ENGINEERING MATHEMATICS - I

Ordinary Differential Equations

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Unit 3 : Ordinary Differential Equations

Session : 6

Sub Topic : Equations Reducible to Exact form

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Equations Reducible to Exact Form

Sometimes a differential equation which is not exact may become so, on multiplication by a suitable function known as the integrating factor (IF).

The integrating factor can be obtained as follows :

- **Case 1** : *If* $\dfrac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{N} = g(x)$ *(function of x alone) then,*

$$IF = e^{\int g(x)dx}.$$

- **Case 2** : *If* $\dfrac{\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}}{M} = h(y)$ *(function of y alone) then,*

$$IF = e^{\int h(y)dy}.$$

Equations Reducible to Exact Form

- **Case 3 :** In the given differential equation $Mdx + Ndy = 0$, if $M(x, y)$ *and* $N(x, y)$ is homogeneous of the same degree then,

  $IF = \frac{1}{Mx+Ny}$, *provided that* $Mx + Ny \neq 0.$

Note : If $Mx + Ny = 0$ *then* $IF = \frac{1}{x^2}$ *or* $\frac{1}{y^2}$ *or* $\frac{1}{xy}$.

Equations Reducible to Exact Form

- **Case 4 :** If the differential equation is of the form $f_1(xy)ydx + f_2(xy)xdy = 0$, then $IF = \frac{1}{Mx-Ny}$, where $M = f_1(xy)y$ & $\mathrm{N} = f_2(xy)x$, provided that $Mx - Ny \neq 0$.

Note : If $Mx - Ny = 0$ then $\frac{M}{N} = \frac{y}{x}$ and the given differential equation reduces to $x\mathrm{dy} + \mathrm{y}\mathrm{d}x = 0$ and its solution is $xy = c$.

Equations Reducible to Exact Form - Problems

1. Solve $[x^2y - 2xy^2]dx - [x^3 - 3x^2y]\mathrm{dy} = 0$

Solution : The given equation is of the form,

$$Mdx + Ndy = 0$$

$M = x^2y - 2xy^2;\ N = -x^3 + 3x^2y$

$\dfrac{\partial M}{\partial y} = x^2 - 4xy; \dfrac{\partial N}{\partial x} = -3x^2 + 6xy$

The given equation is not exact.

Consider, $IF = \frac{1}{Mx+Ny} = \frac{1}{x^3y-2x^2y^2-x^3y+3x^2y^2} = \frac{1}{x^2y^2}$

Equations Reducible to Exact Form – Problems

Contd.....

Multiplying the given equation by the IF, it becomes exact.

Therefore, $M = \frac{1}{y} - \frac{2}{x}$, $N = -\frac{x}{y^2} + \frac{3}{y}$

The solution is $\int M dx + \int N(y) dy = C$

$$\int \frac{1}{y} - \frac{2}{x} dx + \int \frac{3}{y} dy = C$$

$$\frac{x}{y} - 2logx + 3logy = c$$

$$\frac{x}{y} + log\left(\frac{y^3}{x^2}\right) = c$$

Equations Reducible to Exact Form - Problems

2. Solve $(xy + 2x^2y^2)dx + x(xy - x^2y^2)\mathrm{dy} = 0$

Solution : The given equation is of the form,

$$yf(xy)dx + xg(xy)dy = 0$$

$M = xy^2 + 2x^2y^3, N = xy^2 - x^3y^2$

Consider, $IF = \frac{1}{Mx - Ny} = \frac{1}{x^2y^2 + 2x^3y^3 - x^2y^2 + x^3y^3} = \frac{1}{3x^3y^3}$

Multiplying the given equation by the IF we have,

$M = \frac{1}{3x^2y} + \frac{2}{3x}$, $N = \frac{1}{3xy^2} - \frac{1}{3y}$

Equations Reducible to Exact Form

Contd......

The solution is given by

$$\int Mdx + \int N(y)dy = C$$

$$\int \left(\frac{1}{3x^2y} + \frac{2}{3x}\right)dx + \int \left(\frac{1}{3xy^2} - \frac{1}{3y}\right)dy = C$$

$$\frac{-1}{3xy} + \frac{2}{3}logx - \frac{1}{3}logy = c$$

$$\frac{-1}{xy} + log\frac{x^2}{y} = c$$

THANK YOU

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