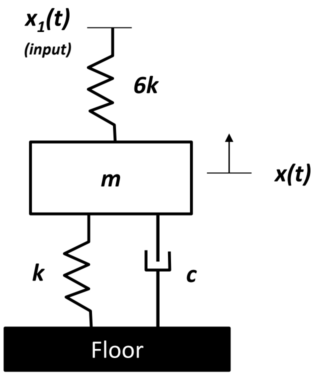
The George Washington University
MAE 3134 – Linear System Dynamics
Spring 2015

Final Exam

<u>Problem # 1</u> [60 points]



For the system shown in the figure,

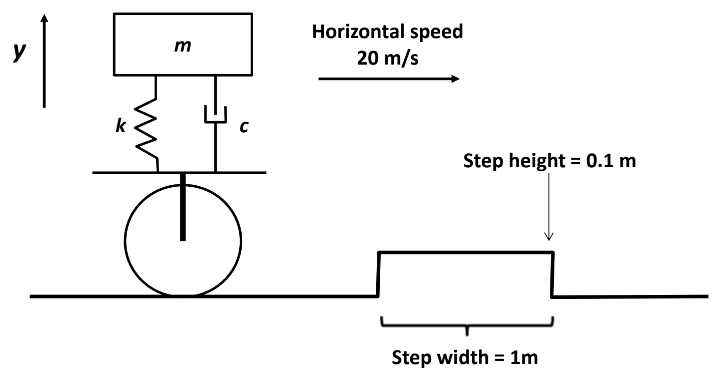
(i) Construct the state-space and output equations in matrix form. Include the position and the velocity of the mass as your two outputs. ***[20 points]***

(ii) Calculate the output matrix in Laplace space for $x_1(t) = t\,u(t)$ and zero initial conditions. ***[20 points]***

(iii) Invert the output matrix to obtain the solutions in time space for k = 2 N/m, c = 0.5 N s/m and m = 1 Kg. ***[20 points]***

<u>Problem # 2</u> [60 points]

Consider the model for a car-tire system in the figure, in which the vertical position of the mass exhibits a time-dependent response due to changes in the road topography. The parameters are m = 2000 Kg, k = 8000 N/m and c = 4000 Ns/m.



(i) Write the equation of motion of the system. ***[7 points]***

(ii) Derive the transfer function of the system for the vertical motion of the mass, taking the road height as the input. ***[10 points]***

(iii) Provide an expression for the response of the mass <u>in the Laplace domain</u>, $Y(s)$ when the input is a unit step (y_{tire} = u(t)) and the initial conditions are zero. ***[8 points]***

(iv) Calculate the time-dependent response for the Laplace-domain expression you derived for $Y(s)$ in step (iii). ***[15 points]***

(v) Calculate the time-dependent vertical response of the mass in the car-tire system to the road feature shown in the figure. Take t = 0 as the instant when the tire encounters the up-step. The initial conditions in the vertical direction $y(0) and\, \dot{y}(0)$ are both zero. The car is traveling at a horizontal speed of 20 m/s. ***[20 points]***

STATE-SPACE SOLUTIONS

Homogeneous Solution:

$\mathbf{y}_{\mathbf{homogeneous}}(t) = \mathcal{L}^{-1}\{ (s\mathbf{I} - \mathbf{A})^{-1} \}\, \mathbf{x}(0)$

Particular Solution:

$\mathbf{y}_{\mathbf{particular}}(t) = \mathcal{L}^{-1}\{ [\mathbf{C}(s\mathbf{I} - A)^{-1}\mathbf{B} + \mathbf{D}]\, \mathbf{U}(s) \}$

Full Solution:

$\mathbf{Y}(t) = \mathbf{y}_{\mathbf{homogeneous}}(t) + \mathbf{y}_{\mathbf{particular}}(t)$

Table of Laplace Transforms

	$f(t)=\mathfrak{L}^{-1}\{F(s)\}$	$F(s)=\mathfrak{L}\{f(t)\}$		$f(t)=\mathfrak{L}^{-1}\{F(s)\}$	$F(s)=\mathfrak{L}\{f(t)\}$
1.	1	$\dfrac{1}{s}$	2.	$\mathbf{e}^{at}$	$\dfrac{1}{s-a}$
3.	$t^n, \quad n=1,2,3,\ldots$	$\dfrac{n!}{s^{n+1}}$	4.	t^p, $p > -1$	$\dfrac{\Gamma(p+1)}{s^{p+1}}$
5.	$\sqrt{t}$	$\dfrac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$	6.	$t^{n-\frac{1}{2}}, \quad n=1,2,3,\ldots$	$\dfrac{1\cdot3\cdot5\cdots(2n-1)\sqrt{\pi}}{2^n s^{n+\frac{1}{2}}}$
7.	$\sin(at)$	$\dfrac{a}{s^2+a^2}$	8.	$\cos(at)$	$\dfrac{s}{s^2+a^2}$
9.	$t\sin(at)$	$\dfrac{2as}{(s^2+a^2)^2}$	10.	$t\cos(at)$	$\dfrac{s^2-a^2}{(s^2+a^2)^2}$
11.	$\sin(at)-at\cos(at)$	$\dfrac{2a^3}{(s^2+a^2)^2}$	12.	$\sin(at)+at\cos(at)$	$\dfrac{2as^2}{(s^2+a^2)^2}$
13.	$\cos(at)-at\sin(at)$	$\dfrac{s(s^2-a^2)}{(s^2+a^2)^2}$	14.	$\cos(at)+at\sin(at)$	$\dfrac{s(s^2+3a^2)}{(s^2+a^2)^2}$
15.	$\sin(at+b)$	$\dfrac{s\sin(b)+a\cos(b)}{s^2+a^2}$	16.	$\cos(at+b)$	$\dfrac{s\cos(b)-a\sin(b)}{s^2+a^2}$
17.	$\sinh(at)$	$\dfrac{a}{s^2-a^2}$	18.	$\cosh(at)$	$\dfrac{s}{s^2-a^2}$
19.	$\mathbf{e}^{at}\sin(bt)$	$\dfrac{b}{(s-a)^2+b^2}$	20.	$\mathbf{e}^{at}\cos(bt)$	$\dfrac{s-a}{(s-a)^2+b^2}$
21.	$\mathbf{e}^{at}\sinh(bt)$	$\dfrac{b}{(s-a)^2-b^2}$	22.	$\mathbf{e}^{at}\cosh(bt)$	$\dfrac{s-a}{(s-a)^2-b^2}$
23.	$t^n\mathbf{e}^{at}, \quad n=1,2,3,\ldots$	$\dfrac{n!}{(s-a)^{n+1}}$	24.	$f(ct)$	$\dfrac{1}{c}F\left(\dfrac{s}{c}\right)$
25.	$u_c(t)=u(t-c)$ Heaviside Function	$\dfrac{\mathbf{e}^{-cs}}{s}$	26.	$\delta(t-c)$ Dirac Delta Function	$\mathbf{e}^{-cs}$
27.	$u_c(t)f(t-c)$	$\mathbf{e}^{-cs}F(s)$	28.	$u_c(t)g(t)$	$\mathbf{e}^{-cs}\mathfrak{L}\{g(t+c)\}$
29.	$\mathbf{e}^{ct}f(t)$	$F(s-c)$	30.	$t^n f(t), \quad n=1,2,3,\ldots$	$(-1)^n F^{(n)}(s)$
31.	$\dfrac{1}{t}f(t)$	$\int_s^\infty F(u)\,du$	32.	$\int_0^t f(v)\,dv$	$\dfrac{F(s)}{s}$
33.	$\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$	34.	$f(t+T)=f(t)$	$\dfrac{\int_0^T \mathbf{e}^{-st}f(t)\,dt}{1-\mathbf{e}^{-sT}}$
35.	$f'(t)$	$sF(s)-f(0)$	36.	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
37.	$f^{(n)}(t)$	$s^nF(s)-s^{n-1}f(0)-s^{n-2}f'(0)\cdots-sf^{(n-2)}(0)-f^{(n-1)}(0)$			