ENGINEERING MATHEMATICS - I

Homogeneous Functions
Euler's Theorem

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UNIT 2 : Partial Differentiation

Session : 6

Sub Topic : Homogeneous Functions and Euler's Theorem

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Homogeneous function

Definition 1: A function $u = f(x, y)$ is said to be a homogeneous function of degree n if it can be expressed in the form $x^n g\left(\frac{y}{x}\right)$ or $y^n g\left(\frac{x}{y}\right)$, g being the arbitrary function.

For example: Consider $u = 3x + 4y$

$$= x(3 + 4(y/x))$$

$$= x^1 g\left(\frac{y}{x}\right)$$

Therefore u is a homogeneous function of degree 1.

Homogeneous function

Definition 2: A function $u = f(x, y, z)$ is said to be a homogeneous function of degree n if it can be expressed in the form $x^n g\left(\frac{y}{x}, \frac{z}{x}\right)$ $or\ y^n g\left(\frac{x}{y}, \frac{z}{y}\right)$, or $z^n g\left(\frac{x}{z}, \frac{y}{z}\right)$, g being the arbitrary function.

For example: Consider $u = x^3 + y^3 + z^3 + 3xyz$

$$= x^3\left(1+\left(\frac{y}{x}\right)^2 + \left(\frac{z}{x}\right)^3 +3\left(\frac{y}{x}\right)\left(\frac{z}{x}\right)\right)$$

$$= x^3 g\left(\frac{y}{x}, \frac{z}{x}\right)$$

Therefore u is a homogeneous function of degree 3.

Homogeneous function

- A constant function is homogeneous of degree 0.
- Linear functions are homogeneous of degree 1.
- The degree of the homogeneous function can be positive, negative or zero.
- How to relate homogeneous functions with their partial derivatives.(Euler’s Theorem).
- Scaling of the given function.
- Extensive applications in Economics like production function and Marginal productivities, Elasticity of demand etc.,
- In Thermodynamics, Extensive variables are homogeneous of degree 1.
- Intensive variables are homogeneous of degree 0.

Euler's Theorem on Homogeneous Functions

Statement 1:If $u = f(x, y)$ is a homogeneous function of degree n then

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu$$

Corollary 1: If $u = f(x, y)$ is a homogeneous function of degree n then

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = n(n-1)u$$

Statement 2:If $u = f(x, y, z)$ is a homogeneous function of degree n then

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = nu$$

https://www.researchgate.net/publication/281457879_EXTENSION_OF_EULER'S_THEOREM_ON_HOMOGENEOUS_FUNCTION_TO_HIGHER_DERIVATIVES

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