ENGINEERING PHYSICS

Sunitha V R

Department of Science & Humanities

- ***Suggested Reading***

  1. *Fundamentals of Physics, Resnik and Halliday, Chapter 41*
  2. *Concepts of Modern Physics, Arthur Beiser, Chapter 9*
  3. *Learning material prepared by the department- Unit III*

- ***Reference Videos***

  1. *Physics Of Materials-IIT-Madras/lecture-07.html*

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Unit III : Application of Quantum Mechanics to Electrical transport in Solids

PES UNIVERSITY ONLINE

Class #23

- *Features of classical free electron model*
- *Expression for electrical conductivity*
- *Failures of classical free electron theory*

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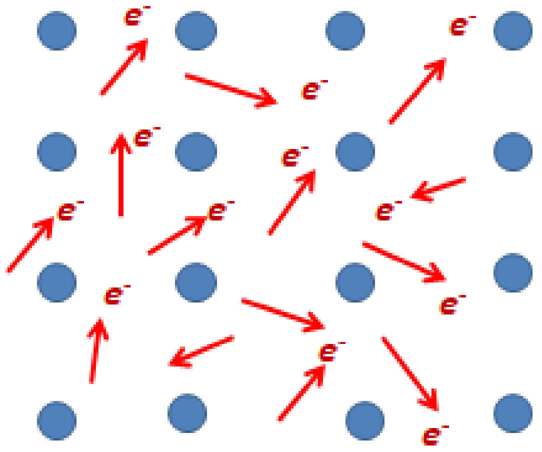
Features of classical free electron model

Drude and Lorentz proposed the classical free electron theory of electrical conductivity of metals in 1900

- *Valence electrons- treated as conduction electrons*
- *Electrostatic interaction- neglected*
- *Electrons in the metal– treated as free electron gas*
- *Free-electron gas- follows Maxwell-Boltzmann statistics*



Thermal Velocity

Thermal energy of the free electrons in the absence of the electric field is given by

$$\frac{1}{2}mv_{th}^2 = \frac{3}{2}kT$$

Therefore average thermal velocity $v_{th} = \sqrt{\frac{3k_BT}{m}}$

- *Random motion of the electrons do not contribute to a net drift of electrons.*
- *no current flows through the material*

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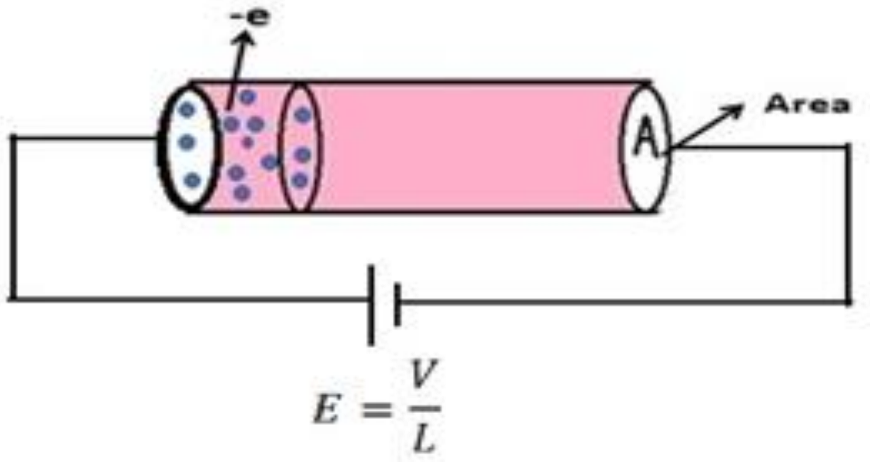
Drift Velocity

Under the influence of electric field E the average velocity experienced by the electrons is called drift velocity

$$v_d = \frac{e\tau E}{m} = \mu E$$

Where $\mu = \frac{e\tau}{m} = \frac{v_d}{E}$ *is the drift velocity per unit electric field*

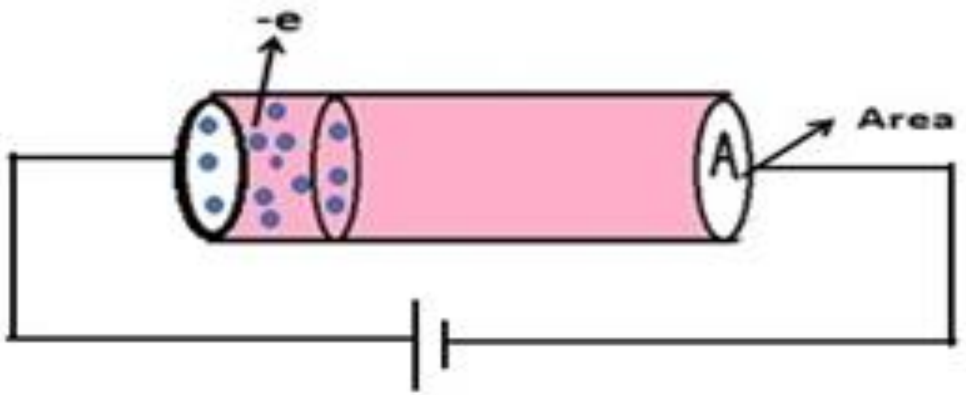


Expression for electrical conductivity

The current through a conductor $I = nev_dA$

Where v_d is the drift velocity given by $v_d = \frac{e\tau E}{m}$

$$\therefore\ I = neA\,\frac{e\tau E}{m}$$



$$E = \frac{V}{L}$$

Expression for electrical conductivity

$$J = \frac{I}{A} = \frac{ne^2\tau}{m}\,E \;= \sigma\, E$$

where σ is a constant for a given metal

$$\sigma = \frac{ne^2\tau}{m} = ne\mu$$

The resistivity of the metal is given by

$$\rho = \frac{1}{\sigma} = \frac{m}{ne^2\tau} = \frac{1}{ne\mu}$$

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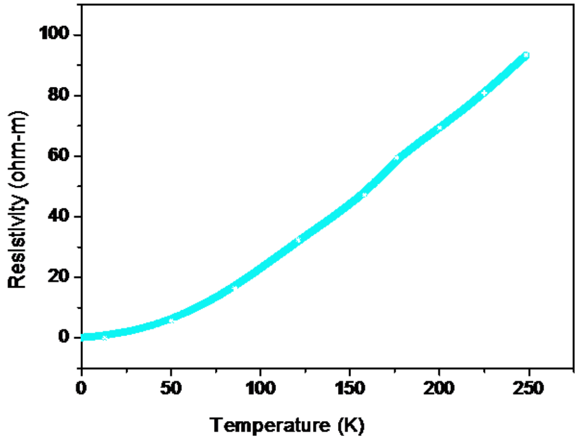
Experimental temperature dependence of resistivity

Total resistivity of the metal is given by $\rho = \rho_i + \rho_{sc}$

Substituting the value of ρ we get $\dfrac{m}{ne^2\tau} = \dfrac{m}{ne^2\tau_{res}} + \dfrac{m}{ne^2\tau_{sc}}$

This relation is called Mattheissen's rule

This gives the effective relaxation time $\dfrac{1}{\tau} = \dfrac{1}{\tau_{res}} + \dfrac{1}{\tau_{sc}}$



Failures of classical free electron theory

- *Temperature dependence of resistivity*
- *Specific heat of electrons in a metal*
- *Conductivity variations with electron concentrations*

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Temperature dependence of resistivity

According to CFET resistivity is given by $\rho = \dfrac{m}{ne^2\tau}$

Temperature dependence of the relaxation time $\tau = \dfrac{\lambda}{v_{th}}$

For a given temperature $v_{th} = \sqrt{\dfrac{3k_BT}{m}}$

Hence $v_{th} \propto \sqrt{T}$ *or* $\tau \propto T^{-\frac{1}{2}}$

Since the resistivity $\rho \propto \dfrac{1}{\tau}$ $\therefore \rho \propto \sqrt{T}$

According to experimental observations $\rho \propto T$

Specific heat of electrons in a metal

Specific heat of the metal is $C_{el} = \dfrac{dU}{dT}$

For one mole of a mono-valent metal $U = \dfrac{3}{2} k_B T . N_A$

Hence the specific heat of the electrons

$$C_{el} = \frac{dU}{dT} = \frac{3}{2} k_B . N_A = \frac{3}{2} R$$

Experimental estimation of the specific heat of electrons is ≈ 1% of the theoretical values

Conductivity variations with electron concentrations

$$\sigma = \frac{ne^2\tau}{m}$$

Free electron concentration of copper and aluminum is $8.4 x 10^{28}/m^3$ and $18.1 x 10^{28}/m^3$

$$n_{cu} < n_{Al}$$

Conductivity of copper and aluminum at 20°C is $5.9 x 10^7$ S/m and $3.5 x 10^7$ S/m

$$\sigma_{cu} > \sigma_{Al}$$

Copper with lesser electronic concentration has a higher electrical conductivity as compared to trivalent Aluminum.

The concepts which are correct are….

1. *Drude model treats the free electrons equivalent to molecules in a gas*
2. *The distribution of energy and velocity is assumed to follow Maxwell-Boltzmann statistics*
3. *The drift velocity of the electron is due to the application thermal energy*
4. *The resistivity in metal due to impurities dependent on temperature*
5. *The classical free electron theory explains the electronic specific heat (C_v) of solid*

THANK YOU

Sunitha VR, Ph.D.

Assistant Professor, Department of Science and Humanities

sunithavr@pes.edu

+91 80 21722683 Extn 716