

1

a) find $\frac{C(s)}{R(s)}$ for $\ddot{c} + 3\dot{c} + 7c + 5c = \ddot{r} + 4\dot{r} + 3r$

$$s^3 C(s) + 3s^2 C(s) + 7s C(s) + 5C(s) = s^2 R(s) + 4s R(s) + 3R(s)$$

$$C(s)(s^3 + 3s^2 + 7s + 5) = R(s)(s^2 + 4s + 3)$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{s^2 + 4s + 3}{s^3 + 3s^2 + 7s + 5}}$$

b) find $\frac{Y(s)}{X(s)}$ for $\ddot{y} + 3\dot{y} + 5y + y = \ddot{x} + 4\dot{x} + 6x + 8x$

$$s^3 Y(s) + 3s^2 Y(s) + 5s Y(s) + Y(s) = s^3 X(s) + 4s^2 X(s) + 6s X(s) + 8X(s)$$

$$Y(s)(s^3 + 3s^2 + 5s + 1) = X(s)(s^3 + 4s^2 + 6s + 8)$$

$$\boxed{\frac{Y(s)}{X(s)} = \frac{s^3 + 4s^2 + 6s + 8}{s^3 + 3s^2 + 5s + 1}}$$

2

a) $G(s) = \frac{C(s)}{R(s)} = \frac{2s+1}{s^2+6s+2} \rightarrow C(s)(s^2 + 6s + 2) = R(s)(2s + 1)$

$$\rightarrow \ddot{c} + 6\dot{c} + 2c = 2\dot{r} + r$$

b) $G(s) = \frac{X(s)}{F(s)} = \frac{1}{s^2+2s+7} \rightarrow X(s)(s^2 + 2s + 7) = F(s)$

$$\rightarrow \ddot{x} + 2\dot{x} + 7x = f$$

c) $G(s) = \frac{X(s)}{F(s)} = \frac{10}{(s+7)(s+8)} \rightarrow X(s)(s^2 + 15s + 56) = 10F(s)$

$$\rightarrow \ddot{x} + 15\dot{x} + 56x = 10f$$

d) $G(s) = \frac{X(s)}{F(s)} = \frac{s+2}{s^3+8s^2+9s+15} \rightarrow X(s)(s^3 + 8s^2 + 9s + 15) = F(s)(s+2)$

$$\rightarrow \ddot{x} + 8\dot{x} + 9x + 15x = f + 2f$$

e) $G(s) = \frac{C(s)}{R(s)} = \frac{s^5+2s^4+4s^3+s^2+1}{s^6+7s^5+3s^4+2s^3+s^2+3}$

$$\rightarrow C(s)(s^6 + 7s^5 + 3s^4 + 2s^3 + s^2 + 3) = R(s)(s^5 + 2s^4 + 4s^3 + s^2 + 3)$$

$$\rightarrow \frac{d^6 c}{dt^6} + 7 \frac{d^5 c}{dt^5} + 3 \frac{d^4 c}{dt^4} + 2 \frac{d^3 c}{dt^3} + \frac{d^2 c}{dt^2} + 3c = \frac{d^5 r}{dt^5} + 2 \frac{d^4 r}{dt^4} + 4 \frac{d^3 r}{dt^3} + \frac{d^2 r}{dt^2} + 3r$$

f) $G(s) = \frac{C(s)}{R(s)} = \frac{s^4+2s^3+5s^2+s+1}{s^5+3s^4+2s^3+4s^2+5s+2}$

$$\rightarrow C(s)(s^5 + 3s^4 + 2s^3 + 4s^2 + 5s + 2) = R(s)(s^4 + 2s^3 + 5s^2 + s + 1)$$

$$\rightarrow \frac{d^5 c}{dt^5} + 3 \frac{d^4 c}{dt^4} + 2 \frac{d^3 c}{dt^3} + 4 \frac{d^2 c}{dt^2} + 5 \frac{dc}{dt} + 2c = \frac{d^4 r}{dt^4} + 2 \frac{d^3 r}{dt^3} + 5 \frac{d^2 r}{dt^2} + \frac{dr}{dt} + 1$$

$$\rightarrow \frac{d^5 c}{dt^5} + 3 \frac{d^4 c}{dt^4} + 2 \frac{d^3 c}{dt^3} + 4 \frac{d^2 c}{dt^2} + 5 \frac{dc}{dt} + 2c = 2(18) + 5(18) + 9t^2 + 1$$

$$\rightarrow \frac{d^5 c}{dt^5} + 3 \frac{d^4 c}{dt^4} + 2 \frac{d^3 c}{dt^3} + 4 \frac{d^2 c}{dt^2} + 5 \frac{dc}{dt} + 2c = 9t^2 + 90t + 37$$

3

$$G(s) = \frac{C(s)}{R(s)} = \frac{s}{(s+4)(s+8)}$$

$$a) C(s) = G(s) R(s) = \frac{s}{s^2(s+4)(s+8)}$$

$$r(t) = + \rightarrow R(s) = \frac{1}{s^2}$$

$$\rightarrow \frac{s}{s^2(s+4)(s+8)} = \frac{A}{s^2} + \frac{B}{s} + \frac{C}{s+4} + \frac{D}{s+8}$$

$$\rightarrow s = A(s+4)(s+8) + B s(s+4)(s+8) + C s^2(s+8) + D s^2(s+4)$$

$$= A(s^2 + 12s + 32) + B(s^3 + 32s^2 + 32s) + C(s^3 + 8s^2) + D(s^3 + 4s^2)$$

$$= s^3(B+C+D) + s^2(A+12B+8C+4D) + s(12A+32B) + 32A$$

$$\rightarrow 32A = 0 \rightarrow A = 0$$

$$12A + 32B = 1 \rightarrow 32B = 1 \rightarrow B = \frac{1}{32}$$

$$A + 12B + 8C + 4D = 0 \rightarrow 8C + 4D = -\frac{1}{32} \Rightarrow C = -\frac{3}{64} - \frac{D}{2} \rightarrow C = -\frac{1}{16}$$

$$B + C + D = 0 \rightarrow \frac{1}{32} - \frac{3}{64} - \frac{D}{2} + D = 0 \rightarrow D = \frac{1}{32}$$

$$\rightarrow C(s) = \frac{0}{s^2} + \frac{\frac{1}{32}}{s} - \frac{\frac{1}{16}(s+4)}{(s+4)^2} + \frac{\frac{1}{32}(s+8)}{(s+8)^2}$$

$$\rightarrow C(t) = \frac{1}{32} - \left(\frac{1}{16}\right)e^{-4t} + \frac{1}{32}e^{-8t}$$

b) See MATLAB plot

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$$F(s) = \frac{10}{s(s+2)(s+3)^3} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{(s+3)^2} + \frac{D}{(s+3)^3} + \frac{E}{s+3}$$

$$\rightarrow 10 = A(s+2)(s+3)^3 + B s(s+3)^3 + C s(s+2) + D s(s+2)(s+3) + E s(s+2)(s+3)^2$$

$$= A(s^4 + 11s^3 + 45s^2 + 81s + 54) + B(s^4 + 9s^3 + 27s^2 + 27s) + C(s^3 + 2s) + D(s^3 + 5s^2 + 6s) + E(s^4 + 8s^3 + 21s^2 + 18s)$$

$$= s^4(A+B+E) + s^3(11A + 9B + D + 8E) + s^2(45A + 27B + C + 5D + 21E) + s(81A + 27B + 2C + 6D + 18E) + (54A)$$

$$\rightarrow 54A = 10 \rightarrow A = \frac{5}{27}$$

$$\text{plug in } -3: C(-3)(-1) = 10 \rightarrow C = \frac{10}{3}$$

$$\text{plug in } -2: B(-2)(1)^3 = 10 \rightarrow B = -5$$

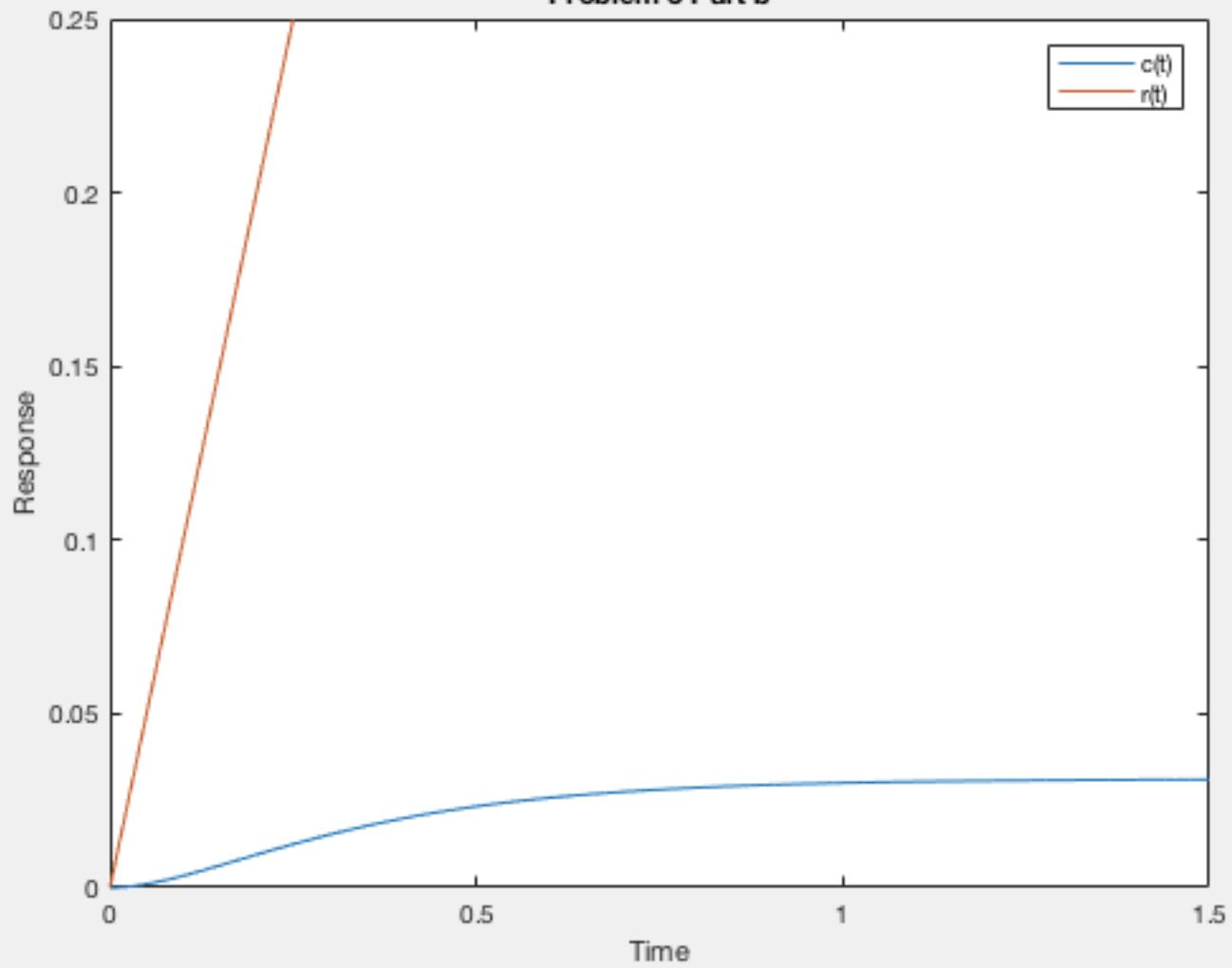
$$A + B + E = 0 \rightarrow \frac{5}{27} - 5 + E = 0 \rightarrow E = \frac{130}{27}$$

$$11A + 9B + D + 8E = 0 \rightarrow 11\left(\frac{5}{27}\right) + 9(-5) + D + 8\left(\frac{130}{27}\right) = 0 \rightarrow D = \frac{40}{9}$$

$$\rightarrow F(s) = \frac{5}{27s} + \frac{-5}{s+2} + \frac{10}{3(s+3)^3} + \frac{40}{9(s+3)^2} + \frac{130}{27(s+3)}$$

$$\rightarrow f(t) = \frac{5}{27} + -5e^{-2t} + \left(\frac{5}{3}\right)e^{-3t} + \frac{2}{9} + \left(\frac{40}{9}\right)e^{-3t} + \left(\frac{130}{27}\right)e^{-3t}$$

Problem 3 Part b



5

$$\cdot \frac{d^2y}{dt^2} + 12 \frac{dy}{dt} + 32y = 32u(t)$$

$$s^2 Y(s) + 12sY(s) + 32Y(s) = 32\left(\frac{1}{s}\right)$$

$$\rightarrow Y(s)(s^2 + 12s + 32) = \frac{32}{s} \rightarrow Y(s) = \frac{\frac{32}{s}}{s(s^2 + 12s + 32)} = \frac{32}{s(s+4)(s+8)}$$

$$\frac{32}{s(s+4)(s+8)} = \frac{A}{s} + \frac{B}{s+4} + \frac{C}{s+8} \rightarrow 32 = A(s^2 + 12s + 32) + B(s^2 + 8s) + C(s^2 + 4s)$$

$$32 = s^2(A+B+C) + s(12A+8B+4C) + (32A)$$

$$\rightarrow 32A = 32 \rightarrow A = 1$$

$$A+B+C=0 \rightarrow 1+B+C=0 \rightarrow B=-1-C \rightarrow B=-2$$

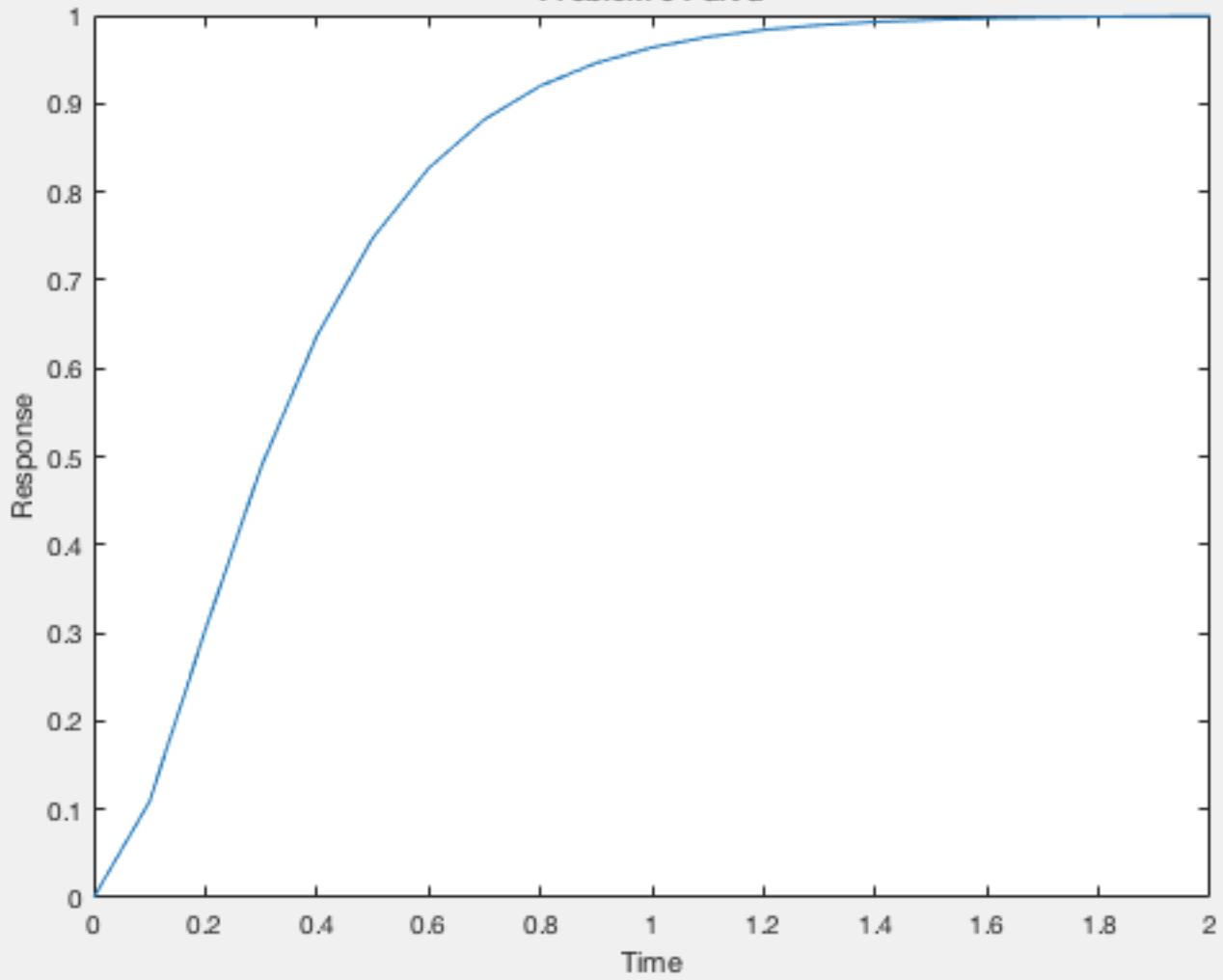
$$12A+8B+4C=0 \rightarrow 12 - 8 - 8C+4C=0 \rightarrow C = \overline{1}$$

$$Y(s) = \frac{1}{s} - \frac{2}{s+4} + \frac{1}{s+8}$$

$$\rightarrow y(t) = 1 - 2e^{-4t} + e^{-8t}$$

a) see MATLAB plot

Problem 5 Part a



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%----- Problem 6 Part a -----
clear

syms t
laplace(5*(t^2)*cos((3*t)+(pi/4)))
% ans: (5*2^(1/2)*(6/(s^2 + 9)^2 - (24*s^2)/(s^2 + 9)^3))/2 - (5*2^(1/2)*((6*s)/(s^2 + 9)^2 - (8*s^3)/(s^2 + 9)^3))/2

%----- Problem 6 Part b -----
clear

syms t
laplace(5*t*exp(-2*t)*sin((4*t)+(pi/3)))
% ans: 10*(2*s + 4)/((s + 2)^2 + 16)^2 - (5*3^(1/2)*(1/((s + 2)^2 + 16) - ((2*s + 4)*(s + 2))/((s + 2)^2 + 16)^2))/2

%----- Problem 6 Part c -----
clear

syms s
ilaplace(((s^2)+(3*s)+7)*(s+2)/((s+3)*(s+4)*((s^2)+(2*s)+100)))
% ans: (11*exp(-4*t))/54 - (7*exp(-3*t))/103 + (4807*exp(-t)*cos(3*11^(1/2)*t) - (4681*11^(1/2)*sin(3*11^(1/2)*t))/52877))/5562

%----- Problem 6 Part d -----
clear

syms s
ilaplace(((s^3)+(4*s^2)+(6*s)+5)/((s+8)*((s^2)+(8*s)+3)*((s^2)+(5*s)+7)))
% ans: (1367*exp(-4*t)*cosh(13^(1/2)*t) - (4895*13^(1/2)*sinh(13^(1/2)*t))/17771)/417 - (272*exp(-(5*t)/2)*cos((3^(1/2)*t)/2) + (29*3^(1/2)*sin((3^(1/2)*t)/2))/102))/4309 - (299*exp(-8*t))/93

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