ENGINEERING PHYSICS

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Class #17

- Bound particle system
- 1D infinite potential well
- Solution of the Schrodinger's wave equation
- Characteristics of the wave function

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- ***Suggested Reading***

  1. *Concepts of Modern Physics, Arthur Beiser, Chapter 5*
  2. *Learning Material prepared by the Department of Physics*

- ***Reference Videos***

  1. *Video lectures : MIT 8.04 Quantum Physics I*
  2. *Engineering Physics Class #12-14*

Particle in an 1D infinite potential well

A particle restricted to move in a small region of space with zero potential – a bound particle system

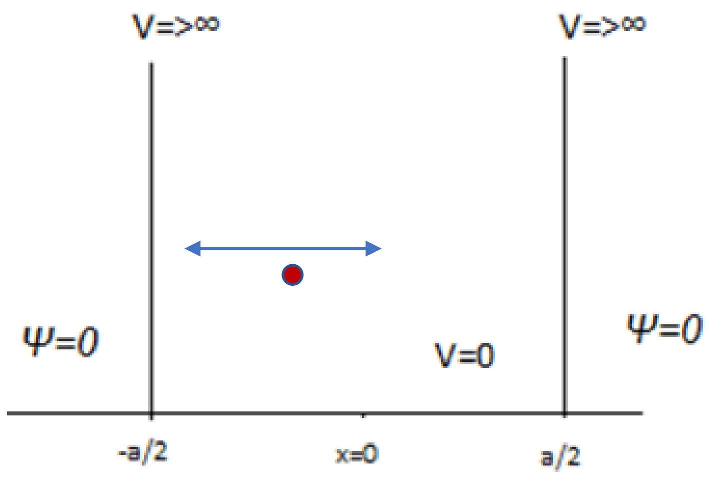
The binding comes from an infinite potential (wall) existing at the boundaries at $x = \pm\frac{a}{2}$

*The particle of mass **m** and energy **E** is in a state of motion between* $-\frac{a}{2}$ *and* $+\frac{a}{2}$

$$V = 0 \quad for -\frac{a}{2} < x < +\frac{a}{2}$$

$$V = \infty \quad at \;\; x = \pm\frac{a}{2}$$

Particle has zero probability of being outside the bound region



1D infinite potential Schrodinger's wave equation

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- ***The Schrodinger's wave equation***

$$\frac{\partial^2\psi(x)}{\partial x^2} + \frac{2m}{\hbar^2}(E - V)\psi(x) = 0$$

- ***Reduces to the free particle wave equation for the region between*** $-\frac{a}{2}$ ***and*** $+\frac{a}{2}$ ***with*** $V = 0$

$e^{i\theta} = \cos\theta + i\sin\theta$

- ***The general solution could be written as***

$$\psi(x) = A\sin(kx) + B\cos(kx)$$

where $k = \sqrt{\frac{2mE}{\hbar^2}}$ ***which gives the energy of the particle***

$$E = \frac{\hbar^2 k^2}{2m}$$

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the boundary conditions for the problem

$$\psi = 0 \quad and \quad d\psi = 0 \quad at\ x = -\frac{a}{2}\ and\ x = \frac{a}{2}$$

Additionally the wave function has to be normalizable

At $x = -\frac{a}{2}$

$$\psi\left(x = -\frac{a}{2}\right) = Asin\left(-k\frac{a}{2}\right) + Bcos\left(k\frac{a}{2}\right) = 0$$

$results\ in\quad -Asin\left(k\frac{a}{2}\right) + Bcos\left(k\frac{a}{2}\right) = 0$ [1]

At $x = \frac{a}{2}$

$$\psi\left(x = \frac{a}{2}\right) = Asin\left(k\frac{a}{2}\right) + Bcos\left(k\frac{a}{2}\right) = 0$$

$results\ in\quad Asin\left(k\frac{a}{2}\right) + Bcos\left(k\frac{a}{2}\right) = 0$ [2]

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[1] and [2] indicate the two distinct possibilities

$$either \quad A = 0 \;\; if\ B \neq 0 \quad or \;\; B = 0 \;\; if\ A \neq 0$$

$if\ A = 0 \quad then\ \ B \neq 0$

[1] leads to $\cos\left(k\frac{a}{2}\right) = 0 \quad k\frac{a}{2} = n\frac{\pi}{2}$

where n is an odd number n=1,3,5,7,…

$$k = n_{odd}\frac{\pi}{a}$$

$if\ \ A \neq 0 \quad then\ \ B = 0$

[2] leads to $\sin\left(k\frac{a}{2}\right) = 0 \quad k\frac{a}{2} = n\pi$

where n is an integer

$$k = n_{even}\frac{\pi}{a}$$

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The general solution …

$$\psi_n(x) = B\cos(kx) = B\cos\left(n\frac{\pi}{a}x\right) \quad for\ n\ odd$$

and

$$\psi_n(x) = A\sin(kx) = A\sin\left(n\frac{\pi}{a}x\right) \quad for\ n\ even$$

n = 1, 2, 3, 4, 5… correspond to the allowed wave functions of the particle

n describes the state of the system

1D infinite potential Schrodinger's wave equation

The constant A and B can be evaluated by normalizing the wave function i.e..,

integrating the wave function between limits of $-\frac{a}{2}$ *and* $\frac{a}{2}$

$$\int \psi^* \psi dx = 1$$

$$\int_{-a/2}^{a/2} \left[A sin(\frac{n\pi}{a}x)\right]^2 dx = \frac{A^2}{2} \int_{-a/2}^{a/2} \left[1 - cos(\frac{2n\pi}{a}x)\right] dx$$

$$\frac{A^2}{2} \left[x - \frac{L}{2n\pi} \sin(\frac{2n\pi}{a}x)\right]_{-a/2}^{a/2} = \frac{A^2}{2} [a - 0] = 1 \quad \Longrightarrow \quad A = \sqrt{\frac{2}{a}}$$

In a very similar way the second wave function can be

normalized to get $B = \sqrt{\frac{2}{a}}$

1D infinite potential Schrodinger's wave equation

The eigen wave function becomes

$$\psi_n(x) = \sqrt{\frac{2}{a}} \cos\left(\frac{n\pi}{a} x\right) \quad for\ n\ odd$$ (even parity)

$$\psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a} x\right) \quad for\ n\ even$$ (odd parity)

The propagation constant $k = \sqrt{\frac{2mE}{\hbar^2}} = \frac{n\pi}{a}$ is quantized and hence the momentum and energy of the particle are also quantized.

$P_n = \hbar k = \hbar \frac{n\pi}{a}$

$\lambda_n = \frac{h}{p}$

The energy of the allowed states

$$E_n = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2 n^2 \pi^2}{2ma^2} = \frac{h^2 n^2}{8ma^2}$$

These are the eigen energy values of the bound particle.

Class #17 …….. Quiz….

The concepts which are true of a particle in a box…

1. Particle in a box is an example of a bound particle system
2. The eigen function of the particle in the second state is an even parity function
3. The propagation constant is determined by the width of the box.
4. The energy of the particle in the ground state is non zero
5. Particle can have any momentum inside the box
6. The de Broglie wavelength depends on the size of the box

THANK YOU

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