ENGINEERING MATHEMATICS - I

Extremas of a function

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UNIT 2 : Partial Differentiation
Session : 11
Sub Topic : Maxima and Minima of a Function of Two Variables

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Maxima and Minima for a Function of Two Variables

Problems:

1. The temperature T at any point (x, y, z) in space is $400xyz^2$. Find the highest temperature on the surface of the unit sphere $x^2 + y^2 + z^2 = 1$.

Solution:

$z^2 = 1 - y^2 - x^2$

$f(x, y) = 400xy(1 - y^2 - x^2)$

f_x=0 implies $y = 0$ or $1 - 3x^2 - y^2 = 0$

f_y=0 implies $y = 0$ or $1 - x^2 - 3y^2 = 0$

On solving, we get $x = \pm\frac{1}{2}; y = \pm\frac{1}{2}$

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Solution:

The stationary points are $(0,0)$, $(\frac{1}{2}, \frac{1}{2})$,$(\frac{1}{2}, -\frac{1}{2})$, $(-\frac{1}{2}, \frac{1}{2})$,$(-\frac{1}{2}, -\frac{1}{2})$

At $(0,0)$, $rt - s^2 = -1600 < 0$(saddle point)

At $(\frac{1}{2}, \frac{1}{2})$, $rt - s^2 = 356000 > 0$(maximum point)

At $(-\frac{1}{2}, -\frac{1}{2})$, $rt - s^2 = 356000 > 0$(maximum point)

T is maximum at $(\frac{1}{2}, \frac{1}{2})$, $(-\frac{1}{2}, -\frac{1}{2})$ and the maximum value is 50.

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2. A Container with an open top is to have 10 m^3 capacity and be made of thin sheet metal. Calculate the dimensions of the box if it is to use the minimum possible amount of metal.

Solution:

Let x, y and z ft. be the edges of the box and A be its surface.

Then $A = 2xy + 2yz + xz$……….(1)

$xyz = 10$………..(2)

Substitute (2) in (1)

$$A = 2xy + 10\left(\frac{2}{x} + \frac{1}{y}\right)$$

$$A_x = 0 \; gives \; 2y - \frac{20}{x^2} = 0 \; and \; y = \frac{10}{x^2}$$

Maxima and Minima for a Function of Two Variables

Contd.....

$$A_y = 0 \; gives \; 2x - \frac{20}{y^2} = 0 \; and \; x = \frac{10}{y^2}$$

$$Solving \; we \; get \; x = \sqrt[3]{10}, y = \sqrt[3]{10}, z = \sqrt[3]{10}$$

$$A_{xx} = \frac{40}{x^3}$$

$$A_{yy} = \frac{40}{y^3}$$

$$A_{xy} = 2$$

$$A_{xx}A_{yy} - \left(A_{xy}\right)^2 > 0 \; and \; A_{xx} > 0$$

Therefore it is minimum and the dimensions of the box are

$x = \sqrt[3]{10}, y = \sqrt[3]{10}, z = \sqrt[3]{10}$

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