

# **ENGINEERING MATHEMATICS - I Ordinary Differential Equations**

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**Unit 3: Ordinary Differential Equations** 

Session: 3

Sub Topic: Bernoulli's Differential Equation

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# **Bernoulli's Differential Equation**



Any differential equation of the form

$$\frac{dy}{dx} + Py = Qy^n$$

where P and Q are functions of x only is called as Bernoulli's Differential equation in y.

To reduce this to the linear equation in standard form,

Step 1 : Divide throughout by  $y^n$  to obtain

$$\frac{1}{y^n}.\frac{dy}{dx} + Py^{1-n} = Q$$

## **Bernoulli's Differential Equation**



$$(1-n)y^{-n}\frac{dy}{dx} = \frac{dz}{dx}$$

and

$$\frac{dz}{dx} + P'z = Q'$$

which is linear in z

Integrating Factor = 
$$IF = e^{\int P'dx}$$

Solution: 
$$z(IF) = \int Q' \cdot (IF) dx + c$$



## **Bernoulli's Differential Equation**



Any differential equation of the form

$$\frac{dx}{dy} + Px = Qx^n$$

where P and Q are functions of y only is called as Bernoulli's Differential equation in x.

To reduce this to the linear equation in standard form,

Step 1 : Divide throughout by  $x^n$  to obtain

$$\frac{1}{x^n}.\frac{dx}{dy}+Px^{1-n}=Q$$

## **Bernoulli's Differential Equation**

■ Take the substitution  $x^{1-n} = z$ , then

$$(1-n)x^{-n}\frac{dx}{dy} = \frac{dz}{dy}$$

and

$$\frac{dz}{dx} + P'z = Q'$$

which is linear in z

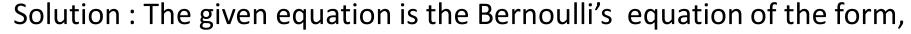
Integrating Factor = 
$$IF = e^{\int P'dy}$$

Solution: 
$$z(IF) = \int Q'.(IF)dy + c$$



## Example 1.

Solve: 
$$\frac{dy}{dx} + \frac{y}{x} = y^2x$$



$$\frac{dy}{dx} + \mathbf{P}y = \mathbf{Q}y^{\mathbf{n}} \quad (\mathbf{Linear in } y)$$

where 
$$P = \frac{1}{x}$$
,  $Q = x$  and  $n = 2$ .

- Dividing throughout by  $y^2$ ,  $\frac{1}{y^2} \frac{dy}{dx} + \left(\frac{1}{xy}\right) = x$
- Taking the substitution,  $y^{-1} = z$ , we obtain  $\frac{dz}{dx} = \frac{-1}{y^2} \cdot \frac{dy}{dx}$

On simplifying, 
$$-\frac{dz}{dx} + \frac{1}{x}$$
  $z = x$  or  $\frac{dz}{dx} - \frac{z}{x} = -x$ 

This is a linear differential equation of the form  $\frac{dz}{dx} + \mathbf{P}'\mathbf{z} = \mathbf{Q}'$ 



## Example 1.

where 
$$P' = -\frac{1}{x}$$
 and  $Q' = x$ 

Integrating Factor = 
$$IF = e^{-\int P'dx} = e^{-\int \frac{1}{x}dx} = \frac{1}{x}$$

General Solution: 
$$z(IF) = \int Q' \cdot (IF) dx + c$$
  

$$\Rightarrow z \cdot \frac{1}{x} = \int -x \cdot \frac{1}{x} dx + c$$

$$\Rightarrow z \cdot \frac{1}{x} = -x + c$$

$$\Rightarrow \frac{1}{xy} = -x + c \text{ is the required solution.}$$



## Example 2.

Solve: 
$$\frac{dr}{d\theta} = rtan\theta - \frac{r^2}{cos\theta}$$

Solution: The given equation can be written as

$$\frac{dr}{d\theta} - rtan\theta = -\frac{r^2}{\cos\theta}$$

This is the Bernoulli's equation linear in r where  $\mathbf{P} = -tan\theta$ ,  $\mathbf{Q} = -1/cos\theta$  and n=2

lacktriangle Dividing throughout by  ${f r^2}$  ,

$$\frac{1}{r^2}\frac{dr}{d\theta} - \frac{1}{r}\tan\theta = -\frac{1}{\cos\theta} = -\sec\theta$$



## Example 2.



■ Taking the substitution, 
$$\frac{1}{r} = t$$
, we obtain  $\frac{dt}{d\theta} = \frac{-1}{r^2} \cdot \frac{dr}{d\theta}$ 

Therefore 
$$-\frac{dt}{d\theta} - t \cdot tan\theta = -sec\theta$$
  
 $\Rightarrow \frac{dt}{d\theta} + t \cdot tan\theta = sec\theta$ 

This is a linear differential equation of the form  $\frac{dt}{dx} + P't = Q'$  where  $P' = tan\theta$  and  $Q' = sec\theta$ 

## Example 2.



Integrating Factor = IF = 
$$e^{\int P'd\theta} = e^{\int tan\theta d\theta} = e^{\log(sec\theta)} = sec\theta$$

General Solution : 
$$t(IF) = \int Q'.(IF)d\theta + c$$

$$\Rightarrow tsec\theta = \int sec^2\theta d\theta + c$$

$$\Rightarrow tsec\theta = tan\theta + c$$

$$\Rightarrow \frac{sec\theta}{r} = tan\theta + c$$
 is the required solution.



# **THANK YOU**

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