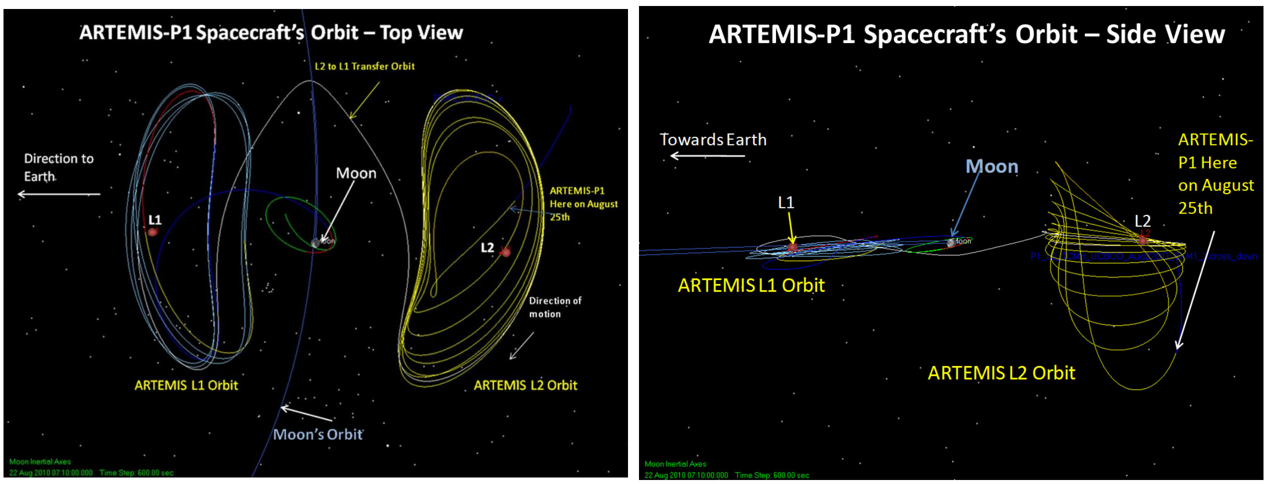
MAE3145: Homework 1

Due date: September 14, 2018

Problem 1. After a complex route to the vicinity of the moon, the two identical ARTEMIS spacecraft (P1 and P2) inserted into orbit near the Moon on August 23 and October 22, 2010, respectively. The spacecraft eventually inserted into lunar orbits on June 27 and July 17, 2011. The trajectories in the lunar vicinity prior to the lunar orbit insertion were influenced significantly by the gravity fields of other bodies, particularly the Earth and the Sun. The P1 path from arrival to the lunar insertion point appears in Fig. 1. Note that it is far from a classical elliptical orbit.



(a) P1 Orbit top

(b) P1 Orbit side

Figure 1: ARTEMIS-P1 orbit

Define a system that is composed of five particles. The law of gravity between each pair is the familiar inverse square law. Obviously, the planets are not truly aligned simultaneously, but assume that the Sun, spacecraft (s/c), and other bodies are collinear and positioned as illustrated in Fig. 2. Assume that the

Figure 2: Planet alignment

spacecraft is instantaneously located such that the distance between the S/C and the Moon is 150 000 km. The total mass of each ARTEMIS space is 130 kg. The distances of the other planets from the Sun are assumed to be equal to the semi-major axis as listed on the constants handout shown on Blackboard.

(a) Locate the center of mass of the system. Identify it on a sketch. Add unit vectors and the appropriate position vectors.

(b) Write the vector differential equation of motion of the S/C with respect to the center of mass, i.e. $\bar{r}_i = \bar{r}_{s/c}$. You should obtain an expression for the accelerations on the S/C, i.e. $\ddot{\bar{r}}_{s/c}$ = sum of four terms. Assuming the alignment given in Fig. 2, compute the accelerations on the S/C due to each of the other bodies. Which body produces the largest accelerations on the S/C? Smallest? What is the descending order of the accelerations due to each body? What is the net acceleration on the spacecraft in $\mathrm{km\,s^{-2}}$?

(c) Compare the relative magnitude of the acceleration terms on the S/C? Is the order of influence what you expected? Which gravity term dominates? Do the acceleration terms seem consistent with your expectations?

Problem 2. Assume that a system is composed of three particles. A Hubble Space Telescope image of Pluto, and its five moons appears in Fig. 3. Let the three particles be Pluto, Charon, and P5 (newly discovered

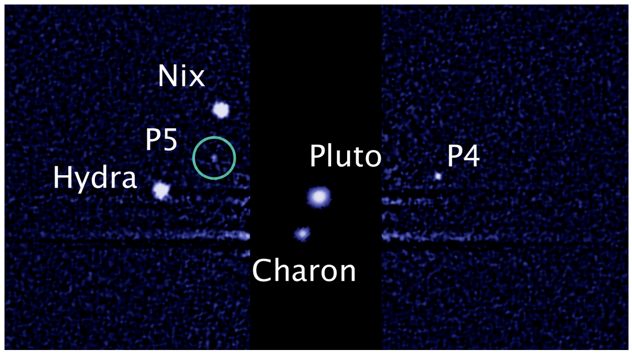


Figure 3: Pluto and moons

in June 2012). If we want to know the orbit of the new moon, we need to write the governing differential equations. The law of gravitation is the familiar inverse square law.

(a) Write each of the vector equations of motion that govern $\bar{r}_P, \bar{r}_C, \bar{r}_{P_5}$. Include a sketch of the arbitrary position vectors.

(b) Add together the three second-order vector equations of motion. Identify the individual terms that cancel; there should be no summation signs left in your expressions. Integrate this result to define the linear momentum integrals $\bar{C}_1, \bar{C}_2$ for this three-body system. Explain how this demonstrates that the velocity of the center of mass of this system is constant.

(c) Apply the cross product of $\bar{r}_P, \bar{r}_C, \bar{r}_{P_5}$ with each equation from (a), respectively. Then sum all the equations together and identify the terms that cancel. Can you integrate the result? This procedure is the derivation of the angular moment integral $\bar{C}_3$.

Note that the expressions for the integrals $\bar{C}_1, \bar{C}_2, \bar{C}_3$ are all functions of $\bar{r}_P, \bar{r}_C, \bar{r}_{P_5}$ and $\frac{d\bar{r}_P}{dt}, \frac{d\bar{r}_C}{dt}, \frac{d\bar{r}_{P_5}}{dt}$.

Problem 3. Assume that a system is composed of three particles. This time, let the particles be the Sun, Earth, and Moon.

(a) Write the **relative** vector equations of motion for the motion of the Moon relative to the Earth.

(b) Assume that the bodies are located such that they create a right triangle as shown in Fig. 4.

Compute the dominant and perturbing accelerations on the Moon. Use the information provided in the table of constants on Blackboard. By comparing the magnitudes, is it reasonable to model the Moon's motion using only the gravity of the Earth (two body problem)? Why or why not? Will the solar gravity have a significant influence?

(c) Is a two-body model (Earth and Moon) reasonable for the motion of the Moon?

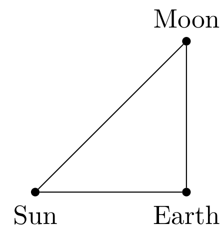


Figure 4: Sun, Earth, Moon three body system

Problem 4. The motion of two point masses acting under their mutual gravity is described, with respect to an inertial frame, by the following set of ordinary differential equations.

$$m_1 \ddot{R}_1 = G\frac{m_1 m_2}{r^2}\hat{u}_r, \tag{1}$$

$$m_2 \ddot{R}_2 = -G\frac{m_1 m_2}{r^2}\hat{u}_r, \tag{2}$$

where $r = R_2 - R_1$, $r = \|r\|$, $\hat{u}_r = \frac{r}{r}$. Suppose that the units are normalized such that $m_1 = 2\,\mathrm{kg}$, $m_2 = 1\,\mathrm{kg}$, $G = 1\,\mathrm{m}^3/\mathrm{kgs}^2$.

The initial conditions are given by

$$R_1(0) = [0,0,0]^T\ (\mathrm{m}), \qquad V_1(0) = [0,0,0]^T\ (\mathrm{m/s}),$$
$$R_2(0) = [1,0,0]^T\ (\mathrm{m}), \qquad V_2(0) = [1,1,0]^T\ (\mathrm{m/s}).$$

We wish to compute the resulting trajectories of m_1 and m_2 using Python.

First, we rewrite the equations of motion as the standard form of $\dot{x} = f(t,x)$. Let the state vector be $x = [R_1^T,\, V_1^T,\, R_2^T,\, V_2^T] \in \Re^{12}$. The equations of motion can be rewritten as

$$\begin{bmatrix} \dot{R}_1 \\ \dot{V}_1 \\ \dot{R}_2 \\ \dot{V}_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ G\frac{m_2}{r^2}\hat{u}_r \\ V_2 \\ -G\frac{m_1}{r^2}\hat{u}_r \end{bmatrix}. \tag{3}$$

(a) Write a Python function, namely `eomTBI` that returns $\dot{x}$ for given (t,x). This function will be used by the `scipy.integrate.odeint` integrator.

You should test that your function is correct. Decide on a sequence of unit tests that will ensure your function is correct. For example, for the initial conditions given above, your function should return

$$\dot{x} = \Big[0,0,0,1,0,0,1,1,0,-2,0,0\Big]. \tag{4}$$

(b) Write a Python script, entitled `simTBI.py` to obtain $R_1(t), R_2(t)$ using `scipy.integrate.odeint`, and plot the trajectories of m_1, m_2 together on a single xy plane (The x axis is for the x-component of R_1, R_2, and the y axis is for their y components. There is no need to plot the z-components, as they are identical to zero). The simulation time is $0 \le t \le 10$ seconds.

(c) The position of the mass center of two point masses is given by

$$R_G = \frac{m_1 R_1 + m_2 R_2}{m_1 + m_2}.$$

Compute the trajectory of the mass center using the results of (d) and plot it on the xy plane.

(d) The relative position of m_2 from m_1 is given by

$$r = R_2 - R_1.$$

Compute the trajectory of the relative motion, and plot it on the xy plane.