

ENGINEERING MATHEMATICS - I Ordinary Differential Equations

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Unit 3: Ordinary Differential Equations

Session: 5

Sub Topic: Equations Reducible to Exact form

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Equations Reducible to Exact Form

Sometimes a differential equation which is not exact may become so, on multiplication by a suitable function known as the integrating factor (IF).

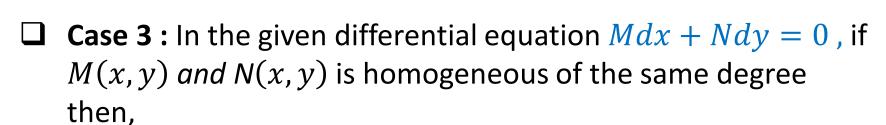


□ Case 1: If
$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = g(x)$$
 (function of x alone) then,
$$IF = e^{\int g(x)dx}.$$

Case 2: If
$$\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = h(y)$$
 (function of y alone) then,
$$IF = e^{\int h(y)dy}.$$



Equations Reducible to Exact Form



$$IF = \frac{1}{Mx+Ny}$$
, provided that $Mx + Ny \neq 0$.

Note: If
$$Mx + Ny = 0$$
 then $IF = \frac{1}{x^2} \text{ or } \frac{1}{y^2} \text{ or } \frac{1}{xy}$.



Equations Reducible to Exact Form



Case 4: If the differential equation is of the form $f_1(xy)ydx + f_2(xy)xdy = 0$, then $IF = \frac{1}{Mx - Ny}$, where $M = f_1(xy)y \& N = f_2(xy)x$, provided that $Mx - Ny \neq 0$.

Note: If Mx - Ny = 0 then $\frac{M}{N} = \frac{y}{x}$ and the given differential equation reduces to x dy + y dx = 0 and its solution is xy = c.

Equations Reducible to Exact Form - Problems



1. Solve
$$(1 + (x + y)tany)\frac{dx}{dy} + 1 = 0$$

Solution:

$$M = 1, N = 1 + (x + y)tany$$

Consider,
$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{M} = \frac{0 - tany}{1} = -tany = g(y)$$
 (function of y only)

Then,
$$IF = e^{-\int g(y)dy} = e^{\int tanydy} = e^{\log(secy)} = secy$$

Multiplying the given equation by the *IF* we have, M = secy, N = secy + (x + y)tanysecy

Equations Reducible to Exact Form



Contd...

The solution is given by

$$\int Mdx + \int N(y)dy = C$$

$$\int secydx + \int secy + ytanysecydy = C$$

$$xsecy + \log(secy + tany) + ysecy - \log(secytany) = c$$

$$xsecy + ysecy = c$$

Equations Reducible to Exact Form - Problems

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2. Solve
$$(2xlogx - xy)dy + 2ydx = 0$$

Solution:

$$M = 2y$$
, $N = 2x log x - xy$

Consider,
$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{2 - (2logx + 2 - y)}{2xlogx - xy} = \frac{-(2logx - y)}{x(2logx - y)} = \frac{-1}{x} = f(x)$$
 (function of x only)

Then,
$$IF = e^{\int \frac{-1}{x} dx} = e^{-logx} = \frac{1}{x}$$

Multiplying the given equation by the IF we have,

$$M = \frac{2y}{x}$$
, $N = 2logx - y$

Equations Reducible to Exact Form



Contd...

The solution is given by

$$\int Mdx + \int N(y)dy = C$$

$$\int \frac{2y}{x} dx - \int y dy = c$$

$$2ylogx - \frac{y^2}{2} = c$$



THANK YOU

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