ENGINEERING MATHEMATICS - I

Ordinary Differential Equations

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Unit 3 : Ordinary Differential Equations

Session : 12

Sub Topic : Application problems

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Applications of first order differential equation :

- Newton's law of cooling
- Laws of natural growth
- Laws of natural decay
- Simple electric circuits(RL-circuit and RC-circuit)

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Newton's law of cooling :

According to this law, the temperature of a body changes at a rate which is proportional to the difference in temperature between that of the surrounding medium and that of the body itself.

If t_2 is the temperature of the surroundings and T that of the body at any time t, then

$$\frac{dT}{dt} = -k(T - t_2)$$

where k is the constant of proportionality.

Note: the negative sign indicates the cooling of the body with the increase of the time.

ODE-Application problems

Every geometrical or physical problem when translated into mathematical symbols gives rise to a differential equation.

Recall!

The study of a differential equation consists of three phases:

- Formulation of differential equation from the given physical situation, called modelling.

- Solutions of this differential equation ,evaluating the arbitrary constants from the given conditions and

- Physical interpretation of the solution

ODE-Application problems - Newton’s law of cooling

- Express Newton’s law of cooling in the form of differential equation and solve it.

Let t_1 °c be the initial temperature of the body and

t_2 °c be the constant temperature of the medium. Further

T °c be the temperature of the body at any time t.

Then by Newton’s law of cooling,

$\frac{dT}{dt} = -k(T - t_2)$ with the condition T(0)= t_1

$$\int \frac{dT}{T-t_2} = \int -k\, dt + \alpha$$

$$\log(T - t_2) = -kt + \alpha$$

$$\text{Or } T - t_2 = e^{-kt+\alpha}$$

$T - t_2 = c\, e^{-kt}$ where c=e^{α}= constant

Applying the initial condition,

$T = t_1$ when t=0, we have

$$t_1 - t_2 = ce^{0} \text{ Or } t_1 - t_2 = c$$

Therefore,

$$T - t_2 = (t_1 - t_2)e^{-kt}$$

$$\boldsymbol{T = t_2 + (t_1 - t_2)e^{-kt}}$$

1. Water at temperature 10˚c takes 5 min to warm up to 20 ˚c in a room at temperature 40 ˚c

A) find the temperature after 20 min; after ½ hr

B) when will the temperature be 25 ˚c

Solution:

Let t_1 ˚c be the initial temperature of the water and

t_2 ˚c be the room temperature.

Further

T ˚c be the temperature of the water at any time t.

Then by Newton's law of cooling,

$$T = t_2 + (t_1 - t_2)e^{-kt} \text{ ....(1)}$$

Given:

$$t_1 = 10\,, t_2 = 40\ \ , T = 20\ \ and\ t = 5\ min$$

Substituting all these values in (1),

we get $k = \frac{-1}{5}\ or\ 0.08109.$

a) Find T when t= 20 min

Substituting $t_1 = 10\,, t_2 = 40\ \ , k = 0.08109\ \ and\ t = 20\, min$ in (1), we have T= 34.073

Find T when t= 30 min

Substituting $t_1 = 10\,, t_2 = 40\ \ , k = 0.08109\ \ and\ t = 30\, min$ in (1), we have T= 37.36

b) Find t when T= 25

Substituting $t_1 = 10, t_2 = 40,$

$k = 0.08109 \;\; and \; T = 25$ in (1),

we have t= 8.5478.

2. A copper ball is heated to a temperature of 100 ˚c. Then at time t=0 it is placed in water which is maintained at a temperature of 30 ˚c. At the end of 3 minutes temperature of the ball is reduced to 70 ˚c. Find the time at which the temperature of the ball drops to 31 ˚c.

Solution:

Let t_1 ˚c be the initial temperature of the copper and t_2 ˚c be the temperature of the medium. Further let T ˚c be the temperature of the copper at any time t.

Then by Newton's law of cooling,

$$T = t_2 + (t_1 - t_2)e^{-kt} \quad \text{........(1)}$$

Given:

$$t_1 = 100\,, t_2 = 30\ \ , T = 70\ \ and\ t = 3\ min$$

Substituting all these values in (1) we get $k = 0.1865$.

To find *t* when *T= 31*

Substituting $t_1 = 100\,, t_2 = 30\ \ , k = 0.1865\ \ and\ T = 31$ in (1), we have *t* = 22.73 min.

THANK YOU

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