ENGINEERING PHYSICS

Radhakrishnan S, Ph.D.

Department of Science and Humanities

PES UNIVERSITY ONLINE

Week #2 Class #8

- Analysis of the wave packet
- Heisenberg’s Uncertainty Principle
- Gamma Ray microscope
- Other applications

PES UNIVERSITY ONLINE

➢ *Suggested Reading*

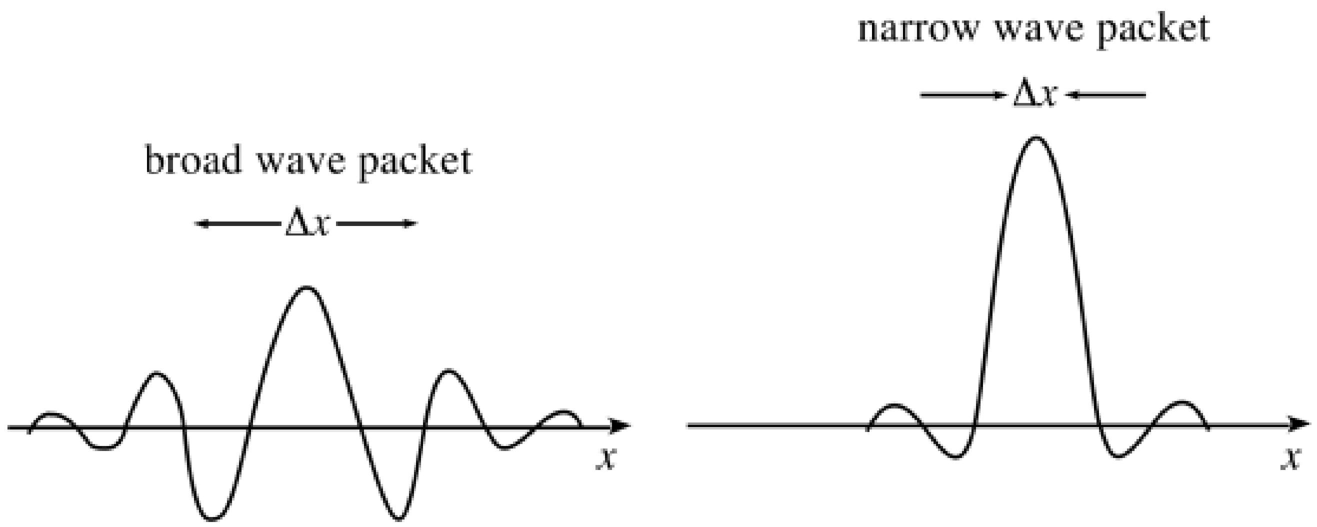
1. *Concepts of Modern Physics, Arthur Beiser, Chapter 3*
2. *Learning Material prepared by the Department of Physics*

➢ *Reference Videos*

1. *Video lectures : MIT 8.04 Quantum Physics I*

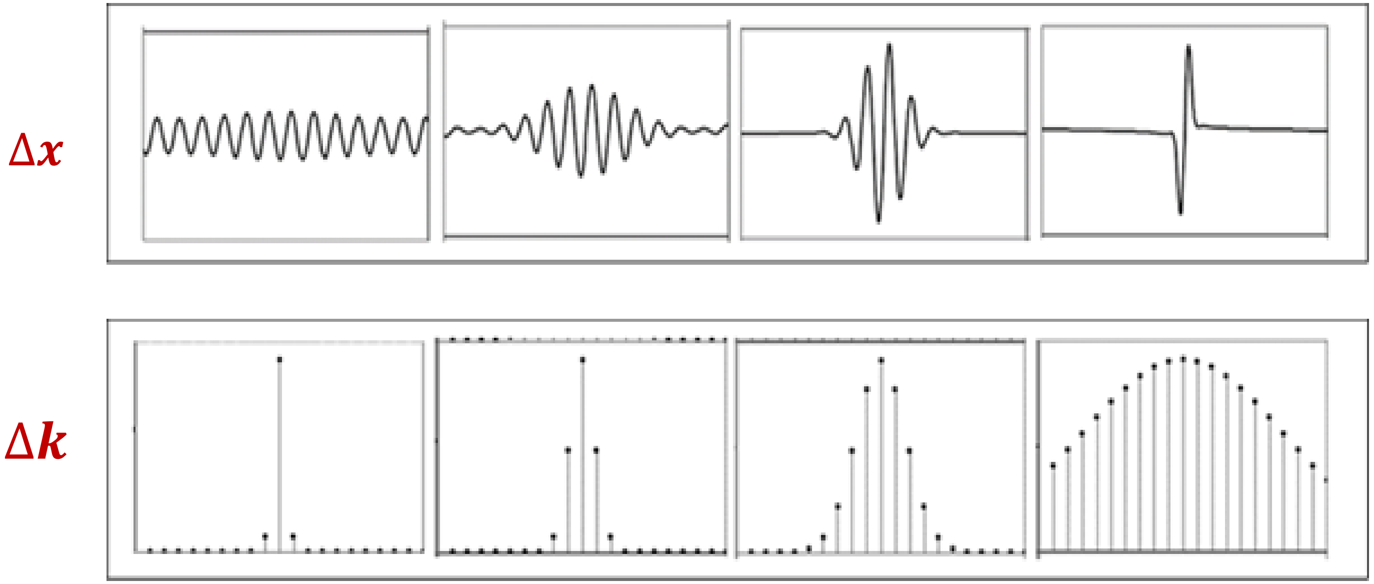
Analysis of Wave Packets

- ***Wave packets are formed by the superposition of waves***
- ***More number of components waves result in a narrow wave packet***
- ***More number of component waves also result in a spread in the propagation constant***



Analysis of Wave Packets

- *Wave packets have an inherent component of uncertainties*
- *A broad wave packet has a high uncertainty in position, but a high accuracy in momentum*
- *A narrow wave packet results in a high accuracy in position and a high uncertainty in the momentum*



Heisenberg's analysis of wave packets

- *The spread in x can be described by* σ_x
- *The spread in k can be described by* σ_k
- *The product of the standard deviations* $\sigma_x . \sigma_k \geq \frac{1}{2}$
- *Replacing the standard deviation by the inaccuracies*

$$\Delta x . \Delta k \geq \frac{1}{2}$$

- *Replacing* $k = \frac{p}{\hbar}$
- *The product of the uncertainties* $\Delta x . \Delta p \geq \frac{\hbar}{2}$

Position momentum uncertainty:

The position and momentum of a particle cannot be determined simultaneously with unlimited precision.

$$\Delta x . \Delta p \geq \frac{\hbar}{2}$$

Δx ***- the uncertainty in the position***

Δp ***- the uncertainty in the momentum***

Heisenberg's Uncertainty Principle

Energy time uncertainty:

The energy and life time of a particle in a state cannot be determined simultaneously with unlimited precision.

ΔE - *the uncertainty in the energy of the particle*

Δt - *the uncertainty in the life time of the particle in the state*

The product of the uncertainties $\Delta E . \Delta t \geq \frac{\hbar}{2}$

Heisenberg's Uncertainty Principle

Uncertainty relation for circular motion:

The angular position and angular momentum of a particle in a circular motion cannot be determined simultaneously with unlimited precision.

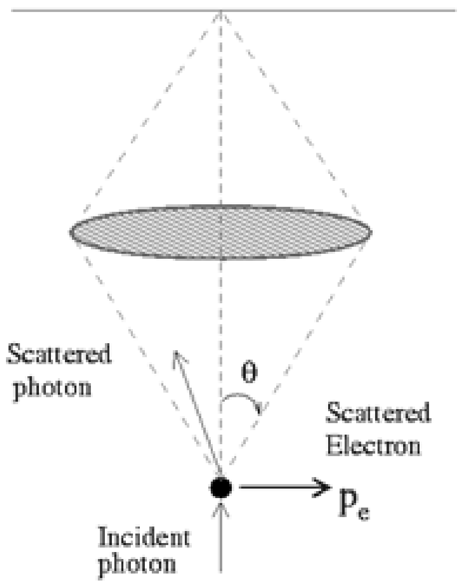
$\Delta\theta$ - *the uncertainty in the angular position*

ΔL - *the uncertainty in the angular momentum*

The product of the uncertainties $\Delta\theta . \Delta L \geq \frac{\hbar}{2}$

Gamma ray microscope a thought experiment

- *Experiment to "measure" the position of an electron*
- *Construct a "microscope" capable of imaging the electron*



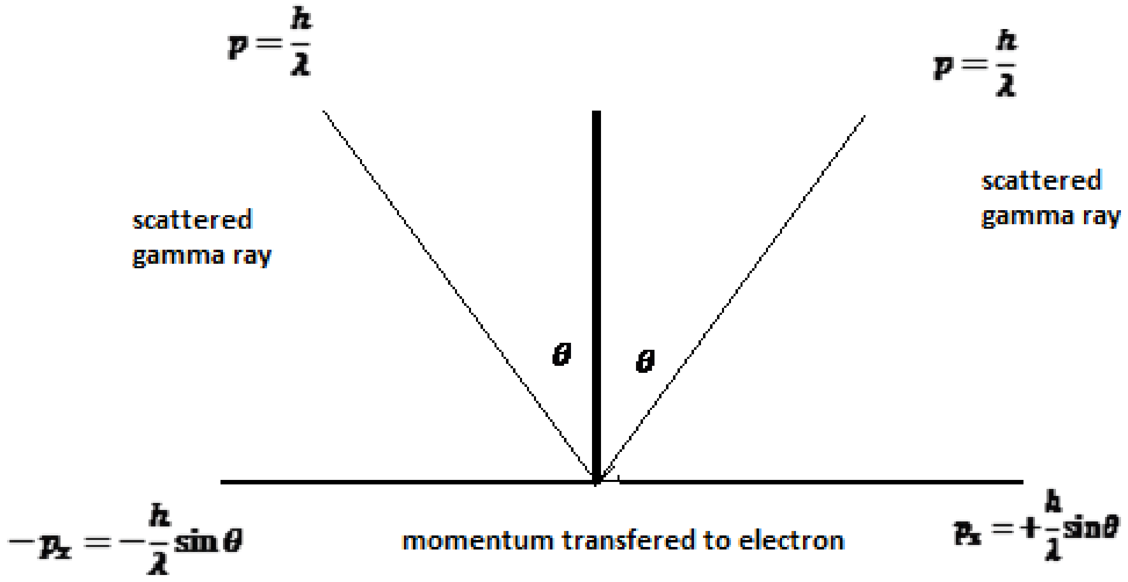
Gamma ray microscope a thought experiment

- *Wavelength of radiation to be comparable to size of electron*
- *Gamma rays wavelengths* $\approx \mathbf{10^{-12}}\ \boldsymbol{m}$
- *Resolution of the microscope =>* $\Delta x \approx \frac{\lambda}{sin\theta}$

Uncertainty in the position of the electron

Gamma ray microscope a thought experiment

- *High energy gamma rays impart momentum to the electrons – Compton effect with gamma rays*
- *Electron momentum along x direction* $\pm \frac{h}{\lambda} sin\theta$
- *Minimum electron momentum uncertainty* $\approx \Delta p = 2\frac{h}{\lambda} sin\theta$



Gamma ray microscope a thought experiment

- *Product of the uncertainties*
- $\Delta x \times \Delta p = 2\frac{hsin\theta}{\lambda}.\frac{\lambda}{sin\theta} = 2h$
- *Greater than* $\frac{h}{4\pi}$ *!*
- *Confirms that the position and momentum cannot be simultaneously determined accurately.*

Other applications – electron confinement in a nucleus

- *In a radioactive β decay an electron is emitted from the nucleus with energies of the order of 8 MeV*
- *Estimate the energy of the electron from the uncertainty principle*
- *Electron if present in the nucleus has to be confined to the diameter of the nucleus*
- *The uncertainty in the position is* $\Delta x \approx 10^{-14} m$

Other applications – electron confinement in a nucleus

- *Uncertainty in the momentum of electron confined to the nucleus*
- $\Delta p = \frac{h}{4\pi\Delta x} = \frac{h}{4\pi\times10^{-14}} = 5.28\ \times 10^{-21}\ kg\ ms^{-1}$
- *Minimum momentum of the electron cannot be lesser than the uncertainty in momentum* $p \approx \Delta p$
- *A simple non relativistic estimation for the energy*
- $E = \frac{p^2}{2m} = \frac{\Delta p^2}{2m} = \frac{1}{2m}\left(\frac{\hbar}{2.\Delta x}\right)^2 \approx 96\ MeV$
- *Relativistic calculations yield energy >20 MeV*
- *Hence electrons cannot be part of the nucleus !*

Summarizing .. .. . .

- *De Broglie hypothesis and Heisenberg's Uncertainty principles …*
  - *two fundamental ideas which cannot be violated in any theory of quantum systems*

PES UNIVERSITY ONLINE

The concepts which true of the uncertainty principle ….

1. Uncertainty principle is based on the measurement accuracies of equipments used.
2. The position of a particle cannot be determined accurately
3. The momentum of a particle always has an uncertainty which is related to the uncertainty in the position of the particle
4. Electrons cannot be confined to a nucleus as it is energetically not feasible
5. Electrons cannot be confined to a nucleus as it’s size is bigger the nuclear diameter

THANK YOU

Radhakrishnan S, Ph.D.

Professor, Department of Science and Humanities

sradhakrishnan@pes.edu

+91 80 21722683 Extn 759