

ENGINEERING MATHEMATICS - I Ordinary Differential Equations

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Unit 3: Ordinary Differential Equations

Session: 6

Sub Topic: Equations Reducible to Exact form

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Equations Reducible to Exact Form

Sometimes a differential equation which is not exact may become so, on multiplication by a suitable function known as the integrating factor (IF).



□ Case 1: If
$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = g(x)$$
 (function of x alone) then,
$$IF = e^{\int g(x)dx}.$$

Case 2: If
$$\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = h(y)$$
 (function of y alone) then,
$$IF = e^{\int h(y)dy}.$$



Equations Reducible to Exact Form



Case 3 : In the given differential equation Mdx + Ndy = 0, if M(x,y) and N(x,y) is homogeneous of the same degree then,

$$IF = \frac{1}{Mx+Ny}$$
, provided that $Mx + Ny \neq 0$.

Note: If
$$Mx + Ny = 0$$
 then $IF = \frac{1}{x^2} \text{ or } \frac{1}{y^2} \text{ or } \frac{1}{xy}$.

Equations Reducible to Exact Form

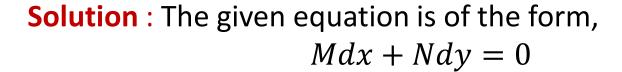


Case 4: If the differential equation is of the form $f_1(xy)ydx + f_2(xy)xdy = 0$, then $IF = \frac{1}{Mx - Ny}$, where $M = f_1(xy)y \& N = f_2(xy)x$, provided that $Mx - Ny \neq 0$.

Note: If Mx - Ny = 0 then $\frac{M}{N} = \frac{y}{x}$ and the given differential equation reduces to x dy + y dx = 0 and its solution is xy = c.

Equations Reducible to Exact Form - Problems

1. Solve
$$[x^2y - 2xy^2]dx - [x^3 - 3x^2y]dy = 0$$



$$M = x^2y - 2xy^2$$
; $N = -x^3 + 3x^2y$

$$\frac{\partial M}{\partial y} = x^2 - 4xy; \frac{\partial N}{\partial x} = -3x^2 + 6xy$$

The given equation is not exact.

Consider,
$$IF = \frac{1}{Mx + Ny} = \frac{1}{x^3y - 2x^2y^2 - x^3y + 3x^2y^2} = \frac{1}{x^2y^2}$$



Equations Reducible to Exact Form – Problems



Contd.....

Multiplying the given equation by the IF, it becomes exact.

Therefore,
$$M = \frac{1}{y} - \frac{2}{x}$$
, $N = -\frac{x}{y^2} + \frac{3}{y}$

The solution is $\int Mdx + \int N(y)dy = C$

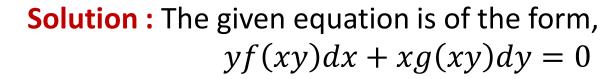
$$\int \frac{1}{y} - \frac{2}{x} dx + \int \frac{3}{y} dy = C$$

$$\frac{x}{y} - 2logx + 3logy = c$$

$$\frac{x}{y} + \log\left(\frac{y^3}{x^2}\right) = c$$

Equations Reducible to Exact Form - Problems

2. Solve
$$(xy + 2x^2y^2)dx + x(xy - x^2y^2)dy = 0$$



$$M = xy^2 + 2x^2y^3$$
, $N = xy^2 - x^3y^2$

Consider,
$$IF = \frac{1}{Mx - Ny} = \frac{1}{x^2y^2 + 2x^3y^3 - x^2y^2 + x^3y^3} = \frac{1}{3x^3y^3}$$

Multiplying the given equation by the IF we have,

$$M = \frac{1}{3x^2y} + \frac{2}{3x}$$
, $N = \frac{1}{3xy^2} - \frac{1}{3y}$



Equations Reducible to Exact Form



Contd.....

The solution is given by

$$\int Mdx + \int N(y)dy = C$$

$$\int \left(\frac{1}{3x^2y} + \frac{2}{3x} \right) dx + \int \left(\frac{1}{3xy^2} - \frac{1}{3y} \right) dy = C$$

$$\frac{-1}{3xy} + \frac{2}{3}\log x - \frac{1}{3}\log y = c$$

$$\frac{-1}{xy} + \log \frac{x^2}{y} = c$$



THANK YOU

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