ENGINEERING PHYSICS

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Week #1

Cl#1 Review of Electric and magnetic fields

Cl#2 EM Wave equation

Cl#3 Energy transported by EM Waves

Cl#4 Max Planck's Black Body Radiation equation

Electromagnetic wave equations

Class #2

- *Maxwell's equation in differential form*
- *Maxwell's Equations in free space*
- *Ideas of Electric and Magnetic waves*
- *Wave equation*

Electromagnetic wave equations

- ***Suggested Reading***
    1. *Fundamentals of Physics, Halliday, Resnik, Chapter 34*
    2. *NCERT Physics Book I grade 12 – Chapter 8*
- ***Reference Videos***
    1. *https://nptel.ac.in/courses/108/106/108106073/*
    2. *Engineering Physics week1 class1*

Electric fields

the divergence of the electric field is proportional to the charge density

$$\nabla.\vec{E} = \frac{\rho}{\varepsilon_o}$$

Magnetic Fields

the divergence of the magnetic flux is uniformly zero

$$\nabla.\vec{B} = 0$$

Maxwell's equations - Faraday's law

Faraday's law of electromagnetic induction

The curl of the induced electric field in a closed loop is proportional to the rate of change of magnetic flux linked with the loop

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Ampere - Maxwell's law

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Ampere - Maxwell circuital law

The curl of the magnetic field in a closed loop is equal to the sum of the current density and the displacement current due to the time varying electric field

$$\nabla \times \vec{B} = \mu_o \vec{j} + \mu_o \varepsilon_o \frac{\partial \vec{E}}{\partial t}$$

Maxwell's equations in free space

Free space implies – charges and currents do not exist

$$\vec{\nabla}.\vec{E} = 0 \qquad 1.$$

$$\vec{\nabla}.\vec{B} = 0 \qquad 2.$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad 3$$

$$\vec{\nabla} \times \vec{B} = +\mu_o \varepsilon_o \frac{\partial \vec{E}}{\partial t} \qquad 4.$$

Electric waves in free space

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The curl of equation 3. $\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla} \times \left(-\frac{\partial \vec{B}}{\partial t}\right)$

$$\vec{\nabla}(\vec{\nabla}.\vec{E}) - \nabla^2\vec{E} = \left(-\frac{\partial \vec{\nabla} \times \vec{B}}{\partial t}\right)$$

For free space $\vec{\nabla}.\vec{E} = 0$

$$-\nabla^2\vec{E} = \left(-\frac{\partial \vec{\nabla} \times \vec{B}}{\partial t}\right)$$

Substituting for curl of B from equation 4.

$$\nabla^2\vec{E} = \left(\mu_o \varepsilon_o \frac{\partial^2 E}{\partial t^2}\right)$$

With $\mu_o \varepsilon_o = \frac{1}{c^2}$

$$\nabla^2\vec{E} = \left(\frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2}\right) \quad ......$$

A wave equation for electric wave propagating in free space !

Magnetic waves in free space

- *The curl of Maxwell's equation 4.*

$$\vec{\nabla} \times \vec{\nabla} \times \vec{B} = \vec{\nabla} \times \left(+\mu_o \varepsilon_o \frac{\partial \vec{E}}{\partial t}\right) \rightarrow$$

$$\nabla^2 \vec{B} = \left(\frac{1}{c^2} \frac{\partial^2 \vec{B}}{\partial t^2}\right) \rightarrow$$

a magnetic wave propagating in free space with speed of light

- *Both electric and magnetic waves propagate with the speed of light*
- *Maxwell's predication of light (radiation) as electromagnetic waves*

Electromagnetic waves in free space

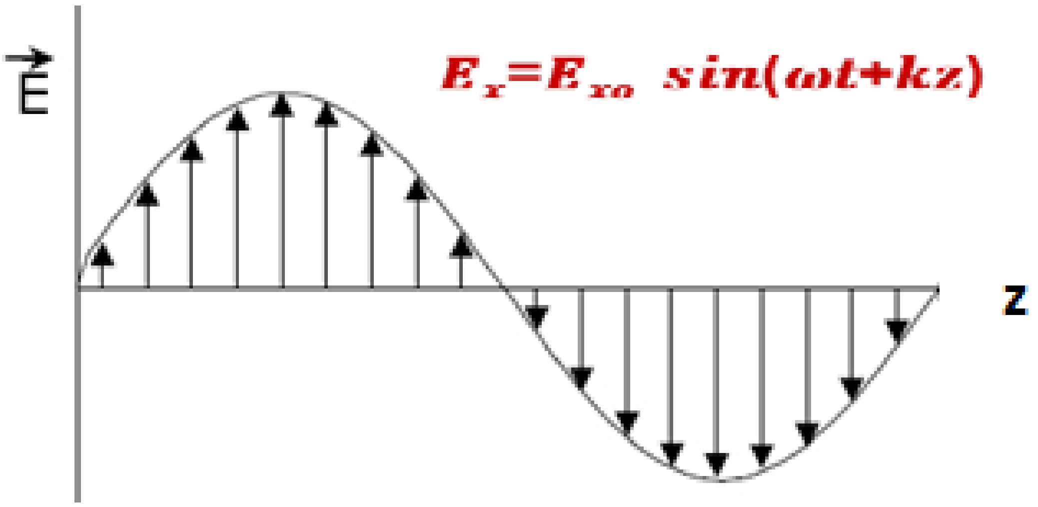
Light waves are transverse waves ---

Electromagnetic waves are transverse waves !

The electric and magnetic fields are perpendicular to the direction of propagation of the electromagnetic wave

Electromagnetic waves in free space

- *Let a plane EM wave be propagating along the z direction* $(E_z = 0)$
- *Instantaneous electric field* $E_x = E_{xo}\, sin(\omega t + kz)$



Electromagnetic waves in free space

- *Plane wave implies* $E_y = 0$
- *From the third Maxwell's equation*
- $$\vec{\nabla} \times \overrightarrow{E_x} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = -\hat{j}\frac{\partial E_x}{\partial z} = -\frac{\partial \vec{B}}{\partial t}$$
- $\frac{\partial \vec{B}}{\partial t}$ *can be evaluated since* E_x *is known*

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Electromagnetic waves in free space

- *Integrate* $\frac{\partial \vec{B}}{\partial t}$ *with respect to time to obtain*
- $B_y = \hat{j}\frac{E_{xo}}{\omega/_k} sin(\omega t + kz)$
- $\frac{\omega}{k} = the\ velocity\ of\ the\ wave = c$
- *magnetic field is oriented in the y direction*
- *perpendicular to the Electric field (x direction)*
- *E and B are mutually perpendicular to each other*

Electromagnetic waves in free space

- $E_x = \hat{\imath} E_{xo} \sin(\omega t + kz)$
- $B_y = \hat{\jmath} \frac{E_{xo} \sin(\omega t + kz)}{\omega / k}$
- *in phase with the electric field variations*
- $\omega/_k = c$ *is the phase velocity of the wave*
- *Thus* B_y *is perpendicular to E and* $|B_y| = \frac{|E_x|}{c}$

Electromagnetic waves in free space

- *Electromagnetic wave*
- *E and B are mutually perpendicular to each other and the direction of propogation*

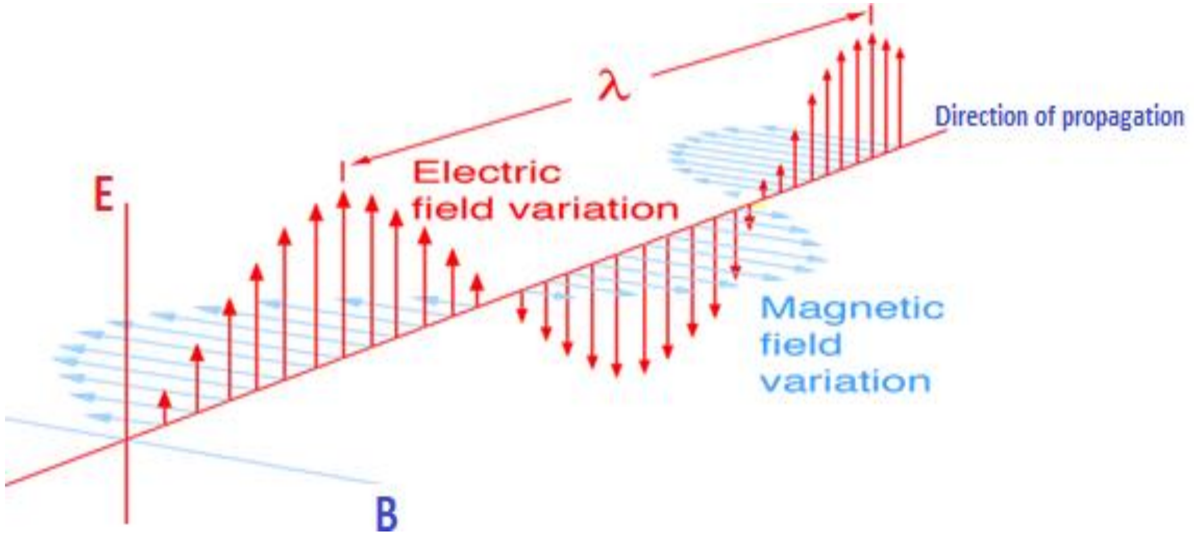

Image courtesy **Wikipedia**

Class 2 . Quiz …

The concepts which are not correct are….

1. Electric waves in free space are longitudinal
2. Magnetic waves in free space are transverse
3. The curl of a magnetic field is uniformly zero
4. The divergence of a magnetic field can be non zero
5. The curl of an electric field is always linked to an time varying magnetic field
6. The divergence of a vector field is a also a scalar

THANK YOU

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