MAE3145: Final Exam

December 14, 2016

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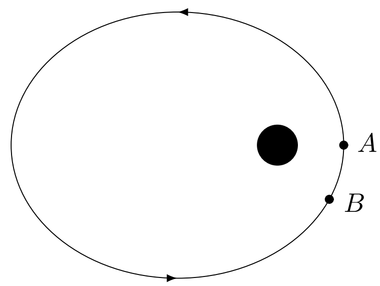
Last Name First Name Student ID

Prob. 1 (20)	Prob. 2 (15)	Prob. 3 (20)	Prob. 4 (15)	Prob. 5 (10)	Total (80)

Problem 1 ***(20pt)*** Mark whether each statement written in *italic font* is True or False.

(a) The orbital radius of the Earth is $149.6 \times 10^6\,\mathrm{km}$, and the orbital radius of the Jupiter is $778.6 \times 10^6\,\mathrm{km}$. *The standard Hohmann transfer from the Earth to the Jupiter is always more efficient that the bi-elliptic Hohmann transfer between them.* [True, False]

(b) Spacecraft A and B are on the same orbit. The chaser spacecraft A executes a phasing maneuver to catch the target spacecraft B after one revolution at the point A. *Then, the chaser spacecraft A should increase its velocity at the beginning of the phasing maneuver.* [True, False]

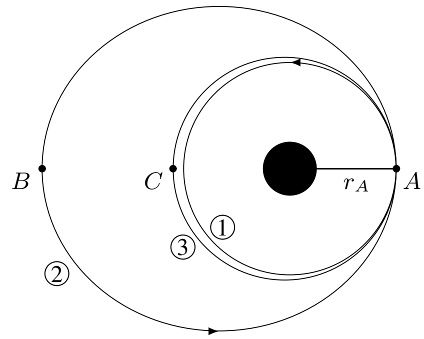


(c) Consider a satellite on a circular orbit around the Earth. *Changing the orbital inclination by $15°$ requires more Δv than that to make it completely escape from the Earth gravitational field.* [True, False]

(d) Consider the circular restricted three body problem for the Earth-Moon system. Let C_i be the value of the Jacobi constant at the i-th Lagrange points for $i \in \{1, 2, \ldots 5\}$. Suppose that a spacecraft is on a near-Earth orbit, and its Jacobi constant C satisfies $C < C_1$. *This spacecraft has an enough energy to be transferred to the Moon without firing a rocket*. [True, False]

Problem 2 ***(20pt)***

Problem 3 ***(15pt)*** Spacecraft A is on a circular orbit ① around the Earth, and Spacecraft B is on an elliptic orbit ② around the Earth.



$$\mu = 398600\,\mathrm{km^3/s^2}, \qquad r_A = 8000\,\mathrm{km}, \qquad e_2 = 0.4.$$

We wish to design a phase maneuver of Spacecraft A such that rendezvous between Spacecraft A and B occurs at the point A after one revolution. After the rendezvous, Spacecraft A is transferred to Orbit ②. Let Orbit ③ be the phasing orbit of Spacecraft A, and let C be the apoapsis of Orbit ③.

(a) Find the time t_{BA} for Spacecraft B to return to the point A.

(b) Find the period T_3 and the apoapsis distance r_C of the phasing orbit ③.

(c) Find the velocity change $\Delta v_A = v_{A_3} - v_{A_1}$ at the beginning of the phasing maneuver.

(d) Find the velocity change $\Delta v_{A'} = v_{A_2} - v_{A_3}$ at the end of the phasing maneuver.

(e) Show that the total velocity change to complete the phasing maneuver is $1.2933\,\mathrm{km/s}$.

Problem 4 ***(20pt)*** Consider a spacecraft on a circular orbit with radius r_P. We wish to rotate the orbital plane by δ without changing the orbit shape and size. In the class, we found that the required velocity change for one-impulse plane change maneuver is given by

$$\Delta v = 2v \sin\frac{\delta}{2}, \qquad \text{where} \quad v = \sqrt{\frac{\mu}{r_P}}. \tag{1}$$

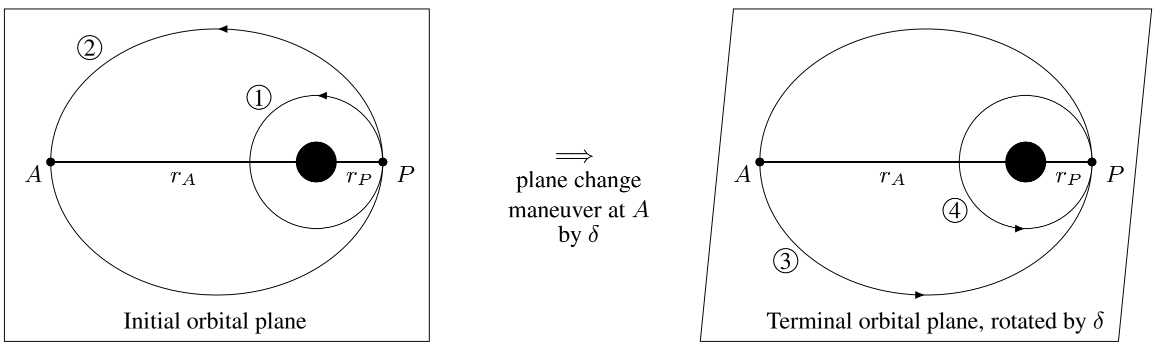
This cost of a plane change is quite high. In this question, we try to reduce the cost of a plane change by designing a series of maneuvers. Let the initial circular orbit be denoted by ①, and the terminal rotated circular orbit be ④. The proposed maneuver from ① to ④ is composed of the following three steps:

– ①→② at P: transfer the spacecraft to an elliptic orbit ②,

– ②→③ at A: perform a plane change maneuver at the apoapsis A of the elliptic orbit ②,

– ③→④ at P: transfer the spacecraft to the rotated circular orbit ④ at the periapsis P.

Let the apoapsis distance of the transferring elliptic orbits be $r_A > r_P$. These orbits are illustrated as follows:



	Description	Inclination	Periapsis	Apoapsis	Velocity at the beginning	Velocity at the end
Orbit ①	Initial circular orbit	0	r_P	r_P	V_{P_1}	-
Orbit ②	Hohmann transfer orbit	0	r_P	r_A	V_{P_2}	V_{A_2}
Orbit ③	Hohmann transfer orbit	δ	r_P	r_A	V_{A_3}	V_{P_3}
Orbit ④	Target circular orbit	δ	r_P	r_P	V_{P_4}	-

(a) Find Δv_P to transfer the spacecraft from the initial circular orbit ① to the elliptic orbit ② at P.

(b) Find Δv_A required to rotate the orbital plane of Orbit ② by δ at A, transferring the spacecraft to Orbit ③.

(c) Find $\Delta v_{P'}$ to transfer the spacecraft from the elliptic orbit ③ to the terminal circular orbit ④ at P.

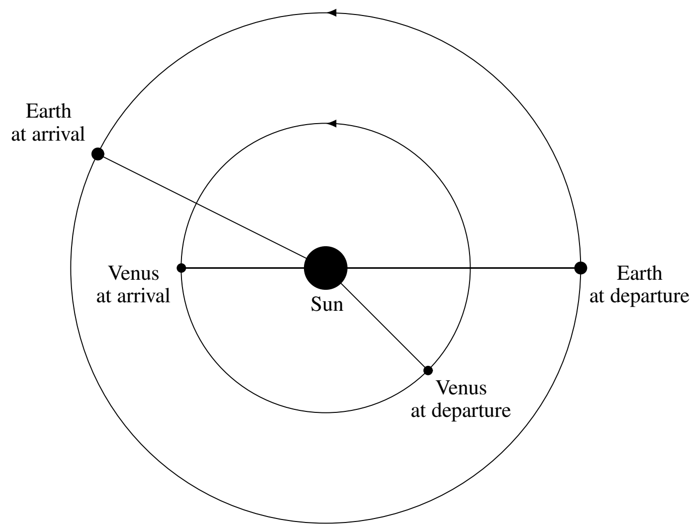
(d) The ratio of the total velocity change $\Delta v_T = |\Delta v_P| + |\Delta v_A| + |\Delta v_{P'}|$ of this maneuver to the velocity change Δv of the one-impulse maneuver at (1) can be written as

$$\frac{\Delta v_T}{\Delta v} = \sqrt{\frac{2}{\rho(1+\rho)}} + \frac{1}{\sin \delta/2}\left\{ \qquad\qquad\qquad \right\}, \tag{2}$$

where $\rho = \dfrac{r_A}{r_P} > 1$. Find the expression in the braces $\{\ \}$. (Hint: $|\Delta v_{P'}| = |\Delta v_P|$).

(e) Suppose that $\rho = 2$, and $\delta = 60°$. Using (2), determine which is more energy efficient between the proposed three-impulse maneuver with the cost of Δv_T, and the one-impulse maneuver with the cost of Δv.

Problem 5 ***(15pt)*** The international Cassini mission to the Saturn made use of gravity assist from the Venus. In this question, we develop the rendezvous condition for a Hohmann transfer from the Earth to the Venus. The locations of the Earth and the Venus at the departure and the arrival are illustrated as follows.



$$\mu_S = 1.327 \times 10^{11}\,\mathrm{km}^3/\mathrm{s}^2, \qquad R_E = 149.6 \times 10^6\,\mathrm{km}, \qquad R_V = 108.2 \times 10^6\,\mathrm{km}.$$

(a) Find the travel time t_{EV} from the Earth to the Venus along the Hohmann transfer orbit between them.

(b) Let ϕ be the phase angle of the Venus relative to the Earth, i.e. $\phi = \theta_V - \theta_E$, and let ϕ_0 be the value of the phase angle at departure. The rotation angle of the Venus during the Hohmann transfer is given by $n_V t_{EV}$. Mark the angles ϕ_0 and $n_V t_{EV}$ in the above diagram.

(c) Find ϕ_0 for a rendezvous between the Venus and the spacecraft to occur at the end of the Hohmann transfer. (Hint: $\phi_0 < 0$)

Problem 6 ***(10pt)*** Consider a circular restricted three body problem. To make the equation of motion simpler, we normalize the units for distance, time, and mass by r_{12}, Ω, and $m_1 + m_2$, respectively. The resulting *dimensionless* Jacobi constant is given as follows:

$$D = \frac{1}{2}(\dot{x}^2 + \dot{y}^2) - \frac{1}{2}(x^2 + y^2) - \frac{1-\alpha}{r_1} - \frac{\alpha}{r_2},$$

where

$$\alpha = \frac{m_2}{m_1 + m_2}, \quad r_1 = \sqrt{(x+\alpha)^2 + y^2}, \quad r_2 = \sqrt{(x-1+\alpha)^2 + y^2}.$$

Suppose that $\alpha = 0.01$ for the Earth-Moon system. The values of the dimensionless Jacobi constant at the five Lagrange Points are given as follows:

$$D_1 = -1.5838, \quad D_2 = -1.5772, \quad D_3 = -1.5050, \quad D_{4,5} = -1.4950, \tag{3}$$

where D_i denotes the dimensionless Jacobi constant at the i-th Lagrange point. (**Note**: these values are *different* from the Jacobi constant C discussed in class!)

Consider the following three spacecraft near the Earth. The initial conditions for these spacecraft and the corresponding Jacobi constants are summarized as follows:

Spacecraft	x	y	$\dot{x}$	$\dot{y}$	D
A	0.50	0.50	0	0	-1.650
B	0.20	0.18	1.121	1.685	-1.580
C	0.18	0.20	1.801	1.000	-1.515

(a) Mark all of spacecraft that is (are) impossible to fly to the Moon without firing a onboard rocket.

[Spacecraft A, Spacecraft B, Spacecraft C].

(b) The locations of Spacecraft B and five Lagrange points are illustrated as follows. Sketch *all* of the forbidden region to which Spacecraft B cannot fly.