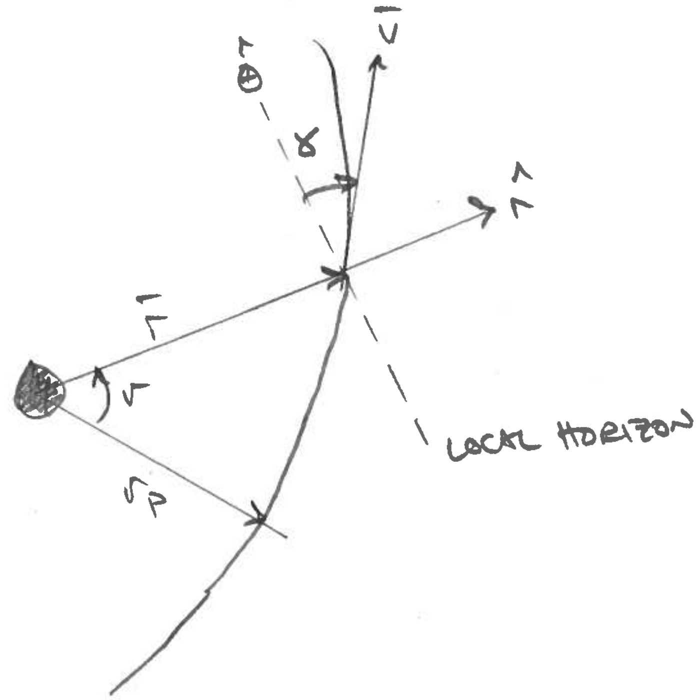
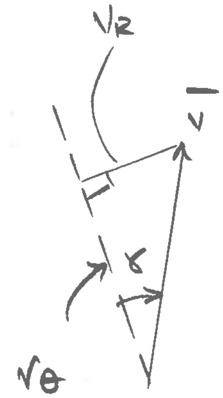
ORBITAL MANEUVERS

- HAVE ONLY CONSIDERED ORBIT PROPERTIES
- ARTIFICIAL SATELLITES CAN CHANGE ORBITS
- OUR APPROACH
    - DEFINE POSITION/VELOCITY IN ORBIT
    - SINGLE IMPULSE ADJUSTMENT
    - MULTIPLE IMPULSE TRANSFERS.





$$V_R = V\sin\gamma$$
$$V_\theta = V\cos\gamma$$

r, v, γ CHARACTERIZE THE ORBIT

$$\bar{h} = \bar{r} \times \bar{v} \Rightarrow h = r V_\theta$$

$$\boxed{V_\theta = \frac{h}{r} = \frac{\mu}{h}(1 + e\cos\nu)} \qquad (1)$$

$$V_R = \dot{r} = \frac{dr}{d\nu}\dot{\nu} = \frac{dr}{d\nu}\frac{h}{r^2}$$

$$= \frac{d}{d\nu}\left(\frac{h^2/\mu}{1 + e\cos\nu}\right)\frac{h}{r^2} \Rightarrow \boxed{V_R = \frac{\mu e}{h}\sin\nu} \qquad (2)$$

REARRANGE ① + ②

$$e\cos\nu = \frac{h v_\theta}{\mu} - 1$$

$$e\sin\nu = \frac{h v_R}{\mu} = \frac{r v_\theta v_R}{\mu}$$

$$e^2 = \left[(e\cos\nu)^2 + (e\sin\nu)^2\right]$$

$$= \left(\frac{h v \cos\gamma}{\mu} - 1\right)^2 + \frac{r v^2 (\cos^2\gamma) v^2 (\sin^2\gamma)^2}{\mu^2}$$

$(1-\cos^2\gamma)$

$$= \frac{r v^2 \cos^4\gamma}{\mu} - \frac{2 r v^2 \cos^2\gamma}{\mu} + 1 + \frac{r v^4 (\cos^2\gamma)(\sin^2\gamma)^2}{\mu^2}$$

$\sin^2\gamma + \cos^2\gamma$

$$\boxed{e^2 = \left(\frac{r v^2}{\mu} - 1\right)^2 \cos^2\gamma + \sin^2\gamma}$$

$$\tan\nu = \frac{r v_\theta v_r/\mu}{\frac{h v_\theta}{\mu} - 1} = \frac{r v^2 \sin\gamma\cos\gamma}{\mu\left(\frac{r v^2 \cos^2\gamma}{\mu} - 1\right)}$$

$$\boxed{\tan\nu = \frac{\left(\frac{r v^2}{\mu}\right)\cos\gamma\sin\gamma}{\left(\frac{r v^2}{\mu}\right)\cos^2\gamma - 1}}$$

SINGLE IMPULSE ADJUSTMENT

ELIMINATE LAUNCH ERRORS

BRING S/C TO A MORE DESIRABLE ORBIT

CORRECT ORBIT

OTHERS ...

NOTE: CANNOT TRANSFER TO A NEW ORBIT WITH A SINGLE IMPULSE UNLESS ORBITS INTERSECT.

ASSUMPTIONS :
1. ONLY IN-PLANE CHANGES
2. IMPULSIVE THRUST
3. CAN DIRECT THRUST IN ANY DIRECTION

EXAMPLE 1

SATELLITE IN EARTH ORBIT $a = 3R_\oplus$ $e = 0.5$ $r_p = 1.5R_\oplus$

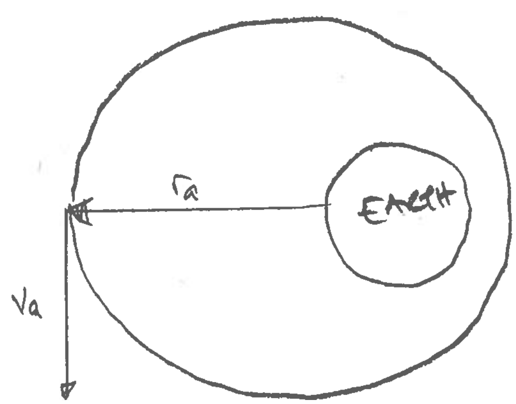
GOAL: CHANGE ORBIT S.T e CONSTANT

$a_N = 4R_\oplus$

ΔV (THRUST) MUST BE APPLIED AT APOAPSIS.

FIND MAGNITUDE + DIRECTION OF $\Delta \bar{V}$ ⟵

SOLUTION



1. ESTABLISH CURRENT ORBIT
2. FIND CONDITIONS PRIOR TO MANEUVER
3. DETERMINE CONDITIONS AFTER MANEUVER
4. VECTOR DIAGRAM

B. CONDITIONS AT THRUST POINT BEFORE MANEUVER

$r_a = a(1+e) = 4.5 R_\oplus$

$$\frac{V_a^2}{2} = \frac{\mu_\oplus}{r_a} - \frac{\mu_\oplus}{2a} \Rightarrow V_a = 2.64 \text{ km/sec}$$

$\gamma = 0, \nu = 180°$

NOTE: TO INCREASE a, LIKELY REQUIRES INCREASE IN V

$$\frac{V^2}{2} = \underset{\substack{\uparrow \\ \text{CONSTANT}}}{\frac{\mu}{r}} - \frac{\mu}{2a}$$

$a \uparrow \rightarrow$ BECOMES LESS NEGATIVE

RHS $\uparrow \Rightarrow V \uparrow$

IF WE INCREASE V AND MAINTAIN r DOES e CHANGE?

FOR SAME e, HIGHER V IN DIFFERENT PART OF ORBIT

DOES θ, r CHANGE?

DOES γ CHANGE?

C. DETERMINE CONDITIONS AFTER MANEUVER (IF POSSIBLE)

$r_N = r_a = 4.5 R_\oplus$

$a_N = 4 R_\oplus$ $\quad e = 0.5$ GIVEN

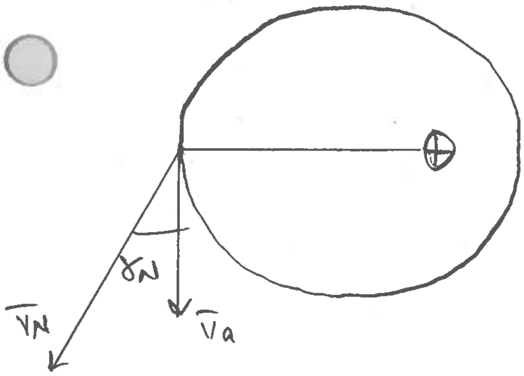
$$\frac{V_N^2}{2} = \frac{\mu_\oplus}{r_N} - \frac{\mu_\oplus}{2a_N} \Rightarrow V_N = 3.49 \frac{\text{km}}{\text{sec}}$$

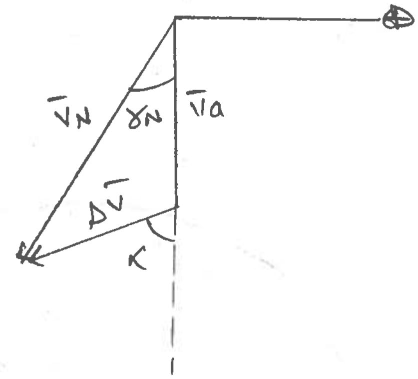
$h = r v \cos\gamma$

$$\sqrt{\mu_\oplus \underset{\substack{\uparrow \\ \text{NEW ORBIT}}}{P_N}} = r_N V_N \cos\gamma_N \Rightarrow \gamma_N = \pm 29.23°$$

D. SKETCH VECTOR DIAGRAM

CHOOSE $\Delta\gamma = +29.23$ FOR NOW





COSINE LAW

$$\Delta V^2 = V_N^2 + V_a^2 - 2 V_N V_a \cos \Delta\gamma_N$$

OR

$$\Delta V = \left[V_N^2 + V_a^2 - 2 V_N V_a \cos \Delta\gamma_N\right]^{1/2}$$

$\boxed{\Delta V = 1.75 \ \frac{km}{sec}}$

USE GEOMETRY TO FIND $\boxed{\alpha = 76^\circ}$ WRT INITIAL VELOCITY

SIN LAW

$$\frac{\sin(\alpha - 180)}{V_N} = \frac{\sin \gamma_N}{\Delta\bar{V}}$$

KNOWN $r_N, V_N, \gamma_N \Rightarrow \nu_N = \tan^{-1}\left(\dfrac{\frac{rv^2}{\mu}\cos\gamma\sin\gamma}{\frac{rv^2}{\mu}\cos^2\gamma - 1}\right)$

$= -48.4^\circ, \boxed{+131.6^\circ}$

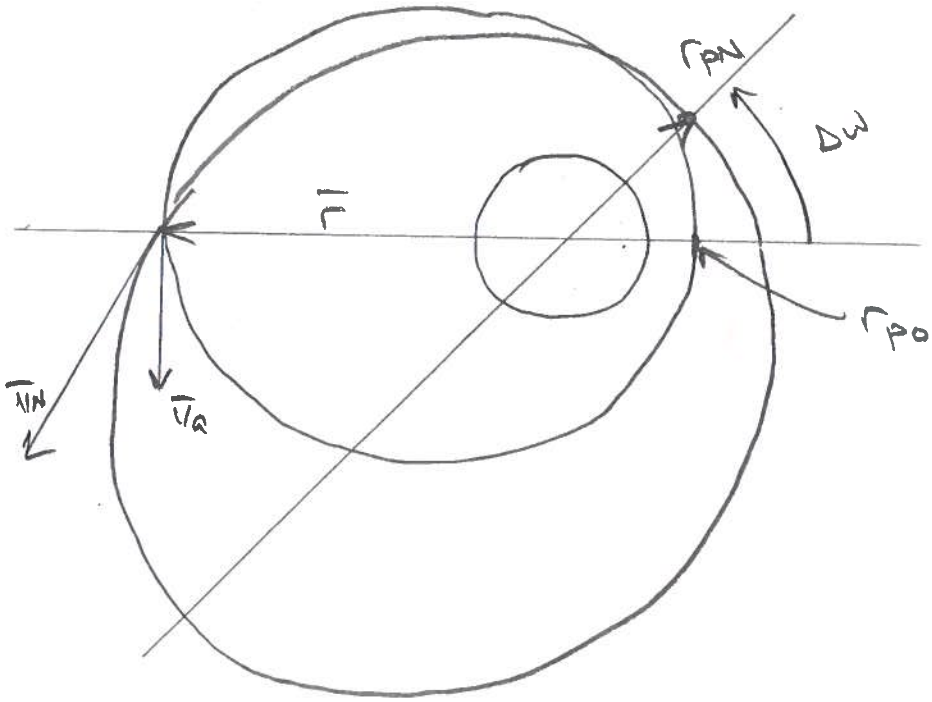
CHECK $\alpha, \gamma_N \rightarrow$ SAME SIGN.

ORIGINALLY $\nu = 180^\circ \Rightarrow$ NOW $\nu = 131.6^\circ \Rightarrow \Delta\omega = 180 - 131.6^\circ$

NEW ORBIT

$r = 4.5 R_\oplus$ $\quad$ $v = 3.49$ km/sec $\quad$ $\gamma = +29.23^\circ$

$e = 0.5$ $\quad$ $a = 4 R_\oplus$ $\quad$ $r_p = 2 R_\oplus \leftarrow$ ALWAYS CHECK

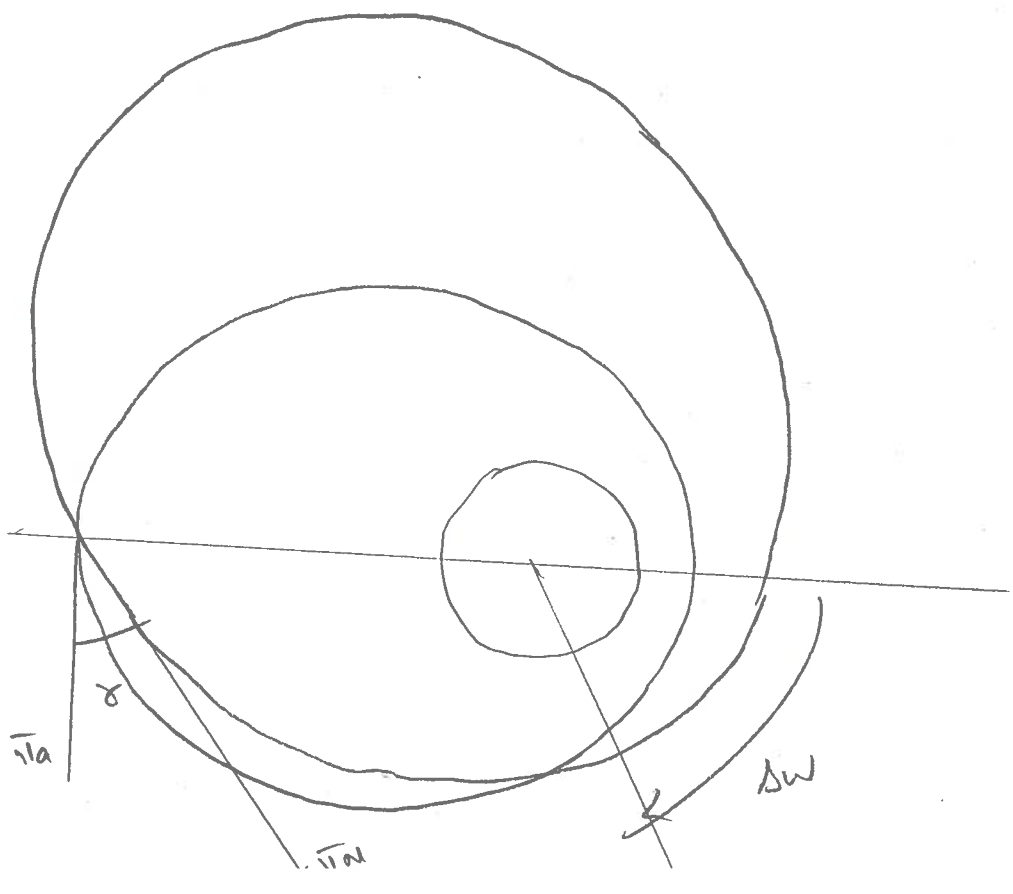


CHANGE IN LINE OF APSIDES

RAISED PERIGEE

ADVANCED PERIGEE BY 48.4°

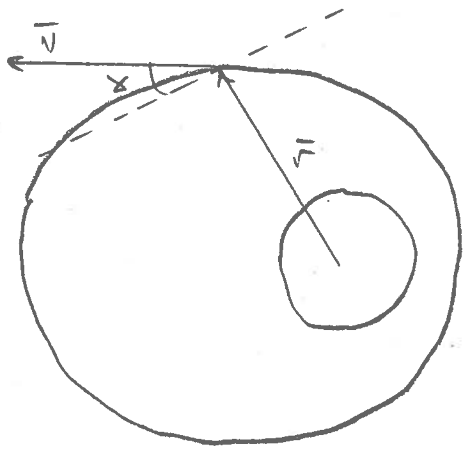
IF $\Delta\gamma = -29.23^\circ$



CAN SAME BE ACCOMPLISHED FOR LOWER COST?

PUT MANEUVER IN DIFFERENT LOCATION?

CURRENTLY $a = 3R_\oplus$ $e = 0.5$ $(r_p = 1.5R_\oplus)$



MANEUVER AT $\nu = 120°$

$e_N = e = 0.5$ CONSTANT

$a_N = 4R_\oplus$

1. CURRENT ORBIT ESTABLISHED

2. CONDITIONS ON ORBIT BEFORE MANEUVER

$r = 3R_\oplus$

$v = 4.5642$ km/sec

$\gamma = 30°$

CONSIDER HOW TO ACCOMPLISH OBJECTIVE

$$\frac{v^2}{2} = \frac{\mu}{r} - \frac{\mu}{2a} \Rightarrow a\uparrow \; v\uparrow$$

INCREASE / DECREASE v?

IS TANGENTIAL Δv POSSIBLE?

$$e^2 = \left(\frac{rv^2}{\mu} - 1\right)^2 \cos^2\gamma + \sin^2\gamma$$

CANNOT KEEP e CONSTANT UNLESS CHANGE IN γ

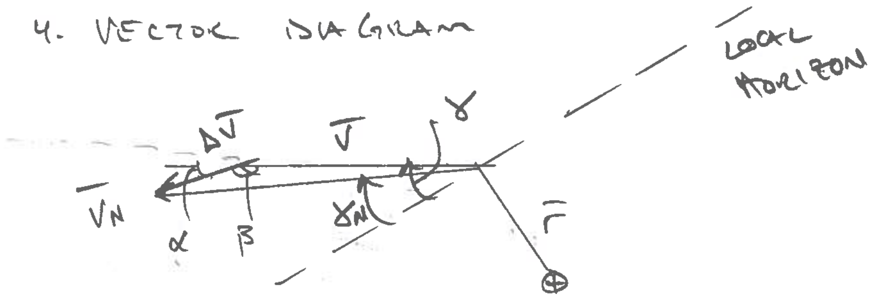
3. CONDITIONS AFTER MANEUVER

$r_N = r_a = 3R_\oplus$

$V_N = 5.1029$ km/sec

$a_N = 4 R_\oplus \qquad e = 0.5$

4. VECTOR DIAGRAM



COSINE LAW

$\Delta V^2 = V_N^2 + V_a^2 - 2 V_N V_a \cos \Delta\gamma_N \quad \longrightarrow \quad \boxed{\Delta V = 0.61145 \text{ km/sec}}$

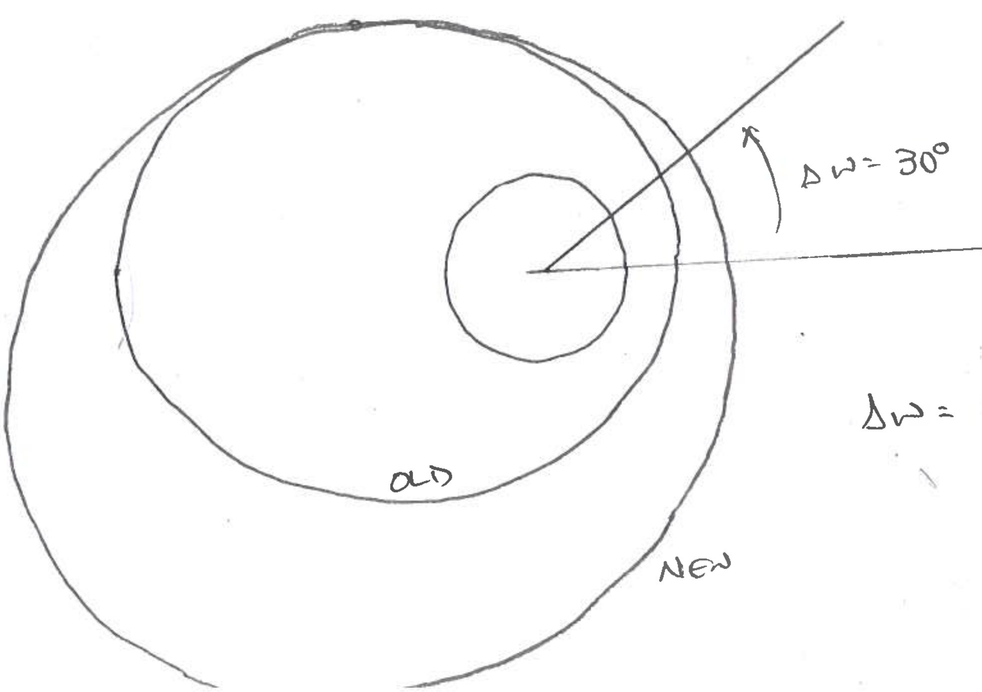
SINE LAW

$$\frac{\Delta V}{\sin \Delta\gamma} = \frac{V_N}{\sin \beta} \implies \beta = 150^\circ$$

$\boxed{\alpha = -30^\circ}$ ← CHECK SIGN

$\nu_N = \pm 90^\circ \implies \nu_N = +90^\circ \qquad \gamma_N > 0$

$\nu_0 = 120^\circ$



$\Delta\omega = \nu_0 - \nu_N$

EXAMPLE

AN EARTH SATELLITE HAS THE FOLLOWING STATE

$r = 1.65\ R_\oplus$

$V = 5.7\ km/sec$

$\gamma = -10.2°$

AT THIS POINT A MAN. $\Delta V = 1.2\ km/sec$, $\alpha = +25°$

→ FIND FINAL ORBIT.

1. ESTABLISH CURRENT ORBIT.

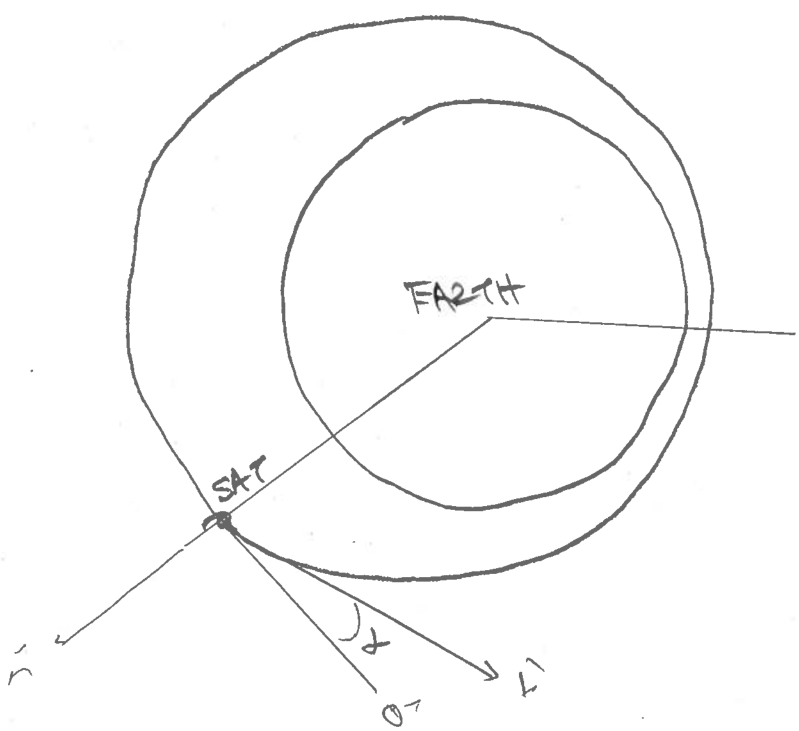
$a = 1.4446\ R_\oplus$ $\quad$ $E = -129.05°$

$e = 0.22571$ $\quad$ $\nu = -138.52°$

$r_p = 1.1185\ R_\oplus$ $\quad$ $\mathbb{P} = 2.4449\ hr$

$r_a = 1.7710\ R_\oplus$ $\quad$ $(t - T) = -0.80823\ hr.$

2. CONDITIONS PRIOR TO ORBIT.

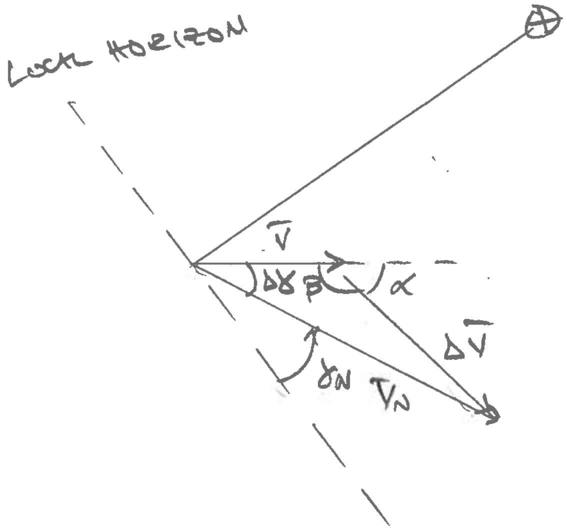


ADD $\Delta \bar{V}$

$|\Delta \bar{V}| = 1.2\ km/sec$

$\alpha = +25°$

3. CONDITIONS AFTER IMPULSE



COSINE LAW

$$V_N^2 = V^2 + \Delta V^2 - 2V\Delta V\cos\beta$$

$$V_N = 6.806 \ \frac{km}{sec}$$

$$\cos\Delta\gamma = \frac{\Delta V^2 - V_N^2 - V^2}{-2V_N V}$$

$$\Delta\gamma = 4.273°$$

$$\gamma_N = -5.927°$$

4. NEW ORBIT.

$a_N = 2.124 \ R_\oplus$

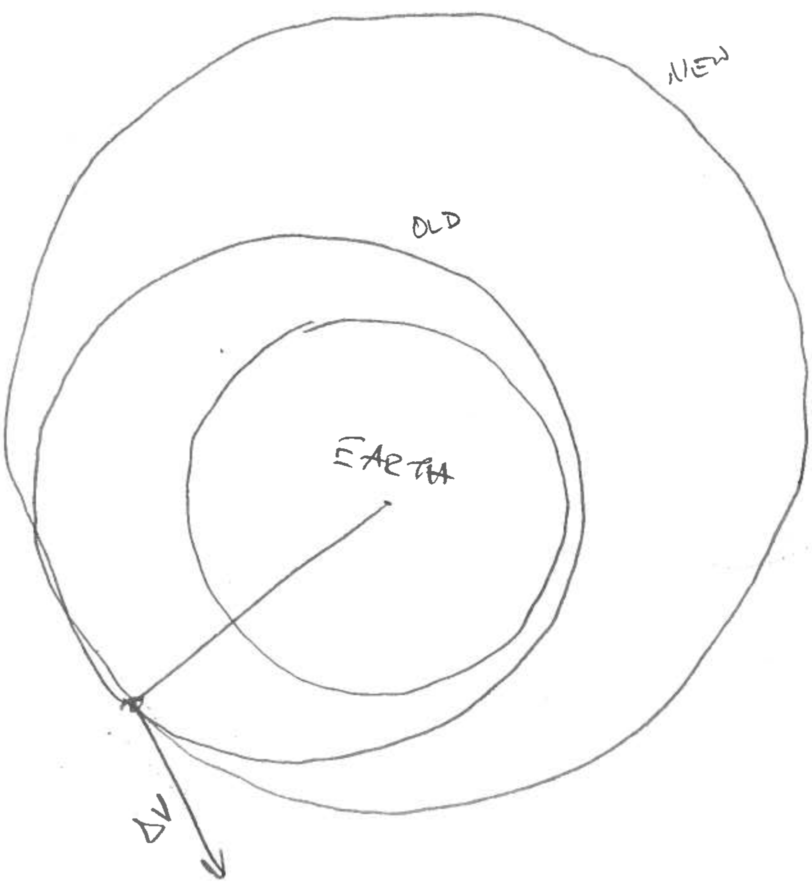
$e = 0.24482$

$r_p = 1.6446 \ R_\oplus$

$E_N = -24.758°$

$\nu_N = -30.874$

$P = 4.3590$ hrs.

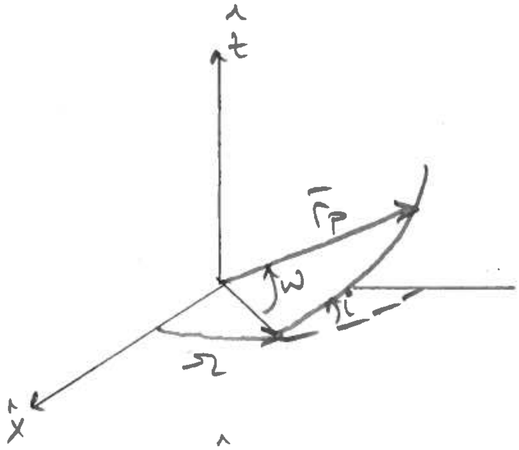


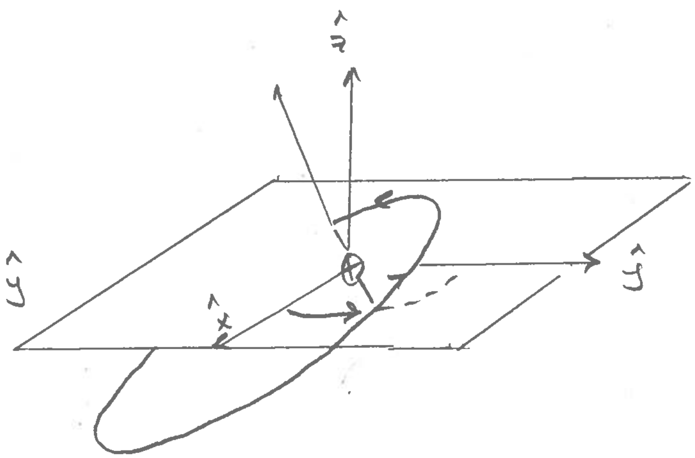
3D EXAMPLE

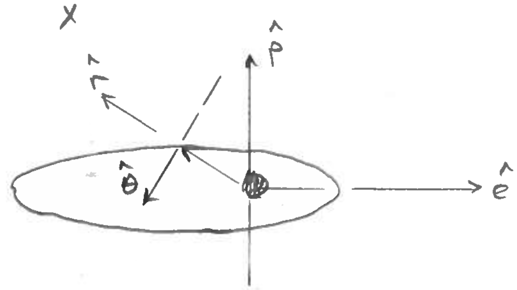
EARTH ORBITING S/C

$a = 8 R_\oplus$ $\quad e = 0.7$ $\quad i = 30°$ $\quad \Omega = 60°$ $\quad \underbrace{\omega = 90° \quad \nu = 90°}_{\text{DESCN. NODE}}$

"EARTH CENTERED MEAN J2000" — ECI / GCRF







DETERMINE $\bar{r}, \bar{v}$ PRIOR TO MANEUVER $\quad$ COE2RV

$$\bar{r} = 26022.80\,\hat{r} \text{ km}$$

$$= -13011.40\,\hat{x} - 22536.40\,\hat{y} \text{ km}$$ ⟵ DESCENDING NODE

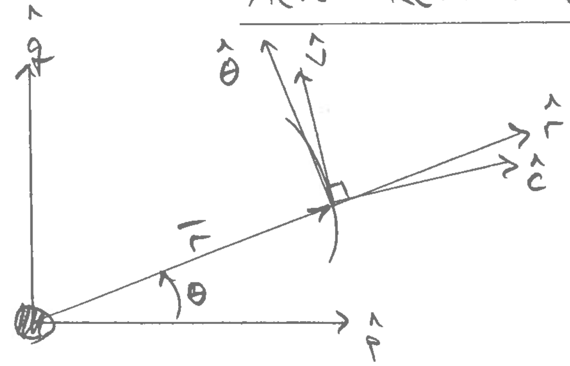
$$|\bar{v}| = 4.777328 \text{ km/sec} \qquad \gamma = 34.992°$$

$$\bar{v} = 2.739616\,\hat{r} + 3.91374\,\hat{\theta}$$

$$= 1.56550\,\hat{x} - 4.06728\,\hat{y} - 1.95687\,\hat{z} \text{ km/sec}$$

BOTH IN INERTIAL FRAME

NEW REF. FRAME - VNC



VNC COORDINATE FRAME

USEFUL FOR DESCRIBING ΔV + MANEUVERS

$\hat{V}$ - PARALLEL TO VELOCITY
TANGENT TO PATH

$\hat{C}$ - NORMAL TO CURVE -
IN PLANE OF MOTION

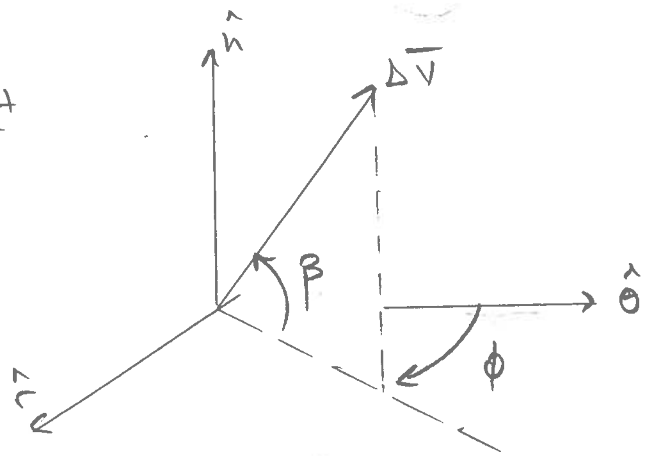
$\hat{N}$ - NORMAL OUT OF PLANE

TANGENT $\quad \hat{V} = \dfrac{\bar{v}}{|\bar{v}|}$

NORMAL $\quad \hat{N} = \dfrac{\bar{r} \times \bar{v}}{|\bar{r} \times \bar{v}|} = \hat{h}$

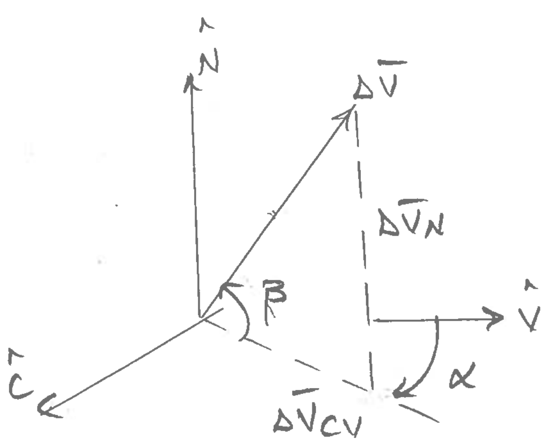
CO-NORMAL $\quad \hat{C} = \hat{V} \times \hat{N}$

LVLH



$$\Delta\bar{V} = \Delta V\left(\cos\beta\cos\phi\,\hat{\theta} + \cos\beta\sin\phi\,\hat{r} + \sin\beta\,\hat{h}\right)$$

VNC



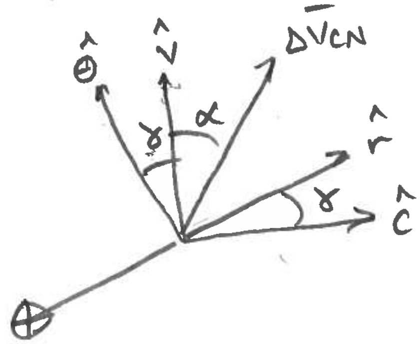
$$\Delta\bar{V} = \Delta V\left(c\beta c\alpha\,\hat{V} + c\beta s\alpha\,\hat{C} + s\beta\,\hat{N}\right)$$

ASSUME AN OUT OF PLANE MANEUVER

$$\Delta V = 2 \text{ km/sec} \quad \alpha = 0 \quad \beta = 150^\circ \quad \phi = \alpha + \gamma = 34.992^\circ$$

$$\Delta \bar{V} = -1.732\, \hat{v} + 1.0\, \hat{n} \quad \text{km/sec}$$

$$= -1.4192\, \hat{\theta} - 0.9928\, \hat{r} + 1\, \hat{h} \quad \text{km/sec}$$



$$\Delta \bar{V} = -0.134551\, \hat{x} + 1.22457\, \hat{y} + 1.57548\, \hat{z} \quad \text{km/sec}$$

ALSO IN INERTIAL FRAME

$$\bar{V}_{NEW} = \bar{V}_{OLD} + \Delta \bar{V}$$

$$= 1.43094\, \hat{x} - 2.84270\, \hat{y} - 0.38139\, \hat{z} \quad \text{km/sec}$$

CHARACTERIZE NEW ORBIT

$$\hat{h}_{NEW} = \hat{r}_{NEW} \times \hat{v}_{NEW} = 0.1229\, \hat{x} - 0.0701\, \hat{y} + 0.98986\, \hat{z}$$

$$\hat{h} = \sin\Omega \sin i\, \hat{x} - \cos\Omega \sin i\, \hat{y} + \cos i\, \hat{z} = \hat{w}$$

FROM WHERE ?

$$R_{\substack{PQW \to \\ ECI}} = \begin{array}{c} \begin{array}{ccc} \hat{p} & \hat{q} & \hat{w} \end{array} \\ \left[\begin{array}{ccc} -s\Omega c i s\omega + c\Omega c\omega & -s\Omega c i c\omega - c\Omega s\omega & s\Omega s i \\ c\Omega c i s\omega + s\Omega c\omega & c\Omega c i c\omega - s\Omega s\omega & -c\Omega s i \\ s i s\omega & s i c\omega & c i \end{array} \right] \end{array} \begin{array}{c} \hat{x} \\ \hat{y} \\ \hat{z} \end{array}$$

$\cos i = 0.9898 \rightarrow \boxed{i = 8.1576^\circ}$

$$\left.\begin{aligned} S\Omega\, Si &= 0.122989 \\ -C\Omega\, Si &= -0.071008 \end{aligned}\right\} \begin{aligned} \Omega &= 60^\circ, 120^\circ \\ \Omega &= \pm 60 \end{aligned} \Bigg\} \boxed{\Omega = 60^\circ}$$

$|\bar{v}| = 3.206887$ km/sec

$$\frac{v^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a} \Longrightarrow \boxed{a = 19576.962 = 3\, R_\oplus}$$

$$\underbrace{\bar{r}\cdot\bar{v}}_{r\dot{r}} = +45476 \ \frac{km^2}{sec} \rightarrow \gamma > 0$$

$$r\dot{r} = rv\sin\gamma \rightarrow \boxed{\gamma = +33.0137}$$

$$e^2 = \left(\frac{rv^2}{\mu} - 1\right)^2 \cos^2\gamma + \sin^2\gamma \rightarrow \boxed{e = 0.610802}$$

$$\tan \nu = \frac{\left(\frac{rv^2}{\mu}\right)\sin\gamma\cos\gamma}{\left(\frac{rv^2}{\mu}\right)\cos^2\gamma - 1} \Rightarrow \nu = -30.143^\circ, \boxed{149.887^\circ}$$

$$\hat{p}\cdot\hat{z} = \sin i \sin\omega \rightarrow$$

$$\hat{p}\cdot\hat{z} = \cos\Omega\cos\omega - S\Omega\, c i\, S\omega \rightarrow \boxed{\omega_{NEW} = 30.11292^\circ}$$

IN PLANE $\rightarrow$ CHANGE a, e, ω

OUT OF PLANE $\Rightarrow$ CHANGE Ω, i $\Big\}$ COUPLED

ORBITAL TRANSFERS

GOAL: SHIFT TO AN ORBIT THAT DOES <u>NOT</u> INTERSECT ORIGINAL ORBIT

→ USE MULTIPLE - IMPULSE TRANSFERS

→ MIN $\Delta V \neq$ MIN NO. OF IMPULSES

TRANSFER PROBLEMS:

1. <u>DEFINE TRANSFER GEOMETRY</u>: ← PART OF CONIC SECTION

   FIND DEPARTURE/ARRIVAL POINTS ON INITIAL/FINAL ORBITS.

2. <u>DEFINE DEPARTURE/ARRIVAL POINTS</u>

   SOLVE FOR THE TRANSFER ARC ← MUCH MORE DIFFICULT.

WE'LL FOCUS ON 1 → SIMPLE

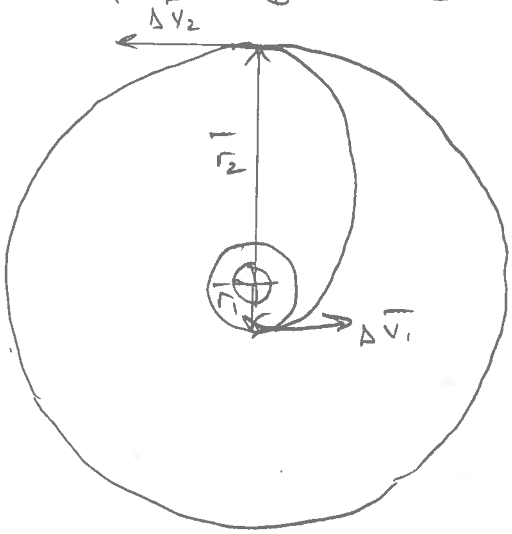
SIMPLEST EXAMPLE → CIRCLE TO CIRCLE TRANSFER

HOHMANN TRANSFER - SIMPLEST TWO-IMPULSE TRANSFER

ALSO MINIMUM ΔV TWO-IMPULSE TRANSFER

EXAMPLE - HOHMANN 2-IMPULSE

$r_1 = 2R_\oplus$ $\quad$ $r_2 = 4R_\oplus$ $\quad$ CO-PLANAR



TRANSFER ANGLE $\Rightarrow 180°$

1. ESTABLISH CURRENT ORBIT

$$a = r_1 = 2R_\oplus \qquad e = 0$$

2. CONDITIONS AT THRUST POINT PRIOR TO MANEUVER

$$r_1 = 2R_\oplus \qquad V_1 = 5.59 \frac{km}{sec} \qquad \gamma_1 = 0°$$

TO CALCULATE $\Delta V \rightarrow$ NEED CONDITIONS ON TRANSFER ELLIPSE

3. TRANSFER ELLIPSE

$$a_T = \frac{1}{2}(r_p + r_a) = 3R_\oplus \qquad r_p = a(1-e) \rightarrow e_T = \frac{1}{3}$$

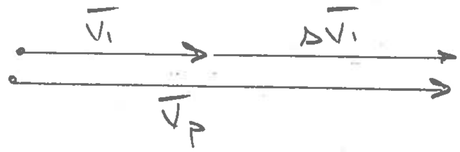
4. CONDITIONS ON THRUST POINT ON TRANSFER ORBIT

$$r = r_1$$

$$\frac{V_p^2}{2} = \frac{\mu}{r_1} - \frac{\mu}{2a_T} \rightarrow V_p = 6.45 \frac{km}{sec}$$

$$\gamma_1 = 0$$

5. VECTOR DIAGRAM → ALWAYS



$\Delta\bar{V}_1 = \bar{V}_p - \bar{V}_1$ VECTOR

$|\Delta\bar{V}_1| = 0.861 \ \frac{km}{sec}$

6. CONDITIONS AT 2nd THRUST POINT BEFORE ΔV_2 (ON TRANSFER ORBIT)

$r_a = r_2 = 4R_\oplus$

$\frac{V_a^2}{2} = \frac{\mu}{r_2} - \frac{\mu}{2a_T} \rightarrow V_a = 3.22 \ \frac{km}{sec}$

$\gamma_2 = 0$ (APOGEE)

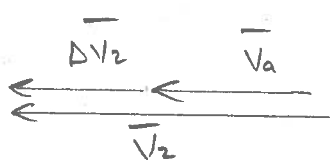
7. CONDITIONS REQUIRED AFTER MANEUVER IN FINAL ORBIT

$r_2 = 4R_\oplus$

$V_2 = \sqrt{\frac{\mu}{r_2}} = 3.95 \ \frac{km}{sec}$

$\gamma = 0$

8. VECTOR DIAGRAM FOR $\Delta\bar{V}_2$



$\Delta\bar{V}_2 = \bar{V}_2 - \bar{V}_a$

$|\Delta\bar{V}_2| = 0.73 \ \frac{km}{sec}$

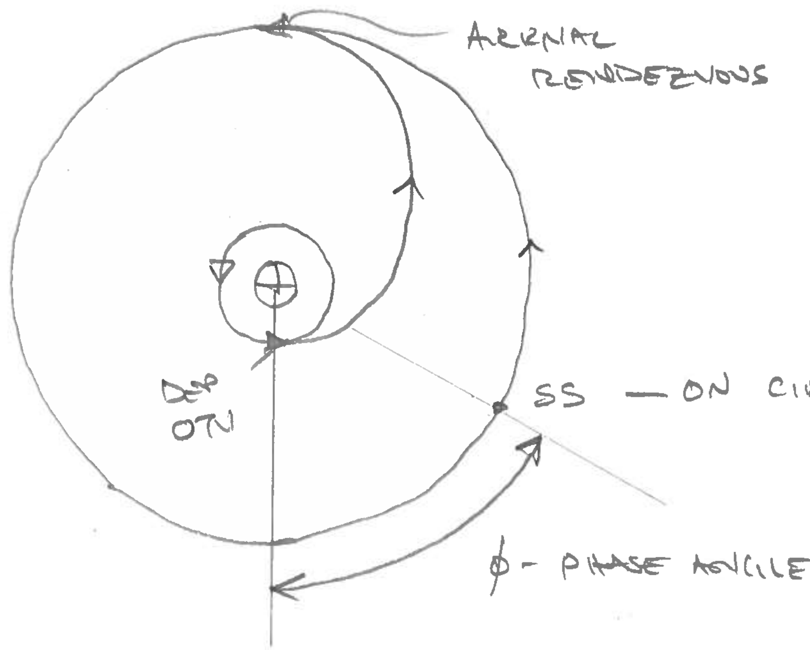
9. COMPUTE TOTAL $\Delta V = |\Delta\bar{V}_1| + |\Delta\bar{V}_2| = 1.59 \ \frac{km}{sec}$

CONDITIONS FOR RENDEZVOUS

ADDITIONAL COMPLEXITY IF WE WANT TO ARRIVE AT A SPECIFIC POINT / TIME

TIMING IS CRITICAL ! ⇒ ISS / RENDEZVOUS

EXAMPLE ⊕ ORBITING OTV DEPARTS LOW EARTH ⊕ ORBIT TO RENDEZVOUS WITH SPACE STATION



TOF - TRANSFER TIME 180°

$$= \frac{P_T}{2}$$

$r_2 = 4R_\oplus$

$a_T = 3R_\oplus$

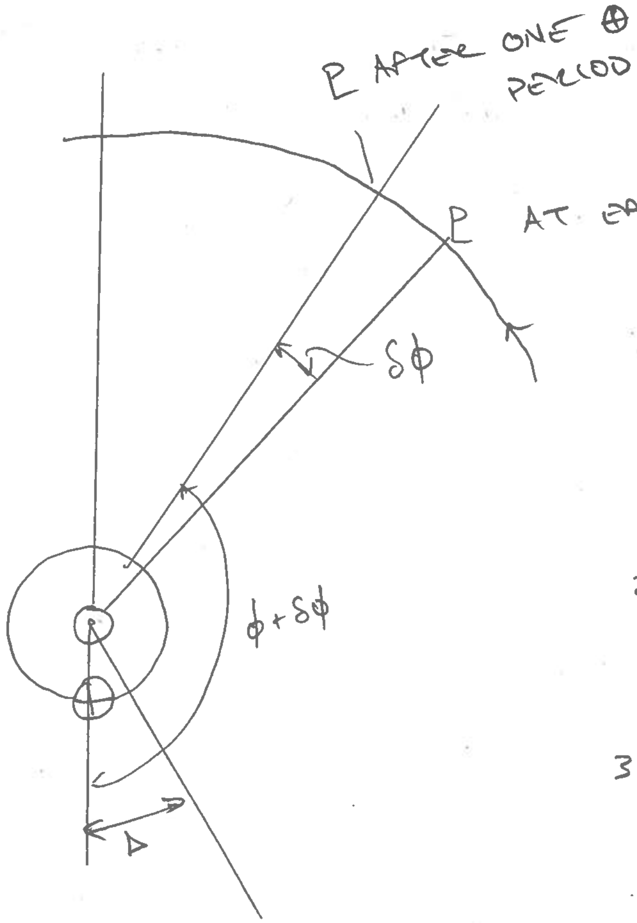
TOF = 3.6 Hrs

ϕ - PHASE ANGLE AT DEPARTURE

$180^\circ - \phi \Rightarrow n_2\,(\text{TOF}) \Rightarrow \boxed{\phi = 63.1^\circ}$ LEAD ANGLE FOR RENDEZVOUS

ONLY TRUE WHEN SS IS ON A CIRCULAR ORBIT.

EXAMPLE HOHMANN EARTH TO PLUTO



1. $P_{PLUTO} = 247$ YRS – P DOES NOT MOVE

2. AFTER ONE P_{EARTH} ANGLE BTWN EARTH + PLUTO $= \phi + \delta\phi$

3. EARTH MOVES FASTER THAN PLUTO ⇒ IF WE LET BOTH MOVE A LITTLE EARTH WILL CATCH UP

$$\mathbb{P} = \frac{2\pi}{n}$$

$$t_s = \mathbb{P} + \Delta$$

$$\left.\begin{aligned} n_{\oplus} t_s &= 2\pi + \Delta \\ n_P t_s &= \Delta \end{aligned}\right\} \text{SUBTRACT}$$

$$t_s (n_{\oplus} - n_P) = 2\pi$$

$$\boxed{t_s = S = \frac{2\pi}{n_1 - n_2}}$$

SYNODIC PERIOD – TIME TO RETURN TO SAME ANGULAR ORIENTATION.

TIME UNTIL ALIGNMENT (ϕ) IS REACHED –

IF WE MISS OPPORTUNITY HOW LONG TO WAIT?

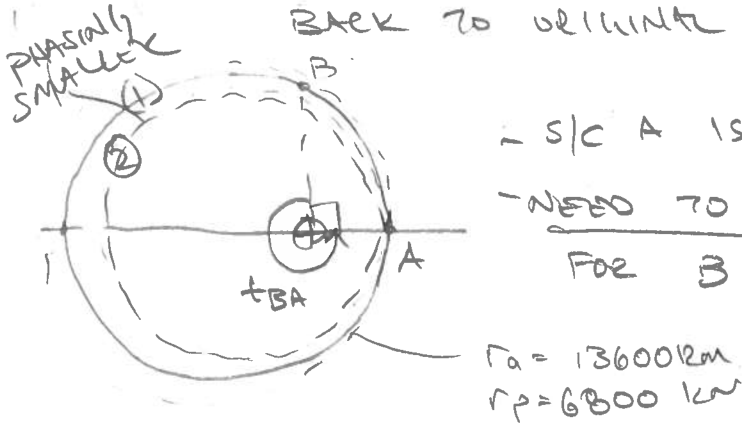
INITIAL POSITION NOT IMPORTANT ⟹ NEED RELATIVE!

PHASING   CURTIS 6.5

- WHAT IF WE NEED TO RENDEZVOUS BUT ALREADY IN THE SAME ORBIT?

- MOVE TO A DIFFERENT LOCATION IN SAME ORBIT
  → GEOSTATIONARY ORBIT.

- GOAL – HOHAMANN TRANSFER TO PHASING ORBIT THEN BACK TO ORIGINAL ORBIT.



- S/C A IS BEHIND/TRAILING S/C B
- NEED TO CATCH UP (INCREASE v) / WAIT FOR B TO CATCH UP TO US.

→ KEY IDEA – NEED TO SPEED UP TO SLOW DOWN!

→ CHASER A WILL CATCH B AT POINT A AFTER ONE PERIOD OF (1)

$t_{BA} = P_2 < P_1$

TRANSFER ORBIT PHASING

FIND TIME FOR B TO GET TO POINT A

$t_{AB} + t_{BA} = P_1 \Rightarrow t_{BA} = P_1 - t_{AB} = 8756.3\ (sec)$

TOF TO GO FROM $\nu = 0° \rightarrow \nu = 90°$

$\nu \rightarrow E \rightarrow M \rightarrow \Delta t$

$$\tan\frac{E}{2} = \sqrt{\frac{1-e_1}{1+e_1}}\tan\frac{\nu}{2} \rightarrow E_B = 1.23\ rad$$

$$e_1 = \frac{r_a - r_p}{r_a + r_p} = 0.3$$

$M_B = E_B - e\sin E_B = 0.192$ rad

$n(t_{AB} - T_1) = M \rightarrow t_{AB} = 1.495 \times 10^3$ SEC. $\quad \nu = 0 \rightarrow \nu = 90^\circ$

↳ TIME FOR A/B TO COVER $\nu = 90^\circ$ IN ORIGINAL ORBIT

$$\mathbb{P}_1 = 2\pi\sqrt{\frac{a^3}{\mu}} = 1.205 \times 10^4 \text{ SEC}$$

$$a_1 = \frac{1}{2}(r_a + r_p) = 10200 \text{ km} \leftarrow \text{ORIGINAL ORBIT}$$

→ DESIGN PHASING / TRANSFER ORBIT.

$$\mathbb{P}_2 = t_{BA} = 2\pi\sqrt{\frac{a^3}{\mu}} \longrightarrow a_p = 9182. \text{ km}$$

$$a_p = \frac{1}{2}(r_p + \underset{\text{UNKNOWN}}{r_a}) \longrightarrow r_a = 11564 \leftarrow$$ LESS THAN ORIGINAL r_a

SMALLER → FASTER ORBIT.

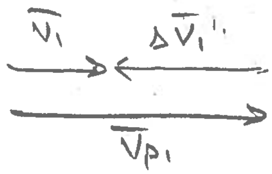
→ NOW CAN COMPUTE ΔV → MOVE FROM (1) TO PHASING ORBIT THEN BACK TO (1)

FIND VELOCITY AT A ON BOTH ORBITS

$r_1 = 6800$ km

$$\frac{V_1^2}{2} = \frac{\mu}{r_1} - \frac{\mu}{2a_1} \rightarrow V_1 = 8.59 \frac{\text{km}}{\text{sec}}.$$

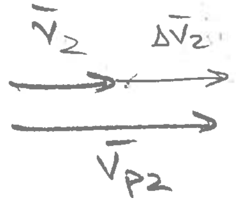
PHASING $$\frac{V_{p1}^2}{2} = \frac{\mu}{r_1} - \frac{\mu}{2a_p} \rightarrow V_{p1} = 8.84 \frac{\text{km}}{\text{sec}}$$



$$\Delta\bar{V}_1 = \bar{V}_{p1} - \bar{V}_1$$

$$\boxed{|\Delta\bar{V}_1| = 0.2485 \frac{\text{km}}{\text{sec}}}$$

SECOND MANEUVER



$$\Delta\bar{V}_2 = \bar{V}_2 - \bar{V}_{p2}$$

$$|\Delta\bar{V}_2| = 0.2485 \ \frac{km}{sec}$$

↑ OPPOSITE DIRECTION

$$|\Delta\bar{V}_T| = |\Delta\bar{V}_1| + |\Delta\bar{V}_2|$$

NON PLANAR TRANSFERS

CO-PLANAR – CHANGE a, e, ω

NON-PLANAR – CHANGE i, Ω ⟵

APPLY AT NODE (AN/DN) → ONLY CHANGE i

APPLY AT INTERSECTION (NEAR MIN/MAX LATITUDE)

→ CHANGE Ω

ANYWHERE ELSE CHANGE BOTH

LAUNCHES WILL ALMOST ALWAYS REQUIRE A PLANE CHANGE

- DIRECT LAUNCH — GROUND INTO ORBIT

LAUNCH SITE LATITUDE ≤ DESIRED INCLINATION

(GEO STATIONARY)

$$\cos i = \cos\phi_{gc} \sin\beta \implies \sin\beta = \frac{\cos i}{\cos\phi_{gc}}$$

DESIRED INCLINATION

WHICH DIRECTION TO LAUNCH (AZIMUTH)

LATITUDE OF LAUNCH SITE

$\phi_{gc} \leq i$

NASA	VANDENBURG	–	34° N, -120° W	$147° \leq \beta \leq 201°$
	KENNEDY	–	28.5° N, -80.5° W	$37° \leq \beta \leq 112°$
FRANCE / ESA	KOUROU	–	5.2° N, -52.8° W	$340° \leq \beta \leq 100°$
RUSSIA	KAPUSTIN YAR	–	48° N, 45° E	$350 \leq \beta \leq 90°$

LAUNCH SITE IS ON A ROTATING EARTH

$\Rightarrow$ INITIAL VELOCITY OF S/C

$$\bar{V}_L = \bar{\omega}_\oplus \times \bar{r}_{SITE} \quad \text{OR} \quad V_L = |\bar{\omega}_\oplus \times \bar{r}_{SITE}|$$

$$= \omega_\oplus r_{SITE} \cos \phi_{gc}$$

AT EQUATOR

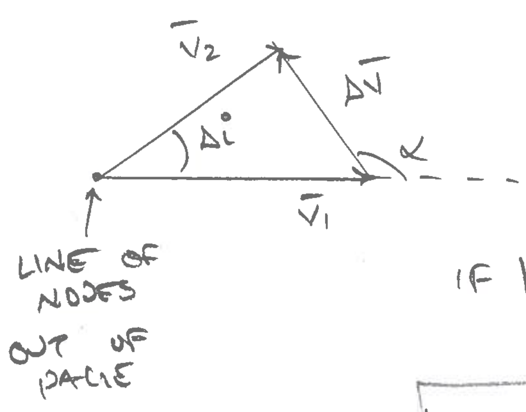
$$V_L = \omega_\oplus r_{SITE} = 7.2921159 \times 10^{-5} \frac{rad}{sec} \cdot 6378.137 \text{ km}$$

$$\boxed{= 0.465 \frac{km}{sec}}$$

$\leftarrow \approx 500$ m/sec FOR FREE AT EQUATOR + NEVER NEED A PLANE CHANGE

- INCLINATION CHANGE MUST OCCUR AT ORBIT INTERSECTION $\Rightarrow$ AN OR DN.
- ORBITS SHOULD INTERSECT AT THIS POINT — SIMPLE PLANE CHANGE !

VIEW DOWN COMMON LINE OF NODES



COSINE LAW

$$\Delta V^2 = V_1^2 + V_2^2 - 2 V_1 V_2 \cos \Delta i$$

$$\cos \Delta i = 1 - 2 \sin^2 \frac{\Delta i}{2}$$

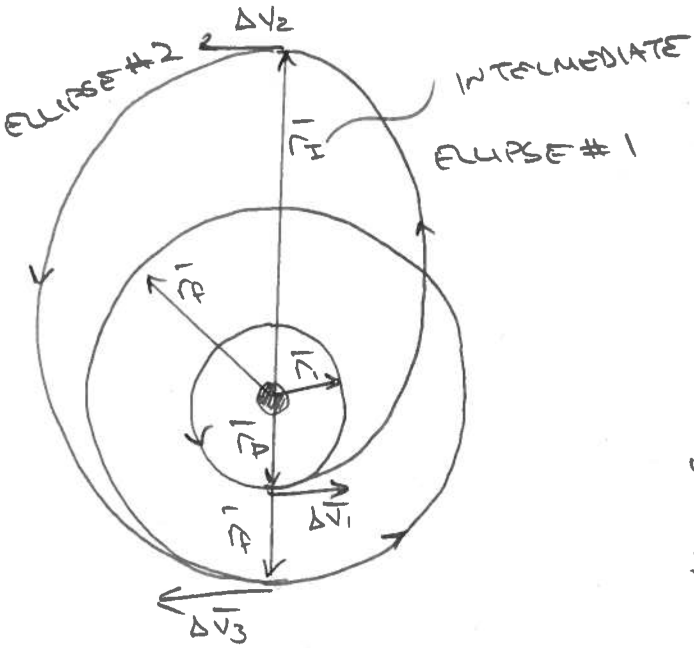
IF $|V_1| = |V_2| \Rightarrow$ SAME ORBIT NEW i

$$\boxed{\Delta V = 2 V \sin \frac{\Delta i}{2}}$$

BI-ELLIPTICAL TRANSFER - LARGE TOF BUT CAN SOMETIMES SAVE PROPELLANT.

- USES THREE TANGENTIAL IMPULSES

ASSUME CIRCULAR + COPLANAR



TRANSFER ANGLE - 360°

TOF $\approx$ 2 $TOF_{HOHMANN}$

1. INITIAL ORBIT CIRCULAR (?)
2. $\Delta\bar{V}_1$ → SHIFT TO ELLIPSE 1
3. APOGEE ON E1 → $r > r_f$

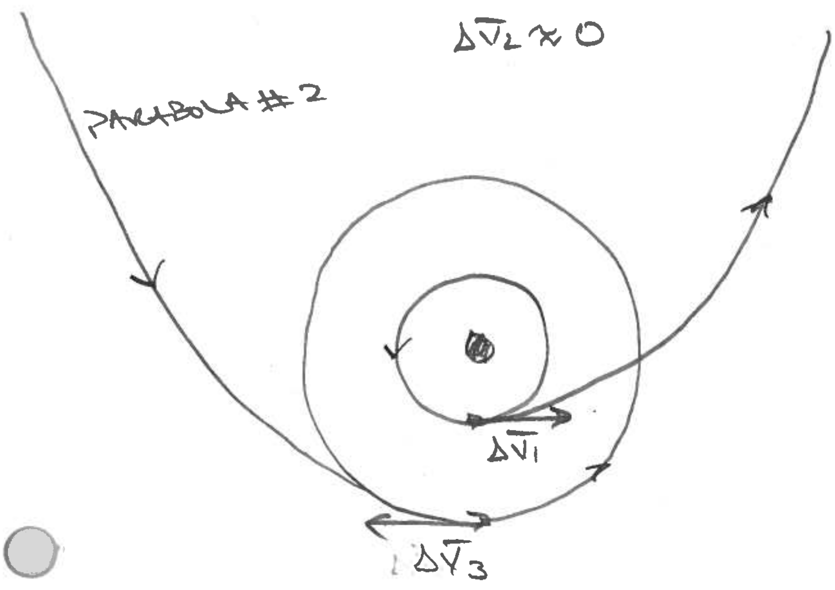
   $\Delta\bar{V}_2$ → TRANSFER FROM $E_1 \to E_2$
4. $\Delta\bar{V}_3$ → TRANSFER FROM $E_2 \to r_f$
5. TOTAL $|\Delta v_T| = |\Delta\bar{V}_1| + |\Delta\bar{V}_2| + |\Delta\bar{V}_3|$

- BI-PARABOLIC - LIMITING CASE

  MOVE INTERMEDIATE RADIUS OUT TO INFINITY $(r \to \infty)$

  PARABOLIC TRANSFER PATHS

  $\Delta V_2 \to 0$ (IDEAL CASE)



- NOT PRACTICAL

  → INFINITE TOF

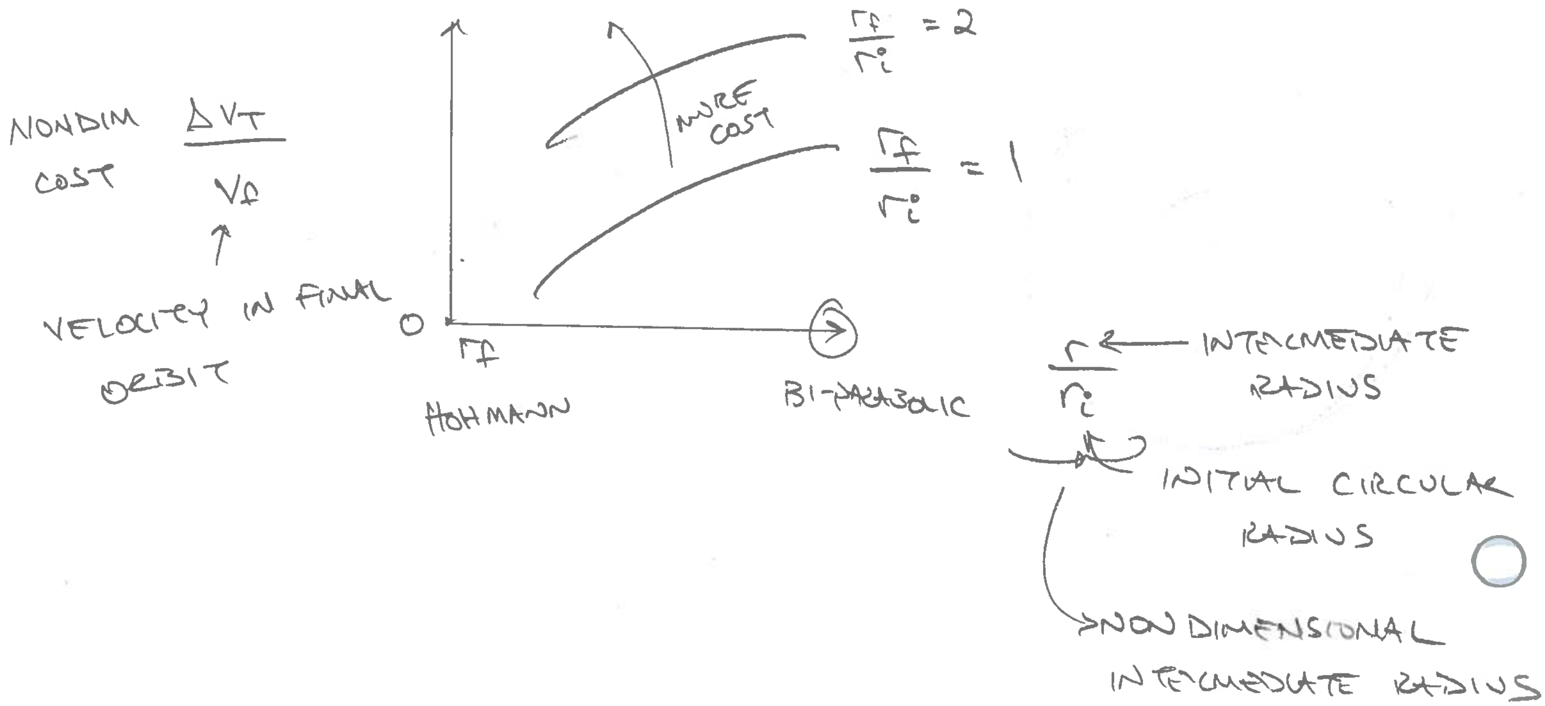
- GAIN OF BI-PARABOLIC SMALL (MAX $\approx$ 10%) SO HOHMANN USED IN PRACTICE

RETURN TO BI-ELLIPTIC

$\Delta V_{TOTAL} = |\Delta \bar{V}_1| + |\Delta \bar{V}_2| + |\Delta \bar{V}_3|$

TOTAL COST IS A FCN OF INTERMEDIATE RADIUS

CONSIDER A PLOT OF ΔV_{TOTAL} AND r FOR CIRCLE TO CIRCLE BI-ELLIPTIC TRANSFERS



FIND CONDITIONS FOR MIN COST

CHECK LIMITS $r = r_f$ ← TWO IMPULSE HOHMANN

$r \to \infty$ ← BIPARABOLIC.

A) $1 \le r_f \le 9$ → ΔV INCREASES → HOHMANN IS MINIMUM

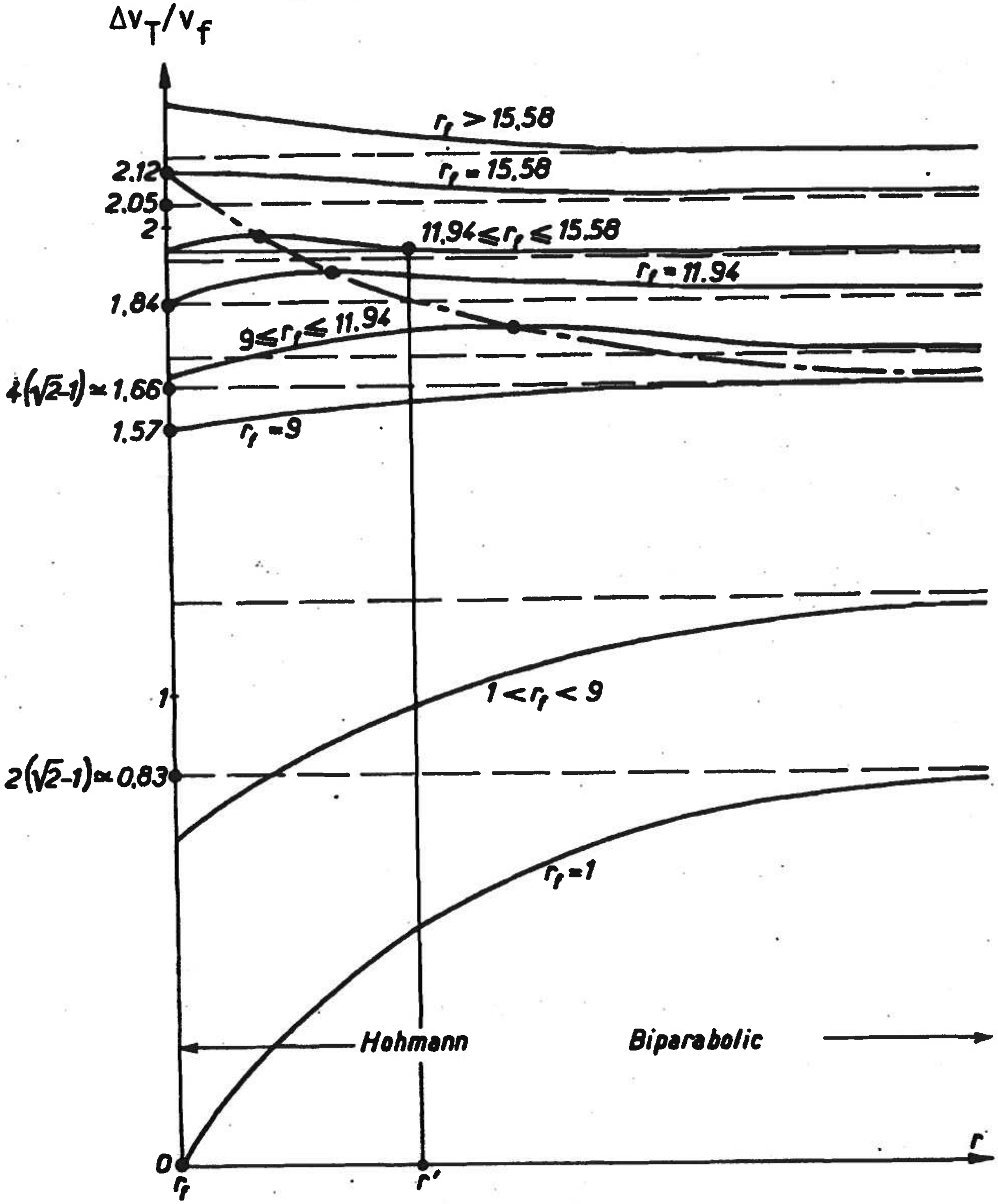
B) $9 \le r_f \le 15.58$ → ΔV LOCAL MINIMUMS EXIST.

$9 \le r_f \le 11.94$ → MIN IS HOHMANN

$11.94 \le r_f \le 15.58$ → MIN IS BI-PARABOLIC !

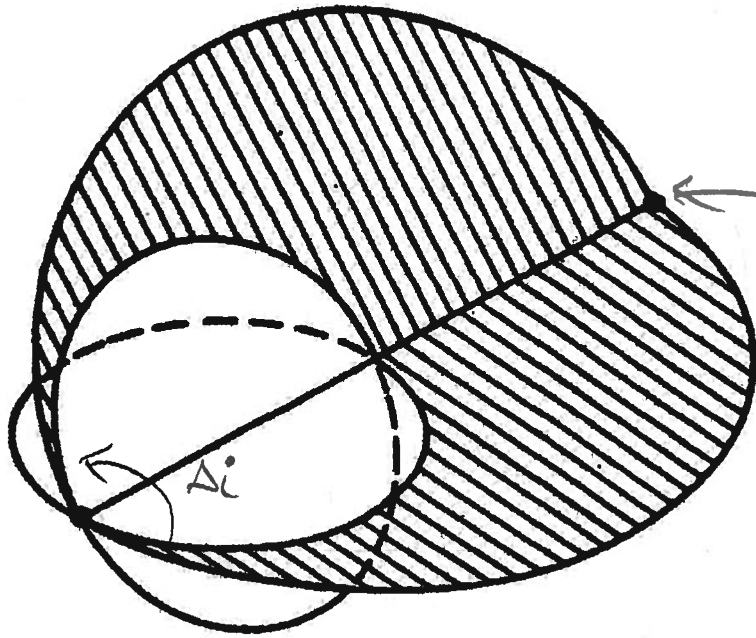
C) $r_f \ge 15.58$ → BI-ELLIPTICAL IS ALWAYS A LOWER ΔV_{TOTAL}

BI-ELLIPTICAL
CIRCLE TO CIRCLE



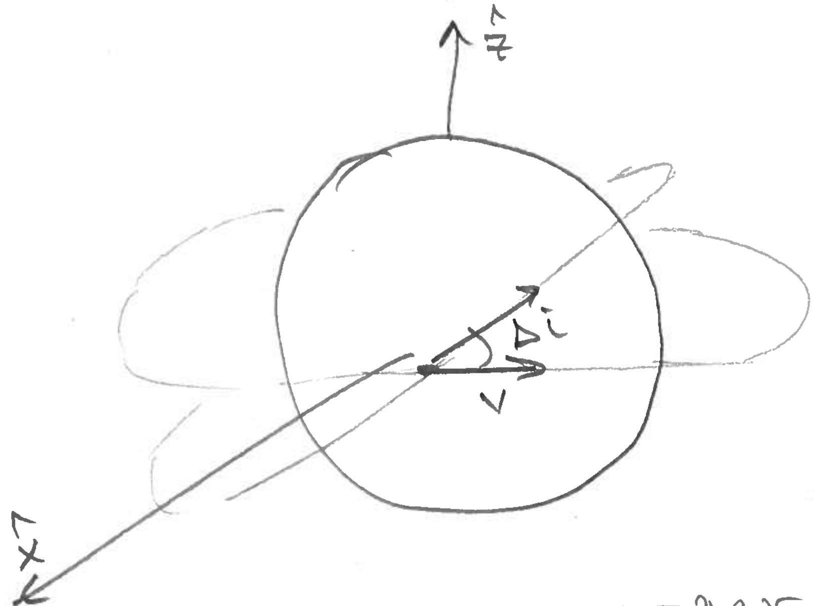
INTERMEDIATE RADIUS

BI-ELLIPTICAL → USEFUL FOR PLANE CHANGE AT INTERMEDIATE DISTANCE



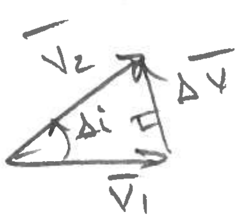
PLANE CHANGE AT APOAPSIS

LARGER r_{INT} → OTHER GRAVITY FIELDS



SIMPLE PLANE CHANGE $- |\bar{V}_1| = |\bar{V}_2|$

$$\Delta V = 2 V_1 \sin \frac{\Delta i}{2}$$ ← SAME ORBIT



ISOSCELES

PROPORTIONAL TO CURRENT ORBITAL VELOCITY

SPHERICAL TRIG

$\cos i = \cos\phi_{gc} \sin\beta$

$\sin\beta = \dfrac{\cos i}{\cos\phi_{gc}}$

$-1 \le 1 \Rightarrow \phi_{gc} \le i$

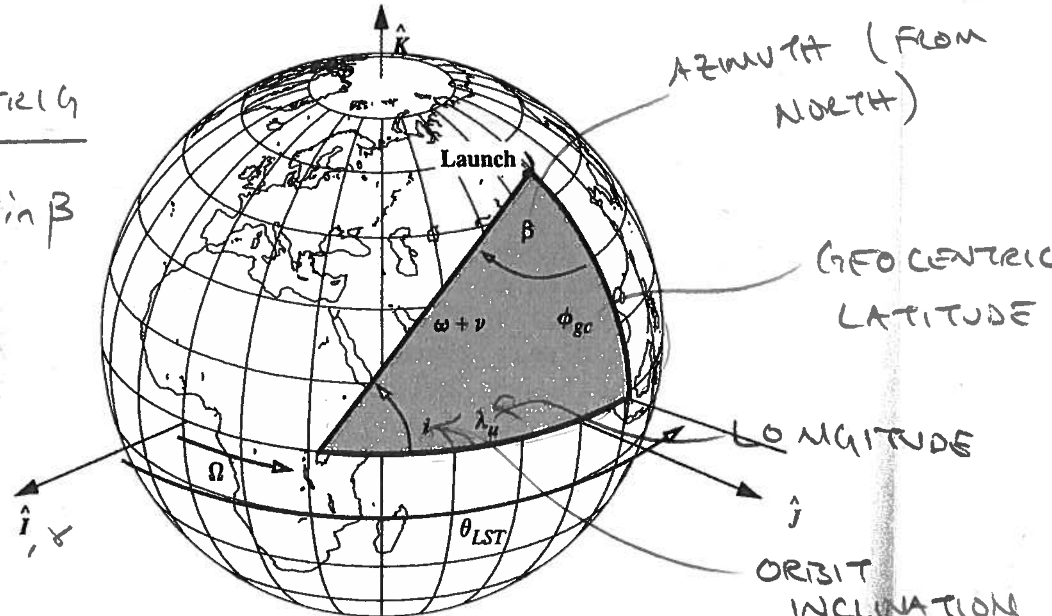


Figure 6-8. **Launch-Window Geometry.** Direct launches can occur only if the launch site's latitude is less than or equal to the satellite's inclination, i. Determining the launch window involves three basic steps: (1) finding the launch azimuth, β, from the site (I've shown an equivalent angle in the triangle because β is measured from north), (2) determining the auxiliary angle, λ_u, and (3) finding the GMST (or LST).

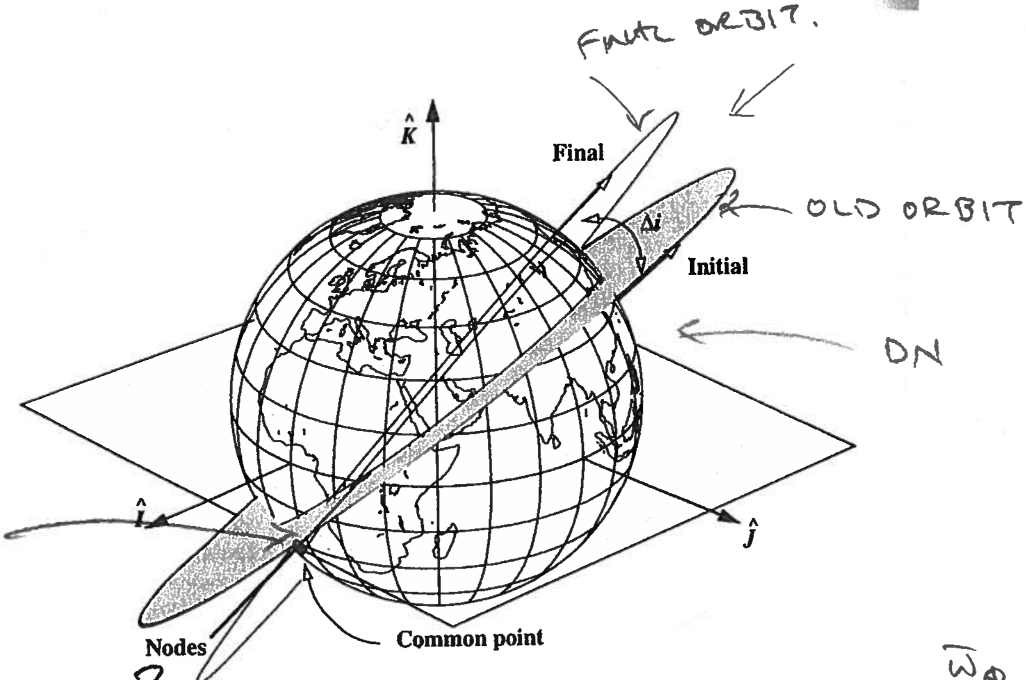


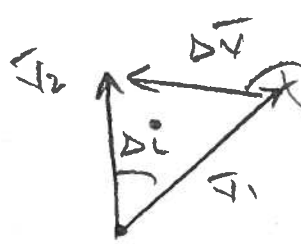
Figure 6-10. **Inclination-Only Changes.** An inclination-only change must occur at one of the nodal crossings because these are the only two points common to *both* orbits. The angular separation is equal to the change in inclination.

VIEW DOWN LINE OF NODES

$\bar{\omega}_\oplus = \bar{V}$

COSINE LAW

$\Delta V^2 = V_1^2 + V_2^2 - 2V_1V_2\cos\Delta i$



$|V_1| = |V_2|$ ← ONLY Δi

$\cos\Delta i = 1 - 2\sin^2\dfrac{\Delta i}{2}$

SIMPLE PLANE CHANGE $\boxed{\Delta V = 2V\sin\dfrac{\Delta i}{2}}$