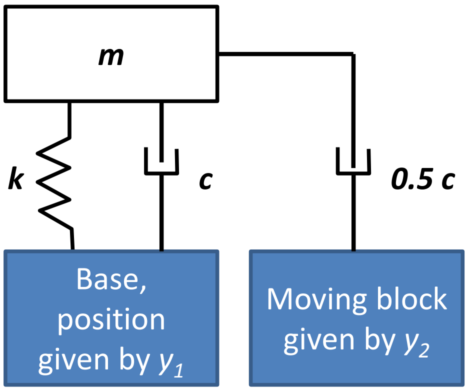
MAE 3134 – Linear System Dynamics

Homework # 6



Consider the system shown in the figure, with m = 5 Kg, k = 2 N/m and c = 2 Ns/m.

1. Derive the equation of motion of the mass around the static equilibrium position, taking the upward direction as the positive y-direction.

2. Derive the transfer function for the case when the position of the mass is the output and the <u>*position*</u> of the base, y_1 is the only input ($y_2 = 0$ in this case).

3. Derive the transfer function for the case when the position of the mass is the output and the <u>*velocity*</u> of the block, $\dot{y}_2$ is the only input ($y_1 = 0$ in this case).

4. Solve for the time-dependent position of the mass when the initial conditions with respect to the static equilibrium position are $y(0) = 1\,m$ and $\dot{y}(0) = -3\frac{m}{s}$, and the position of the base and the block are both kept fixed at $y_1(t) = y_2(t) = 0$.

5. Solve for the time-dependent position of the mass when the initial conditions with respect to the static equilibrium position are zero and the position of the base is prescribed as $y_1(t) = 2\,t$. Assume that the block is kept fixed at $y_2(t) = 0$. Solve this part of the problem <u>using the transfer function you found in part 2</u>.

6. Solve for the time-dependent position of the mass when the initial conditions with respect to the static equilibrium position are zero, the base is kept fixed at $y_1 = 0$, and the <u>*velocity*</u> of the block is prescribed as $\dot{y}_2(t) = \exp(-2t)$. Solve this part of the problem <u>using the transfer function you found in part 3</u>.

7. Solve for the time-dependent position of the mass when,
   a. The initial conditions with respect to the static equilibrium position are $y(0) = -5\,m$ and $\dot{y}(0) = +15\frac{m}{s}$,
   b. The position of the base is prescribed as $y_1(t) = 0.5(t-1)u(t-1)$, and
   c. The velocity of the block is prescribed as $\dot{y}_2(t) = 8\exp(-2t)$.

8. Assuming that the initial conditions are zero and the base is moving according to $y_1 = A\cos(\omega t)$. Provide an equation that describes the motion of the block, $y_2(t)$, such that the mass does not move at all. That is, what should be the motion of the block such that its effect completely cancels out the effect of the motion of the base? (*hint:* if the position of the mass is denoted by $y(t)$ and the mass does not move, then $y(t) = 0$. If $y(t)$ is always zero, then its Laplace transform is also zero).