

Summary of Orbital Properties

Any Type of Orbit

$$\begin{aligned}
 \vec{h} &= \vec{r} \times \vec{v}, & \vec{r} &= r\hat{u}_r, & \vec{v} &= v_r\hat{u}_r + v_\theta\hat{u}_\theta, \\
 h &= rv \cos \gamma = rv_\theta = r^2\dot{\theta}, & r &= \frac{h^2/\mu}{1 + e \cos \theta}, & v_r &= \frac{\mu}{h} e \sin \theta = \dot{r}, \\
 \vec{e} &= \frac{\vec{v} \times \vec{h}}{\mu} - \frac{\vec{r}}{r}, & r_p &= \frac{h^2/\mu}{1 + e}, & v_\theta &= \frac{\mu}{h} (1 + e \cos \theta) = r\dot{\theta}, \\
 \mathcal{E} &= \frac{1}{2}v^2 - \frac{\mu}{r} = -\frac{1}{2}\frac{\mu^2}{h^2}(1 - e^2), & \tan \gamma &= \frac{v_r}{v_\theta} = \frac{e \sin \theta}{1 + e \cos \theta}.
 \end{aligned}$$

Circular Orbits: ($e = 0$)

$$v = \sqrt{\frac{\mu}{r}}, \quad \mathcal{E} = -\frac{\mu}{2r}, \quad T = \frac{2\pi}{\sqrt{\mu}} r^{3/2}.$$

Elliptic Orbits: ($0 < e < 1$)

$$\begin{aligned}
 r &= \frac{a(1 - e^2)}{1 + e \cos \theta}, & a &= \frac{h^2/\mu}{1 - e^2} = \frac{1}{2}(r_a + r_p), & T &= \frac{2\pi}{\sqrt{\mu}} a^{3/2}, \\
 r_p &= \frac{h^2/\mu}{1 + e} = a(1 - e), & b &= a\sqrt{1 - e^2}, & e &= \frac{r_a - r_p}{r_a + r_p}, \\
 r_a &= \frac{h^2/\mu}{1 - e} = a(1 + e), & \mathcal{E} &= -\frac{\mu}{2a}, & h &= \sqrt{\mu a(1 - e^2)}.
 \end{aligned}$$

Parabolic Orbits: ($e = 1$)

$$v = \sqrt{\frac{2\mu}{r}}, \quad \mathcal{E} = 0.$$

Hyperbolic Orbits: ($e > 1$)

$$\begin{aligned}
 r &= \frac{a(e^2 - 1)}{1 + e \cos \theta}, & a &= \frac{h^2/\mu}{e^2 - 1}, & \theta_\infty &= \cos^{-1}(-1/e), \\
 r_p &= a(e - 1), & b &= a\sqrt{e^2 - 1}, & \beta &= \cos^{-1}(1/e), \\
 \mathcal{E} &= \frac{\mu}{2a}, & h &= \sqrt{\mu a(e^2 - 1)}.
 \end{aligned}$$