

MAE3145: Final Exam

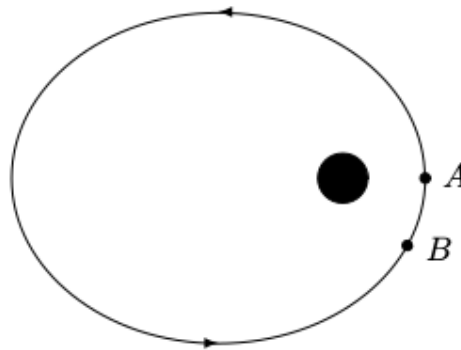
December 14, 2016

Last Name	First Name	Student ID
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Prob. 1 (20)	Prob. 2 (15)	Prob. 3 (20)	Prob. 4 (15)	Prob. 5 (10)	Total (80)

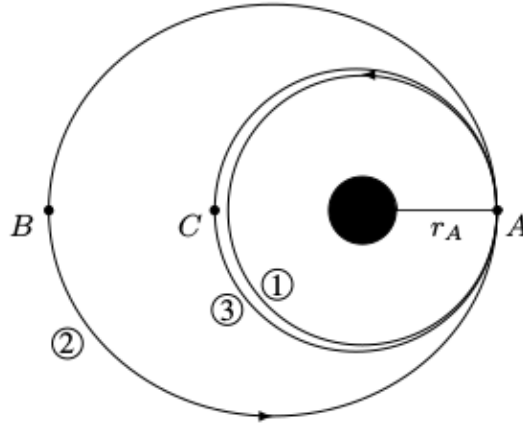
Problem 1 (20pt) Mark whether each statement written in *italic font* is True or False.

- (a) The orbital radius of the Earth is 149.6×10^6 km, and the orbital radius of the Jupiter is 778.6×10^6 km. *The standard Hohmann transfer from the Earth to the Jupiter is always more efficient than the bi-elliptic Hohmann transfer between them.* [True, False]
- (b) Spacecraft *A* and *B* are on the same orbit. The chaser spacecraft *A* executes a phasing maneuver to catch the target spacecraft *B* after one revolution at the point *A*. *Then, the chaser spacecraft *A* should increase its velocity at the beginning of the phasing maneuver.* [True, False]



- (c) Consider a satellite on a circular orbit around the Earth. *Changing the orbital inclination by 15° requires more Δv than that to make it completely escape from the Earth gravitational field.* [True, False]

Problem 2 (15pt) Spacecraft A is on a circular orbit ① around the Earth, and Spacecraft B is on an elliptic orbit ② around the Earth.



$$\mu = 398600 \text{ km}^3/\text{s}^2, \quad r_A = 8000 \text{ km}, \quad e_2 = 0.4.$$

We wish to design a phase maneuver of Spacecraft A such that rendezvous between Spacecraft A and B occurs at the point A after one revolution. After the rendezvous, Spacecraft A is transferred to Orbit ②. Let Orbit ③ be the phasing orbit of Spacecraft A , and let C be the apoapsis of Orbit ③.

(a) Find the time t_{BA} for Spacecraft B to return to the point A .

(b) Find the period T_3 and the apoapsis distance r_C of the phasing orbit ③.

(c) Find the velocity change $\Delta v_A = v_{A_3} - v_{A_1}$ at the beginning of the phasing maneuver.

(d) Find the velocity change $\Delta v_{A'} = v_{A_2} - v_{A_3}$ at the end of the phasing maneuver.

(e) Show that the total velocity change to complete the phasing maneuver is 1.2933 km/s.

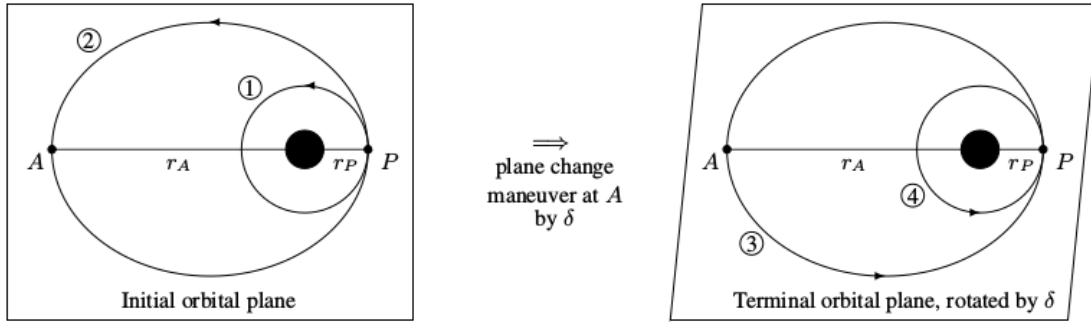
Problem 3 (20pt) Consider a spacecraft on a circular orbit with radius r_P . We wish to rotate the orbital plane by δ without changing the orbit shape and size. In the class, we found that the required velocity change for one-impulse plane change maneuver is given by

$$\Delta v = 2v \sin \frac{\delta}{2}, \quad \text{where} \quad v = \sqrt{\frac{\mu}{r_P}}. \quad (1)$$

This cost of a plane change is quite high. In this question, we try to reduce the cost of a plane change by designing a series of maneuvers. Let the initial circular orbit be denoted by ①, and the terminal rotated circular orbit be ④. The proposed maneuver from ① to ④ is composed of the following three steps:

- ①→② at P : transfer the spacecraft to an elliptic orbit ②,
- ②→③ at A : perform a plane change maneuver at the apoapsis A of the elliptic orbit ②,
- ③→④ at P : transfer the spacecraft to the rotated circular orbit ④ at the periapsis P .

Let the apoapsis distance of the transferring elliptic orbits be $r_A > r_P$. These orbits are illustrated as follows:



	Description	Inclination	Periapsis	Apoapsis	Velocity at the beginning	Velocity at the end
Orbit ①	Initial circular orbit	0	r_P	r_P	V_{P_1}	-
Orbit ②	Hohmann transfer orbit	0	r_P	r_A	V_{P_2}	V_{A_2}
Orbit ③	Hohmann transfer orbit	δ	r_P	r_A	V_{A_3}	V_{P_3}
Orbit ④	Target circular orbit	δ	r_P	r_P	V_{P_4}	-

- (a) Find Δv_P to transfer the spacecraft from the initial circular orbit ① to the elliptic orbit ② at P .

(b) Find Δv_A required to rotate the orbital plane of Orbit ② by δ at A , transferring the spacecraft to Orbit ③.

(c) Find $\Delta v_{P'}$ to transfer the spacecraft from the elliptic orbit ③ to the terminal circular orbit ④ at P .

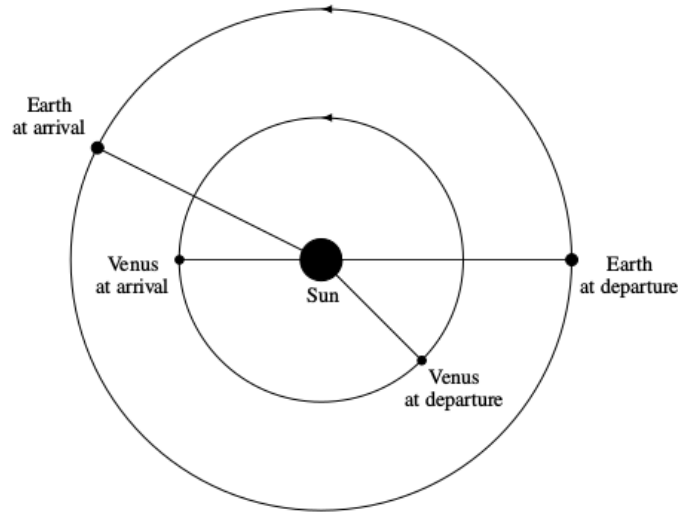
(d) The ratio of the total velocity change $\Delta v_T = |\Delta v_P| + |\Delta v_A| + |\Delta v_{P'}|$ of this maneuver to the velocity change Δv of the one-impulse maneuver at (1) can be written as

$$\frac{\Delta v_T}{\Delta v} = \sqrt{\frac{2}{\rho(1+\rho)}} + \frac{1}{\sin \delta/2} \left\{ \quad \quad \quad \right\}, \quad (2)$$

where $\rho = \frac{r_A}{r_P} > 1$. Find the expression in the braces $\{ \}$. (Hint: $|\Delta v_{P'}| = |\Delta v_P|$).

(e) Suppose that $\rho = 2$, and $\delta = 60^\circ$. Using (2), determine which is more energy efficient between the proposed three-impulse maneuver with the cost of Δv_T , and the one-impulse maneuver with the cost of Δv .

Problem 4 (15pt) The international Cassini mission to the Saturn made use of gravity assist from the Venus. In this question, we develop the rendezvous condition for a Hohmann transfer from the Earth to the Venus. The locations of the Earth and the Venus at the departure and the arrival are illustrated as follows.



$$\mu_S = 1.327 \times 10^{11} \text{ km}^3/\text{s}^2, \quad R_E = 149.6 \times 10^6 \text{ km}, \quad R_V = 108.2 \times 10^6 \text{ km}.$$

- (a) Find the travel time t_{EV} from the Earth to the Venus along the Hohmann transfer orbit between them.

- (b) Let ϕ be the phase angle of the Venus relative to the Earth, i.e. $\phi = \theta_V - \theta_E$, and let ϕ_0 be the value of the phase angle at departure. The rotation angle of the Venus during the Hohmann transfer is given by $n_V t_{EV}$. Mark the angles ϕ_0 and $n_V t_{EV}$ in the above diagram.

- (c) Find ϕ_0 for a rendezvous between the Venus and the spacecraft to occur at the end of the Hohmann transfer. (Hint: $\phi_0 < 0$)