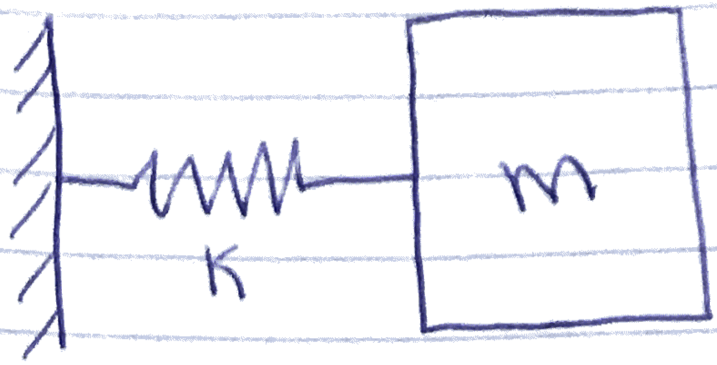
Homework 1 Solutions

1)

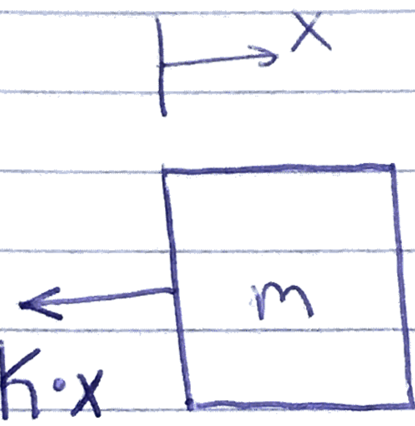


$m = 500g$

$k = 7.5\ N/m$

(a) $\sum F = ma \Rightarrow \sum F = m\ddot{x}(t)$

FBD

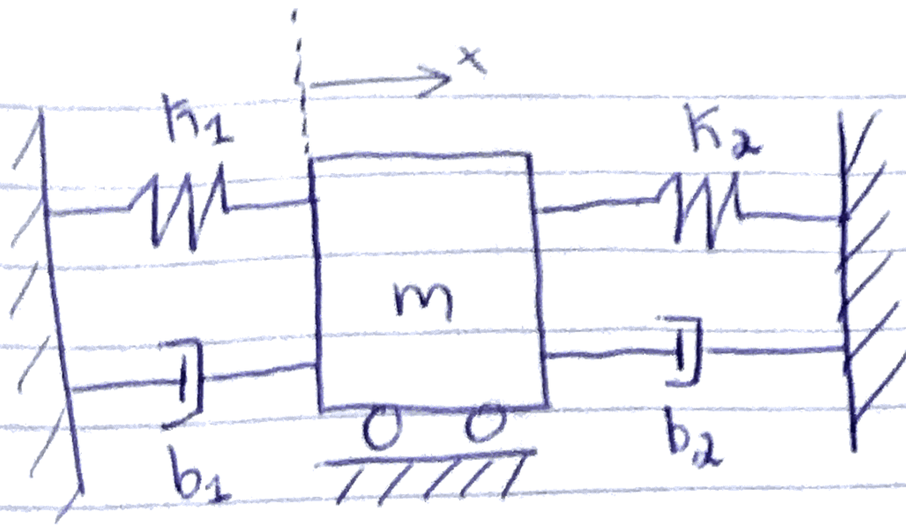


$$m\ddot{x} = -k(x) \Rightarrow 0.5\ddot{x} = -7.5(x)$$

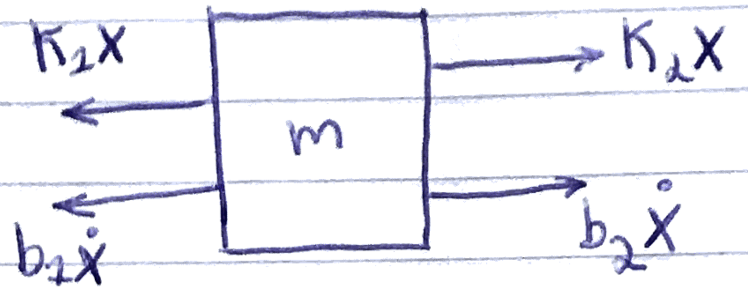
$$0.5\ddot{x} + 7.5x = 0$$

$$\boxed{\ddot{x} + 15x = 0}$$

2)



FBD:



$$\sum F = ma \Rightarrow \sum F = m\ddot{x}$$

$$m\ddot{x} = -k_1 x - k_2 x - b_1\dot{x} - b_2\dot{x}$$

$$m\ddot{x} = -(k_1+k_2)x - (b_1+b_2)\dot{x}$$

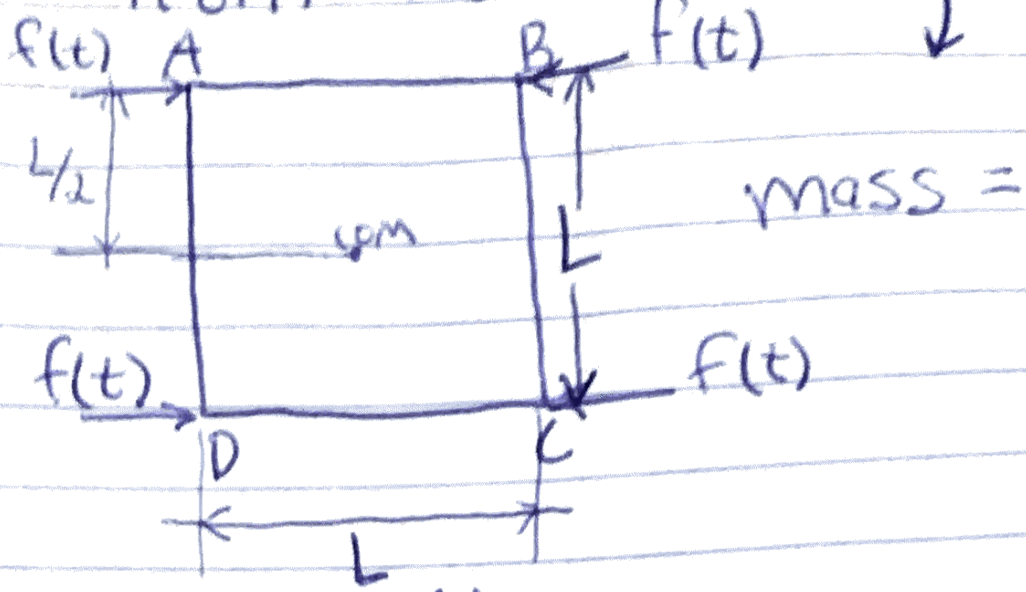
$$m\ddot{x} + (k_1+k_2)x + (b_1+b_2)\dot{x} = \emptyset$$

$$\ddot{x} + \frac{(k_1+k_2)}{m}x + \frac{(b_1+b_2)}{m}\dot{x} = \emptyset$$

$$\boxed{\ddot{x} + \frac{(b_1+b_2)}{m}\dot{x} + \frac{(k_1+k_2)}{m}x = \emptyset}$$

3) FBD

Front view



mass = m

$$\sum F = ma \Rightarrow \sum \overset{M}{\cancel{F}} = J\ddot{\theta}$$

$$J_{cube} = \frac{1}{6} mL^2$$

$$J\ddot{\theta} = +f_A(t)\left(\frac{L}{2}\right) + f_C(t)\left(\frac{L}{2}\right) - f_B(t)\left(\frac{L}{2}\right) - f_D(t)\left(\frac{L}{2}\right)$$

$$J\ddot{\theta} = \frac{L}{2}\left(f_A(t) + f_C(t)\right) - \frac{L}{2}\left(f_B(t) + f_D(t)\right)$$

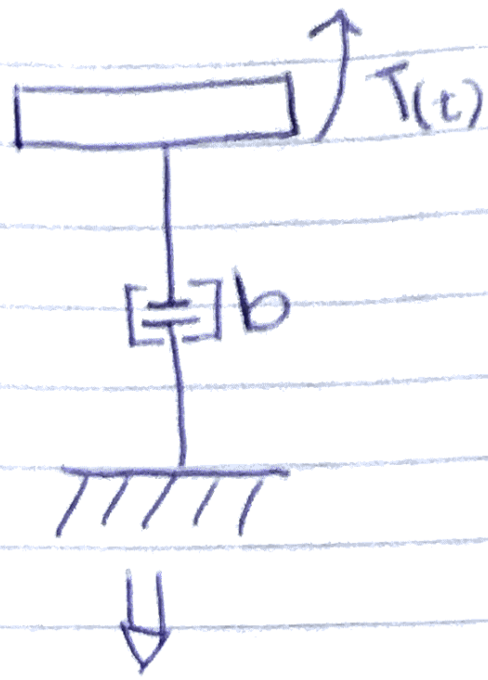
$$J\ddot{\theta} = \frac{L}{2}\left(f_A(t) + f_C(t) - f_B(t) - f_D(t)\right)$$

$$\frac{1}{6} mL^2 \ddot{\theta} = \frac{L}{2}\left(f_A(t) + f_C(t) - f_B(t) - f_D(t)\right)$$

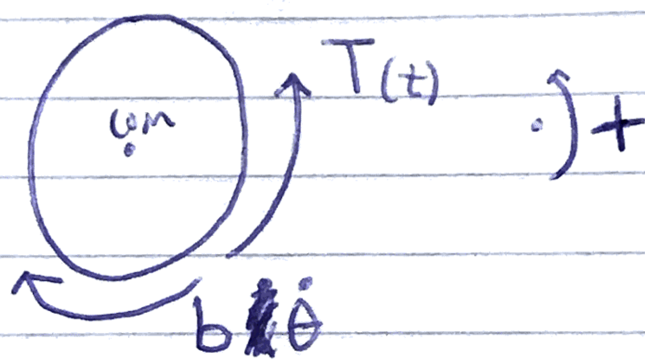
$$\ddot{\theta} = \frac{3}{mL}\left(f_A(t) + f_C(t) - f_B(t) - f_D(t)\right)$$

4)

FBD



Top view

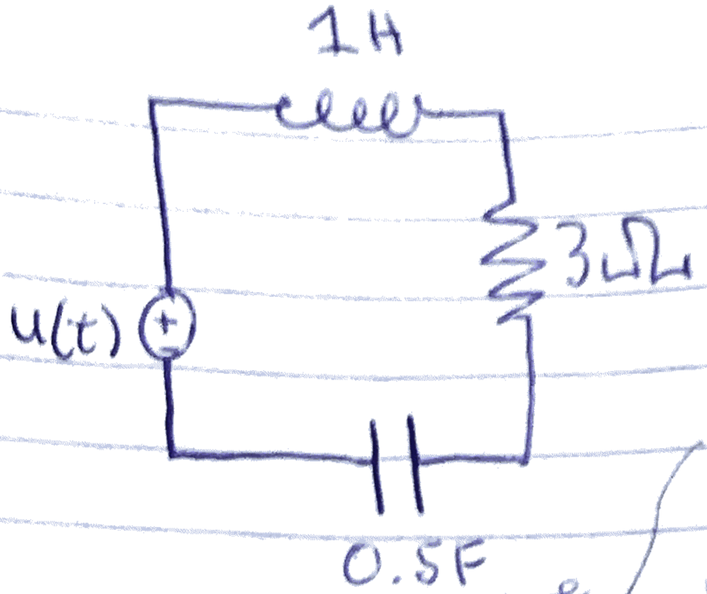


$$\sum F = ma \Rightarrow \sum M = J\ddot{\theta}$$

$$J\ddot{\theta} = T_{(t)} - b\dot{\theta}$$

$$J\ddot{\theta} - b\dot{\theta} = T_{(t)} \Rightarrow \boxed{\ddot{\theta} - \frac{b}{J}\dot{\theta} = \frac{T_{(t)}}{J}}$$

5)



find voltage through each component

$V_{inductor} = L\frac{di}{dt} = L\ddot{q}$

$V_{resistor} = iR = R\dot{q}$

$V_{capacitor} = \frac{1}{C}q$

combine

$$u(t) = L\ddot{q} + R\dot{q} + \frac{1}{C}q$$

$$u(t) = \ddot{q} + 3\dot{q} + \frac{1}{0.5}q$$

$$\boxed{u(t) = \ddot{q} + 3\dot{q} + 2q}$$

$$F_{(s)} = \int_0^\infty f_{(t)} \cdot e^{-st} \, dt$$

6) (a) $e^{4t} + 5$

$$\mathcal{L}[e^{4t} + 5] = \mathcal{L}[e^{4t}] + \mathcal{L}[5]$$

$$\mathcal{L}[e^{4t}] = \int_0^\infty e^{4t} \cdot e^{-st} \, dt = \int_0^\infty e^{(4-s)t} \, dt$$

$$\int_0^\infty e^{(4-s)t} \, dt = -\frac{e^{(4-s)x}}{s-4} \Bigg|_0^\infty$$

$$\frac{-e^{(4-s)\infty}}{s-4} - \frac{e^{(4-s)0}}{s-4} = \boxed{0 + \frac{1}{s-4}}$$

$$\mathcal{L}[5] = \int_0^\infty 5 \cdot e^{-st} \, dt = \frac{-5}{s} e^{-st} \Bigg|_0^\infty$$

$$\frac{-5}{s} e^{-s(\infty)} - \frac{-5}{s} e^{-s(0)} = \boxed{\frac{5}{s}}$$

$$\boxed{\mathcal{L}[e^{4t} + 5] = \frac{1}{s-4} + \frac{5}{s}}$$

b) $\cos(2t) + 7\sin(2t)$

$$\mathcal{L}[\cos(2t)] = \int_0^\infty \cos(2t) e^{-st}\, dt$$

$$= \int_0^\infty \left(\frac{e^{i2t} + e^{-i2t}}{2}\right) e^{-st}\, dt = \frac{1}{2}\int_0^\infty (e^{2it} + e^{-2it}) e^{-st}$$

$$= \frac{1}{2}\int_0^\infty e^{2it} e^{-st} + e^{-2it} e^{-st} = \frac{1}{2}\int_0^\infty e^{-(s \mp 2i)t} + e^{-(s+2i)t}$$

$$= \frac{1}{2}\left[\frac{-e^{-(2i-s)t}}{s-2i} + \frac{-e^{-(s+2i)t}}{s+2i}\right]\Bigg|_0^\infty$$

$$= \frac{1}{2}[\emptyset + \emptyset] - \frac{1}{2}\left[\frac{-1}{s-2i} - \frac{1}{s+2i}\right]$$

$$= \frac{1}{2}\left[\frac{1}{s-2i} + \frac{1}{s+2i}\right] = \frac{1}{2}\frac{1}{s-2i} + \frac{1}{2}\frac{1}{s+2i}$$

$$= \frac{1}{2}\left[\frac{s+2i}{s^2+4} + \frac{s-2i}{s^2+4}\right] = \frac{(s+\cancel{2i}) + (s-\cancel{2i})}{2(s^2+4)}$$

$$= \frac{2s}{2(s^2+4} = \boxed{\frac{s}{s^2+4}}$$

$$\mathcal{L}[7\sin(2t)] = \int_0^\infty 7\sin(2t) e^{-st}\, dt$$

$$=7\int_0^\infty \frac{e^{2it}-e^{-2it}}{2i}e^{-st}\,dt = \frac{7}{2i}\int_0^\infty (e^{2it}-e^{-2it})e^{-st}\,dt$$

$$=\frac{7}{2i}\int_0^\infty e^{-(s-2i)t}-e^{-(s+2i)t}\,dt$$

$$=\frac{7}{2i}\left[-\frac{e^{-(s-2i)t}}{s-2i}+\frac{e^{-(s+2i)t}}{s+2i}\right]\Bigg|_0^\infty$$

$$=\frac{7}{2i}\left[\emptyset+\emptyset\right]-\frac{7}{2i}\left[\frac{-1}{s-2i}+\frac{1}{s+2i}\right]$$

$$=\frac{-7}{2i}\left[\frac{s+\cancel{2i}}{s^2+4}+\frac{s-\cancel{2i}}{s^2+4}\right]=\frac{-7}{\cancel{2i}}\left[\frac{\cancel{2i}(s+1)}{s^2+4}+\frac{\cancel{2i}(s-1)}{s^2+4}\right]$$

$$=\frac{7}{s^2+4}+\frac{7}{s^2+4}=\boxed{\frac{14}{s^2+4}}$$

$$\mathcal{L}\left[\cos(2t)+7\sin(2t)\right]=\boxed{\frac{s}{s^2+4}+\frac{14}{s^2+4}}$$

(c) $\dfrac{s+17}{s^2+4s+13}$

(d) $\dfrac{10s^3+5s^2+2s-24}{s^4}$

(e) $\dfrac{2(s-2)^2}{(s-3)^3}$

(f) $\dfrac{6(s-5)}{(s-2)^2+4}+\dfrac{1}{7-s}$