MAE3145: Midterm Exam

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Last Name First Name Student ID

Prob. 1 (18)	Prob. 2 (40)	Prob. 3 (22)	Prob. 4 (20)	Total (100)

Problem 1. 18pt

Mark whether each statement written in *italic font* is True or False.

(a) The International space station (ISS) is on a circular orbit at the altitude of 422 km, and GPS satellites are on circular orbits at the altitude of 20 200 km. *The specific mechanical energy of ISS is greater than GPS satellites, i.e.* $\mathcal{E}_{ISS} > \mathcal{E}_{GPS}$. [True, False]

(b) *The orbital period of the ISS is greater than that of GPS satellites, i.e.* $\mathbb{P}_{ISS} > \mathbb{P}_{GPS}$. [True, False]

(c) An Earth orbiting satellite has an apogee of $r_a = 25\,000$ km and a perigee of $r_p = 8000$ km. *The orbital velocity at perigee is less than that orbital velocity at apogee, i.e.* $v_p < v_a$. [True, False]

Answer the following short response questions.

(d) Who made this statement? The force of gravity between two bodies is directly proportional to the product of their two masses and inversely proprtional to the square of the distance between them.

(e) Who made this statement? The orbits of the planets are ellipses with the Sun at one focus.

(f) Who defined the theory of epicycles, or described the apparent motion of the planets as composed of circular motion of the planet (epicycle) on a larger circle (deferent) centered on the Earth.

(g) In your own words, write the three laws of planetary motion as described by Johaness Kepler.

(h) What are the equations of motion for the relative motion of two bodies under their mutual gravitational attraction?

(i) What is the analytical solution for the relative two-body problem? What geometric shape does it define?

Problem 2. 40pt Worldwide, space agencies are considering missions to asteroids, even double and triple body systems. Assume we reach a triple system with three asteroids that possess the following gravitational mass parameters:

Body	(Gm)
Spacecraft	≈ 0
Alpha	2μ
Beta	μ
Gamma	μ

At a certain instant of time, assume that the asteroids and spacecraft are positioned at the four corners of a square. The distance along any edge is d.

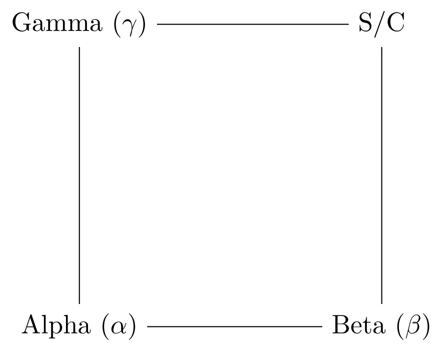

Figure 1: System diagram

(a) Alpha is the primary asteroid; write the relative vector equations of motion for the spacecraft with respect to Alpha.

(b) Determine the magnitudes and directions of the dominant acceleration (A_D), direct (A_{direct}), and indirect ($A_{indirect}$) accelerations on the spacecraft.

(c) Calculate the magnitude and direction of the total acceleration on the spacecraft at this instant. What is the component parallel to the spacecraft–Alpha line.

(d) Is the net perturbing acceleration on S/C instantaneously directed toward or away from the primary asteroid Alpha?

(e) Is it reasonable to design the trajectory assuming relative two-body motion for the S/C and Alpha? Why or why not?

Problem 3. 22pt Consider an elliptical orbit. Define t_{outer} as the time required to move from a point on one end of the minor axis, through apoapsis, to a point on the other end of the minor axis.

(a) Write an expression for the ratio of t_{outer} to the orbital period, i.e. $\frac{t_{outer}}{\mathbb{P}}$.

(b) If $e = \frac{3}{4}$, the time spent in the outer half of the orbit is what percentage of the total period? In other words, find the ratio $\frac{t_{outer}}{\mathbb{P}}$.

Problem 4. 20pt Assume that a spacecraft is in the orbit about some planet of radius R and it is reasonable to model the orbit in terms of the two-body problem. The perifocal set of unit vectors are $\hat{p}$ and $\hat{q}$.

At a given instant, the spacecraft is located at the end of the minor axis such that:

$$\bar{r} = -4R\hat{p} - 4\sqrt{3}R\hat{q}$$
$$\|\bar{v}\| = 3\,\mathrm{rad\,s^{-1}}$$

(a) Determine the following, where a is the semimajor axis, b is the semiminor axis, p is the semilatus rectum, e is eccentricity, γ is the flight path angle, $\mathcal{E}$ is the specific mechanical energy, E is eccentric anomaly, and h is the specific angular momentum.

$$\frac{a}{R}, \quad \frac{b}{R}, \quad \frac{p}{R}, \quad e, \quad \gamma, \quad \nu, \quad E, \quad \mathcal{E}, \quad \frac{h}{R}$$

(b) Sketch the orbit and mark $\bar{r}, \bar{v}, \gamma, \nu, E$ and the local horizontal and local vertical frame.