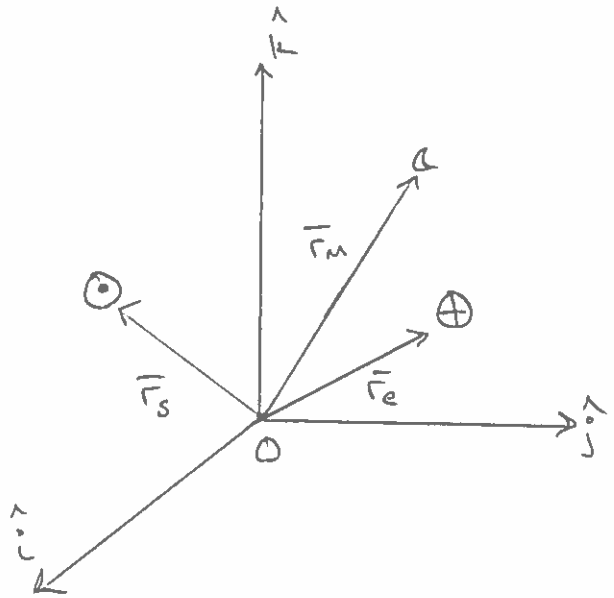
N-BODY EXAMPLE

CONSIDER THREE BODIES: ☉, ⊕, ☾

SUN EARTH MOON



WE WILL ASSUME THAT OUR ORIGIN = SUN !

CENTER OF MASS

$$\left(\sum_{i=1}^{3} M_i\right)\bar{r}_{COM} = \sum_{i=1}^{3} M_i \bar{r}_i$$

$$\bar{r}_{COM} = \frac{M_S \bar{r}_S + M_e \bar{r}_e + M_M \bar{r}_M}{M_S + M_e + M_M}$$ ← PLUG IN VALUES FROM CONSTANTS SHEET

FOR OUR SOLAR SYSTEM + CONSTANTS SHEET

$\bar{r}_S = \bar{0}$ => FOR THIS PROBLEM WE'LL TREAT PT O, OUR ORIGIN AS THE SUN

=> THIS MEANS $\bar{r}_{COM}$ WILL BE A VECTOR FROM THE SUN/ORIGIN TO THE CENTER OF MASS

EQUATIONS OF MOTION OF ☾ WRT CENTER OF MASS

$$M_☾ \ddot{\bar{r}}_☾ = -G \sum_{j=1}^{2} \frac{M_☾ M_j}{r_{j☾}^3} \bar{r}_{j☾}$$

WHERE j = SUN, EARTH

CENTER OF MASS IS AN INERTIALLY FIXED POINT FROM THE CONSERVATION OF LINEAR MOMENTUM

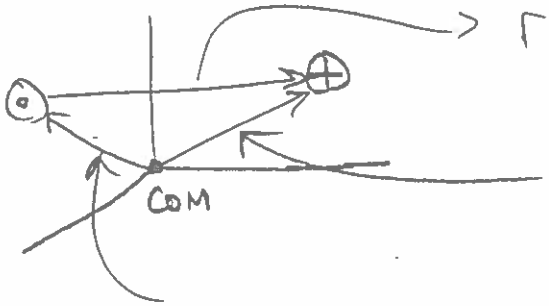
$$M_\oplus \ddot{\bar{r}}_{COM\oplus} = -\frac{G M_\oplus M_\odot}{r_{\odot\oplus}^3}\bar{r}_{\odot\oplus} - \frac{G M_\oplus M_\oplus}{r_{\oplus\oplus}^3}\bar{r}_{\oplus\oplus} \quad (*)$$

↑ CENTER OF MASS TO ⊕

↑ NEED TO FIND VECTOR FROM SUN TO MOON/EARTH

↑

LET'S FIND THE POSITION OF EACH BODY WRT COM.



$\bar{r}_{\odot\oplus}$ = FROM CONSTANTS SHEET

$\bar{r}_{COM\oplus} = \bar{r}_{COM\odot} + \bar{r}_{\odot\oplus}$

↑ SOLVE FOR THIS PART TO USE IN (*)

$\bar{r}_{COM\odot}$ = FROM PART A

QUESTION TO ASK YOURSELF

1. IS THERE A DIFFERENCE IN THESE TWO EQUATIONS

$$\bar{r}_{\odot\oplus} = \bar{r}_{\oplus\oplus} - \bar{r}_{\oplus\odot} \rightarrow (\text{MOON TO EARTH}) - (\text{MOON TO SUN})$$

$$\bar{r}_{\odot\oplus} = \bar{r}_{COM\oplus} - \bar{r}_{COM\odot} \rightarrow (\text{COM TO EARTH}) - (\text{COM TO SUN})$$