

ENGINEERING MATHEMATICS - I Ordinary Differential Equations

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Unit 3: Ordinary Differential Equations

Session: 11

Sub Topic: Non-Linear Differential Equations

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ODE- Non linear Differential Equation

Equations solvable for y



Working Procedure

Step 1. Rewrite the given differential equation f(x, y, p) = 0 in the form y = F(x, p).....(1)

Step 2. **Differentiate (1) w.r.t 'x'** to obtain the equation of the form, $\mathbf{p} = \emptyset\left(\mathbf{x}, \mathbf{p}, \frac{d\mathbf{p}}{d\mathbf{x}}\right)$(2) which is a first order and first degree differential equation in the variable \mathbf{p} .

ODE- Non linear Differential Equation



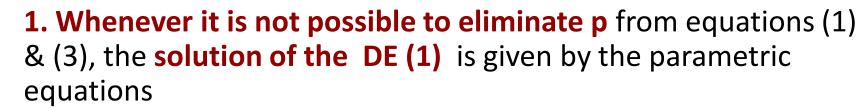
Working Procedure(contd...)

Step 3. Solve the differential equation (2). The solution is of the form G(x, p, c) = 0.....(3)

Step 4. Eliminating p from equations (1) and (3), the required solution of the DE (1).

ODE- Non linear Differential Equation

NOTE:



$$x = x(p,c) \& y = y(p,c).$$

2. When the factor which does not contain *dp/dx* is equated to zero and solved, we obtain another solution called the singular solution of the given differential equation. Observe that the singular solution does not contain any arbitrary constant.



ODE- Non linear Differential Equation - Solvable for y

1. Find the general solution of $3x^4p^2 - xp - y = 0$

Answer: The given equation can be written as,

$$y = 3x^4p^2 - xp$$
....(1)

Differentiating w.r.t 'x',

$$> \frac{dy}{dx} = 12x^3p^2 + 6x^4p\frac{dp}{dx} - x\frac{dp}{dx} - p$$

$$> p = 12x^3p^2 + 6x^4p\frac{dp}{dx} - x\frac{dp}{dx} - p$$

$$> 2p = 12x^3p^2 + 6x^4p\frac{dp}{dx} - x\frac{dp}{dx}$$

$$> 2p - 12x^3p^2 = 6x^4p\frac{dp}{dx} - x\frac{dp}{dx}$$

$$> 2p(1-6x^3p) = -x(1-6x^3p)\frac{dp}{dx}$$



ODE- Non linear Differential Equation - Solvable for y

$$> \left(2p + x\frac{dp}{dx}\right)\left(1 - 6x^3p\right) = 0$$

$$> \frac{dp}{2p} = -\frac{dx}{x}$$
 (By equating the first factor to zero)

$$> \frac{dp}{2p} + \frac{dx}{x} = 0$$

$$> \frac{1}{2} log p + log x = log k$$

$$> px^2 = c \text{ or } p = \frac{c}{x^2}$$

Substituting in (1)

$$3x^4\left(\frac{c}{x^2}\right)^2 - x\left(\frac{c}{x^2}\right) - y = 0$$

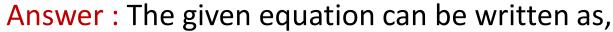
$$> 3c^2 - \frac{c}{r} - y = 0$$

$$\geqslant 3c^2x - c - xy = 0 \quad - \Rightarrow c(3cx - 1) = xy$$



ODE- Non linear Differential Equation - Solvable for y

2. Solve :
$$y + px = p^2x^4$$



$$y = x^4p^2 - xp$$
....(1)

Differentiating w.r.t 'x',

$$> \frac{dy}{dx} = 4x^3p^2 + 2p\frac{dp}{dx} - x\frac{dp}{dx} - p$$

$$p = 4x^3p^2 + 2x^4p\frac{dp}{dx} - x\frac{dp}{dx} - p$$

$$> 2p = 4x^3p^2 + 2x^4p\frac{dp}{dx} - x\frac{dp}{dx}$$

$$> 2p - 4x^3p^2 = 2px^4\frac{dp}{dx} - x\frac{dp}{dx}$$

$$\frac{dy}{dx} = 4x^3p^2 + 2p\frac{dp}{dx} - x\frac{dp}{dx} - p$$

$$p = 4x^3p^2 + 2x^4p\frac{dp}{dx} - x\frac{dp}{dx} - p$$

$$2p = 4x^3p^2 + 2x^4p\frac{dp}{dx} - x\frac{dp}{dx}$$

$$2p - 4x^3p^2 = 2px^4\frac{dp}{dx} - x\frac{dp}{dx}$$

$$2p(1 - 2x^3p) = -x(1 - 2x^3p)\frac{dp}{dx}$$



ODE- Non linear Differential Equation - Solvable for y



$$> \frac{dp}{2p} + \frac{dx}{x} = 0$$

$$> \frac{1}{2}logp + logx = logk$$

$$> px^2 = c \text{ or } p = \frac{c}{x^2}$$

Substituting in (1)

$$y = x^4 \left(\frac{c}{x^2}\right)^{-2} x \left(\frac{c}{x^2}\right)$$

$$>y+\frac{c}{r}=c^2$$

$$> xy + c = c^2 x$$



THANK YOU

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