ENGINEERING MATHEMATICS - I

Lagrange's method of Undertermined Multipliers

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UNIT 2 : Partial Differentiation

Session : 12

Sub Topic : Lagrange’s Method of Undetermined Multipliers

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Lagrange's Method of Undetermined Multipliers

Suppose we need to minimize or maximize a function $f(x, y, z)$ subject to the constraint $\emptyset(x, y, z)$, then we can introduce an additional unknown constant λ known as Lagrange's multiplier to ease the process of finding the extrema's.

Working Procedure

Step 1: Form the auxiliary equation

$$F(x, y, z) = f(x, y, z) + \lambda\emptyset(x, y, z) \ldots (1)$$

Step2: Partially differentiate $F\ in\ (1) w.r.t\ x, y, z$ respectively

Step 3: Solve the four equations $F_x = 0, F_y = 0, F_z = 0$ and the constraint for the Lagrange multiplier λ and stationary values x, y, z

Lagrange's Method of Undetermined Multipliers

Advantages:

1. The stationary points of $f(x, y, z)$ can be determined without determining x, y, z explicitly.
2. This method can be extended to function of several variables subject to many constraints.

Disadvantages:

1. Nature of the stationary points can not be determined.
2. Equations $F_x = 0, F_y = 0, F_z = 0$ are only necessary conditions.

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1. Show that the rectangular solid of maximum volume that can be inscribed in a given sphere is a cube.

Solution:

Let the equation of the sphere be $x^2 + y^2 + z^2 = r^2$.

Let $2x, 2y, 2z$ be the length, breadth and height of the rectangular solid so that its volume

$$v = f(x, y, z) = 8xyz$$

Let $\emptyset = x^2 + y^2 + z^2 - r^2$

Form the auxiliary function

$F = f + \lambda\phi$

$F = 8xyz + \lambda(x^2 + y^2 + z^2 - r^2)$

Lagrange's Method of Undetermined Multipliers

Contd……..

$\frac{\partial F}{\partial x} = 0; \frac{\partial F}{\partial y} = 0; \frac{\partial F}{\partial z} = 0$ gives

$8zy + 2x\lambda=0$ which implies $8xzy + 2x^2\lambda=0$

$8zx + 2y\lambda=0$ which implies $8xzy + 2y^2\lambda=0$

$8xy + 2z\lambda=0$ which implies $8xzy + 2y^2\lambda=0$

$2x^2\lambda = -8xyz = 2y^2\lambda = 2z^2\lambda$

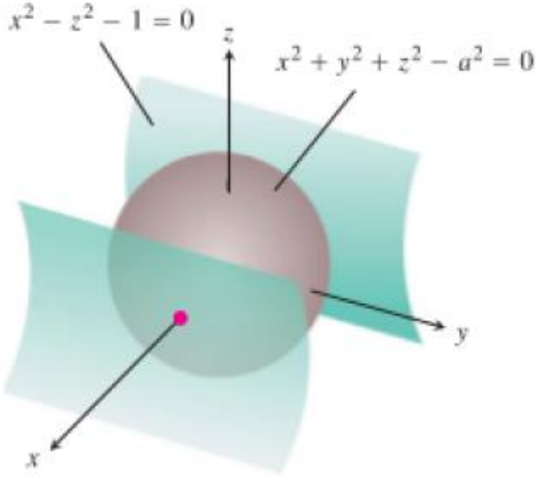
Thus for a maximum volume $x = y = z$.
Therefore the rectangular solid is a cube.

Lagrange's Method of Undetermined Multipliers

2. Find the points closest to the origin on the hyperbolic cylinder $x^2 - z^2 = 1$.

Solution:

To find the points on the cylinder closest to the origin, imagine a small sphere centered at the origin expanding like a soap bubble until it just touches the cylinder $x^2 - z^2 - 1 = 0$.



Lagrange's Method of Undetermined Multipliers

Contd…….

Let $f(x,y,z) = x^2 + y^2 + z^2 - a^2$ and $\emptyset(x,y,z) = x^2 - z^2 - 1.$

Form the auxiliary function

$F = f + \lambda\phi$

$F = x^2 + y^2 + z^2 - a^2 + \lambda(x^2 - z^2 - 1)$

$\frac{\partial F}{\partial x} = 0; \frac{\partial F}{\partial y} = 0; \frac{\partial F}{\partial z} = 0$ gives

$2x + 2x\lambda = 0$ ………………(1)

$2y = 0$ ………………(2)

$2z - 2z\lambda = 0$ ………………(3)

Also consider $x^2 - z^2 - 1 = 0$ ……………(4)

Lagrange’s Method of Undetermined Multipliers

Contd.......

From (2), $y = 0$

Form (1),

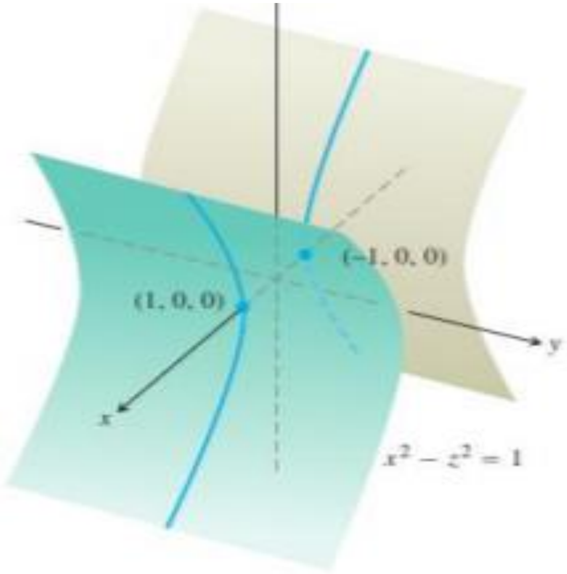
$2x(1+\lambda)$=0 which gives $x = 0$ or $\lambda = -1$

But x cannot be equal to 0 and hence $\lambda = -1$

From (3), we get $z = 0, \lambda = 1$

Substitute $z = 0$ in (4), we get $x = \pm 1$.

Therefore the points closest to the origin on the hyperbolic cylinder are ($\pm$1,0,0).



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