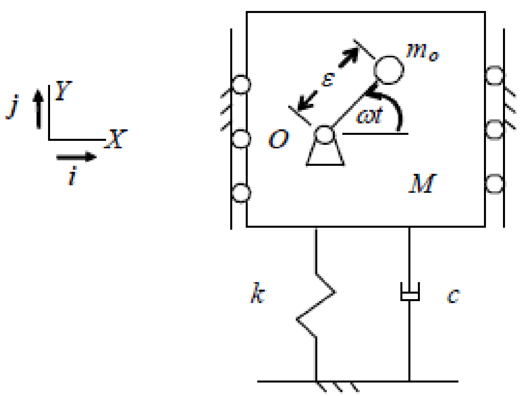
The George Washington University
MAE 3134 – Linear System Dynamics
Spring 2015

Homework # 1

Due on Thursday, January 29th at the beginning of lecture

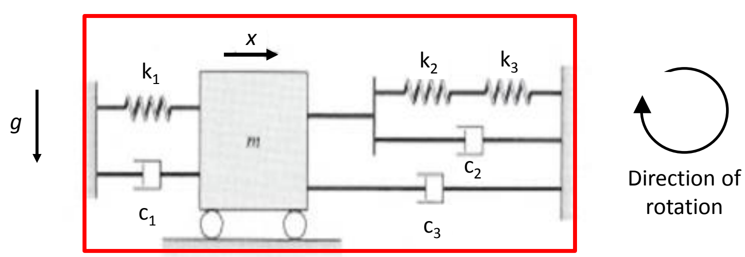
Problem # 1

Determine the equation of motion of the system shown in the figure, for motion in the *Y*-direction. The system consists of a washing machine resting on a spring-damper combination. Inside the washer, there is an imbalance consisting of a small mass m_o which spins with angular velocity ω in the direction shown. The radius of rotation is ε. The total mass of the system is m_o + M (in other words, *M* does *not* include the imbalance mass). Gravity acts in the negative *Y*-direction.



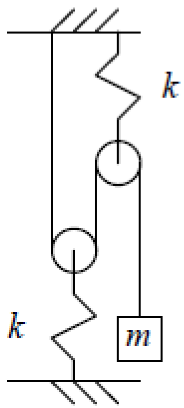
Problem # 2

(a) Derive the equation of motion of the system shown in the figure. The mass can move only in the direction indicated as *x,* which is fixed with respect to the red box, but the entire red box rotates in the direction indicated with respect to the gravity vector (the gravity vector is fixed) with angular velocity ω. The initial position at time zero is as shown in the figure. (b) Provide expressions for the equivalent stiffness and equivalent damping coefficient. Neglect any centrifugal forces due to rotation.



Problem # 3

(a) Derive the equation of motion of the system shown in the figure. (b) What is the equivalent stiffness? (c) What is the natural frequency?



MAE 3134
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HOMEWORK 1 - SOLUTION

PROBLEM 1

The position of m_0 with respect to a fixed reference frame in the y direction is $\epsilon \sin \omega t + y$ where y is the vertical position of the washer. The acceleration acting on m_0 is then

$$\frac{d^2}{dt^2}(\epsilon \sin \omega t + y) = -\epsilon\omega^2 \sin \omega t + \ddot{y}$$

The force acting on m_0 is $m_0(-\epsilon\omega^2 \sin \omega t + \ddot{y})$, so the reaction force transmitted to the washer is:

$$-m_0(-\epsilon\omega^2 \sin \omega t + \ddot{y}) = m_0(\epsilon\omega^2 \sin \omega t - \ddot{y})$$

The weight will be included when we set up the equation of motion next:

$$\sum F_M = M\ddot{y} = -\overbrace{Mg}^{\text{WEIGHT}} - m_0 g + m_0(\epsilon\omega^2 \sin \omega t - \ddot{y}) - c\dot{y} - ky$$

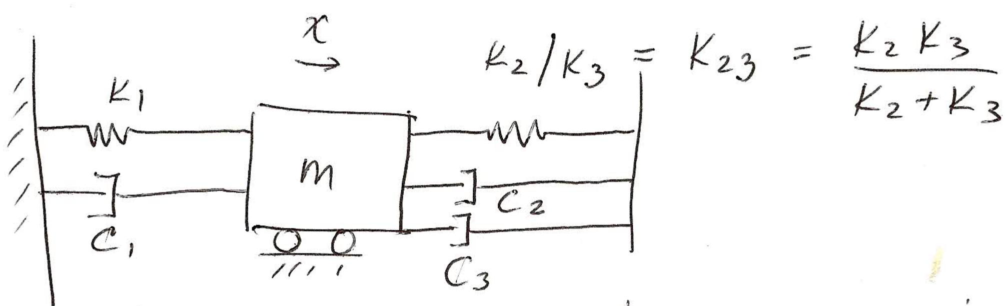
for the large mass

$$\Rightarrow M\ddot{y} = -(M+m_0)g + m_0\epsilon\omega^2 \sin \omega t - m_0\ddot{y} - c\dot{y} - ky$$

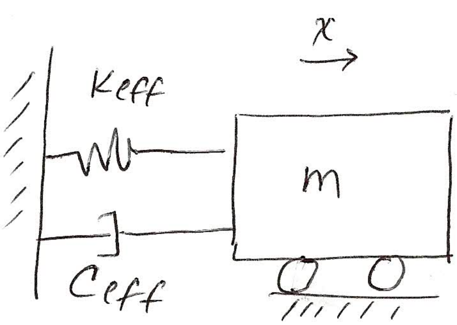
$$\Rightarrow \boxed{(M+m_0)\ddot{y} + c\dot{y} + ky = -(M+m_0)g + \epsilon m_0 \omega^2 \sin \omega t}$$

PROBLEM 2

In this problem it is easier to first calculate the effective stiffness and damping coefficient. We can first compute an effective stiffness for k_2 and k_3 as $k_{23} = \dfrac{k_2 k_3}{k_2 + k_3}$ because these springs are in series. Next we rewrite our system as:



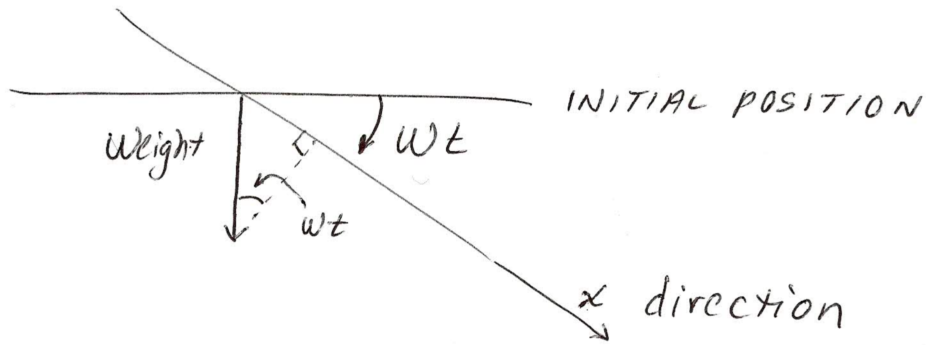
We see that all other elements are in parallel, so we add the constants:



$$\boxed{\begin{aligned} k_{eff} &= k_1 + \frac{k_2 k_3}{k_2 + k_3} \\ c_{eff} &= c_1 + c_2 + c_3 \end{aligned}}$$

(answer to (b))

Now we have to consider the direction of the weight with respect to the direction x. We are told that the system rotates with angular velocity ω:



We see that the component of the weight acting in the x direction is $mg \sin \omega t$.

Thus, our equation of motion is

$$\sum F_x = m\ddot{x} = -K_{eff}\, x - C_{eff}\, \dot{x} + mg \sin \omega t$$

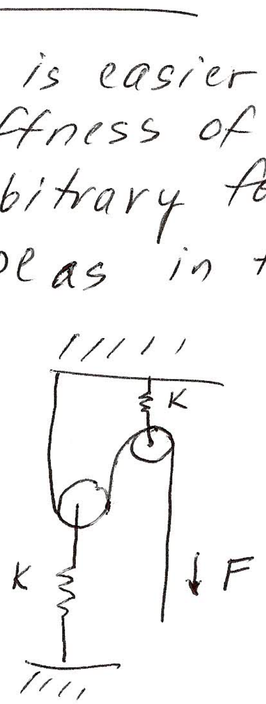
$$\Rightarrow m\ddot{x} + C_{eff}\, \dot{x} + K_{eff}\, x = mg \sin \omega t$$

Substituting C_{eff} and K_{eff} we obtain finally,

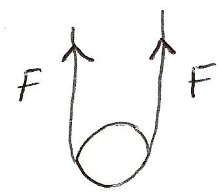
$$\boxed{m\ddot{x} + (C_1 + C_2 + C_3)\dot{x} + \left(K_1 + \frac{K_2 K_3}{K_2 + K_3}\right)x = mg \sin \omega t}$$

PROBLEM 3

It is easier to first calculate the effective stiffness of the system. If we have an arbitrary force F acting at the end of the rope as in the diagram we can see that



the force acting on each pulley will be equal to $2F$:



This is because the force is the same along the rope. Since each pulley is held in place by a spring of stiffness k, then the displacement, according to Hooke's law will be,

$$\text{displacement} = \frac{\text{Force}}{\text{stifness}} = \frac{2F}{k}$$

Since we have two pulleys and the rope displacement is twice the displacement of each pulley, the total rope displacement for the rope is

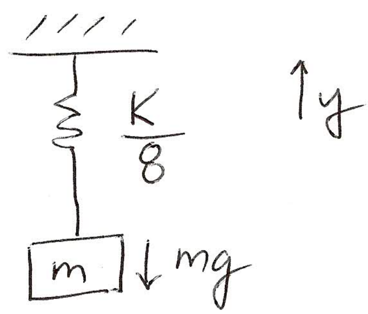
$$\underbrace{2 \times \frac{2F}{K}}_{\text{Twice the displacement of the pulley}} \times \underbrace{2}_{\text{Accounting for two pulleys}} = 8\frac{F}{K}$$

Thus, we see that application of a force F at the end of the rope results in a total displacement of $8\frac{F}{K}$.

The effective stiffness is equal to total force divided by total displacement,

$$K_{eff} = \frac{F}{\left(8\frac{F}{K}\right)} = \frac{K}{8}$$

This is the stiffness that the mass m experiences, and is the answer to part (b). We can now redraw our system as follows:



The equation of motion is then (answer to (a)).

$$\boxed{m\ddot{y} = -\frac{K}{8}y - mg}$$

The natural frequency is $\omega = \sqrt{\frac{K_{eff}}{m}} = \sqrt{\frac{K}{8m}}$

$$\Rightarrow \boxed{\omega = \frac{1}{2}\sqrt{\frac{K}{2m}}}$$