Lesson 26
ASYMPTOTIC BODE PLOT CONSTRUCTION RULES

TRANSFER FUNCTION ELEMENT		GAIN	PHASE
Constant		constant 20 log K_B	constant 0° (-180° if negative gain)
Free "s"	Pole	-20 dB/dec crossing 0 dB @ ω = 1	constant -90°
$j\omega$	Zero	20 dB/dec crossing 0 dB @ ω = 1	constant +90°
1st Order	Pole	constant 0 dB until corner frequency (σ) then -20 dB/dec	starts @ 0° and ends @ -90° -45°/dec starting at 0.1σ and ending at 10σ equals -45° at σ
$\left(\frac{s}{\sigma}+1\right)$	Zero	constant 0 dB until corner frequency (σ) then +20 dB/dec	starts @ 0° and ends @ +90° +45°/dec starting at 0.1 σ and ending at 10 σ equals +45° at σ
2nd Order	Pole	constant 0 dB until corner frequency (ω_n) then -40 dB/dec $M_{\omega_n} = -20\log(2\zeta)$ at ω_n Resonance for $\zeta < 0.707$ such that: $\omega_r = \omega_n\sqrt{1-2\zeta^2}; M_r = -20\log(2\zeta\sqrt{1-\zeta^2})$	starts @ 0° and ends @ -180° -90°/dec starting at 0.1 ω_n and ending at 10 ω_n equals -90° at ω_n
$\left(\frac{s^2}{\omega_n^2}+\frac{2\zeta s}{\omega_n}+1\right)$	Zero	constant 0 dB until corner frequency (ω_n) then +40 dB/dec $M_{\omega_n} = +20\log(2\zeta)$ at ω_n Resonance for $\zeta < 0.707$ such that: $\omega_r = \omega_n\sqrt{1-2\zeta^2}; M_r = +20\log(2\zeta\sqrt{1-\zeta^2})$	starts @ 0° and ends @ +180° +90°/dec starting at 0.1 ω_n and ending at 10 ω_n equals +90° at ω_n

EXAMPLE

1. FIND FREQ. RESP

2. APPROX. BODE PLOT.

$$G(s) = \frac{s+20}{(s+1)(s+7)(s+50)}$$

FREQ. RESP — $G(j\omega) = \dfrac{j\omega+20}{(j\omega+1)(j\omega+7)(j\omega+50)}$

USE ALGEBRA TO GET $G(j\omega) = a + bj$

$$(j\omega+1)(j\omega+7) = j^2\omega^2 + 7\omega j + \omega j + 7$$

$$= -\omega^2 + 8\omega j + 7 = (7-\omega^2) + 8\omega j$$

$$((7-\omega^2)+8\omega j)(j\omega+50) = (7-\omega^2)j\omega + 50(7-\omega^2) + 8\omega^2 j^2 + 50(8\omega j)$$

$$= 7j\omega - \omega^3 j + 50(7) - 50\omega^2 - 8\omega^2 + 50(8)\omega j$$

$$= [50(7) - 50\omega^2 - 8\omega^2] + [7\omega - \omega^3 + 50(8)\omega]j$$

$$= (350 - 58\omega^2) + (407\omega - \omega^3)j \longleftarrow \text{DENOMINATOR}$$

$$G(j\omega) = \frac{20 + j\omega}{(350-58\omega^2)+(393\omega-\omega^3)j} \cdot \frac{(350-58\omega^2)-(393\omega-\omega^3)j}{(350-58\omega^2)-(393\omega-\omega^3)j}$$

DENOMINATOR

$$D = (350-58\omega^2)^2 + (393\omega-\omega^3)^2$$

NUMERATOR

$$N = 20(350 - 58\omega^2) + \omega(393\omega - \omega^3) + [\omega(350 - 58\omega^2) - 20(393\omega - \omega^3)]j$$

$$G(j\omega) = \frac{20(350 - 58\omega^2) + \omega(393\omega - \omega^3)}{D} + \frac{\omega(350 - 58\omega^2) - 20(393\omega - \omega^3)}{D} j$$

APPROXIMATE BODE PLOT

$$G(s) = \frac{20}{(1)(7)(50)} \frac{s/20 + 1}{\left(\frac{s}{1} + 1\right)\left(\frac{s}{7} + 1\right)\left(\frac{s}{50} + 1\right)}$$

1. BREAK FREQ. $\omega = 1, 7, 20, 50$

2. FIND CONTRIBUTION of EACH COMPONENT.

CONSTANT 0° PHASE $20\log\frac{20}{7.50} = -24.86$ dB

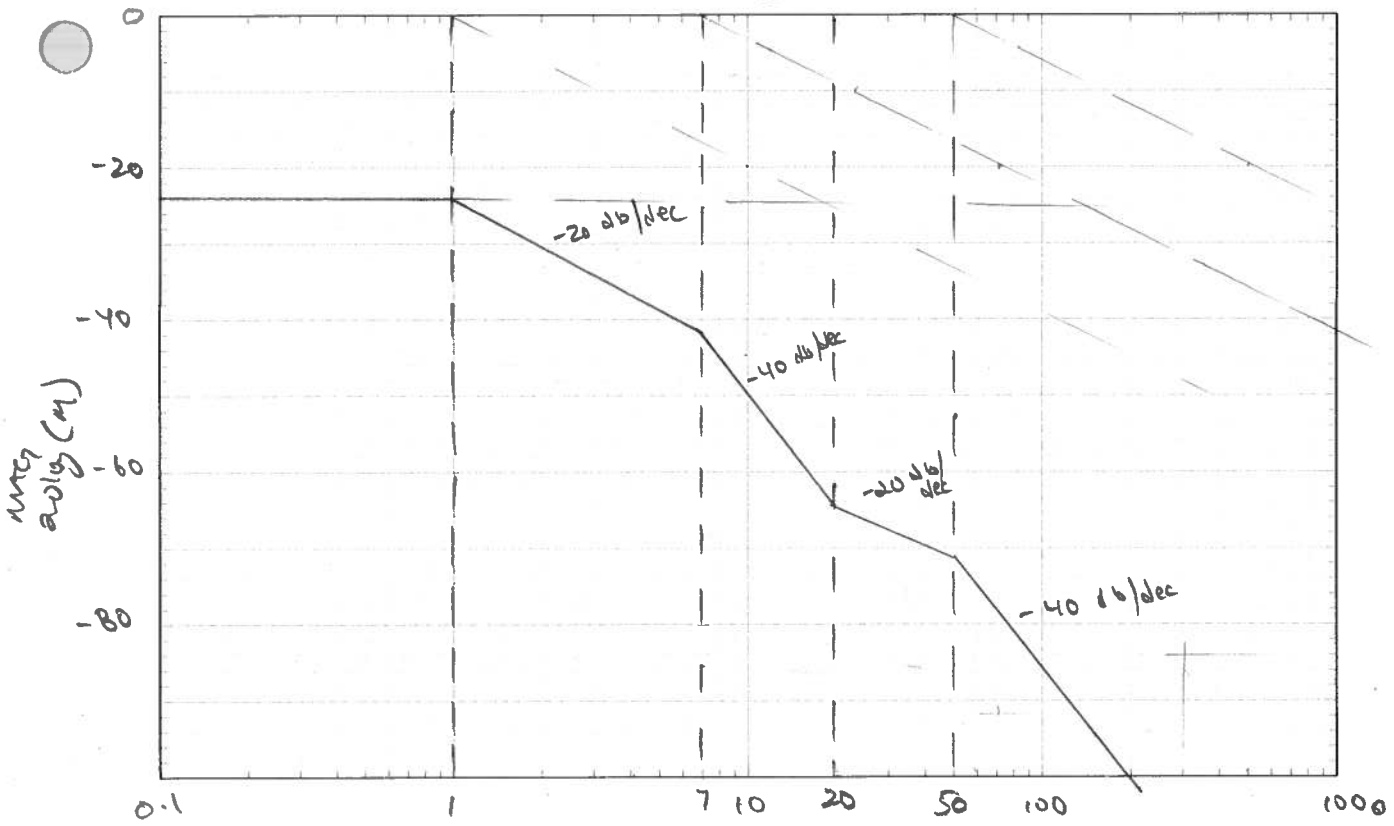
ZERO MAG $+20\frac{dB}{dec}$ @ $\omega = 20$

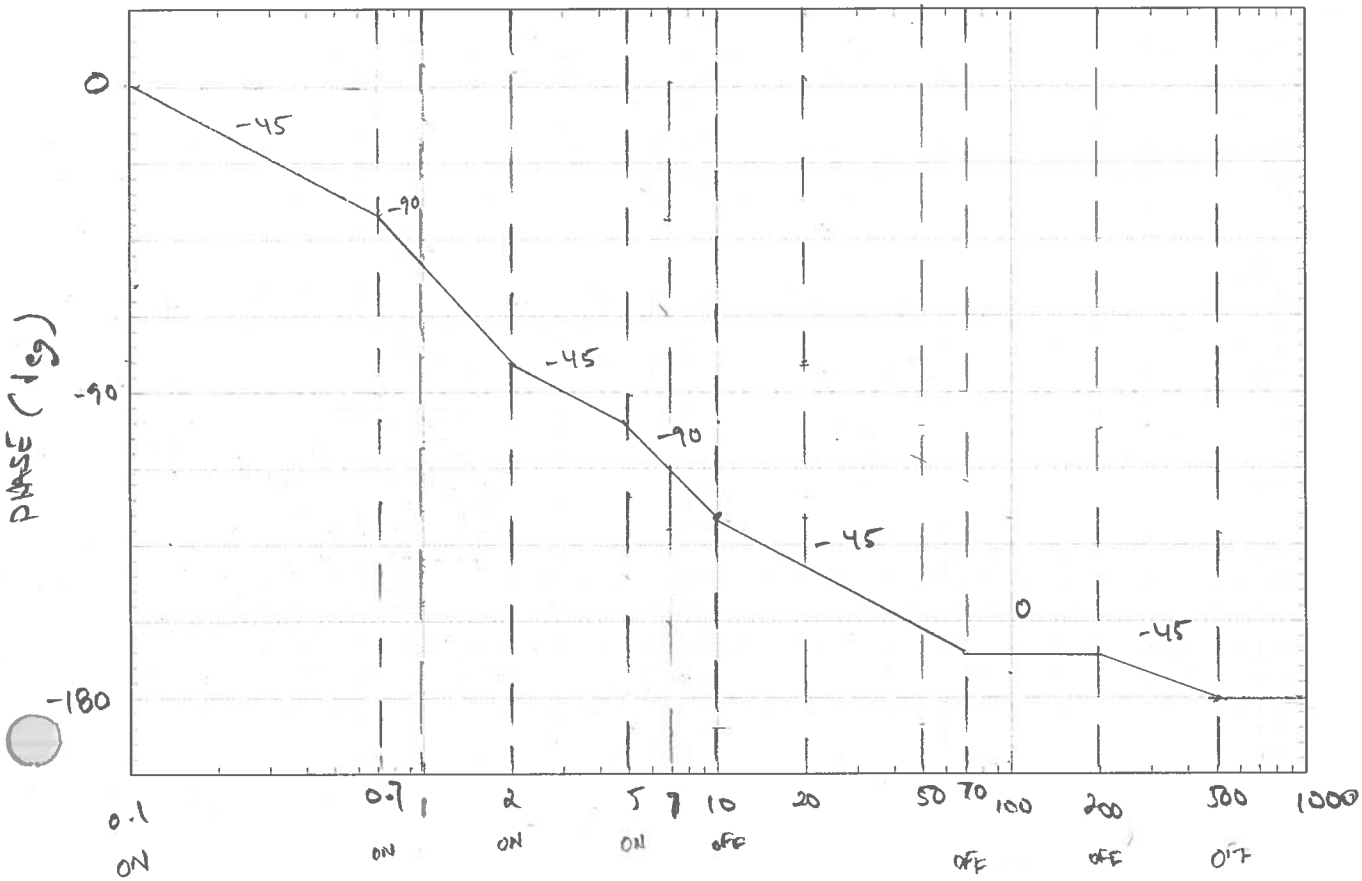
PHASE $0° \rightarrow +90°$ BTWN $\omega = 2 \rightarrow \omega = 200$

POLES MAG $-20\frac{dB}{dec}$ @ $\omega = \omega^*$

PHASE $0° \rightarrow -90°$ @ $\omega = 0.1\omega^* \rightarrow 10\omega^*$

$$G(s) = \frac{s+20}{(s+1)(s+7)(s+50)}$$





BODE PLOT EXAMPLES

CONSIDER $G(s) = \dfrac{10(s+1)}{(s+10)} = \dfrac{10/10\,(s+1)}{\left(\frac{s}{10}+1\right)}$

FREQ. RESPONSE FCN $G(j\omega) = \dfrac{j\omega+1}{\left(\frac{j\omega}{10}+1\right)}$

GENERATE A BODE PLOT (APPROXIMATION)

M { ZERO $\omega = 1$ BREAK FREQ $\longrightarrow$ $+20\ \frac{dB}{dec}$
  { POLES $\omega = 10$ $-20\ \frac{dB}{dec}$

ϕ { ZERO $0 \to +90^\circ$ BTWN $\omega = 0.1$ AND $\omega = 10$
  { POLE $0 \to -90^\circ$ BTWN $\omega = 1$ AND $\omega = 100$

CHECK MAG + PHASE AT LOW + HIGH FREQ.

LOW $\lim_{\omega \to 0} G(j\omega)$

$G(j\omega) \to 1 \to \boxed{0\,dB < 0^\circ}$

HIGH $\lim_{\omega \to \infty} G(j\omega) =$

$G(j\omega) \to \dfrac{j\omega}{j\omega/10} \to 10 \Rightarrow \boxed{20\,dB < 0^\circ}$

$$G(s) = \frac{s+1}{\left(\frac{s}{10}+1\right)}$$