MAE3145: Homework 4

Due date: 2 458 038.197 916 666 5 JD

Problem 1. Halley's comet last passed through perihelion on Feb 9, 1986. The orbit is described with an eccentricity of $e = 0.9671429$ and a semi-major axis of $a = 17.834144$ua. In 1986, the European Space Agency spacecraft Giotto encountered and photographed teh nucleus of the comet as it approached the Sun. Data from Giotto's camera was used to generate the enhanced image shown in Fig. 1. The potatoe shaped nucleus

Figure 1: Halley's Comet as seen by Giotto

measures roughly 15 km across and is composed primarily of water and carbon dioxide ices.

(a) On this date, determine the following additional orbital characteristics associated with Halley's orbit (assume a conic model):

$$r, \quad v, \quad \nu, \quad \gamma, \quad \mathcal{E}, \quad h, \quad \mathsf{P}, \quad r_a, \quad r_p, \quad E$$

Ensure that you list all distances in astronomical units.

You can check the path of 1P/Halley at the JPL Small-Body Data Browser by going to the following link. Observers on Earth can pick up the comet when its true anomaly is approximately 100° prior perihelion.

(a) Determine the same quantities shown in part (a) above, but at the time when the observers expect to pick up the next return.

(b) Determine the time required to go from the last perihelion passage to the location where Earth observers can again pick up the return of Halley's Comet. What is the approximate date? How old will you be?

(c) How many days until the comet passes through perihelion?

Problem 2. A satellite is in orbit about the Earth. Assume that it is reasonable to model the behavior in terms of the relative two-body problem (Earth and Satellite). The orbit is characterized by $r_p = 1.5R_\oplus$ and $r_a = 6.2R_\oplus$. It is currently located, t_0, at the point in the orbit such that $\nu = 135°$.

(a) Determine the following orbit paramterts and satellite state information:

$$a, \quad e, \quad p, \quad \mathsf{P}, \quad \mathcal{E}, \quad r_0, \quad v_0, \quad E_0, \quad \gamma_0, \quad (t_0 - T)\,.$$

List the time in hours and all angles in degrees.

(b) Write $\bar{r}_0, \bar{v}_0$ in terms of components in both the local vertical/local horizontal and perifocal reference frames.

(c) Determine the value of true anomaly in exactly 3 h. Determine the satellite state at the new time, t_1.

$$r_1, \quad v_1, \quad E_1, \quad \gamma_1, (t_1 - T)$$

Determine the $\Delta\nu, \Delta E$ that corresponds to $\Delta t = 3\,\mathrm{h}$.

(d) Write $\bar{r}_1, \bar{v}_1$ in terms of the local vertical/local horizontal and perifocal reference frames.

(e) Plot the orbit using your skills from the previous homeworks. By hand, mark on the plot the location of perigee and location of the satellite at t_0 and t_1. At each location indicate $r, \nu, \bar{v}, E, \gamma$ also sketch the local horizon. Add the auxillary circle and mark $\Delta\nu$ and ΔE.

Problem 3. As part of some interplanetary mission, assume that a spacecraft departs the Earth vicinity along a hyperbola. The hyperbola is defined such that $r_p = 1000\,\mathrm{km}$ and $e = 1.05$.

(a) Determine the following orbital characteristics: $a, p, v_\infty, \mathcal{E}, \delta, \nu_\infty$ and the aiming radius.

(b) When the spacecraft reaches $\nu_1 = 90°$, determine $r_1, v_1, H_1, \gamma_1, (t_1 - T)$. Write $\bar{r}_1, \bar{v}_1$ in terms of the local vertical/local horizontal and perifocal reference frames?

(c) Plot the hyperbolic orbit between $\pm 135°$, ensure you also include asymptotes. Label the appropriate quantities including, $b, \delta, \frac{\delta}{2}, \nu_\infty, a, \gamma, \text{center}, \text{local horizon}$. Add the Earth to scale.

(d) How long until the spacecraft reaches $\nu_2 = 150°$, $t = t_2$? What is the value of r at this time?

Problem 4. An orbit transfer vehicle (OTV) is currently in Earth orbit with the following characteristics (with respect to the Earth Centered Inertial frame).

$$\begin{aligned} a &= 3R_\oplus & \Omega &= 45° \\ e &= 0.40 & \omega &= 90° \\ i &= 28.5° & \nu &= 235° \end{aligned}$$

(a) Determine the state of the satellite and additional orbital parameters: $\bar{r}, \bar{v}, r, v, \gamma, \nu, M, E, (t - T)$. In addition, write $\bar{r}, \bar{v}$ in terms of the local horizontal/local vertical, perifocal, and inertial reference frames.

(b) Plot the orbital plane and mark all of the appropriate quantities. What are the appropriate quantities?

Problem 5. The OTV is now in a new orbit. At a certain time, the following information is given.

$$\begin{aligned} \bar{r}_1 &= 3.0R_\oplus\hat{x} + 5.0R_\oplus\hat{y}\,\mathrm{km} \\ \bar{v}_1 &= -3.2\hat{x} + 2.0\hat{y} + 2.5\hat{z}\,\mathrm{km\,s^{-1}}. \end{aligned}$$

Determine the following and plot the orbit in the orbital plane.

$$a, \quad e, \quad i, \quad \omega, \quad \Omega, \quad r, \quad v, \quad \gamma, \quad \nu, \quad M, \quad E, \quad (t - T)\,.$$