ENGINEERING PHYSICS

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Week #2 Class #7

- Superposition of waves
- Phase and group velocities
- Group velocity relations

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➢ *Suggested Reading*

1. *Concepts of Modern Physics, Arthur Beiser, Chapter 2*
2. *Learning Material prepared by the Department of Physics*

➢ *Reference Videos*

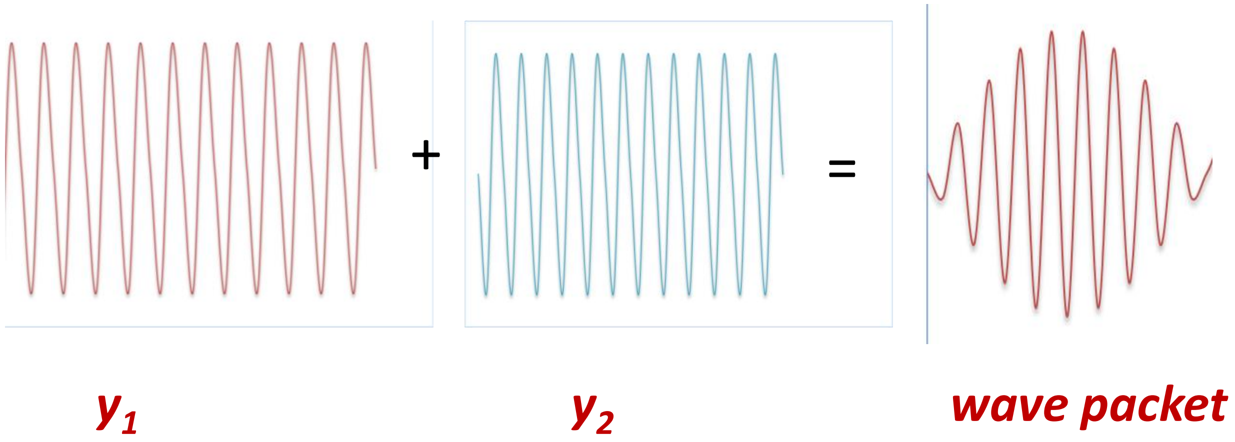
1. *Video lectures : MIT 8.04 Quantum Physics I*
2. *Institute of Sound and Vibrations Research, UK*

Superposition of waves

- *Mathematical wave representation of a moving particle*
  - *Information about position and momentum*
- *Amplitude of the wave should have a defined maximum apart from a defined wavelength*
- *Superposition of two waves*
  - *Wave packets*

Wave Packets

- $y_1 = A sin(\omega t + kx)$
- $y_2 = A\, sin\{(\omega + \Delta\omega)\, t + (k + \Delta k)x\}$
- *Superposition*
- $y = y_1 + y_2 = 2Asin(wt + kx).\cos\left(\frac{\Delta wt + \Delta kx}{2}\right)$

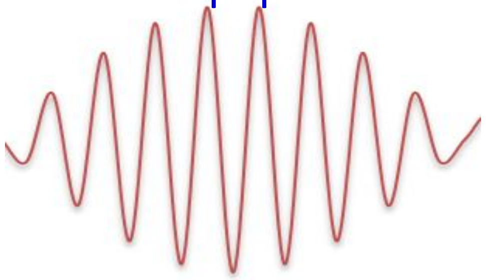


Wave Packets

- ***From k we can infer λ - which defines momentum***
- ***The spread around the central maximum can be the approximate position of the particle***
- ***More defined wave packets can be evolved by wave shaping***

Moving particle

$|\lambda|$

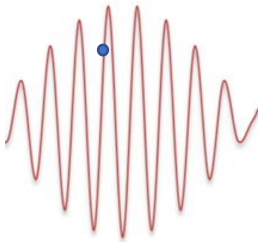


Wave packet ->

<- Δx ->

Phase and group velocities

- $y = y_1 + y_2 = 2A\sin(wt + kx).\cos\left(\frac{\Delta wt+\Delta kx}{2}\right)$
- *The phase velocity of the wave packet is the velocity of a representative point on the wave packet*



$$v_{ph} = \frac{\omega}{k}$$

- *The group velocity of the wave packet is the velocity of common velocity of the superposed wave group*

$$v_g = \frac{d\omega}{dk}$$

Group and particle velocities

- *The energy of the wave packet*

$$E = h\nu = \frac{h\omega}{2\pi} = \hbar\omega$$

- *The momentum of the particle*

$$p = \frac{h}{\lambda} = \frac{h\ .2\pi}{2\pi\ .\lambda} = \hbar k$$

- *Group velocity*

$$v_g = \frac{d\omega}{dk} = \frac{dE}{dp} = \frac{d}{dp}\left(\frac{p^2}{2m}\right) = \frac{p}{m} = v_{particle}$$

- *Group velocity is reflecting the particle velocity*

Group and Phase velocity relation

- *Group velocity*

$$v_g = \frac{d}{dk}(\omega) = \frac{d}{dk}(v_{ph}k)$$

$$= v_{ph} - k\frac{dv_{ph}}{dk} = v_{ph} - k\frac{dv_{ph}}{d\lambda}\frac{d\lambda}{dk}$$

$$= v_{ph} - \lambda\frac{dv_{ph}}{d\lambda}$$

- *Group velocity is dependent on the phase velocity and how the phase velocity changes with wavelength*

Group and Phase velocity relation

Group velocity = Phase velocity $v_g = v_{ph}$

$$-\lambda \frac{dv_{ph}}{d\lambda} = 0$$

- *Phase velocity does not change with wavelength*
- *The medium is non dispersive*
- *A dispersive medium is one in which*

  Group velocity <> Phase velocity

  - $v_g < v_{ph}$
  - $v_g > v_{ph}$

Group and Phase velocity relation

- $v_g < v_{ph}$ *- group velocity is half the phase velocity*
- $v_g = {}^{v_{ph}}/_{2}$
- $\frac{dv_p}{v_{ph}} = \frac{1}{2}\frac{d\lambda}{\lambda}$ on integration yields
- $ln(v_{ph}) \propto ln\sqrt{\lambda}$ or $v_{ph} \propto \sqrt{\lambda}$
- **This implies that the phase velocity is proportional to the square root of the wavelength**

Group and Phase velocity relations

- $v_g > v_{ph}$ - *group velocity is twice the phase velocity*
- $v_g = 2v_{ph}$
- $\frac{dv_p}{v_{ph}} = -\frac{d\lambda}{\lambda}$ on integration yields
- $ln(v_{ph}) \propto ln\frac{1}{\lambda}$ or $v_{ph} \propto \lambda^{-1}$
- This implies that the phase velocity is inversely proportional to the wavelength

Class #7 …….. Quiz….

The concepts which true of matter waves ….

1. Wave packet is a cosine wave
2. The outline connecting the peaks of the wave packet is a low frequency wave
3. Wave packets are longitudinal
4. The energy of the wave is equal to the energy of the particle
5. Wave packets do not disperse in any medium
6. In a non dispersive medium the group velocity is equal to the phase velocity

THANK YOU

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