ENGINEERING PHYSICS

Radhakrishnan S, Ph.D.

Department of Science and Humanities

Class #18

- Eigen functions and eigen values of a particle in a box
- Graphical representation of the wave function and probability density
- Interpretation of the probability density

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- ***Suggested Reading***

  1. ***Concepts of Modern Physics, Arthur Beiser, Chapter 5***
  2. ***Learning Material prepared by the Department of Physics***

- ***Reference Videos***

  1. ***Video lectures : MIT 8.04 Quantum Physics I***
  2. ***Engineering Physics Class #17***

1D infinite potential Schrodinger's wave equation

The eigen wave functions of the particle in box

$$\psi_n(x) = \sqrt{\frac{2}{a}}\cos\left(\frac{n\pi}{a}x\right) \quad for\ n\ odd \qquad \text{(even parity)}$$

$$\psi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right) \quad for\ n\ even \qquad \text{(odd parity)}$$

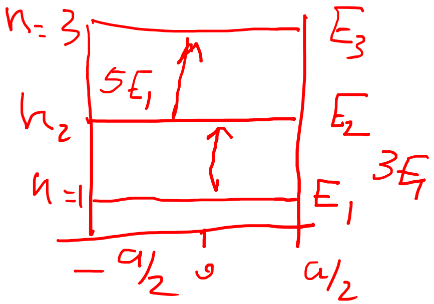
The eigen energy values of the allowed states

$$E_n = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2 n^2 \pi^2}{2ma^2} = \frac{h^2 n^2}{8ma^2}$$

1D infinite potential Schrodinger's wave equation

n=1 corresponds to the lowest allowed energy state which is the ground state of the system

$$E_1 = \frac{h^2 1^2}{8ma^2} = \frac{h^2}{8ma^2} = E_0$$



n=2 corresponds to the first excited energy state

$$E_2 = \frac{h^2 2^2}{8ma^2} = 4E_1 = 4E_0$$

n=3 corresponds to the second excited energy state

$$E_3 = \frac{h^2 3^2}{8ma^2} = 9E_1$$

n=4 corresponds to the third excited energy state

$$E_4 = \frac{h^2 4^2}{8ma^2} = 16E_1$$

1D infinite potential Schrodinger's wave equation

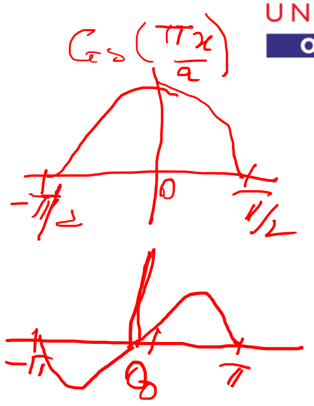
The eigen wave functions of the particle in box in the first four states can be written as

$$\psi_1(x) = \sqrt{\frac{2}{a}}\cos\left(\frac{1\pi}{a}x\right) \quad n = 1$$ **(even parity)**

$-a/2$ $\cos\left(-\frac{\pi}{2}\right)$ $+a/2$ $\cos\left(\frac{\pi}{2}\right)$

$$\psi_2(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{2\pi}{a}x\right) \quad n = 2$$ **(odd parity)**

$\sin(-\pi)$ $-\sin(\pi)$



$$\psi_3(x) = \sqrt{\frac{2}{a}}\cos\left(\frac{3\pi}{a}x\right) \quad n = 3$$ **(even parity)**

$$\psi_4(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{4\pi}{a}x\right) \quad n = 4$$ **(odd parity)**

1D infinite potential Schrodinger's wave equation

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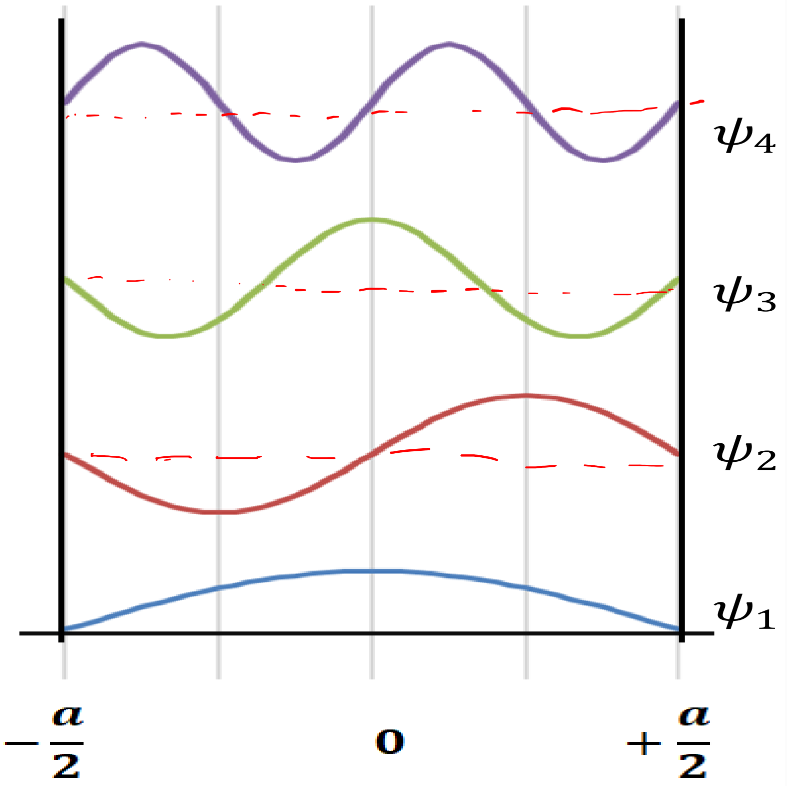
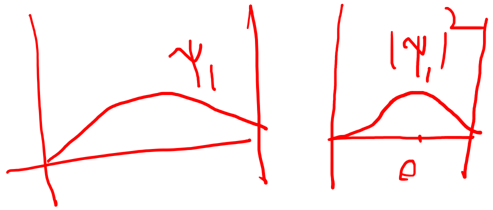
Graphical representation of eigen wave functions of the particle in box in the first four states



1D infinite potential Schrodinger's wave equation

The probability density functions of the particle in box in the first four states can be written as

$$|\psi_1(x)|^2 = \frac{2}{a}cos^2\left(\frac{1\pi}{a}x\right) \quad n = 1$$



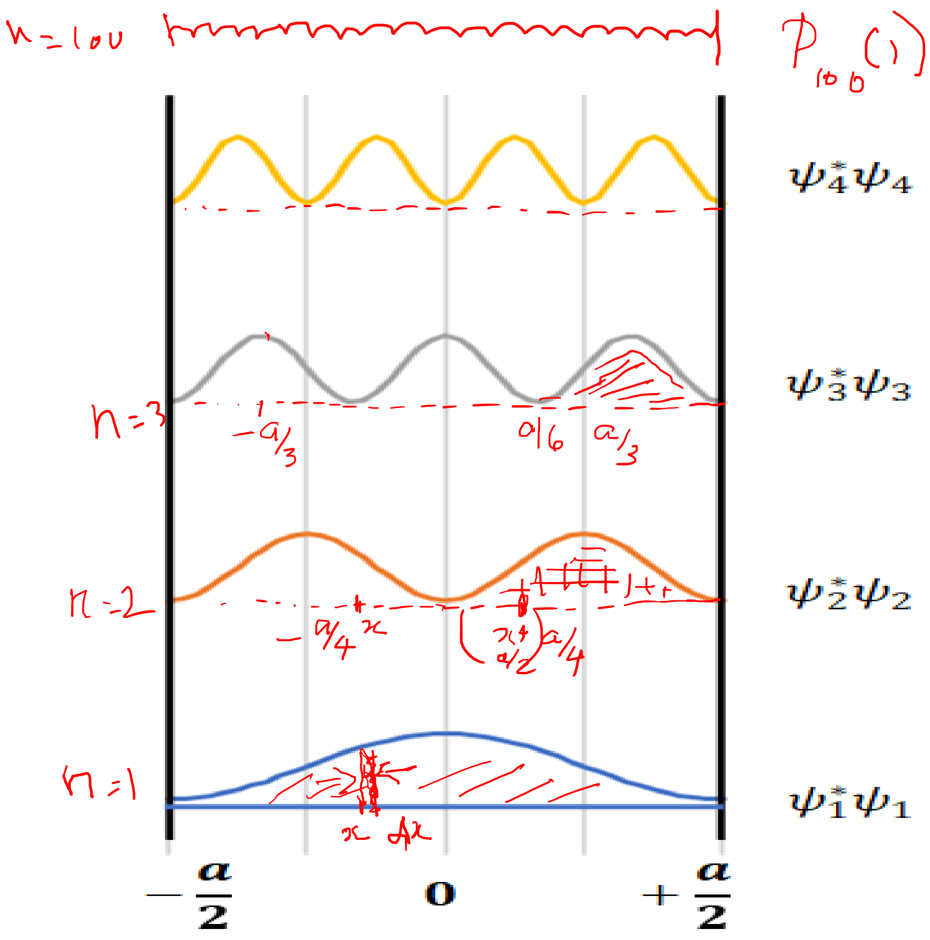
$$|\psi_2(x)|^2 = \frac{2}{a}sin^2\left(\frac{2\pi}{a}x\right) \quad n = 2$$

$$|\psi_3(x)|^2 = \frac{2}{a}cos^2\left(\frac{3\pi}{a}x\right) \quad n = 3$$

$$|\psi_4(x)|^2 = \frac{2}{a}sin^2\left(\frac{4\pi}{a}x\right) \quad n = 4$$

1D infinite potential Schrodinger's wave equation

The probability density functions of the particle in box in the first four states can be written as



$P_{100}(1) = \frac{1}{100}$

n^{th} state

$P_n(1) = \frac{1}{n}$

$\int_x^{x+a/2} \psi^*\psi \, dx = P_2(1) = \frac{1}{2}$

$\int_{a/6}^{a/2} \psi^*\psi \, dx = \frac{1}{3}$ $\quad P_3(1) = \frac{1}{3}$

$\int_0^{a/2} \psi^*\psi \, dx = 0.5 = \frac{1}{2}$ $\quad P_2(1) = \frac{1}{2}$

$\int_{-a/2}^{a/2} \psi^*\psi \, dx = 1$ $\quad P_1 = \frac{1}{1}$

area under the curve.

Class #18 …….. Quiz….

The concepts which are true of a particle in a box…

1. The energy of the particle in a 1D box are equally spaced
2. The probability amplitude of the particle in the state n=3 at the center of the box is high
3. The probability of finding the particle at x=0 in the 3rd excited state is zero
4. For large values of n the probability density is more or less a constant
5. The energy required by a particle in the ground state to be excited to the state n=4 is $4E_o$

THANK YOU

Radhakrishnan S, Ph.D.

Professor, Department of Science and Humanities

sradhakrishnan@pes.edu

+91 80 21722683 Extn 759