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ENGINEERING MATHEMATICS - I

Ordinary Differential Equations

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Unit 3 : Ordinary Differential Equations

Session : 1

Sub Topic : Introduction

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Unit 3 : Contents

- Bernoulli's Linear Differential Equation
- Exact Differential Equations
- Reducible to Exact Differential Equations
- Orthogonal Trajectories
- Solution of first order Non-Linear Differential Equations

  (Equations solvable for p, y and x)
- Application problems on Differential Equations.

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Differential Equations - Introduction

1. What is a Differential Equation ?

2. Why Engineers need Differential Equations ?

What is a Equation ?

Equation : $x^2 - 4x = 4$

Solution of a Equation :

- Single value of x
- Several values of $'x'$
- Interval of $'x'$

Note : In this case $x = 2$ is the solution.

What is a Differential Equation ?

Recall :

An equation that represents the relation between the

- Independent variables
- Dependent variable
- Derivative of the dependent variable w.r.t the independent variables

is called as a ***Differential Equation***.

Examples :

- For example : $\frac{dy}{dx} = y$

  Solution : $y = e^x$

- Suppose we consider, $\frac{dy}{dx} = e^x$

  Solution : $y = e^x$

  or $y = e^x + 4$

  or $y = e^x + 6$

  In general $y = e^x + C$

Another Example :

Consider,

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} - 6y = 0$$

Solution : $y = e^{6x}$

Now, $y' = 6e^{6x}$ and $y'' = 36e^{6x}$

Substituting these in the above example

$36e^{6x} - 30\, e^{6x} - 6e^{6x} = 0$

$\Longrightarrow y = e^{6x}$

Similarly, it can be checked that $y = e^{-x}$ is also a solution.

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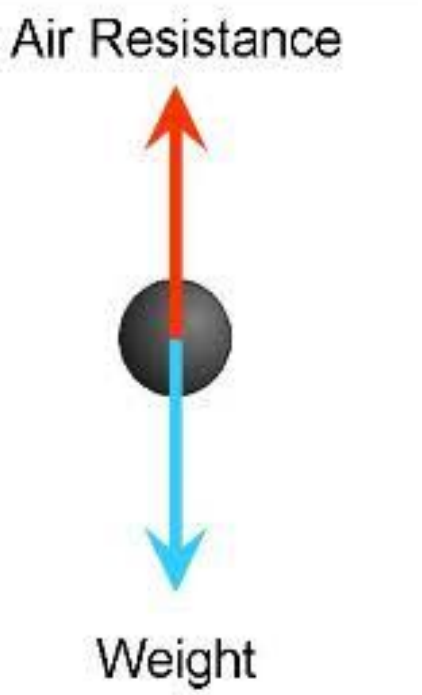
Why Differential Equations ?

Example from Physics :

- Consider an object that is falling from some height.
- What is the velocity and time taken by the object to hit the ground?



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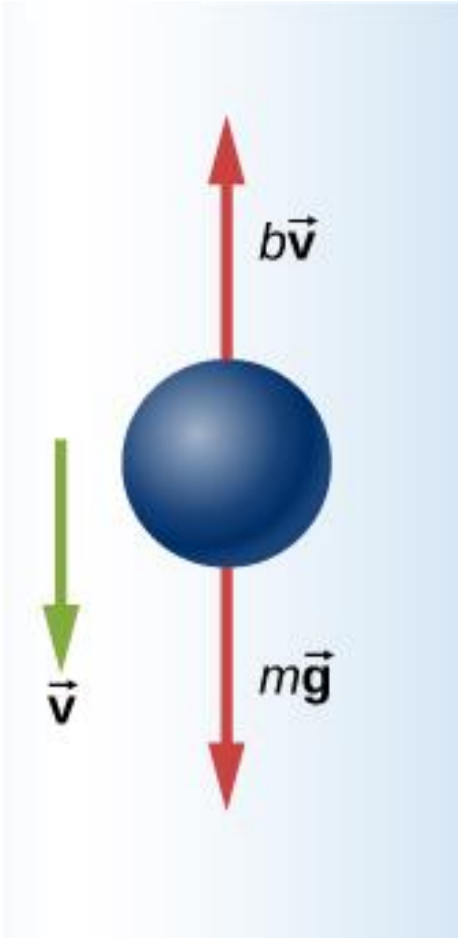
Example from Physics :



- Force due to gravity $= mg$
- Force due to air resistance$= -bv$
- Net Force $= mg - bv$
- By the Newton's second law of motion, $F = ma$
- That is, $mg - bv = m\frac{dv}{dt}$
- Rewriting, $\frac{dv}{dt} + \frac{b}{m}v = g$
- This is the Mathematical representation of the considered problem.

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Forensic Mathematics :

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Forensic Mathematics :

- A police personnel discovers the body of a dead person presumably murdered
- The problem is to estimate the time of murder.

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Forensic Mathematics :

To estimate the time of death, Mathematical modelling is done using

- Differential Equation
- Newton's law of cooling

The time of death can be estimated by solving the resulting Differential Equations.

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Why Differential Equations ?

❖ In Physics :

- Classical Mechanics
- Quantum Mechanics
- Electro Dynamics
- General Relativity
- Radioactive decay
- To describe motion of waves or pendulums or any chaotic processes.

❖ In Chemistry :

- Chemical Kinetics (Rate equation for a chemical reaction)

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Why Differential Equations ?

- In Biology :
  - Predator – Prey equations
  - Population, qrowth and decay equation
  - Molecular Biology

- In Medicine :
  - Modelling cancer growth
  - Modelling the diabetes and glucose metabolism
  - Drug distribution in human body
  - To predict rate of Spread of disease or pandemic like COVID - 19

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Why Differential Equations ?

- In Econonics & Finance
  - To estimate Optimum Investment strategies
  - To solve a simple Gross Domestic Product (GDP) model
  - To predict the changes in Bond Price
  - Consumer's preferences

- Forensic Mathematics
- In Archaeology & Paleontology :
  - Carbon Dating

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Why Differential Equations ?

In Engineering :

- Electronics & Communication Engineering :
  - Controls & systems
  - State space models
  - Kalman Filters
  - Generative models in signal processing
  - Electrical circuits
  - Stabilizing drone flight

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Why Differential Equations ?

- Computer Science Engineering :

  - Digital Image Processing
  - Advanced Machine Learning
  - Robotics
  - Sports Analytics
  - Scientific Computing
  - Collision detection in game programming

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Why Differential Equations ?

❖ Mechanical Engineering :

- Rigid body dynamic analysis
- Design of containers and funnels
- Design of heat spreaders in micro - electronics
- Design of heating and cooling chambers
- To predict the dynamic response of mechanical systems (Fo eg : Missiles)

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Why Differential Equations ?

- Civil Engineering :

  - Axial deformation on bar
  - Elastic beams
  - Torsion of elastic bars
  - Seepage flow in 2D
  - Irrotational fluid flow

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THANK YOU

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