ENGINEERING MATHEMATICS - I

Ordinary Differential Equations

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Unit 3 : Ordinary Differential Equations

Session : 3

Sub Topic : Bernoulli’s Differential Equation

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Bernoulli's Differential Equation

❖ Any differential equation of the form

$$\frac{dy}{dx} + Py = Qy^n$$

where ***P and Q are functions of x only*** is called as **Bernoulli's Differential equation in y**.

To reduce this to the linear equation in standard form,

Step 1 : Divide throughout by y^n to obtain

$$\frac{1}{y^n} \cdot \frac{dy}{dx} + Py^{1-n} = Q$$

Bernoulli's Differential Equation

Step 2 : Take the substitution $y^{1-n} = z$, then

$$(1-n)y^{-n}\frac{dy}{dx} = \frac{dz}{dx}$$

and

$$\frac{dz}{dx} + P'z = \mathrm{Q}'$$

which is linear in z

Integrating Factor = IF = $e^{\int P' dx}$

Solution **:** $z(\mathrm{IF}) = \int \mathrm{Q}'.(\mathrm{IF})dx + \mathrm{c}$

Bernoulli's Differential Equation

❖ Any differential equation of the form

$$\frac{dx}{dy} + Px = Qx^n$$

where ***P and Q are functions of y only*** is called as **Bernoulli's Differential equation in x**.

To reduce this to the linear equation in standard form,

Step 1 : Divide throughout by x^n to obtain

$$\frac{1}{x^n} \cdot \frac{dx}{dy} + Px^{1-n} = Q$$

Bernoulli's Differential Equation

- Take the substitution $x^{1-n} = z$, then

$$(1-n)x^{-n}\frac{dx}{dy} = \frac{dz}{dy}$$

and

$$\frac{dz}{dx} + P'z = Q'$$

which is linear in z

Integrating Factor = IF = $e^{\int P' dy}$

Solution : $z(\mathrm{IF}) = \int Q'.(\mathrm{IF})dy + c$

Example 1.

Solve : $\frac{dy}{dx} + \frac{y}{x} = y^2 x$

Solution : The given equation is the Bernoulli's equation of the form,

$$\frac{dy}{dx} + \mathrm{P}y = \mathrm{Q}y^{\mathrm{n}} \quad (\textbf{Linear in } y)$$

where $\mathrm{P} = \frac{1}{x}$, $\mathrm{Q} = x$ and $\mathrm{n} = 2$.

- Dividing throughout by y^2, $\frac{1}{y^2}\frac{dy}{dx} + \left(\frac{1}{x\mathrm{y}}\right) = x$
- Taking the substitution, $y^{-1} = \mathrm{z}$, we obtain $\frac{dz}{dx} = \frac{-1}{y^2}.\frac{dy}{dx}$

On simplifying, $-\frac{dz}{dx} + \frac{1}{x}.\mathrm{z} = x$ or $\frac{dz}{dx} - \frac{\mathrm{z}}{x} = -x$

This is a linear differential equation of the form $\frac{dz}{dx} + \mathrm{P}'\mathrm{z} = \mathrm{Q}'$

Example 1.

where $\boldsymbol{P' = -\frac{1}{x}}$ **and** $\boldsymbol{Q' = x}$

Integrating Factor = $IF = e^{-\int P' dx} = e^{-\int \frac{1}{x} dx} = \frac{1}{x}$

General Solution : $\boldsymbol{z(IF) = \int Q'.(IF)dx + c}$

$$\Rightarrow z.\frac{1}{x} = \int -x.\frac{1}{x} dx + c$$

$$\Rightarrow z.\frac{1}{x} = -x + c$$

$$\Rightarrow \frac{1}{xy} = -x + c$$ *is the required solution.*

Example 2.

Solve : $\frac{dr}{d\theta} = r\tan\theta - \frac{r^2}{\cos\theta}$

Solution : The given equation can be written as

$$\frac{dr}{d\theta} - r\tan\theta = -\frac{r^2}{\cos\theta}$$

This is the Bernoulli's equation linear in r
where $\mathrm{P} = -\tan\theta, \mathrm{Q} = -1/\cos\theta$ and $n = 2$

- Dividing throughout by $\mathbf{r^2}$,

$$\frac{1}{r^2}\frac{dr}{d\theta} - \frac{1}{r}\tan\theta = -\frac{1}{\cos\theta} = -\sec\theta$$

Example 2.

- Taking the substitution, $\boldsymbol{\frac{1}{r} = t}$, we obtain $\frac{dt}{d\theta} = \frac{-1}{r^2} . \boldsymbol{\frac{dr}{d\theta}}$

Therefore $-\frac{dt}{d\theta} - t.tan\theta = -sec\theta$

$$\Longrightarrow \frac{dt}{d\theta} + t.tan\theta = sec\theta$$

This is a linear differential equation of the form $\boldsymbol{\frac{dt}{dx}} + P't = Q'$

where $\boldsymbol{P' = tan\theta}$ **and** $\boldsymbol{Q' = sec\theta}$

Example 2.

Integrating Factor = $\mathrm{IF} = e^{\int P' d\theta} = e^{\int tan\theta d\theta} = e^{\log(sec\theta)} = sec\theta$

General Solution : $\boldsymbol{t(IF) = \int Q'.(IF)d\theta + c}$

$$\Longrightarrow tsec\theta = \int sec^2\theta d\theta + c$$

$$\Longrightarrow tsec\theta = tan\theta + c$$

$\Longrightarrow \frac{sec\theta}{r} = tan\theta + c$ is the required solution.

THANK YOU

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