ENGINEERING MATHEMATICS - I

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Class Content

- Series of Positive terms
- Conditions for convergence of Series of Positive terms
- Comparison test
- Limit form of comparision test
- P Series (statement)
- Examples

Series of Positive terms

If all the terms of the series $\sum a_n$ are positive *i.e.*, $a_n > 0, \ \forall n$, then the series $\sum a_n$ is called a series of positive terms.

Results on Series of Positive terms:

1. If c is a constant and $\sum a_n$ converges to l, then $\sum(ca_n)$ converges to cl.

2. If $\sum_{n=1}^{n=\infty} a_n$ and $\sum_{n=1}^{n=\infty} b_n$ converges to l and m respectively then

   a. $\sum(a_n + b_n) converges to (l + m)$.

   b. $\sum(a_n - b_n) converges to (l - m)$.

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Conditions for Convergence of a Series of positive terms

Necessary condition for the convergence of a series of positive terms :

If a series of positive terms $\sum a_n$ is convergent then $\lim\limits_{n\to\infty} a_n = 0$

REMARK

1.Converse of the above statement is not true while the contrapositive of the statement is true. i.e if $\lim\limits_{n\to\infty} a_n \neq 0$,
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2. The nature of a series is not altered by adding a finite number of terms to the series or by removing a finitie terms from the beginning of the series .

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Comparision test

TEST (Comparison Test)

Suppose that $\sum_{n=1}^{\infty} a_n$ *and* $\sum_{n=1}^{\infty} b_n$ *are two series of positive terms i.e* $a_n \geq 0$ *and* $b_n \geq 0$. *Then,*

1) If $\sum_{n=1}^{\infty} b_n$ *is* convergent *and* $a_n \leq k\, b_n$ *for all n ,*

where k is a positive constant ,

$\Longrightarrow \quad \sum_{n=1}^{\infty} a_n$ *is* convergent.

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Suppose that $\sum_{1}^{\infty} a_i$ *and* $\sum_{1}^{\infty} b_i$ *are series with all terms positive. 1)*
$\lim_{i\to\infty} \frac{a_i}{b_i} = c > 0 \implies \sum_{i=1}^{\infty} a_i$ **and** ,
$\sum_{i=1}^{\infty} b_i$ *either* both converge, *or* both diverge.

2) $\lim_{i\to\infty} \frac{a_i}{b_i} = 0$ **and** $\sum_{i=1}^{\infty} a_i$ converges
$\implies$ **the series** $\sum_{i=1}^{\infty} b_i$ converges

3) $\lim_{i\to\infty} \frac{a_i}{b_i} = \infty$ **and** $\sum_{i=1}^{\infty} b_i$ diverges
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Remarks

- > To know whether $\sum a_n$ is convergent, we have to compare $\sum a_n$ with another series of the same order and this series can be determined as

$$b_n = \frac{\textbf{Highest degree term in n in the Numerator of } a_n}{\textbf{Highest degree term in n in the Denominator of } a_n}$$

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This is one of the most powerful tests, because it squeezes the two series "in the limit". Just be sure to use it right!

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A useful test

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Problem 1: Discuss the convergence of the series $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \ldots$

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Example for Comparison Test

Soln:

$$\text{Here} \quad a_n = \frac{1}{n(n+1)} = \frac{1}{n^2\left(1+\frac{1}{n}\right)}.$$

Let us compare $\sum a_n$ with $\sum b_n$ where $b_n = \frac{1}{n^2}$. $\frac{a_n}{b_n} = \frac{1}{1+\frac{1}{n}} \therefore \lim\limits_{n\to\infty} \frac{a_n}{b_n} = \lim\limits_{n\to\infty} \frac{1}{1+\frac{1}{n}} = 1$ which is finite and $\neq 0.$, $\therefore \sum a_n$ and $\sum b_n$ behave alike.

Since $\sum b_n = \sum \frac{1}{n^2}$ is of the form $\sum \frac{1}{n^p}$ with $p = 2 > 1$, $\sum b_n$ is convergent
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Example for Comparison Test

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$$\text{Here} \quad a_n = \frac{T_n \text{ of } 1,3,5,\ldots}{n(n+1)(n+2)} = \frac{2n-1}{n(n+1)(n+2)}$$

$$= \frac{n\left(2-\frac{1}{n}\right)}{n^3\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)}.$$

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Example for Comparison Test

$\therefore \sum a_n$ and $\sum b_n$ behave alike.
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Problems :

1. Discuss the convergence or divergence of the following:

1. $$\frac{1}{a\cdot1^2+b}+\frac{1}{a\cdot2^2+b}+\frac{1}{a\cdot3^2+b}+\cdots\infty$$

n^{th} term : $a_n = \dfrac{1}{a\cdot n^2+b} = \dfrac{1}{n^2\left(a+\frac{b}{n^2}\right)}$

$\sum b_n = \sum \dfrac{1}{n^2}$ (p series with $p=2>1$ hence convergent)

$$\lim_{n\to\infty}\frac{a_n}{b_n} = \lim_{n\to\infty}\frac{1}{a+\frac{b}{n^2}} = \frac{1}{a} = \text{finite and} \neq 0$$

$\because \sum b_n$ is convergent $\quad \therefore \sum a_n$ and $\sum b_n$ behave alike

$\therefore \sum a_n$ is convergent.

2. $\frac{1}{1^p} + \frac{1}{3^p} + \frac{1}{5^p} + \frac{1}{7^p} + \cdots\cdots\infty$

$\Rightarrow$ n^{th} term : $a_n = \frac{1}{(2n-1)^p} = \frac{1}{n^p\left(2-\frac{1}{n}\right)^p}$

choosing $b_n = \frac{1}{n^p}$

$\to$ p series

$\sum b_n = \sum \frac{1}{n^p}$ converges $p>1$ and diverges $p \leq 1$

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{1}{\left(2-\frac{1}{n}\right)^p} = \frac{1}{2^p}$ which is finite and non zero

$\because \sum b_n$ converges $\therefore \sum a_n$ converges

$\sum b_n$ diverges $\Rightarrow$ $\sum a_n$ also diverges for $p \le 1$ and

converges $p > 1$

3. $$\sum \frac{\sqrt{n+1} - \sqrt{n}}{n^p}$$

$$\Rightarrow \frac{\sqrt{n+1} - \sqrt{n}}{n^p} \cdot \frac{\sqrt{n+1} + \sqrt{n}}{\sqrt{n+1} + \sqrt{n}} = a_n$$

On simplifying $\dfrac{1}{n^{p+1/2}\left(\sqrt{1+\frac{1}{n}}+1\right)}$

Choose $v_n = \dfrac{1}{n^{p+1/2}}$

Convergent $p + 1/2 > 1$ divergent $p + \frac{1}{2} \le 1$

$$\lim_{n\to\infty} \frac{a_n}{v_n} = \frac{1}{2}$$

$\therefore$ $\sum a_n$ and $\sum v_n$ behave alike

$\sum a_n$ converges for $p+1/2 > 1$

diverges for $p + 1/2 \leq 1$

$\therefore$ $\sum a_n$ is convergent if $p > 1/2$

divergent if $p \leq \frac{1}{2}$

THANK YOU

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