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ENGINEERING MATHEMATICS - I

Ordinary Differential Equations

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Unit 3 : Ordinary Differential Equations

Session : 11

Sub Topic : Non-Linear Differential Equations

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ODE- Non linear Differential Equation

- **Equations solvable for *y***

Working Procedure

Step 1. ***Rewrite the given differential equation*** $f(x, y, p) = 0$ *in the form* $y = F(x, p)$.....(1)

Step 2. ***Differentiate (1) w.r.t 'x'*** *to obtain the equation of the form,* $p = \emptyset\left(x, p, \frac{dp}{dx}\right)$.....(2) *which is a first order and first degree differential equation in the variable p.*

Working Procedure(contd…)

Step 3. ***Solve the differential equation (2)*** *. The solution is of the form* $\boldsymbol{G(x, p, c) = 0}$*……(3)*

Step 4. ***Eliminating p from equations (1) and (3)****, the required solution of the DE (1).*

ODE- Non linear Differential Equation

NOTE :

1. Whenever it is not possible to eliminate p from equations (1) & (3), the **solution of the DE (1)** is given by the parametric equations

$$x = x(p, c) \ \& \ y = y(p, c).$$

2. **When the factor which does not contain *dp/dx* is equated to zero and solved**, we obtain another solution called the **singular solution** of the given differential equation. Observe that the singular solution does not contain any arbitrary constant.

ODE- Non linear Differential Equation - Solvable for y

1. Find the general solution of $3x^4p^2 - xp - y = 0$

Answer : The given equation can be written as,

$$\boldsymbol{y = 3x^4p^2 - xp}\text{.....(1)}$$

Differentiating w.r.t 'x',

- $\boldsymbol{\frac{dy}{dx} = 12x^3p^2 + 6x^4p\frac{dp}{dx} - x\frac{dp}{dx} - p}$
- $\boldsymbol{p = 12x^3p^2 + 6x^4p\frac{dp}{dx} - x\frac{dp}{dx} - p}$
- $\boldsymbol{2p = 12x^3p^2 + 6x^4p\frac{dp}{dx} - x\frac{dp}{dx}}$
- $\boldsymbol{2p - 12x^3p^2 = 6x^4p\frac{dp}{dx} - x\frac{dp}{dx}}$
- $\boldsymbol{2p(1 - 6x^3p) = -x(1 - 6x^3p)\frac{dp}{dx}}$

ODE- Non linear Differential Equation - Solvable for y

- $\left(2p + x\frac{dp}{dx}\right)\left(1 - 6x^3p\right) = 0$
- $\frac{dp}{2p} = -\frac{dx}{x}$ (By equating the first factor to zero)
- $\frac{dp}{2p} + \frac{dx}{x} = 0$
- $\frac{1}{2}logp + logx = logk$
- $px^2 = c$ or $p = \frac{c}{x^2}$

Substituting in (1)

$$3x^4\left(\frac{c}{x^2}\right)^2 - x\left(\frac{c}{x^2}\right) - y = 0$$

- $3c^2 - \frac{c}{x} - y = 0$
- $3c^2x - c - xy = 0 \longrightarrow c(3cx - 1) = xy$

ODE- Non linear Differential Equation - Solvable for y

2. Solve : $y + px = p^2x^4$

Answer : The given equation can be written as,

$y = x^4p^2 - xp$.....(1)

Differentiating w.r.t 'x',

- $\frac{dy}{dx} = 4x^3p^2 + 2p\frac{dp}{dx} - x\frac{dp}{dx} - p$
- $p = 4x^3p^2 + 2x^4p\frac{dp}{dx} - x\frac{dp}{dx} - p$
- $2p = 4x^3p^2 + 2x^4p\frac{dp}{dx} - x\frac{dp}{dx}$
- $2p - 4x^3p^2 = 2px^4\frac{dp}{dx} - x\frac{dp}{dx}$
- $2p(1 - 2x^3p) = -x(1 - 2x^3p)\frac{dp}{dx}$

ODE- Non linear Differential Equation - Solvable for y

- $\left(2p + x\frac{dp}{dx}\right)\left(1 - 2x^3p\right) = 0$
- $\frac{dp}{2p} = -\frac{dx}{x}$ **(Equating the first factor to zero)**
- $\frac{dp}{2p} + \frac{dx}{x} = 0$
- $\frac{1}{2}logp + logx = logk$
- $px^2 = c$ or $p = \frac{c}{x^2}$

Substituting in (1)

$$y = x^4\left(\frac{c}{x^2}\right)^{-2} x\left(\frac{c}{x^2}\right)$$

- $y + \frac{c}{x} = c^2$
- $xy + c = c^2x$

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THANK YOU

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