MAE 3134 – Linear System Dynamics

Homework # 7

Consider an LRC circuit with one inductor, one resistor, one capacitor and one voltage source. Assume that the initial conditions (capacitor charge and current) are zero. Assume also that the above components are arranged clockwise, that the current direction is clockwise and that the voltage is positive for that current direction.

1. Draw the circuit describe above.

2. For each of the three cases below, find $q(t)$ and $i(t)$ using the Laplace transform method (make sure to show all the algebra steps required to reach the solution):

    a. **Case 1:** L = 10 H; R = 20 Ω; C = 0.1 F; V = 2 V
    b. **Case 2:** L = 10 H; R = 40 Ω; C = 0.1 F; V = 2 V
    c. **Case 3:** L = 10 H; R = 5 Ω; C = 0.1 F; V = 2 V

3. Plot $q(t)$ for all three cases of part 2 together on the same graph, plot $i(t)$ for all three cases together on a separate graph (there will be a total of two graphs: one for $q(t)$ for all cases together, and one for $i(t)$ for all cases together) and answer the following questions:

    a. If the responses corresponded to mechanical systems instead of electrical systems, indicate which response would be underdamped, which response would be critically damped and which response would be overdamped, explaining what features of the solutions lead to the conclusions you reached.

    b. If the above responses corresponded to mechanical systems instead of electrical systems, what would be the damping factor ζ in each of the three cases of part 2? Give the equivalent mass, damper constant and force constant in each case.

4. Calculate the steady state solution for $q(t)$ and $i(t)$ for case 3 of part 2 for V = 10 sin(0.05 t). To solve this part, use the equations that were discussed in class for the response of a vibratory mechanical system to a sinusoidal input force.