ENGINEERING PHYSICS

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Class #15

- Barrier potentials of finite widths
- Matter wave incident on a barrier potential $E<V_o$
- Solutions of the SWE
- Interpretation of the wave functions
- Probabilities of barrier penetration

➢ *Suggested Reading*

1. ***Concepts of Modern Physics, Arthur Beiser, Chapter 5***
2. ***Learning Material prepared by the Department of Physics***

➢ *Reference Videos*

1. ***Video lectures : MIT 8.04 Quantum Physics I***
2. ***Engineering Physics Class #14***

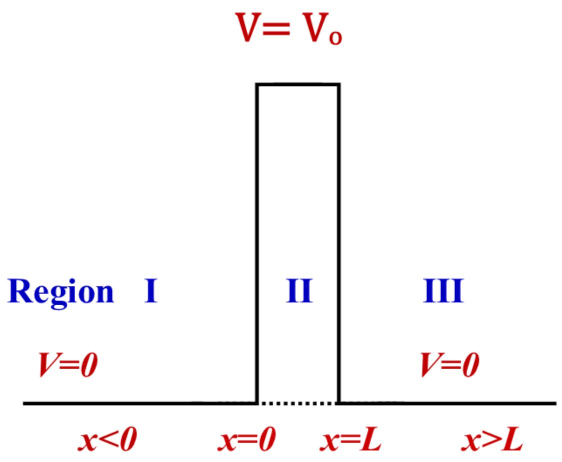
Barrier potential

- *A 1D rectangular barrier potential is defined by*

  Region I $V = 0 \; for \; x < 0$

  Region II $V = V_o \; for \; 0 < x < L$

  Region III $V = 0 \; for \; x > L$



Barrier Potential

Region I $x < 0$ $V = 0$

Standard free particle solution

$$\frac{\partial^2 \psi_I(x)}{\partial x^2} + \frac{2m}{\hbar^2} E\psi_I(x) = 0$$

$$\frac{\partial^2 \psi_I(x)}{\partial x^2} + k_I^2 \psi_I(x) = 0$$

Where $k_I = \sqrt{\frac{2mE}{\hbar^2}}$

The solution is the same as

$$\psi_I(x) = Ae^{ik_I x} + Be^{-ik_I x}$$

Barrier Potential

Region II $x > 0$ $V = V_o > E$ $(E - V_o)$ $is\ negative$

- *The SWE reduces to*

$$\frac{\partial^2 \psi_{II}(x)}{\partial x^2} + \frac{2m}{\hbar^2}(E - V_o)\psi_{II}(x) = 0$$

$$\frac{\partial^2 \psi_{II}(x)}{\partial x^2} - \alpha^2 \psi_{II}(x) = 0$$

where $\alpha = \sqrt{\frac{2m(V_o - E)}{\hbar^2}}$

And the solution is

$\psi_{II}(x) = De^{-\alpha x}$ $\rightarrow$ *exponentially decaying function*

Barrier potentials

Region III $x < L$ $V = 0$

Standard free particle solution

$$\frac{\partial^2\psi_{III}(x)}{\partial x^2} + \frac{2m}{\hbar^2}E\psi_{III}(x) = 0$$

$$\frac{\partial^2\psi_{III}(x)}{\partial x^2} + k_{III}^2\psi_{III}(x) = 0$$

Where $k_{III} = \sqrt{\frac{2mE}{\hbar^2}}$

The solution is

$$\psi_{III}(x) = Ge^{ik_{III}x}$$

Barrier potential

Continuity of wave functions and derivatives

$$\psi_I = \psi_{II} \; at \; x = 0 \quad and \quad \psi_{II} = \psi_{III} \; at \; x = L$$

$$\frac{d\psi_I}{dx} = \frac{d\psi_{II}}{dx} \; at \; x = 0 \quad and \quad \frac{d\psi_{II}}{dx} = \frac{d\psi_{III}}{dx} \; at \; x = L$$

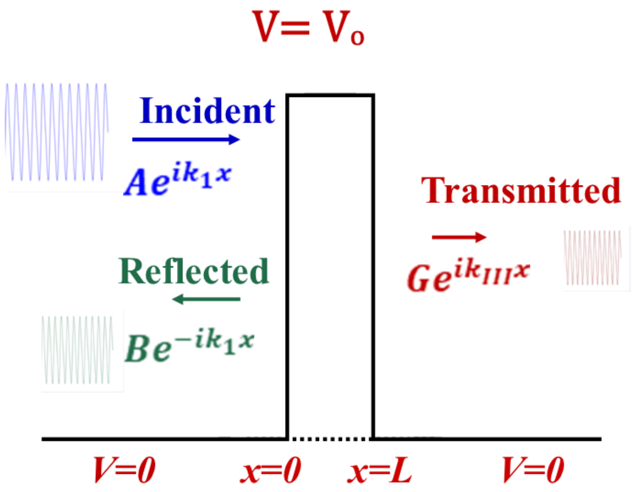
- transmission coefficient $T = \dfrac{transmitted\,flux}{incident\,flux} = \dfrac{G^*G\,v_{III}}{A^*A\,v_I}$
- $k_I = k_{III}$
- T is also the probability of the particles being transferred to the third region even when $E < V_o$
- $T = \dfrac{G^*G}{A^*A} \approx 16\dfrac{E}{V_o}\left[1-\left(\dfrac{E}{V_o}\right)^2\right] e^{-2\alpha L} \cong e^{-2\alpha L}$
- T will be higher if α or L is small

Barrier potential

The transmission probability is higher if the penetration depth is greater than the width of the barrier $\Delta x > L$

The energy of the particle in region II does not allow the particle to be found region II.

Hence the particle is said to tunnel through the barrier.



Summarizing Case II: Barrier tunneling

Region I

- $\psi_I(x) = Ae^{ik_I x} + Be^{-ik_I x}$
- $k_I = \sqrt{\frac{2mE}{\hbar^2}}$
- $E = \frac{\hbar^2 k_I^2}{2m} = KE$
- $P_I = \hbar k_I$
- $\lambda_I = \frac{h}{\sqrt{2mE}}$

Region II

- $\psi_{II}(x) = De^{-\alpha x}$
- $\alpha = \sqrt{\frac{2m(V_o - E)}{\hbar^2}}$
- $\Delta x = \sqrt{\frac{\hbar^2}{2m(V_o - E)}}$
- $KE = E - V_o$ -ve

Region III

- $\psi_{III}(x) = Ge^{ik_{III} x}$
- $k_{III} = \sqrt{\frac{2mE}{\hbar^2}}$
- $E = \frac{\hbar^2 k_{III}^2}{2m} = KE$
- $P_{III} = \hbar k_{III}$
- $\lambda_{III} = \frac{h}{\sqrt{2mE}}$

- *The transmission probability* $T \cong e^{-2\alpha L}$
- *In general potentials are a function space and for most cases inversely proportion to the magnitude of the radial vector* $V \propto \frac{1}{x}$

  Can be approximated to piece wise rectangular potentials

The concepts true of a particle with energy $E < V_o$ approaching a barrier potential …

1. The energy of the particle in region III is less than the energy of the particle in region I
2. The wave function of the particle in region II is an exponential decay
3. The tunneling probability is higher if the energy of the particle E<<Vo
4. The transmission co-efficient is very high if $L << \alpha$
5. The reflection probability of the particle is $\cong 0$
6. The momentum of the particle is the same in regions I & III

THANK YOU

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