## Homework # 8

1. Consider an LRC circuit with one inductor, one resistor, one capacitor and one voltage source. Assume that the initial conditions (capacitor charge and current) are zero. Assume also that the above components are arranged clockwise, that the current direction is clockwise and that the voltage is positive for that current direction. Furthermore, $V(t) = 3 \cos (0.75\ t) + 6 \cosh(1.25\ t)$.

   a. Construct the space state and output equations *in matrix form* for the system, for L = 20 H, R = 10 Ω and C = 0.05 F. Consider the charge and current ($q(t)$ and $i(t)$, respectively) as the outputs and the voltage ($V(t)$) as the input.
   b. Identify the matrices **A**, **B**, **C** and **D**
   c. Calculate the transfer matrix of the system.
   d. Use the transfer matrix to find the charge and current *in Laplace space* (you do <u>not</u> need to invert the Laplace expressions).

2. Consider the system of masses shown in the figure below. If $k = 1$ N/m, $c = 0.5$ Ns/m, $m = 2$ Kg, $f_a(t) = t\ u(t)$ and $f_b(t) = \cos(t)\ u(t)$,
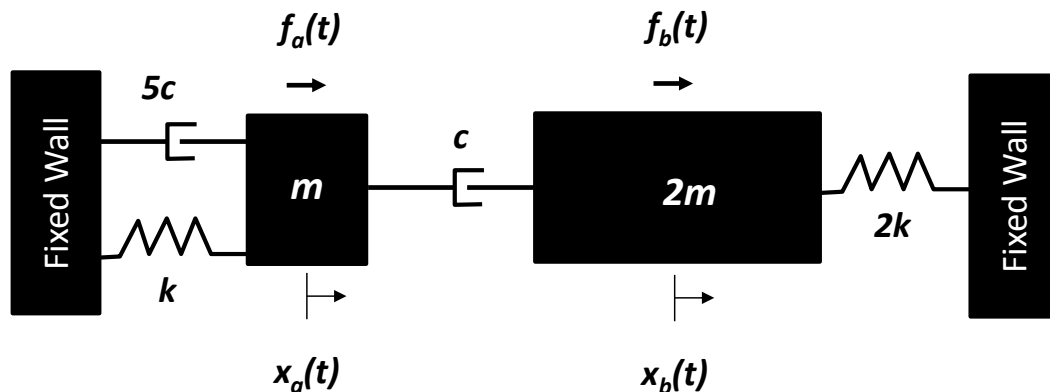
   a. Construct the space state and output equations in matrix form for the system, taking the positions of the two masses and the velocities of the two masses as the outputs, and the two forces as the inputs.
   b. Provide an expression to calculate the transfer matrix of the system. Here you do <u>not</u> need to calculate the inverse of the matrix ($s\mathbf{I} - \mathbf{A}$). You just need to plug in all the values of the matrix elements and simplify $\mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}+\mathbf{D}$ as far as you can.
   c. Use the above expression for the transfer matrix to derive an expression for the output matrix *in Laplace space*. You can leave the matrix operations indicated, simplifying as much as possible, as in the previous step.