$\boxed{1}$

a) $T(s) = \frac{16}{s^2+3s+16}$

1) $\omega_n = \sqrt{16} = 4$

$\zeta = \frac{3}{2\omega_n} = 0.3750$

$T_s = \frac{4}{\zeta\omega_n} = 2.667$

$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 0.8472$

$T_r = \frac{1}{\omega_n}\cdot(1.76\zeta^3 - 0.417\zeta^2 + 1.039\zeta + 1)$

$= 0.3559$

$\%OS = \exp(-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}) \times 100 = 28.06\%$

2) See MATLAB

b) $T(s) = \frac{0.04}{s^2+0.02s+0.04}$

1) $\omega_n = \sqrt{0.04} = 0.2$

$\zeta = \frac{0.02}{2\omega_n} = 0.05$

$T_s = \frac{4}{\zeta\omega_n} = 400$

$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 15.7276$

$T_r = \frac{1}{\omega_n}(1.76\zeta^3 - 0.417\zeta^2 + 1.039\zeta + 1) = 5.2556$

$\%OS = \exp(-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}) = 85.4468\%$

2) See MATLAB

c) $T(s) = \frac{1.05\times10^7}{s^2+1.6\times10^3 s+1.05\times10^7}$

1) $\omega_n = \sqrt{1.05\times10^7} = 3.2404\times10^3$

$\zeta = \frac{1.6\times10^3}{2\omega_n} = 0.2469$

$T_s = \frac{4}{\zeta\omega_n} = 0.005$

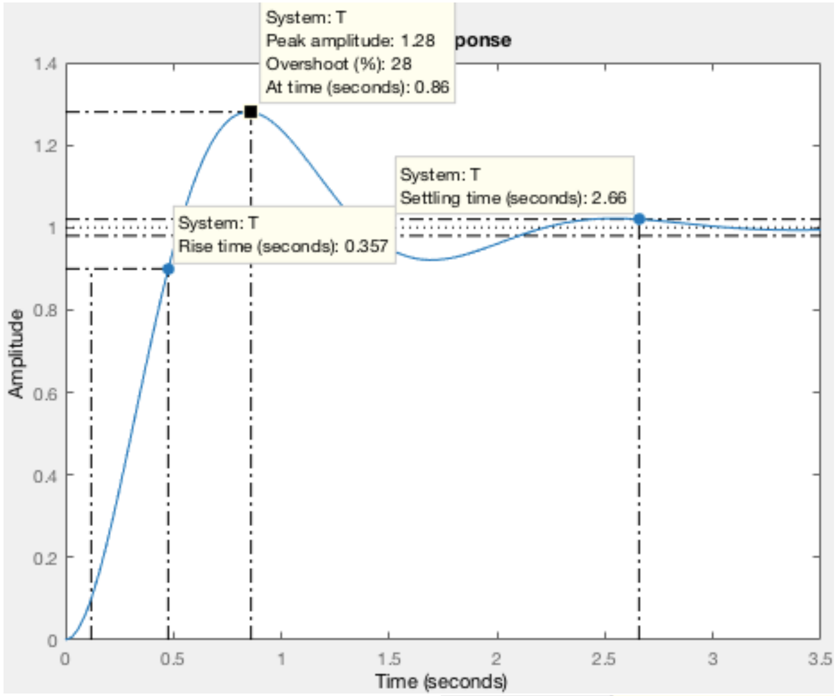
$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 0.001$

$T_r = \frac{1}{\omega_n}(1.76\zeta^3 - 0.417\zeta^2 + 1.039\zeta + 1)$

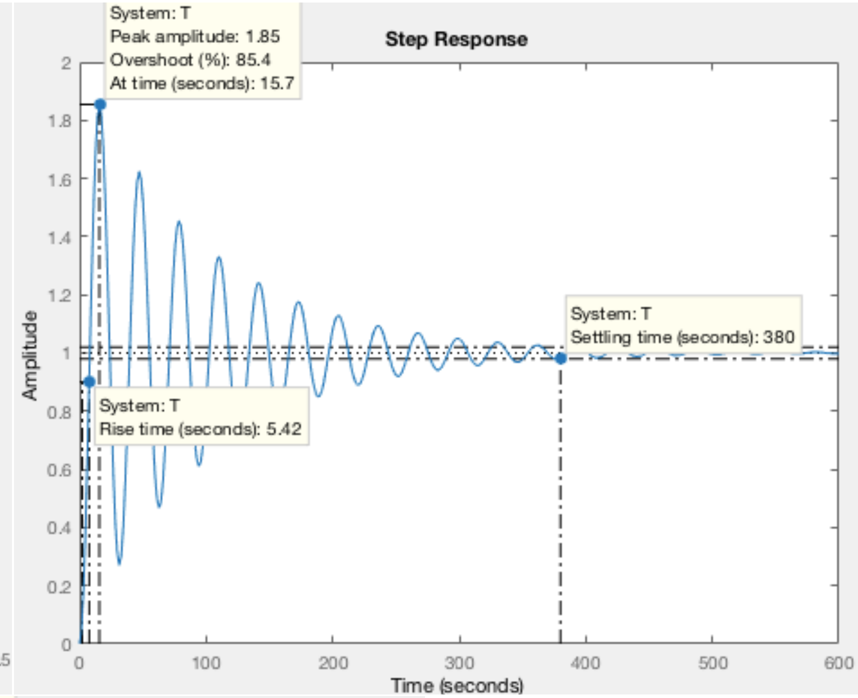
$= 3.8810\times10^{-4}$

$\%OS = \exp(-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}) = 44.9154\%$

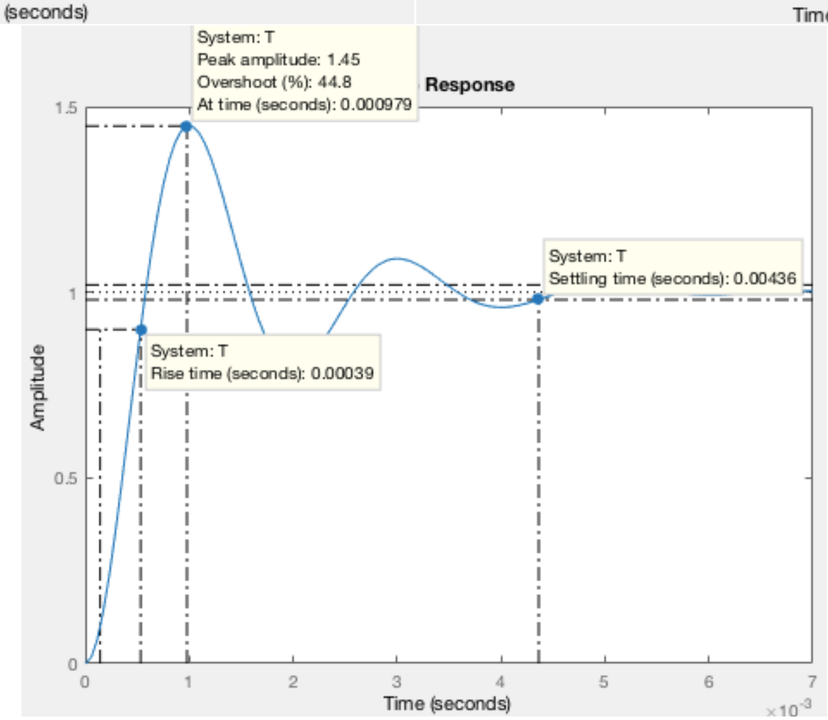
2) See MATLAB



System: T
Peak amplitude: 1.28
Overshoot (%): 28
At time (seconds): 0.86
ponse
System: T
Settling time (seconds): 2.66
System: T
Rise time (seconds): 0.357
Amplitude
Time (seconds)



System: T
Peak amplitude: 1.85
Overshoot (%): 85.4
At time (seconds): 15.7
Step Response
System: T
Settling time (seconds): 380
System: T
Rise time (seconds): 5.42
Amplitude
Time (seconds)

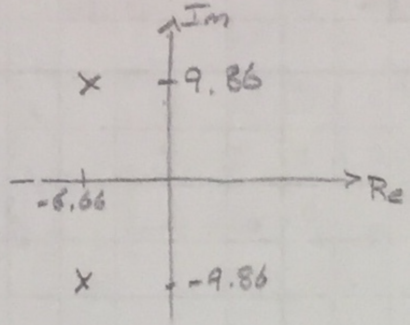


System: T
Peak amplitude: 1.45
Overshoot (%): 44.8
At time (seconds): 0.000979
Response
System: T
Settling time (seconds): 0.00436
System: T
Rise time (seconds): 0.00039
Amplitude
Time (seconds)
×10⁻³

2) $T(s) = \dfrac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

a) $\%OS = 12\%$ $\quad T_s = 0.6s$ $\qquad \zeta = \dfrac{-\ln(\%OS/100)}{\sqrt{\pi^2 + \ln^2(\%OS/100)}} = 0.56$

1)



$\omega_n = \dfrac{4}{\zeta T_s} = 11.9$

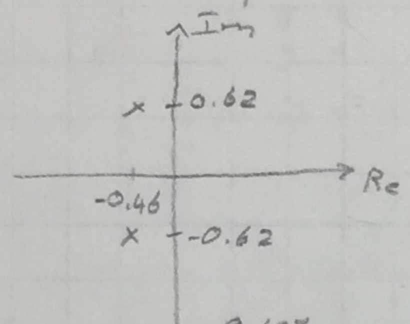
$\omega_d = \omega_n \sqrt{1-\zeta^2} = 9.86$

2) $T(s) = \dfrac{141.6}{s^2 + 13.3s + 141.6}$

3&4) See MATLAB

b) $\%OS = 10\%$ $\quad T_p = 5s$ $\qquad \zeta = \dfrac{-\ln(\%OS/100)}{\sqrt{\pi^2 + \ln^2(\%OS/100)}} = 0.59$

1)



$\omega_n = \dfrac{\pi}{T_p\sqrt{1-\zeta^2}} = 0.779$

$\omega_d = \omega_n\sqrt{1-\zeta^2} = 0.62$

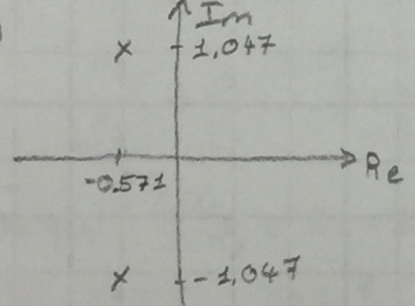
2) $T(s) = \dfrac{0.607}{s^2 + 0.919s + 0.607}$

3&4) see MATLAB

c) $T_s = 7s$ $\quad T_p = 3s$ $\qquad \omega_n = \dfrac{4}{\zeta T_s} \rightarrow \dfrac{4}{\zeta T_s} = \dfrac{\pi}{T_p\sqrt{1-\zeta^2}} \rightarrow 1-\zeta^2 = \dfrac{\pi^2\zeta^2 T_s^2}{T_p^2\, 16}$

$\rightarrow 1 = \zeta^2\left(\dfrac{\pi^2 T_s^2}{16 T_p^2} + 1\right) \rightarrow \zeta = \sqrt{\dfrac{1}{\frac{\pi^2 T_s^2}{16 T_p^2} + 1}} \rightarrow \zeta = 0.479 \rightarrow \omega_n = 1.193$

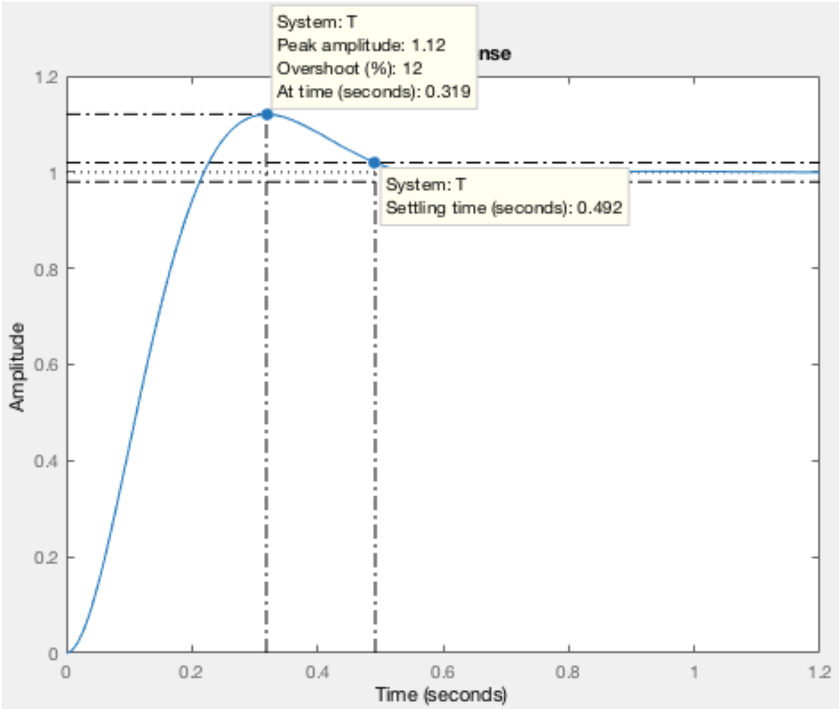
1)

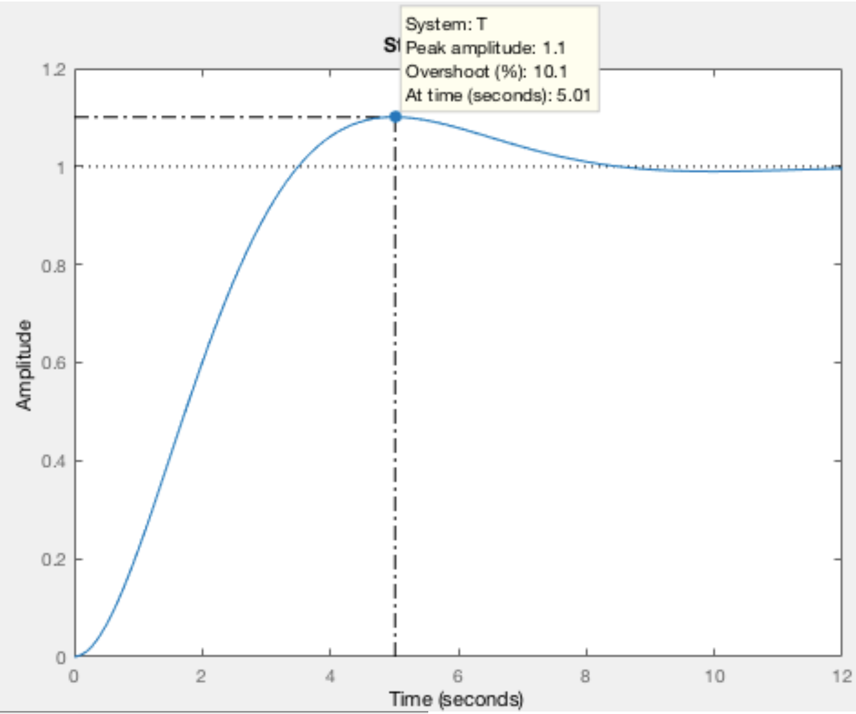


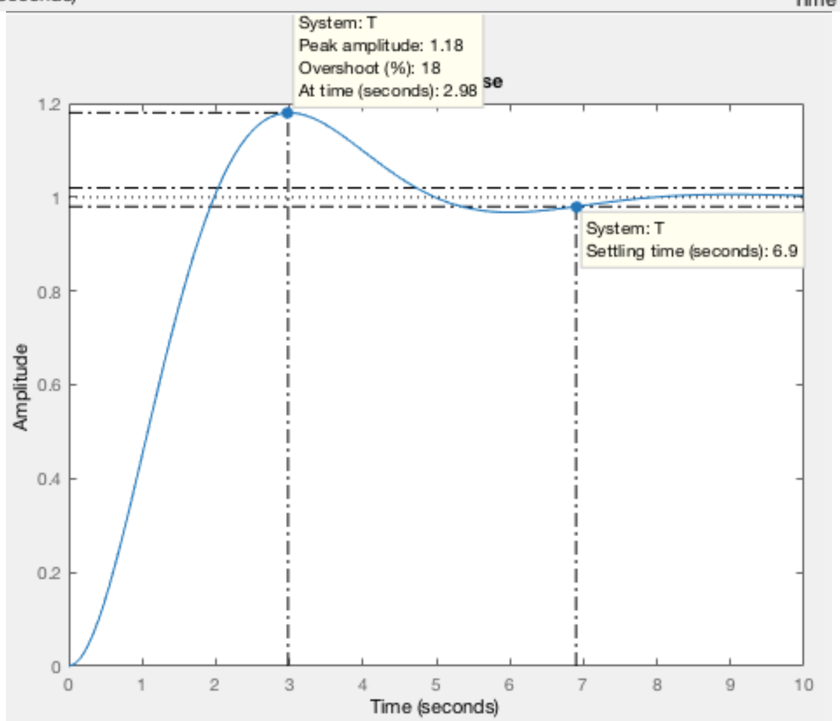
$\omega_d = \omega_n\sqrt{1-\zeta^2} = 1.047$

2) $T(s) = \dfrac{1.423}{s^2 + 1.143s + 1.423}$

3&4) See MATLAB


System: T
Peak amplitude: 1.12
Overshoot (%): 12
At time (seconds): 0.319
nse
System: T
Settling time (seconds): 0.492
1.2
1
0.8
0.6
0.4
0.2
0
Amplitude
0
0.2
0.4
0.6
0.8
1
1.2
Time (seconds)


System: T
Peak amplitude: 1.1
Overshoot (%): 10.1
At time (seconds): 5.01
1.2
1
0.8
0.6
0.4
0.2
0
Amplitude
0
2
4
6
8
10
12
Time (seconds)


System: T
Peak amplitude: 1.18
Overshoot (%): 18
At time (seconds): 2.98
se
System: T
Settling time (seconds): 6.9
1.2
1
0.8
0.6
0.4
0.2
0
Amplitude
0
1
2
3
4
5
6
7
8
9
10
Time (seconds)

3

a) $\frac{C(s)}{R(s)} = \frac{24}{s^3+9s^2+26s+24} \rightarrow \dddot{c}+9\ddot{c}+26\dot{c}+24c = 24r$

choose states: $x_1 = c$ $x_2 = \dot{c}$ $x_3 = \ddot{c}$ $\dot{x}_1 = x_2$ $\dot{x}_2 = x_3$ $\dot{x}_3 = -24x_1 - 26x_2 - 9x_3 + 24r$

$y = x_1 = c$

$$\underbrace{\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -24 & -26 & -9 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 24 \end{bmatrix}}_{B} r$$

$\dot{x} = Ax + Br$ $C = [1\ 0\ 0]$

$y = Cx + Dr$ $D = 0$

b) $\frac{C(s)}{R(s)} = \frac{s^2+7s+2}{s^3+9s^2+26s+24} \rightarrow \frac{C(s)}{R(s)} = \underbrace{\frac{Z(s)}{R(s)}}_{\text{state}} \cdot \underbrace{\frac{C(s)}{R(s)}}_{\text{output}}$

state: $\frac{Z(s)}{R(s)} = \frac{1}{s^3+9s^2+26s+24}$

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -24 & -26 & -9 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r$$

output: $\frac{C(s)}{Z(s)} = s^2+7s+2 \rightarrow C(s) = (s^2+7s+2)Z$

$c = \ddot{z} + 7\dot{z} + 2z$ $x_1 = z$ $x_2 = \dot{z}$ $x_3 = \ddot{z}$ $y = c = x_3 + 7x_2 + 2x_1$

$y = [2\ 7\ 1]x$

c) $G(s) = \frac{2s+1}{s^2+7s+9}$

state: $\frac{Z(s)}{R(s)} = \frac{1}{s^2+7s+9} \rightarrow \ddot{z} + 7\dot{z} + 9z = r$ $x_1 = z$ $x_2 = \dot{z}$ $\dot{x}_1 = x_2$

$\dot{x}_2 = -9x_1 - 7x_2 + r$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -9 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r$$

output: $\frac{C(s)}{Z(s)} = 2s+1 \rightarrow C(s) = (2s+1)Z \rightarrow c = 2\dot{z} + z$

$x_1 = z$ $x_2 = \dot{z}$ $c = y = 2x_2 + x_1$

$y = [1\ 2]x$

$\boxed{4}$

a) $\dot{x} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix}}_{A} x + \underbrace{\begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}}_{B} u \qquad y = \underbrace{\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}}_{C} x$

$D = 0$

$$sI - A = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} = \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 1 & 2 & s+3 \end{bmatrix}$$

$$\Phi = (sI - A)^{-1} = \frac{1}{s^3 + 3s^2 + 2s + 1} \begin{bmatrix} s^2 + 3s + 2 & s+3 & 1 \\ -1 & s(s+3) & s \\ -s & -(2s+1) & s \end{bmatrix}$$

$T(s) = C\Phi B + D$

$$T(s) = \frac{(s^2 + 3s + 2)10}{s^3 + 3s^2 + 2s + 1}$$

b) $\dot{x} = \underbrace{\begin{bmatrix} -4 & -1.5 \\ 4 & 0 \end{bmatrix}}_{A} x + \underbrace{\begin{bmatrix} 2 \\ 0 \end{bmatrix}}_{B} u \qquad y = \underbrace{\begin{bmatrix} 1.5 & 0.625 \end{bmatrix}}_{C} x \qquad D = 0$

$$sI - A = \begin{bmatrix} s+4 & 1.5 \\ -4 & s \end{bmatrix} \qquad \Phi = (sI - A)^{-1} = \frac{1}{s^2 + 4s + 6} \begin{bmatrix} s & -3/2 \\ 4 & s+4 \end{bmatrix}$$

$$T(s) = C\Phi B + D = \frac{3s + 5}{s^2 + 4s + 6}$$

```matlab
%----- Problem 1 Part a -----
clear
close all

s = tf('s');
T = 16/(s^2+3*s+16);

step(T)

wn = sqrt(16);
damp = 3/(2*wn);
Ts = 4/(damp*wn);
Tp = pi/(wn*sqrt(1-damp^2));
Tr = (1/wn)*(1.76*damp^3-0.417*damp^2+1.039*damp+1);
OS = exp(-(damp*pi/sqrt(1-damp^2)))*100;
```

```matlab
%----- Problem 1 Part b -----
clear
close all

s = tf('s');
T = 0.04/(s^2+0.02*s+0.04);

step(T)

wn = sqrt(0.04);
damp = 0.02/(2*wn);
Ts = 4/(damp*wn);
Tp = pi/(wn*sqrt(1-damp^2));
Tr = (1/wn)*(1.76*damp^3-0.417*damp^2+1.039*damp+1);
OS = exp(-(damp*pi/sqrt(1-damp^2)))*100;
```

```matlab
%----- Problem 1 Part c -----
clear
close all

s = tf('s');
T = (1.05*10^7)/(s^2+(1.6*10^3*s)+(1.05*10^7));

step(T)

wn = sqrt(1.05*10^7);
damp = (1.6*10^3)/(2*wn);
Ts = 4/(damp*wn);
Tp = pi/(wn*sqrt(1-damp^2));
Tr = (1/wn)*(1.76*damp^3-0.417*damp^2+1.039*damp+1);
OS = exp(-(damp*pi/sqrt(1-damp^2)))*100;
```

```
%----- Problem 2 Part a -----
clear
close all

s = tf('s');
T = 1/(s^2+13.3*s+141.6);

step(T)
```

```
%----- Problem 2 Part b -----
clear
close all

s = tf('s');
T = 1/(s^2+0.919*s+0.607);

step(T)
```

```
%----- Problem 2 Part c -----
clear
close all

s = tf('s');
T = 1/(s^2+1.143*s+1.423);

step(T)
```

```
%----- Problem 4 Part a -----
clear

syms s

A = [0 1 0; 0 0 1; -1 -2 -3];
B = [10; 0; 0];
C = [1 0 0];
D = 0;

Phi = inv(s*eye(3)-A);

T = C*Phi*B+D;

[n,d] = ss2tf(A, B, C, D);
```

```
%----- Problem 4 Part b -----
clear

syms s

A = [-4 -1.5; 4 0];
B = [2; 0];
C = [1.5 0.625];
D = 0;

Phi = inv(s*eye(2)-A);

T = C*Phi*B+D;

[n,d] = ss2tf(A, B, C, D);
```