MAE 3134 – Linear System Dynamics
Spring 2017

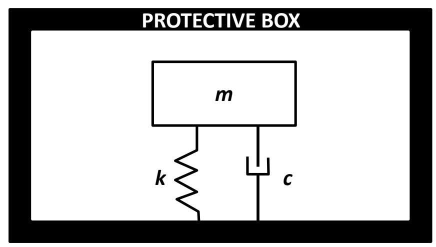
Homework # 3

NOTE: Please make sure to show and explain all the steps followed to arrive at the solution. Otherwise we are unable to assess your understanding of the material and give you credit for your answers.

1. A kitchen door with a mass moment of inertia J_{door} = 20 Kgm^2 is equipped with (i) a damper having a rotational damper constant of 48 Nms/rad and (ii) a rotational spring with stiffness 28.8 Nm/rad. Assuming that the initial position of the door is $\theta = 0$ (closed) and its initial angular speed is 4 rad/s,

(a) Calculate the analytical solution for the door displacement as a function of time.
(b) Plot the analytical solution for the door displacement as a function of time.
(c) Determine *analytically* the maximum displacement of the door with respect to the equilibrium (closed) position.
(d) Calculate *numerically or analytically* the amount of time it takes for the door to return to a displacement equal to 0.01 times the maximum displacement.

2. Consider an electronic component of mass m = 1 Kg packed inside a box that is dropped from a shelf. The box impacts the floor upright with an initial downward velocity of 10 m/s. The electronic component is attached to the box through a spring of force constant k = 9 N/m and a damper with damping constant = 10 Ns/m. Assume that (i) the box does *not* bounce off the floor (i.e., the box sticks to the floor upon impact), (ii) the action of the spring and damper is perfectly aligned with the vertical direction, and (iii) the electronic component is located at $x = -mg/k$ just before the impact, such that we do not need to consider the weight in solving the problem (for simplicity).



(a) Derive an analytical expression for the time-dependent motion of the electronic component.
(b) What is the force experienced by the electronic component upon impact?
(c) How can the parameters of the system be changed to reduce that force, and how do those changes relate to the size of the box that is required to protect the electronic component? Explain your answer mathematically.

3. For the door of problem 1,

(a) Derive the transfer function when the input is an arbitrary torque, $\tau(t)$?
(b) What is the response of the door in the Laplace domain for each of the following three imputs?
    (a) $\tau(t)$ = A δ(t) Nm
    (b) $\tau(t)$ = A cos (ωt)
    (c) $\tau(t)$ = A cosh (ωt)