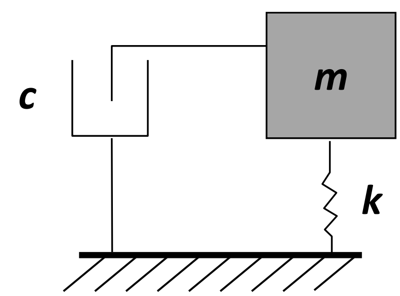
MAE 3134 – Linear System Dynamics

Homework # 5

Problem 1

Consider a vibratory system as shown in the figure, with k = 2 N/m, natural frequency, f_o = 1/π Hz and damping factor ζ = 0.25.



i) If the mass is displaced to an initial position located 1.5 m *above the static equilibrium* position and then released at time t = 0 without any time-dependent forces acting on it, what will be its height after one complete oscillation?

ii) If one wishes to completely stop the oscillation of the mass sometime *before* it completes one full oscillation, and this is to be done by hitting the mass with a hammer, provide a mathematical expression for an impact that will accomplish the objective.

Problem 2

In class we studied the transient and steady state responses of a vibratory system to a force of the form $f(t) = F_o \sin(\omega t)$. Specifically, we saw that the *steady state* response was given by:

$$x(t) = \frac{F_o}{k} A(\omega) \sin[\omega t - \theta(\omega)]$$

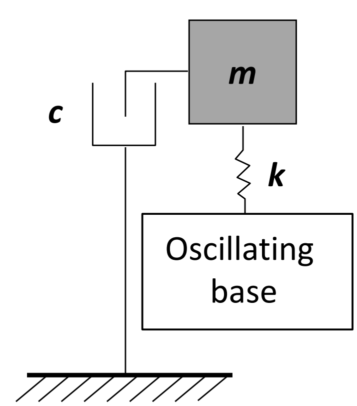
With $A(\omega) = \frac{1}{\sqrt{(1-\left(\frac{\omega}{\omega_o}\right)^2)^2 + (2\zeta\left(\frac{\omega}{\omega_o}\right))^2}}$; $\theta(\omega) = tan^{-1}[2\zeta(\frac{\omega}{\omega_o})/(1 - \left(\frac{\omega}{\omega_o}\right)^2)]$; $\omega_o = \sqrt{\frac{k}{m}}$;

and $\zeta = c/(2\sqrt{k\, m})$

Based on simple mathematical arguments, derive the *steady state* response for the case where the excitation force is of the form $f(t) = F_o$ **cos**(ωt).

Problem 3

Consider the vibratory system shown in the figure, with k = 2 N/m, m = 1 Kg and c = 0.5 N s/m. If a vertical oscillatory force, $f(t)$ = (0.75 N) cos ($\omega_o t$) is applied to the mass and the base position oscillates according to $y(t)$ = (5 m) cos [(2/3) $\omega_o t$],



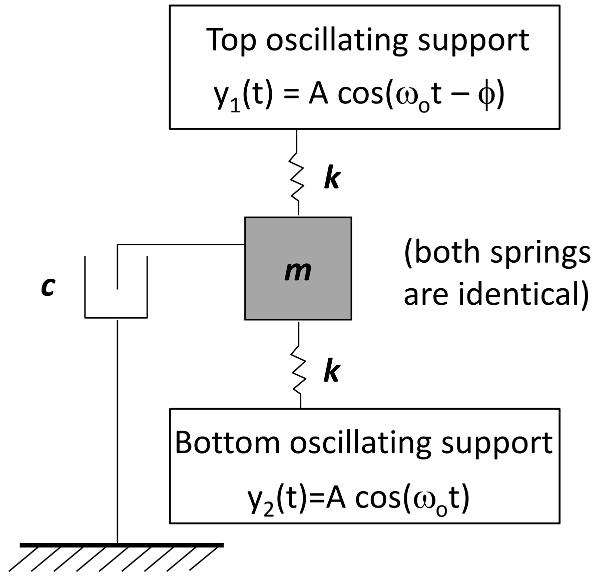
i) Calculate the steady-state response of the mass, $x(t)$.

ii) Calculate the period of oscillation.

ω_o is the natural frequency.

Problem 4

A mass is suspended between two blocks which are sinusoidally oscillating as indicated in the figure.



a) Set up the equation of motion of the system
b) For what value of ϕ will the oscillation amplitude of the mass be greatest? ϕ is a constant phase angle in the expression describing the oscillation of the top block, y_1(t).
c) For what value of ϕ will the oscillation amplitude of the mass be smallest? What is the smallest possible value of the oscillation amplitude at steady state?