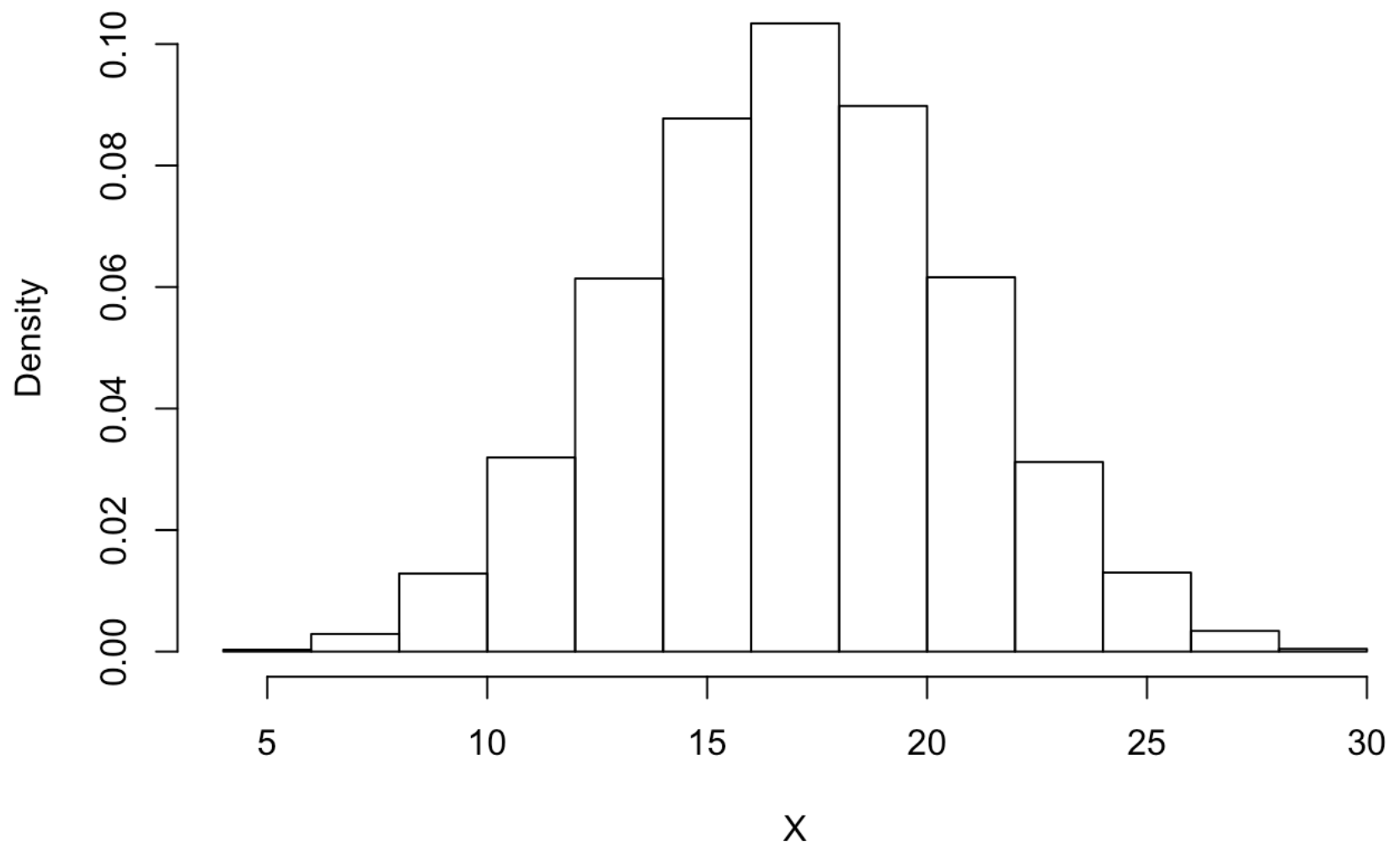# 統模midterm

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2

(a)

```r
set.seed(1)
n=10000
rolls_dice=function(n){
X=rep(NA,n)
for(j in 1:n){
  U=runif(5,0,1)
  X[j]=sum(ceiling(6*U))
}
  return(X)
}
n=10000
X=rolls_dice(n)
mean(X)
```

```
## [1] 17.5053
```

(b)

```r
hist(X,probability = T)
```

Histogram of X


```
length(X[X<=11])/n
```

```
## [1] 0.0602
```

3

(b)

```
sim_3_b=function(n,s){
X=rep(NA,n)
for(j in 1:n){
   U=runif(1,0,1)
   X[j]=-s*log((1-U)/U)
}
   return(X)
}
```
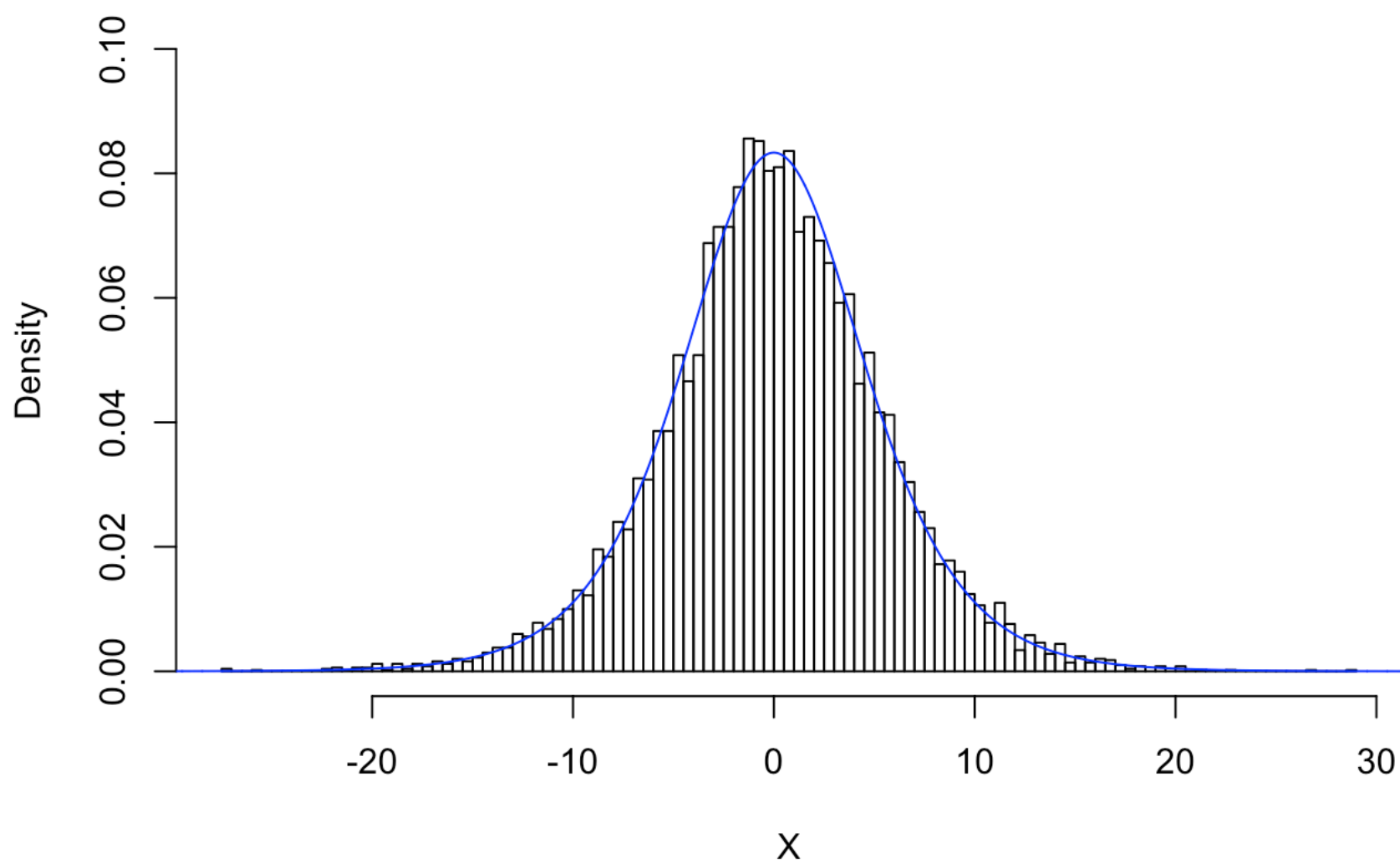
(c)

```
set.seed(1)
n=10000
s=3
X=sim_3_b(n,s=s)
mean(X)
```

```
## [1] -0.00281157
```

```
hist(X,probability = T,ylim=c(0,0.1),breaks=100)

f=function(x,s){
 exp(-x/s)/(s*(1+ exp(-x/s))^2)
}
x=seq(-100,100,0.01)
lines(x,f(x,s),col="blue")
```

Histogram of X

4

(c)

```
hypergeometric=function(rep,N,M,n){
  X=rep(NA,rep)
  for(i in 1:rep){
    U=runif(1,0,1)
    F=0
    j=0
    p=choose(M,0)*choose(N-M,n-0)/choose(N,n)
    F=F+p
    while(U>F){
      p=(M-j)/(j+1)*(n -j)/(N - M - n + j + 1)*p
      F=F+p
      j=j+1
    }
    X[i]=j
  }
  return(X)
}
```
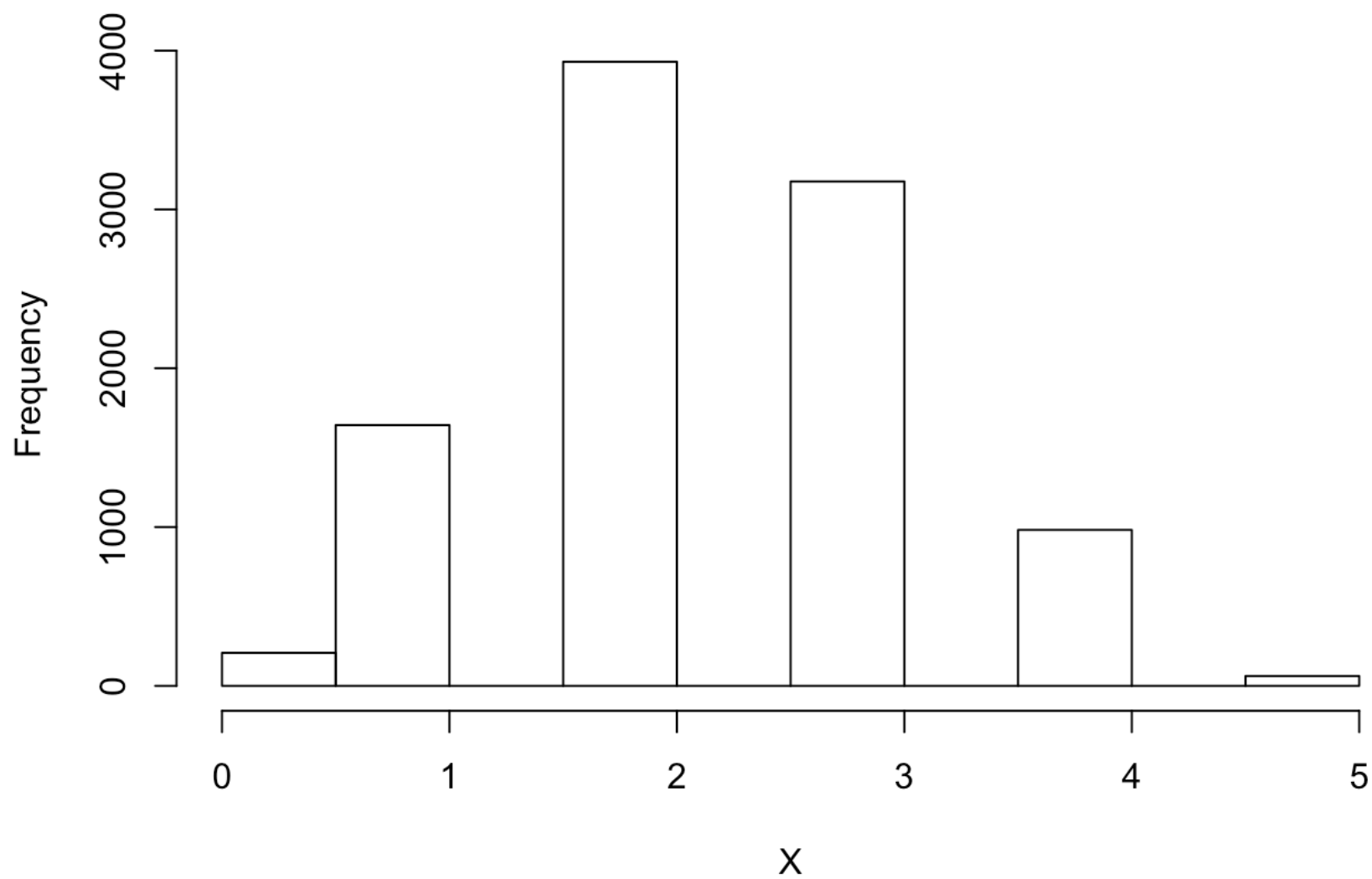
(d)

```
set.seed(1)
rep=10000
N=15
M=7
n=5
X=hypergeometric(rep=rep,N=N,M=M,n=n)
hist(X)
```

Histogram of X


```
#simulation mean
mean(X)
```

```
## [1] 2.3268
```

```
#exact
n*M/N
```

```
## [1] 2.333333
```

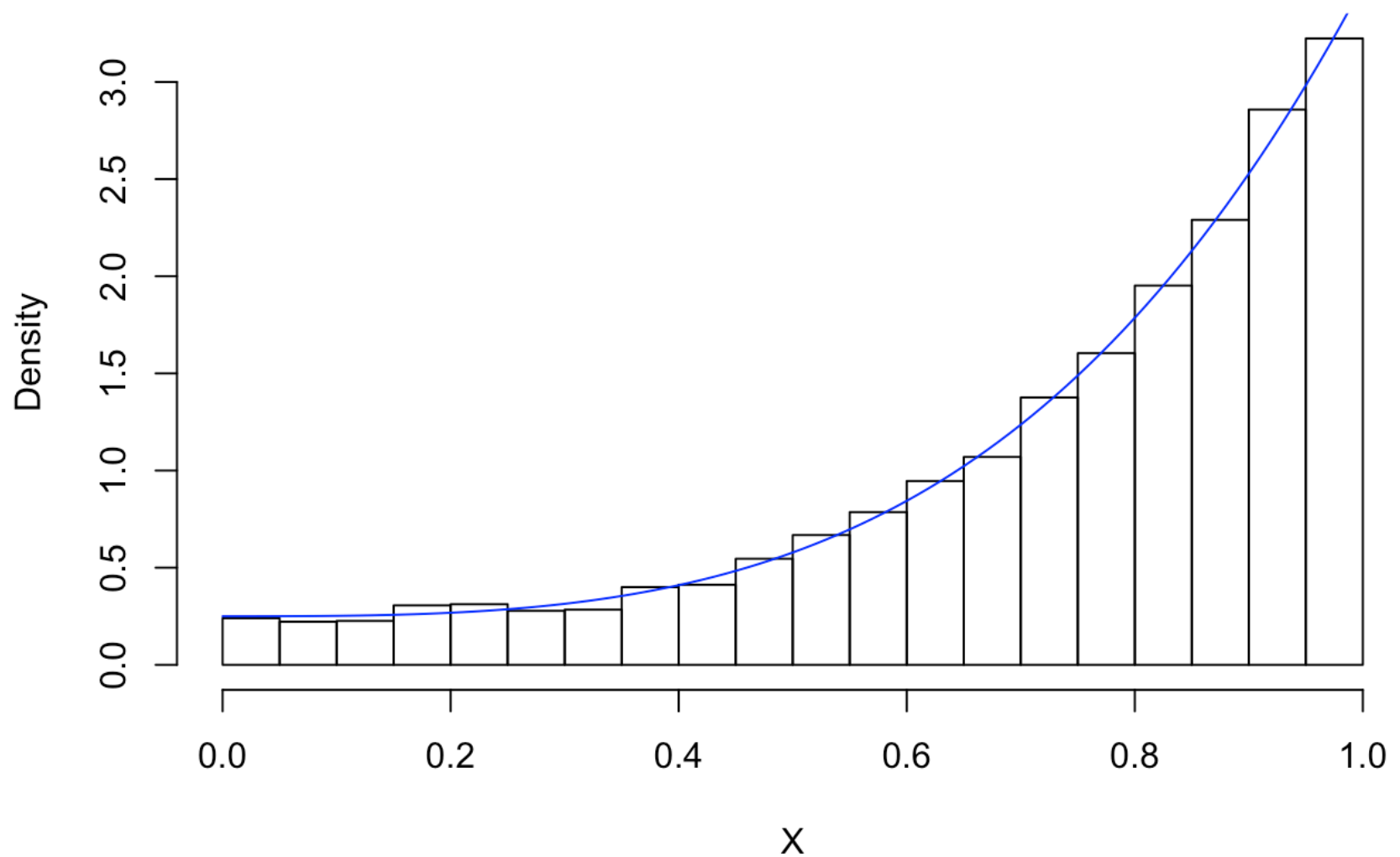# 5

(a)

```
set.seed(1)
sim_5_a=function(n){
  X=rep(NA,n)
  for(i in 1:n){
    U1=runif(1)
    U2=runif(1)
    if(U1<1/4){
      X[i]=U2
    }
    else if(U1<3/4){
      X[i]=(U2)^(1/4)
    }else{
      X[i]=(U2)^(1/5)
    }
  }
  return(X)
}
X=sim_5_a(10000)
hist(X,probability = T)
f=function(x){
  1/4+2*x^3+5/4*x^4
}
x=seq(0,1,0.01)
lines(x,f(x),col="blue")
```


Histogram of X
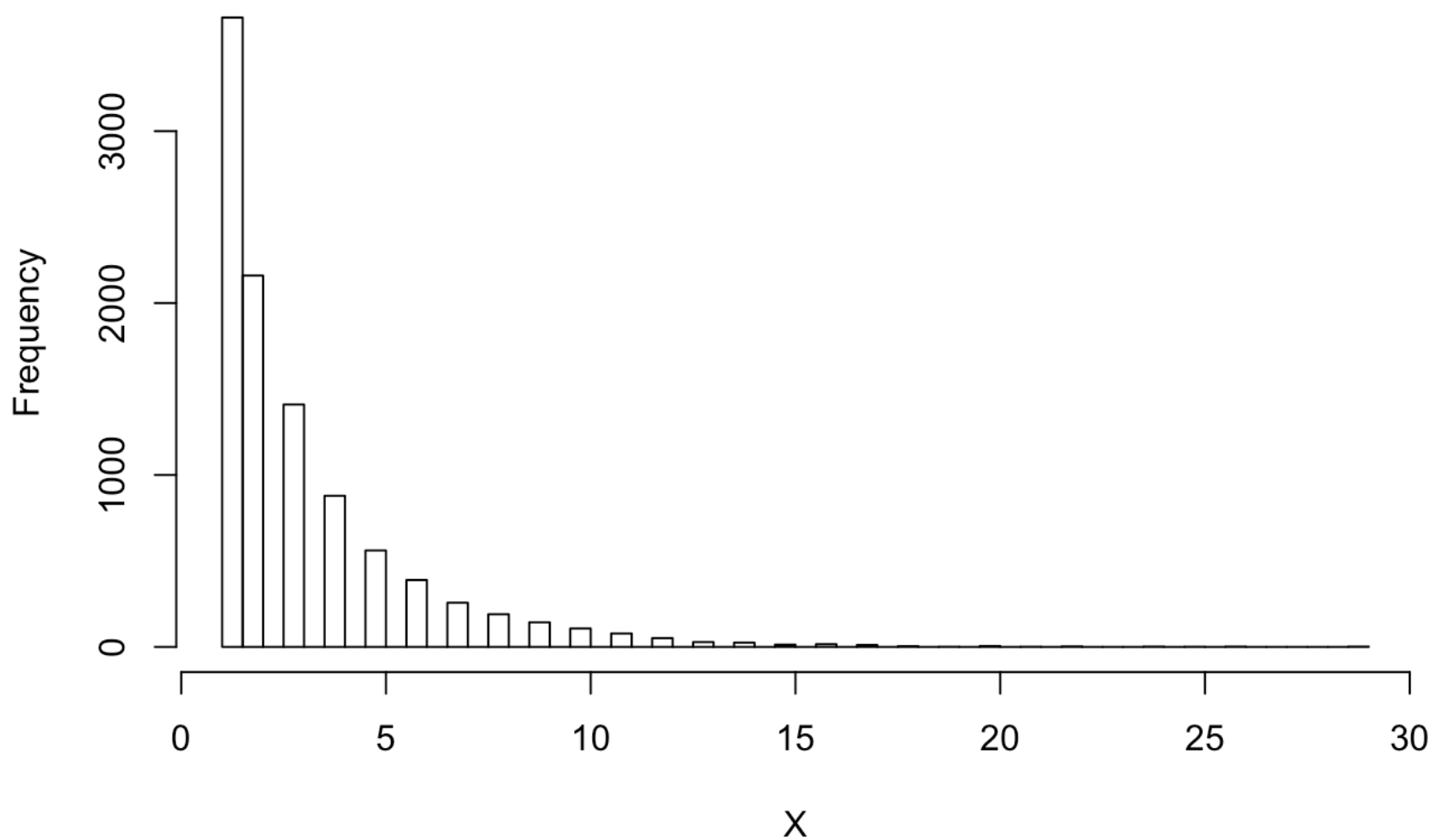
(b)

```
set.seed(1)

sim_5_b=function(n){
  X=rep(NA,n)
  for(i in 1:n){
    U1=runif(1)
    U2=runif(1)
    if(U1<1/3){
      X[i]=ceiling(log(U2)/log(1/2))
    }
    else if(U1<2/3){
      X[i]=ceiling(log(U2)/log(2/3))
    }else{
      X[i]=ceiling(log(U2)/log(3/4))
    }
  }
  return(X)
}
n=10000
X=sim_5_b(n)
hist(X,breaks=50)
```

Histogram of X


```
round(table(X)/10000,2)
```

```
## X
##    1    2    3    4    5    6    7    8    9   10   11   12   13   14   15
## 0.37 0.22 0.14 0.09 0.06 0.04 0.03 0.02 0.01 0.01 0.01 0.01 0.00 0.00 0.00
##   16   17   18   19   20   21   22   24   25   26   29
## 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
```

```
f=function(x){
   1/3*(1/2)^x+(2)^(x-1)/(3)^(x+1)+1/3*(3)^(x-1)/4^x
}
x=seq(1,30,1)
round(f(x),2)
```
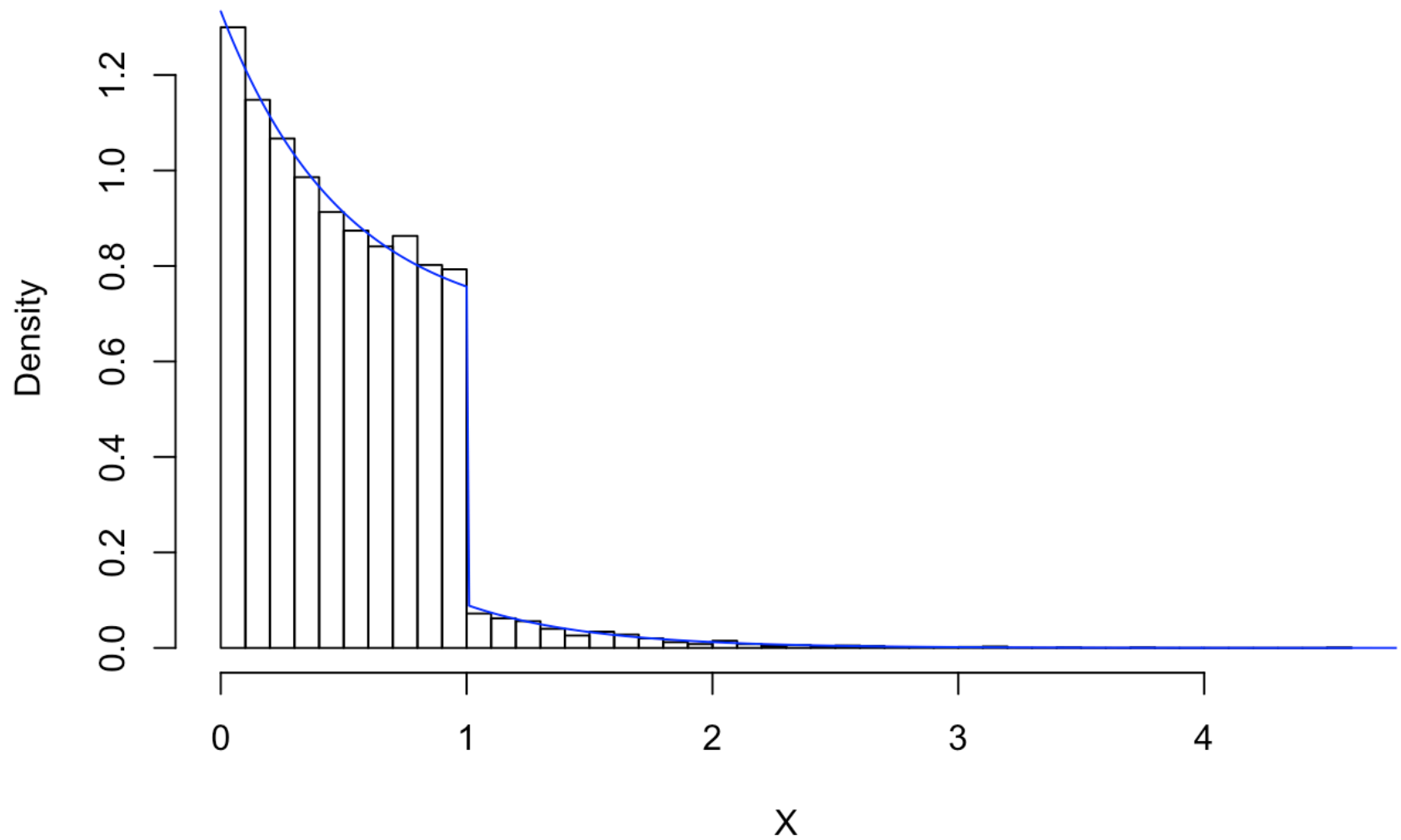
```
##  [1] 0.36 0.22 0.14 0.09 0.06 0.04 0.03 0.02 0.01 0.01 0.01 0.00 0.00 0.00
## [15] 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
## [29] 0.00 0.00
```

(c)

```
set.seed(1)
sim_5_c=function(n){
  X=rep(NA,n)
  for(i in 1:n){
    U1=runif(1)
    U2=runif(1)
    if(U1<1/3){
      X[i]=-log(U2)/2
    }
    else{
      X[i]=U2
    }
  }
  return(X)
}
n=10000
X=sim_5_c(n)
hist(X,breaks=50,probability = T)
x=seq(0,10,0.01)
f=function(x){
  ifelse(x<=1,(2*exp(-2*x)+2)/3,(2*exp(-2*x))/3)
}
lines(x,f(x),col="blue")
```
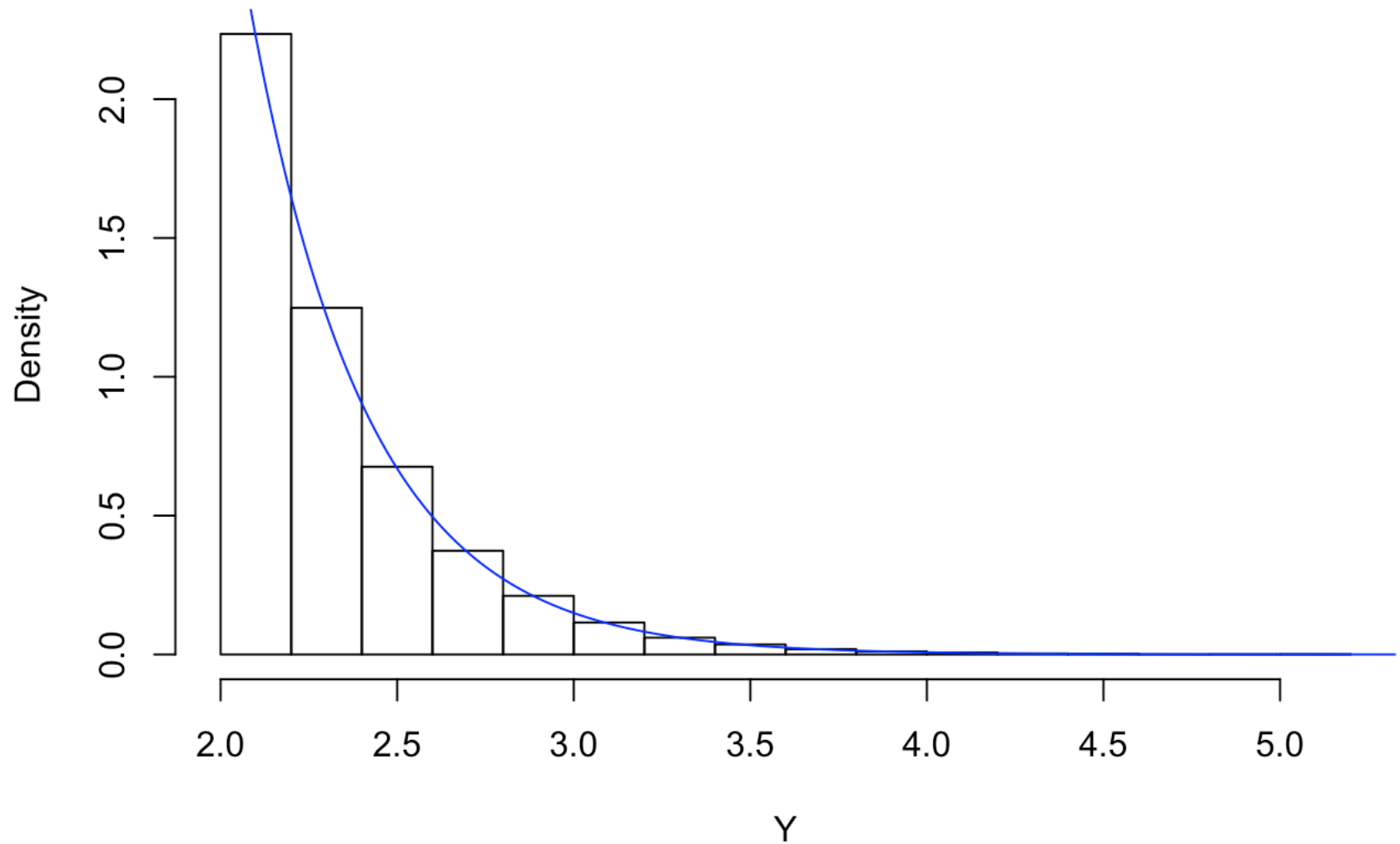
Histogram of X

7

(a)

(i)

```r
set.seed(1)
sim_7_a_i=function(n){
  Y=-log(runif(n))/3+2
  return(Y)
}


Y=sim_7_a_i(10000)
hist(Y,probability = T)
x=seq(2,10,0.01)
f=function(x){
  3*exp(-3*(x-2))
}
lines(x,f(x),col="blue")
```
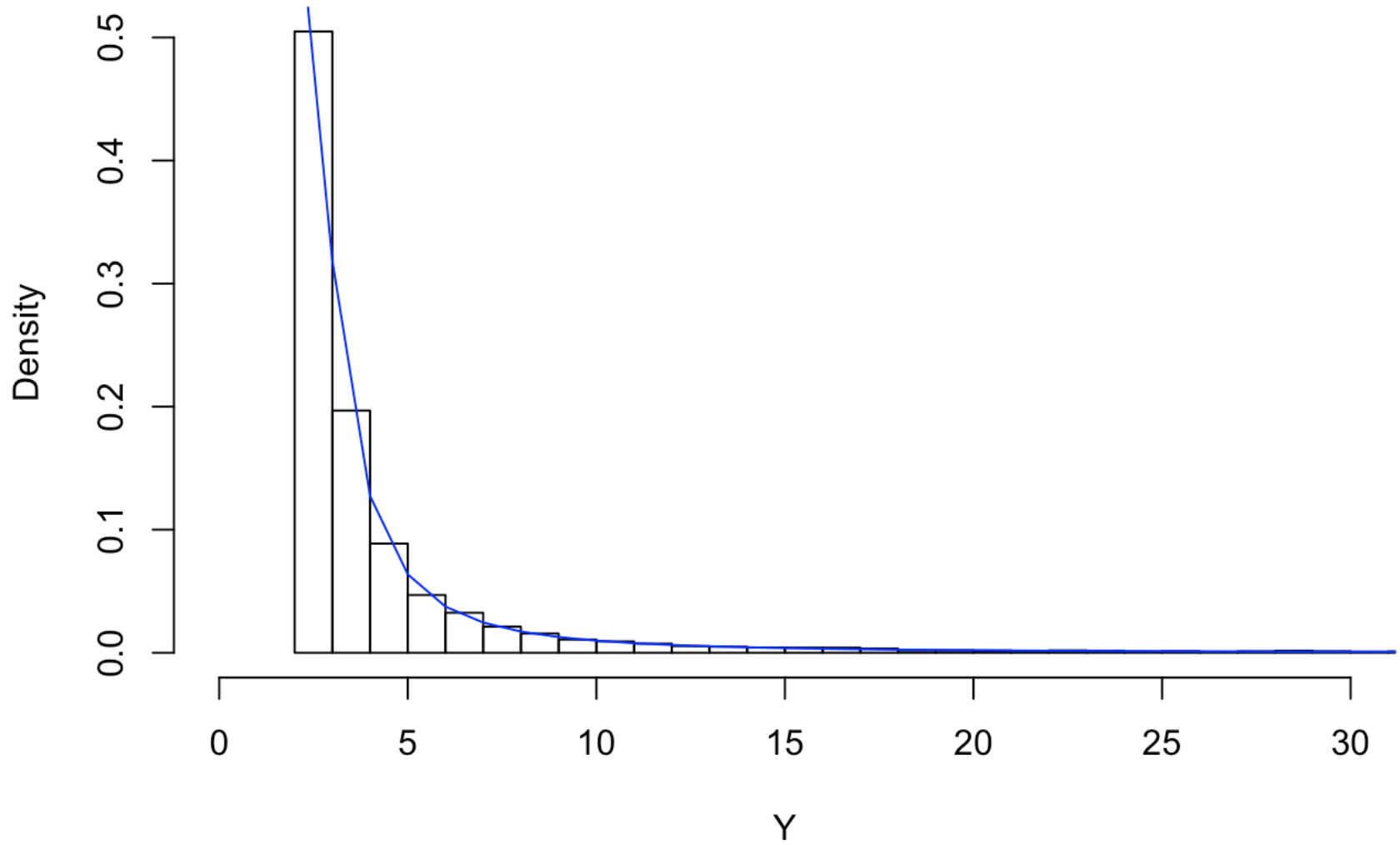
Histogram of Y


```
mean(Y)
```
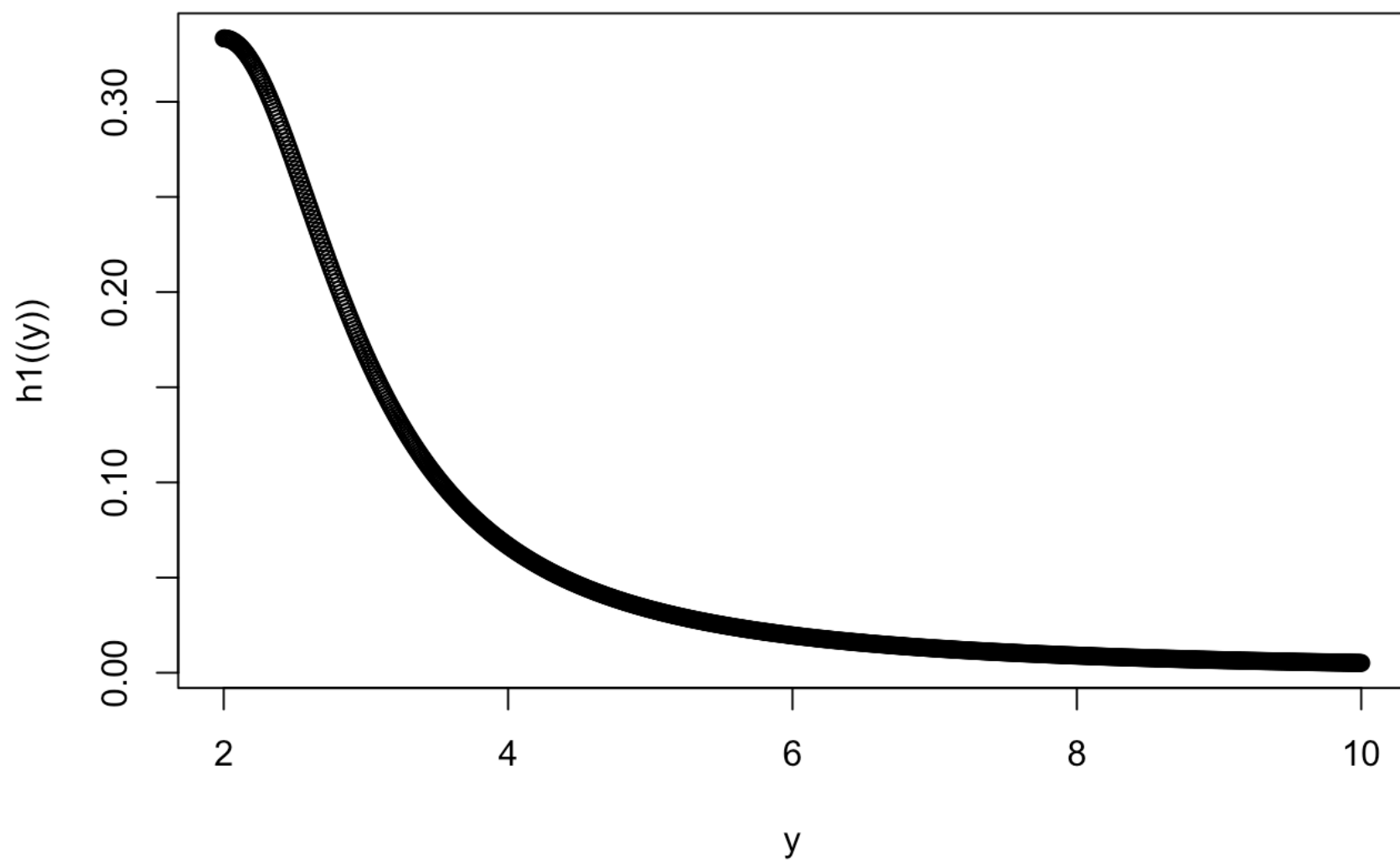
```
## [1] 2.335616
```

(ii)

```
set.seed(1)
sim_7_a_ii=function(n){
  Y=tan(runif(n)*3.14/2)+2
  return(Y)
}
Y=sim_7_a_ii(10000)
hist(Y,probability = T,breaks=1000,xlim=c(0,30))
x=seq(2,100,1)
f=function(x){
 2/3.14/(1+(x-2)^2)
}
lines(x,f(x),col="blue")
```
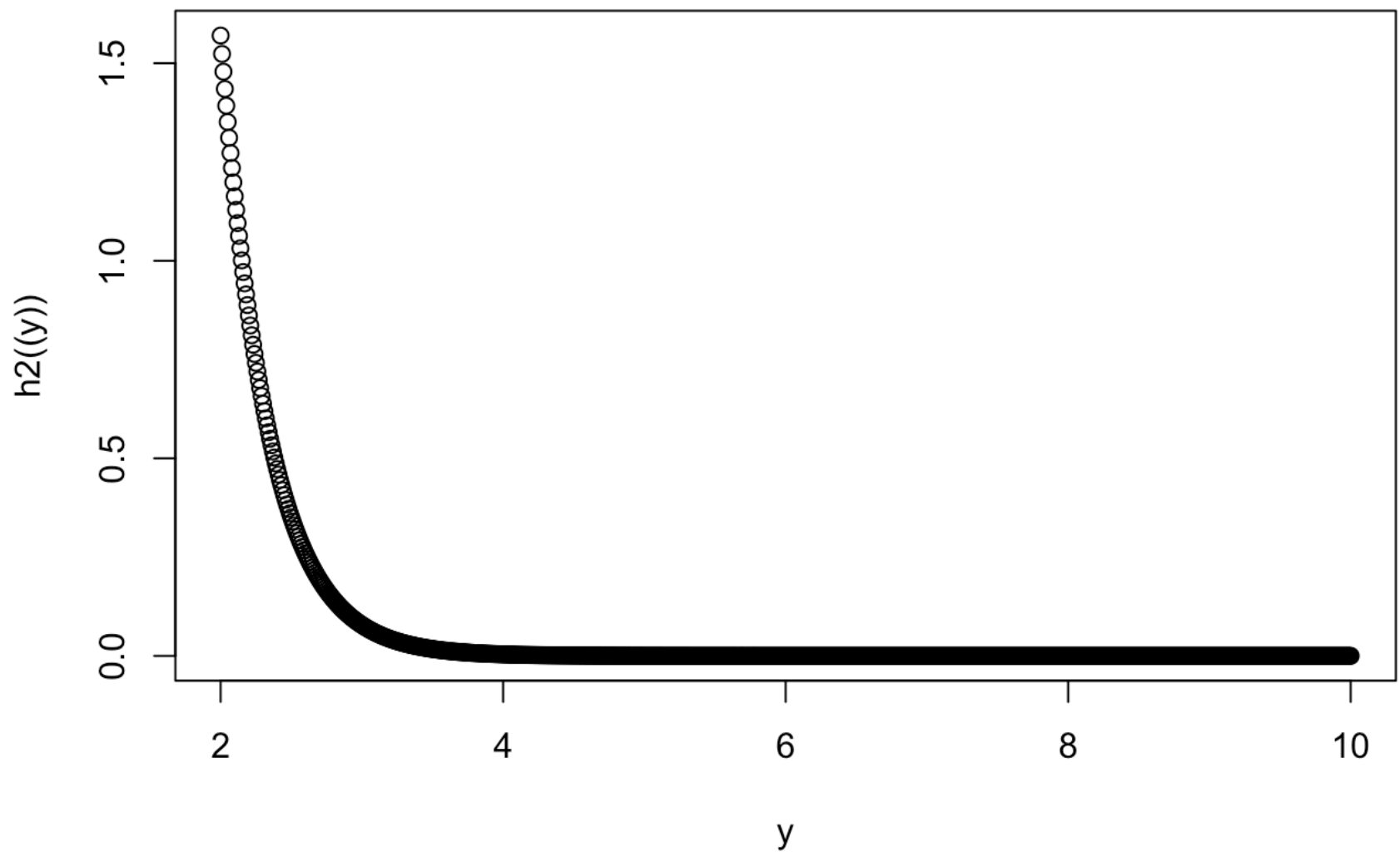
Histogram of Y

(b)

```
h1=function(x){
   1/(1+(x-2)^2)/(3)
}

h2=function(x){
   exp(-3*(x-2))/(2/3.14)
}
y=seq(2,10,0.01)

plot(y,h1((y)))
```

```
plot(y,h2((y)))
```

(c)

```
det.c1=optim(2,h1, lower = 2, upper = 10, method = "L-BFGS-B",control = list(fnsca
le = -1))
det.c2=optim(2,h2, lower = 2, upper = 10, method = "L-BFGS-B",control = list(fnsca
le = -1))
c1=det.c1$value
c2=det.c2$value
c1
```

```
## [1] 0.3333333
```

```
c2
```

```
## [1] 1.57
```

(d)

```
sim_7_d_i=function(n){
  X=rep(NA,n)
  iter=rep(NA,n)
  for(j in 1:n){
    U=runif(1)
    Y=sim_7_a_i(1)
    i=1
    while(U>h1(Y)/c1){
      U=runif(1)
      Y=sim_7_a_i(1)
      i=i+1
    }
    X[j]=Y
    iter[j]=i
  }
  return(list(X=X,iter=iter))
}
```

(e)

```
set.seed(1)
ans1=sim_7_d_i(10000)
X1=ans1$X
iter1=ans1$iter
mean(iter1)
```

```
## [1] 1.1447
```

```
approximate_k=mean(iter1)/c1

f=function(x){
  approximate_k*exp(-3*(x-2))/(1+(x-2)^2)
}
x=seq(2,20,0.01)
hist(X1,probability = T,breaks=100)

lines(x,f(x),col="blue")
```

Histogram of X1