MAE3134: Homework 5

Due date: 29 March 2018

Problem 1. For each of the electrical systems below, find the state space representation.

(a) There is a single voltage source and the output is the voltage difference across the capacitor C_1.

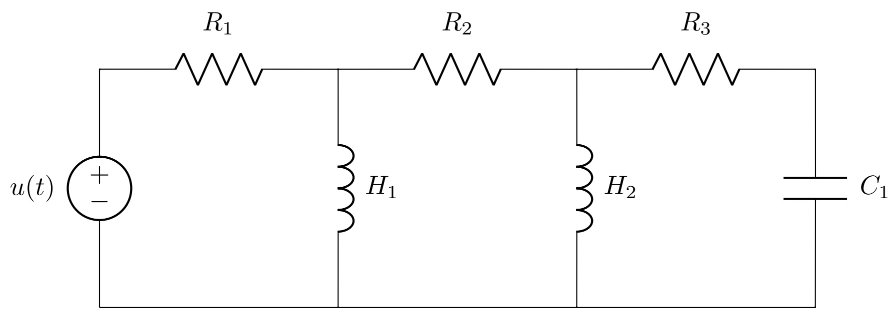
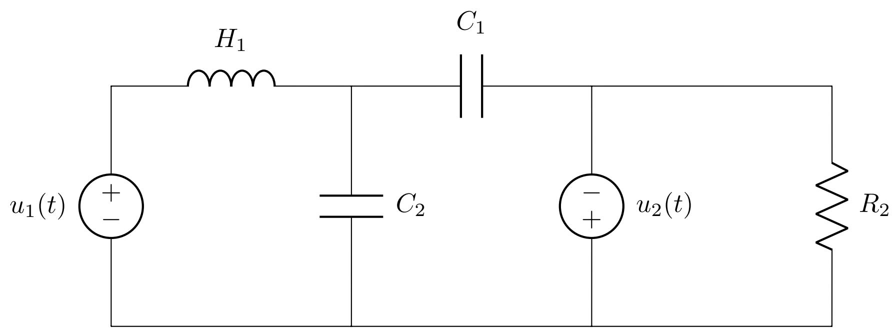


Figure 1: Electrical System

(b) The output is $i(t)$, which defines the current across the resistor R_2.



(c) The output is $v_o(t)$ which defines the voltage across the resistor R_3.

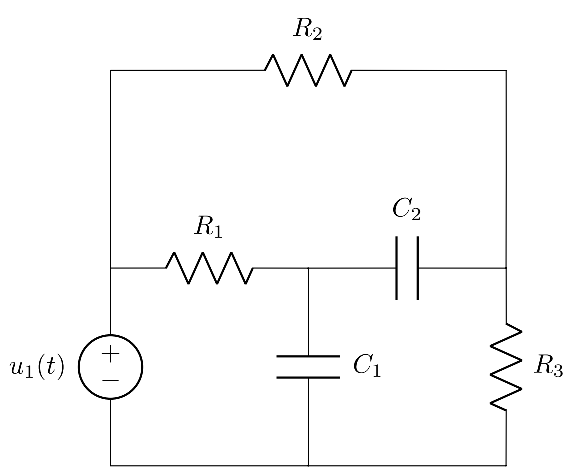


Figure 2: Electrical System

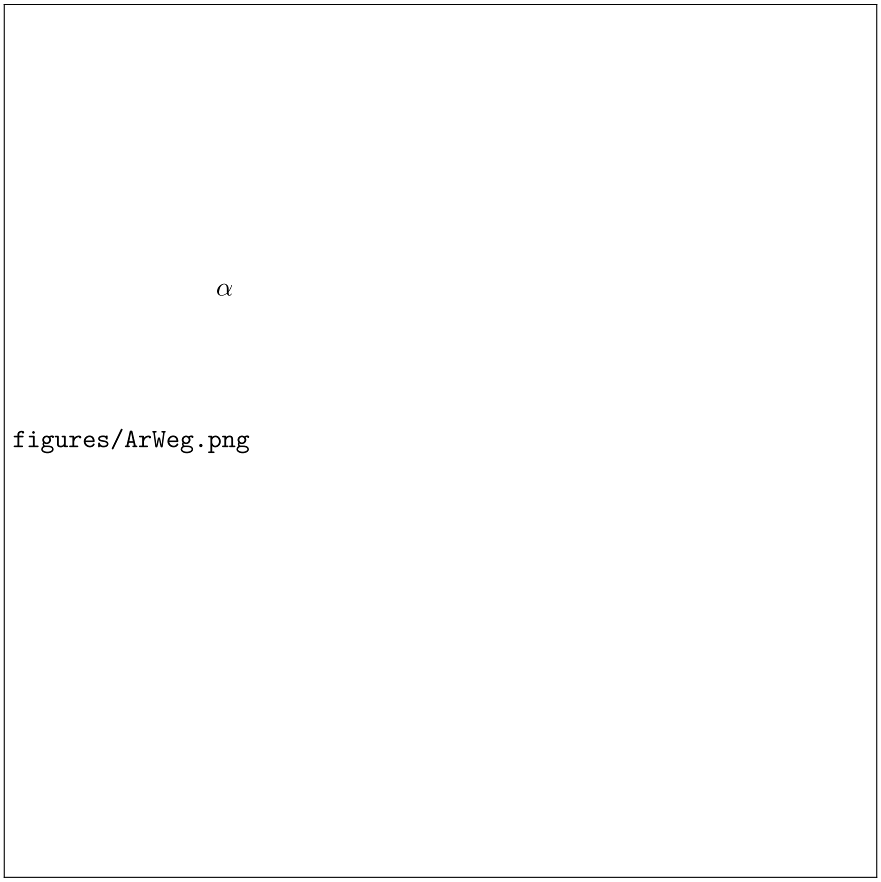


Figure 3: Missile in flight

Problem 2. A missile in flight, as shown in **??**, is subject to several forces: thrust, lift, draft, and gravity. The missile flies at an angle of attack α, from its longitudinal axis, creating lift. For steering, the body angle from vertical, ϕ, is controlled by rotating the engine at the tail. The transfer function relating the body angle, ϕ, to the angular displacement δ of the engine is of the form

$$\frac{\Phi(s)}{\delta(s)} = \frac{K_a s + K_b}{K_3 s^3 + K_2 s^2 + K_1 s + K_0}.$$

Find the representation of the missile steering control in state space.

figures/f4e.png

(a) F-4E with canards

figures/QF-4_Holloman_AFB.jpg

(b) F-4E in flight

Problem 3. The McDonnell Douglas F-4 Phantom II is a tandem two-seat, twin-engine, all-weather, long-range supersonic jet interceptor and fighter-bomber. First entering service in 1960, it proved highly adaptable and was a major part of the air wings of three service components, the US Navy, US Marine Corps and US Air Force. The F-4 was used extensively during the Vietnam War and served as the principal air superiority fighter for both the Navy and Air Force. The F-4 remained in active use through the 1991 Gulf War serving in reconnaissance and Wild Weasel (Suppression of Enemy Air Defenses) roles.

Normal accelerations, a_n, and pitch rate, q, are controlled by elevator deflection, δ_e, on the horizontal stabilizers and by canard deflection, δ_c. A commanded deflection , δ_{com}, is used to effect a change in both δ_e and δ_c. The actuator deflections, combined with the aircraft longitudinal dynamics yield a_n and q. The state equations describing the effect of δ_{com} on a_n and q is given by

$$\begin{bmatrix} \dot{a}_n \\ \dot{q} \\ \dot{\delta}_e \end{bmatrix} = \begin{bmatrix} -1.702 & 50.72 & 263.38 \\ 0.22 & -1.418 & -31.99 \\ 0 & 0 & -14 \end{bmatrix} \begin{bmatrix} a_n \\ q \\ \delta_e \end{bmatrix} + \begin{bmatrix} -272.06 \\ 0 \\ 14 \end{bmatrix} \delta_{com}$$

.

Find the following transfer functions:

(a)

$$G_1(s) = \frac{A_n(s)}{\delta_{com}(s)}$$

(b)

$$G_2(s) = \frac{Q(s)}{\delta_{com}(s)}$$

Problem 4. An autopilot is to be designed for a submarine as shown in **??** to maintain a constant depth under severe wave disturbances. This system has two inputs and two outputs in contrast to classical control

`figures/submarine.png`

Figure 5: Submarine pitch axis control

methods of single input and single output systems. The linearized dynamics under neutral buoyancy and at a constant speed are given by

$$
\begin{aligned}
\dot{\boldsymbol{x}} &= A\boldsymbol{x} + B\boldsymbol{u} \\
\boldsymbol{y} &= C\boldsymbol{x}
\end{aligned}
$$

where

$$
A = \begin{bmatrix} -0.038 & 0.896 & 0 & 0.0015 \\ 0.0017 & -0.092 & 0 & -0.0056 \\ 1 & 0 & 0 & -3.086 \\ 0 & 1 & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} -0.0075 & -0.023 \\ 0.0017 & -0.0022 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}
$$

$$
C = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

.

The state, output and input are defined as

$$
\boldsymbol{x} = \begin{bmatrix} w \\ q \\ z \\ \theta \end{bmatrix} \qquad \boldsymbol{y} = \begin{bmatrix} z \\ \theta \end{bmatrix} \qquad \boldsymbol{u} = \begin{bmatrix} \delta_B \\ \delta_S \end{bmatrix}
$$

where w is the heave velocity, q is the pitch rate, z is the submarine depth, θ is the pitch angle, δ_B is the bow hydroplane angle, and δ_S is the stern hydroplane angle.

(a) Use Matlab/Python to find the transfer function matrix. Recall there should be four possible transfer functions.

(b) Using the previous results write the transfer functions for the following input/output combinations.

$$
\frac{z(s)}{\delta_B(s)}, \quad \frac{z(s)}{\delta_S(s)}, \quad \frac{\theta(s)}{\delta_B(s)}, \quad \frac{\theta(s)}{\delta_S(s)}
$$

Problem 5. Linearize (if possible) the following systems about **EACH** of the their equilibrium states, if not possible state why, and obtain the state matrix, A, for these linearized systems.

(a)

$$\dot{x} = x^3$$

(b)

$$\dot{x} = \sqrt{|x|}$$

(c) The following scalar equations are one form of Euler's equations for the rotational motion of a rigid body. You may assume that the body is non-symmetric, i.e. $I_1 \neq I_2 \neq I_3$.

$$\begin{aligned} I_1\dot{\omega}_1 &= (I_2 - I_3)\omega_2\omega_3, \\ I_2\dot{\omega}_2 &= (I_3 - I_1)\omega_3\omega_1, \\ I_3\dot{\omega}_3 &= (I_1 - I_2)\omega_1\omega_2 \end{aligned}$$

Problem 6. Linearize about each equilibrium point and find the state matrix, A for the state space representation of the linearized system.

(a)

$$\ddot{y} + \left(y^2 - 1\right)\dot{y} + y = 0$$

where $y(t)$ is a scalar.

(b)

$$\ddot{y} + \dot{y} + y - y^3 = 0$$

where $y(t)$ is a scalar.

(c)

$$\begin{aligned} (M+m)\,\ddot{y} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta + ky &= 0, \\ ml\ddot{y}\cos\theta + ml^2\ddot{\theta} + mgl\sin\theta &= 0. \end{aligned}$$

Problem 7. Obtain the transfer function (matrix) for the following system

$$\begin{aligned} \ddot{y}_1 + \ddot{y}_2 + y_1 + y_2 &= u_1 + \dot{u}_2, \\ 2\ddot{y}_1 + 3\ddot{y}_2 + y_1 - y_2 &= 0. \end{aligned}$$

Problem 8. Obtain the transfer function of the system with input u and output y described by

$$\begin{aligned} \ddot{q}_1 + 3\dot{q}_2 + \dot{q}_1 + 2q_2 &= \dot{u} + 4u, \\ \ddot{q}_1 + 4\dot{q}_2 + 3q_2 &= u, \\ y &= q_1 + q_2. \end{aligned}$$

Problem 9. Obtain the transfer function for the following system

$$\begin{aligned} \dot{x}(t) &= -x(t) + 2x(t-h) + u(t), \\ y(t) &= x(t) \end{aligned}$$