# 統計模擬HW3

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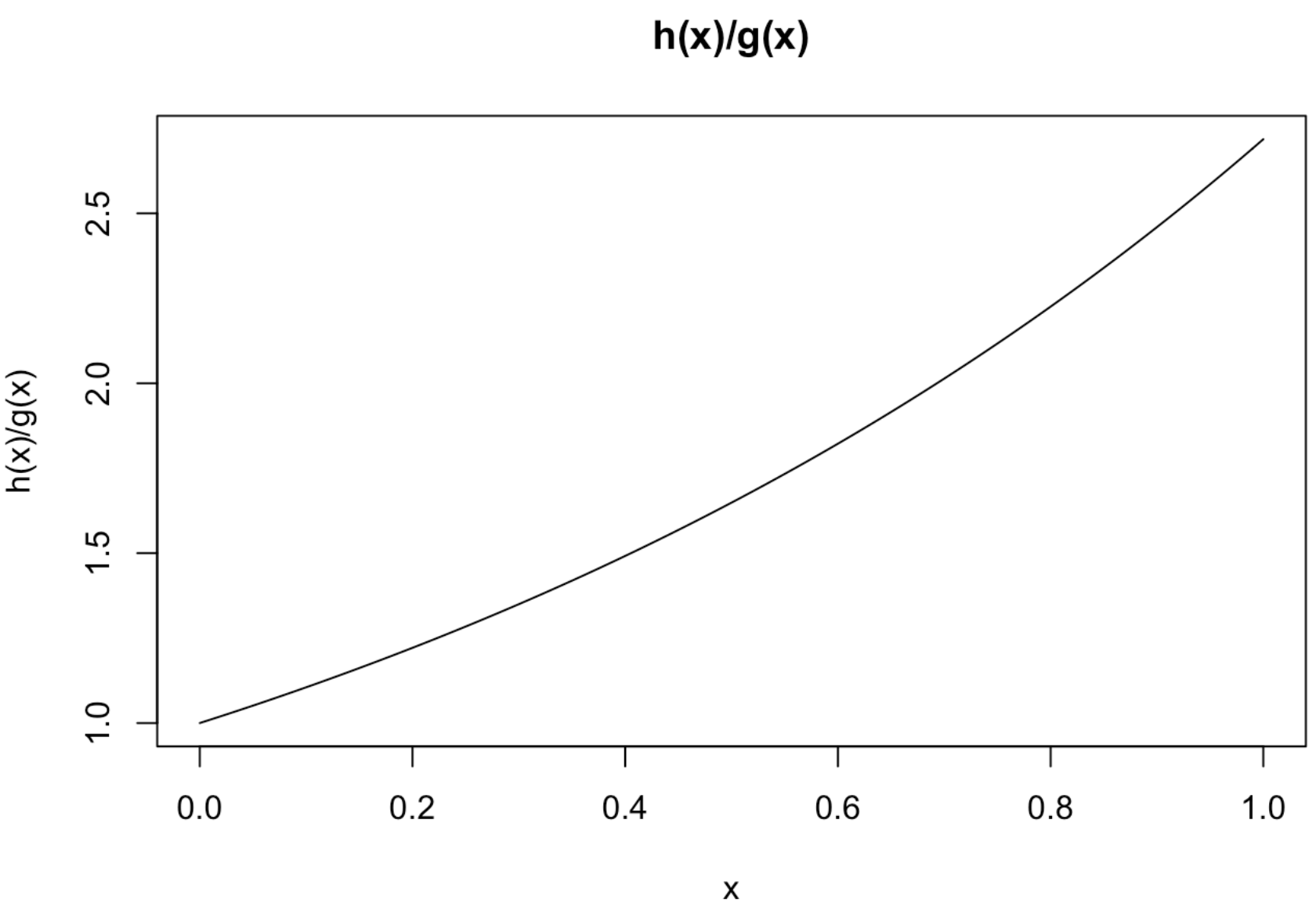
## 1

### (a)

用Y=Unif(0,1) 模擬 g(x)=1 0≤x≤1

### (b)

```
x=seq(0,1,0.01)
h.g=function(x){
  return(exp(x))
}
plot(x,h.g(x),,type="l",main="h(x)/g(x)",,ylab="h(x)/g(x)")
```




```
det.c <- optim(1, h.g, lower = 0, upper = 1, method = "L-BFGS-B",control = list(fn
scale = -1)) ### maximization
det.c$par ### the location of the optimum
```

```
## [1] 1
```

```
det.c$value
```

```
## [1] 2.718282
```

```
c=det.c$value
```

x= 1, h(x)/g(x)= 2.7182818最大，取c=2.7182818

## (c)

```
sim_1=function(n){
  X=rep(NA,n)
  iter=rep(NA,n)
  for(j in 1:n){
    Y=runif(1)
    i=1
    U=runif(1,0)
    while(U>h.g(Y)/c){
      Y=runif(1)
      U=runif(1,0)
      i=i+1
    }
    X[j]=Y
    iter[j]=i
  }
  return(list(X=X,iter=iter))
}

rejection_1=sim_1(100000)
X=rejection_1$X
iter=rejection_1$iter
mean(iter)
```

```
## [1] 1.58015
```

```
k_approximated=mean(iter)/c #k
k_approximated
```

```
## [1] 0.5813047
```

approximated value of k =(average of iteration time)/c=0.5813047

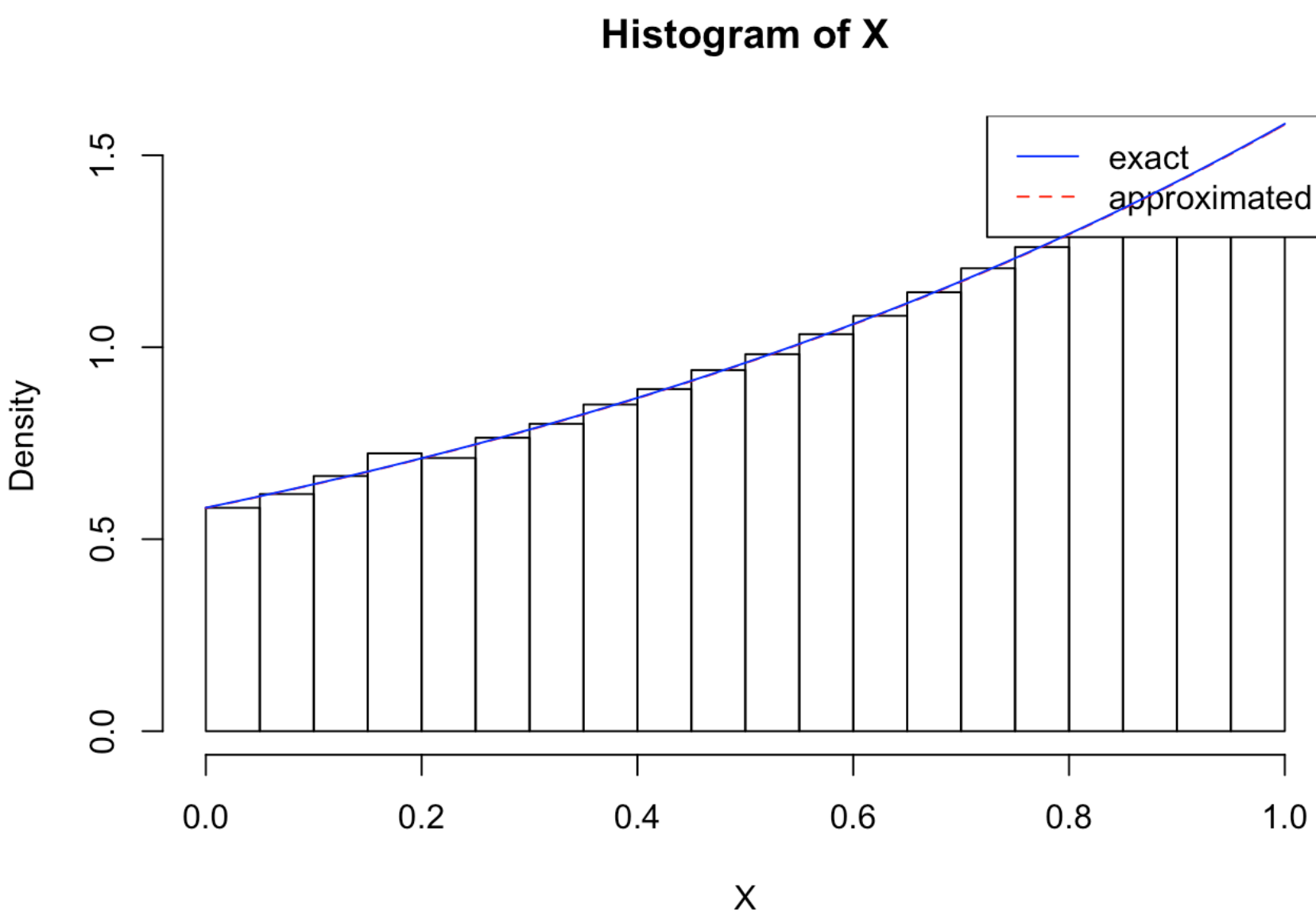
## (d) (e)

```
hist(X,probability = T)
legend("topright", legend = c("exact", "approximated"),,
lty = c(1, 2), col = c("blue", "red"))
lines(x,k_approximated*exp(x),col="red",lty=2)


k=1/(exp(1)-1)
k
```

```
## [1] 0.5819767
```

```
lines(x,k*exp(x),col="blue")
```




兩條線重疊 k可用 (average of iteration time)/c 來估計

# 2

## (a)

**法1用積分機率=1**

$$\int_{0.8}^{1} kx(1-x)^3\,dx = 1$$

$$\Rightarrow k\int_{0.8}^{1} x-3x^2+3x^3-x^4\,dx = 1$$

$$\Rightarrow k\left[\frac{1}{2}x^2 - x^3 + \frac{3}{4}x^4 - \frac{1}{5}x^5 \Big|_{x=0.8}^{x=1}\right] = 1$$

$$\Rightarrow k\left[\left(\frac{1}{2}-1+\frac{3}{4}-\frac{1}{5}\right) - \left(\frac{1}{2}0.8^2 - 0.8^3 + \frac{3}{4}0.8^4 - \frac{1}{5}0.8^5\right)\right] = 1$$

$$k = \frac{1}{\left(\frac{1}{2}-1+\frac{3}{4}-\frac{1}{5}\right) - \left(\frac{1}{2}0.8^2 - 0.8^3 + \frac{3}{4}0.8^4 - \frac{1}{5}0.8^5\right)}$$

```
k=1/((1/2-1+3/4-1/5)-(1/2*0.8^2-0.8^3+3/4*0.8^4-0.8^5/5))
k
```

```
## [1] 2976.19
```

### 法2用條件機率

$$\text{Beta}(2,4)\quad f(x) = \frac{\Gamma(2+4)}{\Gamma(2)\Gamma(4)}x^{2-1}(1-x)^{4-1} = \frac{5!}{1!3!}x(1-x)^3 = 20x(1-x)^3 \quad 0<x<1$$

$$kx(1-x)^3 = P(X \mid 0.8<X<1) = \frac{f(x)}{\int_{0.8}^{1} f(x)\,dx} = \frac{20x(1-x)^3}{\int_{0.8}^{1} 20x(1-x)^3\,dx} = \frac{20x(1-x)^3}{20\times 0.000336}$$

$$0.8<x<1$$

$$= 2976.19 \times x(1-x)^3 \Rightarrow k = 2976.19$$

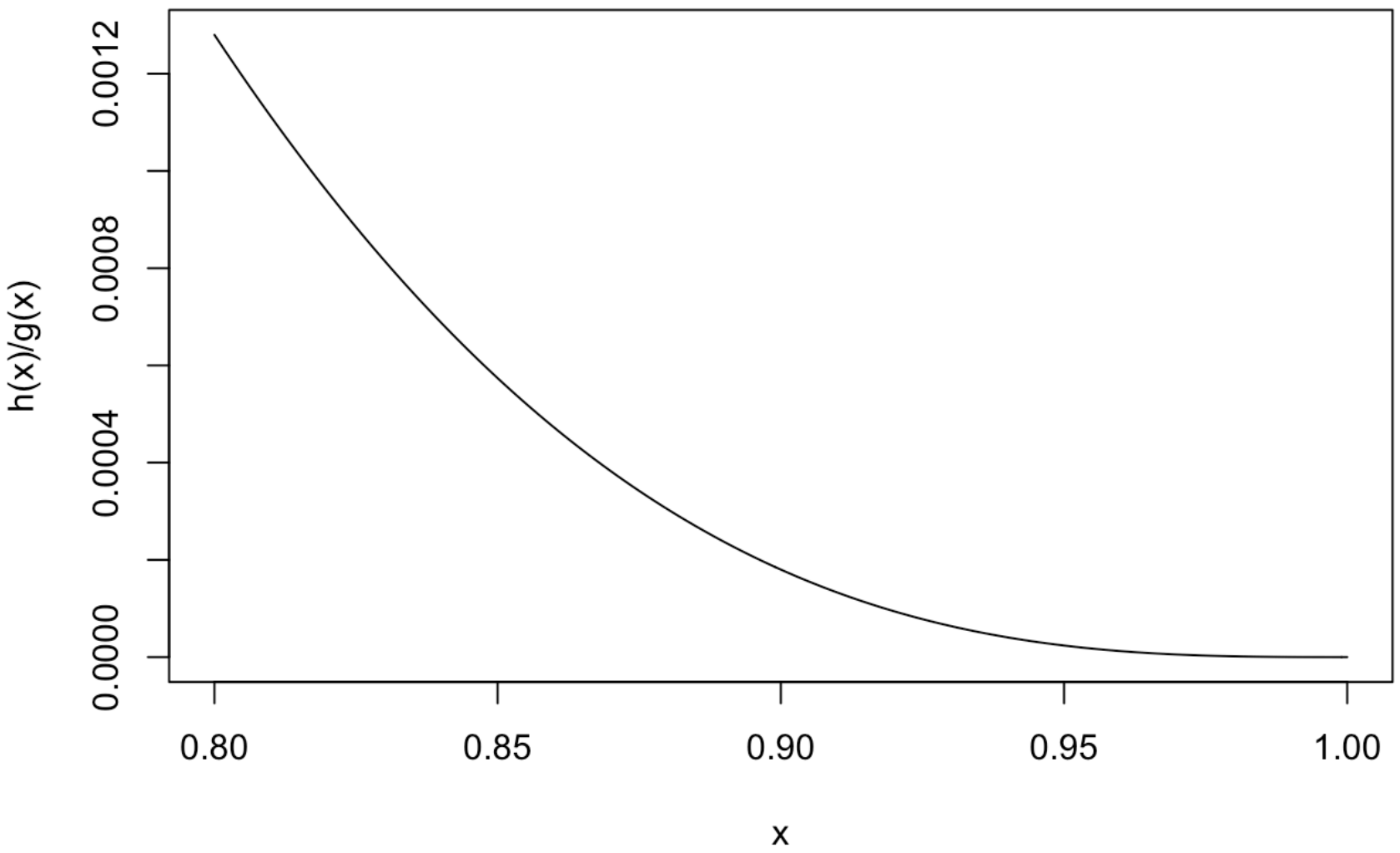
$$\int_{0.8}^{1} 20x(1-x)^3\,dx = 20\times 0.000336$$

## (b)

Y=Unif(0.8,1) g(x)=5 0.8≤x≤1

## (c)

```
x=seq(0.8,1,0.001)
h.g=function(x){
  return(x*(1-x)^3/5)
}
plot(x,h.g(x),main="h(x)/g(x)",,ylab="h(x)/g(x)",type="l")
```




```
det.c <- optim(0.8, h.g, lower = 0.8, upper = 1, method = "L-BFGS-B",control = lis
t(fnscale = -1)) ### maximization
det.c$par  ### the location of the optimum
```

```
## [1] 0.8
```

```
det.c$value
```

```
## [1] 0.00128
```

```
c=det.c$value
```

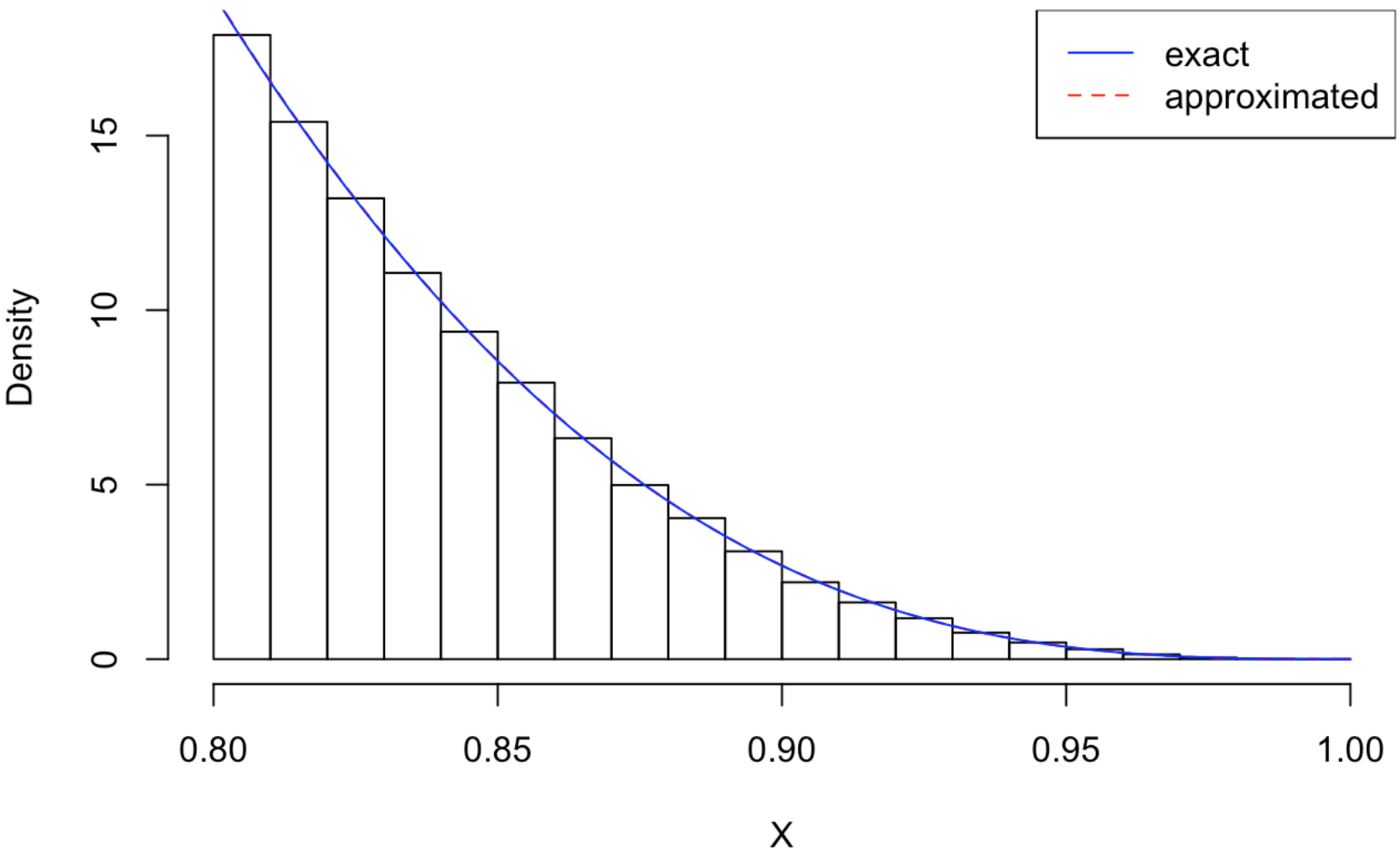
## (d)

```r
n=100000
sim_2=function(n){
  X=rep(NA,n)
  iter=rep(NA,n)
  for(j in 1:n){
    Y=runif(1)*0.2+0.8
    i=1
    U=runif(1,0)
    while(U>h.g(Y)/c){
      Y=runif(1)*0.2+0.8
      U=runif(1,0)
      i=i+1
    }
    X[j]=Y
    iter[j]=i
  }
  return(list(X=X,iter=iter))
}

rejection_2=sim_2(100000)
X=rejection_2$X
iter=rejection_2$iter
hist(X,probability = T)
legend("topright", legend = c("exact", "approximated"),,
lty = c(1, 2), col = c("blue", "red"))

k_approximated=mean(iter)/c #k

lines(x,k_approximated*x*(1-x)^3,col="red",lty=2)
lines(x,k*x*(1-x)^3,col="blue")
```


Histogram of X
exact
approximated
Density
0
5
10
15
0.80
0.85
0.90
0.95
1.00
X

兩條線重疊