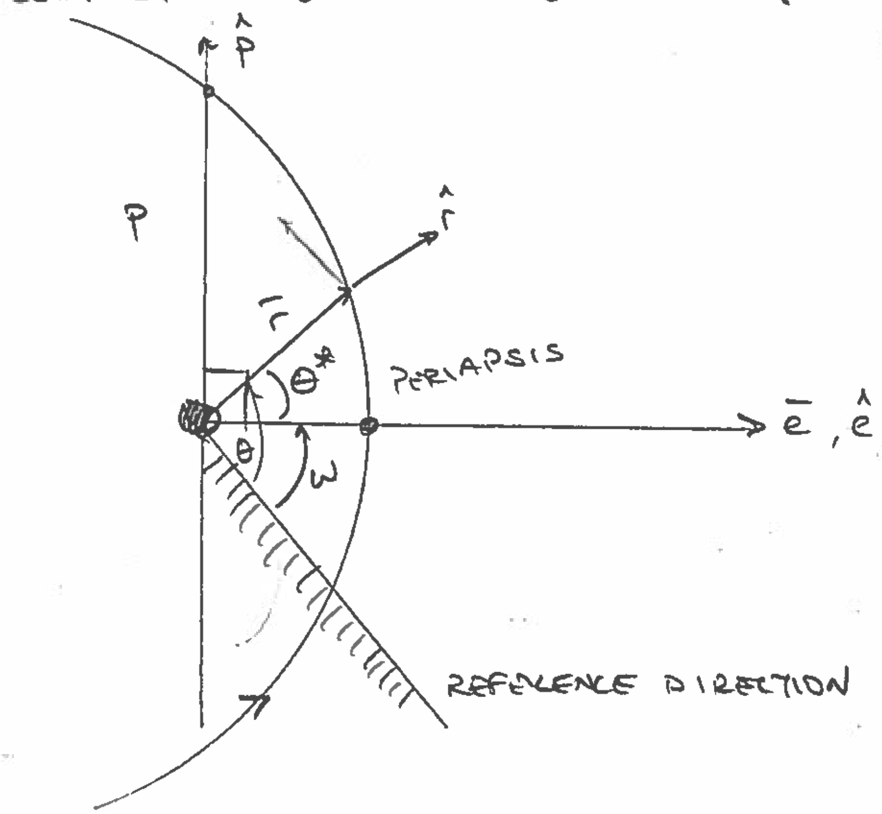
CONIC SECTION

$$r = \frac{h^2/\mu}{1+e\cos\theta} = \frac{p}{1+e\cos\theta}$$

STANDARD POLAR EQ. OF A CONIC SECTION

$$p = \frac{h^2}{\mu} \qquad e = \sqrt{1 + \frac{2\varepsilon h^2}{\mu}}$$

SEMI-LATUS RECTUM   ECCENTRICITY



$2p$ = LATUS RECTUM

θ^* – TRUE ANOMALY (f, ν)

$\theta - \omega = \theta^* = \nu$

$$h = \sqrt{\mu p}$$

$$\varepsilon = \frac{-\mu^2}{2h^2}(1-e^2)$$

DEFINE $a = \dfrac{p}{1-e^2}$ SEMI MAJOR AXIS

$$\boxed{\varepsilon = -\frac{\mu}{2a} = \frac{v^2}{2} - \frac{\mu}{r}}$$

JULIAN DATE – CONTINUOUS COUNT OF DAYS SINCE 1 JAN 4173 B

JD 0 = NOON 1 JAN 4713 BC

JD 2451545 = 12:00 UTC JAN 1 2000

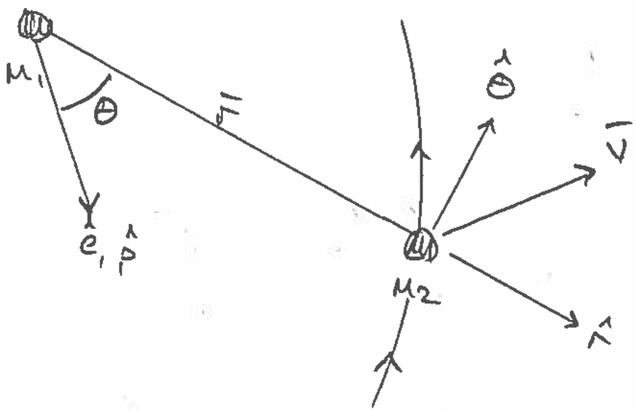
MJD = JD - 2451545

USE ASTRO LIBRARY TO CONVERT JD.

CONIC SECTIONS DEFINED BY TYPES

$0 \le e < 1$	$\mathcal{E} < 0$	ELLIPSE
$e = 1$	$\mathcal{E} = 0$	PARABOLA
$e > 1$	$\mathcal{E} > 0$	HYPERBOLA

CONIC SECTIONS



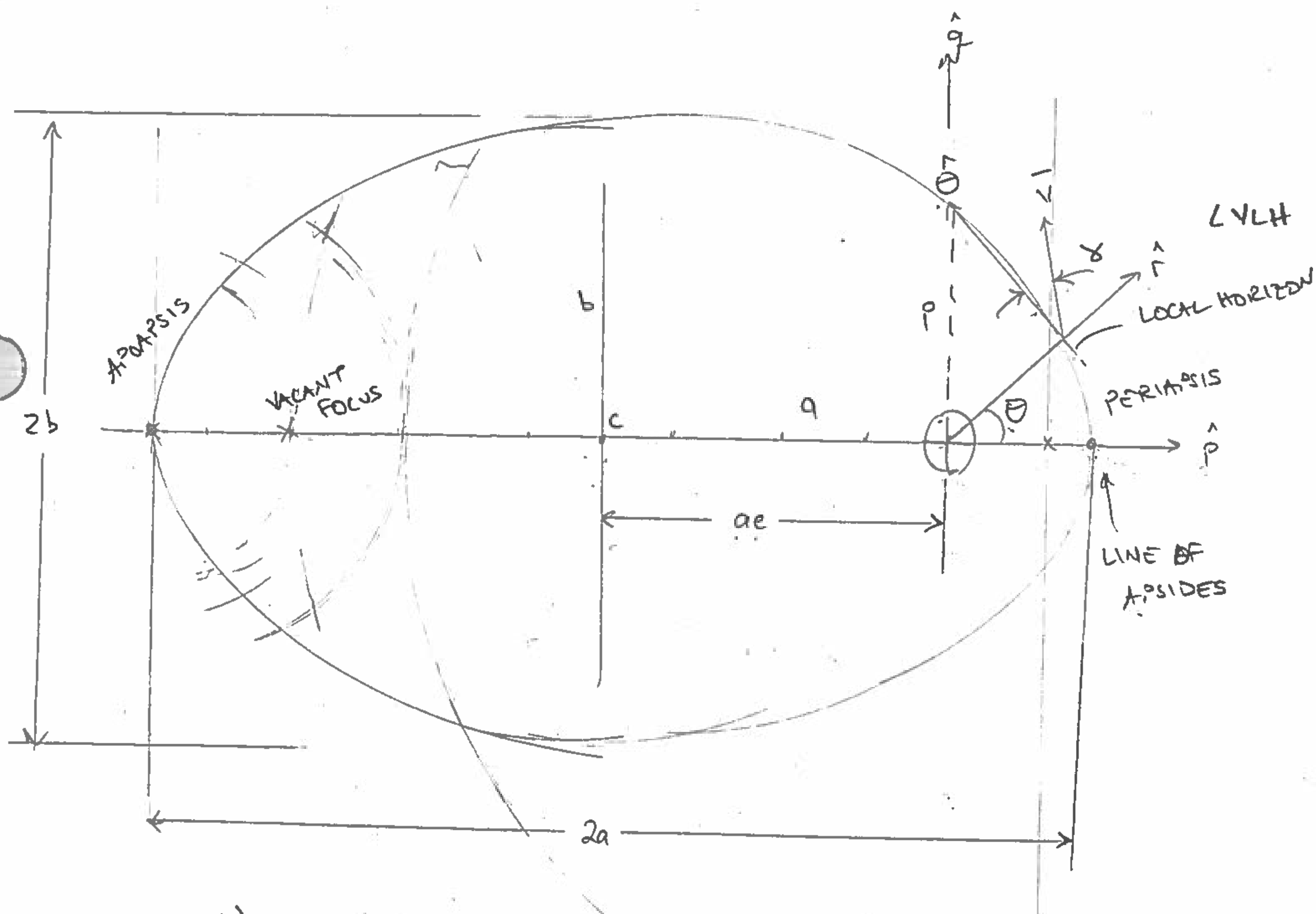
$$r = \frac{h^2/\mu}{1 + e\cos\theta}$$

$$\bar{v} = v_R\,\hat{r} + v_\theta\,\hat{\theta}$$

$$v_R = \frac{\mu}{h}\, e \sin\theta$$

$$v_\theta = \frac{\mu}{h}\left(1 + e\cos\theta\right)$$

ELLIPSE $0 \le e < 1, \quad a > 0, \quad \mathcal{E} < 0$



$p = a(1 - e^2)$

$\theta = 0 \rightarrow$ PERIAPSIS $\quad r_p = \dfrac{p}{1+e} = a(1-e)$

$\theta = 180 \rightarrow$ APOAPSIS $\quad r_a = \dfrac{p}{1-e} = a(1+e)$

$\Bigg\}\ a = \frac{1}{2}(r_p + r_a)$ SEMI MAJOR AXIS

$b = a\sqrt{1 - e^2}$ SEMI MINOR AXIS

γ - FLIGHT PATH ANGLE $\quad 0 < \theta < 180 \rightarrow \gamma > 0$

$180 < \theta < 360 \rightarrow \gamma < 0$

CIRCLE SPECIAL CASE $e = 0$

$a = r = p \quad \boxed{\mathcal{E} = -\frac{\mu}{2a}} = \frac{v^2}{2} - \frac{\mu}{r} \longrightarrow V_c = \sqrt{\frac{\mu}{r}}$ CIRCULAR ORBIT SPEED AT r

GENERAL ELLIPSE

$$\frac{v^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a}$$

$$v^2 = \frac{2\mu}{r} - \frac{\mu}{a} = 2v_c^2 - \frac{\mu}{a}$$

VELOCITY OF ELLIPSE IS ALWAYS LESS THAN $\sqrt{2}$ X CIRCULAR VELOCITY

$\boxed{V < \sqrt{2}\, V_c}$ FIGURE OUT IF ORBIT IS CLOSE

PERIOD

$$\frac{dA}{dt} = \frac{h}{2} \rightarrow dt = \frac{2}{h}\, dA$$

AREA VELOCITY

$$\mathbb{P} = \frac{2}{h}(\pi a b) \leftarrow \text{AREA OF ELLIPSE}$$

$$\begin{cases} b = a(1-e^2)^{1/2} \\ p = a(1-e^2) \\ h = \sqrt{\mu p} \end{cases}$$

$$\boxed{\mathbb{P} = 2\pi\sqrt{\frac{a^3}{\mu}}}$$ PERIOD

BIG P - PERIOD

LITTLE p = SEMI-LATUS RECTUM.

ENERGY

$$\mathcal{E} = -\frac{\mu}{2a} = \frac{v^2}{2} - \frac{\mu}{r} = -\frac{1}{2}\left(\frac{\mu}{h}\right)^2(1-e^2)$$

$e \uparrow \Rightarrow \mathcal{E} \uparrow$

CONIC SECTIONS

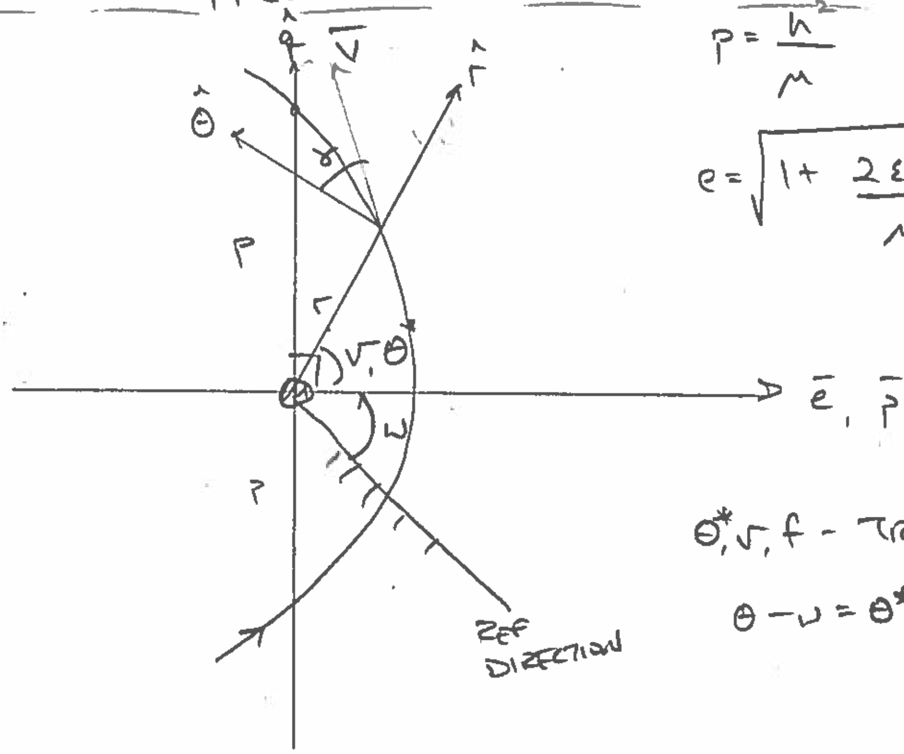
RADIUS / DISTANCE – $r = \dfrac{h^2/\mu}{1 + e\cos\nu} = \dfrac{p}{1 + e\cos\nu}$

CONIC EQUATION

$$\bar{r} = r\,\hat{r}$$

$$\bar{v} = \dot{r}\,\hat{r} + r\dot{\theta}\,\hat{\theta} = \frac{\mu}{h} e\sin\theta\,\hat{r} + \frac{\mu}{h}(1 + e\cos\theta)\,\hat{\theta}$$

$$\tan\gamma = \frac{e\sin\theta}{1 + e\cos\theta}$$



$p = \dfrac{h^2}{\mu}$ SEMI-LATUS RECTUM

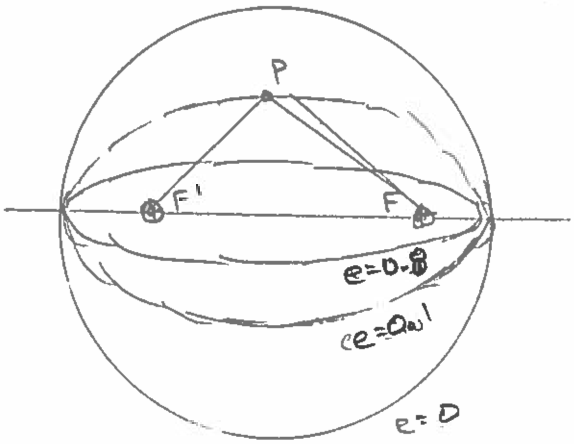
$e = \sqrt{1 + \dfrac{2\varepsilon h^2}{\mu}}$ ECCENTRICITY

θ^*, ν, f – TRUE ANOMALY

$\theta - \omega = \theta^* = \nu$

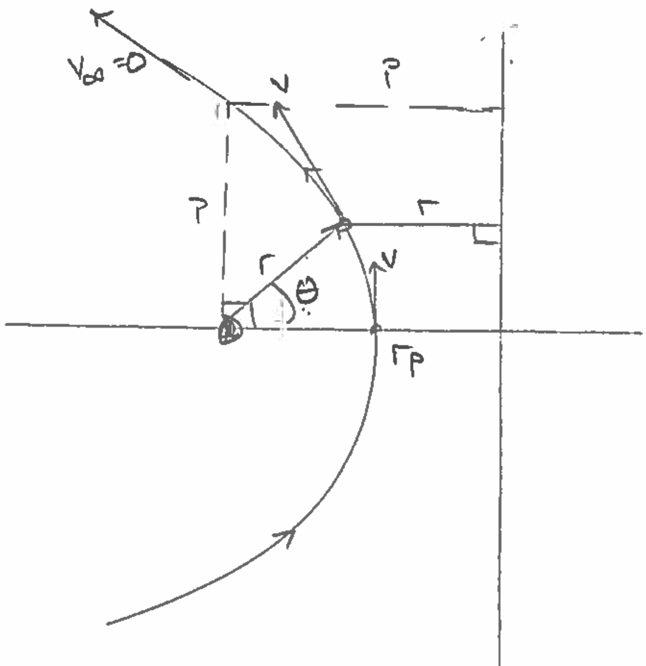
EX ELLIPSES

$\overline{PF} + \overline{PF'} = 2a$ CONSTANT



PARABOLA $e = 1$ $a = \infty$ $\mathcal{E} = 0$ SPECIFIC ENERGY NOT CLOSED

IDEALIZED ORBIT - NOT REALLY FEASIBLE IN REAL LIFE



MINIMUM ENERGY TO ESCAPE CENTRAL BODY

$$-\frac{\mu}{2a} = \mathcal{E} = \frac{V^2}{2} - \frac{\mu}{r} = 0 \longrightarrow V^2 = \frac{2\mu}{r} \Rightarrow \boxed{V_e = \sqrt{2}\, V_c} \qquad V_c = \sqrt{\frac{\mu}{r}}$$

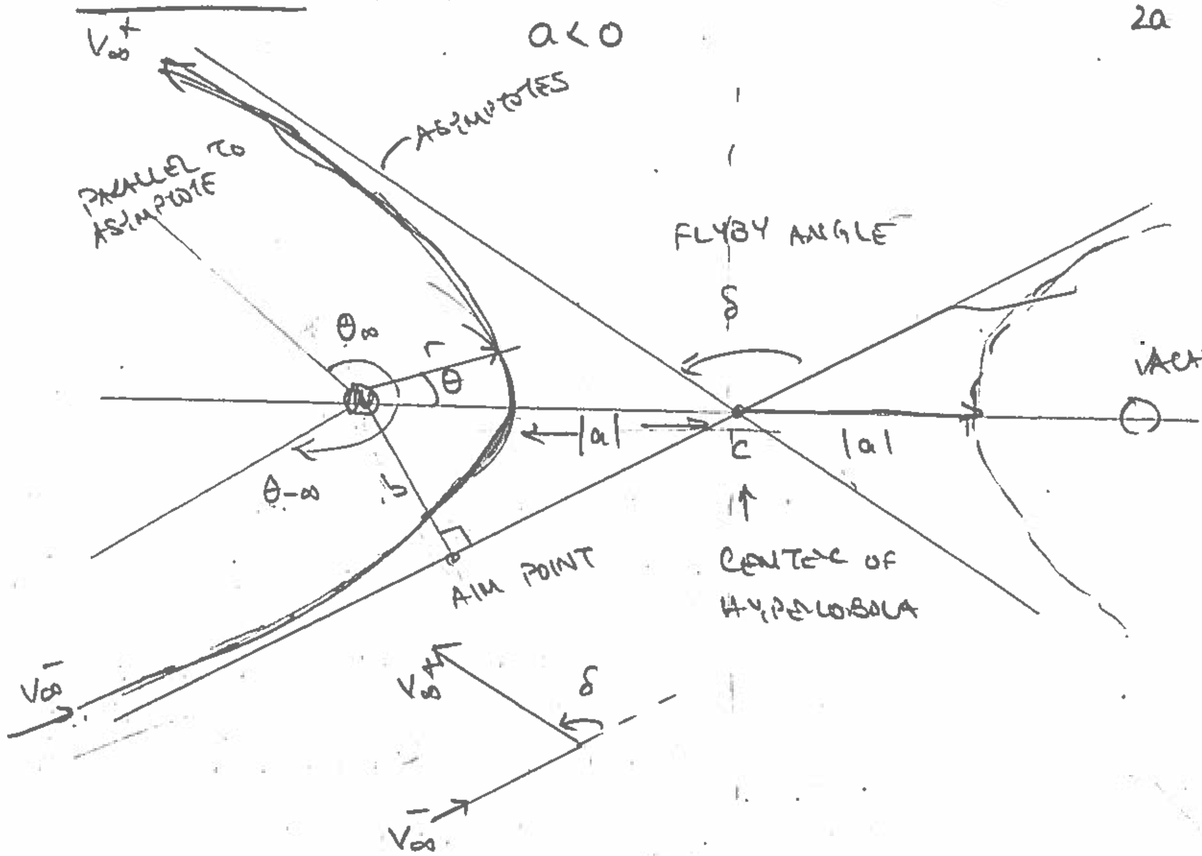
ESCAPE SPEED AT DISTANCE r

$$\frac{V_\infty^2}{2} - \frac{\mu}{r_\infty} = 0 \longrightarrow V_\infty = 0$$

CAN JUST BARELY ESCAPE GRAVITY INFLUENCE

HYPERBOLA $e > 1$ $a < \infty$ $\varepsilon > 0$ $\varepsilon = -\frac{\mu}{2a} = +\frac{\mu}{2|a|}$

$a < 0$



EXCESS RELATIVE VELOCITY AFTER ESCAPE

$$\varepsilon = \frac{\mu}{2|a|} = \frac{v^2}{2} - \frac{\mu}{r} \Rightarrow v^2 = 2\frac{\mu}{r} + \frac{\mu}{|a|}$$

CIRCULAR VELOCITY EXCESS

$\boxed{v > \sqrt{2}\, v_c}$

$$\varepsilon = -\frac{\mu}{2a} = +\frac{\mu}{2|a|} = \frac{v_\infty^2}{2} - \cancel{\frac{\mu}{r_\infty}} = \frac{v_\infty^2}{2} > 0$$

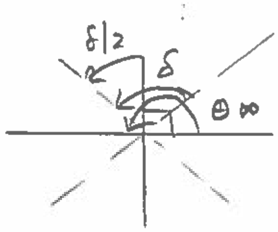
$\boxed{r_p = a(1-e) = |a|(e-1)}$ r_a UNDEFINED

$\boxed{p = a(1-e^2) = |a|(e^2-1)}$ $\boxed{b = |a|\sqrt{e^2-1}}$

$$r_\infty = \frac{|a|(e^2-1)}{1+e\cos\theta_\infty} \rightarrow \boxed{\cos\theta_\infty = -\frac{1}{e}}$$

∞

$$\theta_\infty = 90^\circ + \frac{\delta}{2}$$

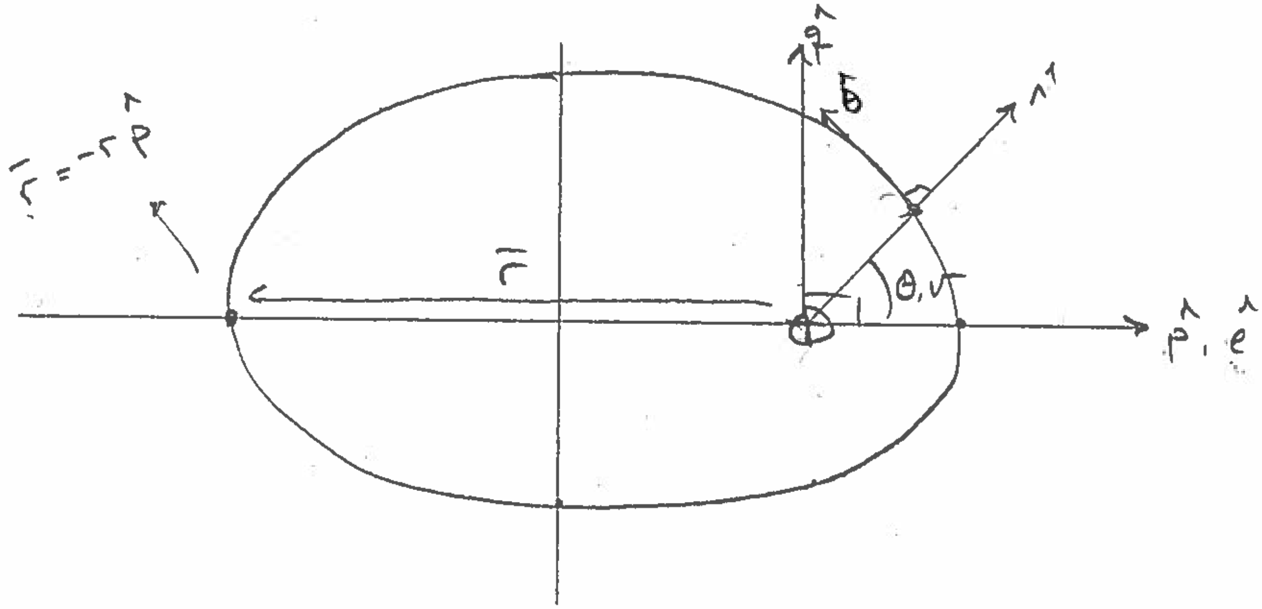


$$\cos\theta_\infty = \cos\left(90^\circ + \frac{\delta}{2}\right)$$

$$\boxed{\sin\frac{\delta}{2} = \frac{1}{e}}$$

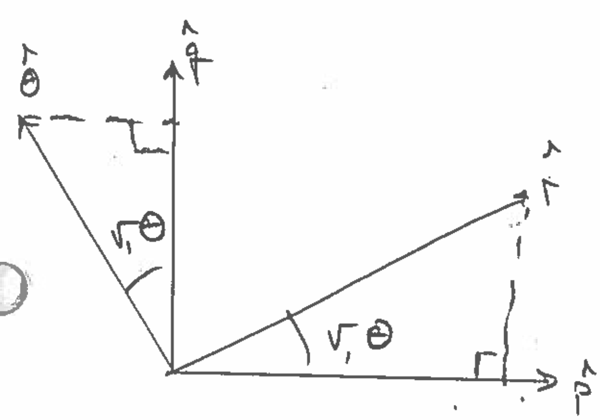
(2) PERIFOCAL REFERENCE FRAME ($\hat{p}\hat{q}\hat{w}$) $\hat{w} = \hat{h}$

- REFERENCE FRAME THAT IS FIXED TO THE ORBIT
- ORIGIN AT ATTRACTING FOCUS OF ORBIT
- $\hat{p}$ - POINTS TOWARDS PERIAPSIS
- $\hat{q}$ - POINTS AT $\nu = 90°$, ALONG SEMI-LATUS RECTUM DIRECTION
- $\hat{w}$ - ALONG ANGULAR MOMENTUM VECTOR



(1) LOCAL VERTICAL | LOCAL HORIZONTAL FRAME

- ROTATING FRAME ATTACHED TO SATELLITE
- $\hat{r}$ - ALONG THE POSITION VECTOR - RADIALLY OUTWARD
- $\hat{\theta}$ - IN ORBIT PLANE - $\perp$ TO $\hat{r}$ - LOCAL HORIZONTAL PLANE
- $\hat{h}$ - ALONG ANGULAR MOMENTUM VECTOR



$R_{LVLH2PQW}$ — $\hat{r}\,\hat{\theta}\,\hat{h} \longrightarrow \hat{p}\,\hat{q}\,\hat{w}$

$$
\begin{array}{c|ccc|}
 & \hat{r} & \hat{\theta} & \hat{h} \\
\hat{p} & c\nu & -s\nu & 0 \\
\hat{q} & s\nu & c\nu & 0 \\
\hat{w} & 0 & 0 & 1
\end{array}
$$

- WE ALREADY HAVE POSITION | VELOCITY IN $\hat{r}\,\hat{\theta}\,\hat{h}$

FROM TRAJECTORY EQ $\qquad r = \dfrac{h^2/\mu}{1 + e\cos\nu}$

- POSITION $\qquad \bar{r} = r\,\hat{r} + 0\,\hat{h}$

VELOCITY $\qquad \bar{v} = \dfrac{\mu}{h} e \sin\theta\,\hat{r} + \dfrac{\mu}{h}(1 + e\cos\theta)\,\hat{\theta} + 0\,\hat{h}$

TRANSFORM TO $\hat{P}\hat{Q}\hat{W}$

$$\bar{r}_{PQW} = R_{\substack{LVLH2\\PQW}} \cdot \bar{r}_{LVLH} = \boxed{r\cos\theta\,\hat{p} + r\sin\theta\,\hat{q} = \bar{r}_{PQW}}$$

$$\bar{v}_{PQW} = R_{\substack{LVLH2\\PQW}} \cdot \bar{v}_{LVLH} =$$

$$= \left[\frac{\mu}{h} e \cancel{\sin\theta} \cdot \cancel{\cos\theta} + \frac{\mu}{h}(1 + e\cancel{\cos\theta}) \cdot -\sin\theta\right]\hat{p}$$

$$+ \left[\frac{\mu}{h} e\sin\theta \cdot \sin\theta + \frac{\mu}{h}(1 + e\cos\theta) \cdot \cos\theta\right]\hat{q}$$

$$+ 0\,\hat{h}$$

$$\boxed{\bar{v}_{pqw} = -\frac{\mu}{h}\sin\theta\,\hat{p} + \frac{\mu}{h}(e + \cos\theta)\,\hat{q}}$$

EXAMPLE FCN - KEPLER MODULE

SPECIFIC MECHANICAL ENERGY

$\mathcal{E} = \frac{V^2}{2} - \frac{\mu}{r}$ $\qquad \mu, V, r \longrightarrow \text{FCN} \longrightarrow \mathcal{E}$

PROBLEM 3

$V = 8.215$

$r = 12172$

$\mathcal{E} = 1 \quad \mu = \mu_\oplus$

TESTS : $V = 0 \quad r = 1 \longrightarrow \mathcal{E} = -\mu$

$V = 1 \quad r = 1 \longrightarrow \mathcal{E} = \frac{1}{2} - \mu$

RADIUS OF PERIAPSIS + APOAPSIS - ELLIPTICAL ORBITS

$r_p = a(1-e)$ $\qquad a, e \longrightarrow \text{FCN} \longrightarrow r_p, r_a$

$r_a = a(1+e)$

TESTS : $a = 6378 \quad e = 0 \Rightarrow r_p = r_a = a$

$a = 8000 \quad e = 0.5 \longrightarrow r_p = 4000, \; r_a = 12000$

PROBLEM 2 $\qquad a = 38500, \; e = 0.8181$

$r_p = 7000 \text{ km} \qquad r_a = 70000 \text{ km}$