FREQUENCY RESPONSE CH 9 | CH 11 4/2

- DEVELOPED IN THE 1930s BY NYQUIST | BODE
- ANOTHER WAY TO VIEW + ANALYZE LINEAR SYSTEMS.
  - USEFUL FOR PHYSICAL SYSTEMS. ←
  - FINDING STABILITY FOR NONLINEAR SYSTEMS.

FREQ. RESPONSE CONCEPT

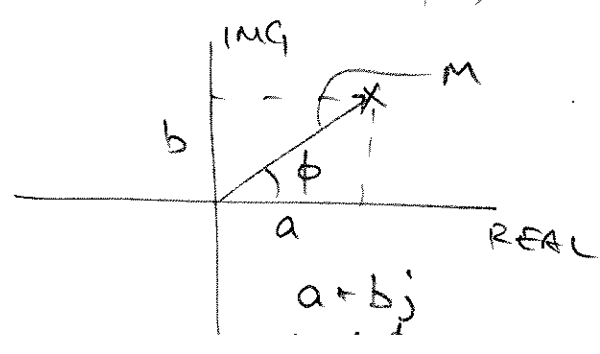
- AT STEADY STATE - SINUSOIDAL INPUT $\xrightarrow{\text{SYS}}$ SINUSOIDAL OUTPUT.
- SAME FREQUENCY BUT DIFFERENCES IN PHASE + AMPLITUDE
- BOTH DIFFERENCES ARE FUNCTIONS OF FREQ.
- WE CAN THINK OF SINUSOIDS AS COMPLEX NUMBERS.

MAGNITUDE ⟶ AMPLITUDE

ANGLE ⟶ PHASE

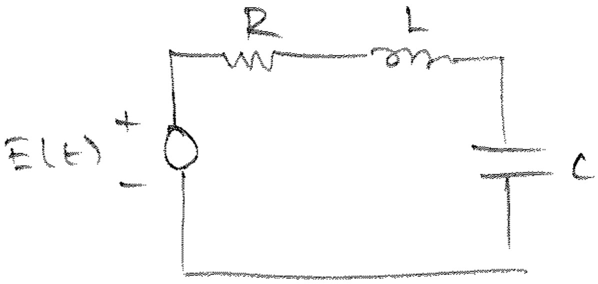
$$M_1 \angle \phi_1 \longrightarrow M_1 \cos(\omega t + \phi_1)$$

FREQUENCY ω IS IMPLICIT



- THE SYSTEM WILL CHANGE THE OUTPUT MAGNITUDE | PHASE $\longrightarrow$ ALSO TREAT AS A COMPLEX NUMBER

ELECTRICAL SYSTEM



SINUSOIDAL INPUT

$E(t) = M_i \cos(\omega t + \phi)$

$\longrightarrow L\ddot{q} + R\dot{q} + \frac{1}{C} q = E(t).$

CAN SOLVE FOR OUTPUT $q(t) = M_o \cos(\omega t + \phi_o)$

$$M_i(\omega) \angle \phi_i(\omega) \longrightarrow \overset{TF}{\boxed{M(\omega) \angle \phi(\omega)}} \longrightarrow M_o(\omega) \angle \phi_o(\omega)$$

COMPLEX NUMBER MULTIPLICATION.

$$M_o(\omega) \angle \phi_o(\omega) = M_i(\omega) M(\omega) \angle \left[\phi_i(\omega) + \phi(\omega) \right]$$

TRANSFORMATION CAUSED BY THE SYSTEM

$$M(\omega) = \frac{M_o(\omega)}{M_i(\omega)} \qquad \phi(\omega) = \phi_o(\omega) - \phi_i(\omega)$$

SYSTEM MAGNITUDE FREQ RESPONSE

SYSTEM PHASE FREQ. RESP.

$$\boxed{M(\omega) \angle \phi(\omega)} \longleftarrow \text{FREQ RESPONSE OF SYSTEM}$$

FREQUENCY RESPONSE OF A SYS. DEFINED BY THE TRANSFER FCN $G(s)$

$$G(j\omega) = G(s)\Big|_{s=j\omega} \longleftarrow \text{FREQ. RESP. FCN.}$$

$$G(j\omega) = M_G(\omega) \angle \phi_G(\omega)$$

TWO COMMON WAYS TO VISUALIZE (PLOTS)

BODE ⟹ 1. FCN OF ω - TWO PLOTS (MAG + PHASE)

NYQUIST 2. POLAR PLOT - MAG + ANGLE.

- WE'LL LOOK AT APPROACH 1.

SEPERATE MAGNITUDE + PHASE PLOTS

MAG IS PLOTTED IN DECIBELS (dB) vs. $\log \omega$

$$dB = 20 \log_{10} M_G$$

PHASE PLOTTED AS ANGLE vs. $\log \omega$

20 Hz ⟶ 20,000 Hz

WE HAVE TO NOW REVIEW LOGARITHMS.

$y = \log_b x \iff b^y = x$ b - BASE

IF $b = 10 \rightarrow$ COMMON (LOG) ← WE'LL USE THIS

IF $b = e \rightarrow$ NATURAL LOG (ln)

LOGARITHMS TURN MULTIPLICATION ⟶ ADDITION
DIVISION ⟶ SUBTRACTION.

EX

$16 \times 8 = 2^4 \times 2^3 = 2^7 = 128$ $16 = 2^4$

$16 \div 8 = 2^4 \div 2^3 = 2^1 = 2$ $8 = 2^3$

PROPERTIES

$\log_b (mn) = \log_b m + \log_b n$

$\log_b \left(\frac{m}{n}\right) = \log_b m - \log_b n$

$\log_b m^r = r \log_b m$

$\log_b b = 1 \quad b^1 = b$

$\log_b 1 = 0 \quad b^0 = 1$

COMMON VALUES

$6\,db \approx 20 \log 2$ $\frac{MO}{Mi} = MG$ $\rightarrow$ 6 dB ⟶ DOUBLE POWER

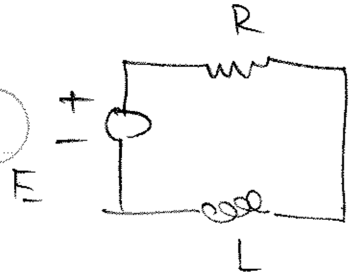
$20\,db \approx 20 \log 10$

$40\,db \approx 20 \log 100$

} DOUBLE dB ⟶ 10 X POWER

EXAMPLE — FIND FREQ. RESP.

— PLOT MAG + PHASE FOR VARIOUS FREQUENCIES (ω)



$$E - iR - L\frac{di}{dt} = 0 \qquad L = 1\ H$$

$$R = 2\ \Omega$$

$$L\frac{di}{dt} + Ri = E(t)$$

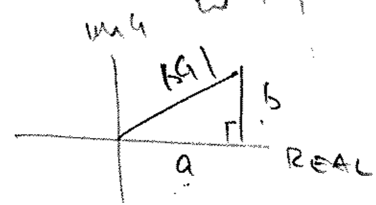
LAPLACE TRANSFORM $\quad G(s) = \dfrac{I(s)}{E(s)} = \dfrac{1}{s+2}$

FREQ. RESP. $\quad G(j\omega) = \dfrac{1}{j\omega+2} = \dfrac{1}{2+j\omega}$

$$\frac{1}{2+j\omega}\cdot\frac{2-j\omega}{2-j\omega} = \frac{2-j\omega}{4+\omega^2} \Rightarrow \boxed{G(j\omega) = \frac{2-j\omega}{\omega^2+4}} = a+bj$$

$$M(\omega) = |G(j\omega)| = \sqrt{\text{REAL}^2 + \text{IMG}^2} \qquad G(j\omega) = \frac{2}{\omega^2+4} + j\,\frac{-\omega}{\omega^2+4}$$

$$\phi(\omega) = \angle G(j\omega) = \tan^{-1}\frac{\text{IMG}}{\text{REAL}}$$



$$M(\omega) = \sqrt{\left(\frac{2}{\omega^2+4}\right)^2 + \left(\frac{-\omega}{\omega^2+4}\right)^2} = \boxed{\frac{1}{\sqrt{\omega^2+4}} = M(\omega)} \quad \text{RATIO } \frac{M_o}{M_i}$$

$$\phi(\omega) = \tan^{-1}\frac{-\omega/\omega^2+4}{2/\omega^2+4} = \tan^{-1}\frac{-\omega}{2} = \boxed{-\tan^{-1}\frac{\omega}{2} = \phi(\omega)}$$

$$\phi = \phi_o - \phi_i$$

PLOT $M(\omega)$ + $\phi(\omega)$ FOR VALUES OF ω

dB

(BODE PLOT)

$$M(\omega) = \frac{1}{\sqrt{\omega^2+4}} \qquad \phi(\omega) = -\tan^{-1}\frac{\omega}{2}$$

ω (RAD/SEC)	$M(\omega)$ (dB)	ϕ (RAD/deg)
0.1	−6 dB	−2.86
1.0	−6.98 dB	−26.56
10	−20 dB	−78.69
100	−40 dB	−88.85

SMALLER

$$M_{dB} = 20 \log M(\omega)$$

$$M(\omega) = \frac{M_o}{M_i} \qquad \phi = \phi_o - \phi_i$$

$$i(t) = M(\omega) M_i \sin\left(\omega t + (\phi + \phi_i)\right)$$

- RESONANCE
- NATURAL FREQUENCY
- HIGH/LOW PASS FILTER