ENGINEERING PHYSICS

Sunitha V R
Department of Science & Humanities

Suggested Reading

1. *Fundamentals of Physics, Resnik and Halliday, Chapter 41*
2. *Solid state Physics, S.O Pillai, Chapter 6*
3. *Learning material prepared by the department-unit III*

Reference Videos

1. *Physics Of Materials-IIT-Madras/lecture-25.html*

ENGINEERING PHYSICS

Unit III : Application of Quantum Mechanics to Electrical transport in Solids

Class #27

- *Expression for Fermi energy*
- *Fermi temperature and Fermi velocity*
- *Average electron energy*

Expression for Fermi energy

Expression for Fermi energy at 0K can be obtained by finding the number of electrons per unit volume occupying energy states from $E = 0$ to $E = E_F$

Number of occupied states is given by $N(E) = g(E) * F(E)$

$$n = \int_0^{E_f} g(E) * F(E)\, dE$$

$$= \frac{\pi}{2}\left(\frac{8m}{h^2}\right)^{\frac{3}{2}} \int_0^{E_f} E^{\frac{1}{2}}\, dE = \frac{\pi}{3}\left(\frac{8m}{h^2}\right)^{\frac{3}{2}} E_f^{3/2}$$

$$E_f = \left(\frac{3}{\pi}\right)^{2/3} \left(\frac{h^2}{8m}\right) n^{2/3}$$

Fermi velocity

The excited electrons have energies very close to the Fermi energy and hence the conduction electrons possess K.E equal to the Fermi energy.

$$E_f = \frac{1}{2} m v_f^2 \quad \text{or} \quad v_f = \sqrt{\frac{2E_f}{m}}$$

where v_f is the Fermi velocity of the conduction electron

For conduction electrons in Copper, Fermi velocity is

$$v_f = 1.06 \times 10^6 \; ms^{-1}$$

This velocity is greater than the thermal velocity of electrons (as per the CFET)

Fermi Temperature

- *Given thermal energy, only those electrons near the Fermi energy are excited into the conduction band.*

- *If T is the temperature of the metal, then k_BT is the thermal energy available to the electrons in the metal.*

- *Hence the thermal energy required to excite the last electron at the bottom of the energy band is* $E_f = k_BT_f$

Where T_f is termed as the Fermi temperature of electrons in the metal.

ENGINEERING PHYSICS

Fermi Temperature

Example:

We will try to evaluate the temperature for Copper with Fermi energy as 7 eV

We know that $$T_F = \frac{E_F}{k_B}$$

This will give the value of the Fermi temperature as ≈ 81000 K

It is obvious that at such high temperatures the metal cannot be in the solid state and hence this is only a representative temperature.

Average energy of electrons in a metal at 0K

- *The distribution of electrons in different energy states shows a non linear variation*
- *This means that the average energy of the electron is not the simple average of the electron's max and min energy.*
- *To obtain the average electron energy at 0K , we have to estimate the total energy of the electron*

ENGINEERING PHYSICS

Average energy of electrons in a metal at 0K

*From the graph of N(E) vs E , we observe that N(E) states have energy E which implies that the total energy of all electrons is filled states up to E_f should be the summation of all N(E)*E*

$$= \int_0^{E_f} g(E)dE * F(E) * E$$

$$< E > = \frac{Total\, energy\, of\, all\, electrons\, in\, different\, energy\, state}{Total\, number\, of\, electrons}$$

$$= \frac{\int_0^{E_f} g(E) * E * F(E)\, dE}{\int_0^{E_f} g(E) * F(E)\, dE}$$

Average energy of electrons in a metal at 0K

$$= \frac{\frac{\pi}{2}\left(\frac{8m}{h^2}\right)^{\frac{3}{2}} \int_0^{E_f} E^{\frac{1}{2}}\, dE * E}{\frac{\pi}{2}\left(\frac{8m}{h^2}\right)^{\frac{3}{2}} \int_0^{E_f} E^{\frac{1}{2}}\, dE}$$

On integration we get average energy of the electron at 0K

$$< E > = \frac{3}{5} E_f$$

ENGINEERING PHYSICS

Class 27 . Quiz …

The concepts which are correct are….

1. *The Fermi energy of metal at absolute zero temperature is proportional to $n^{2/3}$*
2. *Electrons cannot have energy greater than Fermi energy at temperature of 300 K*
3. *Only a small fraction of electrons close to the conduction band can take part in the conduction processes*
4. *Fermi temperature is a representative temperature which highlights that all valence electrons cannot be conduction electrons.*
5. *If the Fermi energy of silver at 0 K is 5 eV, then the mean energy of the electron is 6 eV.*

THANK YOU

Sunitha VR, Ph.D.

Assistant Professor,
Department of Science and Humanities

sunithavr@pes.edu

+91 80 21722683 Extn 716