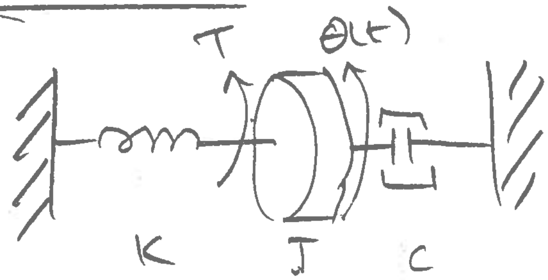
REVIEW

- WE CAN QUALITATIVELY DETERMINE THE RESPONSE BASED ON THE TRANSFER FUNCTION
- WE HAVE PERFORMANCE SPECIFICATIONS FOR FIRST + SECOND ORDER SYSTEMS
- CAN QUICKLY ANALYZE A SYSTEM WITHOUT TAKING THE INVERSE LAPLACE + SOLVING

EXAMPLE



$5 \frac{Nm}{RAD}$

FIND J, C TO YIELD → A 20% OS $T_s = 2$ sec FOR A STEP INPUT

$$J\ddot{\theta} = -K\theta - C\dot{\theta} + T \rightarrow \ddot{\theta} + \frac{C}{J}\dot{\theta} + \frac{K}{J}\theta = \frac{T(t)}{J}$$

$$G(s) = \frac{\theta(s)}{T(s)} = \frac{1/J}{s^2 + \frac{C}{J}s + \frac{K}{J}} \Rightarrow \boxed{\begin{aligned} \omega_n^2 &= \frac{K}{J} \\ 2\zeta\omega_n &= \frac{C}{J} \end{aligned}}$$

$$T_s = 2 = \frac{4}{\zeta\omega_n} \Rightarrow \frac{C}{J} = 4$$

$$\boxed{\zeta\omega_n = 2}$$

ALSO $\zeta = \frac{4}{2\omega_n} = 2\sqrt{\frac{J}{K}}$

20% OS → $\zeta = 0.456$

[illegible]

ADDITIONAL POLES

- PREVIOUS RELATIONSHIPS ARE ONLY VALID FOR SECOND ORDER SYSTEMS WITH COMPLEX POLES + NO ZEROS!

- IN CERTAIN CASES WE CAN APPROXIMATE HIGHER ORDER SYSTEMS AS A DOMINANT SECOND ORDER SYS.

EXTRA POLE

$$G(s) = \frac{K\omega_n^2}{(s+a_r)(s^2+2\xi\omega_n s+\omega_n^2)}$$

STEP RESPONSE (GENERAL)

$$C(s) = \frac{A}{s} + \frac{B(s+\xi\omega_n)+C\omega_d}{(s+\xi\omega_n)^2+\omega_d^2} + \frac{D}{s+a_r}$$

$\downarrow$

OUTPUT IN TIME DOMAIN

$$c(t) = \underbrace{A\,u(t)}_{\text{STEP}} + \underbrace{e^{-\xi\omega_n t}(B\cos\omega_d t + C\sin\omega_d t)}_{\text{SECOND ORDER}} + \underbrace{De^{-a_r t}}_{\text{THIRD}}$$

$t \geq 0$

STEP RESP.

- IF $a_r \gg \xi\omega_n$ THEN $De^{-a_r t}$ WILL DIE OUT QUICKLY $\longrightarrow$ WON'T REALLY AFFECT OUTPUT

$\boxed{a_r\ 5\text{x. GREATER THAN } \xi\omega_n}$

- IF σ_r IS 5X GREATER (FURTHER LEFT) OF DOMINANT SECOND ORDER ($\zeta\omega_n$) THEN WE CAN APPROXIMATE AS A 2ND ORDER SYS.
- EFFECT BECOMES NEGLIBLE THE LARGER $\underline{\sigma_r}$ BECOMES
- EXAMPLE

$$T_1 = \frac{24.542}{s^2+4s+24.542} \qquad \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$$T_2 = \frac{245.42}{(s+10)(s^2+4s+24.542)} \qquad \underline{\text{DC GAIN}} = 1$$

$$T_3 = \frac{73.626}{(s+3)(s^2+4s+24.542)}$$

OUTPUT RESPONSES ⟶ GOOD PRACTICE !

$$c_1(t) = 1 - 1.09e^{-2t}\cos(4.532t - 23.8°)$$

$$c_2(t) = 1 - 0.29e^{-10t} - 1.189e^{-2t}\cos(4.532t - 53.34°)$$

$$c_3(t) = 1 - 1.14e^{-3t} + 0.707e^{-2t}\cos(4.532t + 78.63°)$$

CONVERT TO RADIANS!

C_2 IS CLOSER TO C_1 ⟶ VALID TO APPROXIMATE

C_3 POLE IS TOO CLOSE. (< 5X)

EXAMPLE DETERMINE IF SECOND ORDER APPROX IS VALID.

$$G(s) = \frac{700}{(s+15)(s^2+4s+100)} \qquad -2 \pm 3j$$

VALID

$$G(s) = \frac{360}{(s+4)(s^2+2s+90)}$$

NOT VALID.

$$\approx -1 \pm 9j$$

$$s^2 + 4s + 100$$

$$s^2 + 2\zeta\omega_n s + \omega_n^2$$

$$\boxed{s = -\zeta\omega_n \pm \sqrt{\omega_n^2 - 1}}$$

ADDITIONAL ZEROS

- CONSIDER A TRANSFER FUNCTION $T(s)$

$$T(s) = \frac{1}{(s+a)(s+b)} = \frac{C(s)}{R(s)}$$

NOW ADD A ZERO TO OUTPUT

$$(s+z)\,C(s) = \underbrace{s\,C(s)}_{\text{DERIVATIVE OF ORIGINAL RESPONSE}} + \underbrace{z\,C(s)}_{\text{SCALED RESPONSE}}$$

← ORIGINAL OUTPUT

- IF z IS LARGE THEN OUTPUT IS SIMPLY SCALED VERSION OF ORIGINAL OUTPUT.
- IF z IS SMALL THEN ADDITIONAL DERIVATIVE TERM BECOMES IMPORTANT.
- DERIVATIVE (FOR z IN LHP) IS TYPICALLY POSITIVE → MORE OVERSHOOT FOR 2nd ORDER SYSTEMS

  INCREASING DERIVATIVE.
- ZERO IN RHP → NEGATIVE DERIVATIVE

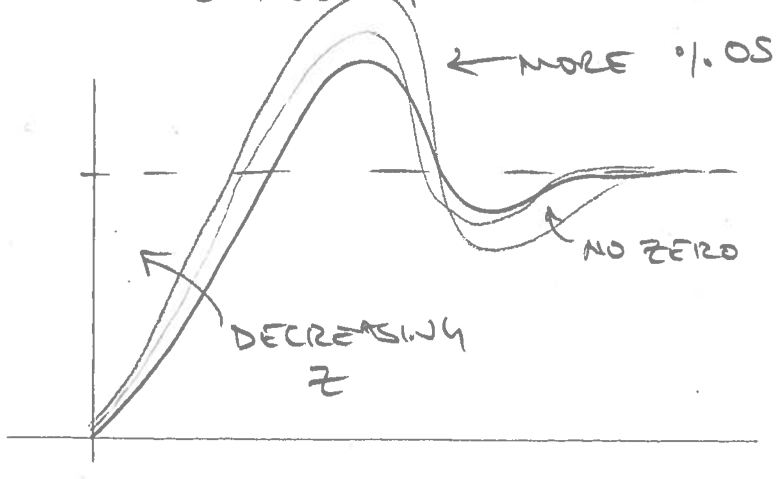
  → NON-MINIMUM PHASE SYSTEM. ←

ZERO

$$T(s) = \frac{9/z\,(s+z)}{s^2+2s+9}$$

LHP ZEROS

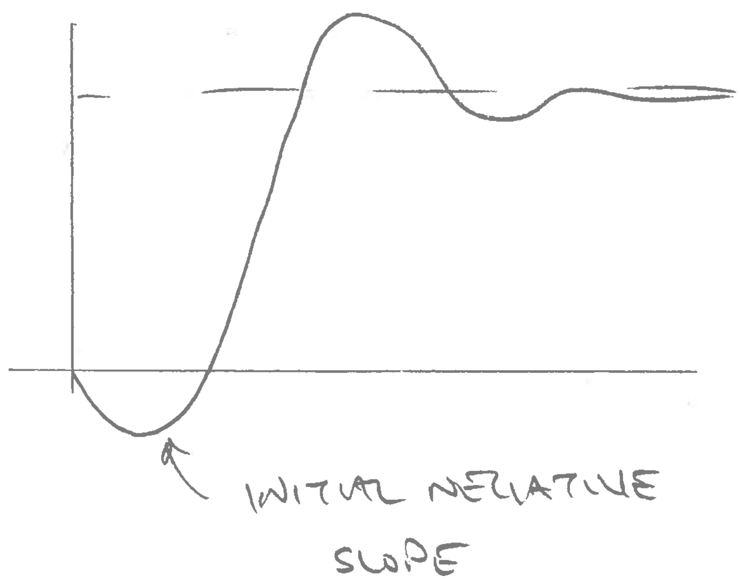
$z = 3,\ 5,\ 10$



$z \rightarrow \quad \%OS \uparrow$

NON-MINIMUM PHASE $\quad z$ IN RHP

SYSTEM INITIALLY MOVES IN OPPOSITE DIRECTION!



RHP.

$$T(s) = \frac{(s+z)}{s^2+2s+9}$$

$z = -2$

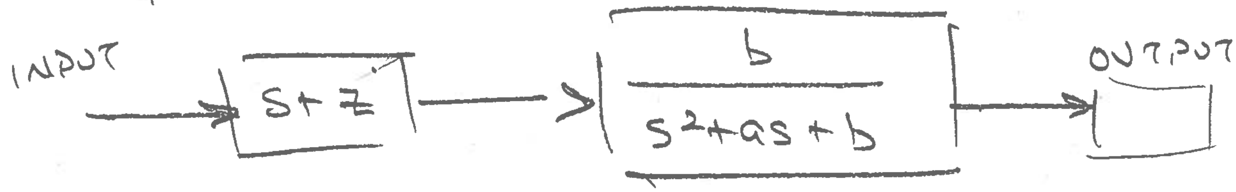
$(s-2)$

- AIRPLANE INITIALLY PITCHES DOWN BEFORE PITCHING UP
- MOTORCYCLE TURNS LEFT BEFORE TURNING RIGHT.

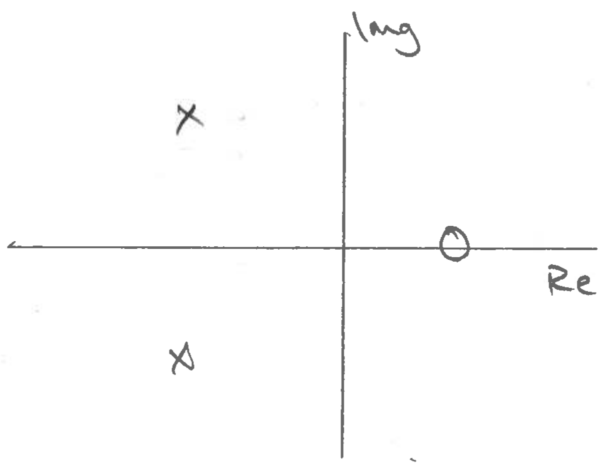
TRYING TO CANCEL POLES + ZEROS

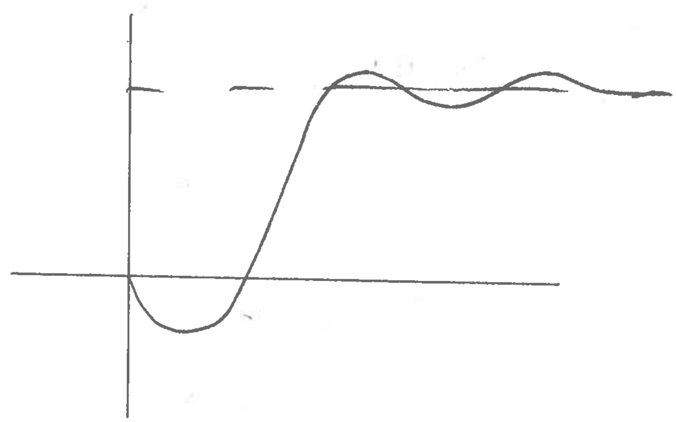
THIS WAS A JOB INTERVIEW QUESTION.

GIVEN: A SYSTEM

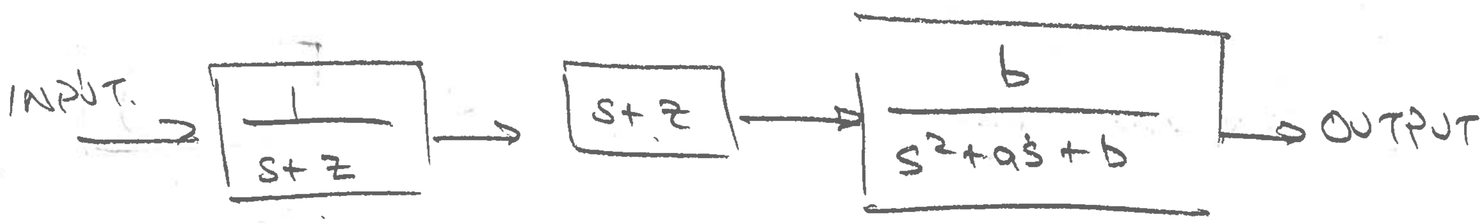


LETS ASSUME THAT THE ZERO IS NON MINIMUM PHASE (RHP) + THE SECOND ORDER SYSTEM IS STABLE.





WE DON'T LIKE THIS BEHAVIOR SO WE ADD A CONTROLLER TO CANCEL THE ZERO



$$G(s) = \frac{\text{OUTPUT}}{\text{INPUT}} = \frac{\cancel{(s+z)}\cdot b}{\cancel{(s+z)}(s^2+as+b)}$$

WILL THIS WORK ? REAL WORLD

- POLE | ZERO CANCELLATION ONLY REALLY WORKS IN THEORY
- REAL WORLD CAUSES POLES | ZEROS TO NOT BE EXACTLY AS PREDICTED
- NOW YOU HAVE A RHP POLE !!