ENGINEERING PHYSICS

Radhakrishnan S, Ph.D.

Department of Science and Humanities

Class #14

- Matter wave incident on a step potential case $E<V_o$
- Solutions of the SWE
- Interpretation of the wave functions
- Probabilities of penetration into the region of the potential

- ***Suggested Reading***

  1. *Concepts of Modern Physics, Arthur Beiser, Chapter 5*
  2. *Learning Material prepared by the Department of Physics*

- ***Reference Videos***

  1. *Video lectures : MIT 8.04 Quantum Physics I*
  2. *Engineering Physics Class #13*

Case II: Particle in a constant potential - $E < V_o$

Region I $x < 0$ $V = 0$

- *The general Schrodinger's wave equation*

$$\frac{\partial^2 \psi_I(x)}{\partial x^2} + \frac{2m}{\hbar^2}(E - V)\psi_I(x) = 0$$

- $V = 0$ *implies a free particle and the SWE reduces to*

$$\frac{\partial^2 \psi_I(x)}{\partial x^2} + \frac{2m}{\hbar^2}E\psi_I(x) = 0$$

$$\frac{\partial^2 \psi_I(x)}{\partial x^2} + k_I^2\psi_I(x) = 0$$

Where $k_I = \sqrt{\frac{2mE}{\hbar^2}}$

Case II: Particle in a constant potential - $E < V_o$

- ***The general solution for the wave function***

$$\psi_I(x) = Ae^{ik_I x} + Be^{-ik_I x}$$

$\rightarrow$ ***forward moving incident wave***

$\leftarrow$ ***backward moving reflected wave***

Case II: Particle in a constant potential - $E < V_o$

Region II $x > 0 \qquad V = V_o > E \;\; (E - V_o)\; is\; negative$

- ***The general Schrodinger's wave equation***

$$\frac{\partial^2 \psi_{II}(x)}{\partial x^2} + \frac{2m}{\hbar^2}(E - V)\psi_{II}(x) = 0$$

- ***the SWE reduces to***

$$\frac{\partial^2 \psi_{II}(x)}{\partial x^2} + \frac{2m}{\hbar^2}(E - V_o)\psi_{II}(x) = 0$$

$$\frac{\partial^2 \psi_{II}(x)}{\partial x^2} - \alpha^2 \psi_{II}(x) = 0$$

where $\alpha = \sqrt{\frac{2m(V_o - E)}{\hbar^2}}$

Case II: Particle in a constant potential - $E > V_o$

- *The general solution for the wave function*

$$\psi_{II}(x) = Fe^{\alpha x} + Ge^{-\alpha x}$$

$\psi_{II}(x)$ *should be finite for all values of x*

Hence the first part of the equation cannot be a part of $\psi_{II}(x)$

Implying $F = 0$

The wave function in the region of constant potential

$\psi_{II}(x) = Ge^{-\alpha x}$ $\rightarrow$ *an exponentially decaying function*

Particle moving into a region of constant potential - $E > V_o$

- *The wave functions* $\psi_I(x)$ *and* $\psi_{II}(x)$ *and their derivatives wrt to* x*,* $d\psi_I(x)$ *and* $d\psi_{II}(x)$ *have to be continuous in the region and at* $x = 0$
- *The flux of the transmitted flux in the second region is zero*
- *The particle is reflected back into the region of zero potential – reflection co-efficient is 1*

Case II: Particle in a constant potential - $E < V_o$

$(E - V_o)$ is the kinetic energy of the particle which is negative – conceptually unacceptable condition

Implying the particle cannot be found in the region $x > 0$

The wave function (the probability amplitude) and the probability density $\psi_{II}{}^{*}\psi_{II}$ are greater than zero

Quantum mechanically the particle may attempt to cross over to region $x > 0$ with an extremely small non-zero probability

Case II: Particle in a constant potential - $E > V_o$

The depth Δx at which the wave function $\psi_{II}(x)$ tends to become insignificant – the penetration depth in region II

$$\psi_{II}(\Delta x) =\cong \frac{1}{e}\psi_{II}(\Delta x = 0)$$

$$Ge^{-\alpha\Delta x} = \frac{1}{e}Ge^{-\alpha 0} = Ge^{-1}$$

$$\Delta x = \frac{1}{\alpha} = \frac{\hbar}{\sqrt{2m(V_o - E)}}$$

Summarizing Case II: E < V$_o$

- *Region I*

Region II

- $\psi_I(x) = Ae^{ik_I x} + Be^{-ik_I x}$
- $k_I = \sqrt{\frac{2mE}{\hbar^2}}$
- $E = \frac{\hbar^2 k_I^2}{2m} = KE$
- $P_I = \hbar k_I$
- $\lambda_I = \frac{h}{\sqrt{2mE}}$

$\psi_{II}(x) = Ge^{-\alpha x}$

$\alpha = \sqrt{\frac{2m(V_o - E)}{\hbar^2}}$

$E = \frac{\hbar^2 \alpha^2}{2m} + V_o$

P_{II} - *undefined*

λ_{II} - *undefined*

Class #14 …….. Quiz….

The concepts true of a particle with energy $E < V_o$ approaching a region of constant potential …

1. The wave function of the particle in the region is a cyclic wave function
2. The wave function gives a non zero probability in the region of constant potential
3. The penetration depth of the wave function is higher if the energy of the particle is higher
4. The reflection probability is less than 1
5. The propagation constant can be defined for the wave function in the region of constant potential

THANK YOU

Radhakrishnan S, Ph.D.

Professor, Department of Science and Humanities

sradhakrishnan@pes.edu

+91 80 21722683 Extn 759