Spring 2017 MAE3134: Final Exam

11 May 2017

Resources allowed: Open notes/book, calculator, ruler. No computers or mobile devices.

Name: __________________________ GWID:__________________________

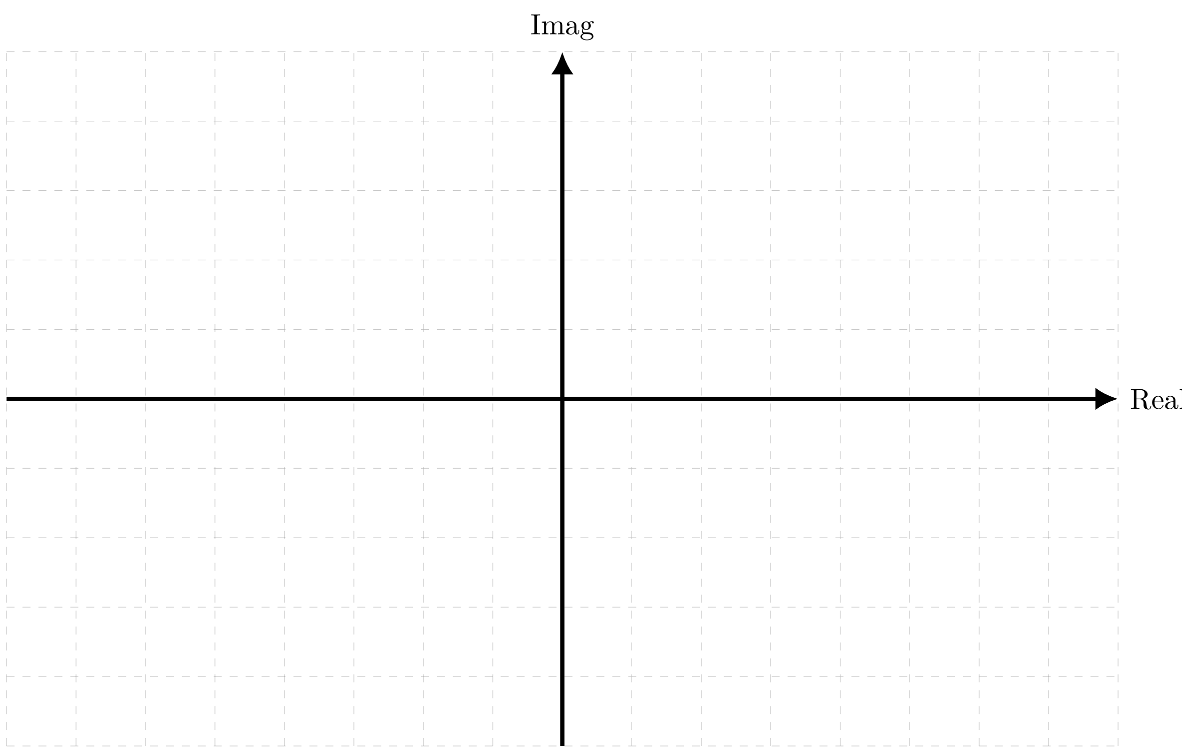
Prob. 1	Prob. 2	Prob. 3	Prob. 4	Prob. 5	Prob. 6	Prob. 7	Prob. 8	Total
10	5	20	20	10	5	10	20	100

Problem 1 Elon Musk, CEO of SpaceX and Tesla Motors, has a background in physics but unfortunately has never passed a Linear Dynamics course. His newest space vehicle must satisfy the following second order time response specifications for a unit step input:

- Percent Overshoot must be less than 5%,
- Rise time less than 1 s,
- Settling Time less than 5 s.

Elon needs your help to choose a set of poles which will satisfy the specifications and save humanity from impending disaster.

1. On the s-plane, or complex plane, map out the acceptable regions where you could locate poles and meet the requirements.
2. Label the specifications lines and show your work.
3. Choose a set of poles that will meet the requirements.
4. Write the transfer function relating the input $C(s)$ to the output $R(s)$ for this system.
5. Draw an electrical circuit which will physically represent your system.



Problem 2 The frequency response of two systems are shown in Fig. 1. Using the plots, circle the correct descriptions:

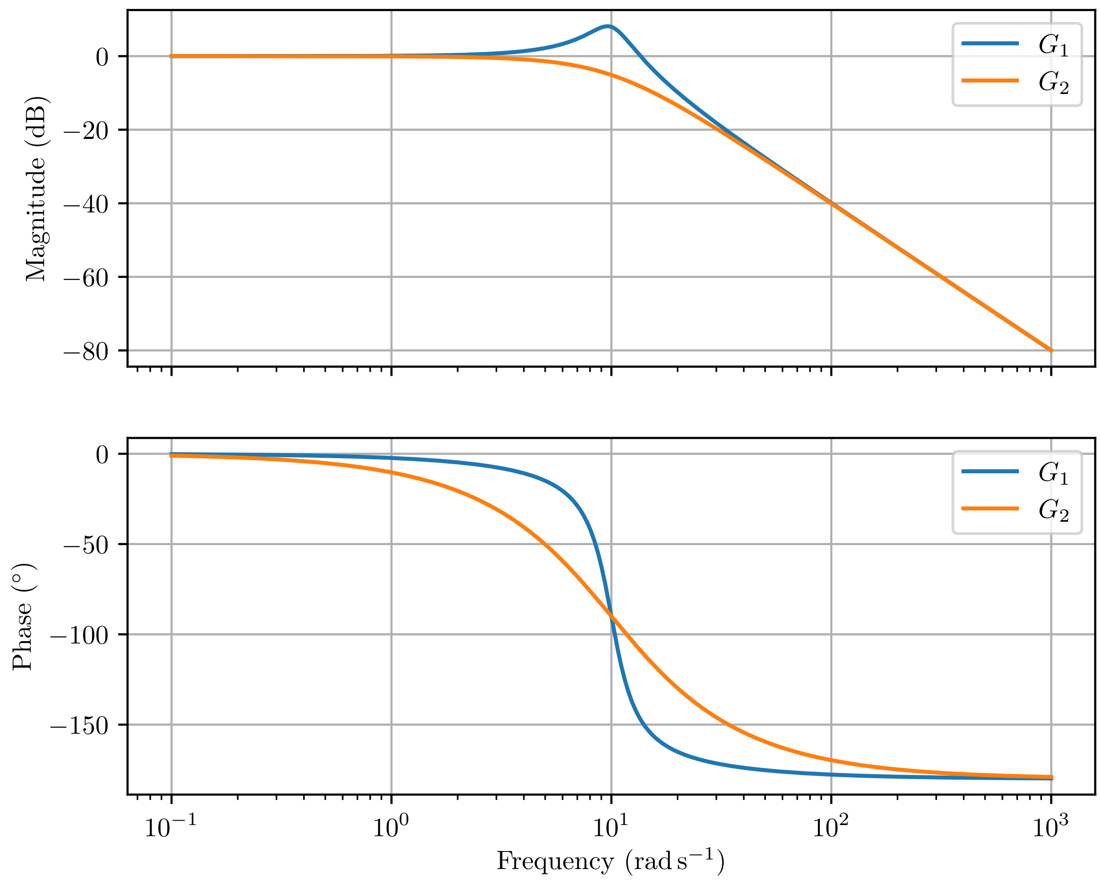


Figure 1: Frequency Response

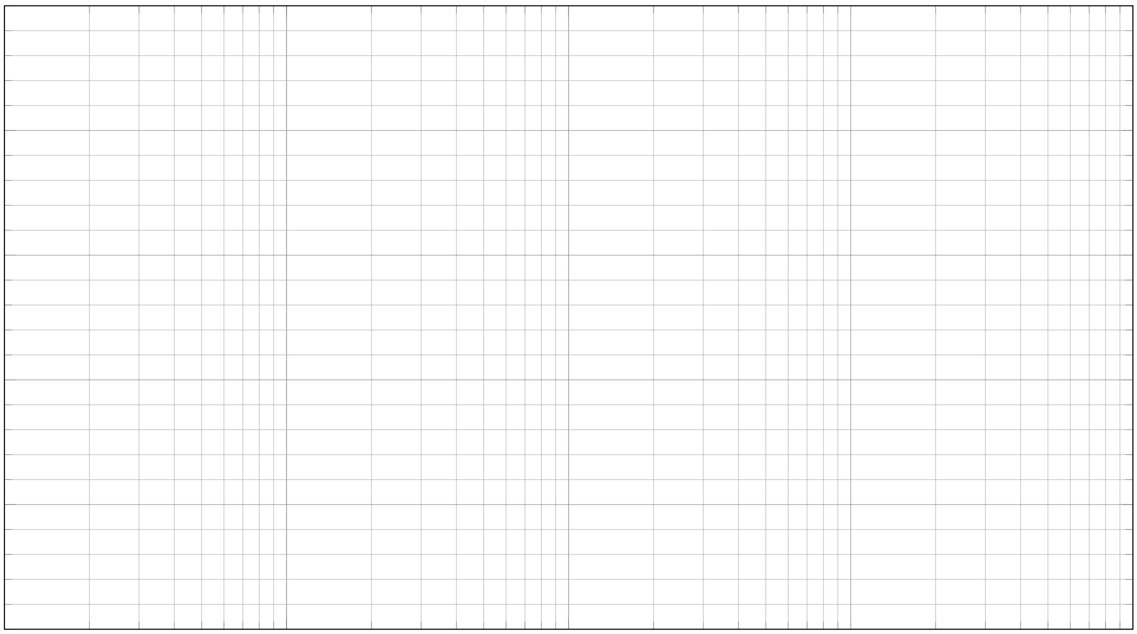
1. Which of the following statements are true about the damping ratios of the two systems?
   (a) The damping coefficients are the same.
   (b) The damping coefficient of G_1 is greater than the damping coefficient of G_2.
   (c) The damping coefficient of G_2 is greater than the damping coefficient of G_1.
   (d) Not enough information to make any statements about the damping ratio.

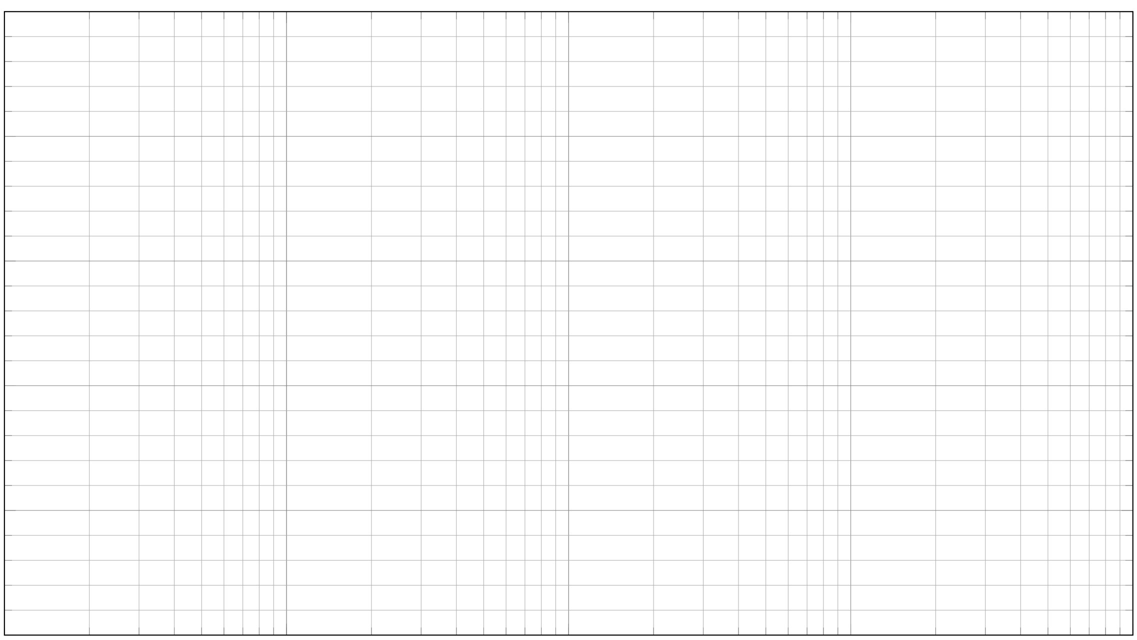
2. Which of the following statements are true about the general form of G_1?
   (a) It is a first order system.
   (b) It must have two free s terms in the denominator since the phase ends at 180°.
   (c) It must have two free s terms in the numerator since the final magnitude slope is 40 dB per decade.
   (d) None of the above.

Problem 3 A transfer function is defined as

$$G(s) = \frac{500(s+100)(s+20)}{s(s^2+8s+25)}.$$

1. Draw the asymptotic Bode plots for this system.
2. What is the steady state output for an input of $u = 5\sin 25t$?





Problem 4 Consider a linear system

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x}$$

with $\boldsymbol{A}$ defined as

$$\boldsymbol{A} = \begin{bmatrix} 2 & 1 & 4 \\ 0 & 2 & 0 \\ 0 & 3 & 1 \end{bmatrix}.$$

1. Find the state transition matrix for this system.
2. Find $\boldsymbol{x}(t)$ for $\boldsymbol{x}(0) = \begin{bmatrix} 1 & 0 & 2 \end{bmatrix}^T$.

Problem 5 For the electrical system in Fig. 3:

1. Find the differential equations of motion for the system.
2. Find the state space representation of the system with your state vector defined as

$$\boldsymbol{x} = \begin{bmatrix} q_1 & i_1 & q_2 & i_2 \end{bmatrix}^T,$$

where q_1, i_1 represent the charge and current in the left loop while q_2, i_2 represent the charge and current in the right loop. The output is defined as

$$y = \begin{bmatrix} q_1 & q_2 \end{bmatrix}^T.$$

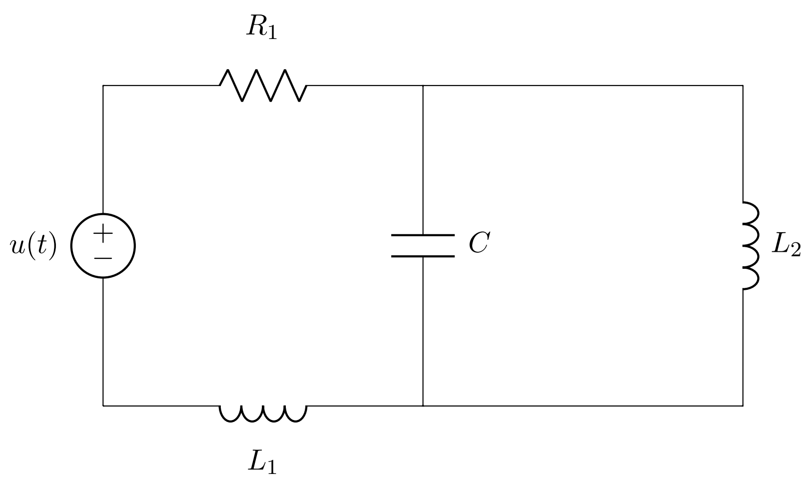


Figure 3: Electrical Circuit

LAPLACE TRANSFORM TABLE

Time Function	LaPlace Transform
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
t	$\frac{1}{s^2}$
$\frac{t^2}{2}$	$\frac{1}{s^3}$
t^{k-1}	$\frac{(k-1)!}{s^k}$
e^{-at}	$\frac{1}{s+a}$
te^{-at}	$\frac{1}{(s+a)^2}$
$t^{k-1}e^{-at}$	$\frac{(k-1)!}{(s+a)^k}$
$1-e^{-at}$	$\frac{a}{s(s+a)}$
$t-\frac{1-e^{-at}}{a}$	$\frac{a}{s^2(s+a)}$
$1-(1+at)e^{-at}$	$\frac{a^2}{s(s+a)^2}$
$e^{-at}-e^{-bt}$	$\frac{b-a}{(s+a)(s+b)}$
$\sin bt$	$\frac{b}{s^2+b^2}$
$\cos bt$	$\frac{s}{s^2+b^2}$
$t \sin bt$	$\frac{2bs}{(s^2+b^2)^2}$
$t \cos bt$	$\frac{s^2-b^2}{(s^2+b^2)^2}$
$e^{-at} \sin bt$	$\frac{b}{(s+a)^2+b^2}$
$e^{-at} \cos bt$	$\frac{s+a}{(s+a)^2+b^2}$