Spring 2017 MAE3134: Final Exam

11 May 2017

Resources allowed: Open notes/book, calculator, ruler. No computers or mobile devices.

Name: ___________________________ GWID:___________________________

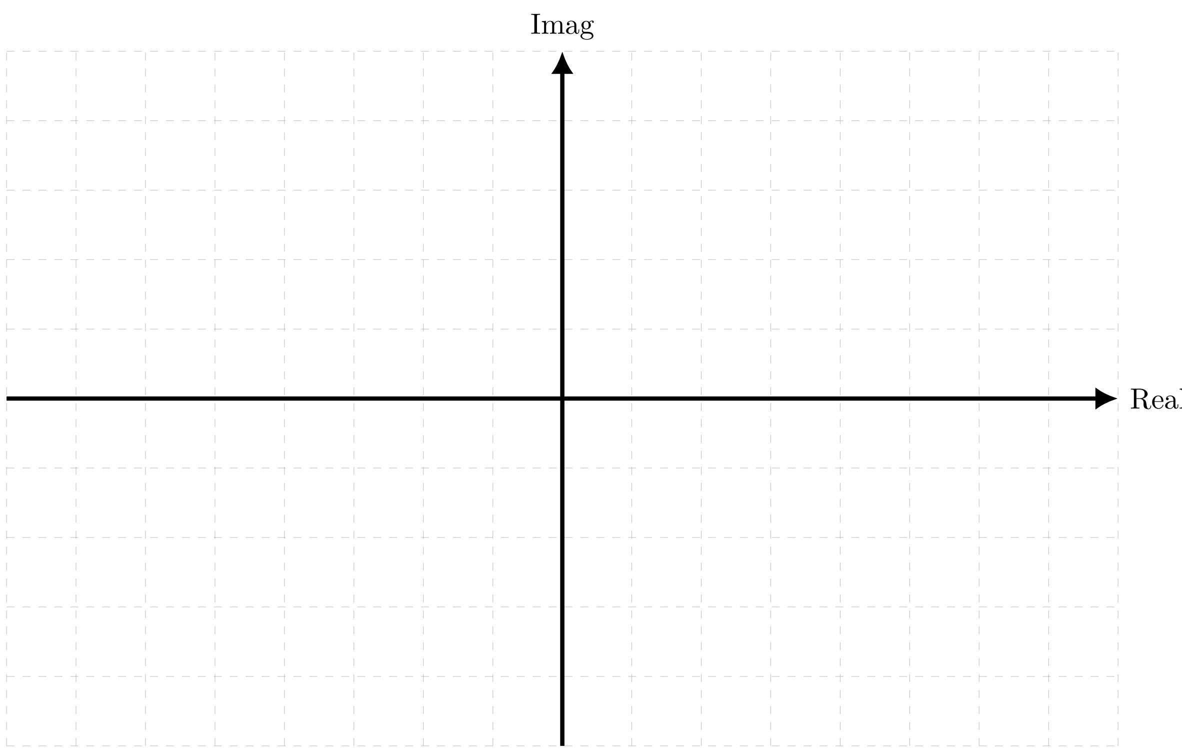
Prob. 1	Prob. 2	Prob. 3	Prob. 4	Prob. 5	Prob. 6	Prob. 7	Prob. 8	Total
10	5	20	20	10	5	10	20	100

Problem 1 Elon Musk, CEO of SpaceX and Tesla Motors, has a background in physics but unfortunately has never passed a Linear Dynamics course. His newest space vehicle must satisfy the following second order time response specifications for a unit step input:

- Percent Overshoot must be less than 5%,
- Peak time less than 1 s,
- Settling Time less than 5 s.

Elon needs your help to choose a set of poles which will satisfy the specifications and save humanity from impending disaster.

1. On the s-plane, or complex plane, map out the acceptable regions where you could locate poles and meet the requirements.
2. Label the specifications lines and show your work.
3. Choose a set of poles that will meet the requirements.



Problem 2 The frequency response of two systems are shown in Figure 1. Using the plots, circle the correct descriptions:

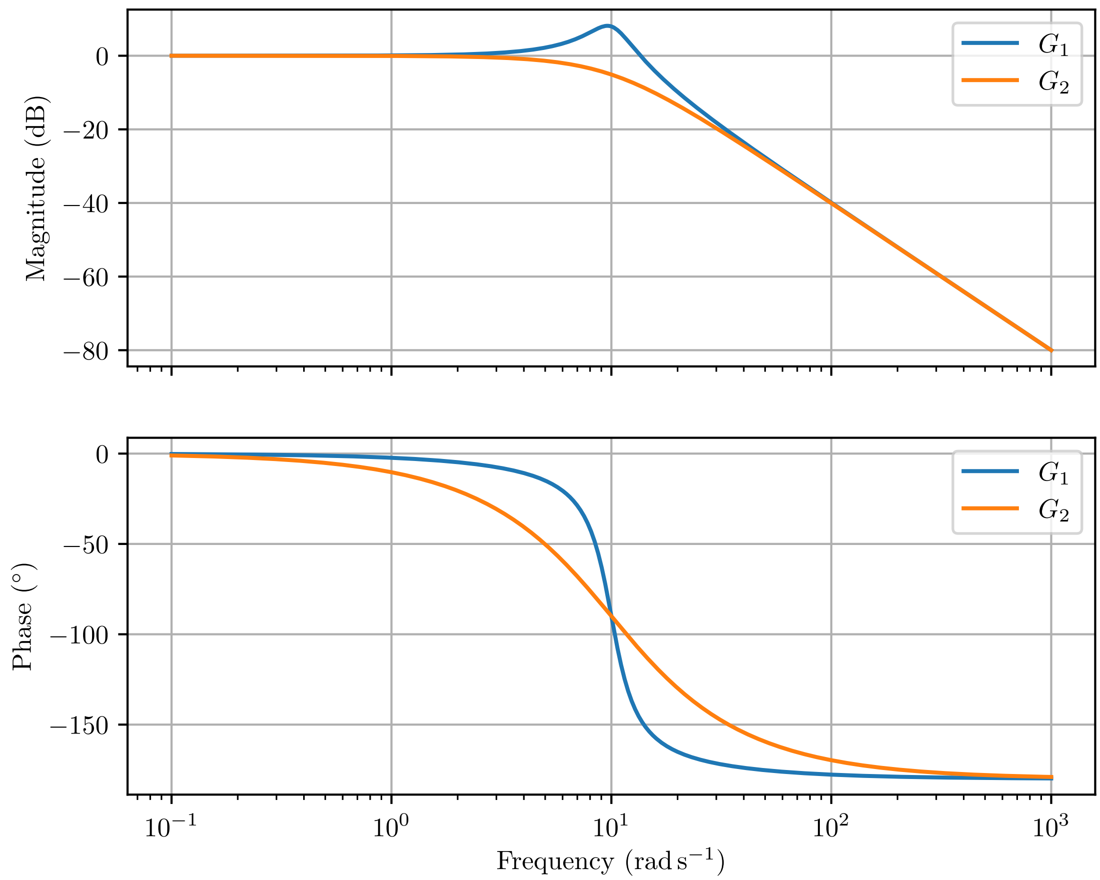


Figure 1: Frequency Response

1. Which of the following statements are true about the damping ratios of the two systems?
   (a) The damping coefficients are the same.
   (b) The damping coefficient of G_1 is greater than the damping coefficient of G_2.
   (c) The damping coefficient of G_2 is greater than the damping coefficient of G_1.
   (d) Not enough information to make any statements about the damping ratio.
2. Which of the following statements are true about the general form of G_1?
   (a) It is a first order system.
   (b) It must have two free s terms in the denominator since the phase ends at 180°.
   (c) It must have two free s terms in the numerator since the final magnitude slope is 40 dB per decade.
   (d) None of the above.

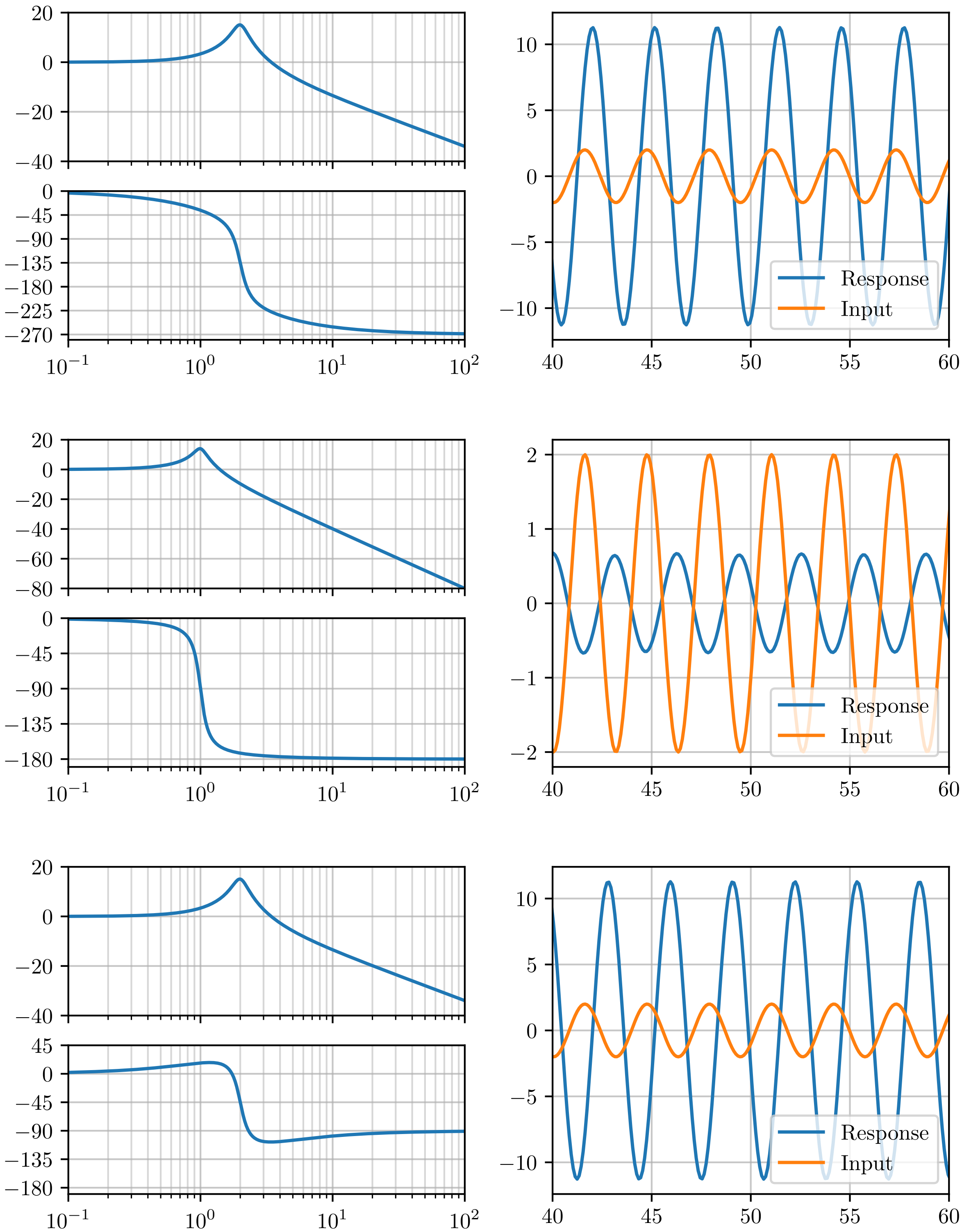
Problem 3 The transfer functions of three systems are given as follows:

$$G_1 = \frac{1}{s^2 + 0.2s + 1}, \qquad G_2 = \frac{2s + 4}{s^2 + 0.5s + 4}, \qquad G_3 = \frac{-2s + 4}{s^2 + 0.5s + 4}.$$

You should accomplish the following tasks:

1. Match each Bode plot shown in Figure 2a with the appropriate transfer function by indicating on each plot the correct transfer function (i.e. G_1, G_2, or G_3). Explain the reasoning that lead to your solution.

2. Match each response plot shown in Figure 2b with the correct transfer function by indicating on each plot the correct transfer function. The same input of $u = 2 \sin 2t$ is applied to each system. Explain the reasoning for your solution. **Note: Figures 2a and 2b are not in the same order.**



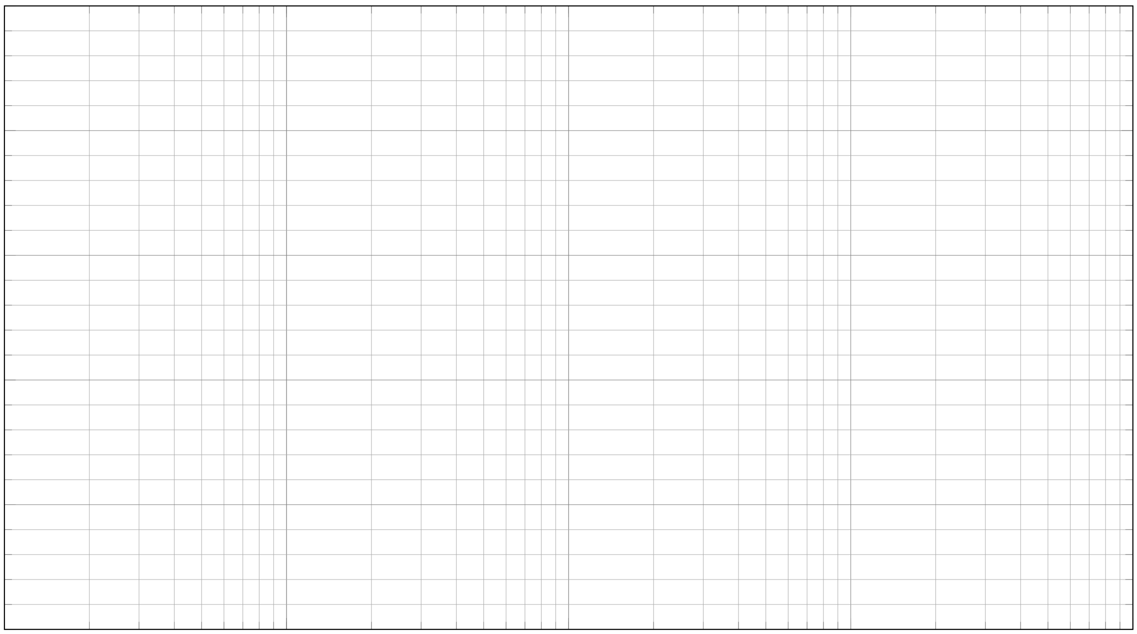
(a) Bode Plots (b) Sinusoidal Response

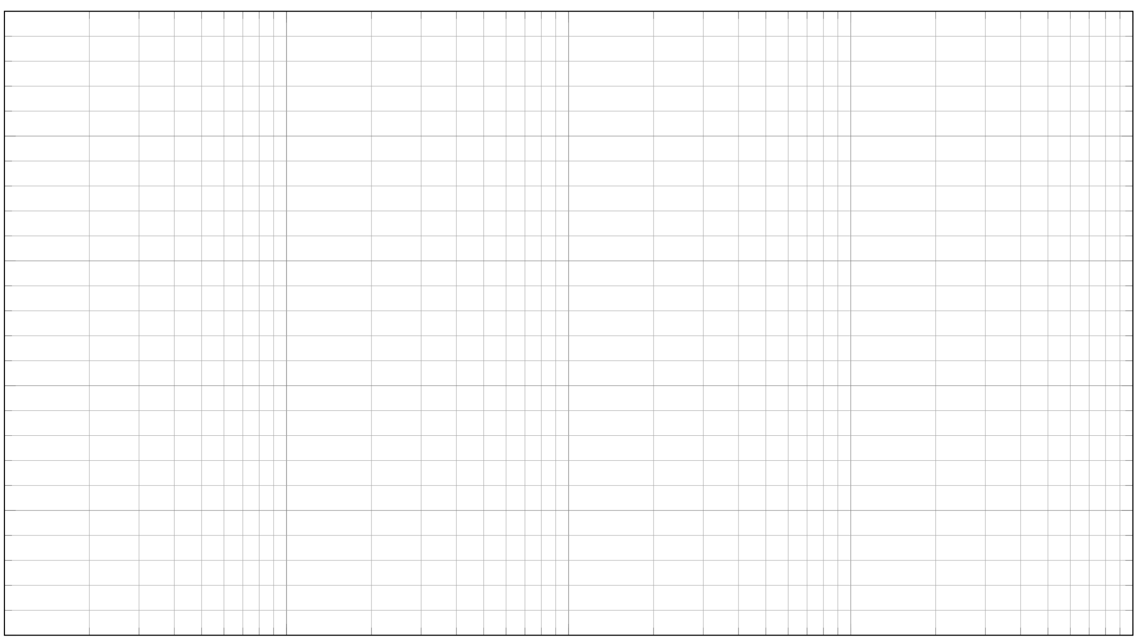
Figure 2: Problem 3 Bode and Sinusoidal Responses

Problem 4 A transfer function is defined as

$$G(s) = \frac{500(s+40)}{s^2 + 8s + 25}.$$

1. Draw the asymptotic Bode plots for this system.
2. What is the steady state output for an input of $u = 5 \sin 25t$?





Problem 5 Given the state space representation defined as

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + B\boldsymbol{u},$$
$$\boldsymbol{y} = C\boldsymbol{x} + D\boldsymbol{u},$$

DERIVE the expression for the transfer function $\frac{\boldsymbol{Y}(x)}{\boldsymbol{U}(s)}$.

Problem 6 List at least two advantages of state-space or "modern control" techniques as compared to "classical control" approaches.

Problem 7 For the electrical system in Figure 4:

1. Find the differential equations of motion for the system.
2. Find the state space representation of the system with your state vector defined as

$$\boldsymbol{x} = \begin{bmatrix} q_1 & i_1 & q_2 & i_2 \end{bmatrix}^T,$$

   where q_1, i_1 represent the charge and current in the left loop while q_2, i_2 represent the charge and current in the right loop. The output is defined as

$$y = \begin{bmatrix} q_1 & q_2 \end{bmatrix}^T.$$

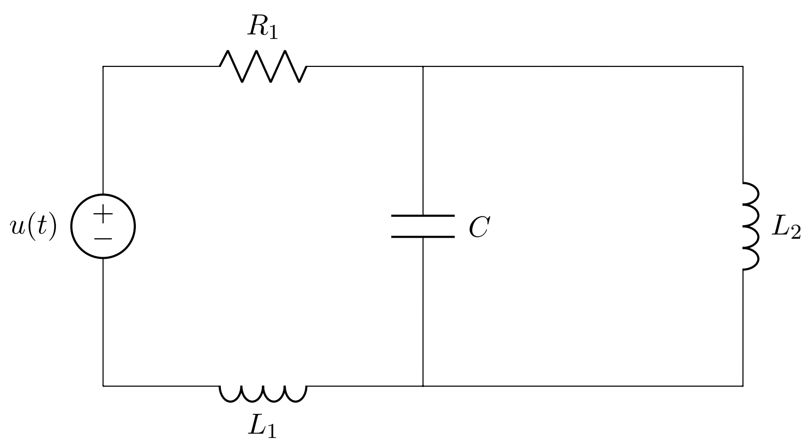


Figure 4: Electrical Circuit

Problem 8 Consider the electrical circuit shown in Figure 5:

1. Find the differential equations which govern the behavior of the electrical system.
2. Construct the state space representation of the system. Assume the desired output is the charge in the system.
3. Find the output response of the system assuming zero initial conditions and a step input of $u(t) = 24\,\mathrm{V}$

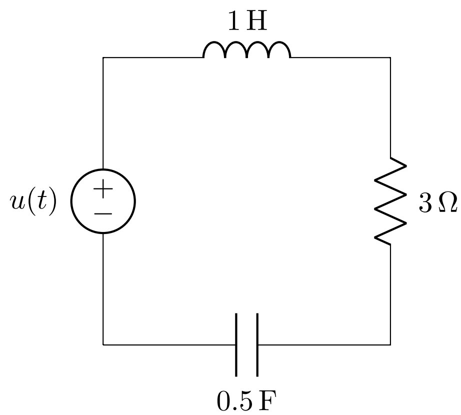


Figure 5: Electrical Circuit

LAPLACE TRANSFORM TABLE

Time Function	LaPlace Transform
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
t	$\frac{1}{s^2}$
$\frac{t^2}{2}$	$\frac{1}{s^3}$
t^{k-1}	$\frac{(k-1)!}{s^k}$
e^{-at}	$\frac{1}{s+a}$
te^{-at}	$\frac{1}{(s+a)^2}$
$t^{k-1}e^{-at}$	$\frac{(k-1)!}{(s+a)^k}$
$1-e^{-at}$	$\frac{a}{s(s+a)}$
$t-\frac{1-e^{-at}}{a}$	$\frac{a}{s^2(s+a)}$
$1-(1+at)e^{-at}$	$\frac{a^2}{s(s+a)^2}$
$e^{-at}-e^{-bt}$	$\frac{b-a}{(s+a)(s+b)}$
$\sin bt$	$\frac{b}{s^2+b^2}$
$\cos bt$	$\frac{s}{s^2+b^2}$
$t \sin bt$	$\frac{2bs}{(s^2+b^2)^2}$
$t \cos bt$	$\frac{s^2-b^2}{(s^2+b^2)^2}$
$e^{-at} \sin bt$	$\frac{b}{(s+a)^2+b^2}$
$e^{-at} \cos bt$	$\frac{s+a}{(s+a)^2+b^2}$