2017 MIDTERM SOLUTION

PROBLEM 1

A $\varepsilon = -\dfrac{\mu}{2a}$

$a_{ISS} = 6378.137 + 422$ km

$a_{GPS} = 6378.137 + 20200$ km

$\varepsilon_{ISS} < \varepsilon_{GPS} \rightarrow \boxed{\text{FALSE}}$ $\quad a \uparrow \rightarrow \varepsilon \uparrow$

B $\mathbb{P} = 2\pi\sqrt{\dfrac{a^3}{\mu}}$ $\quad a \uparrow \rightarrow \mathbb{P} \uparrow$

$P_{ISS} < P_{GPS} \longrightarrow \boxed{\text{FALSE}}$

C NEWTON → UNIVERSAL LAW OF GRAVITATION

D KEPLER → FIRST LAW

E PTOLEMY / COPERNICUS

F

1. THE ORBIT OF EACH PLANET IS AN ELLIPSE WITH THE SUN AT A FOCUS
2. THE LINE JOINING THE PLANET TO THE SUN SWEEPS OUT EQUAL AREAS IN EQUAL TIMES
3. THE SQUARE OF THE PERIOD OF A PLANET IS PROPORTIONAL TO THE CUBE OF ITS MEAN DISTANCE FROM THE SUN.

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$$\ddot{\bar{r}} + \frac{\mu}{r^3}\bar{r} = 0$$

$$r = \frac{a(1-e^2)}{1+e\cos\nu}$$

c

$r_a = 25000$ km

$r_p = 8000$ km

$$\mathcal{E} = -\frac{\mu}{2a} = \frac{v^2}{2} - \frac{\mu}{r}$$

$$v = \sqrt{2\left(\frac{\mu}{r} - \frac{\mu}{2a}\right)}$$

$V_a = 2.8$ km/sec

$V_p = 8.7$ km/sec

$V_p > V_a \longrightarrow \boxed{\text{FALSE}}$

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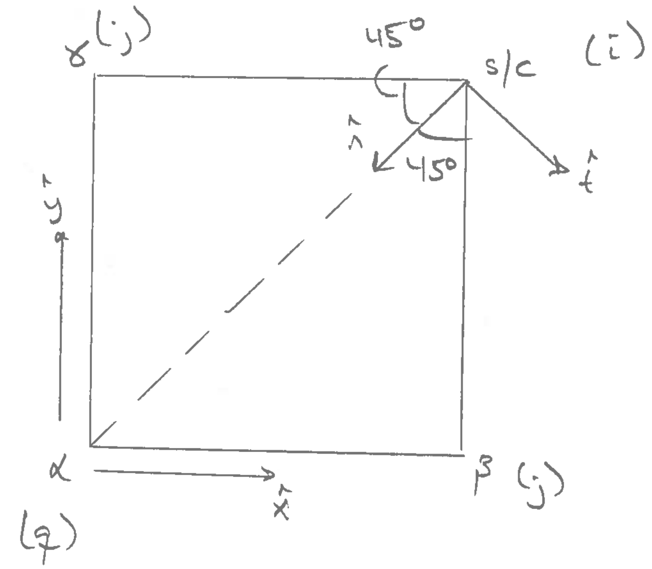

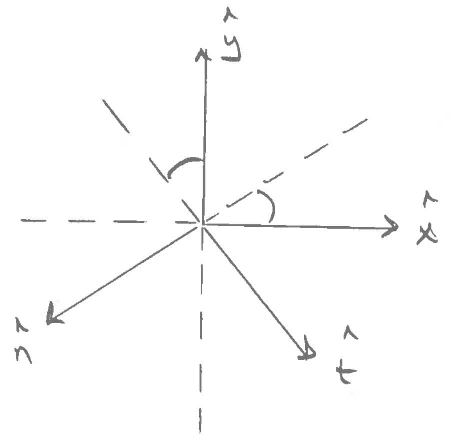

REFERENCE FRAME

$\hat{r}_{\alpha s} = -\hat{n}$

$\hat{r}_{s\beta} = -\hat{y}$

$\hat{r}_{\alpha\beta} = \hat{x}$

$\hat{r}_{s\gamma} = -\hat{x}$

$\hat{r}_{\alpha\gamma} = \hat{y}$

A RELATIVE N-BODY EOMS.

$$\ddot{\bar{r}}_{qi} + \frac{\mu_i + \mu_q}{r_{qi}^2}\hat{r}_{qi} = \sum_{\substack{j=1 \\ j \neq i}}^{n} \mu_j \left(\frac{\hat{r}_{ij}}{r_{ij}^2} - \frac{\hat{r}_{qj}}{r_{qj}^2} \right)$$

MOTION OF S/C WRT ALPHA.

$$\ddot{\bar{r}}_{\alpha s} = -\frac{(\cancelto{0}{\mu_s} + \mu_\alpha)}{r_{\alpha s}^2}\hat{r}_{\alpha s} + \mu_\beta \left(\frac{\hat{r}_{s\beta}}{r_{s\beta}^2} - \frac{\hat{r}_{\alpha\beta}}{r_{\alpha\beta}^2} \right) + \mu_\gamma \left(\frac{\hat{r}_{s\gamma}}{r_{s\gamma}^2} - \frac{\hat{r}_{\alpha\gamma}}{r_{\alpha\gamma}^2} \right)$$

DOMINANT DIRECT INDIRECT DIRECT. INDIRECT.

B DOMINANT ACCEL : $\frac{-2\mu}{(\sqrt{2}d)^2}(-\hat{n}) = \frac{\mu}{d^2}\hat{n} = \bar{A}_D$

DIRECT : $\frac{\mu}{d^2}(-\hat{y}) + \frac{\mu}{d^2}(-\hat{x}) = \bar{A}_{DIRECT}$ $\begin{cases} \text{MAG: } \frac{\sqrt{2}\mu}{d^2} \\ \text{DIRECTION: } +\hat{n} \end{cases}$

INDIRECT ACCEL: $-\frac{\mu}{d^2}(\hat{x}) - \frac{\mu}{d^2}(\hat{y}) = \bar{A}_{INDIRECT} \begin{cases} \text{MAG: } \frac{\sqrt{2}\,\mu}{d^2} \\ \text{DIR: } +\hat{n} \end{cases}$

C TOTAL ACCELERATION:

$$\frac{\mu}{d^2}\hat{n} - \frac{2\mu}{d^2}\hat{x} - \frac{2\mu}{d^2}\hat{y}$$

$$= \left(\underbrace{\frac{\mu}{d^2}}_{\text{DOMINANT}} + \underbrace{\frac{2\sqrt{2}\,\mu}{d^2}}_{\text{PERTURBING}}\right)\hat{n} \quad \begin{cases} \text{MAG: } 3.828\,\frac{\mu}{d^2} \\ \text{DIRECTION: } +\hat{n} \end{cases}$$

D NET PERTURBING ACCEL IN $+\hat{n}$ DIRECTION

$\Longrightarrow$ TOWARDS ALPHA

PERTURBING ACCEL > DOMINANT ACCEL.

$$\frac{\text{PERT}}{\text{DOM}} = 3.828\,!$$

A MODEL COMPRISED OF ONLY TWO BODIES (ALPHA + S/C) IS NOT REASONABLE. NEGLECTING THE PERTURBING ACCELERATION IS NOT A VALID ASSUMPTION.

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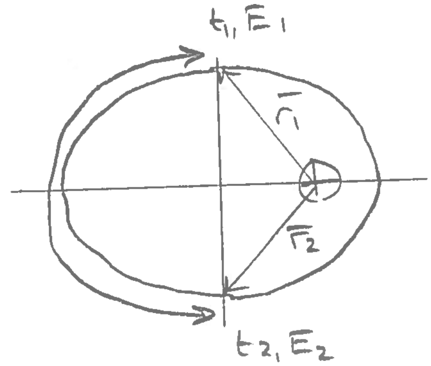
A



$E_1 = 90^\circ \qquad E_2 = 270^\circ$

$n(t - T) = E - e\sin E$

$$\left.\begin{aligned} t_1 - T &= \frac{1}{n}\left(\frac{\pi}{2} - e\right) \\ t_2 - T &= \frac{1}{n}\left(\frac{3\pi}{2} + e\right) \end{aligned}\right\} (t_2 - t_1) = \frac{1}{n}(\pi + 2e)$$

$$P = \frac{2\pi}{n} \Rightarrow \boxed{\frac{t_2 - t_1}{P} = \frac{\pi + 2e}{2\pi}}$$

B

$$e = 0.75 \Rightarrow \frac{t_2 - t_1}{P} = 0.7387 \Rightarrow \boxed{73.9\%}$$

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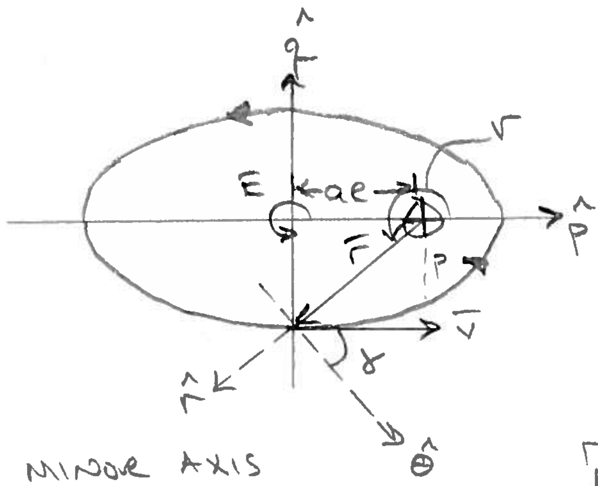

$\bar{r} = -4R\,\hat{p} - 4\sqrt{3}\,\hat{q} \rightarrow r = 8R$

$|\bar{v}| = 3 \ \frac{\text{rad}}{\text{sec}} \qquad \bar{v} = v\,\hat{p}$

MINOR AXIS

$\boxed{r = a = 8R} \qquad \frac{a}{R} = 8$

$v = \sqrt{\frac{\mu}{R}}$

$\boxed{E = 270^\circ = -90^\circ}$

$ae = 4R \Rightarrow \boxed{e = 1/2}$

$\boxed{b = 4\sqrt{3}\,R} \qquad \frac{b}{R} = 4\sqrt{3}$

$p = a(1-e^2) = \boxed{6R = p} \qquad \frac{p}{R} = 6$

$\mathcal{E} = \frac{v^2}{2} - \underset{\uparrow \atop v^2 = 9}{\frac{\mu}{r}} \Rightarrow \mathcal{E} = -\frac{v^2}{2} \Rightarrow \boxed{\mathcal{E} = -4.5 \ \frac{\text{km}^2}{\text{sec}^2}}$

$h^2 = \mu p$ BUT $v^2 = \frac{\mu}{r} = \frac{\mu}{8R} \Rightarrow h^2 = 432\,R^2$

$\boxed{\frac{h}{R} = 12\sqrt{3} \ \frac{\text{km}}{\text{sec}^2}}$

$\boxed{\nu = 180^\circ + 60^\circ = 240^\circ}$

$\tan\gamma = \frac{e}{\sqrt{1-e^2}}$ @ MINOR AXIS

$\boxed{\gamma = -30^\circ \text{ BELOW LOCAL HORIZON}}$