ENGINEERING MATHEMATICS - I

Taylor’s and Maclaurin’s Series

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ENGINEERING MATHEMATICS - I

UNIT 2 : Partial Differentiation

Session : 8

Sub Topic : Taylor's and Maclaurin's Series of a function of Two Variables

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Intuition behind Taylor's and Maclaurin's Series

- Suppose you want to calculate the value of $e^{\tan(\sqrt[3]{3})}$, the first thing you look for is the calculator.
- A calculator expresses any complicated function in terms of its maclaurin's series and find its value.
- Suppose we need to evaluate an integral of the form $\int e^{x^2}\,\mathrm{dx}$ or $\int cos(x^2)\,dx$ all the methods we know so far fails.
- If you want to evaluate the following limit

$$\lim_{x\to 0}\frac{\sin(x)-x}{x^4}$$

Taylor's and Maclaurin's Series for a function of single variable

If a function f(x) has continuous derivatives of order (n+1), then this function can be expressed as a polynomial at a point x=a in the form

$$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!} f''(a) + \cdots + \frac{(x-a)^n}{n!} f^n(a) + \ldots$$

If the function is expanded about the origin we get Maclaurin's Series

$$f(x) = f(0) + (x)f'(0) + \frac{(x)^2}{2!} f''(0) + \cdots + \frac{(x)^n}{n!} f^n(0) + \ldots$$

Standard Maclaurin's Series for a function of single variable

1. $(1+x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2 + \dfrac{n(n-1)(n-2)}{3!}x^3 + \cdots, |x| < 1$

2. $e^x = 1 + \dfrac{x}{1!} + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \cdots, x \in R$

3. $\log_e(1+x) = x - \dfrac{x^2}{2} + \dfrac{x^3}{3} - \dfrac{x^4}{4} + \cdots, |x| < 1$

4. $\sin x = x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \dfrac{x^7}{7!} + \cdots$

5. $\cos x = 1 - \dfrac{x^2}{2} + \dfrac{x^4}{4!} - \dfrac{x^6}{6!} + \cdots$

6. $\sinh x = x + \dfrac{x^3}{3!} + \dfrac{x^5}{5!} + \cdots$

7. $\cosh x = 1 + \dfrac{x^2}{2!} + \dfrac{x^4}{4!} + \cdots$

8. $\tan^{-1} x = x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} + \cdots$

9. $\sin^{-1} x = x + \dfrac{1}{2}\dfrac{x^3}{3} + \dfrac{1\cdot 3}{2\cdot 4}\dfrac{x^5}{5} + \dfrac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\dfrac{x^7}{7} + \cdots, \quad -1 \le x \le 1$

10. $\cos^{-1} x = \dfrac{\pi}{2} - x - \dfrac{1}{2}\dfrac{x^3}{3} - \dfrac{1\cdot 3}{2\cdot 4}\dfrac{x^5}{5} - \dfrac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\dfrac{x^7}{7} - \cdots, -1 \le x \le 1$

Applications of Taylor's and Maclaurin's Series for a function of single variable

1. Expressing any complicated function in terms of a polynomial

2. Evaluating definite integrals which cannot be evaluated by any other methods.

3. Understanding asymptotic behavior of the function

4. Understanding the growth of function

5. Solving differential equations

Applications of Taylor's and Maclaurin's Series for a function of single variable

6. In physics, Maclaurin's series is used to approximate the Lorrentz factor in special relativity.

7. The most famous equation $E = mc^2$ is an approximation for low velocities.

8. In the motion of a pendulum an approximation of $\sin(\theta) \cong \theta$ which comes from Taylor's series.

9.Used in power flow analysis of electrical power system.

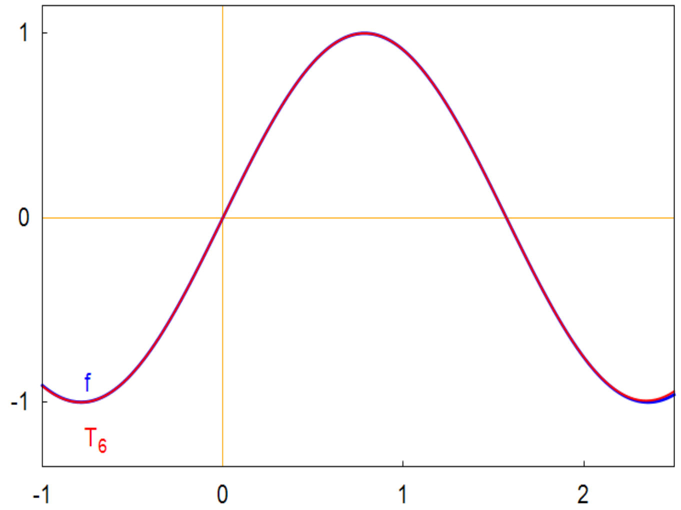
10. Can be used to find the Generating functions.

Source: https://math.stackexchange.com/questions/218421/what-are-the-practical-applications-of-the-taylor-series

Applications of Taylor's and Maclaurin's Series for a function of single variable

n=4

n=6



Taylor's expansion of a function of two variables

Statement:

Taylor's series of $f(x,y)$about (a,b) is given by

$$f(x,y)$$
$$= f(a,b) + \frac{1}{1!}\{(x-a)f_x(a,b) + (y-b)f_y(a,b)\}$$
$$+ \frac{1}{2!}\{(x-a)^2 f_{xx}(a,b) + 2(x-a)(y-b)f_{xy}(a,b)$$

Maclaurin's expansion of a function of two variables

Statement:

Maclaurin's series for a function of two variables is given by

$$f(x,y) = f(0,0) + \frac{1}{1!}\{xf_x(0,0) + yf_y(0,0)\} + \frac{1}{2!}\{x^2 f_{xx}(0,0) + 2xyf_{xy}(0,0) + y^2 f_{yy}(0,0)\} + \ldots..$$

Problems on Taylor's and Maclaurin's expansion

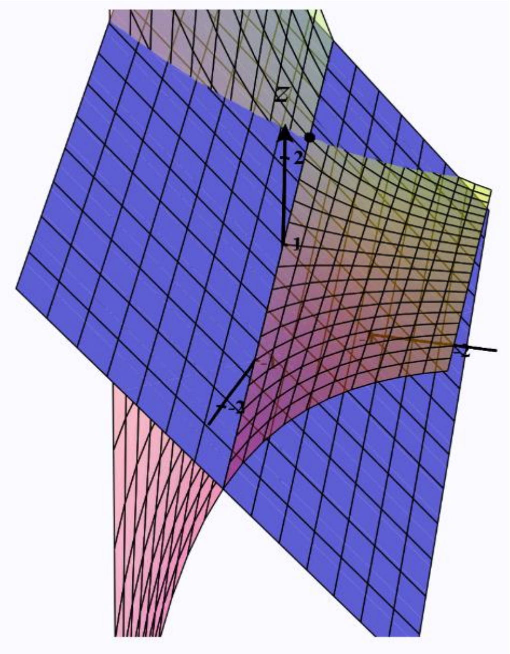


Figure 3: *Graph of* $f(x,y) = xe^y + 1$ *and its* 1^{st} *-degree Taylor polynomial,* $L(x,y) = 1 + x + y$

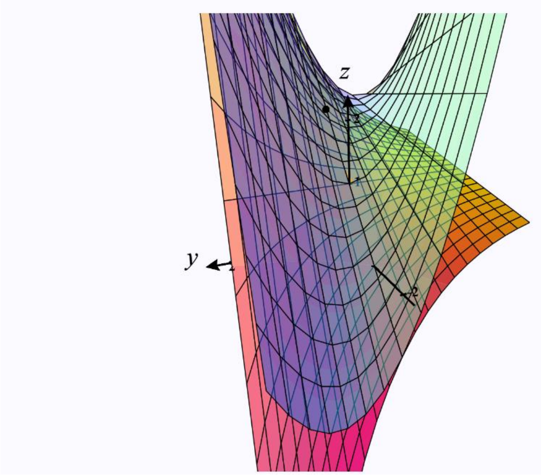


Figure 4: *Graph of* $f(x,y) = xe^y + 1$ *and its* 2^{nd} *-degree Taylor polynomial,* $L(x,y) = 1 + x + xy + \frac{y^2}{2}$

Applications of Euler’s Theorem of a multivariate function

1. Taylor’s series is used in different optimization techniques by expressing it as a linear or quadratic expression.
2. Distances can be measured accurately using sine law and Taylor’s series.
3. To calculate variance of complicated configurations
4. To visualize complicated functions.

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