ENGINEERING MATHEMATICS - I

Ordinary Differential Equations

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Unit 3 : Ordinary Differential Equations

Session : 10

Sub Topic : Non - Linear Differential Equations

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- **Equations solvable for *x* :**

Working procedure:

Step 1. ***Rewrite the given differential equation*** $\boldsymbol{f(x, y, p) = 0}$ *in the form* $\boldsymbol{x} = \boldsymbol{F(y, p)}$*………….(1)*

Step 2. ***Differentiate (1) w.r.t 'y'*** *to obtain the equation of the form,* $\frac{\mathbf{1}}{\boldsymbol{p}} = \emptyset\left(\boldsymbol{y}, \boldsymbol{p}, \frac{\boldsymbol{dp}}{\boldsymbol{dy}}\right)$*…………..(2) which is a first order and first degree differential equation in the variable p.*

Working procedure(Contd…)

Step 3. ***Solve the differential equation (2)*** *. The solution is of the form* $\boldsymbol{G(y, p, c) = 0}$*……………(3)*

Step 4. ***Eliminating p from equations (1) and (3)****, the required solution of the DE (1).*

NOTE :

Whenever it is not possible to eliminate p from equations (1) & (3), **the solution of the DE (1) is given by the parametric equations** $\mathbf{x = x(p, c)}$ & $\mathbf{y = y(p, c)}$

1. Solve $yp^2 - 2xp + y = 0$……………………….(1)

Answer : Solving the given equation for *'x'*

$$x = \frac{yp^2 + y}{2p} = \frac{yp}{2} + \frac{y}{2p}$$

Differentiating w.r.t *'y'*,

- $\frac{dx}{dy} = \frac{1}{2}\left\{p + y\frac{dp}{dy} + \frac{1}{p} - \frac{y}{p^2}\frac{dp}{dy}\right\}$
- $2\frac{1}{p} = p + \frac{dp}{dy} + \frac{1}{p} - \frac{y}{p^2}\frac{dp}{dy}$
- $\frac{1}{p} - p = \left(1 - \frac{y}{p^2}\right)\frac{dp}{dy}$
- $\frac{1}{p} - p = -\frac{y}{p}\left(\frac{1}{p} - p\right)\frac{dp}{dy}$

ODE- Non linear Differential Equation – Solvable for x

- $\left(1+\frac{y}{p}\frac{dp}{dy}\right)\left(\frac{1}{p}-p\right)=0$

Consider $\left(1+\frac{y}{p}\frac{dp}{dy}\right)=0$

- $\frac{dp}{p}+\frac{dy}{y}=0$
- $logp+logy=logc$
- $py=c$
- $p=\frac{c}{y}$

Substituting in (1)

- $y\left(\frac{c^2}{y^2}\right)-2x\left(\frac{c}{y}\right)+y=0$
- $c^2-2cx+y^2=0 \longrightarrow y^2=2cx-c^2$

2. Solve $xp^2 - yp - y = 0$……………………….(1)

Answer : Solving the given equation for ‘x’

$$x = \frac{yp + y}{p^2} = \frac{y}{p} + \frac{y}{p^2}$$

Differentiating w.r.t ‘y’,

- $\frac{dx}{dy} = y\left(\frac{-1}{p^2}\right)\frac{dp}{dy} + \frac{1}{p} + y\left(-\frac{2}{p^3}\right)\frac{dp}{dy} + \frac{1}{p^2}$
- $\frac{1}{p} = \left(\frac{-y}{p^2}\right)\frac{dp}{dy} + \frac{1}{p} + \left(-\frac{2y}{p^3}\right)\frac{dp}{dy} + \frac{1}{p^2}$
- $-\frac{1}{p^2} = \frac{-y}{p^2}\left(1 + \frac{2}{p}\right)\frac{dp}{dy}$
- $\frac{1}{y} = \left(1 + \frac{2}{p}\right)\frac{dp}{dy}$

ODE- Non linear Differential Equation – Solvable for x

- $\frac{dy}{y} = \left(1 + \frac{2p}{y}\right)dp$
- $logy = p + 2logp + c$
- $logy + logc = p + logp^2$
- $log\left(\frac{cy}{p^2}\right) = p$
- $\frac{cy}{p^2} = e^p$ or $y = p^2e^pc$

Substituting in (1)

$$x = \frac{p^2e^pc(1+p)}{p^2} = c(1+p)e^p$$

Therefore,

$x = c(1+p)e^p$, $y = p^2e^pc$ are the required solution in the parametric form.

THANK YOU

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