 1$.

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2. The series $\sum_{n=1}^{n=\infty} \frac{1}{2^n}$ is a geometric series with common ratio $\frac{1}{2} < 1$. Hence it is convergent.

3. The series $\sum_{n=1}^{n=\infty} \frac{7^n}{3^n}$ is a geometric series with common ratio $\frac{7}{3} > 1$. Hence it is divergent.

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Examine the convergence of the series :

1) $\frac{3}{5} + \frac{4}{5^2} + \frac{3}{5^3} + \frac{4}{5^4} + \cdots\cdots\infty \rightarrow$ convergent

Solution :

$\Rightarrow \left(\frac{3}{5} + \frac{3}{5^3} + \frac{3}{5^6} + \cdots\cdots\infty\right) + \left(\frac{4}{5^2} + \frac{4}{5^4} + \cdots\cdots\infty\right)$

$\Rightarrow \quad \Sigma U_n + \Sigma V_n \rightarrow$ sum of convergent series

$\Rightarrow \Sigma U_n$ is Geometric series with common ratio $\frac{1}{5^2} < 1$

$\therefore \quad \Sigma U_n$ is convergent

ΣV_n is Geometric series with common ratio $\frac{1}{5^2} < 1$

$\therefore \quad \Sigma V_n$ is convergent

Examples on Geometric Series

2) $a - b + a^2 - b^2 + a^3 - b^3 + \cdots\cdots \infty$ → convergent whenever $|a| < 1$ and $|b| < 1$

Divergent $a \geq 1$, $b \geq 1$

Solution :

$$(a + a^2 + a^3 + \cdots \infty) - (b + b^2 + b^3 + \cdots \infty)$$

$$\Rightarrow \sum U_n - \sum V_n \rightarrow$$

$\sum U_n$ is Geometric series with common ratio 'a'

$\sum U_n$ is convergent when $|a| < 1$

divergent when $\neq$ a is ≥ 1

$\sum V_n$ is convergent when $|b| < 1$

divergent $b \geq 1$

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Examples on Geometric Series

3) $\frac{1}{2} + \frac{1}{3^2} + \frac{1}{2^3} + \frac{1}{3^4} + \cdots\cdots \infty \rightarrow$ convergent

Solution :

$$\left(\frac{1}{2} + \frac{1}{2^3} + \cdots \infty\right) + \left(\frac{1}{3^2} + \frac{1}{3^4} + \cdots \infty\right)$$

$\Rightarrow$ $\sum U_n$ is Geometric series

$\sum V_n$ is also Geometric series

$\sum U_n \rightarrow$ common ratio $\frac{1}{2^2} < 1$

$\sum V_n \rightarrow$ common ratio $\frac{1}{3^2} < 1$

$\Bigg\}$ convergent

$\sum U_n + \sum V_n$ is convergent

Examples on Geometric Series

THANK YOU

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