統計模擬HW2

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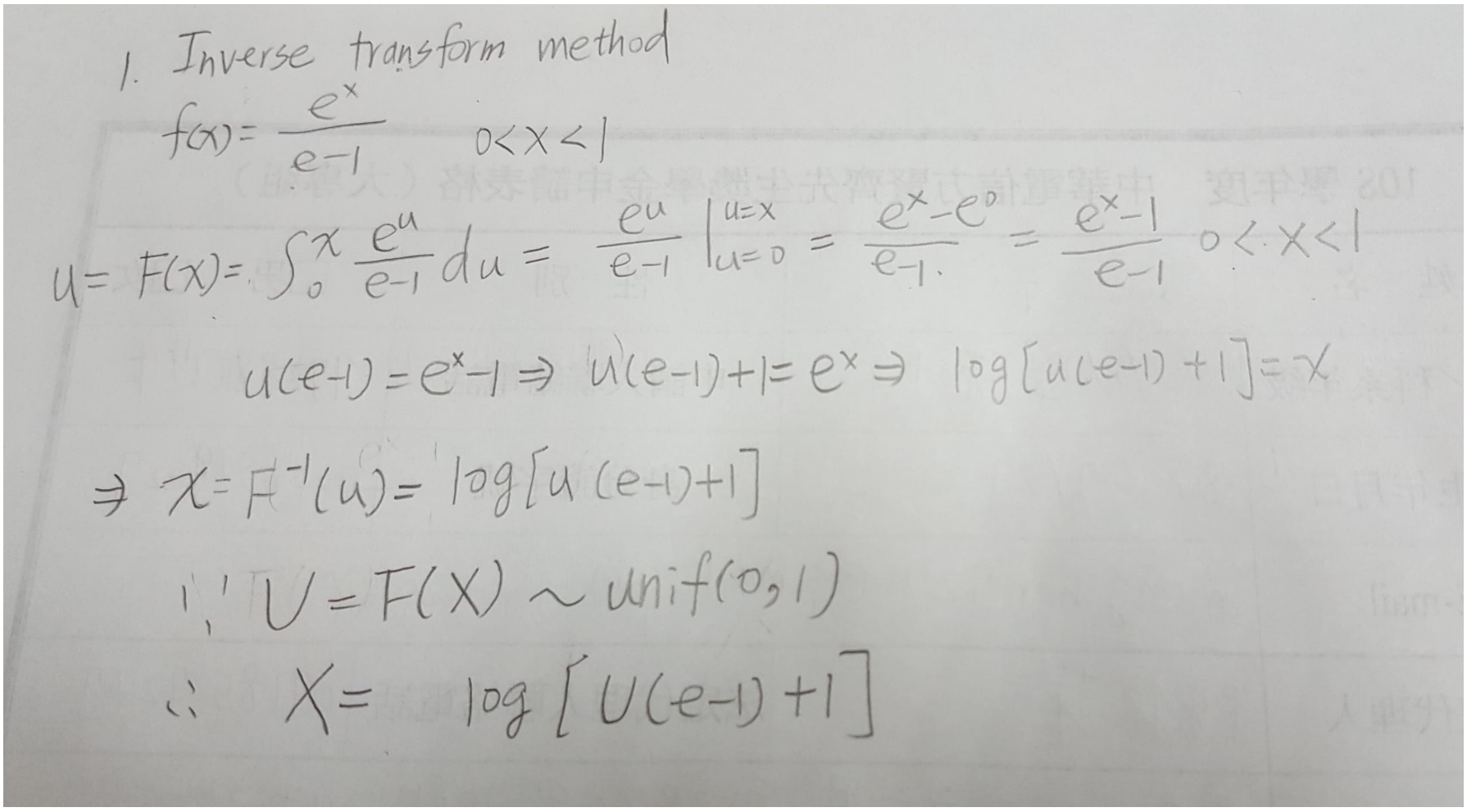
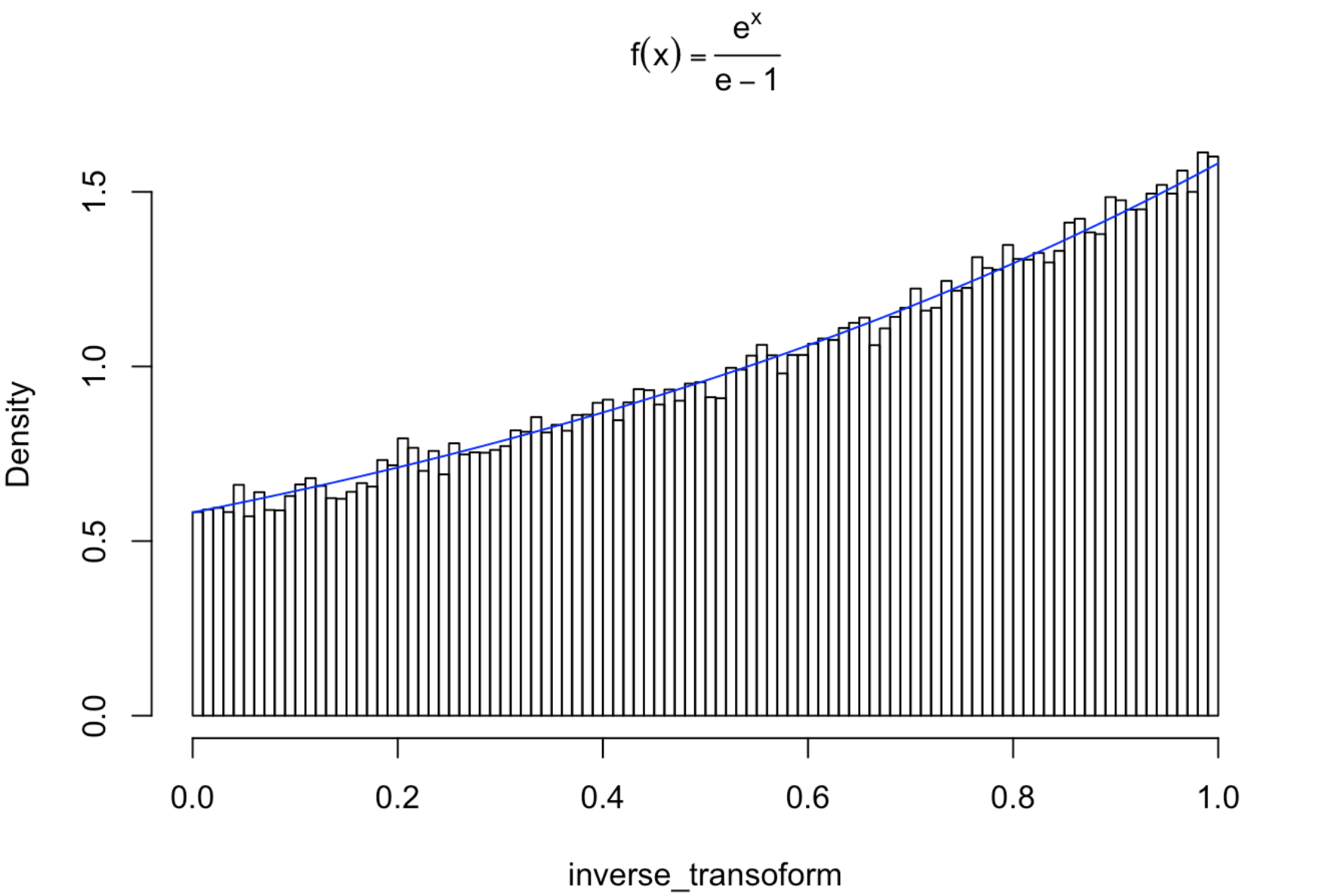
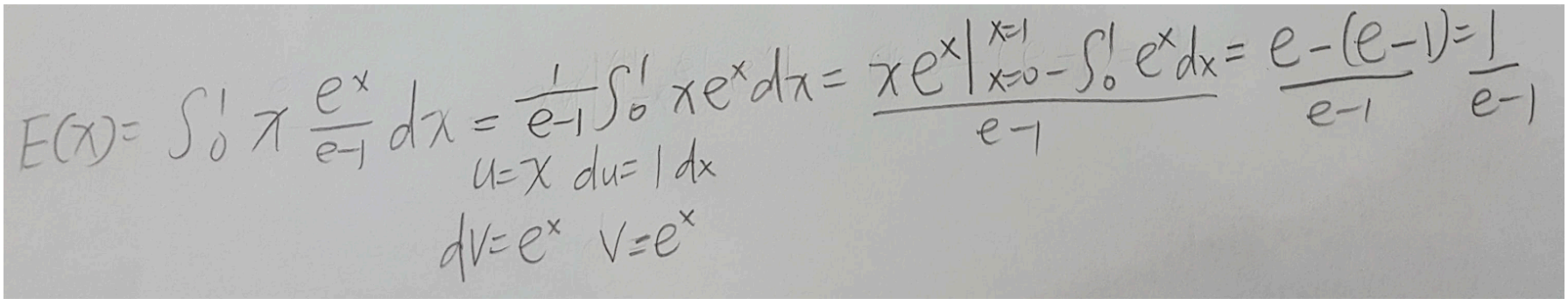
1

inverse transform method




```
inverse_transoform=function(n){
  U=runif(n)
  X=log(U*(exp(1)-1)+1)
  return(X)
}

inverse_transoform=inverse_transoform(100000)
hist(inverse_transoform,probability = T,breaks=100,main=expression(f(x)==frac(e^x,
e-1)),cex.main=1)
x=seq(0,1,0.01)
fx=exp(x)/(exp(1)-1)
lines(x,fx,col="blue")
```






```
mean(inverse_transoform)
```

```
## [1] 0.5832611
```

```
#exact mean
1/(exp(1)-1)
```

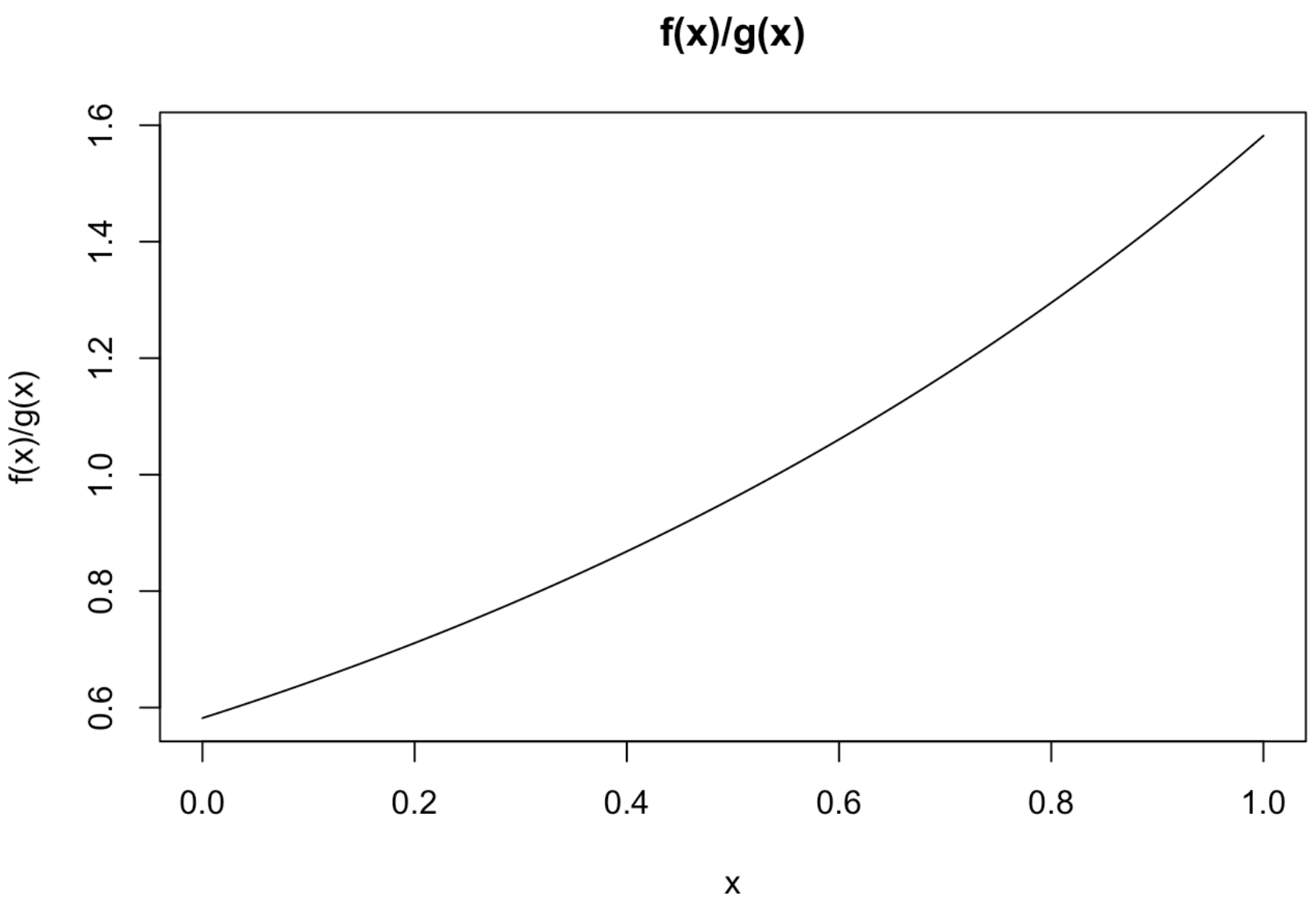
```
## [1] 0.5819767
```

rejection method

Simulate the value of Y having Unif(0,1)

找c

```
f.g <- function(x){
  exp(x)/(exp(1)-1)
}
x=seq(0,1,0.01)
plot(x,f.g(x),type="l",main="f(x)/g(x)",,ylab="f(x)/g(x)")
```




```
det.c <- optim(1, f.g, lower = 0, upper = 1, method = "L-BFGS-B",control = list(fn
scale = -1)) ### maximization
det.c$par ### the location of the optimum
```

```
## [1] 1
```

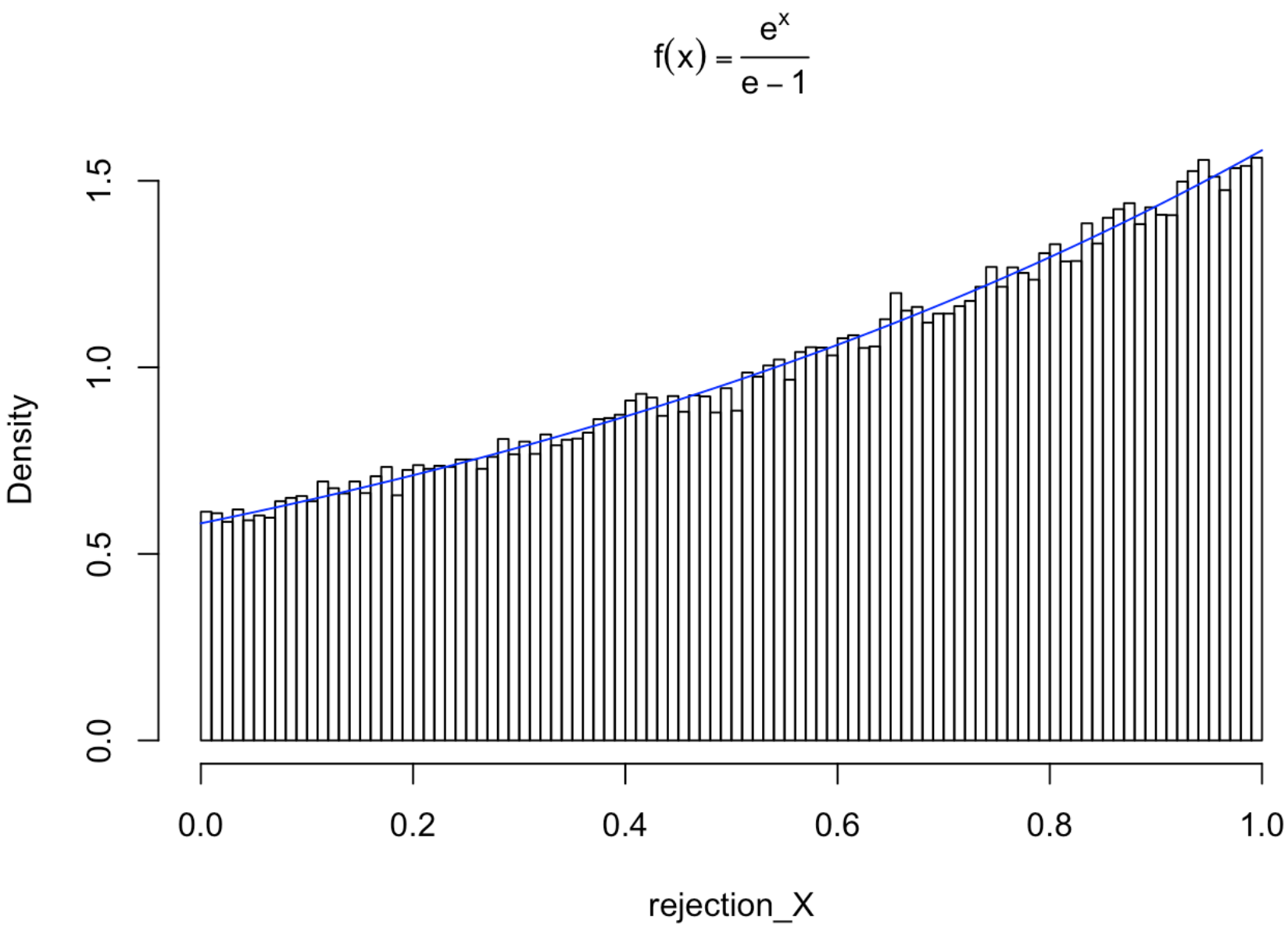
```
det.c$value
```

```
## [1] 1.581977
```

```
c=det.c$value
```

x= 1, f(x)/g(x)= 1.5819767最大，取c=1.5819767

```r
#用unif(0,1)模擬
rejection=function(n){
  X=rep(NA,n)
  iter=rep(NA,n)
  for(j in 1:n){
    U=runif(1)
    Y=runif(1)
    i=1
    while(U>exp(Y)/(exp(1)-1)/c){
      U=runif(1)
      Y=runif(1)
      i=i+1
    }
    X[j]=Y
    iter[j]=i
  }
  return(list(X=X,iter=iter))
}

rejection=rejection(100000)

rejection_X=rejection$X
hist(rejection_X,probability = T,breaks=100,main=expression(f(x)==frac(e^x, e-1)),
cex.main=1)
x=seq(0,1,0.01)
fx=exp(x)/(exp(1)-1)
lines(x,fx,col="blue")
```

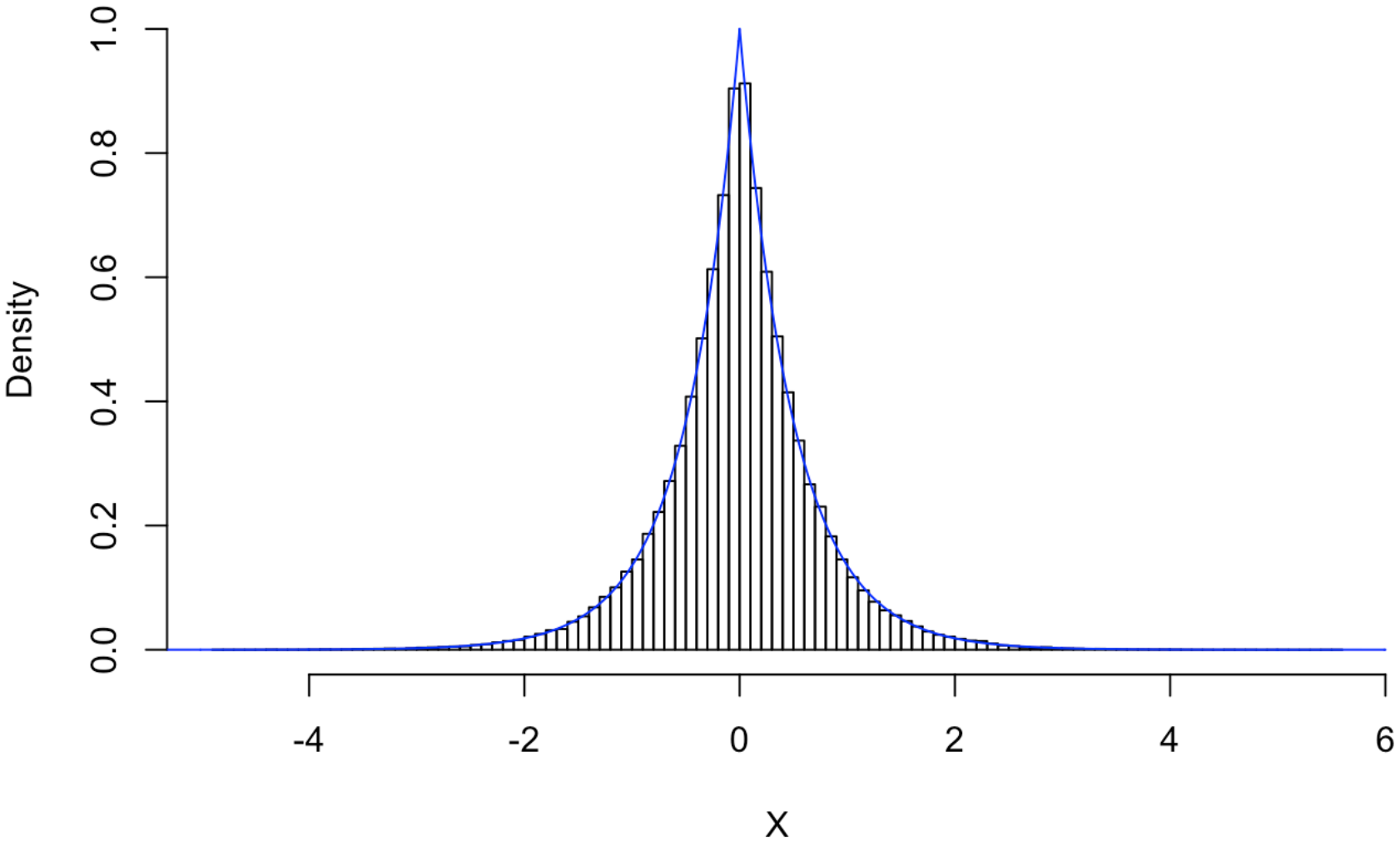



```
mean(rejection$iter)  #平均iter =c
```

```
## [1] 1.58409
```

2

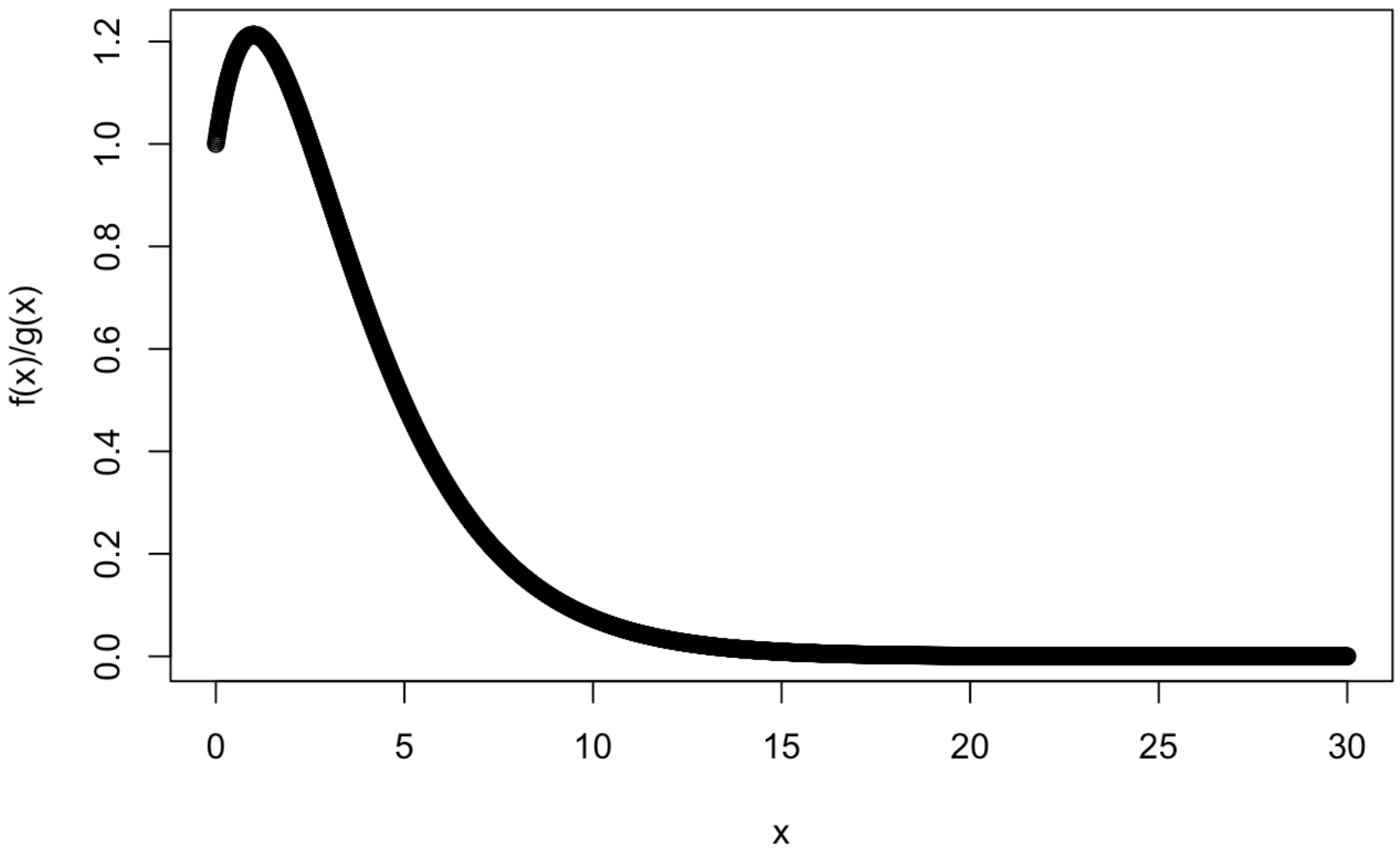
```
sim2=function(n){
  U1=runif(n)
  X=-log(U1)/2 #EXP(2)
  U2=runif(n)
  for(i in 1:n){
    if(U2[i]<0.5){
      X[i]=-X[i]
    }
  }
  return(X)
}

X_2=sim2(100000)
hist(X_2,probability = T,ylim=c(0,1),breaks=100,main="",xlab="X")
x=seq(0,6,0.01)
lines(x,exp(-2*x),col="blue")
y=seq(-6,0,0.01)
lines(y,exp(2*y),col="blue")
```



3

Simulate the value of Y having EXP(1/2)

```
#找c
x=seq(0,30,0.01)
plot(x,(1+x)*exp(-(1/2)*x),main="f(x)/g(x)",,ylab="f(x)/g(x)")
```




```
f.g <- function(x){
  (1+x)*exp(-(1/2)*x)
}
det.c <- optim(1, f.g, lower = 0, upper = 10, method = "L-BFGS-B",control = list(f
nscale = -1)) ### maximization
det.c$par ### the location of the optimum
```

```
## [1] 1
```

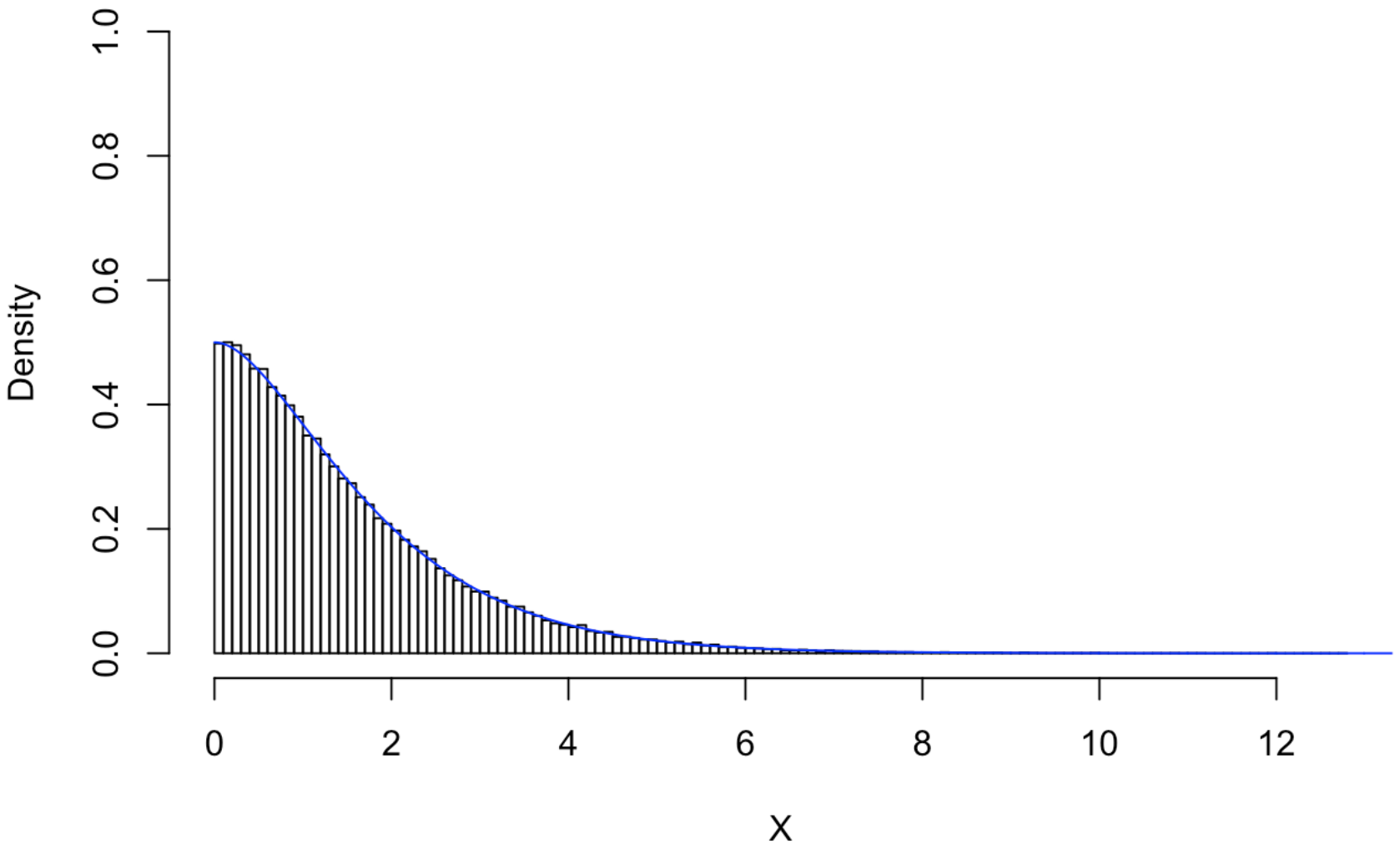
```
det.c$value
```

```
## [1] 1.213061
```

```
c=det.c$value
```

x= 1, f(x)/g(x)= 1.2130613最大，取c=1.2130613

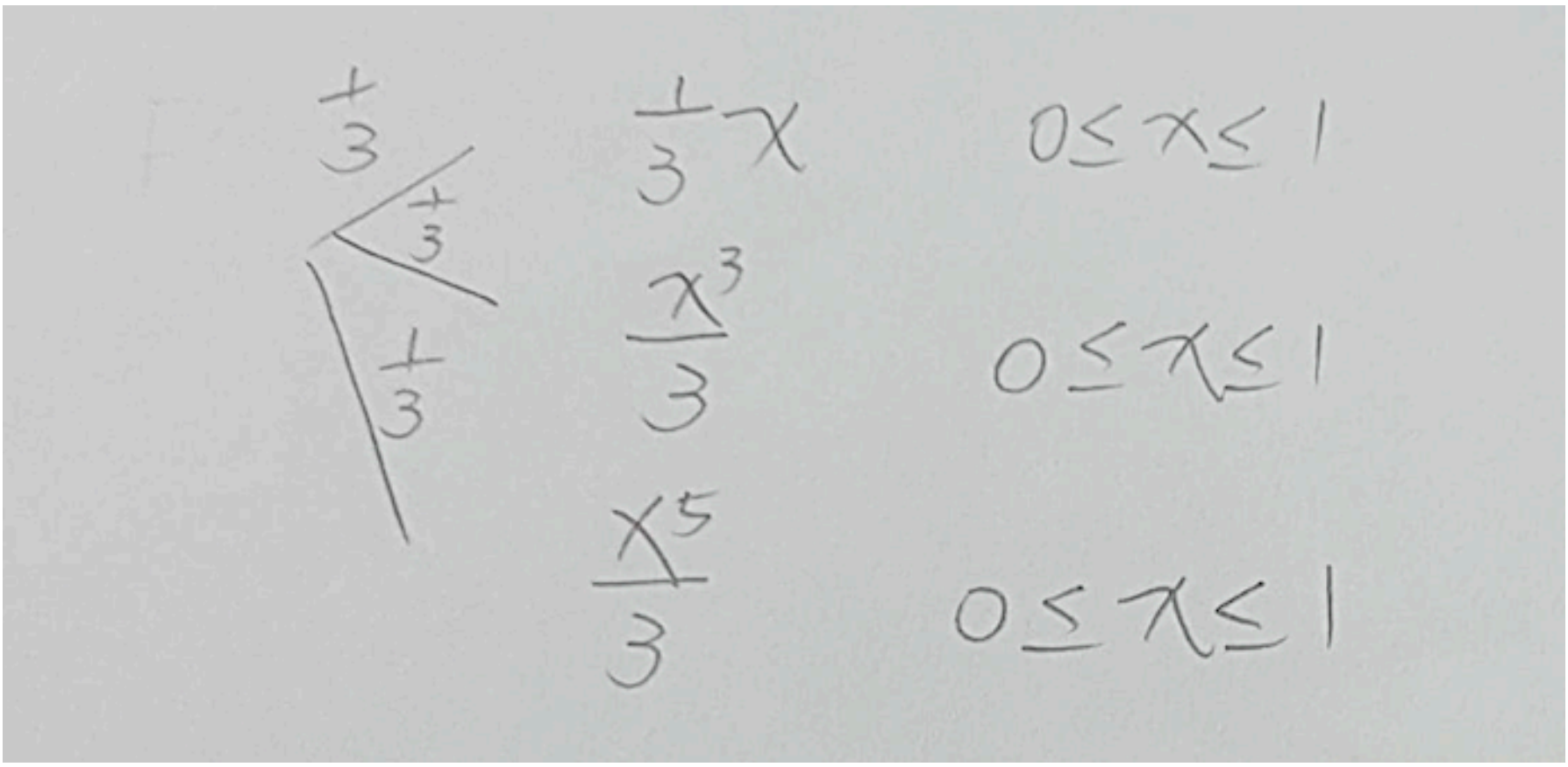
```
rejection_3=function(n){
  X=rep(NA,n)
  iter=rep(NA,n)
  for(j in 1:n){
    Y=-log(runif(1))*2  #Y=Exp(1/2)
    U=runif(1)
    i=1
    fx=1/2*(1+Y)*exp(-Y)
    gx=1/2*exp(-(1/2)*Y)
    while(U>fx/gx/c){
      Y=-log(runif(1))*2
      U=runif(1)
      i=i+1
      fx=1/2*(1+Y)*exp(-Y)
      gx=1/2*exp(-(1/2)*Y)
    }
    X[j]=Y
    iter[j]=i
  }
  return(list(X=X,iter=iter))
}



sim_3=rejection_3(100000)
X=sim_3$X


hist(X,probability = T,ylim=c(0,1),breaks=100,main=expression(f(x)==frac(1, 2)*(1+
x)*e^{-x}),cex.main=1)
x=seq(0,20,0.01)
lines(x,1/2*(1+x)*exp(-x),col="blue")
```



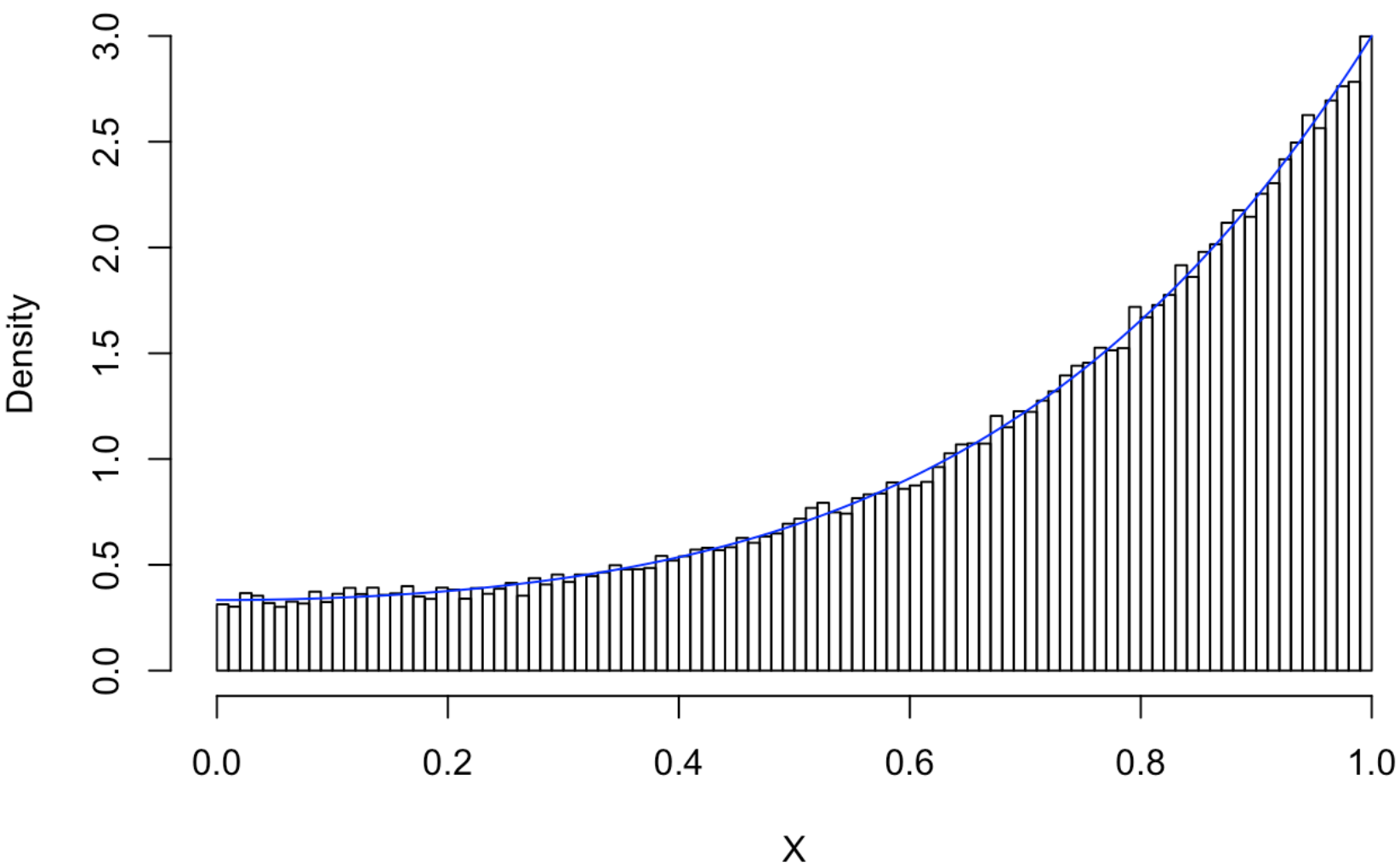

```
mean(sim_3$iter)   #平均iter =c
```

```
## [1] 1.21369
```

4



```
composite=function(n){
  U=runif(n)
  U1=runif(n)
  X=rep(NA,n)
  for(i in 1:n){
    if(U[i]<1/3){
      X[i]=U1[i]
    }
    else if(U[i]<2/3){
      X[i]=(U1[i])^(1/3)
    }
    else{
      X[i]=(U1[i])^(1/5)
    }
  }
  return(X)
}
X=composite(100000)

hist(X,probability = T,breaks=100,main=expression(f(x)==frac(x+x^3+x^5, 3)),cex.ma
in=1)
x=seq(0,1,0.01)
lines(x,(1+3*x^(2)+5*x^(4))/3,col="blue")
```



$F(x) = \frac{x+x^3+x^5}{3} \quad 0 \le x \le 1$

$f(x) = \frac{1+3x^2+5x^4}{3} \quad 0 \le x \le 1$

$E(x) = \int_0^1 x \frac{1+3x^2+5x^4}{3} dx = \int_0^1 \frac{x+3x^3+5x^5}{3} dx = \frac{1}{3}\left[\frac{1}{2}x^2 + \frac{3}{4}x^4 + \frac{5}{6}x^6\right]\Big|_{x=0}^{x=1}$

$= \frac{1}{3}\left[\frac{1}{2} + \frac{3}{4} + \frac{5}{6}\right] = \frac{25}{36}$

```
mean(X)
```

```
## [1] 0.6945766
```

```
#exact mean
25/36
```

```
## [1] 0.6944444
```