

MAE3134: Homework 4

Due date: 6 March 2018

Problem 1. For each second order system below accomplish the following:

1. Find $\zeta, \omega_n, T_s, T_p, T_r$, and $\%OS$.
2. Use Python/Matlab to generate a plot of the response and mark the response specifications on your plots.

You will need to estimate the rise time from a plot of the unit step response.

(a)

$$T(s) = \frac{16}{s^2 + 3s + 16}$$

(b)

$$T(s) = \frac{0.04}{s^2 + 0.02s + 0.04}$$

(c)

$$T(s) = \frac{1.05 \times 10^7}{s^2 + 1.6 \times 10^3 s + 1.05 \times 10^7}$$

Problem 2. For each pair of second order system specifications accomplish the following:

1. Find the regions on the complex plane that will meet the system specifications, i.e. draw the complex plane and identify the locations that will meet the requirements.
2. Write the transfer function that will satisfy the specifications.
3. Verify that your system will meet the specifications by generating the response to a unit step input.
4. On the plots mark the various response specifications.

(a) $\%OS = 12\%, T_s = 0.6 \text{ s}$

(b) $\%OS = 10\%, T_p = 5 \text{ s}$

(c) $T_s = 7 \text{ s}, T_p = 3 \text{ s}$

Problem 3. Find the state space representation for the following transfer functions.

(a)

$$\frac{C(s)}{R(s)} = \frac{24}{s^3 + 9s^2 + 26s + 24}$$

(b)

$$\frac{C(s)}{R(s)} = \frac{s^2 + 7s + 2}{s^3 + 9s^2 + 26s + 24}$$

(c)

$$G(s) = \frac{2s + 1}{s^2 + 7s + 9}$$

Problem 4. Find the transfer function for the following systems. Assume the output is $Y(s)$ and the input is $U(s)$.

(a)

$$\begin{aligned}\dot{\mathbf{x}} &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix} u \\ y &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \mathbf{x}\end{aligned}$$

(b)

$$\begin{aligned}\dot{\mathbf{x}} &= \begin{bmatrix} -4 & -1.5 \\ 4 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 2 \\ 0 \end{bmatrix} u \\ y &= \begin{bmatrix} 1.5 & 0.625 \end{bmatrix} \mathbf{x}\end{aligned}$$