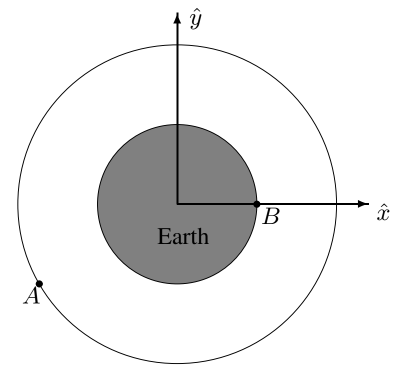
MAE3145: Homework 5

Due date: December 7, 2016

Problem 1 Consider a reentry vehicle at the point A on a circular orbit around the Earth. We wish to design a reentry orbit such that the vehicle arrives at B along a trajectory tangent to the surface of the Earth at B, i.e. the point B is the periapsis of the reentry orbit.



The initial circular orbit is referred to as Orbit 1, and the reentry orbit is referred to as Orbit 2. We do not consider the atmospheric drag effects. Assume that

$$R_1 = 7000\,\text{km}, \quad R_E = 6378\,\text{km}, \quad \mu = 398600\,\text{km}^3/\text{s}^2, \quad \theta_A = 210°.$$

Recall that the velocity vector of a mass located at θ on an orbit with h, e is given by

$$\vec{v} = \frac{\mu}{h}[-\sin\theta\hat{x} + (e + \cos\theta)\hat{y}].$$

(a) Find the velocity vector $\vec{v}_{A_1}$ of the reentry vehicle at the point A on Orbit 1.

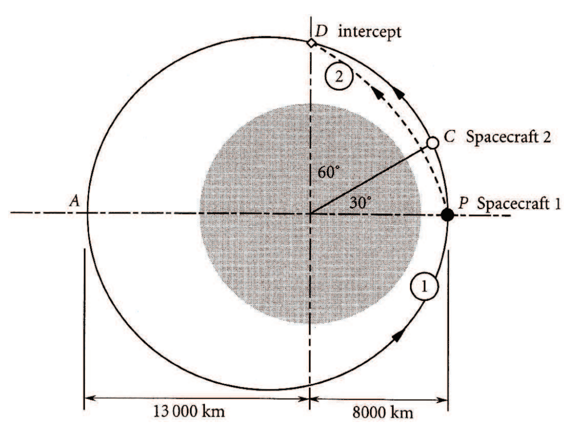
(b) Find the eccentricity e_2 and the specific angular momentum h_2 of Orbit 2.

(c) Find the velocity vector $\vec{v}_{A_2}$ of the reentry vehicle at the point A on Orbit 2.

(d) Find the required velocity change $\vec{v}_A$ at the point A.

(e) Find the resulting velocity $\vec{v}_{B_2}$ of the reentry vehicle at the surface of the Earth.

Problem 2 We solve Exercise 6.21 of the textbook. Two spacecraft are on the same elliptic orbit with $r_p = 8000\,\text{km}$, $r_a = 13000\,\text{km}$. Currently, Spacecraft 1 is at the point P ($\theta_P = 0$), and Spacecraft 2 is at the point C ($\theta_C = 30°$). We will find the velocity change for Spacecraft 1 to intercept and rendezvous with Spacecraft 2 at the point D ($\theta_D = 90°$).



Recall that the position vector of a mass located at θ on an orbit with h, e is given by

$$\vec{r} = r[\cos\theta\hat{x} + \sin\theta\hat{y}], \qquad \text{where} \quad r = \frac{h^2}{\mu}\frac{1}{1+e\cos\theta}.$$

(a) Find the position vector $\vec{r}_P$ and the velocity vector $\vec{v}_{P_1}$ of Spacecraft 1 at the point P of Orbit 1.

(b) Find the position vector $\vec{r}_D$ and the velocity vector $\vec{v}_{D_1}$ of Spacecraft 1 at the point D of Orbit 1.

(c) Show that the time required for Spacecraft 2 to move from C to D is $t_{CD} = 1.3304 \times 10^3$ seconds.

(d) The Matlab function `LambertProb.m` posted at Black Board finds the solution of a Lambert problem. It has the following input and output variables:

```
function [vA_vec vB_vec]=LambertProb(rA_vec,rB_vec,tAB,mu)
% Input 1: rA_vec the position vector to the initial point A
% Input 2: rB_vec the position vector to the terminal point B
% Input 3: tAB transfer time from A to B
% Input 4: mu gravitational parameter
% Output 1: vA_vec the velocity vector at A on the transfer orbit
% Output 2: vB_vec the velocity vector at B on the transfer orbit
```

Using this function, find the velocity vector $\vec{v}_{P_2}$ and $\vec{v}_{D_2}$ of Spacecraft 1 on the transfer orbit, Orbit 2.

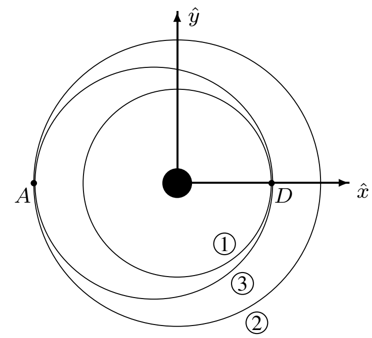
(e) Find the required velocity change $\Delta\vec{v}_P$ and $\Delta\vec{v}_D$ of Spacecraft 1 at P and D

(f) Show that the total velocity change is $\Delta v_{total} = 6.239\,\mathrm{km/s}$.

We wish to design a spacecraft mission to explore the Mars. Suppose that

$$R_E = 149.6 \times 10^6\,\mathrm{km}, \quad R_M = 227.9 \times 10^6\,\mathrm{km},$$
$$\mu_E = 398600\,\mathrm{km}^3/\mathrm{s}^2, \quad \mu_M = 42830\,\mathrm{km}^3/\mathrm{s}^2, \quad \mu_S = 1.3271 \times 10^{11}\,\mathrm{km}^3/\mathrm{s}^2,$$
$$m_E = 5.974 \times 10^{24}\,\mathrm{kg}, \quad m_M = 6.419 \times 10^{23}\,\mathrm{kg}, \quad m_S = 1.989 \times 10^{30}\,\mathrm{kg}.$$

Problem 3 The orbits ①, ② of the Earth and the Mars around the Sun, and the Hohmann transfer orbit ③ from the Earth to the Mars are shown as follows.



The location of the Earth at departure, and the location of the Mars at arrival are denoted by the point D, and A, respectively.

(a) Find the velocity V_{D_1} and V_{D_3} with respect to the Sun.

(b) Find the velocity V_{A_3} and V_{A_2} with respect to the Sun.

(c) Find the travel time t_{DA}.

Problem 4 In this problem, we determine the possible launch date after December 1, 2016 as follows. On August 27, 2003, the true anomalies of the Mars and the Earth were as follows:

$$\theta_M(0) = 358.13^\circ, \quad \theta_E(0) = 230.81^\circ.$$

The time duration between those two dates is 4845 days [1].

The relative phase between the Mars and the Earth at t, namely $\phi(t) = \theta_M(t) - \theta_E(t)$ can be written as

$$\phi(t) = \phi(0) + (n_M - n_E)t,$$

where t represents time since August 27, 2003, i.e., $t = 0$ on August 27, 2003.

(a) Find $\phi(0)$, $n_M - n_E$ in the above equation.

(b) Show that the initial phase angle for the Hohmann transfer from the Earth to the Mars is $\phi_0 = 44.3292^\circ$ from Rendezvous conditions.

(c) The launch date can be determined from the following equation:

$$\phi_0 = \phi(0) + (n_M - n_E)t_d + 2\pi k,$$

where k is an integer, and t_d denotes the time at which the spacecraft exits the sphere of influence of the Earth, measured from August 27, 2003. Find the earliest possible time t_d after December 1, 2016, i.e., choose the integer k such that t_d is greater than 4845 days.

[1] http://www.timeanddate.com

(d) The departure date can be found by adding t_d to August 27, 2003. Convert t_d into days, and find the departure date by using the following web site for date calculation:

`http://www.timeanddate.com/date/dateadd.html`.

Problem 5 In this problem, we design a departure hyperbolic orbit. Suppose that the spacecraft is on a circular orbit around the Earth with an orbital radius of $r = 9000\,\mathrm{km}$.

(a) Find the radius of the sphere of influence of the Earth, r_{SOI_E}.

(b) Using your answers to Problem 1, find the hyperbolic excess speed v_∞ required for the Hohmann transfer.

(c) Find the specific angular momentum h, and the eccentricity e of the departure hyperbolic orbit.

(d) Show that the required velocity change Δv of the spacecraft at the initial circular parking orbit is $\Delta v = 3.2061\,\mathrm{km/s}$.

(e) Find the location of the rocket firing at the initial circular parking orbit. Answer in terms of the angle measured from the line connecting the Earth and the Sun counterclockwise.

Problem 6 In this problem, we design an arrival hyperbolic orbit. Suppose that the arrival hyperbolic orbit is tangent to the surface of the Mars at the periapsis, i.e. the periapsis radius is equal to the radius of the Mars $r_M = 3396\,\mathrm{km}$.

(a) Find the radius of the sphere of influence of the Mars, r_{SOI_M}.

(b) Using your answers to Problem 1, find the hyperbolic excess speed v_∞ resulting from the Hohmann transfer.

(c) Find the specific angular momentum h, and the eccentricity e of the arrival hyperbolic orbit.

(d) Show that the velocity of the spacecraft at the surface of the Mars is $v_p = 5.6776\,\mathrm{km/s}$. Ignore the atmospheric drag.

(e) Find the location of the arrival point on the surface of the Mars. Answer in terms of the angle measured from the line connecting the Mars and the Sun counterclockwise.

(f) Find the aiming radius Δ of the arrival hyperbolic orbit at the surface of the sphere of influence.