ENGINEERING MATHEMATICS - I

Ordinary Differential Equations

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Unit 3 : Ordinary Differential Equations

Session : 7

Sub Topic : Orthogonal trajectories

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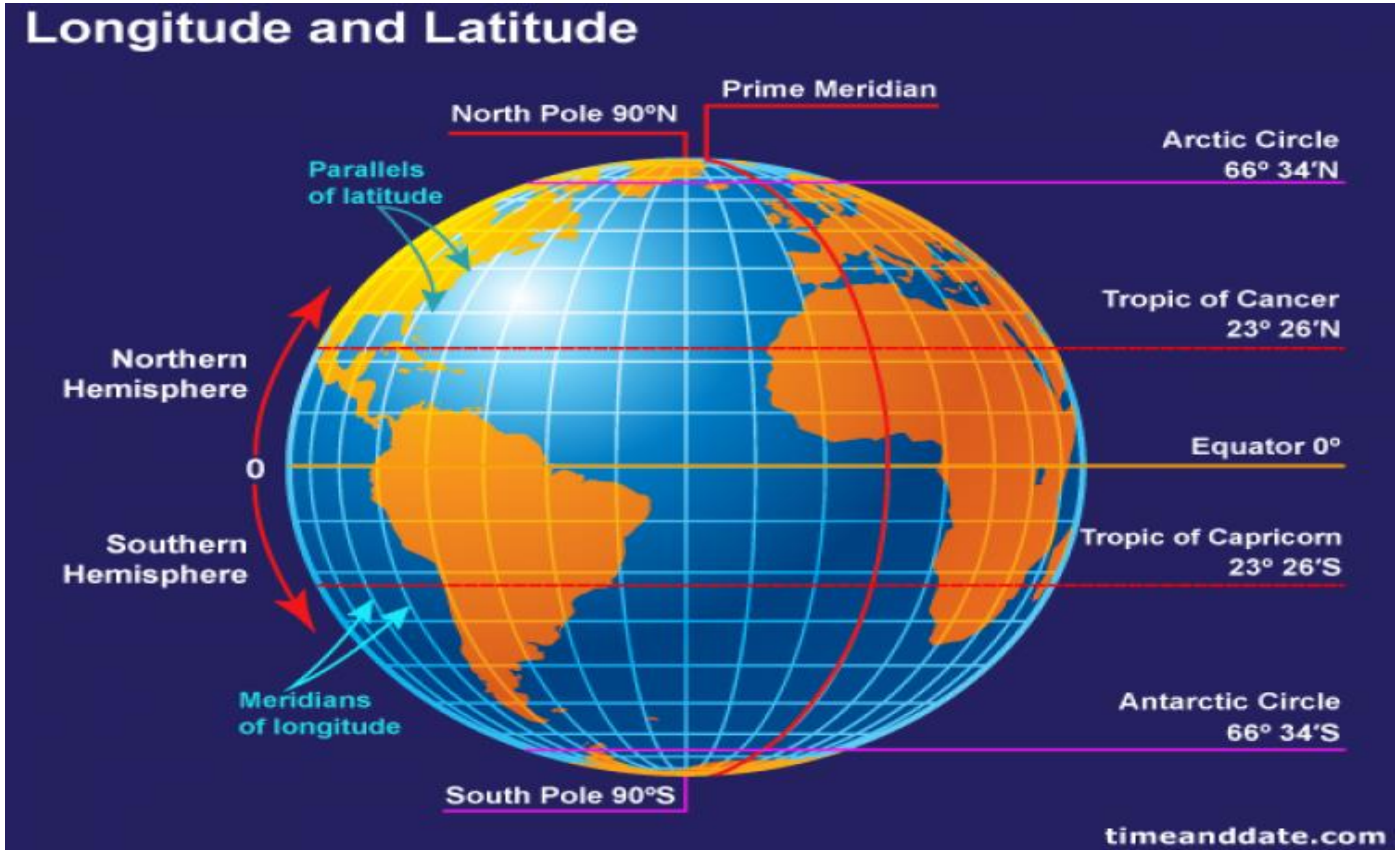
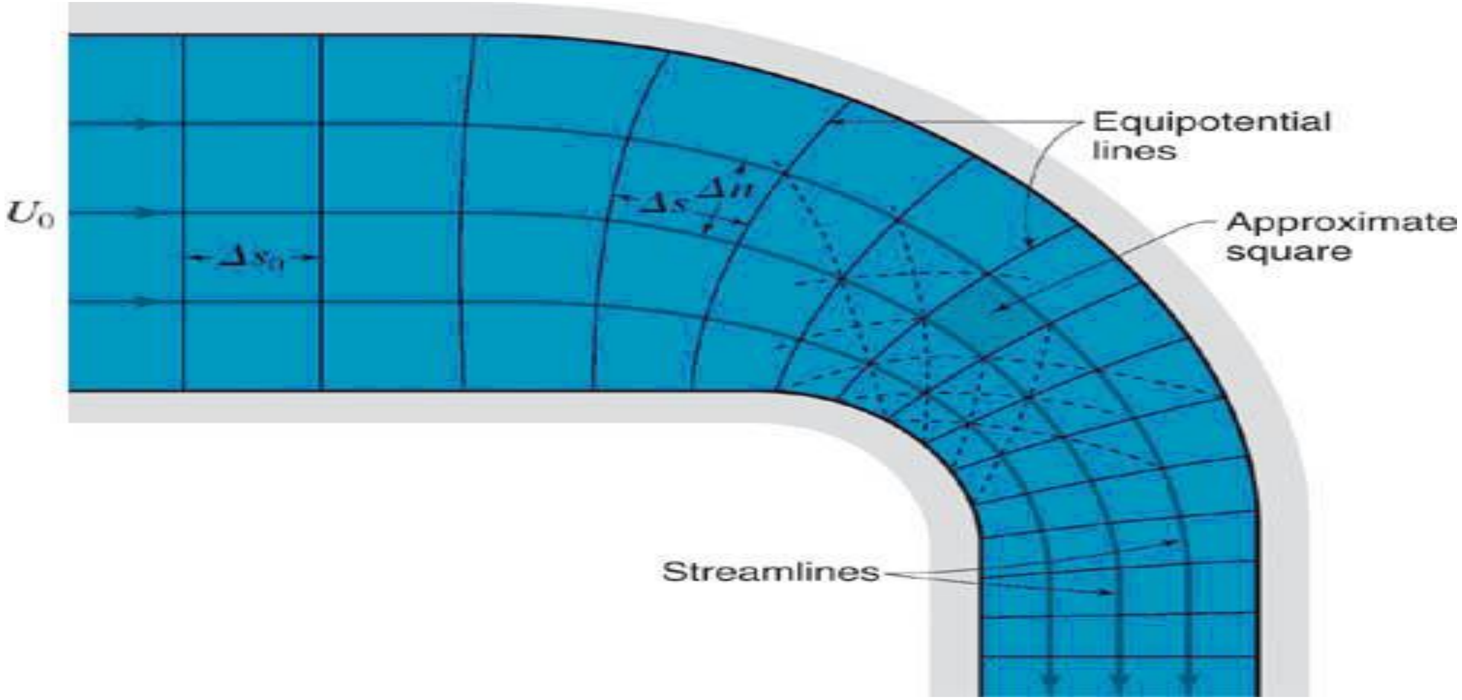
What are Orthogonal Trajectories ???

1. The Meridian & Parallels on the world globe are orthogonal trajectories of each other!



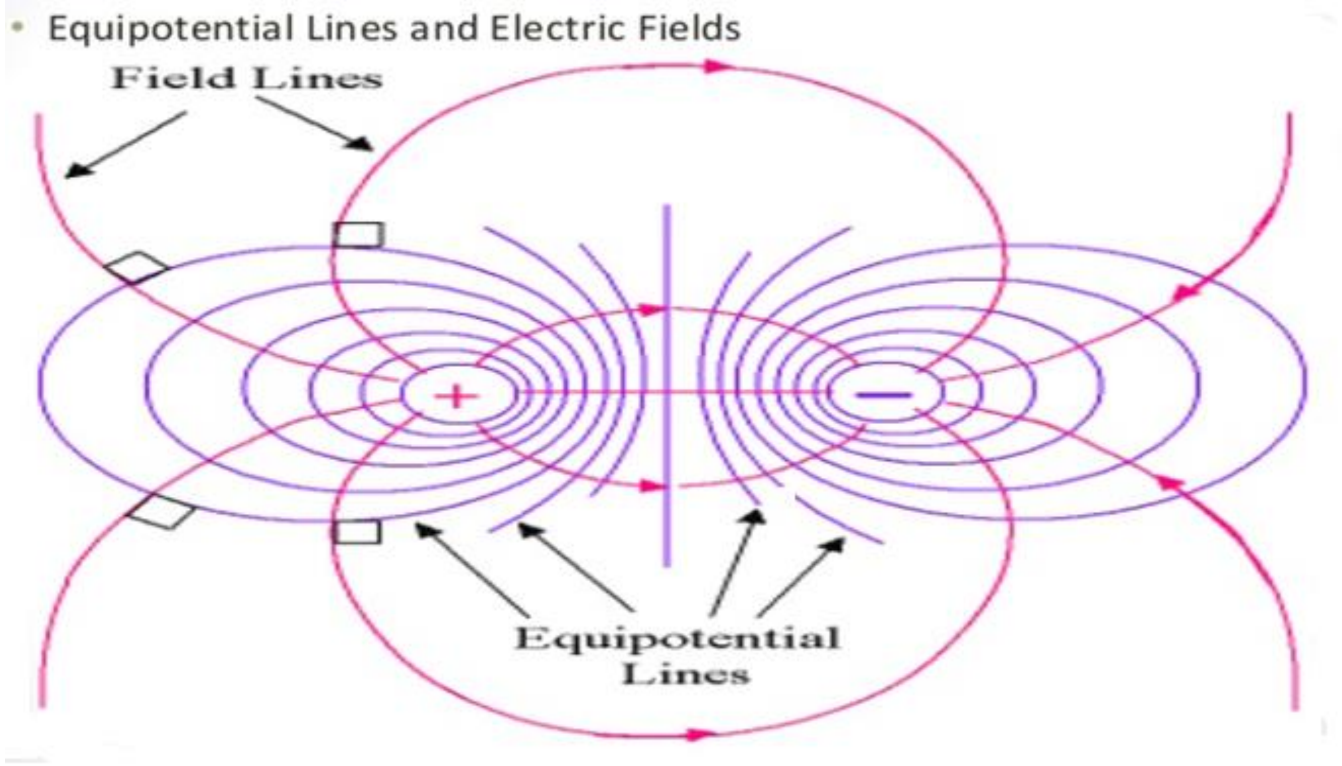
Orthogonal Trajectories - Examples

2. In fluid flow, the Stream lines and equipotential lines (of constant velocity potential) are orthogonal trajectories.



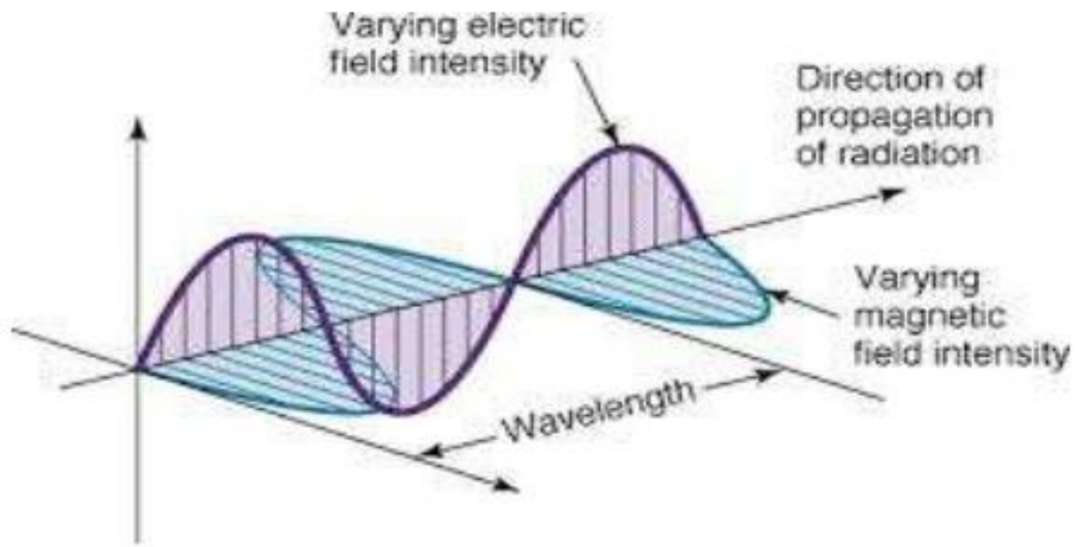
Orthogonal Trajectories - Examples

3. The path of an electric field is perpendicular to the equipotential curves.



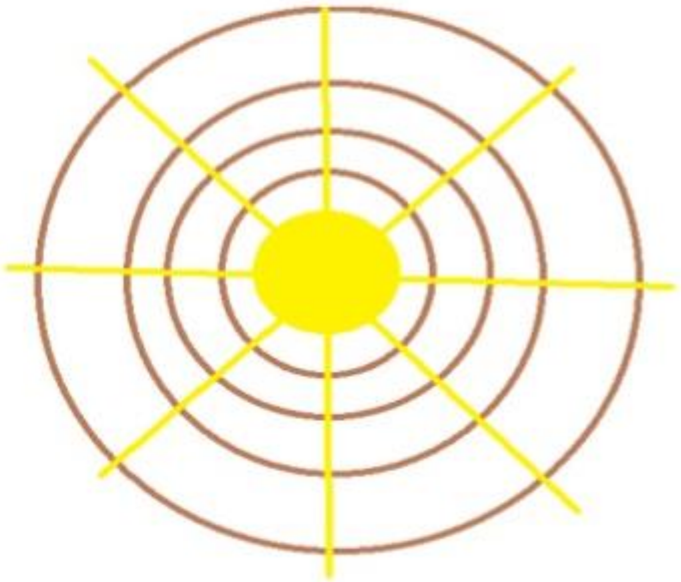
Orthogonal Trajectories - Examples

4. Electromagnetic waves consist of both electric and magnetic field waves. These waves oscillate in perpendicular planes with respect to each other.



Orthogonal Trajectories - Examples

5. The light rays of sun that passes through the orbits of the planets.

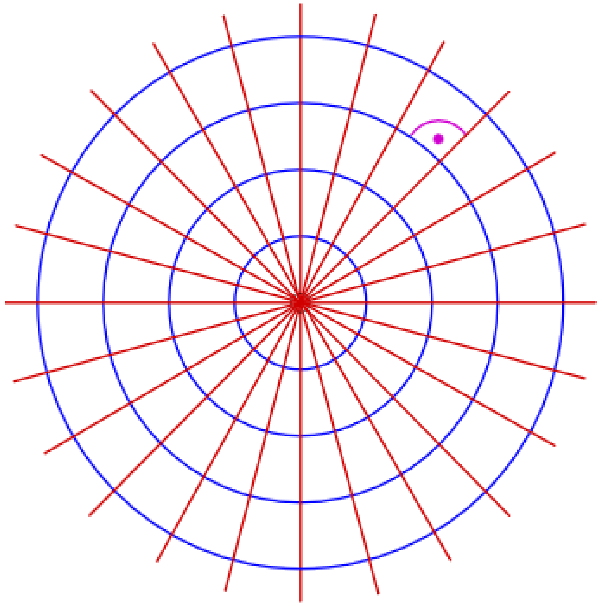


Orthogonal Trajectories – Cartesian Form

Definition : Two families of curves are said to be orthogonal trajectories of each other if every member of one family cuts every other member of the other family orthogonally.

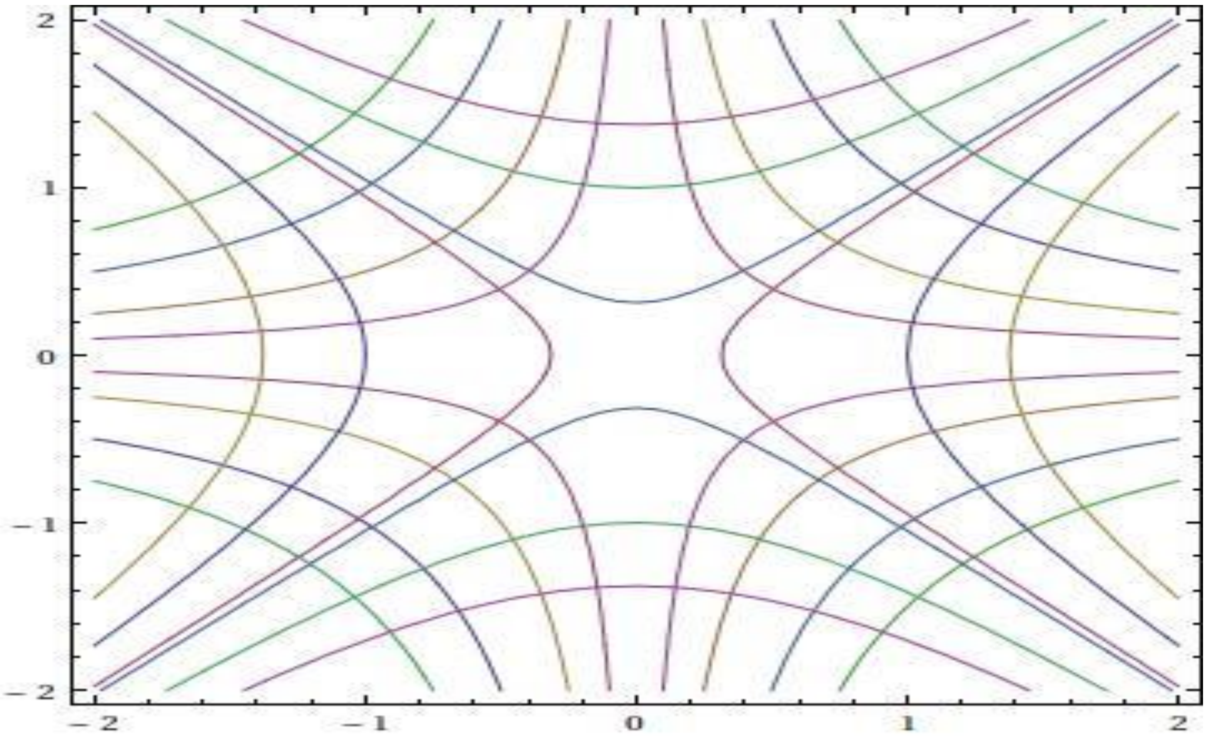
Examples :

1. Family of ***straight lines*** $y = mx$
& the family of ***circles*** $x^2 + y^2 = a^2$
are orthogonal trajectories
of each other.



Orthogonal Trajectories – Cartesian Form

2. Family of curves $xy = c$ & the family of curves $y^2 = x^2 + c$ are orthogonal trajectories of each other.



Orthogonal Trajectories – Cartesian Form

Working Procedure: For finding the Orthogonal Trajectory of Cartesian family of Curves

Step 1 : *Form the differential equation for the given family of curves* $\boldsymbol{F(x, y, c) = 0}$ *in the form* $\boldsymbol{f(x, y, dy/dx) = 0}$.

Step 2 : ***Replace*** $\boldsymbol{\frac{dy}{dx}}$ ***by*** $\boldsymbol{-\frac{dx}{dy}}$ to obtain the ***differential equation of the required orthogonal family of curves***.

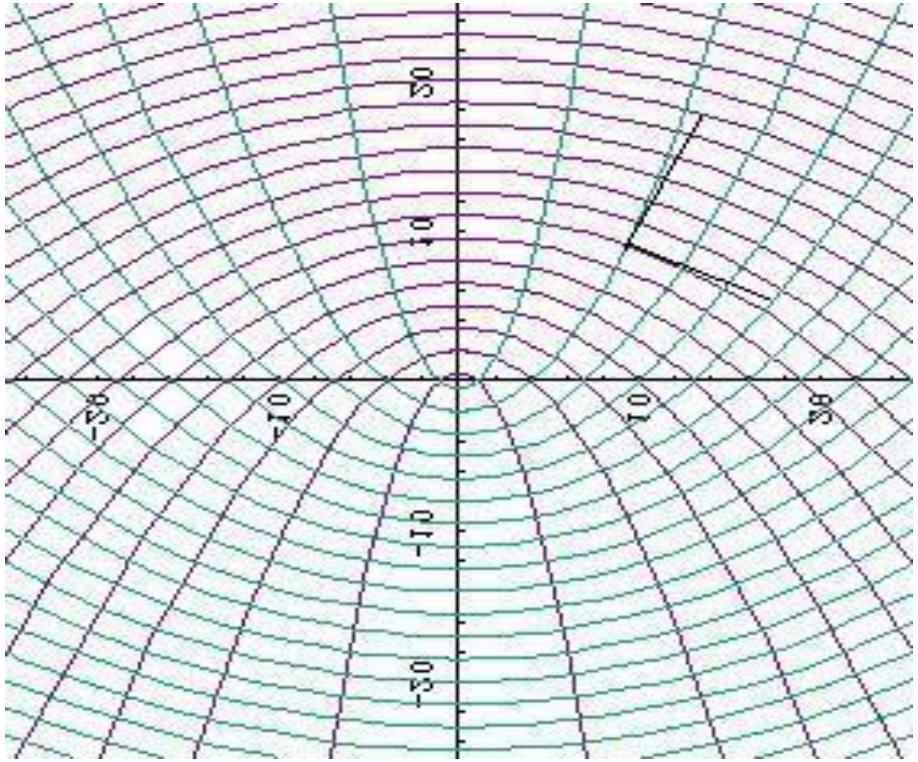
Step 3 : *Solving this differential equation, the orthogonal family of curves can be obtained.*

Orthogonal Trajectories – Cartesian Form

Definition : *A given family of curves is said to be self Orthogonal if its family of Orthogonal Trajectories are the same as the given family of curves.*

Example :

The family of curves $x^2 = 4c(y + c)$ is self orthogonal.



Orthogonal Trajectories – Cartesian Form

Points To Remember!

- If the **differential equation of the given family** is the same as the **differential equation of the orthogonal family**, then the given family is said to be Self Orthogonal.

- In other words, if the **differential equation remains the same after replacing $\frac{dy}{dx}$ by $-\frac{dx}{dy}$**, then the given family is said to be **Self Orthogonal**.

Orthogonal Trajectories – Cartesian Form

1.Find the Orthogonal Trajectories of the family of circles
$x^2 + y^2 = c^2$.

Solution :

Consider $\boldsymbol{x^2 + y^2 = c^2}$

Differentiating with respect to x, we have

$$2x + 2y\frac{dy}{dx} = 0$$

DE of the given family :

$$\boldsymbol{\frac{dy}{dx} = -\frac{x}{y}}$$

Orthogonal Trajectories – Cartesian Form

Replace $\frac{dy}{dx}$ *by* $-\frac{dx}{dy}$ to obtain the *differential equation of the required orthogonal family of curves.*

DE of the Orthogonal family :

$$\frac{dy}{y} = \frac{dx}{x}$$

Solving this we obtain,

$y = cx$ ***which is the required solution.***

Orthogonal Trajectories – Cartesian Form

2. Prove that the system of confocal and coaxial parabolas $y^2 = 4a(x+a)$ is self orthogonal. (a is a parameter)

Solution:

Consider $y^2 = 4a(x+a)$

Differentiating with respect to x, we have
DE of the given family :

$$\boldsymbol{y = 2x\frac{dy}{dx} + y\left(\frac{dy}{dx}\right)^2} \quad \text{................}(1)$$

Orthogonal Trajectories – Cartesian Form

Replacing $\frac{dy}{dx}$ **by** $-\frac{dx}{dy}$

we obtain $y = -2x\frac{dx}{dy} + y\left(\frac{dx}{dy}\right)^2$

Dividing throughout by $\left(\frac{dx}{dy}\right)^2$ *we have,*

$y = 2x\frac{dy}{dx} + y\left(\frac{dy}{dx}\right)^2$ *…………(2) which is the DE of the Orthogonal family .*

Since equations (1) & (2) are the same, the given family of parabolas are self Orthogonal.

THANK YOU

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