

Sunitha V R

Department of Science & Humanities

Unit III: Application of Quantum Mechanics to Electrical transport in Solids



- > Suggested Reading
 - 1. Fundamentals of Physics, Resnik and Halliday, Chapter 41
 - 2. Solid state Physics, S.O Pillai, Chapter 6
 - 3. Learning material prepared by the department-unit III

- > Reference Videos
 - 1. Physics Of Materials-IIT-Madras/lecture-25.html

Unit III: Application of Quantum Mechanics to Electrical transport in Solids



Class #27

- Expression for Fermi energy
- > Fermi temperature and Fermi velocity
- Average electron energy

Expression for Fermi energy



Expression for Fermi energy at 0K can be obtained by finding the number of electrons per unit volume occupying energy states from E = 0 to $E = E_E$

Number of occupied states is given by N(E) = g(E) * F(E)

$$n = \int_{0}^{E_{f}} g(E) * F(E) dE$$

$$= \frac{\pi}{2} \left(\frac{8m}{h^{2}}\right)^{\frac{3}{2}} \int_{0}^{E_{f}} E^{\frac{1}{2}} dE = \frac{\pi}{3} \left(\frac{8m}{h^{2}}\right)^{\frac{3}{2}} E_{f}^{3/2}$$

$$E_{f} = \left(\frac{3}{\pi}\right)^{2/3} \left(\frac{h^{2}}{8m}\right) n^{2/3}$$

Fermi velocity



The excited electrons have energies very close to the Fermi energy and hence the conduction electrons possess K.E equal to the Fermi energy.

$$E_f = \frac{1}{2}mv_f^2$$
 or $v_f = \sqrt{\frac{2E_f}{m}}$

where v_f is the Fermi velocity of the conduction electron

For conduction electrons in Copper, Fermi velocity is

$$v_f = 1.06 \times 10^6 \ ms^{-1}$$

This velocity is greater than the thermal velocity of electrons (as per the CFET)

Fermi Temperature



- ➤ Given thermal energy, only those electrons near the Fermi energy are excited into the conduction band.
- \triangleright If **T** is the temperature of the metal, then k_B **T** is the thermal energy available to the electrons in the metal.
- ightharpoonup Hence the thermal energy required to excite the last electron at the bottom of the energy band is $E_f = k_B T_f$

Where T_f is termed as the Fermi temperature of electrons in the metal.

Fermi Temperature



Example:

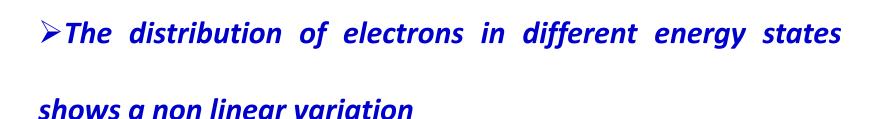
We will try to evaluate the temperature for Copper with Fermi energy as 7 eV

We know that
$$T_F = \frac{E_F}{k_B}$$

This will give the value of the Fermi temperature as ≈ 81000 K

It is obvious that at such high temperatures the metal cannot be in the solid state and hence this is only a representative temperature.

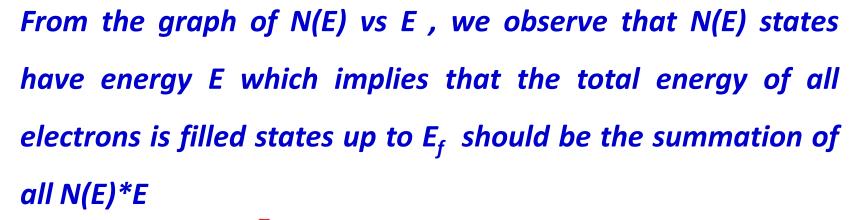
Average energy of electrons in a metal at 0K



- This means that the average energy of the electron is not the simple average of the electron's max and min energy.
- ➤ To obtain the average electron energy at OK , we have to estimate the total energy of the electron



Average energy of electrons in a metal at 0K



$$=\int_0^{E_f} g(E)dE * F(E) * E$$

 $< E> = \frac{Total\,energy\,of\,\,all\,electrons\,in\,different\,energy\,state}{Total\,number\,of\,\,electrons}$

$$=\frac{\int_0^{E_f} g(E) * E * F(E) dE}{\int_0^{E_f} g(E) * F(E) dE}$$



Average energy of electrons in a metal at 0K



$$=\frac{\frac{\pi}{2}\left(\frac{8m}{h^2}\right)^{\frac{3}{2}}\int_0^{E_f} E^{\frac{1}{2}} dE * E}{\frac{\pi}{2}\left(\frac{8m}{h^2}\right)^{\frac{3}{2}}\int_0^{E_f} E^{\frac{1}{2}} dE}$$

On integration we get average energy of the electron at OK

$$\langle E \rangle = \frac{3}{5}E_f$$

Class 27. Quiz ...

The concepts which are correct are....

- 1. The Fermi energy of metal at absolute zero temperature is proportional to $n^{2/3}$
- 2. Electrons cannot have energy greater than Fermi energy at temperature of 300 K
- 3. Only a small fraction of electrons close to the conduction band can take part in the conduction processes
- 4. Fermi temperature is a representative temperature which highlights that all valence electrons cannot be conduction electrons.
- 5. If the Fermi energy of silver at 0 K is 5 eV, then the mean energy of the electron is 6 eV.





THANK YOU

Sunitha VR, Ph.D.

Assistant Professor,
Department of Science and Humanities

sunithavr@pes.edu

+91 80 21722683 Extn 716