HW 0 - SOLUTION

PROBLEM 1

a) $a = [\sqrt{5} \quad \sqrt{3} \quad 1]$ $\quad \|a\| = \sqrt{5 + 3 + 1} = \sqrt{9} = \boxed{3}$

b) $b = [-2 \quad 4 \quad -4]$ $\quad \|b\| = \sqrt{4 + 16 + 16} = \sqrt{36} = \boxed{6}$

c) $c = [0 \quad 0 \quad -9]$ $\quad \|c\| = \sqrt{81} = \boxed{9}$

PROBLEM 2

$a = [0 \quad 3000 \quad 0]$

$b = [4000 \quad 0 \quad 0]$

a) $\bar{c} = a + b = \boxed{[4000 \quad 3000 \quad 0]}$

b) $\|c\| = \sqrt{4000^2 + 3000^2} = \boxed{5000}$

c) $\|a\| + \|b\| = 3000 + 4000 = \boxed{7000}$

d) $\boxed{\text{TRUE}}$ $\quad \|a\| + \|b\| > \|c\|$ $\quad$ TRIANGLE INEQUALITY

PROBLEM 3

a) $a \cdot b = 0 \Rightarrow ab\cos\theta = 0 \Rightarrow \cos\theta = 0 \Rightarrow \boxed{\theta = \pm 90^\circ}$

b) $a = [1 \quad 3 \quad -2]$, $b = [-4, -1, -2]$

$a \cdot b = 1 \cdot -4 + -1 \cdot 3 + -2 \cdot -2 = -4 - 3 + 4 = \boxed{-3 = a \cdot b}$

c) $\bar{a} \cdot \bar{b} = \|a\|\|b\|\cos\theta = \sqrt{14}\sqrt{21}\cos\theta$

$\cos\theta = \dfrac{-3}{\sqrt{14}\sqrt{21}} \Rightarrow \boxed{\theta = 100^\circ}$

d) $a \cdot b < 0 \Rightarrow$ ANGLE BETWN $\bar{a}, \bar{b}$ MUST BE LARGER THAN 90°

PROBLEM 4

a) $\bar{a} \times \bar{b} = 0 \Rightarrow ab \sin\theta = 0 \Rightarrow \sin\theta = 0 \Rightarrow \boxed{\theta = \pm 0^\circ, 2\pi}$

$\bar{a} = [1 \quad 3 \quad -2] \qquad b = [-4 \quad -1 \quad -2]$

b) $c = \bar{a} \times \bar{b} = (1\hat{i} + 3\hat{j} + -2\hat{k}) \times (-4\hat{i} - 1\hat{j} - 2\hat{k})$

$$= -4\cancel{\hat{i} \times \hat{i}} - 1\hat{i} \times \hat{j} - 2\hat{i} \times \hat{k}$$
$$-12\hat{j} \times \hat{i} - 3\cancel{\hat{j} \times \hat{j}} - 6\hat{j} \times \hat{k}$$
$$+ 8\hat{k} \times \hat{i} + 2\hat{k} \times \hat{j} + 4\cancel{\hat{k} \times \hat{k}}$$

$$= -\hat{k} + 2\hat{j} + 12\hat{k} - 6\hat{i} + 8\hat{j} - 2\hat{i}$$

$\boxed{\bar{c} = -8\hat{i} + 10\hat{j} + 11\hat{k}} \qquad \boxed{\|c\| = \sqrt{64 + 100 + 121} = 16.8}$

c) FIND ANGLE BTWN $\bar{a}, \bar{b}$

$a \cdot b = -4 - 3 + 4 = -3 = \|a\| \|b\| \cos\theta \Rightarrow \theta = 100^\circ$

$\|\bar{a} \times \bar{b}\| = \|a\| \|b\| \sin\theta = \sqrt{14}\sqrt{21} \sin 100^\circ = \boxed{16.8}$ ✓

d) $\bar{a} \times \bar{c} = (1\hat{i} + 3\hat{j} - 2\hat{k}) \times (-8\hat{i} + 10\hat{j} + 11\hat{k})$

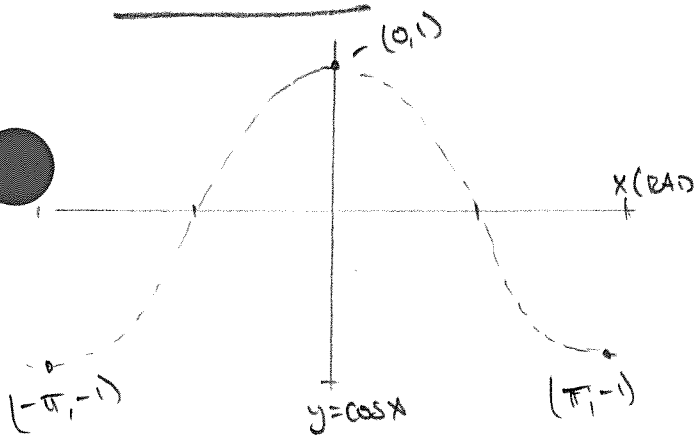
$$= -8\cancel{\hat{i} \times \hat{i}} + 10\hat{i} \times \hat{j} + 11\hat{i} \times \hat{k}$$
$$-24\hat{j} \times \hat{i} + 3\cancel{0\hat{j} \times \hat{j}} + 33\hat{j} \times \hat{k}$$
$$+16\hat{k} \times \hat{i} - 20\hat{k} \times \hat{j} - 22\cancel{\hat{k} \times \hat{k}} \quad = 10\hat{k} - 11\hat{j} + 24\hat{k} + 33\hat{i} + 16\hat{j} + 20\hat{i}$$

$\bar{a} \times \bar{c} = 53\hat{i} + 5\hat{j} + 34\hat{k} \qquad \|\bar{a} \times \bar{c}\| = \sqrt{53^2 + 5^2 + 34^2} = 63.16$

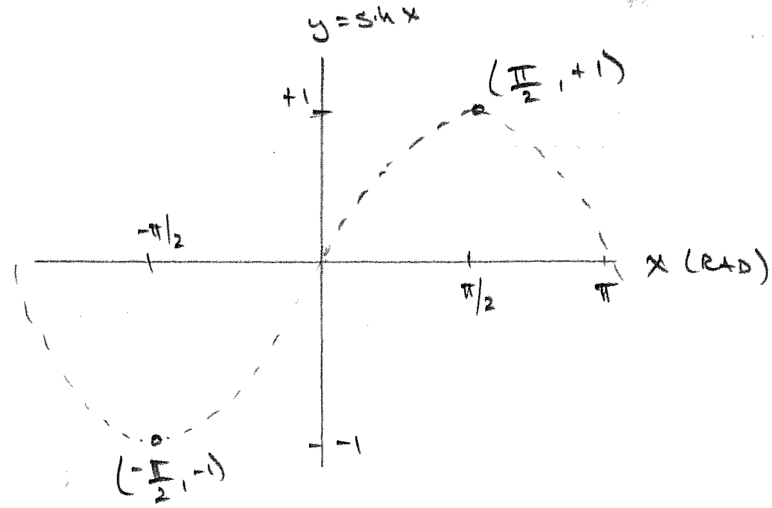
$\|\bar{a} \times \bar{c}\| = 63.16 = \|a\|\|c\| \sin\theta = \sqrt{14} \cdot 16.8 \sin\theta \Rightarrow \theta = 90^\circ$

ORTHOGONAL VECTORS

PROBLEM 5



$y = \cos x$



$y = \sin x$

PROBLEM 6

$A^2 = B^2 + C^2 - 2AB\cos\alpha$ WHEN $\theta = 90 \Rightarrow \cos\theta = 0$

$\Rightarrow A^2 = B^2 + C^2$ ← PYTHAGOREAN

a) $B^2 = A^2 + C^2 - 2AC\cos\beta$

$C^2 = A^2 + B^2 - 2AB\cos\gamma$

$\bar{a} = [3 \;\; 4 \;\; 5]$, $\bar{b} = [0 \;\; 4 \;\; 0]$, $\bar{c} = [-3 \;\; 0 \;\; 4]$

a) $\bar{b} = \bar{a} + \Delta$ $\quad \Delta_{ab} = \bar{b} - \bar{a} = [-3 \;\; 0 \;\; -5]$ $\quad |\Delta_{ab}| = \sqrt{9+25} = \sqrt{34}$

$a = \sqrt{50}$ $\quad b = 4$ $\quad c = 5$

$\Delta_{ab} = \sqrt{34}$ $\quad \Delta_{bc} = \sqrt{41}$ $\quad \Delta_{ac} = \sqrt{53}$

$\Delta_{ab}^2 = a^2 + b^2 - 2ab\cos\alpha$ $\quad \boxed{\alpha = 55.5^\circ}$

b) $\Delta_{bc}^2 = b^2 + c^2 - 2bc\cos\beta$ $\quad \boxed{\beta = 90^\circ}$

c) $\Delta_{ac}^2 = a^2 + c^2 - 2ac\cos\gamma$ $\quad \boxed{\gamma = 71.8^\circ}$

PROBLEM 7

$Ax = b \qquad b \in \mathbb{R}^{n \times 1} \qquad A \in \mathbb{R}^{n \times n} \qquad x \in \mathbb{R}^{n \times 1}$

PROBLEM 8

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

a) $$A^{-1} = \frac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

b) $$A^{-1}A = I = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} da-bc & db-bd \\ -ca+ac & -cb+ad \end{bmatrix} \cdot \frac{1}{ad-bc}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

c) $AA^{-1} = I \qquad A^{-1}A = I$ FROM ABOVE

PROBLEM 9

$x = \begin{bmatrix} 6 & \frac{\pi}{2} & \pi \end{bmatrix}$ SPHERICAL $\begin{bmatrix} r & \phi & \theta \end{bmatrix}$

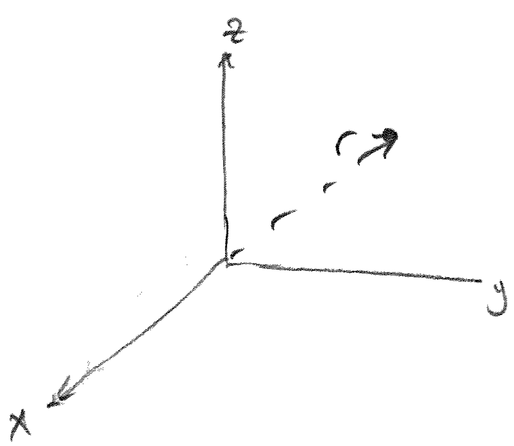
INC. AZIMUTH

SPHERICAL → CARTESIAN

$$x = r \sin\phi \cos\theta = 6 \sin\frac{\pi}{2} \cos\pi = -6$$

$$y = r \sin\phi \sin\theta = 6 \sin\frac{\pi}{2} \sin\pi = 0$$

$$z = r \cos\phi = 6 \cos\frac{\pi}{2} = 0$$



MAGNITUDE OF 6

PROBLEM 10

$y = e^x$

a) TAYLOR SERIES

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}$$

ESTIMATE OF $y(0.1) = 1 + 0.1 + 0.005 + 1.\bar{6} \times 10^{-4}$

$= 1.105166$

TRUE $e^{0.1} = 1.1051709$

– ESTIMATE OF ERROR $\approx |1.105166 - 1.151709| = 4.918 \times 10^{-6}$

PROBLEM 11

a) $x \cdot (x \times y) = y \cdot (\bar{x} \times \bar{x}) = y \cdot (\bar{0}) = 0$

b) $|\bar{q}| = 1$

$$-(q \times (q \times x)) = -((q \cdot x) q - (q \cdot q) x)$$

$$= -(q \cdot x) q + x = \boxed{x - (\bar{q} \cdot \bar{x}) q}$$

PROBLEM 12

a) $|\bar{r}_A| = \sqrt{3741.7^2 + 5612.5^2 + 1870.8^2} = 7000.026$ km

$|\bar{r}_B| = \sqrt{-441.8^2 + 6627.5^2 + 2209.2^2} = 6999.9$ km

$\Big\}$ 7000 km

b) $\bar{r}_A \cdot \bar{r}_b = |r_b| |r_a| \cos\theta \qquad \theta = 50.13^\circ$

c) $$\bar{h} = \frac{\bar{r}_A \times \bar{r}_b}{|r_a| |r_b|} = [0.506, -0.151, 0.556]$$

PROBLEM 13

$$\varepsilon = \frac{v^2}{2} - \frac{\mu}{r}$$

a) $\frac{v^2}{2} = \varepsilon + \frac{\mu}{r}$ $\qquad v = \sqrt{2\varepsilon + \frac{2\mu}{r}}$

b) $v = \sqrt{\cancel{2}\left(\frac{-\mu}{\cancel{2}r}\right) + \frac{2\mu}{r}} = \sqrt{\frac{\mu}{r}}$

c) $v = \sqrt{2\varepsilon + \frac{2\mu}{r}} = \sqrt{\frac{2\mu}{r}}$

d) $r = \infty \Rightarrow \varepsilon = \frac{v^2}{2} - \cancel{\frac{\mu}{\infty}} = \frac{v^2}{2}$

e) $\frac{6.5^2}{2} - \frac{398600.5}{20000} = \frac{v_2^2}{2} - \cancel{\frac{398600.5}{\infty}}$

$$v_2 = \sqrt{6.5^2 - \frac{2(398600.5)}{20000}} = 1.54594 \ \frac{km}{sec}$$