MAE3145: Midterm Exam

October 25, 2017

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Last Name First Name Student ID

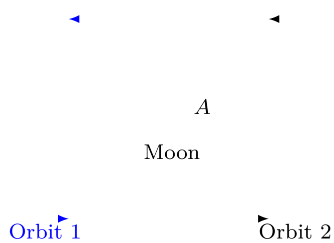
Prob. 1 (15)	Prob. 2 (12)	Prob. 3 (16)	Prob. 4 (16)	Prob. 5 (15)	Total (74)

Problem 1 ***(15pt).*** Mark whether each statement written in *italic font* is True or False.

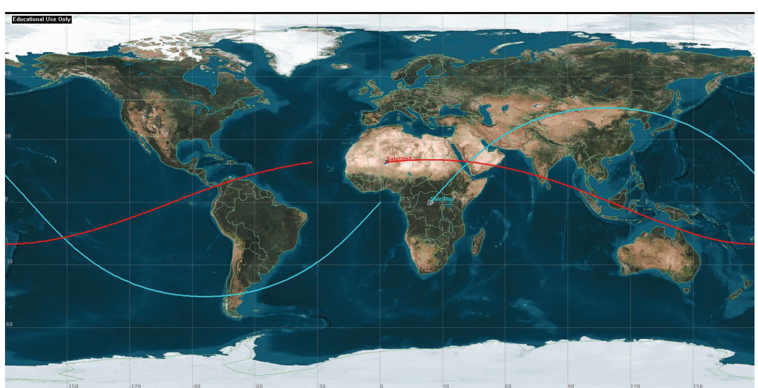
(a) International space station (ISS) is on a circular orbit at the altitude of 422 km, and GPS satellites are on circular orbits at the altitude of 20200 km. *The specific orbital energy of ISS is greater than GPS satellites, i.e.* $\mathcal{E}_{ISS} > \mathcal{E}_{GPS}$, [True, False]

(b) *The orbital period of ISS is greater than GPS satellites, i.e.* $T_{ISS} > T_{GPS}$, [True, False]

(c) The Aitken basin is the largest crater on the far side of the Moon. The following two lunar orbits, namely Orbit 1 and Orbit 2 are proposed to generate a topographic map of the Aitken basin, which is denoted by A below. The size and the shape of two orbits are identical, i.e., $a_1 = a_2$, $e_1 = e_2$, and $T_1 = T_2$. Assume that the Moon is not rotating: A is stationary with respect to both orbits. Then, *spacecraft on Orbit 1 can take images of A for a longer time period per each revolution than another spacecraft on Orbit 2.* [True, False]



(d) Ground track of a satellite is the projection of the orbit of the satellite onto the surface of the Earth. Ground tracks for two satellites are illustrated as follows. *The inclination of Satellite 1 is greater than Satellite 2, i.e.* $i_1 > i_2$. [True, False]

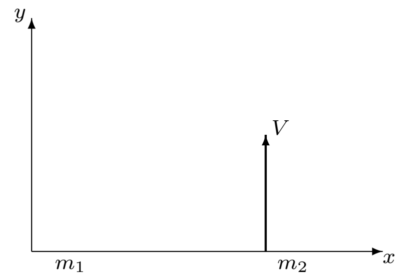


Satellite 1 (blue)

Satellite 2 (red)

(e) Since the Earth is rotating, ground track depends on the spin rate of the Earth. *Assuming the ground tracks at (d) are illustrated for one revolution of each satellite, the orbital period of Satellite 1 is greater than Satellite 2, i.e.,* $T_1 > T_2$. [True, False]

Problem 2 ***(15pt).*** **(Two-body problem with respect to the inertial frame)** We consider a planar motion of two masses acting under their mutual gravitational potential. A mass m_1 is initially at rest with respect to an inertial frame. Another mass m_2 is moving with a velocity V as follows:



More explicitly, the initial position vector and the initial velocity vector of m_1, m_2 in the inertial frame are given by

$$\vec{R}_1(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \text{m}, \quad \dot{\vec{R}}_1(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \text{m/s}, \qquad \vec{R}_2(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{m}, \quad \dot{\vec{R}}_2(0) = \begin{bmatrix} 0 \\ V \end{bmatrix} \text{m/s}.$$

Assume that $m_1 = 2\,\text{kg}$, $m_2 = 1\,\text{kg}$, $\underline{\mu = 1\,\text{m}^3/\text{s}^2}$.

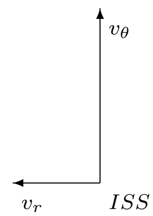
(a) Suppose that $V = 1\,\text{m/s}$. What is the location of the mass center $\vec{R}_G$ at $t = 3$ seconds (specify the units).

(b) Suppose that $V = 1\,\text{m/s}$. What is the type of the orbit for the relative motion $\vec{r} = \vec{R}_2 - \vec{R}_1$. (Hint: compute $\mathcal{E}$ and h, then use the following equation to determine e, $\quad \mathcal{E} = -\frac{1}{2}\frac{\mu^2}{h^2}(1 - e^2)$.)

(c) What is the minimum value of V that makes the distance between two masses approaches infinity, i.e. a parabolic orbit (specify the units).

Problem 3 *(16pt)*. (Properties of orbit in 2D) International Space Station is on a circular orbit at the altitude of $h = 422\,\text{km}$. A bullet is fired from ISS toward the center of the Earth at the velocity of $v_r = -0.3\,\text{km/s}$. We wish to determine whether the bullet hits the surface of the Earth or not. Assume that

$$R_E = 6378\,\text{km}, \quad \mu = 398{,}600\,\text{km}^3/\text{s}^2.$$

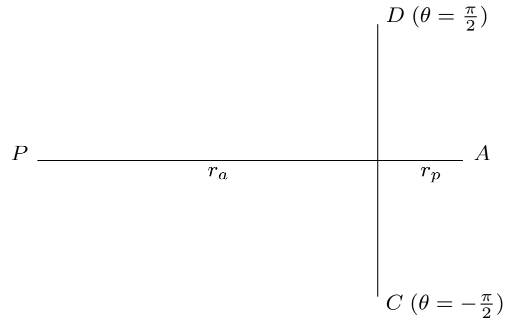


(a) Show that the specific energy of the bullet is given by $\mathcal{E} = -29.2638\,\text{km}^2/\text{s}^2$.
(Hint: $\vec{v} = v_r \hat{u}_u + v_\theta \hat{u}_\theta$)

(b) Show that the eccentricity of the bullet is given by $e = 0.0392$.

(c) Determine whether the bullet hits the surface or the Earth or not.

Problem 4 *(16pt)*. (Orbital position as a function of time) Consider a spacecraft in an elliptic orbit around the Earth.



We observe that the maximum distance r_a, and the minimum distance r_p to the center of the Earth are given by

$$r_a = 32000\,\text{km}, \qquad r_p = 8000\,\text{km}.$$

Assume that the gravitational parameter of the Earth is given by $\mu = 398{,}600\ \text{km}^3/\text{s}^2$.

(a) Find the eccentricity e and the semi-major axis a (specify the units).

(b) Find the specific energy $\mathcal{E}$ and the angular momentum h (specify the units).

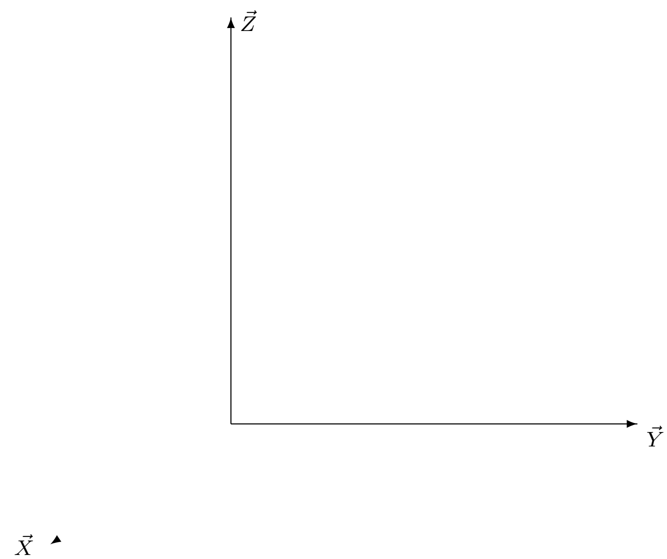
(c) Show the time required for the spacecraft to move from A to P through D, namely t_{ADP} is 3.9095 hours.

(d) Show the time required for the spacecraft to move from C to D through A, namely t_{CAD} is 1.1133 hours.

Problem 5 *(15pt)*. (Geometry of orbit in 3D) The orbital elements for a spacecraft orbiting around the Earth are given as follows:

$$(e = 1.2, \quad \theta = 90^\circ, \quad i = 5^\circ, \quad \Omega = 180^\circ, \quad \omega = 90^\circ).$$

The following figure illustrates the geocentric equatorial frame and the Earth equatorial plane.



Sketch the orbit of this spacecraft according to the following steps.

(a) Draw the node vector $\vec{N}$, and specify the angle between $\vec{N}$ and $\vec{X}$.

(b) Draw the direction of the angular momentum vector $\vec{h}$. Specify the angle between the orbital plane and the equatorial plane.

(c) Draw the eccentricity vector $\vec{e}$, and specify the angle between $\vec{N}$ and $\vec{e}$.

(d) Sketch the orbit. Mark the periapsis by P.

(e) Mark the location of the spacecraft on the orbit by S.

Problem 6 ***(15pt).*** Worldwide, space agencies are considering missions to asteroids, even double and triple body systems. Assume we reach a triple system with three asteroids that possess the following gravitational mass parameters:

Body	(Gm)
Spacecraft	≈ 0
Alpha	2μ
Beta	μ
Gamma	μ

At a certain instant of time, assume that the asteroids and spacecraft are positioned at the four corners of a square. The distance along any edge is d.

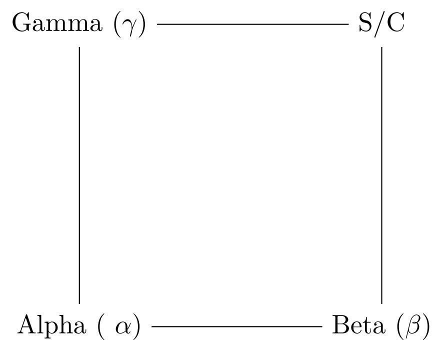


Figure 1: System diagram

(a) Alpha is the primary asteroid; write the relative vector equations of motion for the spacecraft with respect to Alpha.

(b) Determine the magnitudes and directions of the dominant acceleration (A_D), direct (A_{direct}), and indirect ($A_{indirect}$) accelerations on the spacecraft.

(c) Calculate the magnitude and direction of the total acceleration on the spacecraft at this instant. What is the component parallel to the spacecraft–Alpha line.

(d) Is the net perturbing acceleration on S/C instaneously directed toward or away from the primary asteroid Alpha?

(e) Is it reasonable to design the trajectory assuming relative two-body motion for the S/C and Alpha? Why or why not?

Problem 7 ***(15pt).*** Consider an elliptical orbit. Define t_{outer} as the time required to move from a point on one end of the minor axis, through apoapsis, to a point on the other end of the minor axis.

(a) Write an expression for teh ratio of t_{outer} to the orbital period, i.e. $\frac{t_{outer}}{\mathbb{P}}$.

(b) If $e = \frac{3}{4}$, the time spent in the outer half of the orbit is what percentage of the total period? In other words, find the ratio $\frac{t_{outer}}{\mathbb{P}}$.

Problem 8 ***(15pt).*** Assume that a spacecraft is in the orbit about some planet of radius R and it is reasonable to model the orbit in terms of the two-body problem. The perifocal set of unit vectors are $\hat{p}$ and $\hat{q}$.

At a given instant, the spacecraft is located at the end of the minor axis such that:

$$\bar{r} = -4R\hat{p} - 4\sqrt{3}R\hat{q}$$
$$\|\bar{v}\| = 3\,\mathrm{rad\,s^{-1}}$$

(a) Determine the following, where a is the semimajor axis, b is the semiminor axis, p is the semilatus rectum, e is eccentricity, γ is the flight path angle, $\mathcal{E}$ is the specific mechanical energy, E is eccentric anomaly, and h is the specific angular momentum.

$$\frac{a}{R}, \quad \frac{b}{R}, \quad \frac{p}{R}, \quad e, \quad \gamma, \quad \nu, \quad E, \quad \mathcal{E}, \quad \frac{h}{R}$$

(b) Sketch the orbit and mark $\bar{r}, \bar{v}, \gamma, \nu, E$ and the local horizontal and local vertical frame.