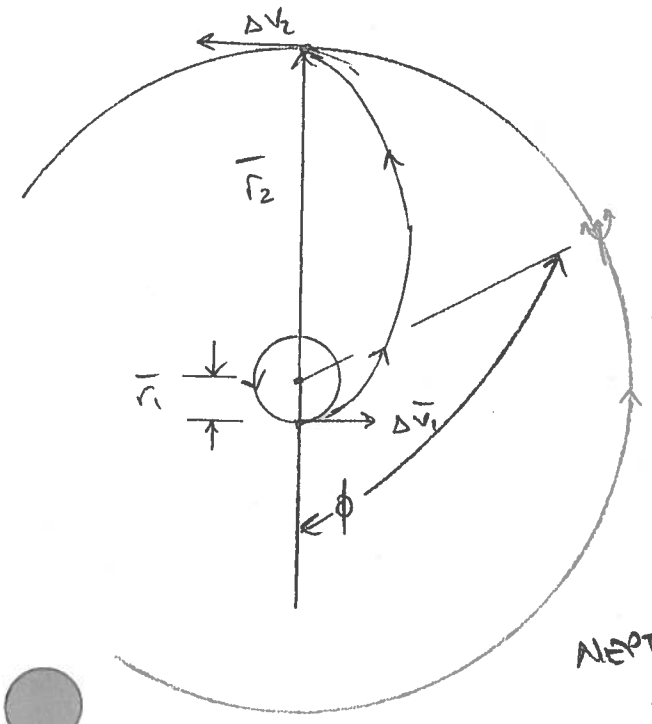
HOMEWORK 6 SOLUTION

PROBLEM 1

EARTH-TO-NEPTUNE HOHMANN TRANSFER

ASSUME: COPLANAR, CIRCULAR, NO LOCAL GRAVITY FIELDS



A. CIRCULAR ORBIT ABOUT THE SUN

EARTH
- $r_1 = 1\ AU$
- $V_1 = \sqrt{\frac{\mu_\odot}{r_1}} = 29.78545\ km/sec$
- $\gamma_1 = 0^\circ$

NEPTUNE
- $r_2 = 30.07057\ AU$
- $V_2 = 5.43167\ km/sec$
- $\gamma_2 = 0^\circ$

TRANSFER ELLIPSE

$a_T = \frac{1}{2}(r_1 + r_2) = 2.323 \times 10^9\ km = 15.535\ AU$

$e_T = 1 - \frac{r_p}{a_T} = 0.935630$

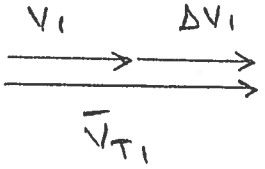
$V_{pT} = 41.439\ km/sec$

$V_{aT} = 1.378\ km/sec$

$\gamma_{T1} = 0^\circ$

$\gamma_{T2} = 0^\circ$

VECTOR DIAGRAM - DEPARTURE MANEUVER

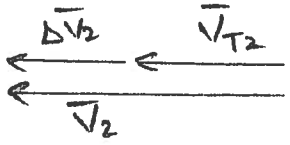


$$\Delta\bar{V}_1 = \bar{V}_{T_1} - \bar{V}_1$$

$$\Delta V_1 = 11.65413 \text{ km/sec}$$

$$\alpha_1 = 0^\circ$$

VECTOR DIAGRAM - ARRIVAL MANEUVER



$$\Delta V_2 = \bar{V}_2 - \bar{V}_{T_2}$$

$$\Delta V_2 = 4.05359 \text{ km/sec}$$

$$\alpha_2 = 0$$

$$TOF = \frac{P}{2} = \frac{2\pi}{2}\sqrt{\frac{a_T^3}{\mu_\odot}} \Rightarrow$$

$$\Delta V_T = 15.707 \text{ km/sec}$$

$$TOF = 9.661 \times 10^8 \text{ sec}$$

$$= 11182.33 \text{ DAYS}$$

$$= 30.63 \text{ YRS}$$

BOTH ΔV_T AND TOF ARE VERY LARGE. TRAVEL TIME IS 30 YEARS! ALSO THE $\Delta V_T \cong$ HALF OF EARTH VELOCITY ABOUT THE SUN!

VOYAGER 2 LAUNCH 8/20/77 - JD 2443376

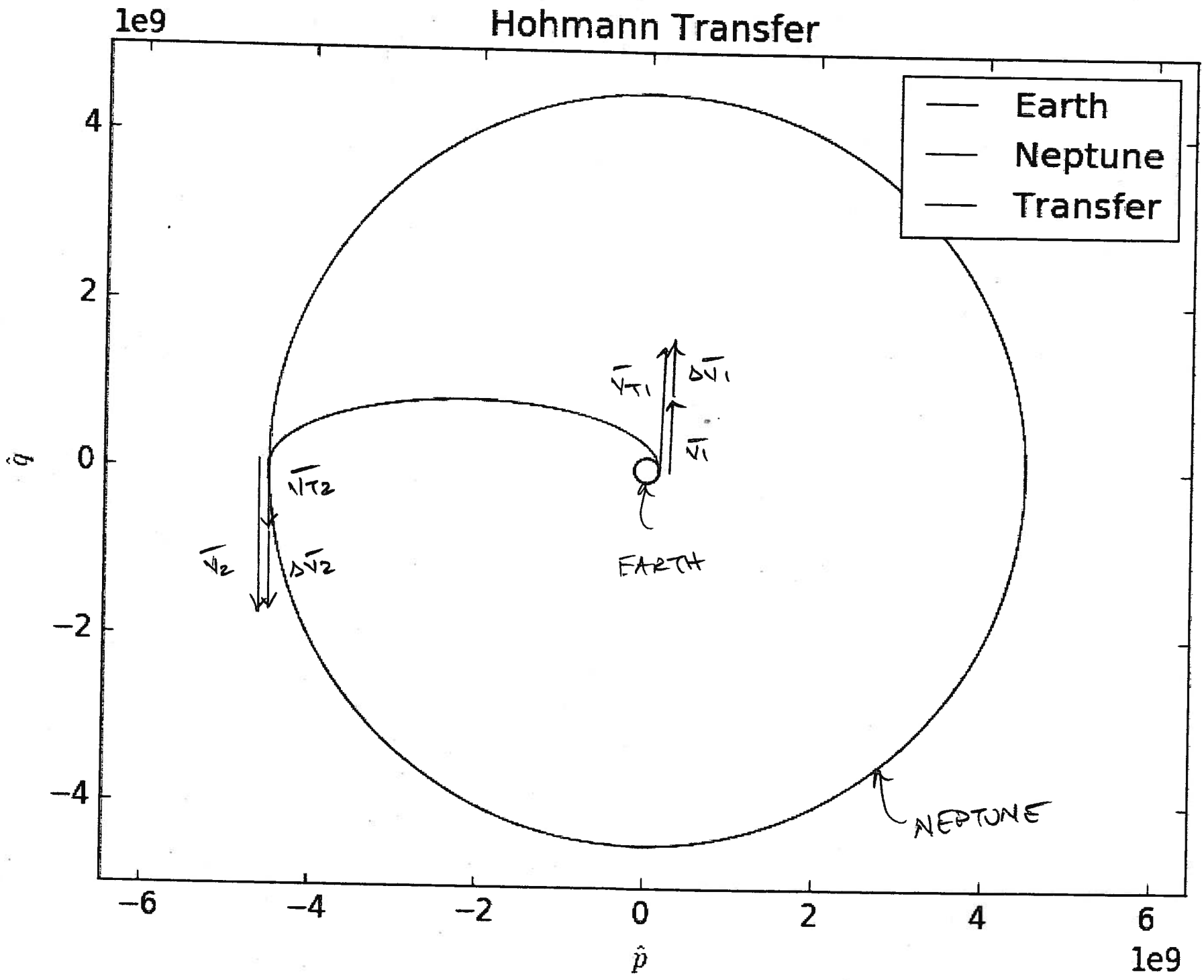
NEPTUNE 8/25/89 - JD 2447764

$$TOF = 4388 \text{ DAYS}$$

$$= 12.02 \text{ YRS}$$

n_Ψ $TOF = 180^\circ - \phi \Rightarrow$ $\phi = 113.153^\circ$

$1.207507 \times 10^{-9} \frac{\text{RAD}}{\text{SEC}}$


Hohmann Transfer
1e9
Earth
Neptune
Transfer
$\bar{V}_{T1}$
$\Delta\bar{V}_1$
$\bar{V}_1$
$\bar{V}_{T2}$
$\bar{V}_2$
$\Delta\bar{V}_2$
EARTH
NEPTUNE
$\hat{q}$
$\hat{p}$
1e9
4
2
0
−2
−4
−6
−4
−2
0
2
4
6

Initial Orbit Velocity : 29.78544591774653 km/sec
Final Orbit Velocity : 5.431669076417027 km/sec
Transfer SMA : 2323919900.0 km Eccentricity : 0.9356303975881441
Transfer Periapsis velocity : 41.43957791013828 km/sec
Transfer Apoapsis velocity : 1.3780777350440716 km/sec
Delta V1 : 11.654131992391754 km/sec
Delta V2 : 4.053591341372963 km/sec
TOF : 966109845.4311681 sec = 11181.826914712594 day = 30.61417362002079 yr
Phase : 113.15962858746643 deg

PROBLEM 2 HOMEWORK 6 SOLUTION

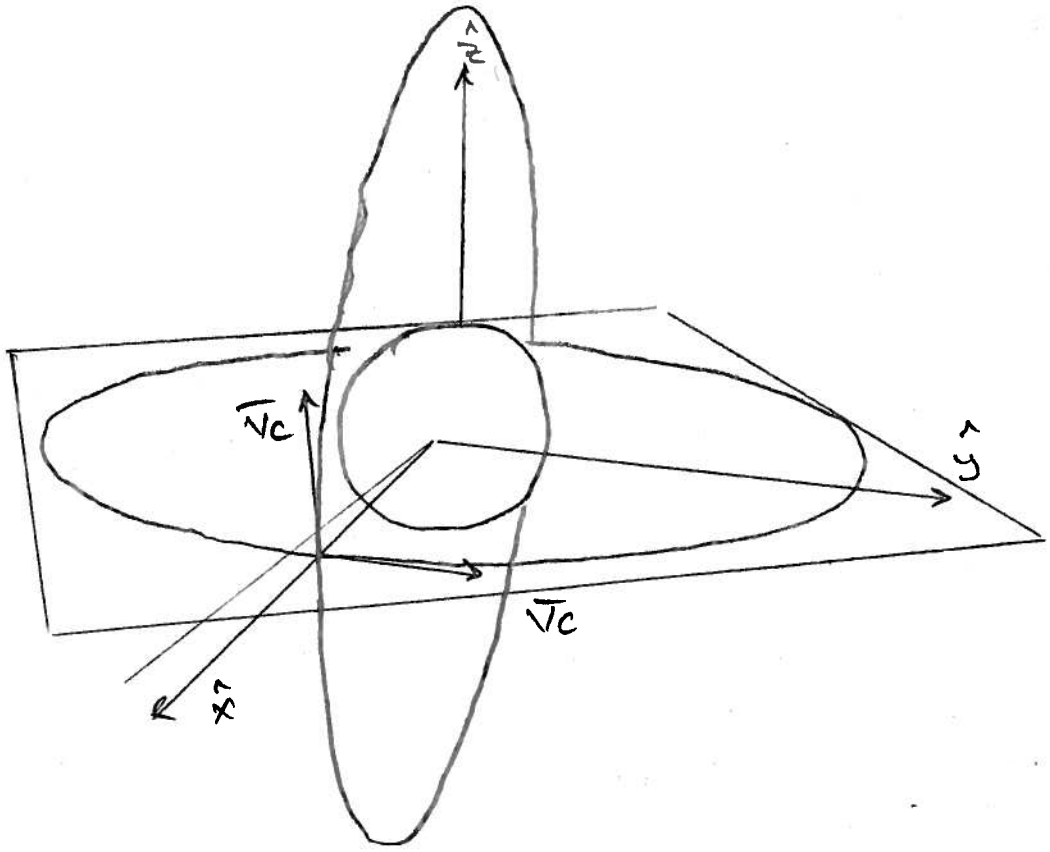
PLANE CHANGE MANEUVERS : LUNAR PLANE CHANGE BY 90°

A. SINGLE MANEUVER

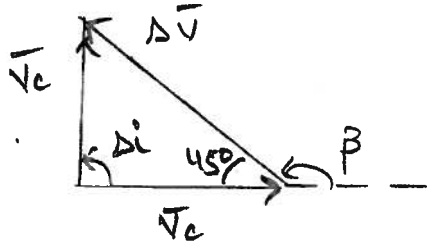
CURRENT LUNAR ORBIT - CIRCULAR $i = 0°$

$r = R_a + 100\ km = 1837.5\ km$

$V_c = \sqrt{\frac{\mu_a}{r}} = 1.63346\ km/sec$



VIEW IN $\hat{y}$-$\hat{z}$ PLANE

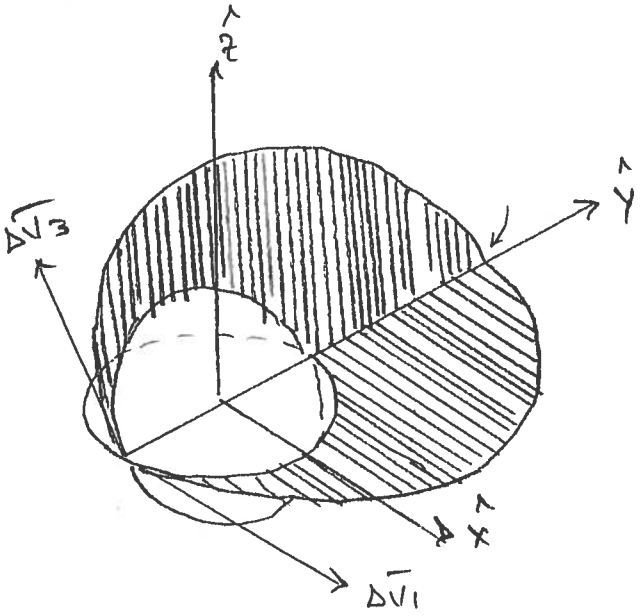


$$\Delta V = 2 V_c \sin \frac{\Delta i}{2}$$

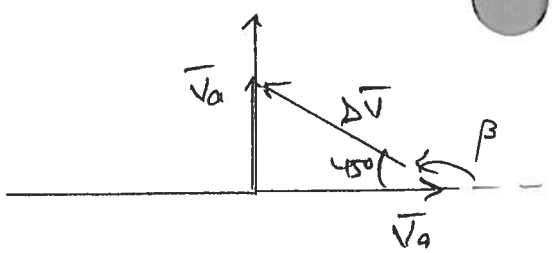
$\Delta V = 2.31006\ \frac{km}{sec}$

$\alpha = 0°$ $\beta = 135°$

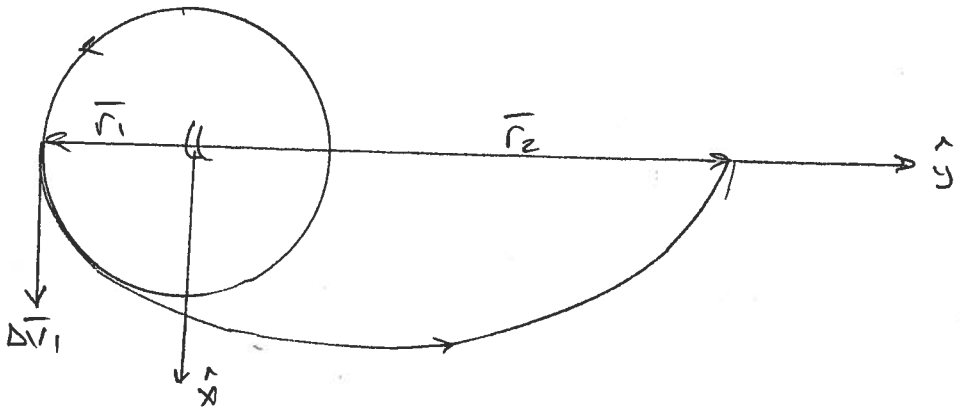
BI-ELLIPTIC MANEUVER



VIEW DOWN $\hat{y}$ - AXIS



MANEUVER #1 — TRANSFER ELLIPSE #1



TRANSFER ELLIPSE #1

$$a_T = \frac{r_1 + r_2}{2} = 9418.75 \text{ km}$$

$V_p = 2.194 \frac{km}{sec}$

$e_T = 0.8049$ $\quad P = \frac{2\pi}{n}$

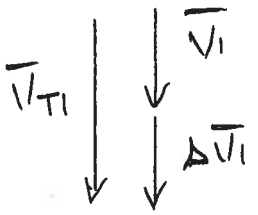
$V_a = 0.237 \frac{km}{sec}$

$P = 82025 \text{ sec}$

$= 22.785 \text{ hr}$

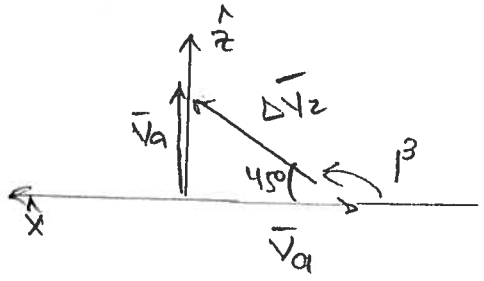
$7.66 \times 10^{-5} \frac{km^3}{sec}$

$\bar{V}_1 + \Delta\bar{V}_1 = \bar{V}_{T1}$



$$\boxed{\Delta V_1 = 0.561 \text{ km/sec} \qquad \alpha_1 = 0° \qquad \beta = 0}$$

MANEUVER #2 – PLANE CHANGE

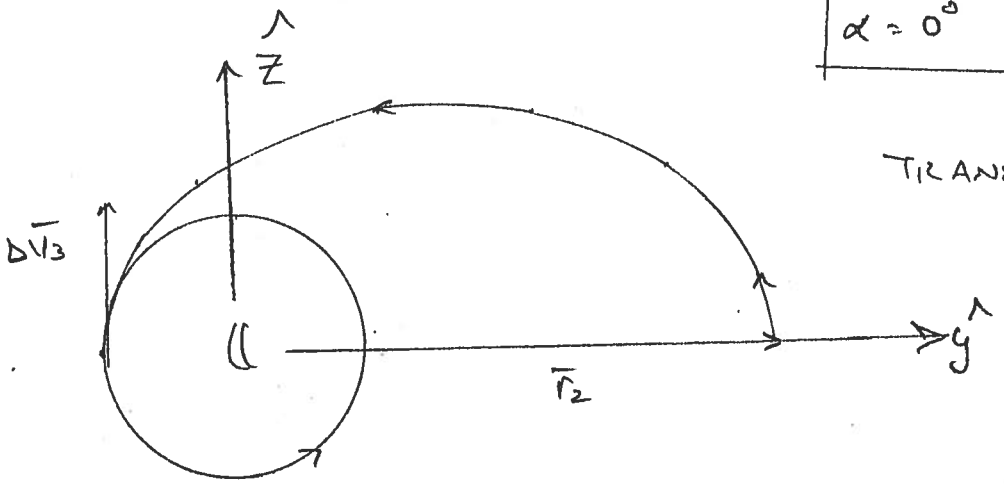


VECTOR DIAGRAM

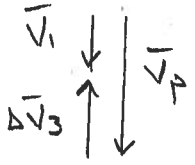
$$\Delta V_2 = 2 V_a \sin \frac{\Delta i}{2}$$

$\Delta V_2 = 0.33545$ km/sec

$\alpha = 0^\circ \qquad \beta = 135^\circ$



TRANSFER ELLIPSE #2



$$\Delta\bar{V}_3 = \bar{V}_1 - \bar{V}_p$$

$\Delta V_3 = 0.56104$ km/sec

$\alpha_3 = 180^\circ \qquad \beta = 0^\circ$

$\Delta V_{TOTAL} = 1.457$ km/sec

TOF = 22.785 hrs

CONSIDERABLE PROPELLANT SAVINGS AT A REASONABLE TIME PENALTY

Simple plane change at current altitude
DV : 2.3100608114032304 km/sec

Bielliptical Plane change
DV1 : 0.5610436476308001 km/sec
DV2 : 0.3354514433852929 km/sec
DV3 : 0.5610436476308001 km/sec
TOF : 82025.27293115648 sec = 22.784798036432356 hr

HOMEWORK 6 SOLUTION

PROBLEM 3

INITIAL ORBIT: $e = 0.4 \quad a = 6 R_\oplus$

MANEUVER: $|\Delta V| = 0.75 \frac{km}{sec} \quad \alpha = -60^\circ \quad @ \quad \nu = 90^\circ$

DEFINE INITIAL ORBIT AT $\nu = 90^\circ$

$$p = a(1-e^2) = 32145.81 \text{ km}$$

$$r = \frac{p}{1+e\cos\nu} = \boxed{32145.81 \text{ km}}$$

$\nu = 90^\circ \rightarrow$ SEMI-LATUS RECTUM

$$\frac{-\mu}{2a} = \frac{v^2}{2} - \frac{\mu}{r} \rightarrow \boxed{v = 3.793 \frac{km}{sec}}$$

$$h = \sqrt{\mu p} \qquad h = rv\cos\gamma \rightarrow \boxed{\gamma = +21.801^\circ}$$

TRANSFORM ΔV TO VNC, LVLH, PQW

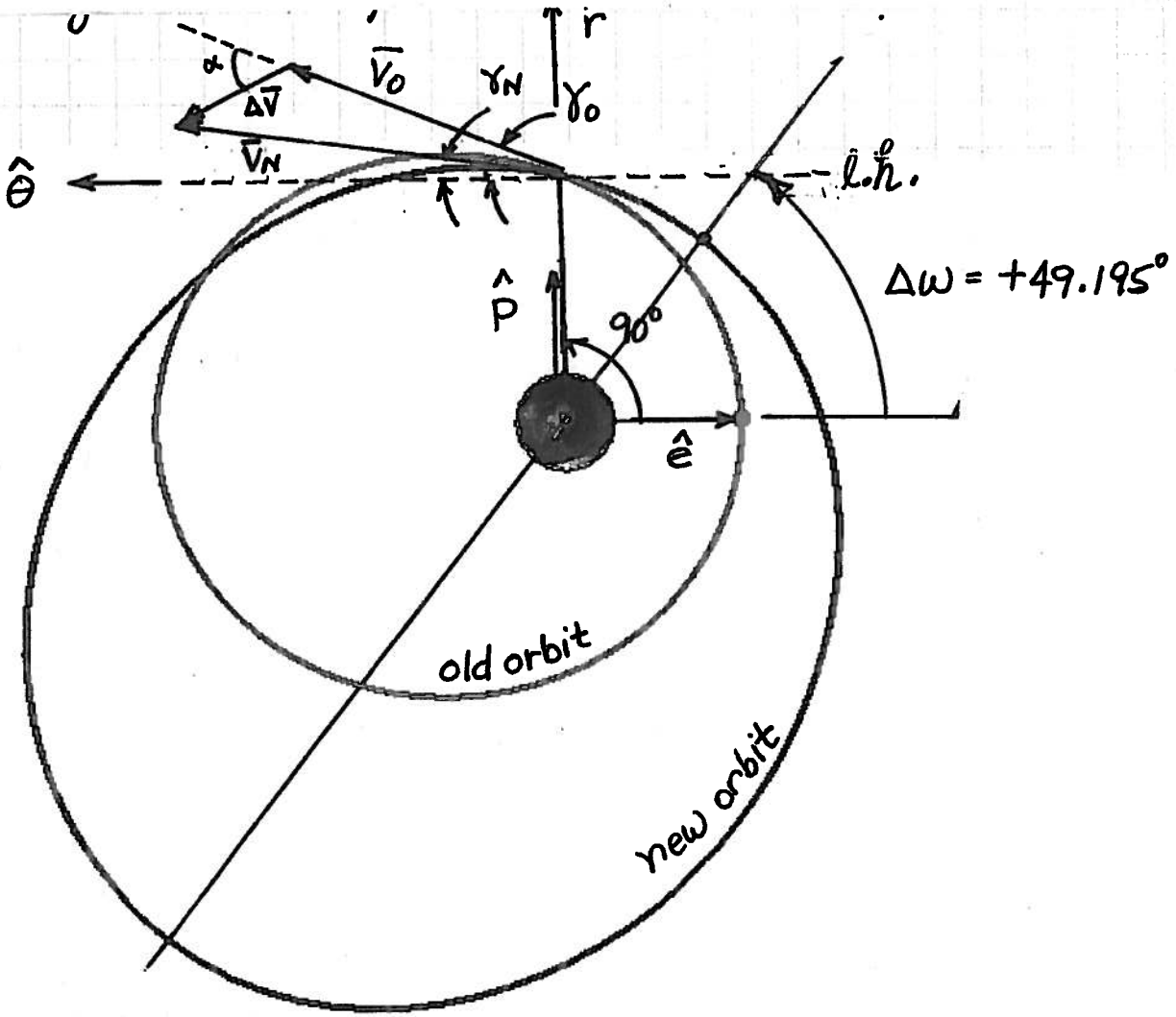
$$R_{LVLH2PQW} = \begin{bmatrix} c\nu & -s\nu & 0 \\ s\nu & c\nu & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Delta\bar{V} = \Delta V(c\beta c\alpha \hat{v} + c\beta s\alpha \hat{c} - s\beta \hat{n}) \qquad VCN$$

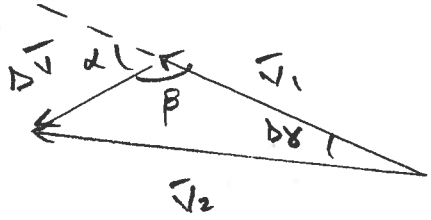
$$= \Delta V(c\beta s\phi \hat{r} + c\beta c\phi \hat{\theta} + s\beta \hat{h}) \qquad LVLH$$

$$\Delta\bar{V} = R_{LVLH2PQW}\, \Delta\bar{V}_{LVLH} \qquad PQW$$

$$\Delta\bar{V} = 0.375 \hat{v} - 0.649 \hat{c}$$
$$= -0.463 \hat{r} + 0.589 \hat{\theta}$$
$$= -0.589 \hat{p} - 0.463 \hat{q} \quad \} \ km/sec.$$



DEFINE NEW ORBIT AFTER MANEUVER (IN PLANE MANEUVER)



$$V_2^2 = \Delta V^2 + V_1^2 - 2 V_1 \Delta V \cos(180 - 60^\circ)$$

$$\boxed{V_2 = 4.21790 \text{ km/sec}}$$

$$\underline{r_2 = r_1 = 5.04\, R_\oplus}$$

$$\frac{\sin \Delta\gamma}{\Delta V} = \frac{\sin\beta}{V_2} \Rightarrow \boxed{\gamma_2 = \gamma_1 + \Delta\gamma = 12.94^\circ}$$

FIND PROPERTIES of NEW ORBIT AFTER MANEUVER

$-\frac{\mu}{2a} = \frac{V^2}{2} - \frac{\mu}{r} \rightarrow a_2 = 56970.93 \text{ km}$ $\quad \varepsilon = -3.504 \frac{\text{km}^2}{\text{sec}^2}$

$h = \sqrt{\mu p} \quad p = a(1-e^2) \rightarrow p = 43807.62 \text{ km}$

$P = 2\pi\sqrt{\frac{a^3}{\mu}} \rightarrow P = 37.492 \text{ hr}$

$r_p = a(1-e) \rightarrow r_p = 29614 \text{ km}$

$r_a = a(1+e) \quad r_a = 84127.5 \text{ km}$

$r = \frac{p}{1+e\cos\theta} \rightarrow \theta_2 = 40.805^\circ$

$r = a(1-e\cos E) \rightarrow E_2 = 24.989^\circ$

$n(t-T) = E_2 - e\sin E_2 \rightarrow (t-T) = 1.388 \text{ hr}$

$\Delta w = \theta_1 - \theta_2 = +49.195^\circ$

PART D

TRAVEL FROM $\nu = 90°$ TO $E = 270°$ BEFORE MANEUVER

FIND TOF FROM $\nu = 90°$ TO

$\nu_1 \rightarrow E_1 \rightarrow M_1$
$\nu_2 \rightarrow E_2 \rightarrow M_2$ } Δt $\quad \Delta t = M_2 - M_1$

$M_1 = 45.417°$ $\quad M_2 = 292.918°$ $\rightarrow$ $\boxed{\Delta t = 15.474 \text{ hr}}$

PROPERTIES AT $E = 270°$ (INITIAL ORBIT)

$a_1 = 6R_\oplus$ $\quad e = 0.4$

$E = 270 \rightarrow \nu_1 = 246.4218°$

$r_1 = 38268.8$ km
$V_1 = 3.2273$ km/sec
$\gamma_1 = -23.578°$

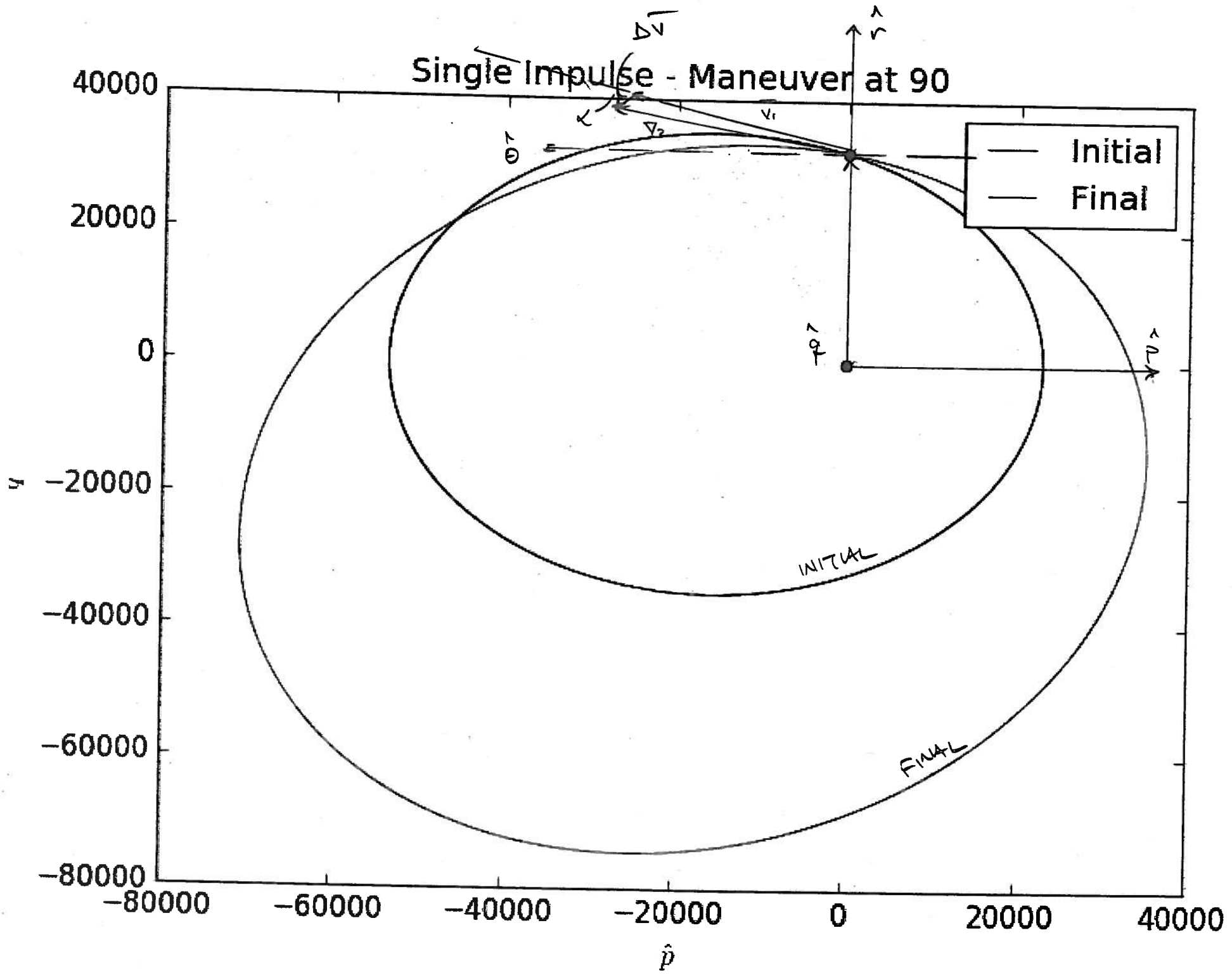
$$\sin\nu = \frac{\sin E \sqrt{1-e^2}}{1 - e\cos E} \qquad \cos\nu = \frac{\cos E - e}{1 - e\cos E}$$

CONVERT ΔV TO VNC, LVLH, POW

$\Delta \bar{V} = 0.375 \hat{V} - 0.649 \hat{C}$ km/sec

$= -0.745 \hat{r} + 0.0838 \hat{\theta}$ km/sec

$= 0.375 \hat{\rho} + 0.649 \hat{\phi}$ km/sec


Single Impulse - Maneuver at 90
Initial
Final
INITIAL
FINAL
40000
20000
0
−20000
−40000
−60000
−80000
−80000
−60000
−40000
−20000
0
20000
40000
$\hat{p}$
$\hat{r}$
$\hat{q}$
$\hat{\theta}$
$\Delta\vec{V}$
V_1
V_2

Initial Orbit

Satellite State
Position and Velocity in LVLH frame
r_hat: 32145.81048 km rd_hat: 1.40853200938761 km/sec
t_hat: 0 km td_hat: 3.52133002346901 km/sec
h_hat: 0 km hd_hat: 0 km/sec

Position and Velocity in EPH/PQW frame
e_hat: 1.96836319551647e-12 km ed_hat: -3.52133002346901 km/sec
p_hat: 32145.81048 km pd_hat: 1.40853200938761 km/sec
h_hat: 0 km hd_hat: 0 km/sec

Position and Velocity in IJK frame
i_hat: 1.96836319551647e-12 km id_hat: -3.52133002346901 km/sec
j_hat: 32145.81048 km jd_hat: 1.40853200938761 km/sec
k_hat: 0 km kd_hat: 0 km/sec

RAD_MAG : 32145.81048 km = 0.000214881471774966 AU
VEL_MAG : 3.79258850333829 km/sec

Orbital Elements
sma: 38268.822 km raan: 0 deg
ecc: 0.4 arg_p: 0 deg
inc: 0 deg nu: 90 deg

Elliptic Orbital Parameters
P : 32145.81048 km = 0.000214881471774966 AU
ANG MOM : 113196.007571969 km^2/sec
PERIOD : 74503.8283888311 sec = 20.6955078857864 hr
ENGERGY : -5.2079013563574 km^2/sec^2
RAD_PER : 22961.2932 km = 0.000153486765553547 AU
RAD_APO : 53576.3508 km = 0.00035813578629161 AU

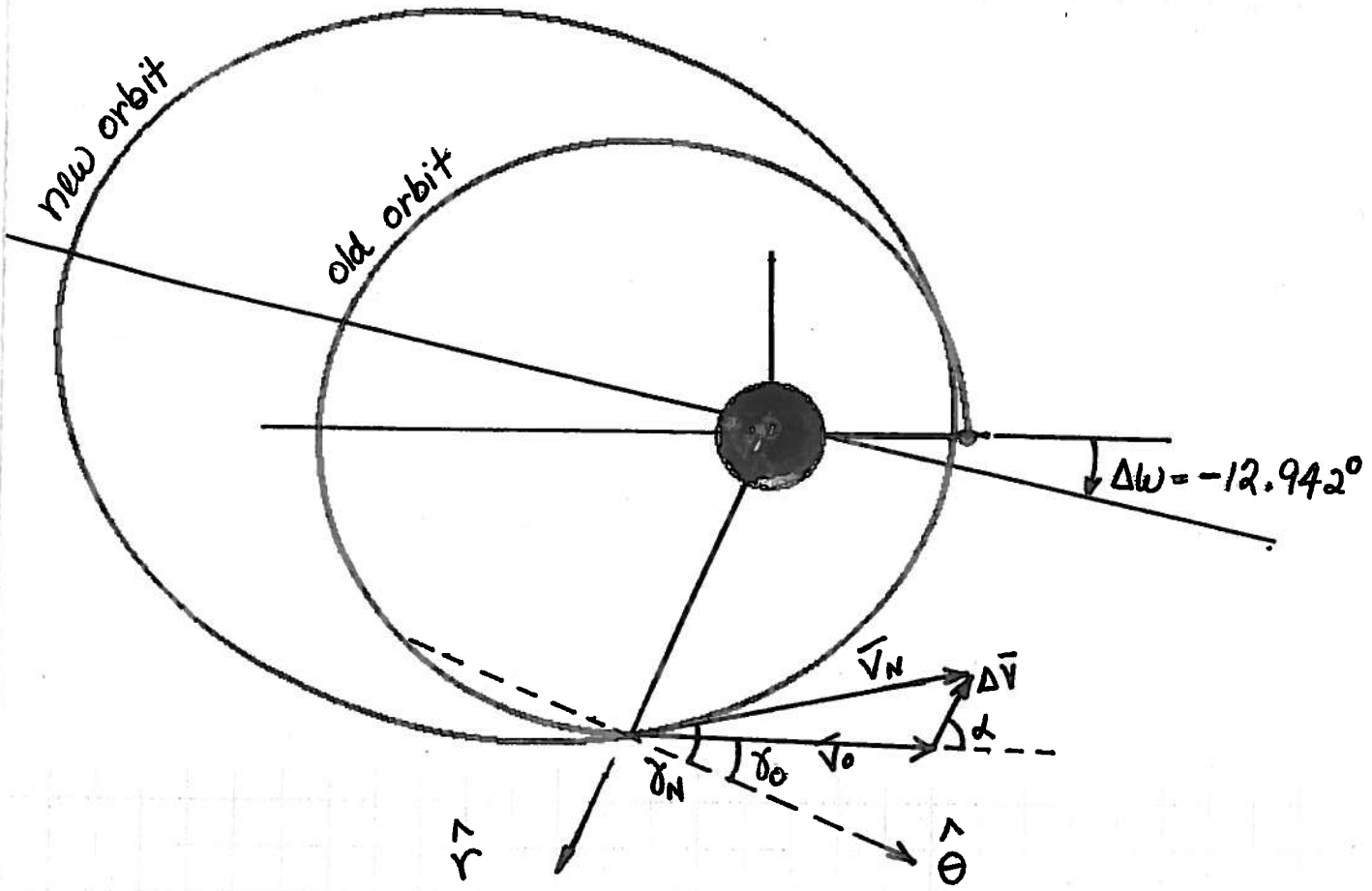
VEL_CIRC : 3.52133002346901 km/sec
VEL_ESC : 4.97991267678145 km/sec
TRUE_ANOM : 90 deg
FPA : 21.8014094863518 deg
ECC_ANOM : 66.4218215217982 deg
MEAN_ANOM : 45.4168417921822 deg
MEAN_MOT : 0.00483196646112172 deg/sec

T_PAST_PER: 9399.2460745762 sec = 2.61090168738228 hr

DV : 0.3750000000000001 V -0.649519052838329 C 0.0 N km/sec
DV : -0.4637917972134706 R 0.5894040794204761 T 0.0 H km/sec
DV : -0.5894040794204761 P -0.4637917972134706 Q 0.0 W km/sec
V2 : 4.217898639507296 km/sec
FPAf : 12.94311865525731 deg
Final orbit after maneuver

Satellite State

VECTOR DIAGRAM → SAME AS BEFORE !



$V_2 = 3.6$ km/sec

$r_2 = 5.04\ 12\theta$

$\gamma_2 = -33.79°$

$\theta_2 = 8.41\ 2\theta$

$e_2 = 0.605$

$\nu_2 = 259.36°$

$E_2 = 298.25°$

$(t_2 - T) = 31.35$ hr

$P = 34.33$ hr

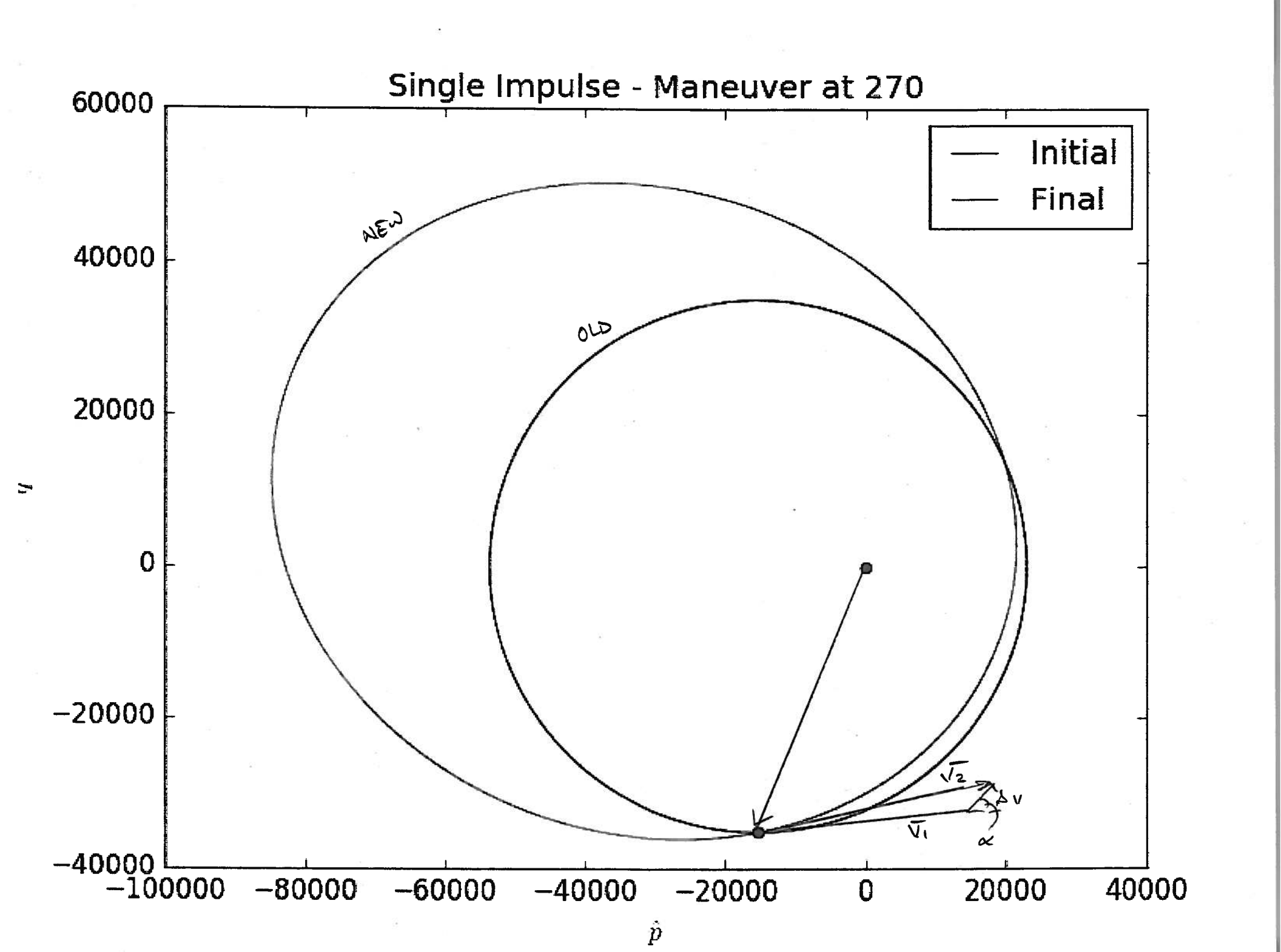
$\mathcal{E} = -3.716\ \frac{km^2}{sec^2}$

$p = 33994.95$ km

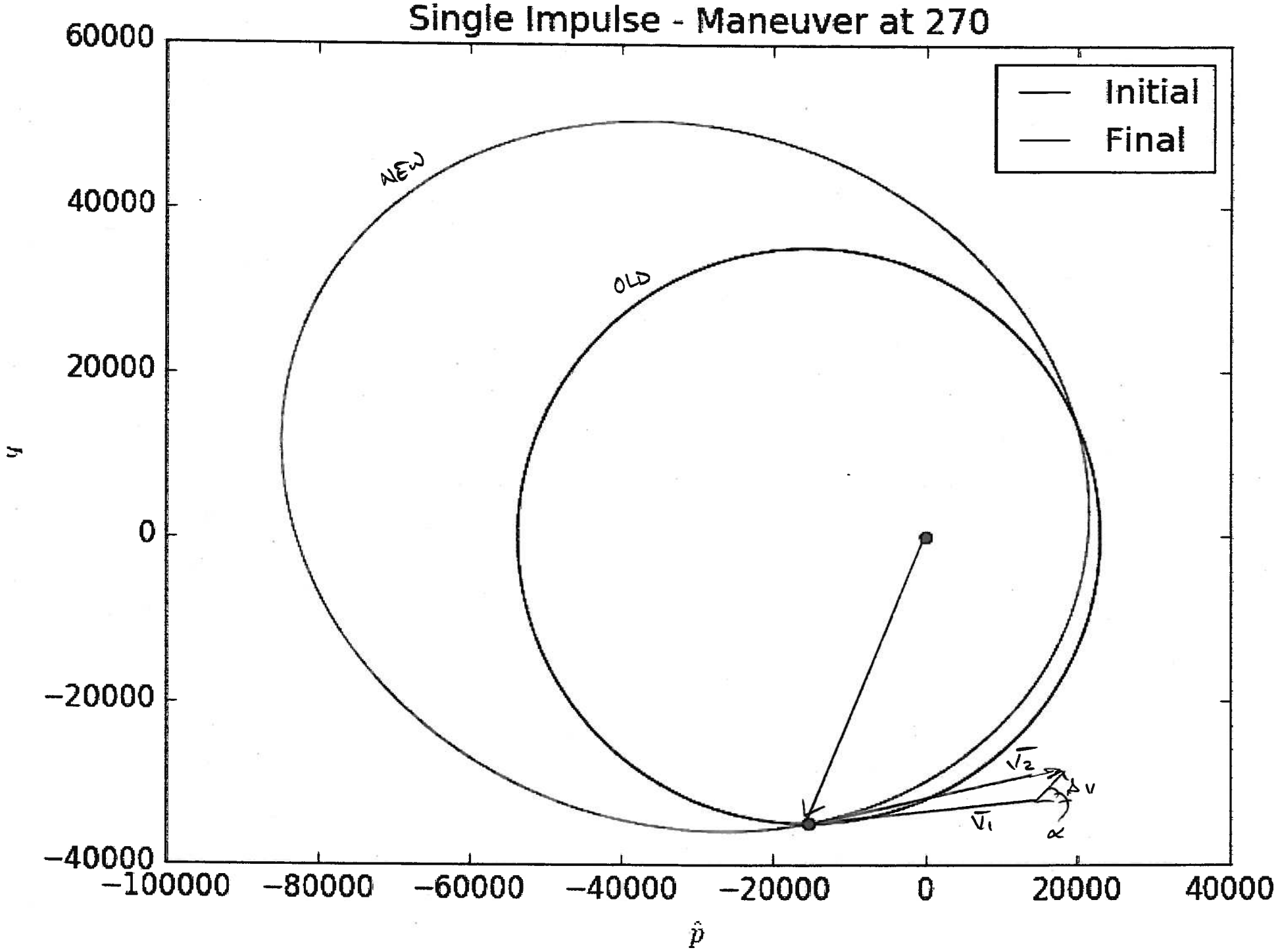
$r_p = 21179.9$ km

$r_a = 86074.67$ km

$\Delta\omega = -12.942$


Single Impulse - Maneuver at 270
Initial
Final
NEW
OLD
60000
40000
20000
0
−20000
−40000
−100000
−80000
−60000
−40000
−20000
0
20000
40000
$\dot{h}$
$\dot{p}$
$\bar{V}_2$
ΔV
$\bar{V}_1$
α

```
e_hat:          -7063.6292443128 km ed_hat:          3.3653840794258 km/sec
p_hat:         -37611.2732989269 km pd_hat:         1.43979405387792 km/sec
h_hat:                         0 km hd_hat:                        0 km/sec

Position and Velocity in IJK frame
i_hat:          -7063.6292443128 km id_hat:          3.3653840794258 km/sec
j_hat:         -37611.2732989269 km jd_hat:         1.43979405387792 km/sec
k_hat:                         0 km kd_hat:                        0 km/sec

RAD_MAG   :             38268.822 km = 0.000255811275922578 AU
VEL_MAG   :      3.66043944351425 km/sec

Orbital Elements
sma:            53627.3162742685 km raan:                          0 deg
ecc:           0.605052768804427    arg_p:                         0 deg
inc:                           0 deg nu:          259.363399105571 deg

Elliptic Orbital Parameters
P          :      33994.9535679206 km =  0.00022724222990555 AU
ANG MOM    :      116406.208982382 km^2/sec
PERIOD     :      123591.910108743 sec =     34.3310861413174 hr
ENGERGY    :     -3.71639425289735 km^2/sec^2
RAD_PER    :      21179.9600789716 km = 0.000141579289056533 AU
RAD_APO    :      86074.6724695654 km = 0.000575373649835826 AU

VEL_CIRC   :      3.22735227589348 km/sec
VEL_ESC    :      4.56416535912423 km/sec
TRUE_ANOM  :      259.363399105571 deg
FPA        :     -33.799030727331 deg
ECC_ANOM   :      298.251047270564 deg
MEAN_ANOM  :      328.788560170234 deg
MEAN_MOT   :  0.00291281200916187 deg/sec

T_PAST_PER:       112876.68381484 sec =     31.3546343930112 hr
```


Single Impulse - Maneuver at 270
Initial
Final
NEW
OLD
60000
40000
20000
0
−20000
−40000
−100000
−80000
−60000
−40000
−20000
0
20000
40000
$\hat{y}$
$\hat{p}$
$\bar{V}_2$
ΔV
$\bar{V}_1$
α

```
Position and Velocity in LVLH frame
r_hat:              32145.81048 km rd_hat:       0.944740212174134 km/sec
t_hat:                         0 km td_hat:       4.11073410288949 km/sec
h_hat:                         0 km hd_hat:                      0 km/sec

Position and Velocity in EPH/PQW frame
e_hat:         24332.3808099028 km ed_hat:       -1.97120111888527 km/sec
p_hat:         21006.8649621488 km pd_hat:        3.72894557241896 km/sec
h_hat:                         0 km hd_hat:                      0 km/sec

Position and Velocity in IJK frame
i_hat:         24332.3808099028 km id_hat:       -1.97120111888527 km/sec
j_hat:         21006.8649621488 km jd_hat:        3.72894557241896 km/sec
k_hat:                         0 km kd_hat:                      0 km/sec

RAD_MAG   :           32145.81048 km = 0.000214881471774966 AU
VEL_MAG   :      4.2178986395073 km/sec

Orbital Elements
sma:           56870.9353682763 km raan:                        0 deg
ecc:          0.479271345335067    arg_p:                       0 deg
inc:                           0 deg nu:          40.8050154970124 deg

Elliptic Orbital Parameters
P         :     43807.6233659673 km = 0.000292835876379572 AU
ANG MOM   :     132142.879405158 km^2/sec
PERIOD    :     134972.877159369 sec =     37.4924658776024 hr
ENGERGY   :    -3.50443066760554 km^2/sec^2
RAD_PER   :     29614.3256638588 km = 0.000197959540893589 AU
RAD_APO   :     84127.5450726937 km = 0.000562357907052378 AU

VEL_CIRC  :     3.52133002346902 km/sec
VEL_ESC   :     4.97991267678145 km/sec
TRUE_ANOM :     40.8050154970124 deg
FPA       :     12.9431186552573 deg
ECC_ANOM  :     24.8891127500627 deg
MEAN_ANOM :     13.3321075595306 deg
MEAN_MOT  :  0.00266720253414271 deg/sec

T_PAST_PER:     4998.53587752225 sec =     1.38848218820063 hr


--------------------MANEUVER AT 270----------------

TOF from 90 to 270 : [ 55705.33623968] sec = [ 15.47370451] hr
Initial Orbit

Satellite State
Position and Velocity in LVLH frame
r_hat:              38268.822 km rd_hat:       -1.29094091035739 km/sec
t_hat:                         0 km td_hat:       2.95791721971397 km/sec
```

```
h_hat:                          0 km hd_hat:                          0 km/sec

Position and Velocity in EPH/PQW frame
e_hat:               -15307.5288 km ed_hat:          3.22735227589348 km/sec
p_hat:         -35073.9547143583 km pd_hat:     -2.7366231685514e-15 km/sec
h_hat:                          0 km hd_hat:                          0 km/sec

Position and Velocity in IJK frame
i_hat:               -15307.5288 km id_hat:          3.22735227589348 km/sec
j_hat:         -35073.9547143583 km jd_hat:     -2.7366231685514e-15 km/sec
k_hat:                          0 km kd_hat:                          0 km/sec

RAD_MAG   :             38268.822 km = 0.000255811275922578 AU
VEL_MAG   :     3.22735227589348 km/sec

Orbital Elements
sma:                    38268.822 km raan:                          0 deg
ecc:                          0.4    arg_p:                          0 deg
inc:                            0 deg nu:             246.421821521798 deg

Elliptic Orbital Parameters
P         :           32145.81048 km = 0.000214881471774966 AU
ANG MOM   :     113196.007571969 km^2/sec
PERIOD    :     74503.8283888311 sec =     20.6955078857864 hr
ENGERGY   :     -5.2079013563574 km^2/sec^2
RAD_PER   :           22961.2932 km = 0.000153486765553547 AU
RAD_APO   :           53576.3508 km =  0.00035813578629161 AU

VEL_CIRC  :     3.22735227589348 km/sec
VEL_ESC   :     4.56416535912423 km/sec
TRUE_ANOM :     246.421821521798 deg
FPA       :    -23.5781784782018 deg
ECC_ANOM  :                  270 deg
MEAN_ANOM :     292.918311805233 deg
MEAN_MOT  :  0.00483196646112172 deg/sec

T_PAST_PER:     60620.9323185644 sec =     16.8391478662679 hr

DV : 0.3750000000000001 V -0.649519052838329 C 0.0 N km/sec
DV : -0.7452940449895329 R 0.08388555598635647 T 0.0 H km/sec
DV : 0.37500000000000067 P 0.6495190528383287 Q 0.0 W km/sec
V2 : 3.6604394435142487 km/sec
FPAf : -33.79903072733104 deg
Final orbit after maneuver

Satellite State
Position and Velocity in LVLH frame
r_hat:                  38268.822 km rd_hat:         -2.03623495534692 km/sec
t_hat:                          0 km td_hat:          3.04180277570033 km/sec
h_hat:                          0 km hd_hat:                          0 km/sec

Position and Velocity in EPH/PQW frame
```

```
e_hat:          -7063.6292443128 km ed_hat:          3.3653840794258 km/sec
p_hat:         -37611.2732989269 km pd_hat:         1.43979405387792 km/sec
h_hat:                         0 km hd_hat:                        0 km/sec

Position and Velocity in IJK frame
i_hat:          -7063.6292443128 km id_hat:          3.3653840794258 km/sec
j_hat:         -37611.2732989269 km jd_hat:         1.43979405387792 km/sec
k_hat:                         0 km kd_hat:                        0 km/sec

RAD_MAG   :             38268.822 km = 0.000255811275922578 AU
VEL_MAG   :      3.66043944351425 km/sec

Orbital Elements
sma:            53627.3162742685 km raan:                          0 deg
ecc:           0.605052768804427    arg_p:                         0 deg
inc:                           0 deg nu:            259.363399105571 deg

Elliptic Orbital Parameters
P          :      33994.9535679206 km =  0.00022724222990555 AU
ANG MOM    :      116406.208982382 km^2/sec
PERIOD     :      123591.910108743 sec =     34.3310861413174 hr
ENGERGY    :     -3.71639425289735 km^2/sec^2
RAD_PER    :      21179.9600789716 km = 0.000141579289056533 AU
RAD_APO    :      86074.6724695654 km = 0.000575373649835826 AU

VEL_CIRC   :      3.22735227589348 km/sec
VEL_ESC    :      4.56416535912423 km/sec
TRUE_ANOM  :      259.363399105571 deg
FPA        :     -33.799030727331 deg
ECC_ANOM   :      298.251047270564 deg
MEAN_ANOM  :      328.788560170234 deg
MEAN_MOT   :  0.00291281200916187 deg/sec

T_PAST_PER:       112876.68381484 sec =     31.3546343930112 hr
```

HOMEWORK 6 SOLUTION

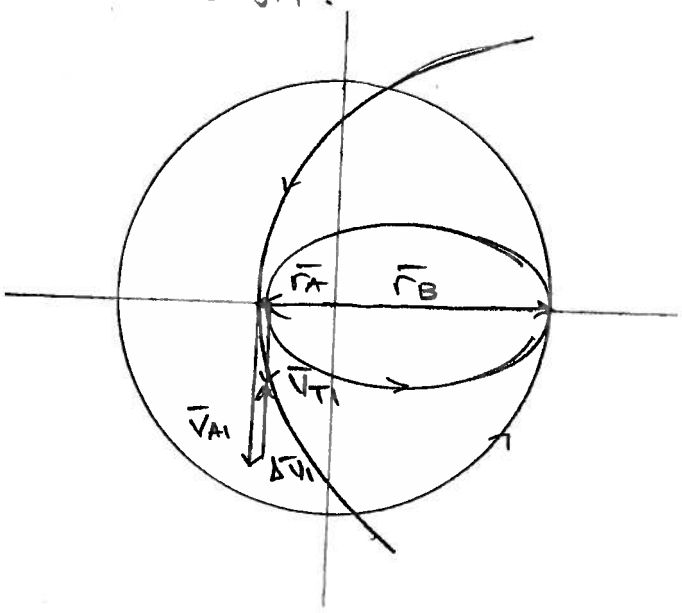
PROBLEM 4

TRANSFER FROM HYPERBOLIC ORBIT TO CIRCULAR ORBIT, THEN RENDEZVOUS WITH ISS IN CIRCULAR ORBIT.

$r_A = 7000$ km $r_B = 14000$ km $V_{A1} = 12$ km/sec $\mu = 398600 \frac{km^3}{sec^2}$

PART A

HOHMANN TRANSFER FROM HYPERBOLIC ORBIT TO CIRCULAR ORBIT.



TRANSFER ELLIPSE

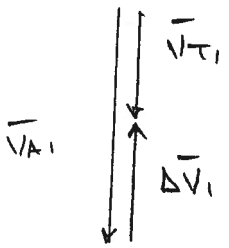
$$a_T = \frac{1}{2}(r_A + r_B) = 10499.9 \text{ km}$$

$$r_p = a(1 - e_T) \rightarrow e_T = 0.\overline{33}$$

VELOCITY IN TRANSFER ELLIPSE AT r_A

$$-\frac{\mu}{2a_T} = \frac{V_{T1}^2}{2} - \frac{\mu}{r_A}$$

$$V_{T1} = 8.713 \text{ km/sec}$$



$$\Delta V_1 = V_{A1} - V_{T1}$$

$\alpha = 180°$

$$\Delta V_1 = 3.2865 \text{ km/sec}$$

PART B

TA = 180° ⟶ TOF IS $\frac{P_T}{2} = \pi\sqrt{\frac{a_T^3}{\mu}}$

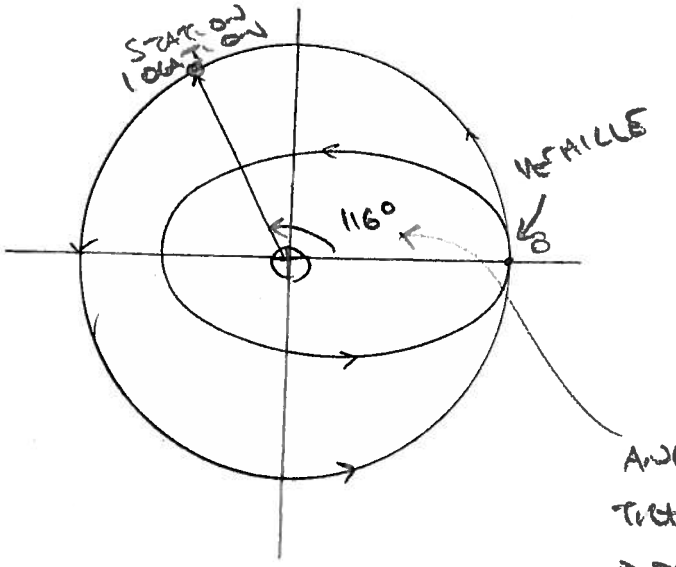
$$\boxed{TOF = 5353.83 \text{ sec}}$$

PART C

S.S STARTS AT POINT B

AFTER TOF FROM PART B IT WILL MOVE THROUGH AN ANGLE $\theta = n_2 \, TOF = 116°$

$$n_2 = \sqrt{\frac{\mu}{a_2^3}} = 3.811 \times 10^{-4} \frac{\text{rad}}{\text{sec}} \qquad P_2 = 2\pi\sqrt{\frac{a_2^3}{\mu}} = 16485.53 \text{ sec}$$



TIME FOR STATION TO RETURN TO B

$P_2 - TOF = 11131.7$ sec

↑ PERIOD OF PHASING ORBIT

ANGLE TRAVELLED DURING HOHMANN

PART D

$$P_p = 2\pi\sqrt{\frac{a_p^3}{\mu}} \longrightarrow a_p = 10775.406 \text{ km}$$

$$a_p = \frac{1}{2}(r_c + r_B) \longrightarrow r_c = 7550.8 \text{ km}$$

PART E

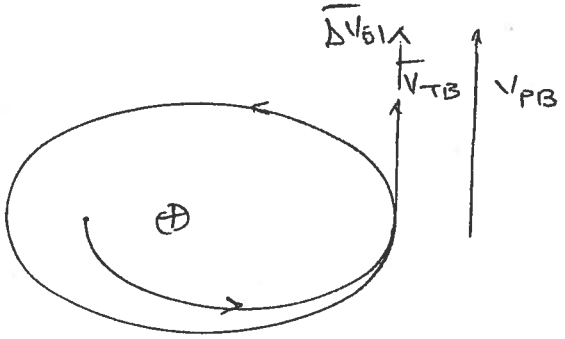
TRANSFER FROM HOHMANN ELLIPSE TO PHASING ORBIT.

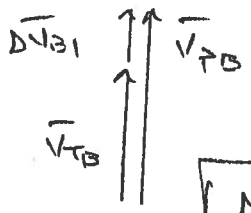
$$-\frac{\mu}{2a_P} = \frac{V_{PB}^2}{2} - \frac{\mu}{r_B} \Rightarrow V_{PB} = 4.46 \text{ km/sec}$$

VELOCITY AT B IN PHASING ORBIT

$$-\frac{\mu}{2a_T} = \frac{V_{TB}^2}{2} - \frac{\mu}{r_B} \Rightarrow V_{TB} = 4.3567 \text{ km/sec}$$

VEL AT B IN HOHMANN ELLIPSE



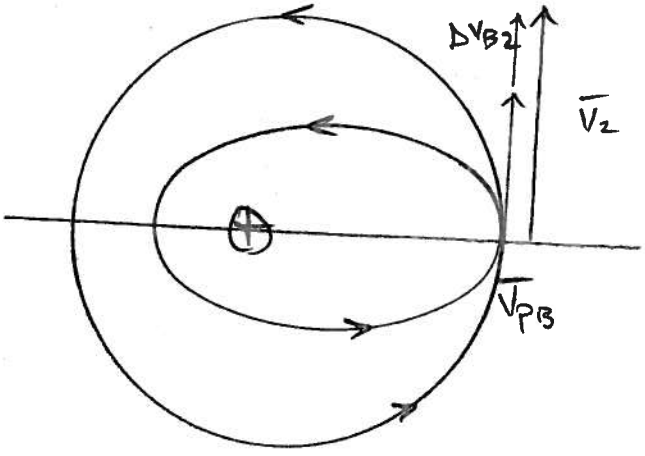


$\alpha = 0^\circ$

$$\Delta V_{B_1} = 0.109 \frac{\text{km}}{\text{sec}}$$

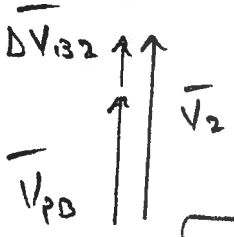
PART F

TRANSFER FROM PHASING TO FINAL ORBIT + RENDEZVOUS



$$V_2 = \sqrt{\frac{\mu}{r_B}} = 5.33 \text{ km/sec}$$

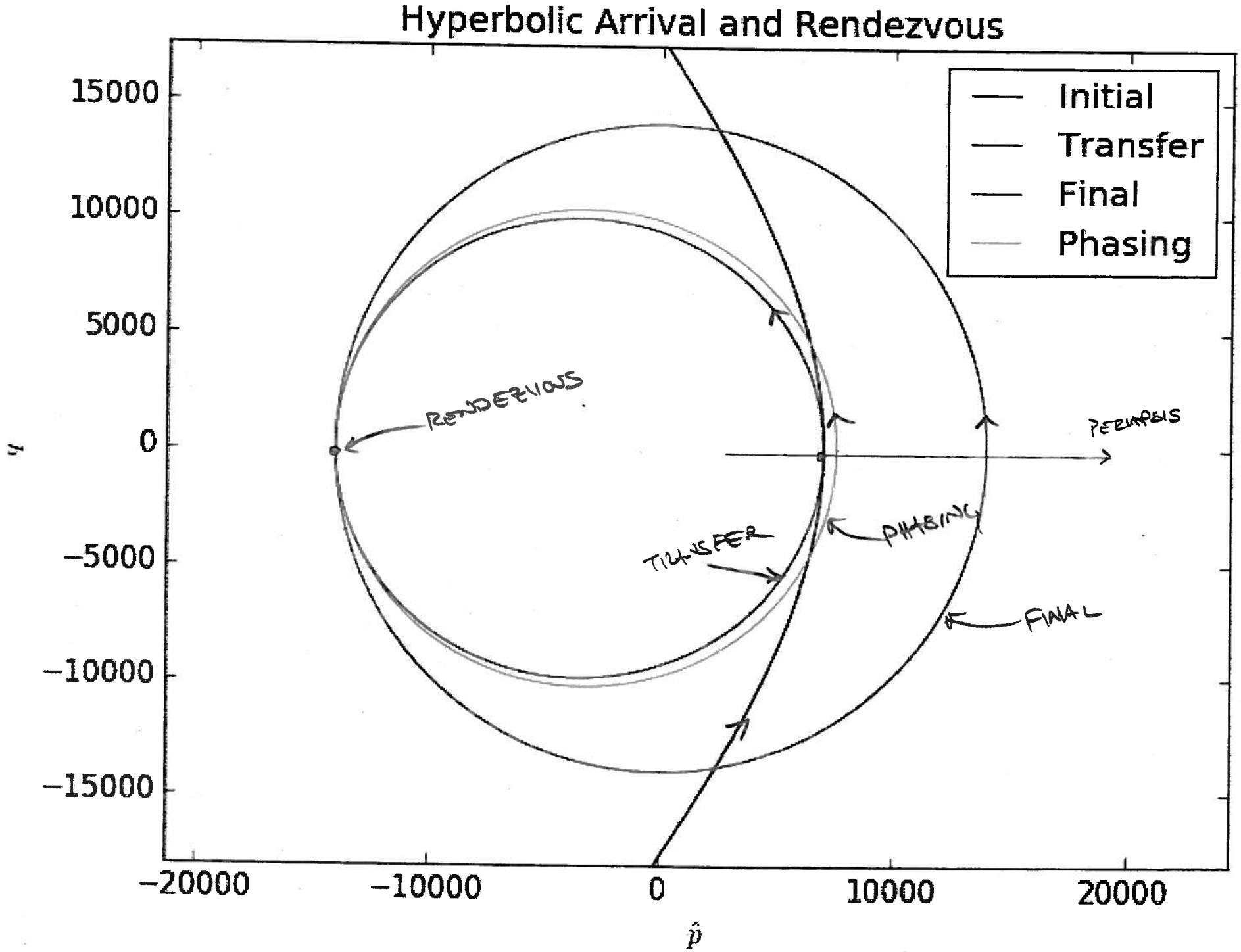
$V_{PB} = 4.46$ km/sec



$\alpha = 0^\circ$

$$\Delta V_{B_2} = 0.869 \text{ km/sec}$$

TOTAL $\Delta V = \Delta V_1 + \Delta V_{B_1} + \Delta V_2 = 4.2657$ km/sec


Hyperbolic Arrival and Rendezvous
Initial
Transfer
Final
Phasing
RENDEZVOUS
PERIAPSIS
TRANSFER
PHASING
FINAL
15000
10000
5000
0
−5000
−10000
−15000
−20000
−10000
0
10000
20000
$\hat{p}$
$\hat{q}$

Transfer ellipse : a = 10499.999999999998 km, e = 0.3333333333333333
VT1 : 8.713432432853587 km/sec
V1 : 12 km/sec
DV1 : 3.286567567146413 km/sec
TOF : 5353.834004010886 sec = 1.487176112225246 hr

Phasing Orbit
Period Phasing : 11131.699347521157 sec
Phasing orbit : a = 10775.406915076717 km, ecc = 0.29925487829248487

Transfer from hohmann ellipse to phasing orbit
DV2 : 0.10996484517633398 km/sec

Transfer from phasing orbit to final orbit
DV3 : 0.869184780574157 km/sec

HOMEWORK 6 SOLUTION

PROBLEM 5

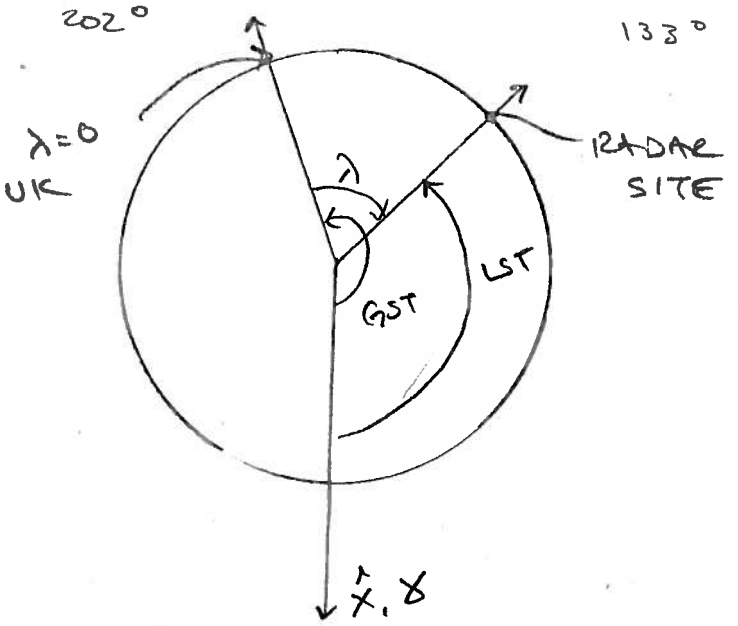
RADAR $\lambda = -68.5$ $\phi = 77.7°$ $h = 0.050$ km

JD 2454154.6376157

FIND: GST, LST. USING time.gstlst(jd)

gst, lst = time.gstlst(jd)

GST = 3.5287 rad LST = 2.332 rad



PROBLEM 6

CONVERT SITE LOCATION. TO ECI + ECEF.

SITE ↓

$$\vec{r}_{ECEF} = \begin{bmatrix} (N+H)\cos\lambda\cos\phi \\ (N+H)\sin\lambda\cos\phi \\ (N(1-e^2)+H)\sin\phi \end{bmatrix} = \begin{bmatrix} 499.5815 \\ -1268.2615 \\ 6209.935 \end{bmatrix} \begin{matrix} \hat{X} \\ \hat{Y} \\ \hat{Z} \end{matrix} \text{ km}$$

$$N = \frac{a_e}{1 - e^2 \sin^2\phi}$$

$$R_{ECEF2ECI} = \begin{bmatrix} -0.925 & 0.377 & 0 \\ -0.377 & -0.925 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\bar{r}_{ECI} = R\,\bar{r}_{ECEF} = -941.485\,\hat{i} + 985.735\,\hat{j} + 6209.935\,\hat{k} \text{ km}$$

PROBLEM 7

FIND $\rho, \dot{\rho}$ IN ECI. FIRST FIND IN SEZ THEN ROTATE TO ECI

$$\bar{\rho}_{SEZ} = \begin{bmatrix} -\rho\cos\alpha\cos\beta \\ \rho\sin\alpha\cos\beta \\ \rho\sin\beta \end{bmatrix} = \begin{bmatrix} -1636.402 \\ 562.20 \\ 1227.409 \end{bmatrix} \text{ km}$$

$$\dot{\bar{\rho}}_{SEZ} = \begin{bmatrix} -\dot{\rho}\cos\alpha\cos\beta + \rho\dot{\alpha}\sin\alpha\cos\beta + \rho\dot{\beta}\cos\alpha\sin\beta \\ \dot{\rho}\sin\alpha\cos\beta + \rho\dot{\alpha}\cos\alpha\cos\beta - \rho\dot{\beta}\sin\alpha\sin\beta \\ \dot{\rho}\sin\beta + \rho\dot{\beta}\cos\beta \end{bmatrix} = \begin{bmatrix} 5.936 \\ -4.493 \\ 4.229 \end{bmatrix} \frac{\text{km}}{\text{sec}}$$

$$R_{SEZ2ECEF} = ROT_3(\lambda)\,ROT_2(\pi/2 - \phi) = \begin{bmatrix} 0.358 & 0.930 & 0.078 \\ -0.909 & 0.366 & -0.198 \\ -0.213 & 0 & 0.977 \end{bmatrix}$$

$$\bar{\rho}_{ECEF} = \begin{bmatrix} 32.935 \\ 1450.354 \\ 1547.838 \end{bmatrix} \text{ km} \qquad \dot{\bar{\rho}}_{ECEF} = \begin{bmatrix} -1.715 \\ -7.878 \\ 2.867 \end{bmatrix} \frac{\text{km}}{\text{sec.}}$$

$$\bar{\rho}_{ECI} = \begin{bmatrix} 517.147 \\ -1355.422 \\ 1547.838 \end{bmatrix} \text{ km} \qquad \dot{\bar{\rho}}_{ECI} = \begin{bmatrix} -1.386 \\ 7.942 \\ 2.867 \end{bmatrix} \text{ km/sec.}$$

PROBLEM 8

SATELLITE POSITION + VELOCITY IN ECI

$$\bar{r}_{SAT}^{ECI} = \bar{r}_{SITE}^{ECI} + \bar{\rho}_{ECI} = \begin{bmatrix} -424.338 \\ -369.687 \\ 7757.774 \end{bmatrix} \text{ km}$$

$$\bar{v}_{SAT}^{ECI} = \dot{\bar{\rho}}_{ECI} + \bar{\omega}_{\oplus} \times \bar{r}_{SAT}^{ECI} = \begin{bmatrix} -1.359 \\ 7.911 \\ 2.867 \end{bmatrix} \text{ km/sec.}$$

```
JD : 2454154.6376157
GST : 3.5287889766597287 rad = 202.18471515489122 deg
LST : 2.3332384390436127 rad = 133.6847151548912 deg
Site ECEF : [  499.58152988 -1268.26159955  6209.93598973] km
ECEF TO ECI :
[[-0.92597135  0.37759378  0.        ]
 [-0.37759378 -0.92597135  0.        ]
 [ 0.          0.          1.        ]]
Site ECI : [ -941.48587201   985.73502661  6209.93598973] km

RHO SEZ : [-1636.40221683   562.20006375  1227.40914278] km
DRHO SEZ: [ 5.93668714 -4.48333195  4.22954416] km/sec
SEZ TO ECEF:
[[ 0.3580884   0.93041757  0.0780759 ]
 [-0.90906037  0.36650123 -0.19820721]
 [-0.21303039  0.          0.97704557]]
RHO ECEF: [   32.93523275  1450.35406659  1547.83806733] km/sec
DRHO ECEF: [-1.71528654 -7.87827981  2.86776265] km/sec

RHO ECI: [  517.14758966 -1355.42245022  1547.83806733] km/sec
DRHO ECI: [-1.38648325  7.94274291  2.86776265] km/sec

SAT POS ECI : [ -424.33828235  -369.6874236   7757.77405706] km
SAT VEL ECI : [-1.35952522  7.91179967  2.86776265] km/sec
```

HOMEWORK 6 SOLUTION

PROBLEM 9

TWO ELLIPTICAL ORBITS

$r_A = 25000$ km $r_B = 40000$ km $r_C = 10000$ km $r_D = 55000$ km

A TO C

DEFINE INITIAL ORBIT.

$a_1 = \frac{1}{2}(r_A + r_B) = 32500$ km

$r_A = a_1(1 - e_1^2) \Rightarrow e_1 = 0.2307$

$p_1 = a_1(1 - e_1^2) = p_1 = 30769$ km

FINAL ORBIT

$a_2 = \frac{1}{2}(r_C + r_D) = 32500$

$r_C = a_2(1 - e_2^2) \Rightarrow e_2 = 0.692$

$p_2 = a_2(1 - e_2^2) = 16923$ km

TRANSFER ELLIPSE A TO C

$a_T = 17500$ km

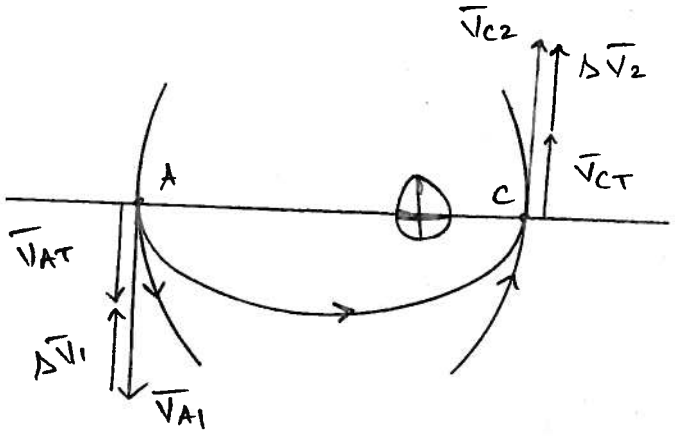
$p_{T_1} = 14285$ km

$e_{T_1} = 0.4285$

VELOCITY AT r_A, r_C ON ALL ORBITS. $\swarrow$ $-\frac{\mu}{2a} = \frac{v^2}{2} - \frac{\mu}{r}$

$V_{A_1} = 4.429$ km/sec $V_{C_2} = 8.213$ km/sec

$V_{AT} = 3.018$ km/sec $V_{CT} = 7.546$ km/sec



$\alpha = 180°$ $\alpha = 0°$

$|\Delta V_1| = 1.411$ km/sec

$|\Delta V_2| = 0.667$ km/sec

$|\Delta V_T| = 2.078$ km/sec.

B TO D

TRANSFER ELLIPSE B TO D

$a_T = 47500$ km

$P_T = 46315.78$ km

$e_T = 0.157$

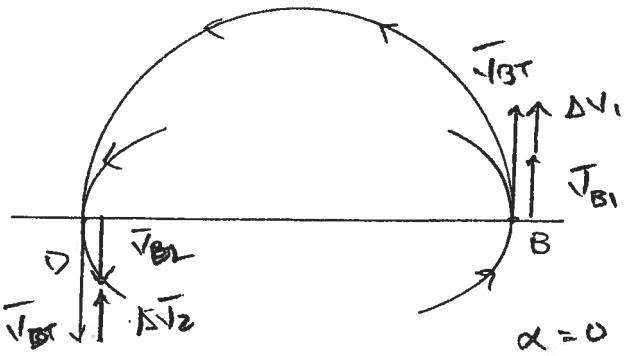
VELOCITY AT r_B, r_D

$V_{B1} = 7.768$ km/sec

$V_{BT} = 3.396$ km/sec

$V_{D2} = 1.493$ km/sec

$V_{DT} = 2.470$ km/sec



$|\Delta V_1| = 0.628$ km/sec

$|\Delta V_2| = 0.977$ km/sec

$|\Delta V_T| = 1.605$ km/sec

TOF FOR B TO D

$$TOF = \frac{P_{aT2}}{2} = 5.513 \text{ sec.}$$

A to C hohmann transfer
Semimajor axis : 17500.0 km
Eccentricity : 0.42857142857142855

V1 at A : 4.429831738426329 km/sec
VT1 at A : 3.01842153640418 km/sec
DV1 : -1.4114102020221488 km/sec

V2 at C : 8.213127859151424 km/sec
VT2 at C : 7.54605384101045 km/sec
DV2 : 0.6670740181409736 km/sec
TOF : 11519.616631051744 sec
B to D hohmann transfer
Semimajor axis : 47500.0 km
Eccentricity : 0.15789473684210525

V1 at B : 2.7686448365164558 km/sec
VT1 at B : 3.3968272735474483 km/sec

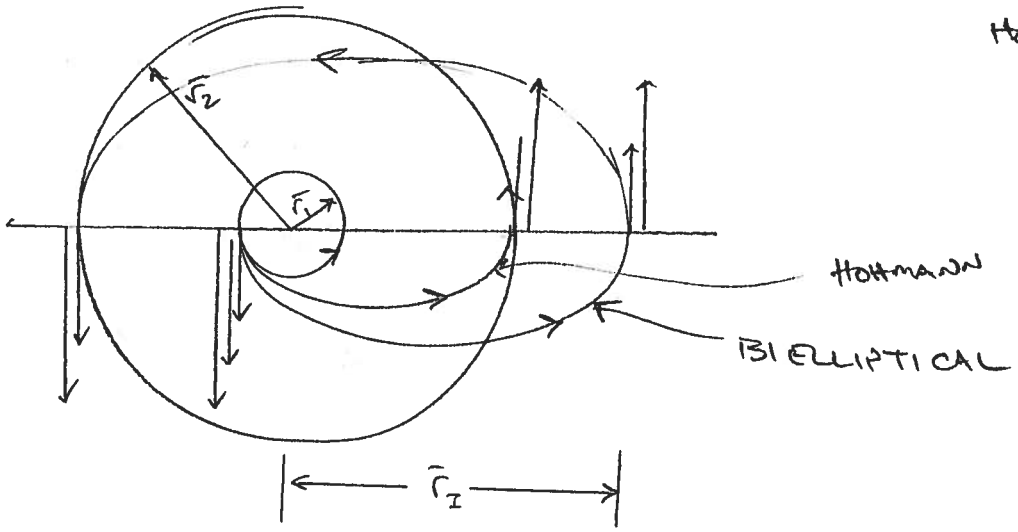
V2 at D : 1.4932959743911676 km/sec
VT2 at D : 2.470419835307235 km/sec
DV1 : 0.628182437030993 km/sec
DV2 : -0.9771238609160668 km/sec
TOF : 51513.53668204253 sec

HOMEWORK 6 SOLUTION

PROBLEM 10

CIRCULAR COPLANAR ORBITS $r_1 = 8000$ km $r_2 = 120000$ km

TRANSFER ELLIPSE - $r_I = 28000$ km ← BIELLIOTICAL VS. STANDARD HOHMANN



BIELLIPTICAL

VELOCITY IN INITIAL + FINAL ORBITS

$$V_1 = \sqrt{\frac{\mu}{r_1}} = 7.0586 \text{ km/sec}$$

$$V_2 = \sqrt{\frac{\mu}{r_2}} = 1.822 \text{ km/sec}$$

OUTBOUND ELLIPSE

$$a_{B1} = \frac{1}{2}(r_1 + r_I) = 14400 \text{ km}$$

$$P_{B1} = a_{B1}(1 - e_{B1}^2) = 15555.56 \text{ km}$$

$$r_1 = a_{B1}(1 - e_{B1}) \rightarrow e_{B1} = 0.944$$

IN BOUND ELLIPSE

$a_{B2} = 200{,}000$ km

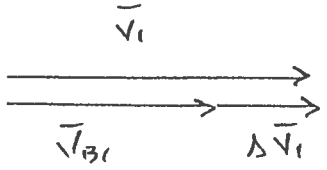
$P_{B2} = 168000$ km

$e_{B2} = 0.4$

FIRST MANEUVER

$$-\frac{\mu}{2a_{B1}} = \frac{V_{B1}^2}{2} - \frac{\mu}{r_1} \rightarrow V_{B1} = 9.842 \text{ km/sec}$$

$$|\Delta V_1| = 2.784 \text{ km/sec}$$



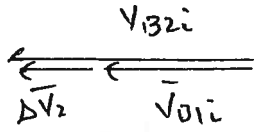
$\alpha = 0^\circ$

SECOND MANEUVER

$$V_{B1i} = 0.281 \text{ km/sec}$$

$$V_{B2i} = 0.924 \text{ km/sec}$$

$$|\Delta V_2| = 0.643 \text{ km/sec}$$



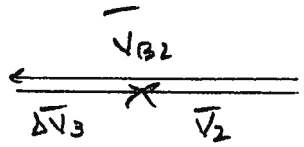
$\alpha = 0^\circ$

THIRD MANEUVER

$$V_{B2} = 2.156 \text{ km/sec}$$

$$V_2 = 1.822 \text{ km/sec}$$

$$|\Delta V_3| = 0.334 \text{ km/sec}$$



$\alpha = 180^\circ$

$$\text{TOTAL} = |\Delta V_T| = \Delta V_1 + \Delta V_2 + \Delta V_3 = 3.761 \text{ km/sec}$$

$$\text{TOF} = \pi\sqrt{\frac{a_{B1}^3}{\mu}} + \pi\sqrt{\frac{a_{B2}^3}{\mu}} = 716977 \text{ sec}$$

$$= 199 \text{ hrs.}$$

$\frac{1}{2}P_{B1}$ $\frac{1}{2}P_{B2}$

HOHMANN TRANSFER

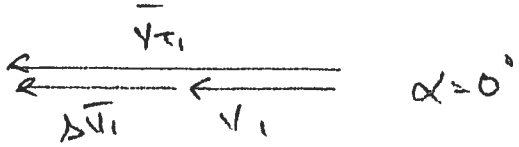
$a_T = \frac{1}{2}(r_1 + r_2) = 64000$ km

$p_T = 15000$ km

$e_T = 0.875$

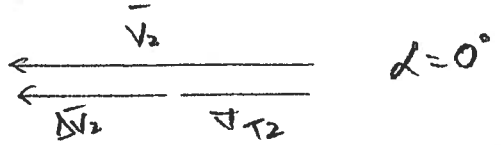
$V_{T_1} = 9.66$ km/sec

$V_{T_2} = 0.644$ km/sec



$\Delta V_1 = 2.607$ km/sec

$\Delta V_2 = 1.178$ km/sec



$TOF = \pi\sqrt{\frac{a_T^3}{\mu}} = 80565$ sec $= 22.38$ hrs.

BIELLIPTICAL — $\Delta V_T = 3.761$ km/sec

$TOF = 199$ hrs

HOHMANN — $\Delta V_T = 3.785$ km/sec

$TOF = 22.38$ hrs

$\approx$ 20 m/sec SAVINGS AT THE COST OF $\approx$ 170 hrs

DOESN'T MAKE MUCH SENSE FOR MOST MISSIONS

COST OF BATTERIES / TIME FAR OUTWEIGHS SMALL SAVINGS IN FUEL.

V1 : 7.058687023802657 km/sec
V2 : 1.8225451526185388 km/sec

Bielliptical transfer
VT1a : 9.842868787717443 km/sec
DV1 : 2.784181763914786 km/sec

VT2a : 0.2812248225062107 km/sec
VT2b : 0.924199074102235 km/sec
DV2 : 0.6429742515960243 km/sec

VT3b : 2.156464506238548 km/sec
DV3 : 0.33391935362000935 km/sec

TOF : 716977.260191342 sec

Hohmann Transfer
VT1 : 9.6655052732643 km/sec
VT2 : 0.6443670182176195 km/sec
DV1 : 2.606818249461643 km/sec
DV2 : 1.1781781344009192 km/sec
TOF : 80565.83528446438 sec

HOMEWORK 16 SOLUTION

PROBLEM 11

$r_1 = 10000$ km

$r_2 = 42160$ km

TRANSFER ORBIT

$e_T = 0.75$

$p_T = 15000$ km

VELOCITY IN INITIAL/FINAL ORBIT

$$V_1 = \sqrt{\frac{\mu}{r_1}} = 6.313 \text{ km/sec}$$

$$V_2 = \sqrt{\frac{\mu}{r_2}} = 3.075 \text{ km/sec.}$$

FIND PROPERTIES OF TRANSFER ELLIPSE @ r_1

$$Q_T = \frac{p_T}{1 - e_T^2} = 34285.71 \text{ km}$$

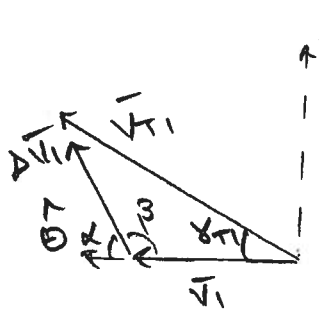
$r_{T1} = r_1 = 10{,}000$ km

$$r_1 = \frac{p_T}{1 + e\cos\nu_1} \rightarrow \nu_1 = \pm 48.189^\circ \rightarrow \text{CHOOSE } +48.189^\circ$$

$$-\frac{\mu}{2a_T} = \frac{V_{T1}^2}{2} - \frac{\mu}{r_1} \rightarrow V_{T1} = 8.252 \text{ km/sec.}$$

$$h = \sqrt{\mu p_T} = r_1 V_{T1} \cos\gamma_1 \Rightarrow \gamma_{T1} = 20.439^\circ$$

SINGLE IMPULSE MANEUVER



$$\Delta V_1^2 = V_1^2 + V_{T1}^2 - 2V_1 V_{T1}\cos\gamma_{T1}$$

$$\boxed{\Delta V_1 = 3.212 \text{ km/sec}}$$

$$\frac{\sin\pi - \alpha}{V_{T1}} = \frac{\sin\gamma_{T1}}{\Delta V_1} \rightarrow \boxed{\alpha = 63.78^\circ}$$

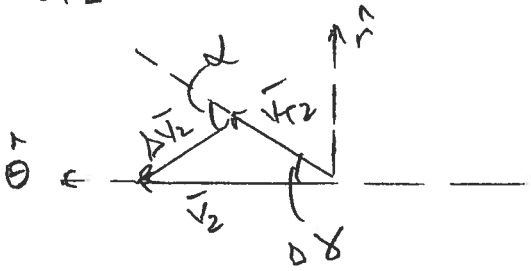
CONDITIONS ON TRANSFER ORBIT @ r_2

$r_2 = r_{T2}$

$V_{T2} = 2.6987$ km/sec

$\gamma_{T2} = \pm 149.199^\circ \Rightarrow$ CHOOSE $+149.199^\circ$

$\gamma_{T2} = +47.187^\circ$



$\Delta V_2 = 2.336$ km/sec

$\alpha = 105.111^\circ$

TOF $\rightarrow$ FROM r_{T1} TO r_{T2} ON TRANSFER ELLIPSE

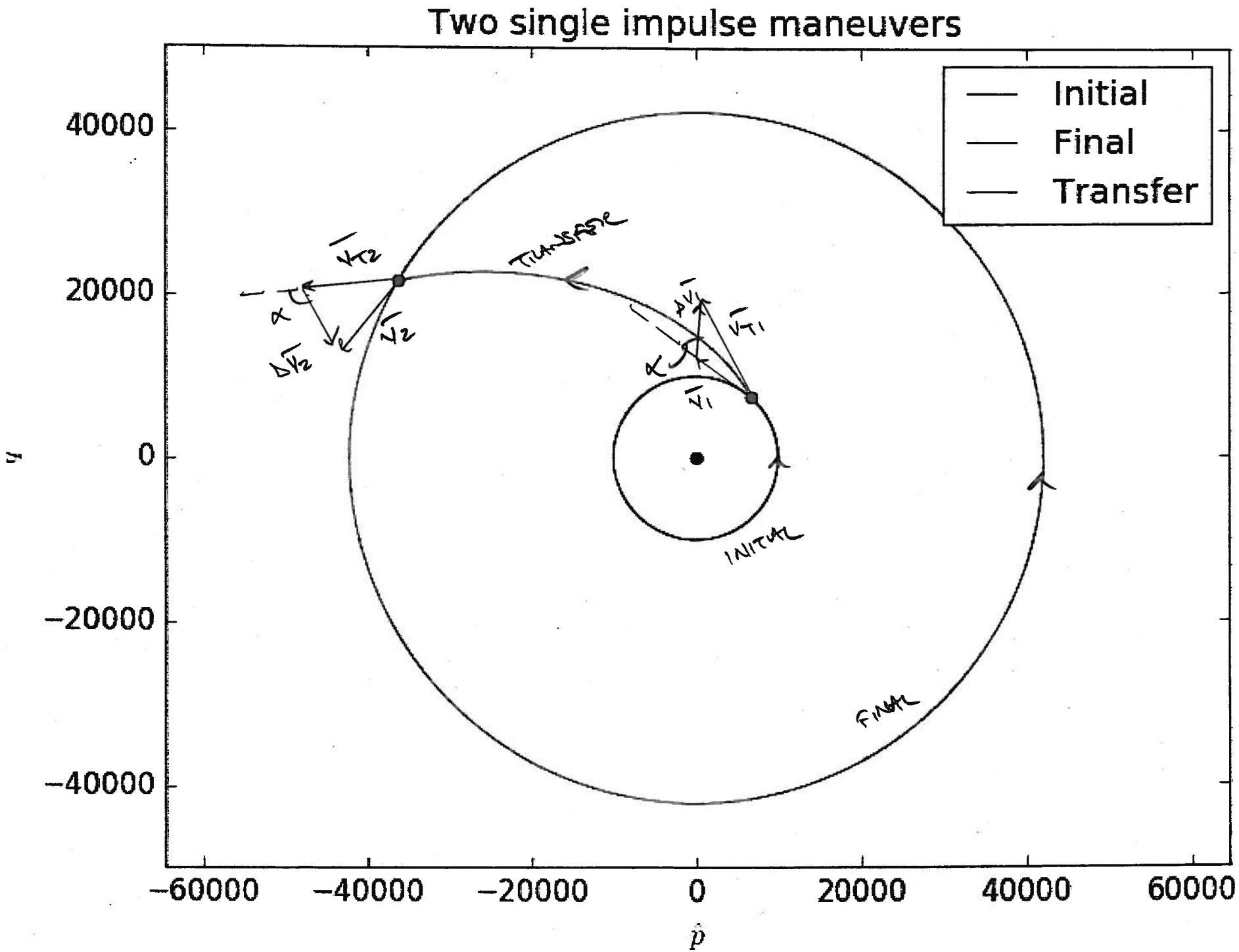
$r_{T1} \rightarrow E_1 \rightarrow M_1$

$r_{T2} \rightarrow E_2 \rightarrow M_2$

$\Big\} \Delta t$

$\Delta t = 10856$ sec

$= 3.016$ hr


Two single impulse maneuvers
Initial
Final
Transfer
40000
20000
0
−20000
−40000
−60000
−40000
−20000
0
20000
40000
60000
$\hat{p}$
$\hat{q}$
TRANSFER
INITIAL
FINAL

Transfer ellipse properties
a : 34285.71428571428 km, ecc : 0.75
V1 : 6.3134816068473665 km/sec
V2 : 3.0748123621805927 km/sec

First maneuver
True anomaly of transfer ellipse : 48.1896851042214 deg
VT1 : 8.251924144303153 km/sec
FPAT1 : 20.43931757325825 deg
DV1 : 3.2120894230249433 km/sec
alpha1 : 63.7847642878842 deg

Second maneuver
True anomaly of transfer ellipse : 149.19889612520493 deg
VT2 : 2.6987208467291546 km/sec
FPAT2 : 47.18728712054306 deg
DV2 : 2.336401926366451 km/sec
alpha2 : 105.1107332688757 deg

TOF : [10856.40635347] sec = [3.01566843] hr

HOMEWORK 6 SOLUTION

PROBLEM 12

A $V_c = \sqrt{\frac{\mu}{r}}$ $\quad V_E = \sqrt{2}\, V_c$ $\quad \Delta V = V_E - V_c = V_c(\sqrt{2} - 1)$

$$V_c(\sqrt{2}-1) = 2 V_c \sin\frac{\Delta i}{2}$$

$$\Delta i = 2\sin^{-1}\left(\frac{\sqrt{2}}{2} - \frac{1}{2}\right) = 123.9^\circ$$ MAXIMUM PLANE CHANGE

B $V_c = \Delta V = 2 V_c \sin\frac{\Delta i}{2} \Rightarrow \Delta i = 60^\circ$

C MINIMUM DELTA V → RAISE ORBIT + PLANE CHANGE

$r_1 = 8000$ km $\quad i_1 = 35^\circ \quad e_1, e_2 = 0$

$r_2 = 42160$ km $\quad i_2 = 0$

$\frac{r_2}{r_1} = 5.27$ → HOHMANN TRANSFER IS BEST.

APPROACH → ΔV_1 TRANSFER ONTO ELLIPSE

ΔV_2 COMBINE PLANE CHANGE + CIRCULARIZE.

TRANSFER ELLIPSE

$a_T = 25080$ km

$p_T = 13448$ km

$e_T = 0.68$

VELOCITIES

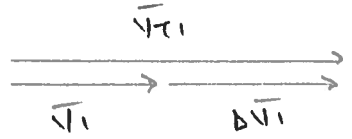
$V_1 = 7.0586$ km/sec

$V_{T_1} = 9.151$ km/sec

$V_2 = 3.075$ km/sec

$V_{T_2} = 1.736$ km/sec.

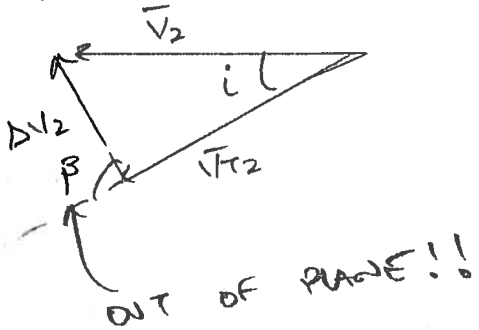
FIRST MANEUVER



$\Delta V_1 = 2.093$ km/sec.

$\alpha = 0°$

SECOND MANEUVER



$\Delta V_2 = 1.79$ km/sec

$\beta = 58.93°$ ← OUT OF PLANE

$\alpha = 0°$

$\Delta V_T = \Delta V_1 + \Delta V_2 = \underline{3.883}$ km/sec

Combined plane change at apoapsis
V1 : 7.058687023802657 , V2 : 3.0748123621805927 km/sec
VT1 : 9.151883036731967 , VT2 : 1.7366001967233344 km/sec
DV1 : 2.093196012929308 km/sec
DV2 : 1.794877543292608 km/sec
Beta (outofplane) : 58.931632657244315 deg