

ENGINEERING MATHEMATICS - I Ordinary Differential Equations

Dr. Karthiyayini

Department of Science and Humanities



Unit 3: Ordinary Differential Equations

Session: 4

Sub Topic: Exact Differential Equations

Dr. Karthiyayini

Department of Science & Humanities

Exact Differential Equations



Definition:

A differential equation of the form

$$M(x,y)dx + N(x,y)dy = 0$$

is said to be exact if its left hand member is the exact differential of some function u(x, y).

That is,
$$d\mathbf{u} = M(x, y)dx + N(x, y)dy = 0$$

Therefore, its solution is u(x, y) = c

Exact Differential Equations



Note that

$$d(xy) = ydx + xdy = 0$$

Therefore, the solution of equation (1) is

$$xy = c$$

Exact Differential Equations



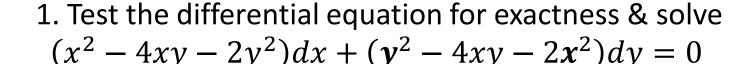
Theorem : The necessary and sufficient condition for the differential equation M(x,y)dx + N(x,y)dy = 0 to be exact is $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

Note: The solution of an exact differential equation is given by,

$$\int_{y \ constant} M dx + \int N(y) dy = c$$

where N(y) = terms of N which contain y alone.

Exact Differential Equations – Problems





$$Mdx + Ndy = 0$$
, where

$$M = x^2 - 4xy - 2y^2$$
 and $N = y^2 - 4xy - 2x^2$.

Then,
$$\frac{\partial M}{\partial y} = -4x - 4y$$
; $\frac{\partial N}{\partial x} = -4x - 4y$

Since
$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$
 the given equation is exact.

The solution is
$$\int M dx + \int N(y) dy = C$$

 $\int x^2 - 4xy - 2y^2 dx + \int y^2 dy = C$
 $\frac{x^3}{3} - \frac{4x^2y}{2} - 2xy^2 + \frac{y^3}{3} = c$



Exact Differential Equations - Problems

2. Solve :
$$(x^3 - 3xy^2)dx + (y^3 - 3x^2y)dy = 0$$



$$Mdx + Ndy = 0$$
, where

$$M = x^3 - 3xy^2$$
 and $N = (y^3 - 3x^2y)$

Then,
$$\frac{\partial M}{\partial y} = -6xy$$
; $\frac{\partial N}{\partial x} = -6xy$

Since
$$\frac{\partial M}{\partial v} = \frac{\partial N}{\partial x}$$
 the given equation is exact.

The solution is
$$\int Mdx + \int N(y)dy = C$$
$$\int (x^3 - 3xy^2)dx + \int y^3dy = C$$
$$x^4 - 6x^2y^2 + y^4 = 4c$$





THANK YOU

Dr. Karthiyayini

Department of Science & Humanities

Karthiyayini.roy@pes.edu

+91 80 6618 6651