MAE3145: Homework 4

Due date: 2 458 397.666 JD

Problem 1. Halley's comet last passed through perihelion on Feb 9, 1986. The orbit is described with an eccentricity of $e = 0.9671429$ and a semi-major axis of $a = 17.834144$au, where 1 au (astronomical unit) is equal to the semi-major axis of the Earth, $1.495\,898 \times 10^8$ km. In 1986, the European Space Agency spacecraft Giotto encountered and photographed the nucleus of the comet as it approached the Sun. Data from Giotto's camera was used to generate the enhanced image shown in Fig. 1. The potatoe shaped nucleus measures

Figure 1: Halley's Comet as seen by Giotto

roughly 15 km across and is composed primarily of water and carbon dioxide ices.

(a) On this date, determine the following additional orbital characteristics associated with Halley's orbit (assume a conic model):

$$r, \quad v, \quad \nu, \quad \gamma, \quad \mathcal{E}, \quad h, \quad \mathbb{P}, \quad r_a, \quad r_p, \quad E, \quad (t_0 - T)$$

Ensure that you list all distances in astronomical units and times in a reasonable unit, seconds probably do not make sense for Halley's comet. Note: 1 au = semi-major axis of the Earth = $1.495\,898 \times 10^8$ km

You can check the path of 1P/Halley at the JPL Small-Body Data Browser by going to the following link. Observers on Earth can pick up the comet when its true anomaly is approximately 100° prior perihelion.

(b) Determine the same quantities shown in part (a) above, but at the time when the observers expect to pick up the next return.

(c) Determine the time required to go from the last perihelion passage to the location where Earth observers can again pick up the return of Halley's Comet. What is the approximate date? How old will you be?

(d) How many days until the comet passes through perihelion?

Problem 2. As part of some interplanetary mission, assume that a spacecraft departs the Earth vicinity along a hyperbola. The hyperbola is defined such that $r_p = 1000\,\mathrm{km}$ and $e = 1.05$.

(a) Determine the following orbital characteristics: $a, p, v_\infty, \mathcal{E}, \delta, \nu_\infty$ and the aiming radius.

(b) When the spacecraft reaches $\nu_1 = 90°$, determine $r_1, v_1, H_1, \gamma_1, (t_1 - T)$. Write $\bar{r}_1, \bar{v}_1$ in terms of the local vertical/local horizontal and perifocal reference frames?

(c) Plot the hyperbolic orbit between $\pm 135°$, ensure you also include asymptotes. Label the appropriate quantities including, $b, \delta, \frac{\delta}{2}, \nu_\infty, a, \gamma, \text{center}, \text{local horizon}$. Add the Earth to scale.

(d) How long until the spacecraft reaches $\nu_2 = 150°$, $t = t_2$? What is the value of r at this time?

Problem 3. An orbit transfer vehicle (OTV) is currently in Earth orbit with the following characteristics (with respect to the Earth Centered Inertial frame).

$$\begin{aligned} a &= 3R_\oplus & \Omega &= 45° \\ e &= 0.40 & \omega &= 90° \\ i &= 28.5° & \nu &= 235° \end{aligned}$$

(a) Determine the state of the satellite and additional orbital parameters: $\bar{r}, \bar{v}, r, v, \gamma, \nu, M, E, (t - T)$. In addition, write $\bar{r}, \bar{v}$ in terms of the local horizontal/local vertical, perifocal, and inertial reference frames.

(b) Plot the orbital plane and mark all of the appropriate quantities. What are the appropriate quantities?

Problem 4. The OTV is now in a new orbit. At a certain time, the following information is given.

$$\begin{aligned} \bar{r}_1 &= 3.0R_\oplus \hat{x} + 5.0R_\oplus \hat{y}\,\mathrm{km} \\ \bar{v}_1 &= -3.2\hat{x} + 2.0\hat{y} + 2.5\hat{z}\,\mathrm{km\,s^{-1}}. \end{aligned}$$

Determine the following and plot the orbit in the orbital plane.

$$a, \quad e, \quad i, \quad \omega, \quad \Omega, \quad r, \quad v, \quad \gamma, \quad \nu, \quad M, \quad E, \quad (t - T)\,.$$