

MAE3134: Homework 5

Due date: 29 March 2018

Problem 1. For each of the electrical systems below, find the state space representation.

- (a) There is a single voltage source and the output is the voltage difference across the capacitor C_1 .

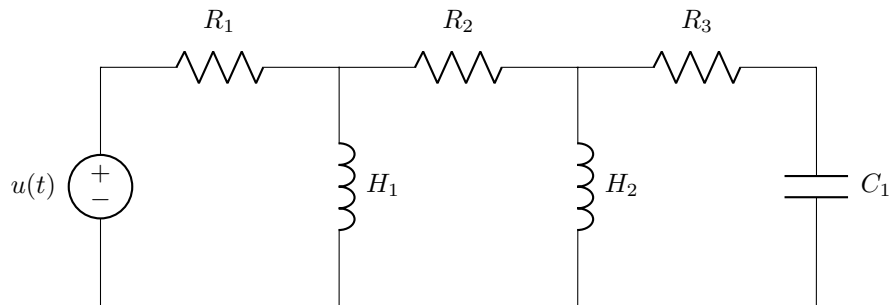
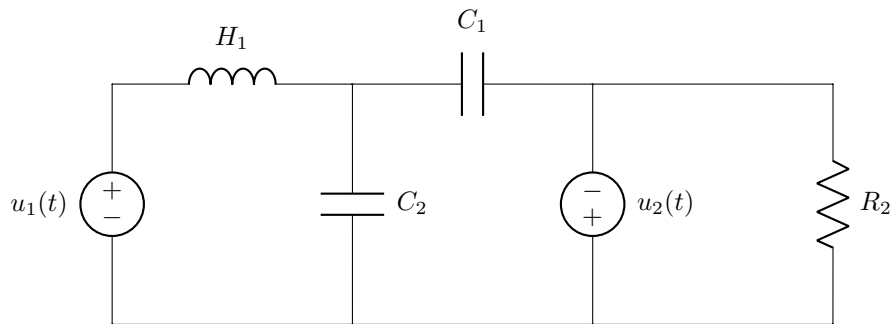


Figure 1: Electrical System

- (b) The output is $i(t)$, which defines the current across the resistor R_2 .



- (c) The output is $v_o(t)$ which defines the voltage across the resistor R_3 .

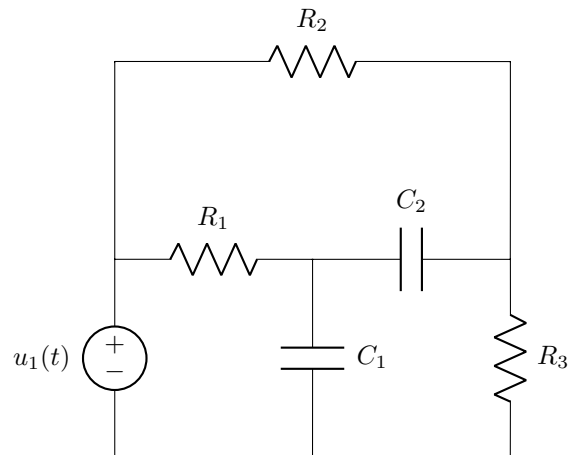


Figure 2: Electrical System

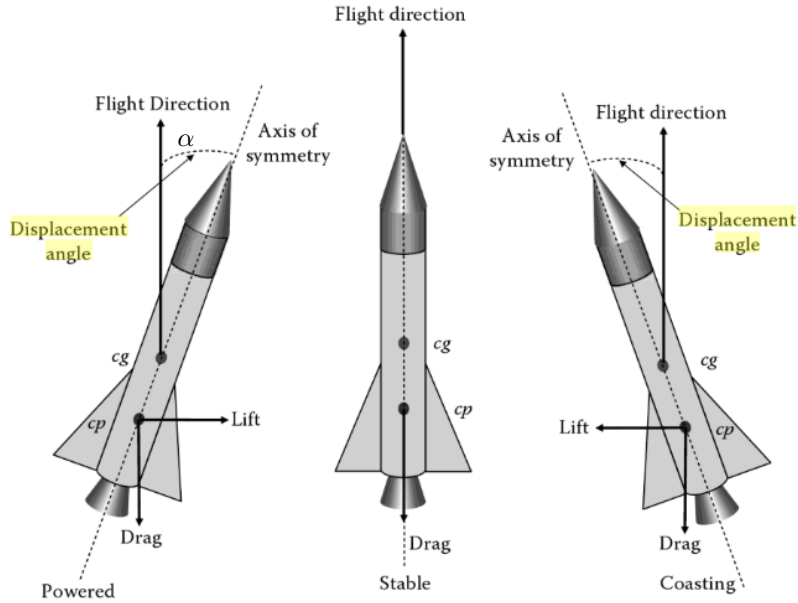


FIGURE 3.15
Three modes of rocket flight stability are illustrated.

Figure 3: Missile in flight

Problem 2. A missile in flight, as shown in Fig. 3, is subject to several forces: thrust, lift, draft, and gravity. The missile flies at an angle of attack α , from its longitudinal axis, creating lift. For steering, the body angle from vertical, ϕ , is controlled by rotating the engine at the tail. The transfer function relating the body angle, ϕ , to the angular displacement δ of the engine is of the form

$$\frac{\Phi(s)}{\delta(s)} = \frac{K_a s + K_b}{K_3 s^3 + K_2 s^2 + K_1 s + K_0}.$$

Find the representation of the missile steering control in state space.

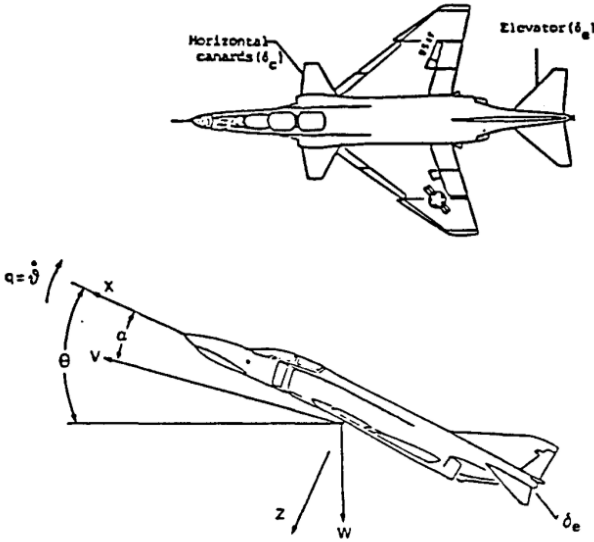


Fig. 2 F4-E with canards.



(a) F4-E with canards

(b) F4-E in flight

Problem 3. The McDonnell Douglas F-4 Phantom II is a tandem two-seat, twin-engine, all-weather, long-range supersonic jet interceptor and fighter-bomber. First entering service in 1960, it proved highly adaptable and was a major part of the air wings of three service components, the US Navy, US Marine Corps and US Air Force. The F-4 was used extensively during the Vietnam War and served as the principal air superiority fighter for both the Navy and Air Force. The F-4 remained in active use through the 1991 Gulf War serving in reconnaissance and Wild Weasel (Suppression of Enemy Air Defenses) roles.

Normal accelerations, a_n , and pitch rate, q , are controlled by elevator deflection, δ_e , on the horizontal stabilizers and by canard deflection, δ_c . A commanded deflection, δ_{com} , is used to effect a change in both δ_e and δ_c . The actuator deflections, combined with the aircraft longitudinal dynamics yield a_n and q . The state equations describing the effect of δ_{com} on a_n and q is given by

$$\begin{bmatrix} \dot{a}_n \\ \dot{q} \\ \dot{\delta}_e \end{bmatrix} = \begin{bmatrix} -1.702 & 50.72 & 263.38 \\ 0.22 & -1.418 & -31.99 \\ 0 & 0 & -14 \end{bmatrix} \begin{bmatrix} a_n \\ q \\ \delta_e \end{bmatrix} + \begin{bmatrix} -272.06 \\ 0 \\ 14 \end{bmatrix} \delta_{com}$$

Find the following transfer functions:

(a)

$$G_1(s) = \frac{A_n(s)}{\delta_{com}(s)}$$

(b)

$$G_2(s) = \frac{Q(s)}{\delta_{com}(s)}$$

Problem 4. An autopilot is to be designed for a submarine as shown in Fig. 5 to maintain a constant depth under severe wave disturbances. This system has two inputs and two outputs in contrast to classical control

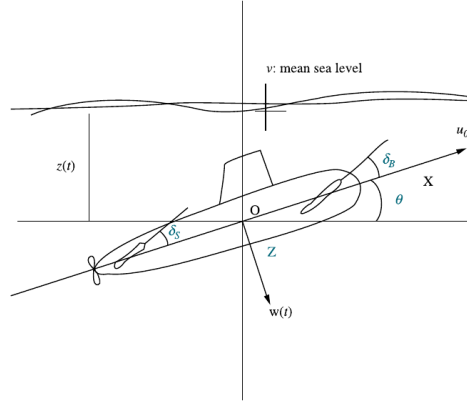


Figure 5: Submarine pitch axis control

methods of single input and single output systems. The linearized dynamics under neutral buoyancy and at a constant speed are given by

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \\ \mathbf{y} &= \mathbf{C}\mathbf{x}\end{aligned}$$

where

$$\begin{aligned}\mathbf{A} &= \begin{bmatrix} -0.038 & 0.896 & 0 & 0.0015 \\ 0.0017 & -0.092 & 0 & -0.0056 \\ 1 & 0 & 0 & -3.086 \\ 0 & 1 & 0 & 0 \end{bmatrix} & \mathbf{B} &= \begin{bmatrix} -0.0075 & -0.023 \\ 0.0017 & -0.0022 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \\ \mathbf{C} &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\end{aligned}$$

The state, output and input are defined as

$$\mathbf{x} = \begin{bmatrix} w \\ q \\ z \\ \theta \end{bmatrix} \quad \mathbf{y} = \begin{bmatrix} z \\ \theta \end{bmatrix} \quad \mathbf{u} = \begin{bmatrix} \delta_B \\ \delta_S \end{bmatrix}$$

where w is the heave velocity, q is the pitch rate, z is the submarine depth, θ is the pitch angle, δ_B is the bow hydroplane angle, and δ_S is the stern hydroplane angle.

- Use Matlab/Python to find the transfer function matrix. Recall there should be four possible transfer functions.
- Using the previous results write the transfer functions for the following input/output combinations.

$$\frac{z(s)}{\delta_B(s)}, \quad \frac{z(s)}{\delta_S(s)}, \quad \frac{\theta(s)}{\delta_B(s)}, \quad \frac{\theta(s)}{\delta_S(s)}$$

Problem 5. Linearize (if possible) the following systems about **EACH** of their equilibrium states, if not possible state why, and obtain the state matrix, A , for these linearized systems.

(a)

$$\dot{x} = x^3$$

(b)

$$\dot{x} = \sqrt{|x|}$$

(c) The following scalar equations are one form of Euler's equations for the rotational motion of a rigid body. You may assume that the body is non-symmetric, i.e. $I_1 \neq I_2 \neq I_3$.

$$I_1 \dot{\omega}_1 = (I_2 - I_3) \omega_2 \omega_3,$$

$$I_2 \dot{\omega}_2 = (I_3 - I_1) \omega_3 \omega_1,$$

$$I_3 \dot{\omega}_3 = (I_1 - I_2) \omega_1 \omega_2$$

Problem 6. Linearize about each equilibrium point and find the state matrix, A for the state space representation of the linearized system.

(a)

$$\ddot{y} + (y^2 - 1) \dot{y} + y = 0$$

where $y(t)$ is a scalar.

(b)

$$\ddot{y} + \dot{y} + y - y^3 = 0$$

where $y(t)$ is a scalar.

(c)

$$(M + m) \ddot{y} + ml \ddot{\theta} \cos \theta - ml \dot{\theta}^2 \sin \theta + ky = 0,$$

$$ml \ddot{y} \cos \theta + ml^2 \ddot{\theta} + mgl \sin \theta = 0.$$

Problem 7. Obtain the transfer function (matrix) for the following system

$$\ddot{y}_1 + \ddot{y}_2 + y_1 + y_2 = u_1 + \dot{u}_2,$$

$$2\ddot{y}_1 + 3\ddot{y}_2 + y_1 - y_2 = 0.$$

Problem 8. Obtain the transfer function of the system with input u and output y described by

$$\ddot{q}_1 + 3\dot{q}_2 + \dot{q}_1 + 2q_2 = \dot{u} + 4u,$$

$$\ddot{q}_1 + 4\dot{q}_2 + 3q_2 = u,$$

$$y = q_1 + q_2.$$

Problem 9. Obtain the transfer function for the following system

$$\dot{x}(t) = -x(t) + 2x(t - h) + u(t),$$

$$y(t) = x(t)$$