

ENGINEERING MATHEMATICS - I Ordinary Differential Equations

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Unit 3: Ordinary Differential Equations

Session: 12

Sub Topic: Application problems

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ODE-Applications

Applications of first order differential equation :

- Newton's law of cooling
- Laws of natural growth
- Laws of natural decay
- Simple electric circuits(RL-circuit and RC-circuit)



ODE-Application problems

Newton's law of cooling:

According to this law, the temperature of a body changes at a rate which is proportional to the difference in temperature between that of the surrounding medium and that of the body itself.

If t_2 is the temperature of the surroundings and T that of the body at any time t, then

$$\frac{dT}{dt} = -k(T - t_2)$$

where k is the constant of proportionality.

Note: the negative sign indicates the cooling of the body with the increase of the time.



ODE-Application problems



Every geometrical or physical problem when translated into mathematical symbols gives rise to a differential equation.

Recall!

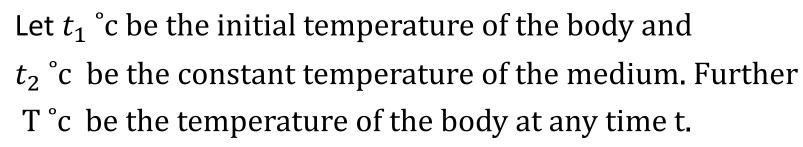


The study of a differential equation consists of three phases:

- Formulation of differential equation from the given physical situation, called modelling.
- ➤ Solutions of this differential equation , evaluating the arbitrary constants from the given conditions and
- ➤ Physical interpretation of the solution

ODE-Application problems - Newton's law of cooling

Express Newton's law of cooling in the form of differential equation and solve it.



Then by Newton's law of cooling,

$$\frac{dT}{dt} = -k(T - t_2) \text{ with the condition T(0)} = t_1$$

$$\int \frac{dT}{T - t_2} = \int -k \ dt + \alpha$$



ODE-Application problems - Newton's law of cooling



$$\log(T - t_2) = -kt + \alpha$$
Or $T - t_2 = e^{-kt + \alpha}$

$$T - t_2 = c e^{-kt}$$
 where $c = e^{\alpha} = \text{constant}$

Applying the initial condition,

 $T = t_1$ when t=0, we have

$$t_1$$
- t_2 = c e^0 Or t_1 - t_2 = c

Therefore,

$$T - t_2 = (t_1 - t_2)e^{-kt}$$

$$T = t_2 + (t_1 - t_2)e^{-kt}$$

ODE-Application problems - Newton's law of cooling

- 1. Water at temperature 10°c takes 5 min to warm up to 20°c in a room at temperature 40°c
- A) find the temperature after 20 min; after ½ hr
- B) when will the temperature be 25 °c

Solution:

Let t_1 °c be the initial temperature of the water and t_2 °c be the room temperature.

Further

T°c be the temperature of the water at any time t.



ODE-Application problems - Newton's law of cooling



Then by Newton's law of cooling,

$$T = t_2 + (t_1 - t_2)e^{-kt}$$
(1)

Given:

$$t_1 = 10$$
, $t_2 = 40$, $T = 20$ and $t = 5$ min

Substituting all these values in (1),

we get
$$k = \frac{-1}{5}$$
 or 0.08109.

ODE-Application problems - Newton's law of cooling



a) Find T when t= 20 min

Substituting
$$t_1=10$$
 , $t_2=40$, $k=0.08109$ and $t=20$ min in (1), we have T= 34.073

Find T when t= 30 min

Substituting $t_1=10$, $t_2=40\,$, $k=0.08109\,$ and $t=30\,min\,$ in (1), we have T= 37.36

ODE-Application problems - Newton's law of cooling



b) Find t when T= 25

Substituting
$$t_1 = 10$$
, $t_2 = 40$, $k = 0.08109 \ and \ T = 25$ in (1), we have t= 8.5478.

ODE-Application problems - Newton's law of cooling

2. A copper ball is heated to a temperature of 100 °c. Then at time t=0 it is placed in water which is maintained at a temperature of 30 °c. At the end of 3 minutes temperature of the ball is reduced to 70 °c. Find the time at which the temperature of the ball drops to 31 °c.

Solution:

Let t_1 °c be the initial temperature of the copper and t_2 °c be the temperature of the medium. Further let T °c be the temperature of the copper at any time t.

Then by Newton's law of cooling,

$$T = t_2 + (t_1 - t_2)e^{-kt}$$
(1)



ODE-Application problems - Newton's law of cooling



Given:

$$t_1 = 100$$
, $t_2 = 30$, $T = 70$ and $t = 3$ min

Substituting all these values in (1) we get k = 0.1865.

To find t when T=31

Substituting $t_1 = 100$, $t_2 = 30$, k = 0.1865 and T = 31 in (1), we have t = 22.73 min.



THANK YOU

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