ENGINEERING MATHEMATICS - I

Extremas of a function

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UNIT 2 : Partial Differentiation
Session : 10
Sub Topic : Maxima and Minima of a Function of Two Variables

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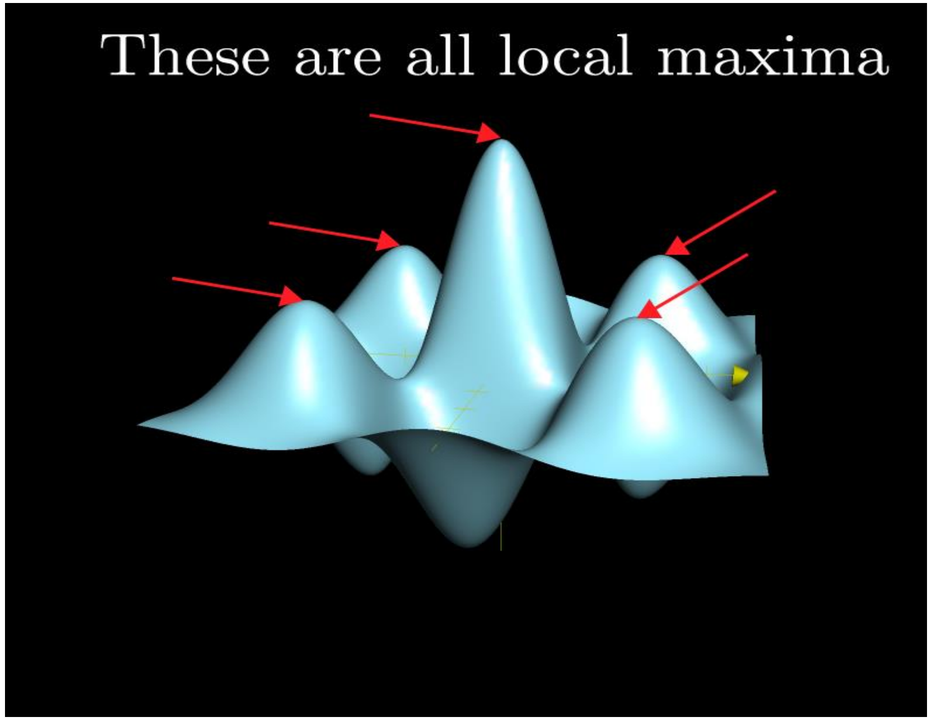
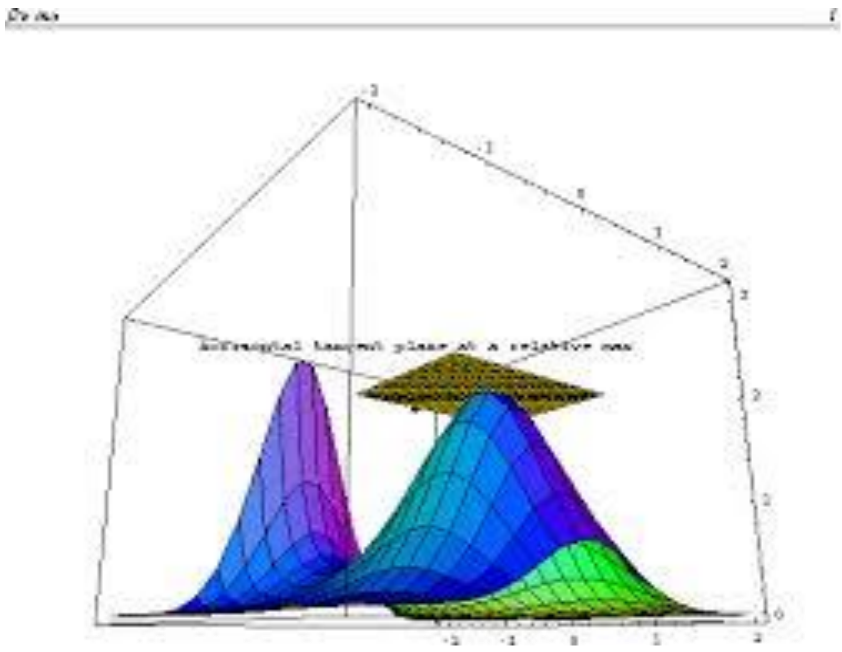
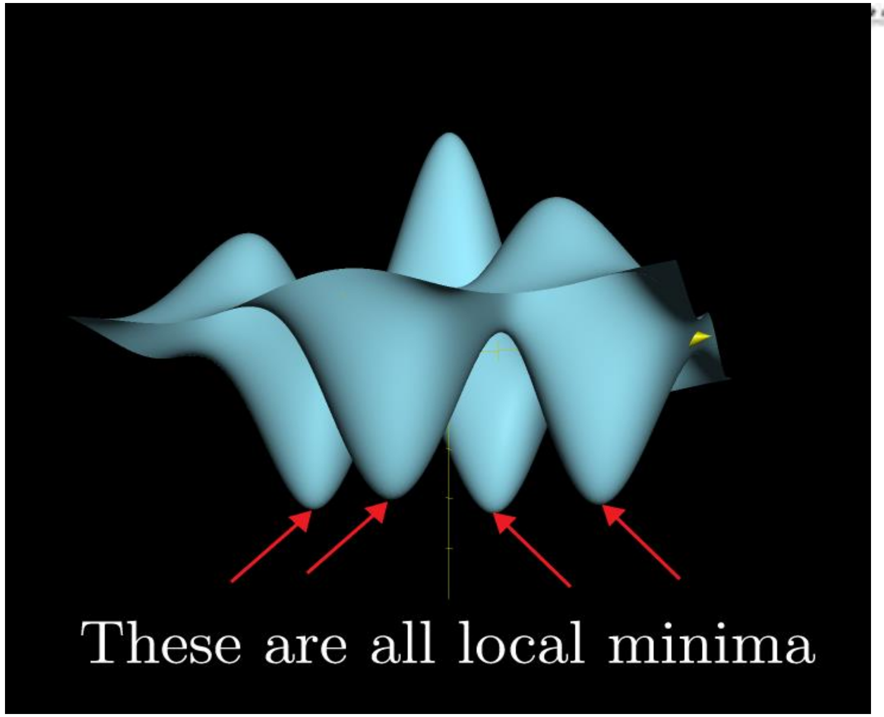
Maxima and Minima for a Function of Two Variables

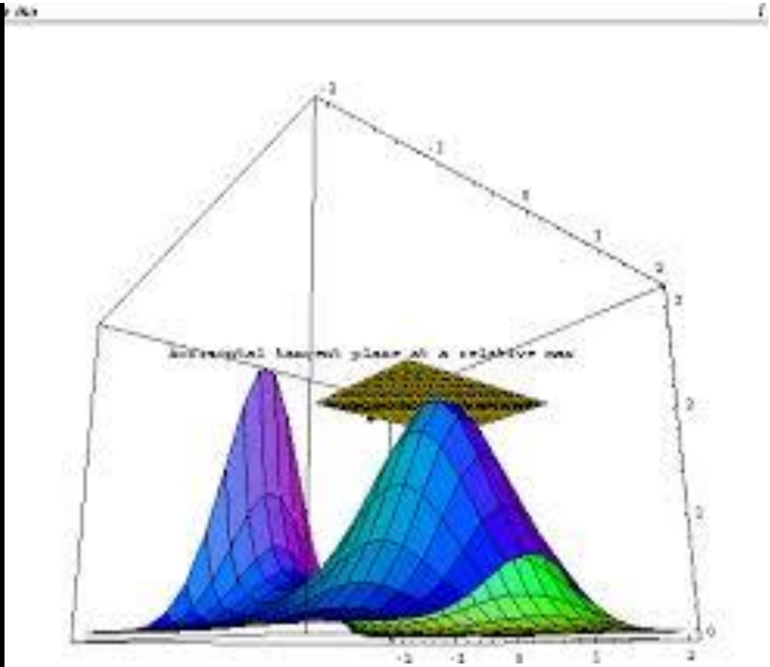
Maxima of a function



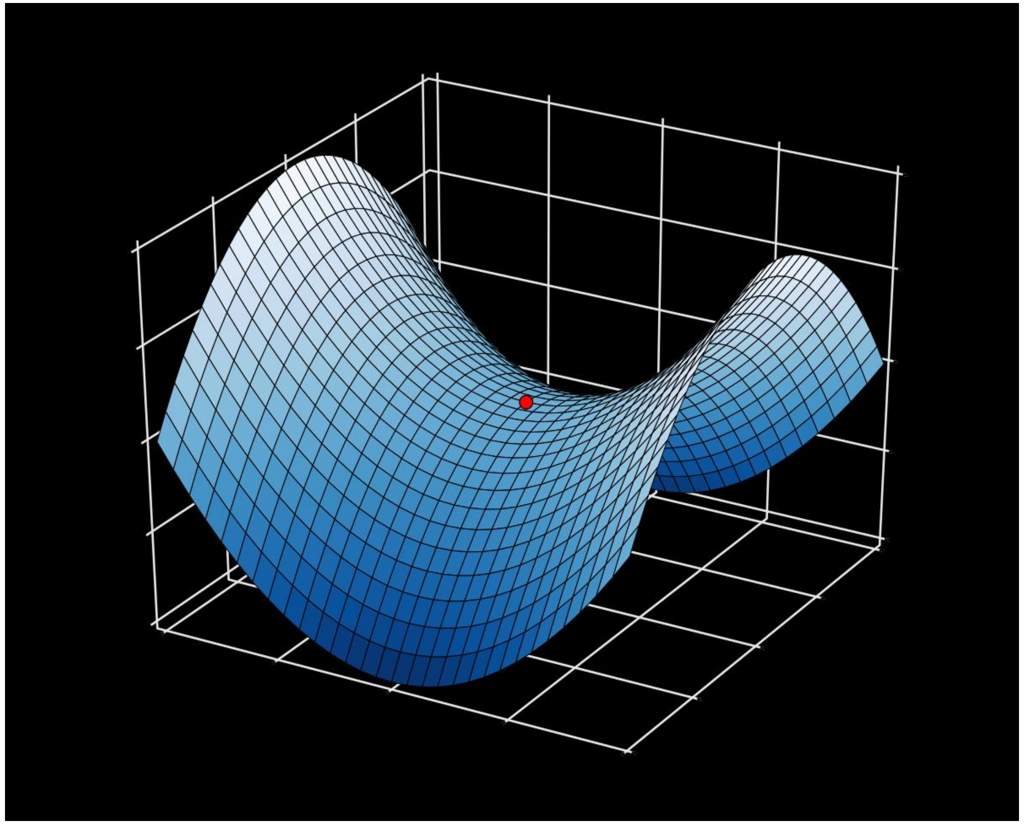


Minima of a function





Saddle Point



Maxima and Minima for a Function of Two Variables

- A function $f(x, y)$ is said to be have a relative maximum at a point (a, b) if there exists a neighborhood of the point (a, b) say $(a + h, b + k)$, h *and* k are small such that $f(a, b) > f(a + h, b + k)$.
- If $f(a, b) < f(a + h, b + k)$ then $f(x, y)$ is said to have relative minimum at (a, b). Also $f(a, b)$ is said to be an extreme value of $f(a, b)$ if it is either a maximum or a minimum.
- In other words if $\Delta = f(a + h, b + k) - f(a, b)$ is of the same sign for all small values of h, k and if this sign is negative then $f(a, b)$ is maximum & if this sign is positive then $f(a, b)$ is a minimum.

Maxima and Minima for a Function of Two Variables

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Necessary and Sufficient Condition:

The necessary and sufficient condition for $f(x,y)$ to have a maximum value at (a,b) is $f_x(a,b)=0, f_y(a,b)=0$ and $rt-s^2, A>0$. where $\left[r=f_{xx}; s=f_{xy}; t=f_{yy}\right]$

$rt-s^2$ is the determinant of Hessian matrix given by

$$\begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix}$$

Note: If $rt-s^2<0$, $f(a,b)$ is not an extreme value. i.e $f(x,y)$ is neither a maximum nor a minimum at the point(a,b) and the point is called a saddle point.

If $rt-s^2=0$, then the test provides no information about the nature of the point.

Maxima and Minima for a Function of Two Variables

Working Rule:

1. Find $\frac{\partial f}{\partial x}$ & $\frac{\partial f}{\partial y}$ and equate each to zero. Solve these simultaneous equations in x &y. Let $(a, b), (c, d) \ldots.$ be the pairs of values.

2. Calculate the value of $r = \frac{\partial^2 f}{\partial x^2}, s = \frac{\partial^2 f}{\partial x \partial y}, t = \frac{\partial^2 f}{\partial y^2}$ for each pair of values.

3. If $rt - s^2 > 0$ and $r < 0$ at $(a, b), f(a, b)$ is a maximum value.

4. If $rt - s^2 > 0$ and $r > 0$ at $(a, b), f(a, b)$ is a minimum value.

Maxima and Minima for a Function of Two Variables

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5. If $rt - s^2 < 0$ at (a, b), $f(a, b)$ is not an extreme value, i.e. (a, b) is a saddle point.

6. If $rt - s^2 = 0$ at (a, b) the case is doubtful and needs further Investigation.

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