# MAE3145: Homework 4

Due date: November 9, 2016

**Problem 1** A satellite is on a circular orbit with the orbital radius of $r_P = 7000\,\mathrm{km}$. We wish to design an impulsive maneuver such that the orbital radius is increased to $r_A = 9000\,\mathrm{km}$. Assume $\mu = 398600\,\mathrm{km}^3/\mathrm{s}^2$.

(a) Show that the initial impulse for the Hohman transfer is $\Delta v_P = 0.4577\,\mathrm{km/s}$.

(b) Show that the terminal impulse for the Hohman transfer is $\Delta v_A = 0.4298\,\mathrm{km/s}$.

(c) Find the total velocity change $\Delta v_{total}$ and the transfer time $t_{PA}$.

**Problem 2** We wish to simulate the above Hohmann transfer using STK. You may follow the steps described below, and upload the resulting `jpg` file to Blackboard.

### First Impulse

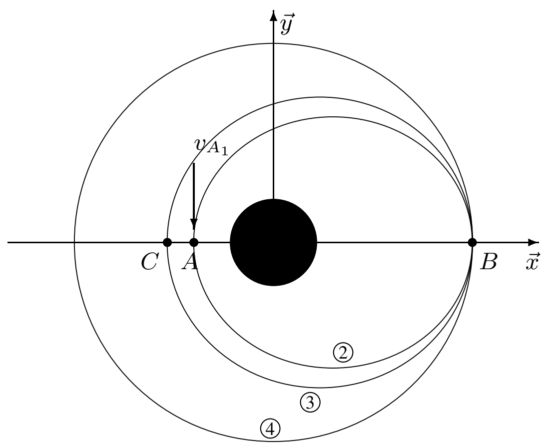
- Download *Hohmann.zip* from Black Board, and extract it to a folder.

  (Note: You SHOULD EXTRACT all of the files to your folder! Double-clicking the scenario file from the zip folder prevents loading predefined satellite.)
- Open *Hohmann.sc*
- Double-click *GWU_Sat*, then the *Basic Orbit* window appears
- Click the icon for *Insert Segment*, then the *Segment Section* window appears
- Select *Maneuver* and click *Ok*
- Type *0.4577* at *Delta V Magnitude* and click *Apply*
- Click the icon for *Insert Segment*, then the *Segment Section* window appears
- Select *Propagate* and click *Ok*
- Uncheck *Duration*
- Click *Insert* and choose *Apoapsis*
- Click the icon for *Run Entire Mission Control Sequence*
- Watch the *2D Graphics Window* and the *3D Graphics Window*

### Second Impulse

- Double-click *GWU_Sat*, then the *Basic Orbit* window appears
- Click the icon for *Insert Segment*, then the *Segment Section* window appears
- Select *Maneuver* and click *Ok*
- Type *0.4298* at *Delta V Magnitude* and click *Apply*
- Click the icon for *Insert Segment*, then the *Segment Section* window appears
- Select *Propagate* and click *Ok*
- Click the icon for *Run Entire Mission Control Sequence*
- Watch the *2D Graphics Window* and the *3D Graphics Window*
- Click the icon for *Snap Frame* at the *3D Graphics Window*, and type the file name and save it as jpg
- Upload your jpg file to Black Board

**Problem 3** A spacecraft is returning from an interplanetary mission along a hyperbolic orbit, and a space station on a circular orbit, namely Orbit ④, around the Earth counter-clockwise. Currently, the spacecraft is at the periapsis of the hyperbolic orbit, the point $A$, and the space station is at the point $B$. At the current time, we set the absolute time $t = 0$.




$$r_A = 7000\,\mathrm{km}, \quad r_B = 14000\,\mathrm{km}, \quad v_{A_1} = 12\,\mathrm{km/s}, \quad \mu = 398600\,\mathrm{km^3/s^2}.$$

We wish to design an orbital maneuver of the spacecraft such that a rendezvous between the spacecraft and the space station occurs at the point B. The maneuver of the spacecraft is composed of the following orbits:

| | Description | Periapsis | Apoapsis | Velocity at the beginning | Velocity at the end |
|---|---|---|---|---|---|
| Orbit ① | Hyperbolic return orbit | $A$ | - | - | $v_{A_1}$ $(t = 0)$ |
| Orbit ② | Hohmann transfer from $A$ to $B$ | $A$ | $B$ | $v_{A_2}$ $(t = 0)$ | $v_{B_2}$ $(t = t_1)$ |
| Orbit ③ | Phasing orbit from $B$ to $B$ | $C$ | $B$ | $v_{B_3}$ $(t = t_1)$ | $v_{B_3}$ $(t = t_2)$ |
| Orbit ④ | Target circular orbit (counter-clockwise) | $B$ | $B$ | $v_{B_4}$ $(t = t_2)$ | - |

(a) Find the velocity change at point $A$, namely $\Delta V_A = v_{A_2} - v_{A_1}$, to transfer the spacecraft from Orbit ① to Orbit ② at $t = 0$.

(b) Compute the absolute time $t_1$ when the spacecraft arrives at $B$ from Orbit ②.

(c) Find the location of the space station when $t = t_1$ (answer in terms of the angle measured from the $\vec{x}$ axis counter-clockwise). What is the *absolute* time $t_2$ when the space station returns to $B$.

(d) The period of the phasing orbit ③ should be $T_3 = t_2 - t_1$. Find the semi-major axis $a_3$, and distance to the apoapsis $r_C$ of the phasing orbit ③.

(e) Find the velocity change at point $B$, namely $\Delta V_{B_1} = v_{B_3} - v_{B_2}$, to transfer the spacecraft from Orbit ② to Orbit ③ at $t = t_1$.

(f) Find the velocity change at point $B$, namely $\Delta V_{B_2} = v_{B_4} - v_{B_3}$, to transfer the spacecraft from Orbit ③ to Orbit ④ at $t = t_2$.

(g) Show that the total velocity change is $\Delta V_{total} = |\Delta V_A| + |\Delta V_{B_1}| + |\Delta V_{B_2}| = 4.2657\,\mathrm{km/s}$.