MAE3145: Midterm Exam (Practice)

October 26, 2016

_____, _____.

Last Name First Name Student ID

Prob. 1 (3)	Prob. 2 (16)	Prob. 3 (15)	Total (34)

Problem 1 ***(3pt)*** Mark whether each statement written in *italic font* is True or False.

(a) *The orbital speed of the Mars around the Sun is greater than that of the Earth.* [True, False]

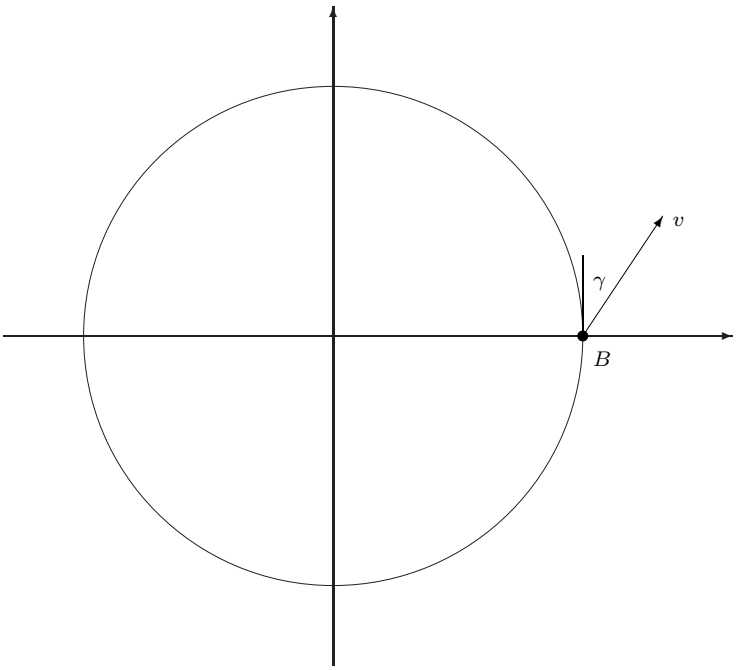
Problem 2 ***(16pt)*** **(Properties of orbit in 2D)** A rocket is fired at the point B on the surface of the Earth. The velocity and the flight path angle of the rocket is given by

$$v = 5\,\mathrm{km/s}, \quad \gamma = 30^\circ.$$

We wish to determine where the rocket would hit on the surface of the Earth. The radius of the Earth and the gravitational parameter are given by

$$R_E = 6378\,\mathrm{km}, \quad \mu = 398{,}600\,\mathrm{km}^3/\mathrm{s}^2.$$

Throughout this question, ignore the atmospheric drag or the rotation of the Earth.



(a) Show that $\mathcal{E} = -49.9961\mathrm{km}^2/\mathrm{s}^2$, and $h = 2.7618 \times 10^4\,\mathrm{km}^2/\mathrm{s}$.

(b) Show that the eccentricity of the rocket is given by $e = 0.7211$.

(c) Find the true anomaly θ_B of the rocket, when it is fired.

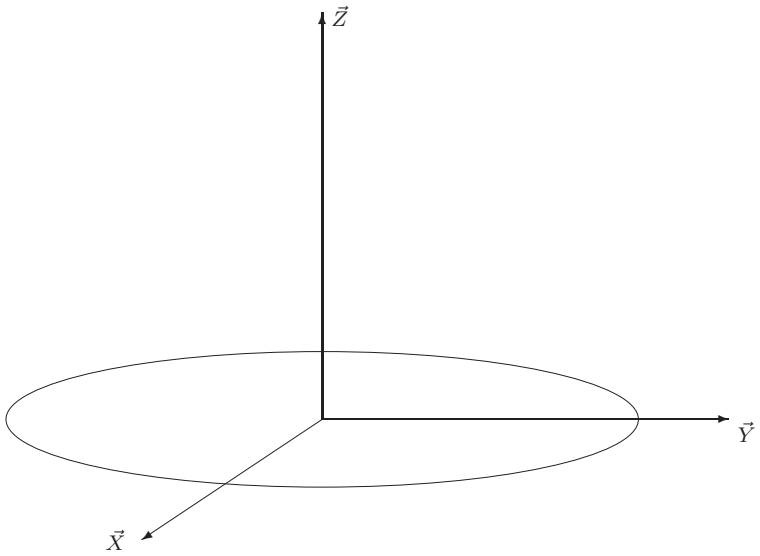
(d) Find the true anomaly θ_H of the rocket, when it hits the surface of the Earth.

(e) Based on the above results, (i) mark the periapsis P, (ii) mark the point H on the surface of the Earth that the rocket hits, and (iii) sketch the trajectory of the rocket between B and H at the diagram of the previous page.

Problem 3 ***(15pt)*** **(Geometry of orbit in 3D)** The orbital elements for a spacecraft orbiting around the Earth are given as follows:

$$(e = 0.6, \quad \theta = 270^\circ, \quad i = 90^\circ, \quad \Omega = 45^\circ, \quad \omega = 180^\circ).$$

The following figure illustrates the geocentric equatorial frame and the Earth equatorial plane.



Sketch the orbit of this spacecraft according to the following steps.

(a) Draw the node vector $\vec{N}$, and specify the angle between $\vec{N}$ and $\vec{X}$.

(b) Draw the direction of the angular momentum vector $\vec{h}$. Specify the angle between the orbital plane and the equatorial plane.

(c) Draw the eccentricity vector $\vec{e}$, and specify the angle between $\vec{N}$ and $\vec{e}$.

(d) Sketch the orbit. Mark the periapsis by P.

(e) Mark the location of the spacecraft on the orbit by S.