MAE3134: Homework 6

Due date: 12 April 2018

Problem 1. Find $Y(s)$ for the following state space system

$$\dot{x} = \begin{bmatrix} 1 & 2 \\ -3 & -1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \sin 3t$$

$$y = \begin{bmatrix} 1 & 2 \end{bmatrix} x$$

with initial condition x_0

$$x(0) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

Problem 2. Find $Y(s)$ for the following state space system

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ -2 & -4 & 1 \\ 0 & 0 & -6 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} e^{-t}$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x$$

with initial condition x_0

$$x(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Problem 3. Find $y(t)$ for the following state space system, where $u(t)$ is the unit step

$$\dot{x} = \begin{bmatrix} -2 & 0 \\ -1 & -1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t)$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} x$$

with initial condition x_0

$$x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Problem 4. Solve for the state transition matrix, the state vector, and the output of the state space system where $u(t)$ is the unit step

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

$$y = \begin{bmatrix} 3 & 2 \end{bmatrix} x$$

with initial condition

$$x(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Problem 5. Solve for the state transition matrix, the state vector, and the output of the state space system where $u(t)$ is the unit step

$$\dot{\boldsymbol{x}} = \begin{bmatrix} -2 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -6 & -1 \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u(t)$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \boldsymbol{x}$$

with initial condition

$$\boldsymbol{x}(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Problem 6. Compute e^{At} for the following matrices.

$$A = \begin{bmatrix} 3 & -5 \\ -5 & 3 \end{bmatrix},$$

$$A = \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix},$$

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},$$

$$A = \begin{bmatrix} -11 & 20 \\ -6 & 11 \end{bmatrix}.$$