MAE3145: Solution for Homework 5, Question 3

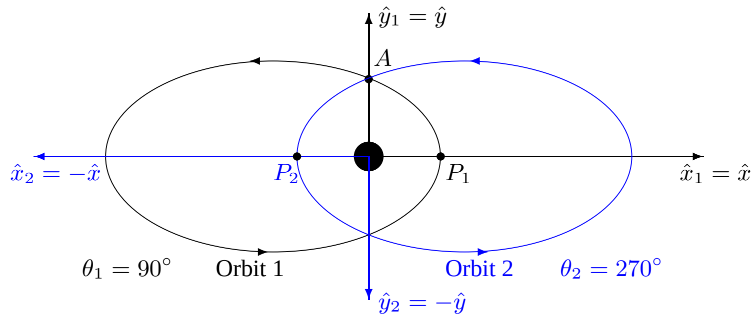
Problem 3 Consider the following two elliptic orbits that have the same eccentricity e and specific angular momentum h. The gravitational parameter is given by μ.

(a) Assume that $\vec{h}_1 = \vec{h}_2 = h\hat{z}$, i.e. in both orbits, the spacecraft rotates counter-clockwise. Find the magnitude of the required velocity change at A on Orbit 1 to transfer the spacecraft to Orbit 2.

Solution: The velocity vector of the i-th spacecraft can be written as

$$\vec{v}_i = \frac{\mu}{h}[-\sin\theta_i\hat{x}_i + (e + \cos\theta_i)\hat{y}_i],$$

where θ_i is measured along the direction of movement from the periapsis, $\hat{x}_i$ points toward the periapsis, and $\hat{y}_i$ points toward $\hat{h}_i \times \hat{x}_i$, or $\theta_i = 90°$. The unit-vectors $\hat{x}_i, \hat{y}_i$ and the true anomaly θ_i for both spacecraft are illustrated as follows:

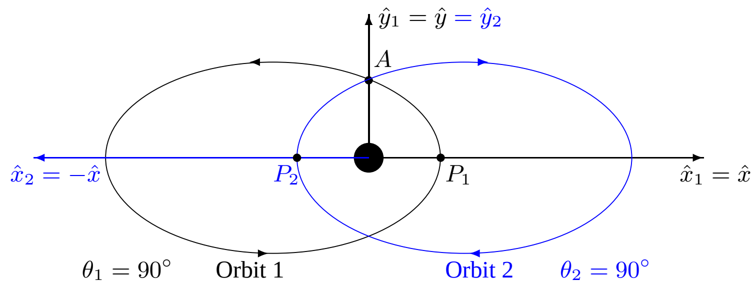


Therefore, the velocity vectors are given by

$$\begin{aligned}
\vec{v}_1 &= \frac{\mu}{h}[-\sin\theta_1\hat{x}_1 + (e + \cos\theta_1)\hat{y}_1] = \frac{\mu}{h}[-\sin 90°\hat{x} + (e + \cos 90°)\hat{y}] = \frac{\mu}{h}[-\hat{x} + e\hat{y}],\\
\vec{v}_2 &= \frac{\mu}{h}[-\sin\theta_2\hat{x}_2 + (e + \cos\theta_2)\hat{y}_2] = \frac{\mu}{h}[-\sin 270°(-\hat{x}) + (e + \cos 270°))(-\hat{y})] = \frac{\mu}{h}[-\hat{x} - e\hat{y}],\\
\Delta\vec{v} &= \vec{v}_2 - \vec{v}_1 = -\frac{2\mu e}{h}\hat{y}.
\end{aligned}$$

(b) Assume that $\vec{h}_1 = -\vec{h}_2 = h\hat{z}$, i.e. the spacecraft rotates counter-clockwise on Orbit 1, and it rotates clockwise on Orbit 2. Find the magnitude of the required velocity change at A on Orbit 1 to transfer the spacecraft to Orbit 2.

Solution: Similarly, we have



Therefore, the velocity vectors are given by

$$\begin{aligned}
\vec{v}_1 &= \frac{\mu}{h}[-\sin\theta_1\hat{x}_1 + (e+\cos\theta_1)\hat{y}_1] = \frac{\mu}{h}[-\sin 90^\circ\hat{x} + (e+\cos 90^\circ)\hat{y}] = \frac{\mu}{h}[-\hat{x} + e\hat{y}], \\
\vec{v}_2 &= \frac{\mu}{h}[-\sin\theta_2\hat{x}_2 + (e+\cos\theta_2)\hat{y}_2] = \frac{\mu}{h}[-\sin 90^\circ(-\hat{x}) + (e+\cos 90^\circ)\hat{y}] = \frac{\mu}{h}[\hat{x} + e\hat{y}], \\
\Delta\vec{v} &= \vec{v}_2 - \vec{v}_1 = \frac{2\mu}{h}\hat{x}.
\end{aligned}$$