LSN 4 – INTRO TO PROGRAMMING

INTRODUCE PYTHON

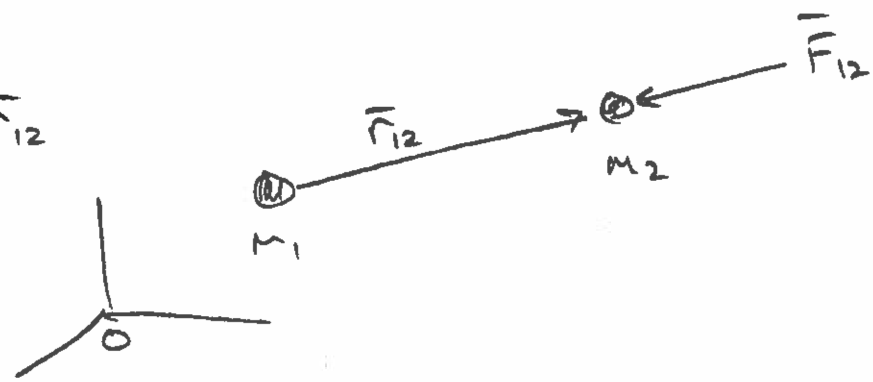
- USE SLIDES – INTRO, INSTALLATION, VARIABLES, FUNCTIONS, CODE STRUCTURE, PLOTTING
- MAKE THEM DOWNLOAD ASTRO LIBRARY
- SHOW DEMO FOR ODEINT + PLOTTING + GRAVITY FCN

CODE – RELATIVE 2-BP

```
- sol = scipy.integrate.odeint(eoms, initial_state, t_vec)
- def eoms(state, t):


      return xdot

- fig, ax = plt.subplots()
  ax.plot(x, y, label='title')
  ax.set_ylabel(          )
  ax.set_xlabel(          )
  ax.set_title(           )
  plt.show()
```

GRAVITY

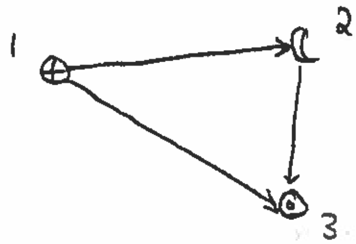
$$\bar{F}_{12} = -\frac{G m_1 m_2}{r_{12}^3}\bar{r}_{12}$$



$$\ddot{\bar{r}}_{12} + \frac{\mu}{r_{12}^3}\bar{r}_{12} = 0$$

```
import numpy as np
import scipy.integrate
import matplotlib.pyplot as plt
import pdb

G = 1
m1 = 2
m2 = 1
                 , m1, m2, G
def eomTBI(state, t):
    """EOMS
    """
    r1 = state[0:3]
    v1 = state[3:6]
    r2 = state[6:9]
    v2 = state[9:12]


    r = r2-r1

    R1_dot = v1
    V1_dot = G*m2/np.linalg.norm(r)**3 *r
    R2_dot = v2
    V2_dot = -G*m1/np.linalg.norm(r)**3 *r

    X_dot = np.concatenate((R1_dot, V1_dot, R2_dot, V2_dot))

    return X_dot

def sim():
    # simulate over ten seconds
    r10 = [0, 0, 0]
    r20 = [1, 0, 0]
    v10 = [0, 0 ,0]
    v20 = [1, 1, 0]
    initial_state = np.hstack((r10, v10, r20, v20))
    t = np.linspace(0, 10, 1000)
                                                              , args = (m1, m2, G)
    sol = scipy.integrate.odeint(eomTBI,initial_state , t)

    # extract out the states
    r1 = sol[:, 0:3]
    v1 = sol[:, 3:6]
    r2 = sol[:, 6:9]
    v2 = sol[:, 9:12]

    # compute the COM
    rcom = (m1*r1 + m2*r2) / (m1 + m2)
    r = r2 - r1

    traj_fig,  traj_ax = plt.subplots()
    traj_ax.plot(r1[:, 0], r1[:, 1], label=r'$r_1$')
```

```
    traj_ax.plot(r2[:, 0], r2[:, 1], label=r'$r_2$')
    traj_ax.plot(rcom[:, 0], rcom[:, 1], label=r'$r_c$')
    traj_ax.grid()
    traj_ax.set_title('Inertial Frame')
    traj_ax.set_xlabel('X axis')
    traj_ax.set_ylabel('Y axis')
    plt.legend()

    rel_fig, rel_ax = plt.subplots()
    rel_ax.plot(r[:, 0], r[:, 1])
    rel_ax.plot([0], [0], 'bo', markersize=20)
    rel_ax.set_title('Relative motion of $m_2$ wrt $m_1$')
    rel_ax.set_ylabel('Y axis')
    rel_ax.set_xlabel('X axis')
    rel_ax.grid()
    plt.show()

if __name__ == '__main__':
    sim()
```



HOW DOES $\leftmoon$ MOVE RELATIVE TO $\oplus$?

$\Longrightarrow \ddot{\vec r}_{\oplus\leftmoon}$ $\boxed{\text{CANNOT SOLVE}}$

$$\ddot{\vec r}_{\oplus\leftmoon} + \frac{G(m_\oplus + m_\leftmoon)}{r_{\oplus\leftmoon}^3}\vec r_{\oplus\leftmoon} = G m_\odot \left(\frac{\vec r_{\leftmoon\odot}}{r_{\leftmoon\odot}^3} - \frac{\vec r_{\oplus\odot}}{r_{\oplus\odot}^3}\right)$$

$\odot$ $\oplus$ DOM ← $\leftmoon$ → NET PERT.

DOMINANT	$2.73 \times 10^{-6}\ \frac{km}{sec^2}$	
DIRECT	$5.90 \times 10^{-6}\ \frac{km}{sec^2}$	NET PERTURBING ACCEL $3 \times 10^{-8}\ \frac{km}{sec^2}$
INDIRECT	$5.93 \times 10^{-6}\ \frac{km}{sec^2}$	

$\odot$ PERT. ← $\leftmoon$ → DOM $\oplus$

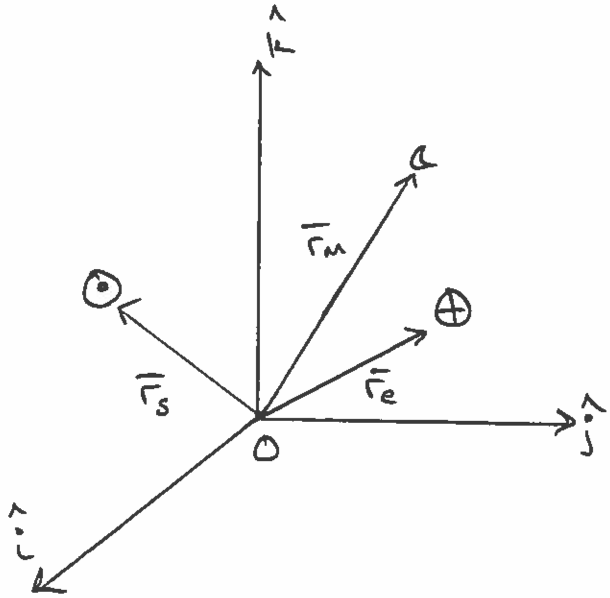
DOMINANT	$2.73 \times 10^{-6}\ \frac{km}{sec^2}$	
DIRECT	$\leftarrow 5.96 \times 10^{-6}\ \frac{km}{sec^2}$	NET PERTURBING ACCEL $\sim 3 \times 10^{-8}\ \frac{km}{sec}$
INDIRECT	$5.93 \times 10^{-6}\ \frac{km}{sec^2}$	

$\boxed{\text{PAY ATTENTION TO VECTOR DIRECTION}}$

N-BODY EXAMPLE

CONSIDER THREE BODIES: $\odot$, $\oplus$, $☾$

SUN EARTH MOON



WE WILL ASSUME THAT OUR ORIGIN = SUN !

CENTER OF MASS

$$\left(\sum_{i=1}^{3} m_i\right) \bar{r}_{com} = \sum_{i=1}^{3} m_i \bar{r}_i$$

$$\bar{r}_{com} = \frac{m_s \bar{r}_s + m_e \bar{r}_e + m_m \bar{r}_m}{m_s + m_e + m_m}$$

← PLUG IN VALUES FROM CONSTANTS SHEET

FOR OUR SOLAR SYSTEM + CONSTANTS SHEET

$\bar{r}_s = \bar{0}$ => FOR THIS PROBLEM WE'LL TREAT PT O, OUR ORIGIN AS THE SUN

=> THIS MEANS $\bar{r}_{com}$ WILL BE A VECTOR FROM THE SUN | ORIGIN TO THE CENTER OF MASS

EQUATIONS OF MOTION OF $☾$ WRT CENTER OF MASS

$$\boxed{m_☾ \ddot{\bar{r}}_☾ = -G \sum_{j=1}^{2} \frac{m_☾ m_j}{r_{j☾}^3} \bar{r}_{j☾}}$$

WHERE j = SUN, EARTH

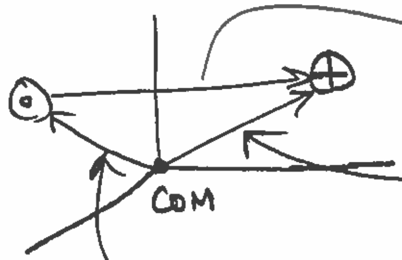
CENTER OF MASS IS AN INERTIALLY FIXED POINT FROM THE CONSERVATION OF LINEAR MOMENTUM

$$M_{☾}\,\ddot{\bar{r}}_{COM☾} = -\frac{G M_{☾} M_{\odot}}{r_{\odot☾}^{3}}\,\bar{r}_{\odot☾} - \frac{G M_{\oplus} M_{☾}}{r_{\oplus☾}^{3}}\,\bar{r}_{\oplus☾} \qquad (*)$$

CENTER OF MASS TO ☾

NEED TO FIND VECTOR FROM SUN TO MOON/EARTH

LET'S FIND THE POSITION OF EACH BODY WRT COM.



$\bar{r}_{\odot\oplus}$ = FROM CONSTANTS SHEET

$\bar{r}_{COM\oplus} = \bar{r}_{COM\odot} + \bar{r}_{\odot\oplus}$

SOLVE FOR THIS PART, TO USE IN (*)

$\bar{r}_{COM\odot}$ = FROM PART A

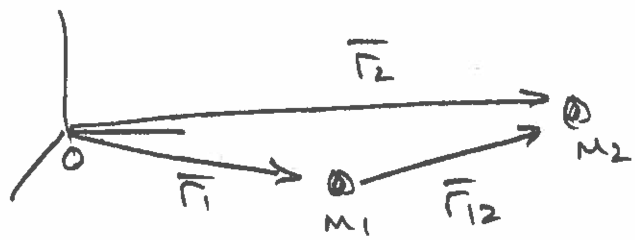
QUESTION TO ASK YOURSELF

1. IS THERE A DIFFERENCE IN THESE TWO EQUATIONS

$\bar{r}_{\odot\oplus} = \bar{r}_{☾\oplus} - \bar{r}_{☾\odot} \rightarrow$ (MOON TO EARTH) $-$ (MOON TO SUN)

$\bar{r}_{\odot\oplus} = \bar{r}_{COM\oplus} - \bar{r}_{COM\odot} \rightarrow$ (COM TO EARTH) $-$ (COM TO SUN)

2BP - CONSTANTS OF MOTION EXAMPLE



$\bar{r}_{12} = \bar{r}_2 - \bar{r}_1$

$\bar{r}_{21} = \bar{r}_1 - \bar{r}_2 = -\bar{r}_{12}$

VECTOR ODE FOR m_1, m_2

$$m_1 \ddot{\bar{r}}_1 = -\frac{G m_1 m_2}{r_{21}^3} \bar{r}_{21} \qquad m_2 \ddot{\bar{r}}_2 = -\frac{G m_1 m_2}{r_{12}^3} \bar{r}_{12}$$

ADD THE EQNS — LINEAR MOMENTUM

$$m_1 \ddot{\bar{r}}_1 + m_2 \ddot{\bar{r}}_2 = -\frac{G m_1 m_2}{r_{21}^3} \bar{r}_{21} - \frac{G m_1 m_2}{r_{12}^3} \bar{r}_{12}$$

$$m_1 \ddot{\bar{r}}_1 + m_2 \ddot{\bar{r}}_2 = \bar{0}$$

INTEGRATE

$$m_1 \dot{\bar{r}}_1 + m_2 \dot{\bar{r}}_2 = \bar{C}_1 \quad \leftarrow$$

$$m_1 \bar{r}_1 + m_2 \bar{r}_2 = \bar{C}_1 t + \bar{C}_2$$

COM VELOCITY DEFINED AS $\Rightarrow \dot{\bar{r}}_{com} = \dfrac{m_1 \dot{\bar{r}}_1 + m_2 \dot{\bar{r}}_2}{m_1 + m_2} = \dfrac{\bar{C}_1}{m_1 + m_2}$

CONSTANT

CROSS PRODUCT – SYSTEM ANGULAR MOMENTUM

$$m_1 \ddot{\bar{r}}_1 \times \bar{r}_1 = \frac{G m_1 m_2}{r_{12}^3}(\bar{r}_2 - \cancel{\bar{r}_1}) \times \bar{r}_1 \qquad m_2 \ddot{\bar{r}}_2 \times \bar{r}_2 = \frac{G m_1 m_2}{r_{21}^3}(\bar{r}_1 - \cancel{\bar{r}_2}) \times \bar{r}_2$$

$$m_1 \ddot{\bar{r}}_1 \times \bar{r}_1 + m_2 \ddot{\bar{r}}_2 \times \bar{r}_2 = \frac{G m_1 m_2}{r_{12}^3}\left(\bar{r}_2 \cancel{\times \bar{r}_1} + \cancel{\bar{r}_1 \times} \bar{r}_2\right)$$

$$m_1 \ddot{\bar{r}}_1 \times \bar{r}_1 + m_2 \ddot{\bar{r}}_2 \times \bar{r}_2 = \bar{0}$$

$$m_i \frac{d}{dt}\left(\dot{\bar{r}}_i \times \bar{r}_i\right) = m_i\left(\ddot{\bar{r}}_i \times \bar{r}_i\right) + m_i\left(\cancel{\dot{\bar{r}}_i \times \dot{\bar{r}}_i}\right)$$

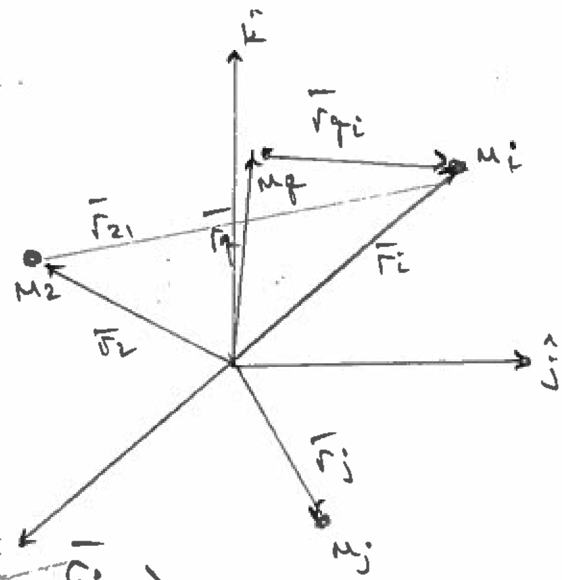
$$\sum m_i \frac{d}{dt}\left(\dot{\bar{r}}_i \times \bar{r}_i\right) = 0$$

INTEGRATE

$$\sum m_i \dot{\bar{r}} \times \bar{r}_i = \bar{C}_3$$ ← SYSTEM ANGULAR MOMENTUM IS CONSTANT.

REVIEW

$$\bar{F} = -\frac{G m_1 m_2}{r_{21}^2} \frac{\bar{r}_{21}}{r_{21}} \qquad \text{UNIVERSAL GRAVITATION}$$



$$m_i \ddot{\bar{r}}_i = -G \sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{m_i m_j}{r_{ji}^3} \bar{r}_{ji} \qquad \text{N-BODY}$$

$$\ddot{\bar{r}}_{qi} + \frac{G(m_q + m_i)}{r_{qi}^3} \bar{r}_{qi} = G \sum_{\substack{j=1 \\ j \neq q,i}}^{n} \left(\frac{\bar{r}_{ij}}{r_{ij}^3} - \frac{\bar{r}_{qj}}{r_{qj}^3} \right) \qquad \text{RELATIVE N-BODY}$$

DOMINANT   DIRECT   INDIRECT

$$\ddot{\bar{r}} + \frac{\mu}{r^3} \bar{r} = \bar{0} \qquad \text{RELATIVE 2BP} \qquad \text{MOTION OF } m_i \text{ WRT } m_q$$

CONSTANTS OF MOTION

6 LINEAR MOMENTUM

3 ANGULAR MOMENTUM

1 ENERGY