# MAE3145: Final Exam

December 14, 2016

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Last Name First Name Student ID

| Prob. 1 (20) | Prob. 2 (15) | Prob. 3 (20) | Prob. 4 (15) | Prob. 5 (10) | Total (80) |
|---|---|---|---|---|---|
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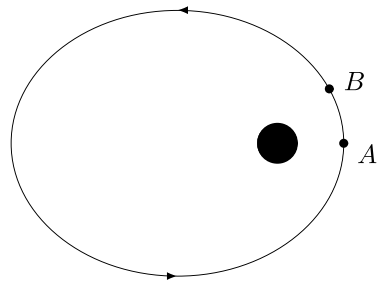
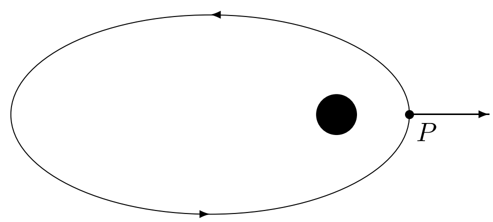
**Problem 1** ***(20pt)*** Mark whether each statement written in *italic font* is True or False (for (a)-(d)), or answer the question shortly (for (e)).

(a) Spacecraft $A$ and $B$ are on the same orbit. The chaser spacecraft $A$ executes a phasing maneuver to catch the target spacecraft $B$ after one revolution at the point $A$. *Then, the chaser spacecraft A should increase its velocity at the beginning of the phasing maneuver.* [True, False]




(b) A spacecraft is on an elliptic orbit. When it passes through the periapsis $P$, its rocket is fired along the outward radial direction as shown below. Then, *the eccentricity vector $\vec{e}$ rotates clockwise*. [True, False]

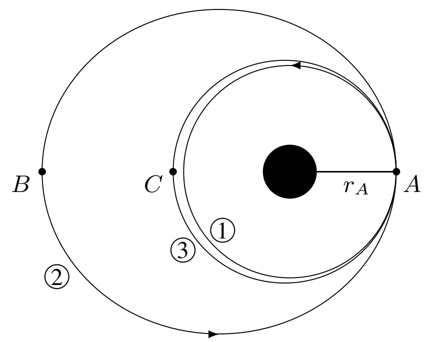



(c) A spacecraft is on a circular orbit around the Earth. *Escaping from the Earth gravitational field completely requires a larger velocity increment than rotating its orbital plane by* $30°$ *without changing the orbital shape and size*. [True, False]

(d) *Among the five Lagrange points in the Earth-Moon system, the first Lagrange point $L_1$ has the lowest value of the Jacobi constant, i.e. $L_1$ has the minimum energy.* [True, False]

(e) Kennedy Space Center has been NASA's primary launch center of human spaceflight since December 1968. Describe *shortly* one of the *engineering* benefits of locating a lauch center in the southern east coast, compared with other places like Portland, OR. (Please, exclude political or climate-related issues.)

**Problem 2** ***(15pt)*** Spacecraft $A$ is on a circular orbit ① around the Earth, and Spacecraft $B$ is on an elliptic orbit ② around the Earth.




$$\mu = 398600\,\mathrm{km}^3/\mathrm{s}^2, \qquad r_A = 8000\,\mathrm{km}, \qquad e_2 = 0.4.$$

We wish to design a phase maneuver of Spacecraft $A$ such that rendezvous between Spacecraft $A$ and $B$ occurs at the point $A$ after one revolution. After the rendezvous, Spacecraft $A$ is transferred to Orbit ②. Let Orbit ③ be the phasing orbit of Spacecraft $A$, and let $C$ be the apoapsis of Orbit ③.

(a) Find the time $t_{BA}$ for Spacecraft B to return to the point $A$.

(b) Find the period $T_3$ and the apoapsis distance $r_C$ of the phasing orbit ③.

(c) Find the velocity change $\Delta v_A = v_{A_3} - v_{A_1}$ at the beginning of the phasing maneuver.

(d) Find the velocity change $\Delta v_{A'} = v_{A_2} - v_{A_3}$ at the end of the phasing maneuver.

(e) Show that the total velocity change to complete the phasing maneuver is $1.2933\,\mathrm{km/s}$.

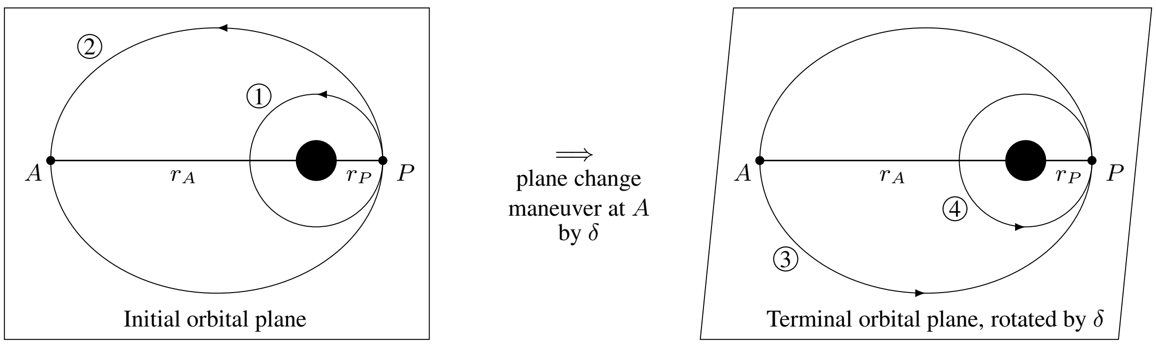
**Problem 3** ***(20pt)*** Consider a spacecraft on a circular orbit with radius $r_P$. We wish to rotate the orbital plane by $\delta$ without changing the orbit shape and size. In the class, we found that the required velocity change for one-impulse plane change maneuver is given by

$$\Delta v = 2v \sin\frac{\delta}{2}, \qquad \text{where} \quad v = \sqrt{\frac{\mu}{r_P}}. \tag{1}$$

This cost of a plane change is quite high. In this question, we try to reduce the cost of a plane change by designing a series of maneuvers. Let the initial circular orbit be denoted by ①, and the terminal rotated circular orbit be ④. The proposed maneuver from ① to ④ is composed of the following three steps:

- ①→② at $P$: transfer the spacecraft to an elliptic orbit ②,
- ②→③ at $A$: perform a plane change maneuver at the apoapsis $A$ of the elliptic orbit ②,
- ③→④ at $P$: transfer the spacecraft to the rotated circular orbit ④ at the periapsis $P$.

Let the apoapsis distance of the transferring elliptic orbits be $r_A > r_P$. These orbits are illustrated as follows:




| | Description | Inclination | Periapsis | Apoapsis | Velocity at the beginning | Velocity at the end |
|---|---|---|---|---|---|---|
| Orbit ① | Initial circular orbit | 0 | $r_P$ | $r_P$ | $V_{P_1}$ | - |
| Orbit ② | Hohmann transfer orbit | 0 | $r_P$ | $r_A$ | $V_{P_2}$ | $V_{A_2}$ |
| Orbit ③ | Hohmann transfer orbit | $\delta$ | $r_P$ | $r_A$ | $V_{A_3}$ | $V_{P_3}$ |
| Orbit ④ | Target circular orbit | $\delta$ | $r_P$ | $r_P$ | $V_{P_4}$ | - |

(a) Find $\Delta v_P$ to transfer the spacecraft from the initial circular orbit ① to the elliptic orbit ② at $P$.

(b) Find $\Delta v_A$ required to rotate the orbital plane of Orbit ② by $\delta$ at $A$, transferring the spacecraft to Orbit ③.

(c) Find $\Delta v_{P'}$ to transfer the spacecraft from the elliptic orbit ③ to the terminal circular orbit ④ at $P$.

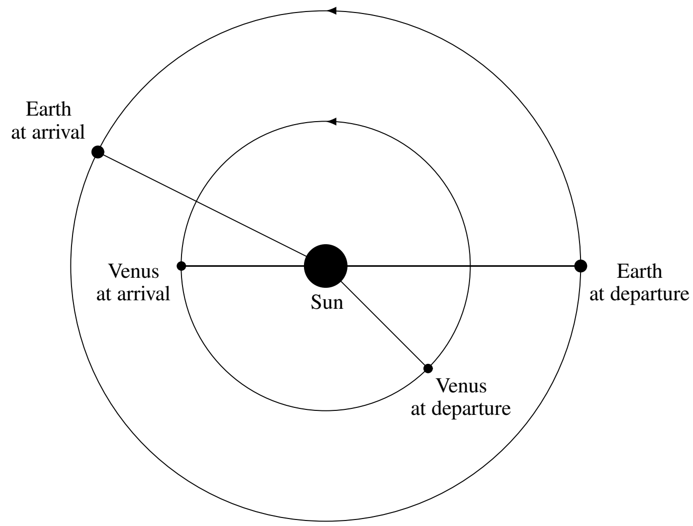
(d) The ratio of the total velocity change $\Delta v_T = |\Delta v_P| + |\Delta v_A| + |\Delta v_{P'}|$ of this maneuver to the velocity change $\Delta v$ of the one-impulse maneuver at (1) can be written as

$$\frac{\Delta v_T}{\Delta v} = \sqrt{\frac{2}{\rho(1+\rho)}} + \frac{1}{\sin \delta/2}\left\{ \qquad\qquad\qquad \right\}, \tag{2}$$

where $\rho = \dfrac{r_A}{r_P} > 1$. Find the expression in the braces { }. (Hint: $|\Delta v_{P'}| = |\Delta v_P|$).

(e) Suppose that $\rho = 2$, and $\delta = 60°$. Using (2), determine which is more energy efficient between the proposed three-impulse maneuver with the cost of $\Delta v_T$, and the one-impulse maneuver with the cost of $\Delta v$.

**Problem 4** ***(15pt)*** The international Cassini mission to the Saturn made use of gravity assist from the Venus. In this question, we develop the rendezvous condition for a Hohmann transfer from the Earth to the Venus. The locations of the Earth and the Venus at the departure and the arrival are illustrated as follows.




$$\mu_S = 1.327 \times 10^{11}\,\mathrm{km}^3/\mathrm{s}^2, \qquad R_E = 149.6 \times 10^6\,\mathrm{km}, \qquad R_V = 108.2 \times 10^6\,\mathrm{km}.$$

(a) Find the travel time $t_{EV}$ from the Earth to the Venus along the Hohmann transfer orbit between them.

(b) Let $\phi$ be the phase angle of the Venus relative to the Earth, i.e. $\phi = \theta_V - \theta_E$, and let $\phi_0$ be the value of the phase angle at departure. The rotation angle of the Venus during the Hohmann transfer is given by $n_V t_{EV}$. Mark the angles $\phi_0$ and $n_V t_{EV}$ in the above diagram.

(c) Find $\phi_0$ for a rendezvous between the Venus and the spacecraft to occur at the end of the Hohmann transfer. (Hint: $\phi_0 < 0$)

**Problem 5** ***(10pt)*** Consider a circular restricted three body problem. To make the equation of motion simpler, we normalize the units for distance, time, and mass by $r_{12}$, $\Omega$, and $m_1 + m_2$, respectively. The resulting *dimensionless* Jacobi constant is given as follows:

$$D = \frac{1}{2}(\dot{x}^2 + \dot{y}^2) - \frac{1}{2}(x^2 + y^2) - \frac{1-\alpha}{r_1} - \frac{\alpha}{r_2},$$

where

$$\alpha = \frac{m_2}{m_1 + m_2}, \quad r_1 = \sqrt{(x+\alpha)^2 + y^2}, \quad r_2 = \sqrt{(x - 1 + \alpha)^2 + y^2}.$$

Suppose that $\alpha = 0.01$ for the Earth-Moon system. The values of the dimensionless Jacobi constant at the five Lagrange Points are given as follows:

$$D_1 = -1.5838, \quad D_2 = -1.5772, \quad D_3 = -1.5050, \quad D_{4,5} = -1.4950, \tag{3}$$

where $D_i$ denotes the dimensionless Jacobi constant at the $i$-th Lagrange point. (**Note**: these values are *different* from the Jacobi constant $C$ discussed in class!)

Consider the following three spacecraft near the Earth. The initial conditions for these spacecraft and the corresponding Jacobi constants are summarized as follows:

| Spacecraft | $x$ | $y$ | $\dot{x}$ | $\dot{y}$ | $D$ |
|---|---|---|---|---|---|
| $A$ | 0.50 | 0.50 | 0 | 0 | $-1.650$ |
| $B$ | 0.20 | 0.18 | 1.121 | 1.685 | $-1.580$ |
| $C$ | 0.18 | 0.20 | 1.801 | 1.000 | $-1.515$ |

(a) Mark all of spacecraft that is (are) impossible to fly to the Moon without firing a onboard rocket.

[Spacecraft $A$, Spacecraft $B$, Spacecraft $C$].

(b) The locations of Spacecraft $B$ and five Lagrange points are illustrated as follows. Sketch *all* of the forbidden region to which Spacecraft $B$ cannot fly.