MAE3134: Homework 0 - Skills Review

Due date: TBD

Problem 1. Consider the general n-th order ordinary differential equation

$$F(t, y(t), y'(t), \ldots, y^{(n)}(t)) = 0.$$

(a) What general form must F have for the equation to be linear?

Classify the following equations as linear or non-linear, state their order, and identify the dependent and independent variables as well as any non-linear terms:

(b) $t\frac{d^2y}{dt^2} + t^2\frac{dy}{dt} + t^3y = \cos t$

(c) $t\frac{d^3y}{dt^3} + t^2\frac{dy}{dt} + t^3y = \cos y$

(d) $\frac{dy}{dx} = \frac{2y-3}{2x+2}$

(e) $(\cos t)\frac{d^2y}{dt^2} + (\sin 2t)\, y = 0 \quad y = y(t)$

Classify the following equations as **ordinary** or **partial** differential equations, also indicate the dependent and independent variables:

(f) $\frac{dx}{dt} + \frac{dy}{dt} + x + y = 0 \quad x = x(t) \quad y = y(t)$

(g) $\frac{df}{dx} + \frac{df}{dy} + x + y = 0 \quad f = f(x, y)$

(h) $\frac{d}{dt}\left[\frac{df}{dx}\right] = 0 \quad f = x^2 + \frac{dx}{dt}$

(i) $\frac{df}{dx} = x \quad f = y^2(x) + \frac{dy}{dx}$

Classify the following linear differential equations as either **time-invariant** or **time-variable**. Indicate any time-variable terms.

(j) $\frac{d^2y}{dt^2} + 2y = 0$

(k) $\frac{d}{dt}\left(t^2y\right) = 0$

(l) $\left(\frac{1}{t+1}\right)\frac{d^2y}{dt^2} + \left(\frac{1}{t+1}\right)y = 0$

(m) $\frac{d^2y}{dt^2} + (\cos t)\, y = 0$

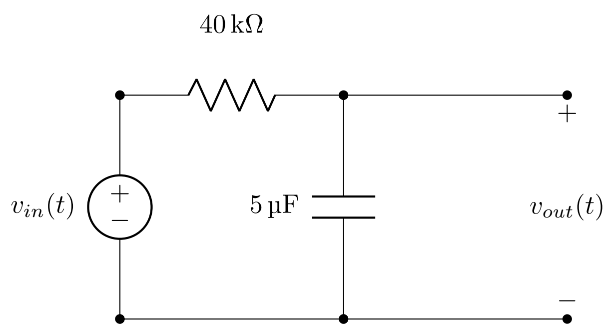

Figure 1: Electrical System

Problem 2. The differential equation relating $v_{out}(t)$ to $v_{in}(t)$ for this circuit is given by:

$$\frac{dv_{out}}{dt} + 5v_{out} = 5v_{in}(t).$$

(a) If $v_{in}(t) = 2\,\mathrm{V}$ and $v_{out}(0) = 0\,\mathrm{V}$, find $v_{out}(t)$ using either the method of undetermined coefficients or the Laplace transform (show your work).

If $v_{out}(t) = 2\left(1 - e^{-5t}\right)$:

(b) What is the steady-state value (value at $t \to \infty$) of v_{out}?

(c) When does v_{out} reach 10 % of its steady-state value?

(d) When does v_{out} reach 90 % of its steady-state value?

(e) When does v_{out} reach 98 % of its steady-state value?

Problem 3. The motion of a particle is described by :

$$y = 0.7 \cos\left(\frac{\pi}{3}t + \frac{\pi}{6}\right)$$

where y represents the position of the particle in meters and t is in seconds.

(a) What is the value of the initial displacment?

(b) What is the value of the initial velocity?

(c) What is the initial acceleration?

(d) What is the maximum velocity?

(e) What is the value of t when y reaches the first maximum (the first positive peak)?

(f) Using the programming language of your choice (i.e. Matlab, Python etc), generate a plot of the motion for $t \in [0, 20]$s.

Problem 4. A linear system's time response is given by

$$x(t) = 0.1e^{-5t}.$$

By hand, draw an accurate approximation for the plot of $x(t)$ versus t. Label your axes.

Problem 5. Given the following matrices:

$$A = \begin{bmatrix} 0 & 1 \\ -10 & 2 \end{bmatrix} \qquad [sI - A] = \begin{bmatrix} s & -1 \\ 10 & s-2 \end{bmatrix}$$

Determine the following.

(a) Find the eigenvalues of A.

(b) Find the inverse of $sI - A$ analytically, and validate your answer.

Problem 6. Given the complex numbers $a = -2 + 0.5j$ and $\lambda = -1 + 3j$.

(a) What are the complex conjugates of a, λ, i.e a^*, λ^*?

(b) Express a, a^* in polar form. Recall: the polar form of a complex number a is $\|a\| e^{j\phi}$ where ϕ is the angle of a expressed in radians.

(c) Find the complex number $b = a + \lambda$.

(d) Find the complex number $c = a\lambda$ (multiplication).

(e) We define the complex plane as the two dimensional plane with the real axis along the horizontal direction and the imaginary axis along the vertical direction. For the four complex numbers (a, b, c, λ computed above, plot their location on the complex plane. In addition, mark the angle and radius of each vector on your plot.