統計模擬HW1

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use simulation to approximate the following integrals. Compare your estimate with the exact answer if known.

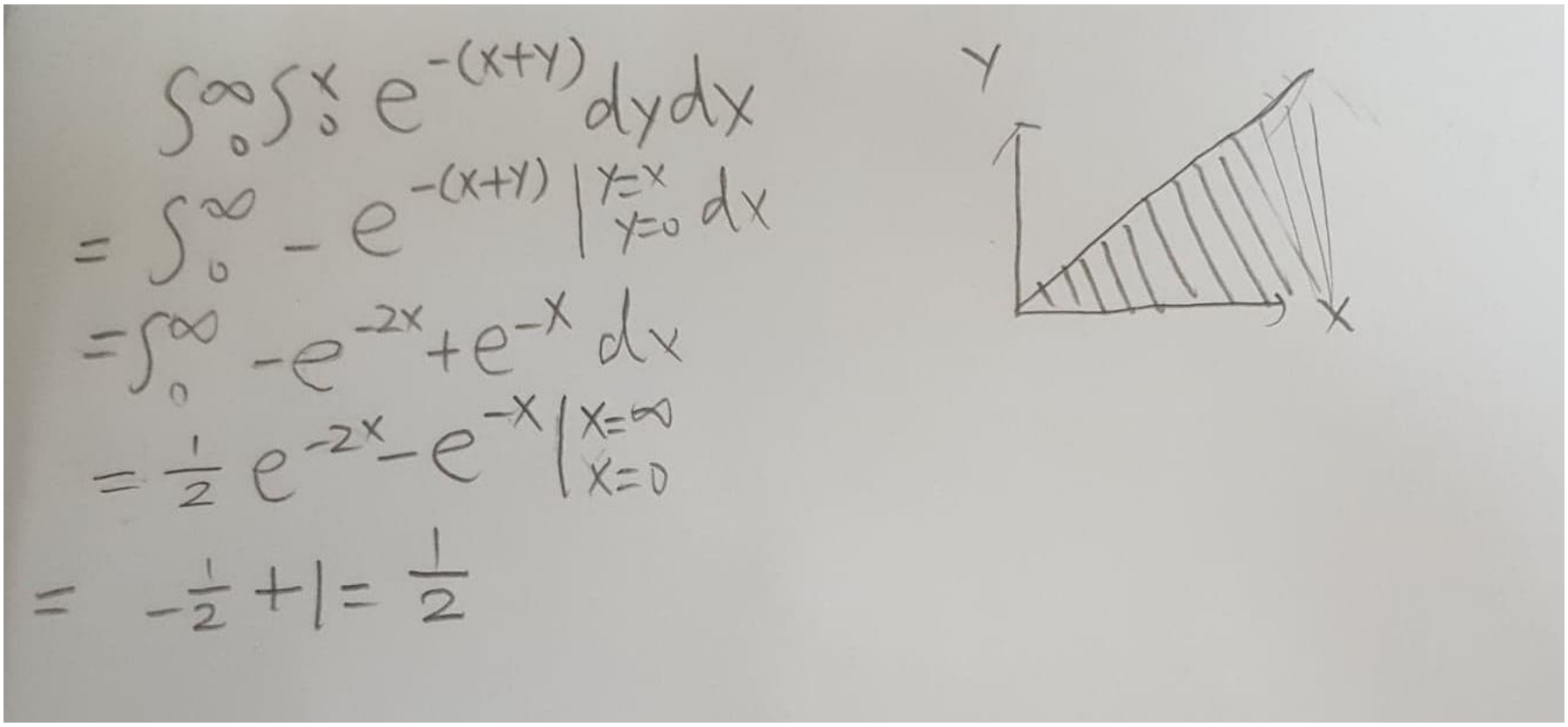
$$\int_0^{\infty}\int_0^{x} e^{-(x+y)}dydx = \frac{1}{2}$$

```
#simulation
integral<-function(n){
  ux<-runif(n)
  uy<-runif(n)
  I<-function(x,y) {ifelse(y<=x,1,0)}
  x=(1-ux)/ux
  y=(1-uy)/uy
  return(mean(exp(-(x+y))*I(x,y)/ux^2/uy^2))
}

ans=integral(100000)
ans
```

```
## [1] 0.5033593
```

```
#exact answer
#1/2
```



$\int_0^{\infty}\int_0^{x} e^{-(x+y)}dydx = \frac{1}{2} \approx$ **0.5033593**

13. Let $U_i, i \geqslant 1$, be random numbers. Define N by

$$N = \text{Maximum}\left\{n: \prod_{i=1}^{n} U_i \geqslant e^{-3}\right\}$$

where $\prod_{i=1}^{0} U_i \equiv 1$.

(a) Find $E[N]$ by simulation.
(b) Find $P\{N = i\}$, for $i = 0, 1, 2, 3, 4, 5, 6$, by simulation.

(a)

```
simulation_13<-function(n){
N<-rep(NA,n)
for(j in 1:n){
U<-runif(100)
multiple<-1
for(i in 1:100){
  multiple<-multiple*U[i]
  if(multiple<exp(-3)){
      N[j]<-i-1
    break
  }
}
}
return(N)
}
mean=mean(simulation_13(100000))
mean
```

```
## [1] 3.00705
```

$E(N)$ **is 3.00705**

(b)

```
N<-simulation_13(100000)
p0<-sum(N==0)/length(N) #P(N=0)
p1<-sum(N==1)/length(N) #P(N=1)
p2<-sum(N==2)/length(N) #P(N=2)
p3<-sum(N==3)/length(N) #P(N=3)
p4<-sum(N==4)/length(N) #P(N=4)
p5<-sum(N==5)/length(N) #P(N=5)
p6<-sum(N==6)/length(N) #P(N=6)
```

$P(N = 0)$ is 0.04952

$P(N = 1)$ is 0.14737

$P(N = 2)$ is 0.22386

$P(N = 3)$ is 0.22617

$P(N = 4)$ is 0.17091

$P(N = 5)$ is 0.09883

$P(N = 6)$ is 0.04956

14. With $x_1 = 23, x_2 = 66$, and

$$x_n = 3x_{n-1} + 5x_{n-2} \mod(100), \quad n \geqslant 3$$

we will call the sequence $u_n = x_n/100, n \geqslant 1$, the *text's random number sequence*. Find its first 14 values.

```
x=c(23,66,rep(NA,12))
for(i in 3:14){
  x[i]<-(3*x[i-1]+5*x[i-2])%%100
}
u=x/100
u
```

```
##  [1] 0.23 0.66 0.13 0.69 0.72 0.61 0.43 0.34 0.17 0.21 0.48 0.49 0.87 0.06
```

first 14 values : 0.23, 0.66, 0.13, 0.69, 0.72, 0.61, 0.43, 0.34, 0.17, 0.21, 0.48, 0.49, 0.87, 0.06.