Statistical Simulation

Part 1 of Midterm 1

Due data: 00:00am, October 20, 2019

In the following, you should provide your codes to the questions 1.(a), (c), (d), (e), 2., 3.(a), (b), (c), and (d), and provide the expected values by simulation ways for the questions 1, 2, and 3.

1. Let X be the negative binomial (NB) random variable with its probability mass function (pmf) having parameters (r, p), where r is the total number of successes and p is the probability of a success. The pmf is given by
$$p_j = P\{X = j\} = \frac{(j-1)!}{(j-r)!(r-1)!} p^r (1-p)^{j-r}, j = r, r+1, \ldots.$$
   (a) Use the relationship between negative binomial and geometric random variables to develop an algorithm for simulating from this distribution.
   (b) Verify the relation
$$p_{j+1} = \frac{j(1-p)}{j+1-r} p_j.$$
   (c) Use the relation in (b) to give a second algorithm for generating the negative binomial random variable.
   (d) Use independent random variables from a Bernoulli distribution with success probability p to generate the samples from an NB(r, p). (Hint: the relationship between the Bernoulli(p) and NB(r, p) distributions.)
   (e) Use above three simulation methods to evaluate $E(X)$ with $r = 5$ and $p = 0.3$. What are the values of $E(X)$? Are these expected values by the three simulation methods closed?

2. A pair of fair dice are to be continually rolled until all the possible outcomes 2, 3, ..., 12 have occurred at least once. Develop a simulation study to estimate the expected number of dice rolls that are needed.

3. Suppose that the discrete random variable X can take on any of the values 5, ..., 14 with the following probabilities:
$$p_j = \begin{cases} 0.11 & \text{when } j \text{ is odd and } 5 \le j \le 13 \\ 0.09 & \text{when } j \text{ is even and } 6 \le j \le 14 \end{cases}$$
   (a) Use the inverse transform method to generate the random variable.
   (b) Use the Acceptance-Rejection (AR) method to generates the value of x. Write the code for the AR method. What is the value of c in the AR method? What is the average of iteration number of your AR method?
   (c) Use the composite method to generate the random variable.
   (d) Use the above three simulation methods to evaluate $E(X)$. What are the values of $E(X)$? Are these expected values by the three simulation methods closed?