ENGINEERING MATHEMATICS - I

Ordinary Differential Equations

Dr. Karthiyayini
Department of Science and Humanities

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Unit 3 : Ordinary Differential Equations

Session : 8

Sub Topic : Orthogonal Trajectories

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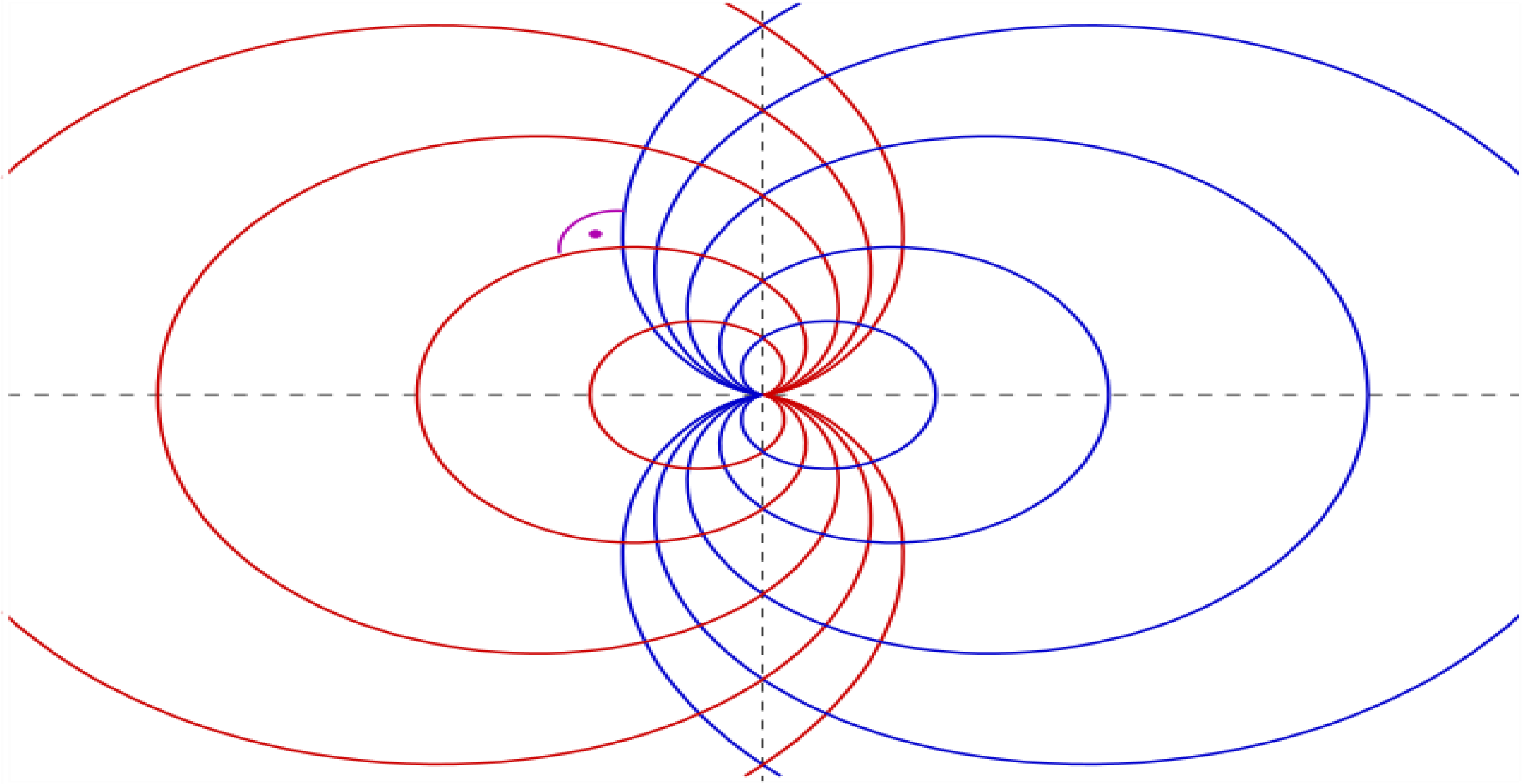
Orthogonal Trajectories – Polar Form-Example

Family of curves $r = a(1 + cos\theta)$ & the family of curves $r = b(1 - cos\theta)$ are orthogonal trajectories of each other.



Orthogonal Trajectories – Polar Form

For any polar curve $r = f(\theta)$, $tan\varphi = r\frac{d\theta}{dr}$ where φ is the ***angle between the radius vector and the tangent*** at any point *P.*

If c_1 ***and*** c_2 ***are 2 polar curves*** and φ_1, φ_2 are the angle between the radii vector and tangent to the curves at their point of intersection c_1 and c_2, then ***the two curves intersect orthogonally if*** $|\varphi_1 - \varphi_2| = \frac{\pi}{2}$.

Also, $tan\varphi_2 = tan\left(\varphi_1 + \frac{\pi}{2}\right) = -cot\varphi_1 = -\frac{1}{tan\varphi_1}$

Therefore $tan\varphi_1 . tan\varphi_2 = -1$

Orthogonal Trajectories – Polar Form

Working Procedure: For finding the Orthogonal Trajectory of Polar Family of Curves

Step 1 : *Form the differential equation for the given family of curves* $\boldsymbol{F(r,\theta,c)=0}$ *in the form* $\boldsymbol{f(r,\theta,dr/d\theta)=0}$.

Step 2 : ***Replace*** $\boldsymbol{\frac{dr}{d\theta}}$ ***by*** $\boldsymbol{-r^2\frac{d\theta}{dr}}$ *to obtain the* ***differential equation of the required orthogonal family of curves.***

Step 3 : *Solving this differential equation, the orthogonal family of curves can be obtained.*

Orthogonal Trajectories – Polar Form

Note:

In case of Polar family of curves, if the **differential equation remains the same after replacing $\frac{dr}{d\theta}$ by $-r^2\frac{d\theta}{dr}$,** then the given family is said to be **Self Orthogonal.**

Orthogonal Trajectories – Problems

1. Find the Orthogonal Trajectories of the curves $r^2 = a^2 cos2\theta$.

Solution :

Consider $\boldsymbol{r^2 = a^2 cos2\theta}$

Differentiating with respect to θ, $we\ have$

$$2r\frac{dr}{d\theta} = -a^2(2sin2\theta)$$

Therefore,

DE of the given family : $\boldsymbol{\frac{1}{r}\frac{dr}{d\theta} = -tan2\theta}$

Orthogonal Trajectories – Polar Form

Replace $\frac{dr}{d\theta}$ *by* $-r^2\frac{d\theta}{dr}$ *to obtain the differential equation of the required orthogonal family of curves*

Hence, DE of the Orthogonal family :

$$\frac{\boldsymbol{dr}}{\boldsymbol{r}} - \boldsymbol{cot\theta d\theta} = \mathbf{0}$$

Solving this we obtain,

$r^2cosec2\theta = c^2$ *or* $r^2 = c^2sin2\theta$ *which is the required solution.*

2. Find the Orthogonal Trajectories of the curves $r = \frac{2a}{1-cos\theta}$.

Solution:

Consider

$$r = \frac{2a}{1 - cos\theta}$$

Differentiating with respect to $\boldsymbol{\theta}$, we have

DE of the given family :

$$\frac{1}{r}\frac{dr}{d\theta} = \frac{sin\theta}{1-cos\theta} = tan\frac{\theta}{2}$$

Orthogonal Trajectories – Polar Form

Replace $\frac{dr}{d\theta}$ *by* $-r^2\frac{d\theta}{dr}$ *to obtain the differential equation of the required orthogonal family of curves*

Hence, DE of the Orthogonal family :

$$\frac{dr}{r} + \tan\frac{\theta}{2}\, d\theta = 0$$

Solving this we obtain,

$r\cos^2(\theta/2) = c^2$ which is the required solution.

THANK YOU

Dr. Karthiyayini
Department of Science & Humanities

Karthiyayini.roy@pes.edu

+91 80 6618 6651