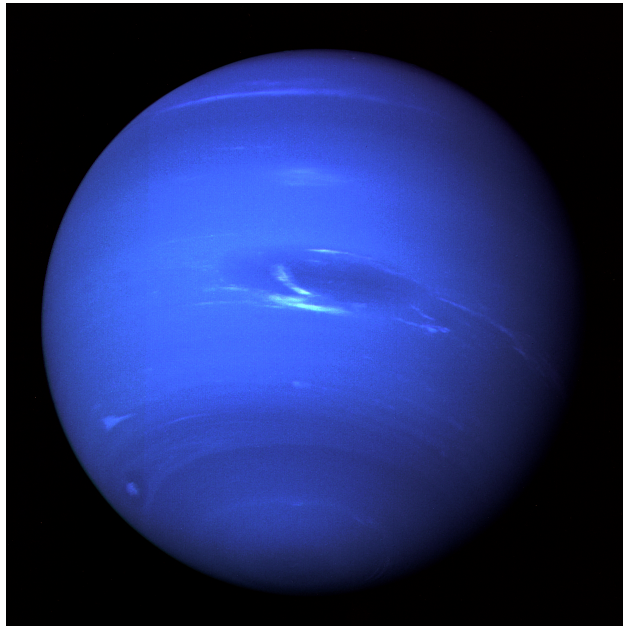


## MAE3145: Homework 6

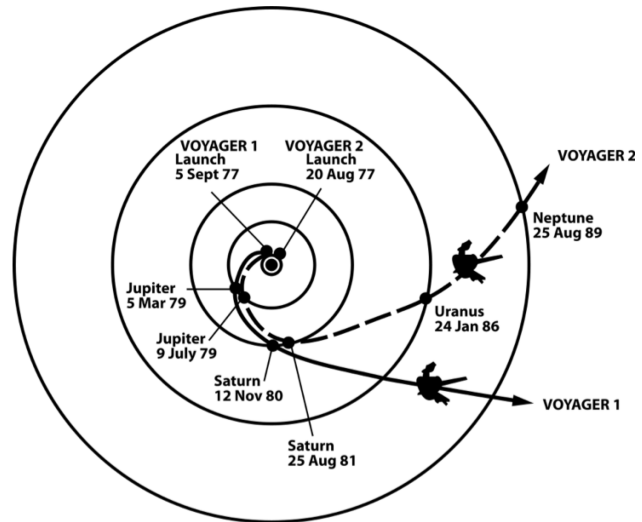
Due date: 2 456 069.417 JD

**Problem 1.** Neptune is now the furthest “planet” in our solar system (since Pluto is classified as a dwarf planet). Voyager 2 passed by Neptune in 1989 but there have not been other spacecraft missions to Neptune. Consider a Neptune mission by doing a few preliminary calculations.

- (a) Begin by examining a Hohmann transfer from the Earth to Neptune. Assume that planetary orbits are coplanar and circular. Compute the total  $\|\Delta\vec{v}_T\|$  and the TOF (time of flight in years). Ensure you draw proper vector diagrams, and compute  $\|\Delta\vec{v}\|$  and  $\alpha$  for each maneuver.
- (b) What is  $\|\Delta\vec{v}_1\|$ , i.e. the maneuver necessary at Earth departure? What is  $\|\Delta\vec{v}_2\|$  to remain in the Neptune system?
- (c) Discuss the feasibility of this mission. Is the total cost ( $\|\Delta\vec{v}_T\|$ ) “a lot”? Is the time of flight reasonable? Even though the Hohmann transfer is the minimum two-impulse transfer, is it likely that we could use this transfer to get to Neptune?
- (d) Compare the time of flight you calculated to the actual Voyager 2 transfer. You can use the Julian date functions, `time.date2jd(yr, mo, day, hr, min, sec)`.
- (e) Compute the phase angle required at departure for this circle-to-circle transfer as seen in the heliocentric view.



(a) Voyager 2 Image of Neptune



(b) Voyager 2 Trajectory

Figure 1: Voyager 2

**Problem 2.** In NASA's original plan for a crewed lunar base (Orion), a ground facility near the Moon's south pole was envisioned, necessitating a polar orbit. The lunar south pole offers areas of continual sunlight, which are ideal locations for continuous power generation, the so called "peaks of eternal light". Thus, the trajectory design ( both arrival at the Moon and the Earth return) included a  $90^\circ$  plane change. Consider the plane change maneuver. Assume that the spacecraft arrives in the plane of the lunar equator and is currently in a circular orbit at 100 km altitude. Two options existed for the plane change to the polar orbit.

1. A single maneuver at the current altitude to shift the orbit to an inclination of  $90^\circ$ .
  2. A bi-elliptic strategy that includes three maneuvers: A maneuver to raise apoapsis to 17 000 km, followed by a plane change maneuver at apoapsis, and a final maneuver to insert back into the 100 km altitude polar orbit.
- (a) Compute and compare the cost, i.e.  $\|\Delta\vec{v}\|$ , for a  $90^\circ$  plane change accomplished with the two approaches. Assume the single plane change is accomplished instantaneously.
  - (b) How much time (TOF) is devoted to the completion of the bi-elliptic option? How does this compare with the single maneuver at the current altitude.

**Problem 3.** A vehicle is launched successfully into an orbit with  $e = 0.4$  and  $a = 6R_\oplus$ . A single in-plane maneuver will be implemented when  $\nu = 90^\circ$  (true anomaly). Let the maneuver be defined as  $\|\Delta\vec{v}\| = 0.75 \text{ km s}^{-1}$ , and  $\alpha = -60^\circ$ .

- (a) Express the  $\Delta\vec{v}$  in terms of the rotating local vertical/local horizontal frame  $(\hat{r}, \hat{\theta})$ , perifocal frame  $(\hat{p}, \hat{q})$ , and VNC reference frames  $(\hat{v}, \hat{c})$ .
- (b) Determine the  $r, v, \gamma$  in the new orbit immediately after the maneuver. Also compute the following characteristics of the new orbit:

$$a \quad e \quad \mathbb{P} \quad \mathcal{E} \quad r_p \quad r_a \quad \nu \quad E \quad (t - T) \quad p \quad \Delta\omega$$

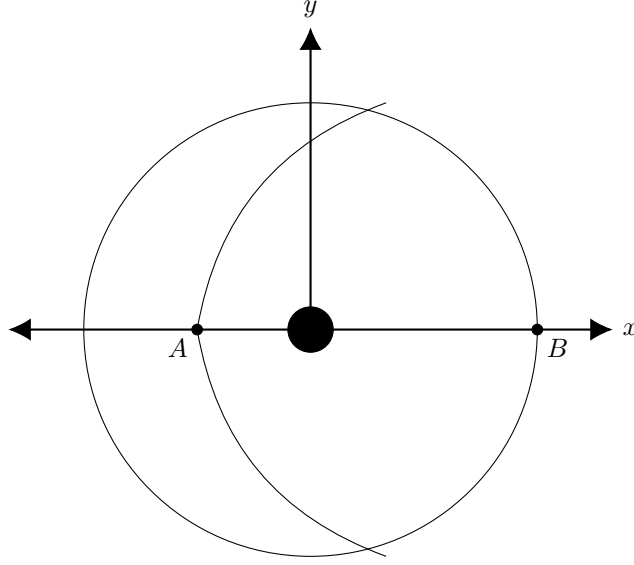
Ensure you include a proper vector diagram.

- (c) Generate a plot of both the old and new orbits. Mark on your plot the vector diagram associated with this maneuver.
- (d) As an alternative, wait until the vehicle reaches the end of the minor axis and is descending and then implement the same maneuver. What is the "wait time" to travel from  $\nu = 90^\circ$  to the end of the minor axis?
  - (a) How do you determine the orbital characteristics at the maneuver point, i.e.  $r^-, v^-, \gamma^-$ .
  - (b) Determine the following orbital characteristics immediately following the maneuver:

$$a \quad e \quad \mathbb{P} \quad \mathcal{E} \quad r_p \quad r_a \quad \nu \quad E \quad (t - T) \quad p \quad \Delta\omega$$

- (c) Plot the old and new orbit and the appropriate quantities on the plot.

**Problem 4.** A spacecraft is returning from an interplanetary mission along a hyperbolic orbit and it is required to rendezvous with a space station already in Earth orbit. Currently, the spacecraft is at  $\nu = 0^\circ$  (at periapsis) in the hyperbolic orbit and approaching periapsis. The space station is located at point  $B$  in the desired final orbit. Both spacecraft are moving in the same direction, such that the angular momentum vectors are aligned and along the  $z$  axis (out of the page). A figure illustrating the problem is shown below.



$$r_A = 7000 \text{ km}, \quad r_B = 14000 \text{ km}, \quad v_{A1} = 12 \text{ km/s}, \quad \mu = 398600 \text{ km}^3/\text{s}^2.$$

We wish to design an orbital maneuver of the spacecraft such that a rendezvous between the spacecraft and the space station occurs at point  $B$ . The maneuver of the spacecraft is composed of the following orbits:

- Hohmann transfer from the hyperbolic orbit to the circular orbit
  - A phasing orbit to ensure a rendezvous at  $B$  on the circular orbit
- (a) Find the velocity change at point  $A$ , namely  $\Delta V_A$ , to transfer the spacecraft from the hyperbolic orbit to a transfer ellipse between  $A$  and  $B$ .
  - (b) Compute the time required to transfer from  $A$  to  $B$  during the Hohmann transfer.
  - (c) Find the location of the space station when the spacecraft arrives at point  $B$ . You can assume the true anomaly for the space station is measured from the positive  $x$  axis. How much time is required for the space station to return to  $B$ ?
  - (d) Find the period of the phasing orbit such that both the spacecraft and space station will arrive at point  $B$  at the same time. Find the semi-major axis  $a_p$ , and distance to the periapsis  $r_C$  of the phasing orbit.
  - (e) Find the velocity change at point  $B$ , namely  $\Delta V_{B1}$ , to transfer the spacecraft from the Hohmann transfer ellipse to phasing orbit.
  - (f) Find the velocity change at point  $B$ , namely  $\Delta V_{B2}$ , to transfer the spacecraft from the phasing orbit onto the target circular orbit.
  - (g) Show that the total velocity change is  $\Delta V_{total} = |\Delta V_A| + |\Delta V_{B1}| + |\Delta V_{B2}| = 4.2657 \text{ km/s}$ .

## COMFIX Hand Calculations

The following problems should help you get started on the COMFIX project. All of the problems are referencing the first satellite observation from `comfix.dat`.

**Problem 5.** Recall the radar site and observation data for satellite 1001 from `comfix.dat`, i.e. latitude =  $77.7^\circ N$ , longitude =  $68.5^\circ W$ , altitude = 50 m, Observation time = 2 454 154.637 615 7 JD. Using this observation (the first one from the file), complete the following.

- Compute Greenwich Sidereal Time -  $GST$
- Compute Local Sidereal Time -  $LST$
- Draw a sketch of the problem using a view from the north pole and identify the location of the inertial  $x$  axis and the angles  $GST, LST$ .

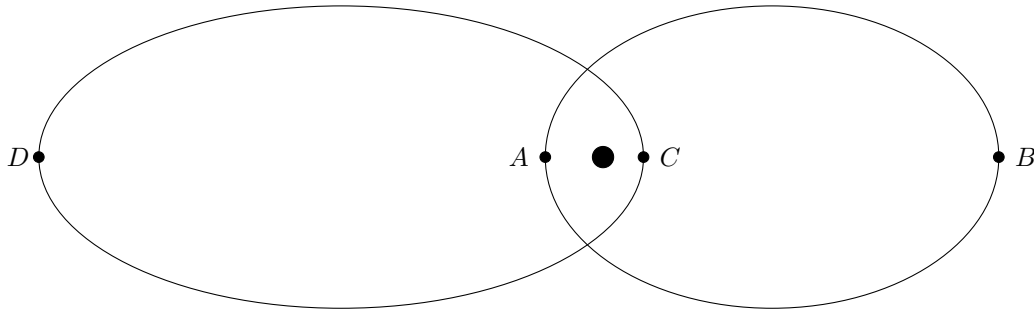
**Problem 6.** Using the same site and time information from the previous problem, compute the position of the observation site  $\vec{r}_{site}$  in both the inertial frame (ECI) and the Earth centered Earth fixed frame (ECEF).

**Problem 7.** Using the same site and observation data, find the vectors  $\vec{\rho}_{ECI}, \dot{\vec{\rho}}_{ECI}$  which define the position and velocity of the satellite relative to the observation site in the Earth Centered Inertial frame.

**Problem 8.** Finally, compute the position and velocity of the satellite,  $\vec{r}_{ECI}, \vec{v}_{ECI}$ , in the Inertial frame and compute the classical orbital elements for this satellite.

## Extra Credit

**Problem 9.** Consider the following orbits: with



$$r_A = 25\,000 \text{ km} \quad r_B = 40\,000 \text{ km} \quad r_C = 10\,000 \text{ km} \quad r_D = 55\,000 \text{ km}$$

- Find the minimum energy ( $\Delta V$ ) transfer ellipse by comparing a transfer from  $A$  to  $C$  vs. a transfer from  $B$  to  $D$ .
- Sketch each transfer ellipse.
- Calculate the time of flight for the minimum energy transfer.

**Problem 10.** Given two circular coplanar orbits with  $r_i = 8000 \text{ km}$  and  $r_f = 120\,000 \text{ km}$ , find the  $\Delta V$  required and time of flight for a bi-elliptic transfer with an intermediate radius of  $r_{int} = 280\,000 \text{ km}$ . Compare this to the direct Hohmann Transfer maneuver and discuss the trade-offs between the two methods.

**Problem 11.** Calculate the total  $\Delta V$ , time of flight, and firing angles ( $\alpha$ ) to transfer between circular coplanar orbits with  $r_i = 10\,000$  km and  $r_f = 42\,160$  km using a transfer ellipse having the parameters  $e = 0.75$  and  $p = 15\,000$  km.

**Problem 12.** Solve the following plane change problems.

- (a) Show that the maximum obtainable simple plane change for a circular orbit where  $\Delta v = \Delta v_{escape}$  is  $\Delta i = 23.9^\circ$ . The velocity change is defined as the **difference** in velocity between the circular orbit velocity and the velocity to escape from the circular orbit.
- (b) Show that the maximum obtainable simple plane for a circular orbit with  $\Delta v = v_{circular}$  is  $\Delta i = 60^\circ$ .
- (c) Determine the minimum  $\Delta v$  required to complete the following transfer:

$$r_i = 8000 \text{ km}$$

$$e_i = 0$$

$$i_i = 30^\circ$$

$$r_f = 42\,160 \text{ km}$$

$$e_f = 0$$

$$i_f = 0^\circ$$