MAE3134: Homework 2

Due date: 15 February 2018

Problem 1. Find the transfer function for the following ordinary differential equations.

(a) Find: $\frac{C(s)}{R(s)}$ for

$$\dddot{c} + 3\ddot{c} + 7\dot{c} + 5c = \ddot{r} + 4\dot{r} + 3r$$

(b) Find: $\frac{Y(s)}{X(s)}$ for

$$\dddot{y} + 3\ddot{y} + 5\dot{y} + y = \dddot{x} + 4\ddot{x} + 6\dot{x} + 8x$$

Problem 2. Find the ordinary differential equation for the given transfer functions

(a)

$$G(s) = \frac{C(s)}{R(s)} = \frac{2s+1}{s^2+6s+2}$$

(b)

$$G(s) = \frac{X(s)}{F(s)} = \frac{1}{s^2+2s+7}$$

(c)

$$G(s) = \frac{X(s)}{F(s)} = \frac{10}{(s+7)\,(s+8)}$$

(d)

$$G(s) = \frac{X(s)}{F(s)} = \frac{s+2}{s^3+8s^2+9s+15}$$

(e) Find the differential equation for the system in Fig. 1.

$$R(s) \;\longrightarrow\; \boxed{\frac{s^5+2s^4+4s^3+s^2+3}{s^6+7s^5+3s^4+2s^3+s^2+3}} \;\longrightarrow\; C(s)$$

Figure 1: Block Diagram

(f) Find the ordinary differential equation for the transfer function between input $r(t) = 3t^3$ and the output $C(s)$ shown in Fig. 2.

$$R(s) \;\longrightarrow\; \boxed{\frac{s^4+2s^3+5s^2+s+1}{s^5+3s^4+2s^3+4s^2+5s+2}} \;\longrightarrow\; C(s)$$

Figure 2: Block Diagram

Problem 3. Given the following transfer function:

$$G(s) = \frac{C(s)}{R(s)} = \frac{s}{(s+4)(s+8)},$$

(a) Determine the output response, $c(t)$, to a ramp input, $r(t) = t$.

(b) Create a plot containing both the input and output responses.

Problem 4. Find the inverse Laplace transform of $F(s)$.

$$F(s) = \frac{10}{s\left(s+2\right)\left(s+3\right)^3}$$

Problem 5. Given the following differential equation, use the Laplace transform to solve for $y(t)$ if all initial conditions are zero. You can assume that $u(t)$ is the unit step function.

$$\frac{d^2y}{dt^2} + 12\frac{dy}{dt} + 32y = 32u(t)$$

(a) Create a plot of the output response $y(t)$.

Problem 6. Use Python/Matlab or some other tool to find the Laplace/Inverse Laplace transforms for the following functions.

(a)

$$f(t) = 5t^2 \cos\left(3t + \frac{\pi}{4}\right)$$

(b)

$$f(t) = 5te^{-2t} \sin\left(4t + \frac{\pi}{3}\right)$$

(c)

$$G(s) = \frac{(s^2+3s+7)(s+2)}{(s+3)(s+4)(s^2+2s+100)}$$

(d)

$$G(s) = \frac{(s^3+4s^2+6s+5)}{(s+8)(s^2+8s+3)(s^2+5s+7)}$$