MAE3145: Homework 5

Due date: 2 458 064.197 916 JD

Problem 1. Observations at a certain instant indicate the following data for an Earth satellite:

$$r_c = 4R_\oplus \quad v_c = 4.54\,\mathrm{km\,s^{-1}} \quad \gamma_c = -40°$$

(a) Determine the following orbital characteristics of the original orbit:

$$a \quad e \quad \mathbb{P} \quad \mathcal{E} \quad r_p \quad r_a \quad \nu_c \quad E_c \quad (t_c - T)$$

(b) In exactly 8.5 h, a manuever will be implemented. What are the orbital characteristics in 8.5 h at the manuever point. In other words, find the orbital properties, r_m, v_m^-, γ_m^- immediately before applying the maneuver.

(c) The manuever is defined as $\|\Delta v\| = 1200\,\mathrm{m\,s^{-1}}$ (a very large manuever) directed such that $\alpha = 30°$ with respect to the original velocity vector at the maneuver point. Determine the following properties immediately after the impulsive maneuver:

$$a^+ \quad e^+ \quad \mathbb{P}^+ \quad \mathcal{E}^+ \quad r_p^+ \quad r_a^+ \quad \nu_c^+ \quad E_c^+ \quad (t_c - T)^+ \quad \Delta\omega$$

Ensure you draw a proper vector diagram!

(d) Create a plot with both the old and new orbit. Mark the appropriate quantities on the plot.

Problem 2. A vehicle has been successfully launched into an orbit such that $e = 0.5$ and $a = 6.0R_\oplus$. Currently at $t = t_0$ the vehicle is located at perigee. A single in-plane maneuver is employed to circularize the orbit at geosynchronous altitude $r = 6.6R_\oplus$.

(a) Determine $\vec{r}_1^-, \vec{v}_1^-, \gamma_1^-$ at the maneuver point. These are the conditions on the orbit prior to the manuever. The manuever occurs at what value of ν_1^-? What is the "wait time" until the manuever, i.e. how long to go from perigee to the required manuever point?

(b) Compute the required maneuver ($\|\Delta\vec{v}\|, \alpha$. Ensure you include a proper vector diagram. What are the conditions on the orbit immediately following the maneuver, i.e. find $\vec{r}_1^+, \vec{v}_1^+, \gamma_1^+$?

(c) Plot the old and new orbits (together). On the plot, mark the following:

$$\vec{r}_0 \quad \vec{r}_1 \quad \vec{v}_1^- \quad \nu_1^- \quad \text{local horizon} \quad \gamma_1^1 \quad \vec{v}_1^+ \quad \gamma_1^+ \quad \Delta\vec{v} \quad \alpha$$

Problem 3. A vehicle is currently in Earth orbit such that $e = 0.75$ and $a = 4.5R_\oplus$. A single in-plane maneuver will be used to raise perigee and lower apogee. New values are specified as $r_p = 2.0R_\oplus$ and $r_a = 6.0R_\oplus$. It is also required that perigee advance by 35°, i.e. $\Delta\omega = +35°$.

(a) At what location ν in the original orbit should the maneuver be implemented? Determine $\vec{r}_1, \vec{v}_1^-, \gamma_1^-$ at the manuever point.

(b) Determine the manuever ($\|\Delta\vec{v}\|, \alpha, \beta)$ to accomplish the objective. If there are two possibilities, **ALWAYS** choose the one with lowest cost. Do not forget the vector diagrams! Can you deduce the lowest cost option from the vector diagrams? What are the values of $\vec{v}_1^+, \gamma^+$?

(c) Define the manuever in the VNC reference frame, i.e. write down the vector $\Delta\vec{v} = a\hat{v} + b\hat{n} + c\hat{c}$. Create a plot with the old and new orbit as well as the properties:

$$\vec{r}_0 \quad \vec{r}_1 \quad \vec{v}_1^- \quad \nu_1^- \quad \text{local horizon} \quad \gamma_1^1 \quad \vec{v}_1^+ \quad \gamma_1^+ \quad \Delta\vec{v} \quad \alpha$$

(d) Determine the position and velocity at $\nu = 250°$ in the new orbit and mark this location on the plot. Determine the amount of time to go from the maneuver to $\nu = 250°$ in the new orbit.

Problem 4. As part of of an interplanetary mission, a spacecraft is in the following orbit around Mars (relative to a Mars-centered inertial coordinate frame):

$$a = 5R_{♂} \quad e = 0.5 \quad i = 30° \quad \Omega = 45° \quad \omega = -60°$$

At $\nu = 120°$, the following maneuver is implemented

$$\Delta\vec{v} = 0.1\hat{x} - 0.25\hat{y} + 0.2\hat{z}\,\mathrm{km\,s^{-1}}.$$

(a) Transform the $\Delta\vec{v}$ into the $\hat{r}, \hat{\theta}, \hat{h}$ components corresponding to the original orbit. How much of the $\Delta\vec{v}$ is out of the plane? Define this out of plane component as Δv_h and find the percentage as compared to the total $\Delta\vec{v}$.

(b) Define $\Delta\vec{v}_{r\theta}$ as the projection of $\Delta\vec{v}$ in the orbital plane. Determine $\|\Delta\vec{v}_{r\theta}\|, \beta, \phi$. Determine α between the velocity vector in the original orbit and $\Delta\vec{v}_{r\theta}$. Sketch this in-plane projection of components of the vector diagram. Add the unit vectors $\hat{r}, \hat{\theta}$ and $\hat{v}, \hat{c}$ to the sketch.

(c) To apply the maneuver, all position and velocities must be written in the same set of unit vectors, such as the inertial unit vectors $\hat{x}, \hat{y}, \hat{z}$. Determine the new $\vec{r}^+, \vec{v}^+$ immediately after the manuever.

(d) Determine the orbital elements of the new orbit. Compare $\hat{r}^-, \hat{\theta}^-, \hat{h}^-$ (pre-maneuver in the original orbit) and $\hat{r}^+, \hat{\theta}^+, \hat{h}^+$ (post-maneuver in the new orbit).