ENGINEERING PHYSICS

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Class #20

- Particle in an 1D Finite potential well
- Qualitative discussion on nature of solution
- Energy of a particle in a finite potential well

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Suggested Reading

1. *Concepts of Modern Physics, Arthur Beiser, Chapter 5*
2. *Learning Material prepared by the Department of Physics*

Reference Videos

1. *Video lectures : MIT 8.04 Quantum Physics I*
2. *Engineering Physics lectures of Class #14 and #17*

Particle in an 1D finite potential well

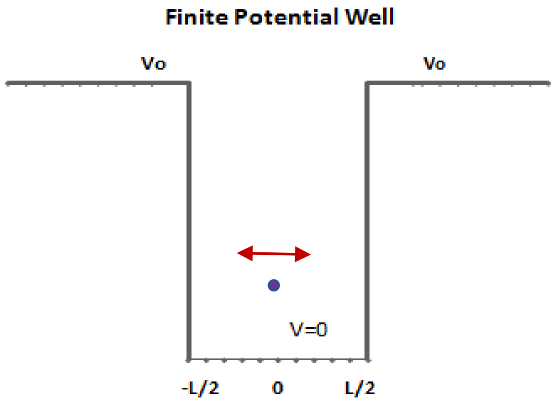
A finite potential well is defined by three regions of potentials defined by

Region I $V = V_o \quad for \quad x < -\frac{L}{2}$

Region II $V = 0 \quad for \quad -\frac{L}{2} < x < +\frac{L}{2}$

Region III $V = V_o \quad for \quad x < +\frac{L}{2}$

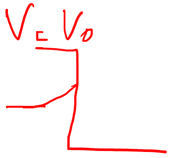
A particle of mass m and energy E<V_o is bound by the finite potential steps $V = V_o$ at the boundaries $\pm\frac{L}{2}$



Particle in an 1D finite potential well

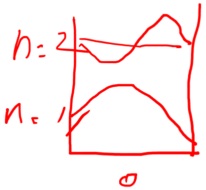
Considering the three regions, the wave functions could be written as

$\psi_1 = De^{\alpha x}$ **where** $\alpha = \sqrt{\frac{2m(V_o - E)}{\hbar^2}}$ **for the region** $x < {}^{-L}/_{2}$



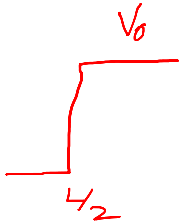
$\psi_2 \cong A\cos(k_2 x)$ $for\ odd\ values\ of\ n$ **or**

$\psi_2 = A\sin(k_2 x)$ $for\ even\ values\ of\ n$



where $k_2 = \sqrt{\frac{2mE}{\hbar^2}}$ **for the region** ${}^{-L}/_{2} < x < {}^{L}/_{2}$

$\psi_3 = Ge^{-\alpha x}$ **where** $\alpha = \sqrt{\frac{2m(V_o - E)}{\hbar^2}}$ **for the region** $x > {}^{L}/_{2}$



However, it is to be noted that the wave function ψ_2 would not be zero at the boundaries of $-\frac{L}{2}$ and $+\frac{L}{2}$

Particle in an 1D finite potential well

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The continuity of the wave functions and their derivatives at the boundaries $\pm\frac{L}{2}$ can be established

$\psi_1 = \psi_2$ and $d\psi_1 = d\psi_2$ @ $\pm L/2$

for the even parity functions (n_{odd}) we obtain

$$\alpha = k_2 \tan(k_2\frac{L}{2}) \quad \text{or} \quad \sqrt{(V_o - E)} = \sqrt{E}\tan\left(\sqrt{\frac{2mE}{\hbar^2}} * \frac{L}{2}\right)$$

And for the odd parity functions (n_{even}) we obtain

$$\alpha = -k_2 \cot(k_2\frac{L}{2}) \quad \text{or} \quad \sqrt{(V_o - E)} = -\sqrt{E}\cot\left(\sqrt{\frac{2mE}{\hbar^2}} * \frac{L}{2}\right)$$

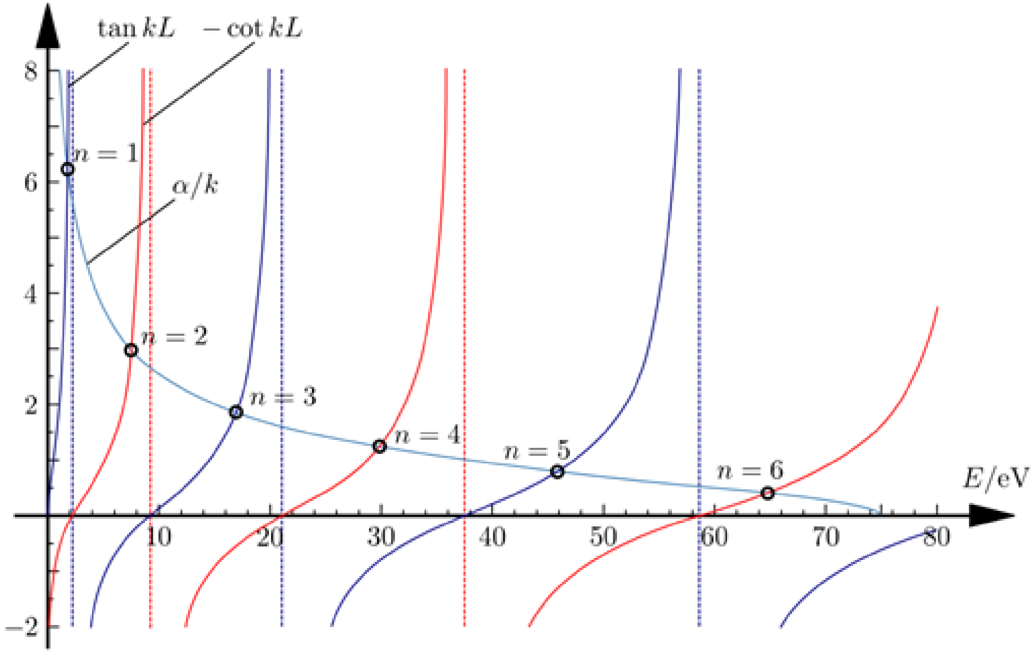
These are transcendental equations which do not yield exact solutions.

Approximate solutions can be obtained using numerical methods or graphical methods

Particle in an 1D finite potential well

Draw a graph with E as the variable and the left and right hand sides of the equation on the same graph

The points of intersection of the LHS and RHS plots are the solutions as the values of E satisfy both the equations.
Thus the different energy states could be evaluated.

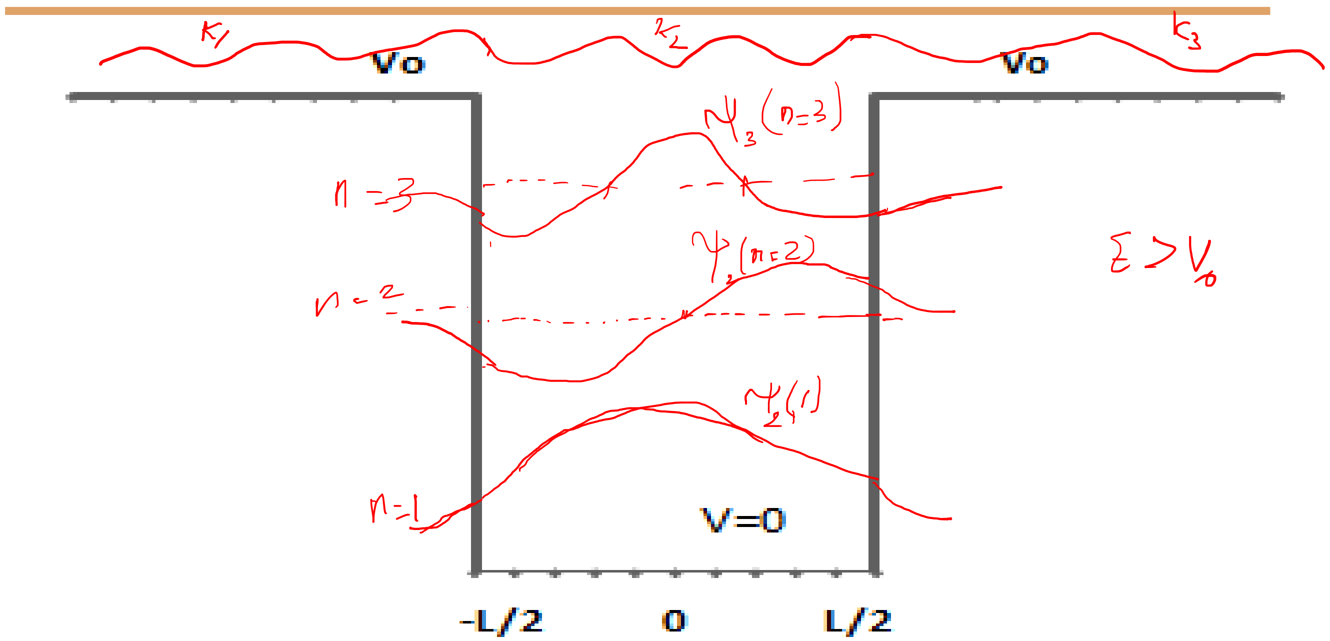


Wave functions of a Particle in an 1D finite potential well

Graphically the wave functions could be drawn with the condition that

$$\psi_1 = \psi_2 \; at \; x = -\frac{L}{2} \quad and \quad \psi_2 = \psi_3 \; at \; x = \frac{L}{2}$$

and the functions ψ_1 *and* ψ_3 *are exponential decaying functions*



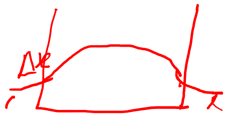
Particle in an 1D finite potential well

The energy of the particle in a finite potential well can be inferred from the concept of energy of a particle in an infinite potential well.

$\psi = 0$ at $\pm L/2$

$$E_{infinite} = \frac{\hbar^2\pi^2n^2}{2mL^2} = \frac{\hbar^2\pi^2n^2}{2m(width\ of\ the\ well\ where\ \psi \to 0)^2}$$

The wave function ψ_1 and ψ_3 decay down in a distance Δx in the regions beyond $|x| > \frac{L}{2}$.



$L + 2\Delta x$ $\psi \to 0$

The energy of the particle in the finite potential well can be approximated as

$$E_{finite} = \frac{\hbar^2\pi^2n^2}{2m(width\ of\ the\ well\ where\ \psi \to 0)^2} = \frac{\hbar^2\pi^2n^2}{2m(L+2\Delta x)^2}$$

$$E_{finite} < E_{infinite}$$

Class #20 …….. Quiz….

The concepts which are true of a finite potential well ….

1. Finite potential energy is of the order of few eV
2. Inside the potential well the potential is zero implies the particle is a free particle
3. If the energy of the particle is greater than the potential then the particle is a free particle (in a limited sense)
4. The energy of the particle in the finite potential well are equally spaced
5. The energy of the particle in the finite potential well is always less than that of a particle in the infinite potential well in identical states

THANK YOU

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