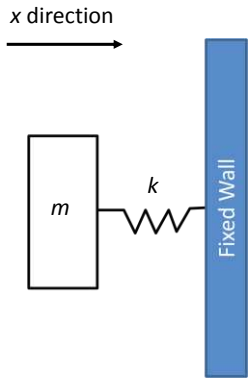
The George Washington University
MAE 3134 – Linear System Dynamics

Midterm Exam

Problem # 1 [25 points]

An engineer is working with a vibratory system and has determined that its transfer function is $G(s) = \frac{X(s)}{F(s)} = \frac{1}{(s+1)^2}$. Calculate the time-dependent response of this system to an input consisting of a double hammer impact. The first impact is given at *t* = 1 s and the second at *t* = 1.25 s. Both impacts have identical magnitude γ Kg m/s. The initial velocity and position (before the impacts are given) are zero.

Problem # 2 [25 points]

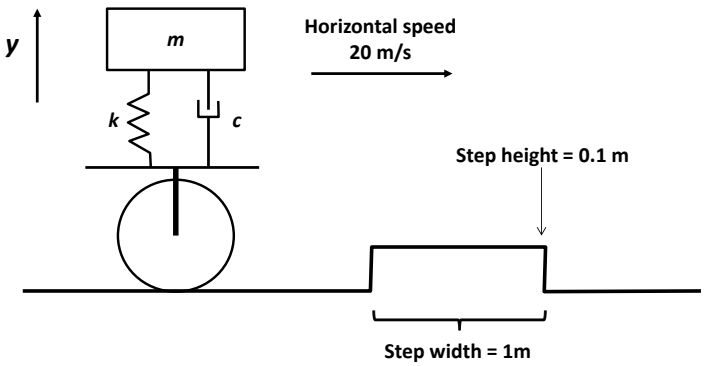


A scientist is conducting experiments with the undamped oscillator shown in the figure. The oscillator is set into motion with the initial conditions, $x(0) = 2\,m$, and $\dot{x}(0) = 0$. *m* = 4 Kg and *k* = 9 N/m.

(i) Derive an expression for the time-dependent motion of the mass, *x*(t). ***[10 points]***

(ii) The scientist wishes to stop the motion of the mass through a single impact, which is to be given when the mass crosses the equilibrium position for the *second* time. Provide an expression for the required *f*(t). ***[15 points]***

Problem # 3 [50 points]



Consider the model for a car-tire system in the figure, in which the vertical position of the mass exhibits a time-dependent response due to changes in the road topography. The parameters are *m* = 1000 Kg, *k* = 4000 N/m and *c* = 2000 Ns/m.

(i) Write the equation of motion of the system. ***[5 points]***

(ii) Derive the transfer function of the system for the vertical motion of the mass, taking the road height as the input. ***[10 points]***

(iii) Provide an expression for the response of the mass in the Laplace domain, *Y*(*s*) when the input is a unit step (y_{tire} = u(t)) and the initial conditions are zero. ***[5 points]***

(iv) Calculate the time-dependent response for the Laplace-domain expression you derived for *Y*(*s*) in step (iii). ***[15 points]***

(v) Calculate the time-dependent vertical response of the mass in the car-tire system to the road feature shown in the figure. Take *t* = 0 as the instant when the tire encounters the up-step. The initial conditions in the vertical direction $y(0) and\ \dot{y}(0)$ are both zero. The car is traveling at a horizontal speed of 20 m/s. ***[15 points]***