Transformation between Orbital Elements and $(\vec{r}, \vec{v})$

Given $(\vec{r}, \vec{v})$, find the orbital elements $(h, e, \theta, \Omega, i, \omega)$

$$
\begin{aligned}
r &= |\vec{r}|, \\
\vec{h} &= \vec{r} \times \vec{v}, \qquad h = |\vec{h}|, \\
\vec{e} &= \frac{1}{\mu}\vec{v} \times \vec{h} - \frac{\vec{r}}{r}, \qquad e = |\vec{e}|, \\
\vec{N} &= \hat{z} \times \vec{h}, \\
i &= \cos^{-1}\left(\frac{\vec{h} \cdot \hat{z}}{h}\right) \quad (0 \le i \le \pi), \\
\Omega &= \tan^{-1}\left(\frac{\hat{y} \cdot \vec{N}}{\hat{x} \cdot \vec{N}}\right) = \texttt{atan2}(\hat{y} \cdot \vec{N}, \hat{x} \cdot \vec{N}), \\
\omega &= \tan^{-1}\left(\frac{\vec{h} \cdot (\vec{N} \times \vec{e})}{h(\vec{N} \cdot \vec{e})}\right) = \texttt{atan2}(\vec{h} \cdot (\vec{N} \times \vec{e}), h(\vec{N} \cdot \vec{e})), \\
\theta &= \tan^{-1}\left(\frac{\vec{h} \cdot (\vec{e} \times \vec{r})}{h(\vec{e} \cdot \vec{r})}\right) = \texttt{atan2}(\vec{h} \cdot (\vec{e} \times \vec{r}), h(\vec{e} \cdot \vec{r})).
\end{aligned}
$$

(Use the Matlab `atan2` function to compute $\tan^{-1}$, i.e. $\tan^{-1}(y/x) = \texttt{atan2(y,x)}$).

Given the orbital elements $(h, e, \theta, \Omega, i, \omega)$, find $(\vec{r}, \vec{v})$

$$
\begin{aligned}
\hat{N} &= \cos\Omega\,\hat{x} + \sin\Omega\,\hat{y}, \\
\hat{h} &= \sin i \sin\Omega\,\hat{x} - \sin i \cos\Omega\,\hat{y} + \cos i\,\hat{z}, \\
\hat{N}_t &= -\sin\Omega \cos i\,\hat{x} + \cos\Omega \cos i\,\hat{y} + \sin i\,\hat{z}, \\
\hat{u}_r &= \cos(\theta + \omega)\,\hat{N} + \sin(\theta + \omega)\,\hat{N}_t, \\
\hat{u}_\theta &= -\sin(\theta + \omega)\,\hat{N} + \cos(\theta + \omega)\,\hat{N}_t, \\
r &= \frac{h^2}{\mu}\frac{1}{1 + e\cos\theta}, \\
\mathcal{E} &= -\frac{1}{2}\frac{\mu^2}{h^2}(1 - e^2), \\
v &= \sqrt{2\left(\mathcal{E} + \frac{\mu}{r}\right)}, \\
\gamma &= \tan^{-1}\left(\frac{e\sin\theta}{1 + e\cos\theta}\right), \\
\vec{r} &= r\,\hat{u}_r, \\
\vec{v} &= v\cos\gamma\,\hat{u}_\theta + v\sin\gamma\,\hat{u}_r.
\end{aligned}
$$