Linear System Dynamics Homework 0

1

$$F(t, y(t), y'(t), \ldots, y^{(n)}(t)) = 0$$

a) $a_n(x)y^{(n)} + a_{n-1}(x)y^{(n-1)} + \ldots + a_1(x)y' + a_0(x)y = F(x)$

b) $t\frac{d^2y}{dt^2} + t^2\frac{dy}{dt} + t^3 y = \cos t$

- linear in y
- second order from d^2y/dt^2
- dependent variable: y; independent variable: t

c) $t\frac{d^3y}{dt^3} + t^2\frac{dy}{dt} + t^3 y = \cos y$

- non-linear: cos y is a non-linear term
- third order from d^3y/dt^3
- dependent variable: y; independent variable: t

d) $\frac{dy}{dx} = \frac{2y-3}{2x+2}$ $\rightarrow$ $(2x+2)\frac{dy}{dx} + 3 - 2y = 0$

- linear
- first order from dy/dt
- dependent variable: y; independent variable: x

e) $(\cos t)\frac{d^2y}{dt^2} + (\sin 2t)y = 0$ $\quad y = y(t)$

- linear
- second order from d^2y/dt^2
- dependent variable: y; independent variable: t

f) $\frac{dx}{dt} + \frac{dy}{dt} + x + y = 0$ $\quad x = x(t)$ $\quad y = y(t)$

- ordinary differential equation
- dependent variable: x, y; independent variable: t

g) $\frac{df}{dx} + \frac{df}{dy} + x + y = 0$ $\quad f = f(x,y)$

- partial differential equation
- dependent variable: f; independent variable: x, y

h) $\frac{d}{dt}\left[\frac{df}{dx}\right] = 0$ $\quad f = x^2 + \frac{dx}{dt}$

- ordinary differential equation
- dependent variable: f, x; independent variable: t

1 (cont.)

i) $\frac{df}{dx} = x \quad f = y^2(x) + \frac{dy}{dx}$

- ordinary differential equation
- dependent variable: f, y; independent variable: x

j) $\frac{d^2y}{dt^2} + 2y = 0$

- time-invariant; does not explicitly depend on t

k) $\frac{d}{dt}(t^2 y) = 0$

- time-variable; $t^2 y$

l) $\left(\frac{1}{t+1}\right)\frac{d^2y}{dt^2} + \left(\frac{1}{t+1}\right) = 0 \rightarrow \frac{d^2y}{dt^2} = 0$

- time-Invariant

m) $\frac{d^2y}{dt^2} + (\cos t)y = 0$

- time-variable; cos t

2

$$\frac{dv_{out}}{dt} + 5v_{out} = 5v_{in}(t)$$

a) $v_{in}(t) = 2\,V \quad v_{out}(0) = 0\,V \quad v_{out}(t) = ?$

$\mathcal{L}\left[\frac{dv_{out}}{dt} + 5v_{out} = 10\,V\right] \rightarrow s\,V_{out}(s) - \overset{0V}{\cancel{v_{out}(0)}} + 5V_{out}(s) = \frac{10}{s}$

$\rightarrow V_{out}(s) = \frac{10}{s(s+5)} = \frac{a}{s} + \frac{b}{s+5} \rightarrow 10 = a(s+5) + bs \rightarrow 10 = (a+b)s + 5a \rightarrow a = 2 \rightarrow b = -2$

$\rightarrow V_{out}(s) = \frac{2}{s} + \frac{-2}{s+5} \rightarrow$ inverse Laplace $\rightarrow v_{out}(t) = 2 - 2e^{-5t} \rightarrow \boxed{v_{out}(t) = 2(1-e^{-5t})}$

b) $v_{out}(t) = 2(1-e^{-5t}) \qquad v_{out}(t) = ?$ as $t \rightarrow \infty$

$v_{out}(\infty) = 2(1-e^{-5(\infty)}) = 2(1-0) = \boxed{2\,V}$

c) $t = ?$ when $v_{out}(t) = 0.1\,v_{out}(\infty) = 0.2V$

$v_{out}(t) = 0.2V = 2(1-e^{-5t}) \rightarrow 0.1 - 1 = -e^{-5t} \rightarrow \ln(0.9) = -5t$

$\rightarrow \boxed{t = 0.021s}$

d) $t = ?$ when $v_{out}(t) = 0.9\,v_{out}(\infty) = 1.8\,V$

$v_{out}(t) = 1.8V = 2(1-e^{-5t}) \rightarrow 0.9 - 1 = -e^{-5t} \rightarrow \ln(0.1) = -5t$

$\rightarrow \boxed{t = 0.461s}$

2 (cont.)

e) $t = ?$ when $V_{out}(t) = 0.98\, V_{out}(\infty) = 1.96V$

$V_{out}(t) = 1.96\,V = 2(1-e^{-5t}) \rightarrow 0.98 - 1 = -e^{-5t} \rightarrow \ln(0.02) = -5t$

$\rightarrow \boxed{t = 0.782s}$

3

$y = 0.7\cos\left(\frac{\pi}{3}t + \frac{\pi}{6}\right)$

a) $y(t=0) = 0.7\cos\left(\frac{\pi}{3}(0) + \frac{\pi}{6}\right) \rightarrow y(0) = 0.7\cos\left(\frac{\pi}{6}\right) = \boxed{0.606\ m}$

b) $\dot{y}(t=0) = ?$ $\quad \dot{y} = 0.7 \cdot \frac{\pi}{3} \cdot -\sin\left(\frac{\pi}{3}t + \frac{\pi}{6}\right) = -0.733\sin\left(\frac{\pi}{3}t + \frac{\pi}{6}\right)$

$\dot{y}(t=0) = -0.733\sin\left(\frac{\pi}{3}(0) + \frac{\pi}{6}\right) \rightarrow \dot{y}(0) = -0.733\sin\left(\frac{\pi}{6}\right) = \boxed{-0.367\ m/s}$

c) $\ddot{y}(t=0) = ?$ $\quad \ddot{y} = -0.733 \cdot \frac{\pi}{3} \cdot \cos\left(\frac{\pi}{3}t + \frac{\pi}{6}\right) = -0.768\cos\left(\frac{\pi}{3}t + \frac{\pi}{6}\right)$

$\ddot{y}(t=0) = -0.768\cos\left(\frac{\pi}{3}(0) + \frac{\pi}{6}\right) = -0.768\cos\left(\frac{\pi}{6}\right) = \boxed{-0.665\ m/s^2}$

d) max velocity = ? $\quad \dot{y}_{max}$ at $\ddot{y} = 0$

$\ddot{y}(t) = 0 = -0.768\cos\left(\frac{\pi}{3}t + \frac{\pi}{6}\right) \rightarrow \cos\left(\frac{\pi}{3}t + \frac{\pi}{6}\right) = 0 \rightarrow \cos^{-1}(0) = \frac{\pi}{3}t + \frac{\pi}{6} \rightarrow t = 1s$

$\dot{y}(1) = -0.733\sin\left(\frac{\pi}{3}(1) + \frac{\pi}{6}\right) = \boxed{-0.733\ m/s}$

e) $t = ?$ when $y = max$ $\quad y_{max}$ at $\dot{y} = 0$

$\dot{y}(t) = 0 = -0.733\sin\left(\frac{\pi}{3}t + \frac{\pi}{6}\right) \rightarrow \sin\left(\frac{\pi}{3}t + \frac{\pi}{6}\right) = 0 \rightarrow \sin^{-1}(0) = \frac{\pi}{3}t + \frac{\pi}{6} \rightarrow t = 2.5, 5.5$

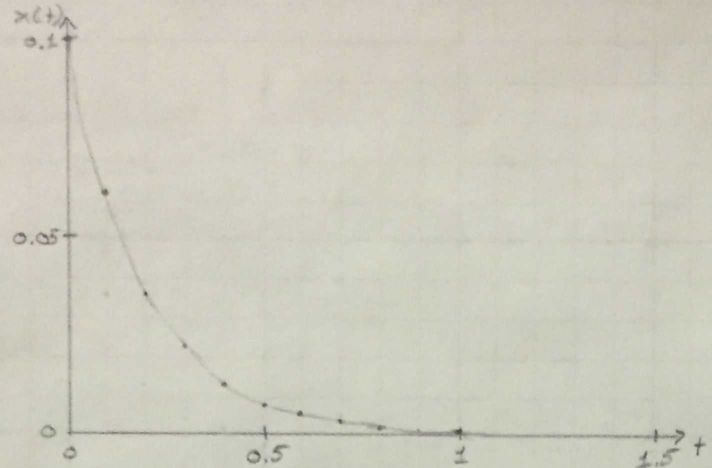
$y(2.5) = 0.7\cos\left(\frac{\pi}{3}(2.5) + \frac{\pi}{6}\right) = -0.7$ (not positive)

$y(5.5) = 0.7\cos\left(\frac{\pi}{3}(5.5) + \frac{\pi}{6}\right) = 0.7 \rightarrow \boxed{t = 5.5s}$

f)

4

$x(t) = 0.1e^{-5t}$



5

$$A = \begin{bmatrix} 0 & 1 \\ -10 & 2 \end{bmatrix} \qquad [sI - A] = \begin{bmatrix} s & -1 \\ 10 & s-2 \end{bmatrix}$$

a) eigenvalues of $A = ?$

$$\det(\lambda I - A) = 0 \rightarrow \det\left(\lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -10 & 2 \end{bmatrix}\right) = 0$$

$$\rightarrow \det\left(\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -10 & 2 \end{bmatrix}\right) = 0 \rightarrow \det\left(\begin{bmatrix} \lambda & -1 \\ 10 & \lambda-2 \end{bmatrix}\right) = 0$$

$$\rightarrow \lambda(\lambda-2) - 10(-1) = 0 \rightarrow \lambda^2 - 2\lambda + 10 = 0$$

$$\rightarrow \boxed{\lambda = 1 \pm i3}$$

b) $[sI - A]^{-1} = ?$

$$\begin{bmatrix} s & -1 \\ 10 & s-2 \end{bmatrix}^{-1} = \frac{1}{s(s-2)+10} \begin{bmatrix} s-2 & 1 \\ -10 & s \end{bmatrix} = \frac{1}{s^2-2s+10} \begin{bmatrix} s-2 & 1 \\ -10 & s \end{bmatrix} = \begin{bmatrix} \frac{s-2}{s^2-2s+10} & \frac{1}{s^2-2s+10} \\ \frac{-10}{s^2-2s+10} & \frac{s}{s^2-2s+10} \end{bmatrix}$$

$$[sI - A] \cdot [sI - A]^{-1} = I$$

$$\begin{bmatrix} s & -1 \\ 10 & s-2 \end{bmatrix} \cdot \frac{1}{s^2-2s+10} \begin{bmatrix} s-2 & 1 \\ -10 & s \end{bmatrix} \rightarrow \frac{1}{s^2-2s+10} \begin{bmatrix} s^2-2s+10 & s-s \\ 10(s-2)-10(s-2) & 10+s^2-2s \end{bmatrix}$$

$$\rightarrow \frac{1}{s^2-2s+10} \begin{bmatrix} s^2-2s+10 & 0 \\ 0 & s^2-2s+10 \end{bmatrix} \rightarrow \frac{s^2-2s+10}{s^2-2s+10} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \rightarrow I \checkmark$$

6

$a = -2 + 0.5j \qquad \lambda = -1 + 3j$

a) $\boxed{a^* = -2 - 0.5j}$ $\boxed{\lambda^* = -1 - 3j}$

b) $\|a\| = \sqrt{2^2 + 0.5^2} = 2.062 \qquad \phi = \angle a = \tan^{-1}\left(\frac{0.5}{-2}\right) = 2.897\,\text{rad} = 166.0°$

$\rightarrow \boxed{a = 2.062e^{2.897j}}$

$\|a^*\| = \sqrt{2^2 + 0.5^2} = 2.062 \qquad \phi = \angle a^* = \tan^{-1}\left(\frac{-0.5}{-2}\right) = -2.897\,\text{rad} = -166.0°$

$= 3.386\,\text{rad} = 194.0°$

$\rightarrow \boxed{a^* = 2.062\,e^{3.386j}}$

c) $b = a + \lambda$

$b = -2 + 0.5j + (-1 + 3j) = -2 - 1 + 0.5j + 3j \rightarrow \boxed{b = -3 + 3.5j}$

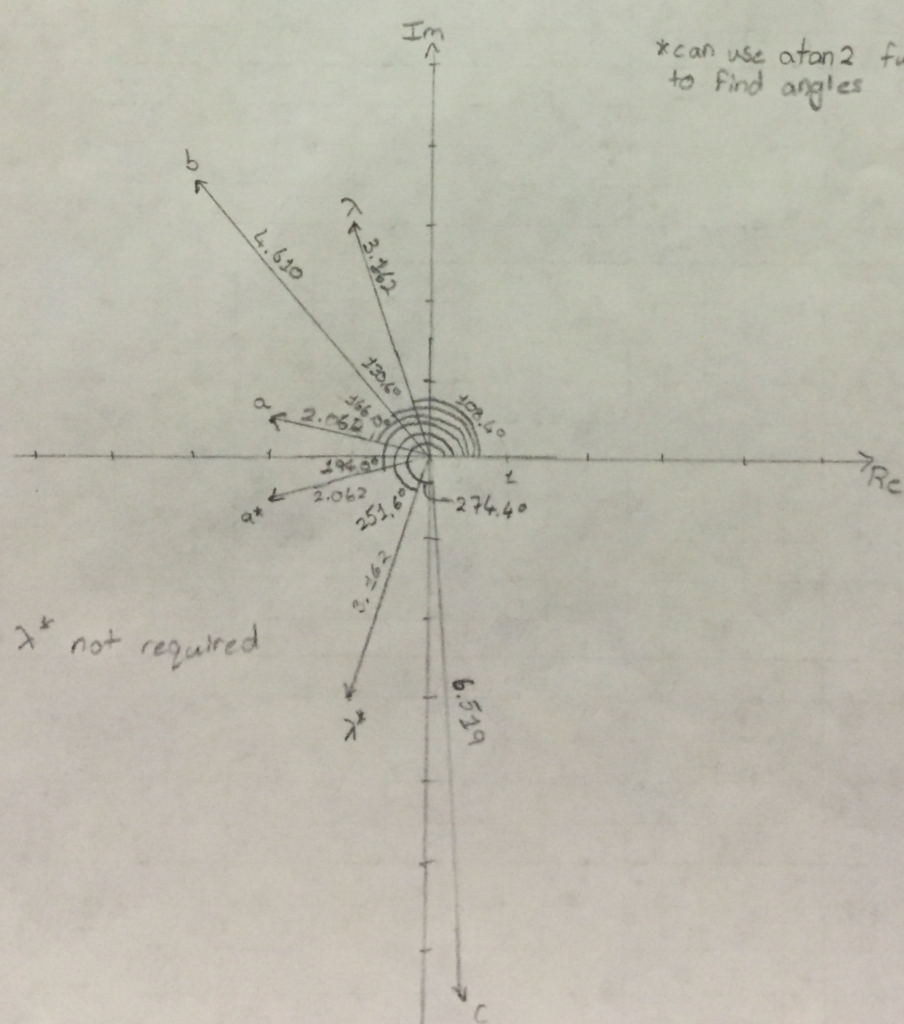
d) $c = a\lambda$

$c = (-2 + 0.5j)(-1 + 3j) = 2 - 6j - 0.5j - 1.5 = 2 - 1.5 - 6j - 0.5j$

$\rightarrow \boxed{c = 0.5 - 6.5j}$

e)

*can use atan2 function in MATLAB to find angles



a^*, λ^* not required