ENGINEERING PHYSICS

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Week #3 Class #10

- Concept of observables of the state of a system
- Operators in quantum mechanics
- Expectation values of observables

- ***Suggested Reading***

  1. *Concepts of Modern Physics, Arthur Beiser, Chapter 5*
  2. *Learning Material prepared by the Department of Physics*

- ***Reference Videos***

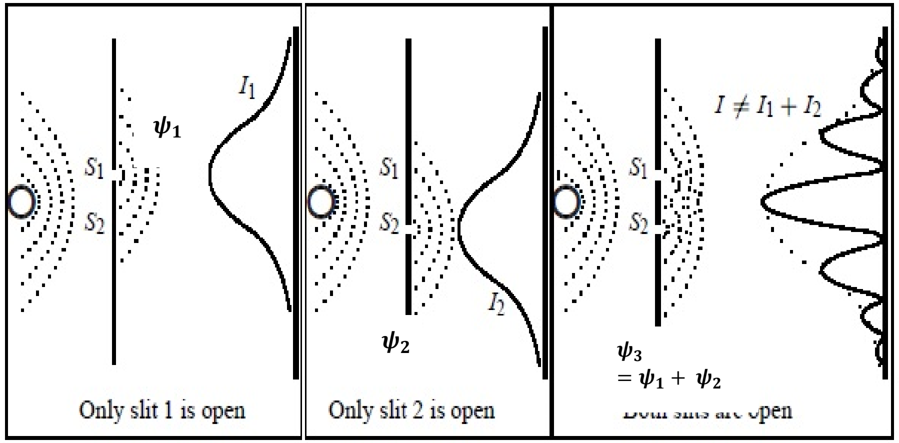
  1. *Video lectures : MIT 8.04 Quantum Physics I*

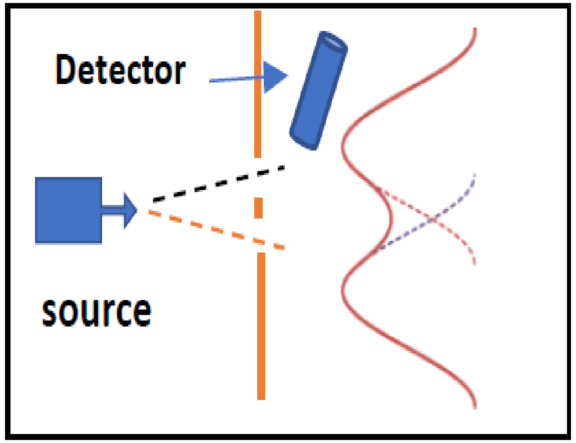
Observables

- *All experimentally measurable parameters of a physical system are observables*
  - *Position*
  - *momentum*
  - *Energy of a state*
  - *life time of electrons*
  - *Spin of a system*
- *Multiple measurements yield average values of the parameters*

Double slit experiment revisited

- ***Quantum mechanically observations on physical systems do not yield information about the system***
- ***Observations lead to a collapse of the wave function - double slit experiment with an "observer"***
- ***Observables in quantum mechanics have to be estimated probabilistically***





Operators

- ***Wave functions contain information about the quantum system***
- ***Mathematical operators can be used to extract information about the physical state in terms of the observables***

Operators

- *A normalized wave function contain information about the quantum system* $\psi = Ae^{\frac{i}{\hbar}(px-Et)}$ *- eigen function*
- *A mathematical operator* $\hat{G}$ *operating on the wavefunction can result in the eigen value* G *of the observable*
- *The eigen value equation* $\hat{G}\psi = G\psi$

Operators

Momentum operator -

- *The partial derivative of ψ with respect to position yields*

$$\frac{\partial\psi}{\partial x} = \left(\frac{ip}{\hbar}\right)\psi$$

$$\left\{-i\hbar\frac{\partial}{\partial x}\right\}\psi = p\,\psi$$

- *The momentum operator* $\hat{p} = \left\{-i\hbar\frac{\partial}{\partial x}\right\}$

  Operating on the eigen function yields the momentum eigen value

Operators

Kinetic energy operator -

- ***The second derivative of ψ with respect to position yields***

$$\frac{\partial^2\psi}{\partial x^2} = \left(\frac{ip}{\hbar}\right)^2 \psi$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} = \frac{p^2}{2m}\psi = KE\psi$$

The kinetic energy operator $\widehat{KE} = \left\{-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\right\}$

operating on the eigen function yields the eigen value of the kinetic energy of quantum system

Operators

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Total energy operator -

- *The derivative of ψ with respect to time yields*

$$\frac{\partial\psi}{\partial t} = \left(-\frac{iE}{\hbar}\right)\psi$$

$$\left\{i\hbar\frac{\partial}{\partial t}\right\}\psi = E\psi$$

The total energy operator $\hat{E} = \left\{i\hbar\frac{\partial}{\partial t}\right\}$

Operating on the eigen function yields the eigen value of the total energy of quantum system

This is also called as the Hamiltonian operator $\hat{H}$

Operators

Position operator -

- *The position operator has to be discussed in the momentum space*
- *The position operator* $\hat{x}$ *operating on* ψ

$$\hat{x}\psi = x\psi$$

- *yields the eigen value of position of the quantum system*

Operators

Potential energy operator -

- *Potential energy operator is not explicitly described*
- *The eigen value of the potential energy can be inferred as the difference of the total energy and the kinetic energy*
- *The eigen value equation for the potential energy is*

$$\hat{V}\psi = V\psi$$

Expectation values of observables

Quantum mechanics predicts only the most probable values of the observables of a physical system

- the expectation values $\equiv$ the average of repeated measurements on the system.

The eigen value equation for momentum

$$\hat{p}\psi = p\psi$$

The operation $\psi^*\hat{p}\psi = \psi^* p\psi = p\psi^*\psi$

$\psi^*\psi$ is the probability density

$p\psi^*\psi$ should be the probability of the eigen value

Expectation values of observables

The spread in the wave packet can yield a range of $p\psi^*\psi$

Integrated over the range of x for the extend of the wave packet

$$\int \psi^*\hat{p}\psi\, dx = \int \psi^* p\psi\, dx = \langle p\rangle \int \psi^*\psi\, dx$$

$\langle p\rangle$ is the most probable value of the momentum.

Thus the expectation value of the momentum is written as

$$\langle p\rangle = \frac{\int \psi^*\hat{p}\psi\, dx}{\int \psi^*\psi\, dx}$$

Expectation values of observables

In general an operator $\hat{G}$ *of the observable* g

Gives the expectation value of the observable

$$\langle g \rangle = \frac{\int \psi^* \hat{G} \psi \, dx}{\int \psi^* \psi \, dx}$$

And in three dimensional space

$$\langle g \rangle = \frac{\int \Psi^* \hat{G} \Psi \, dV}{\int \Psi^* \Psi \, dV}$$

Class #10 …….. Quiz….

Understanding concepts of observables, operators and expectation values ……….

1. Observables in quantum mechanics are obtained as results of experiments
2. The total energy operator is $\left\{ i\hbar \frac{\partial}{\partial t} \right\}$
3. The position operator as obtained from the momentum space would be $\left\{ -i\hbar \frac{\partial}{\partial p} \right\}$
4. The total energy operator is Hamiltonian operator
5. Expectation value of position in 1d is $\langle x \rangle = \frac{\int \psi^* \hat{x} \psi dx}{\int \psi^* \psi dx}$

THANK YOU

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