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Unit II: Quantum Mechanics of simple systems



Class #11

- One dimensional Schrodinger's time dependent wave equation
- Time dependent and position dependent wave functions
- Schrodinger's time independent wave equation

Unit I: Review of concepts leading to Quantum Mechanics



> Suggested Reading

- 1. Concepts of Modern Physics, Arthur Beiser, Chapter 5
- 2. Learning Material Unit II prepared by the Department of Physics

> Reference Videos

- 1. Video lectures: MIT 8.04 Quantum Physics I
- 2. Engineering Physics Class #10

Schrodinger's wave equation

- Schrodinger's formalism of a wave equation
- The total energy of a system E = KE + V
- This equation remains invariant when multiplied by $\psi(x,t)$

$$E\psi(x,t) = KE\psi(x,t) + V\psi(x,t)$$

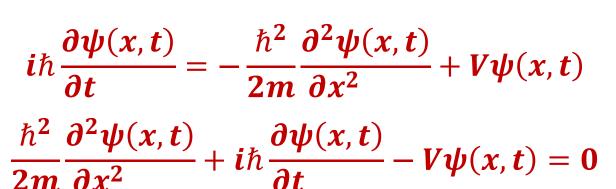
The terms in the equations can be rewritten in terms of operators

$$\widehat{E}\psi(x,t) = \widehat{KE}\psi(x,t) + V\psi(x,t)$$

$$\left\{i\hbar\frac{\partial}{\partial t}\right\}\psi(x,t) = \left\{-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\right\}\psi(x,t) + V\psi(x,t)$$



Schrodinger's wave equation





The solution of this differential equation yields the wave function and its time evolution



Schrodinger's wave equation

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In three dimensions the Schrodinger's time dependent wave equation

$$\frac{\hbar^2}{2m} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \psi(r, t) + i\hbar \frac{\partial \psi(r, t)}{\partial t} - V\psi(r, t) = 0$$

$$\frac{\hbar^2}{2m} \nabla^2 \psi(r, t) + i\hbar \frac{\partial \psi(r, t)}{\partial t} - V\psi(r, t) = 0$$

Schrodinger's time independent wave equation

- Steady state systems observables are time invariant
- The wave function $\psi = Ae^{\frac{i}{\hbar}(px-Et)}$ can be expressed as

$$\psi(x,t) = Ae^{\frac{i}{\hbar}(px)}e^{-\frac{i}{\hbar}Et} = \psi(x)\varphi(t)$$

where $\psi(x)=Ae^{rac{i}{\hbar}(px)}$ is the space dependent component

and $\varphi(t) = e^{-\frac{i}{\hbar}Et}$ is the time dependent component



Schrodinger's time independent wave equation

• Substituting for $\psi(x,t)$ in the time dependent Schrodinger's equation

$$\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2}.\varphi(t)+i\hbar\frac{\partial\varphi(t)}{\partial t}\psi(x)-V\psi(x).\varphi(t)=0$$

- the time derivative of $\varphi(t)$ yields $-\frac{i}{\hbar}E.\varphi(t)$
- Substituting back into the above equation

$$\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2}\cdot\varphi(t)+E\cdot\varphi(t)\cdot\psi(x)-V\psi(x)\cdot\varphi(t)=0$$



Schrodinger's time independent wave equation

Rewriting the equation

$$\left[\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2}+E.\psi(x)-V\psi(x)\right].\varphi(t)=0$$

- the product of two functions is zero implies that either of the terms is zero
- $\varphi(t)$ cannot be zero
- Hence

$$\frac{\hbar^2}{2m}\frac{\partial^2 \psi(x)}{\partial x^2} + (E - V)\psi(x) = 0$$

which is the Schrodinger's time independent wave equation



Schrodinger's time independent wave equation

Writing the equation in the standard form of a differential equation

$$\frac{\partial^2 \psi(x)}{\partial x^2} + \frac{2m}{\hbar^2} (E - V)\psi(x) = 0$$

- The solution of this equation gives the wave function of a steady state system
- The region in which the particle is moving can be defined by the potential function
- The nature of the solution is very much dependent on the potential function



Schrodinger's time independent wave equation

In the case when (E-V) is positive and constant

$$\frac{\partial^2 \psi(x)}{\partial x^2} + k^2 \psi(x) = 0$$

• where
$$k^2 = \frac{2m}{\hbar^2}(E-V)$$
 or $k = \sqrt{\frac{2m}{\hbar^2}(E-V)}$

- This is the familiar equation for a simple harmonic motion
- The solution describes a wave motion



Framework for solving Schrodinger's wave equation

- 1. Problem statement
 - a) The particle and its energy
 - b) The potential energy of the particle
 - c) The range in which the particle is can be found
- 2. Write the Schrodinger's wave equation relevant to the problem
- 3. Obtain solution of the SWE $\psi(x)$
- 4. Verify whether $\psi(x)$ is an acceptable function
 - a) $\psi(x)$ and it's derivatives are finite, continuous and single valued
 - b) $\psi(x)$ is normalized



Class #11 Quiz....

The concepts which are true of Schrodinger's wave equations ...

- 1. Standard wave equations involve second order derivative in space and time
- 2. Schrodinger's wave equation is a standard wave equation
- 3. The operator for the kinetic energy is $\left\{-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\right\}$
- 4. At t=0, $\varphi(t)$ is zero
- 5. The time independent wave equation can be used to evaluate steady state systems
- 6. The wave function is always an imaginary function





THANK YOU

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