ENGINEERING PHYSICS

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Department of Science & Humanities

- ***Suggested Reading***

  1. ***Fundamentals of Physics, Resnik and Halliday, Chapter 41***
  2. ***Solid state Physics, S.O Pillai, Chapter 6***
  3. ***Concepts of Modern Physics, Arthur Beiser, Chapter 9***
  4. ***Learning materials prepared by the department-unit III***

- ***Reference Videos***

  1. ***Physics Of Materials-IIT-Madras/lecture-24.html***

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Unit III : Application of Quantum Mechanics to Electrical transport in Solids

PES UNIVERSITY ONLINE

Class# 25

- ***Density of states derivation***

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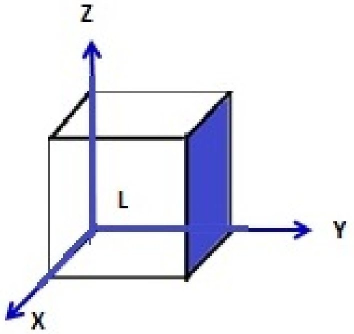
Density of states derivation



The x component of the particle motion described by the SWE $\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + E\psi(x) = 0$

Eigen energy value for the x component $E_{n_x} = \frac{h^2 {n_x}^2}{8mL^2}$

Similarly other two dimensions can be evaluated as

$E_{n_y} = \frac{h^2 n_y^2}{8mL^2}$ *and* $E_{n_z} = \frac{h^2 n_z^2}{8mL^2}$

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Density of states derivation

The total energy of the electron is given by

$$E_n = \frac{h^2}{8mL^2}\left({n_x}^2 + {n_y}^2 + {n_z}^2\right) = E_o R^2$$

Where $R^2 = {n_x}^2 + {n_y}^2 + {n_z}^2$ *and* $E_o = \frac{h^2}{8mL^2}$

The number of states with energy E can be evaluated by varying the combinations of n_x , n_y and n_z

Example :

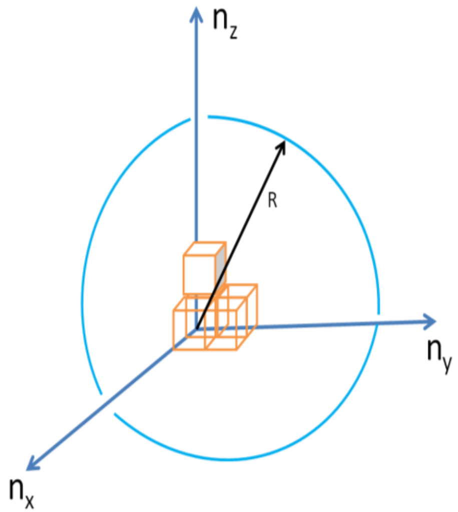
If n_x, n_y and n_z is (111) , then $E_{111} = 3E_o$, similarly

If n_x, n_y and n_z is (121) , (112), (211) all these states have energy equal to $6E_o$

Density of states derivation

- *The distribution of energy states depend on the combinations of n_x, n_y and n_z and can be evaluated by analyzing the n space formed by n_x, n_y and n_z all positive.*

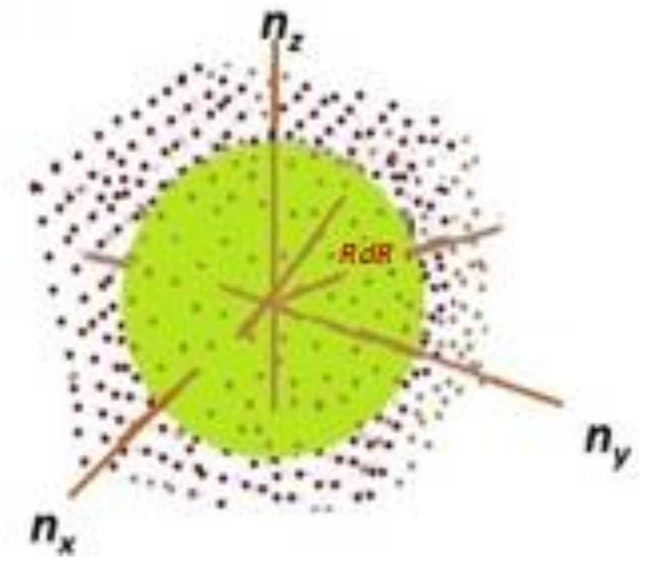
- *Every point in n-space given by the co-ordinates (n_x, n_y, n_z) and the points are at a distance R from the origin.*



- *Every combination of n_x , n_y and n_z result in an additional unit volume in n space .*

- *Hence evaluating the number of states is equivalent to evaluating the volume of the n space.*

Density of states derivation



- *Energy states with the same energy values lie on the surface of an octant of a sphere*
- *Hence it is sufficient to find the volume of sphere of radius R to evaluate the number of energy states up to R*

The sensitivity of increase in the states with increasing n_x, n_y and n_z can be found from the change in the volume of the octant if the radius changes from R to R+dR

$$V = \frac{1}{8}\left(\frac{4}{3}\pi(R + dR)^3 - \frac{4}{3}\pi R^3\right)$$

Where R >> dR

$$dV = \frac{\pi R^2 dR}{2}$$

This gives the number of energy states available between R and R+dR

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Density of states derivation

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The energy expression is given by $E_n = E_o R^2$ *where* $E_o = \frac{h^2}{8mL^2}$

We can write $R^2 = \frac{E_n}{E_o}$ *and* $dR = \frac{dE}{2\{E_n E_o\}^{1/2}}$

Therefore the number of energy states between E and E+dE is given by

$$\frac{\pi R^2 dR}{2} = \frac{\pi}{4}\frac{E_n}{E_o}\frac{dE}{\{E_n E_o\}^{1/2}}$$

$$= \frac{\pi}{4} \cdot \frac{E_n^{1/2}}{E_o^{3/2}} \, dE$$

Density of states derivation

Substituting for $E_o = \frac{h^2}{8mL^2}$

we get the number of energy states between E and E+dE as

$$= \frac{\pi}{4} \cdot \left(\frac{8mL^2}{h^2}\right)^{3/2} E^{1/2} . dE$$

Now we have to apply a factor of 2 for the spin factor

$$= 2 . \frac{\pi}{4} \left(\frac{8m}{h^2}\right)^{\frac{3}{2}} E^{\frac{1}{2}} \, dE$$

Therefore density of states per unit volume is given by

$$g(E)dE = \frac{\pi}{2} \left(\frac{8m}{h^2}\right)^{\frac{3}{2}} E^{\frac{1}{2}} \, dE$$

This shows that the distribution of electrons in energy states vary non- linearly with increasing energy E.

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Class 25 . Quiz …

The concepts which are correct are….

1. *When n_x, n_y and n_z have values 1,2,3, the degree of degeneracy of this level is 3.*

2. *If the Eigen value of energy of the particle in a cubical box is $11h^2/8mL^2$, then the quantum numbers of the states are (3 1 1) .*

3. *The degenerate states with the same energy values lie on the surface of a sphere.*

4. *The density of states for electrons in a metal gives the number of electron states per unit volume with energy E.*

THANK YOU

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