ENGINEERING MATHEMATICS - I

Problems on Taylor's and Maclaurin's Series

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UNIT 2 : Partial Differentiation

Session : 9

Sub Topic : Problems on Taylor’s and Maclaurin’s Series of a Function of Two Variables

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Problems on Taylor's and Maclaurin's expansion

1. Expand $e^x cosy$ in powers of $(x - 1)$ and $(y - \frac{\pi}{4})$ using Taylor's series.

Solution:

$f(x, y) = e^x\ cosy$; $\quad f(1, \frac{\pi}{4}) = \frac{e}{\sqrt{2}}$

$f_x = e^x cosy$; $\quad f_x(1, \frac{\pi}{4}) = \frac{e}{\sqrt{2}}$

$f_{xx} = e^x cosy$; $\quad f_{xx}(1, \frac{\pi}{4}) = \frac{e}{\sqrt{2}}$

$f_y = -e^x siny$; $\quad f_y(1, \frac{\pi}{4}) = -\frac{e}{\sqrt{2}}$

$f_{xy} = -e^x siny$; $\quad f_{xy}(1, \frac{\pi}{4}) = -\frac{e}{\sqrt{2}}$

$f_{yy} = -e^x cosy$; $\quad f_{yy}(1, \frac{\pi}{4}) = -\frac{e}{\sqrt{2}}$

Problems on Taylor's and Maclaurin's expansion

Therefore Taylor's series is given by

$$e^{x}cosy = \frac{e}{\sqrt{2}} + \frac{1}{1!}\left\{(x-1)\frac{e}{\sqrt{2}} + (y-\frac{\pi}{4})(-\frac{e}{\sqrt{2}})\right\} +$$
$$\frac{1}{2!}\left\{(x-1)^2\frac{e}{\sqrt{2}} + 2(x-1)\left(y-\frac{\pi}{4}\right)\left(-\frac{e}{\sqrt{2}}\right) + \left(y-\right.\right.$$

Problems on Taylor's and Maclaurin's expansion

2. Expand $f(x,y) = \log(2x + y + 1)$about the point (0,0) upto three terms.

Solution:

$f(x,y) = \log(2x + y + 1);\qquad f(0,0) = 0$

$f_x = \frac{2}{2x+y+1};\qquad f_x(0,0) = 2$

$f_y = \frac{1}{(2x+y+1)};\qquad f_y(0,0) = 1$

$f_{xx} = \frac{-4}{(2x+y+1)^2};\qquad f_{xx}(0,0) = -4$

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$$f_{yy}=\frac{-1}{(2x+y+1)^2};\qquad f_{yy}(0,0)=-1$$

$$f_{xy}=\frac{-2}{(2x+y+1)^2};\qquad f_{xy}(0,0)=-2$$

Therefore Maclaurin's series is given by

$$f(x,y)=\frac{1}{1!}\{2x+y\}+\frac{1}{2!}\{x^2(-4)+2xy(-2)+y^2(-1)\}+\cdots$$

Problems on Taylor's and Maclaurin's expansion

3. Find Taylor's series expansion of $f(x, y) = tan^{-1}(xy)$. Hence compute an approximate value of f(0.9, -1.2).

Solution:

We expand $f(x, y) = tan^{-1}(xy)$ near the point (1,-1).

$f(x, y) = tan^{-1}(xy)$; $\quad f(1, -1) = \frac{-\pi}{4}$

$f_x = \frac{y}{1+x^2y^2}$; $\quad f_x(1, -1) = \frac{-1}{2}$

$f_y = \frac{x}{1+x^2y^2}$; $\quad f_y(1, -1) = \frac{1}{2}$

Problems on Taylor's and Maclaurin's expansion

$$f_{xx} = \frac{-y(2y^2x)}{(1+x^2y^2)^2}\,; \qquad f_{xx}(1,-1) = \frac{1}{2}$$

$$f_{yy} = \frac{-x(2x^2y)}{(1+x^2y^2)^2}; \qquad f_{yy}(1,-1) = \frac{1}{2}$$

$$f_{xy} = \frac{1-x^2y^2)}{(1+x^2y^2)^2}; \qquad f_{xy}(1,-1) = 0$$

$$tan^{-1}(xy)$$
$$= \frac{-\pi}{4} + \frac{1}{2}[-(x-1)+(y+1)] + \frac{1}{4}[(x-1)^2+(y+1)^2]$$
$$+ \cdots$$

Problems on Taylor's and Maclaurin's expansion

$$tan^{-1}(xy) = -\frac{\pi}{4} + (x-1)\left(-\frac{1}{2}\right) + (y+1)\left(\frac{1}{2}\right) + \cdots$$

Substituting $x = 0.9$ and $y = -1.2$ in the above equation, we get

$$tan^{-1}(0.9 * (-1.2)) \approx \frac{-\pi}{4} - \frac{0.1}{2}$$

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