MAE3145: Homework 3

Due date: October 19, 2016

Problem 1 Consider Asteroid 5 discussed at Question 3 of HW#2. Its specific energy and specific angular momentum are given by $\mathcal{E} = 10\,\mathrm{km}^2/\mathrm{s}^3$, and $h = 8\times 10^4\,\mathrm{km}^2/\mathrm{s}$. We want to determine the time after periapsis passage t when the true anomaly is $\theta = 100°$.

(a) Compute the semi-major axis a, and the eccentricity e.

(b) Compute the maximum true anomaly θ_∞. Is $\theta < \theta_\infty$?

(c) Compute the hyperbolic eccentric anomaly F, and the hyperbolic mean anomaly M_e.

(d) Show that the time after the periapsis passage is given by $t = 0.6979\,\mathrm{hrs}$.

Problem 2 An Earth-orbiting satellite has a period of $T = 15.743$ hours and a periapsis radius $r_p = 12756\,\mathrm{km}$. We want to determine the location of this satellite at time $t = 1$ hour after periapsis passage.

(a) Compute the semi-major axis a, and the eccentricity e.

(b) Compute the mean anomaly M_e.

(c) Write a Matlab program to compute the eccentric anomaly E.

(d) Show that the true anomaly is given by $\theta = 84.2850°$.

(Hint: if you want to verify your code, check that your code gives $\theta = \pi$ when $t = T/2$.)

Problem 3 We observed the position and the velocity of a spacecraft orbiting the Earth as follows:

$$\vec{r}_0 = [6000, 6000, 6000]\,\mathrm{km}, \quad \vec{v}_0 = [-5, -5, 0]\,\mathrm{km/s}.$$

Assume that $\mu = 398600\,\mathrm{km}^3/\mathrm{s}^2$.

(a) Using the Matlab `rv2oe.m` posted to Blackboard, find the orbital elements $(h, e, \theta, \Omega, \omega, i)$.

(b) Write a Matlab function `oe2rv.m` that computes the position and the velocity vector for given orbital elements. The first few lines of this file are as follows:

```
function [r_vec, v_vec]=oe2rv(h,e,theta,Omega,i,omega)
xhat=[1;0;0];
yhat=[0;1;0];
zhat=[0;0;1];
mu=398600;
N_hat=
```

(Hint: if you want to verify your code, check that your code returns $\vec{r}_0, \vec{v}_0$ when the orbital elements are equal to your answer to (a).)

(c) Evaluate the function `oe2rv.m` for varying `theta=linspace(0,2*pi,200)`. The other five orbital elements $(h, e, \Omega, \omega, i)$ are fixed at your solution of (a). Plot the position and the velocity vector in a three-dimensional space. The structure of the Matlab file is as follows:

```
h=
e=
Omega=
omega=
i=
theta=linspace(0,2*pi,200);
for k=1:200
      [r_vec_theta(:,k),v_vec_theta(:,k)]=oe2rv(h,e,theta(k),
end
plot3(r_vec_theta(1,:),
figure;
plot3(v_vec_theta(1,:),
```

(d) Check that $\vec{r}_0$ and $\vec{v}_0$ are on your curves at (c).

Problem 4 A satellite satisfies the following condition at the current time.

- $\vec{r} = [-6634.2, -1261.8, -5230.9]\,\mathrm{km}$
- $\vec{e} = [-0.40907, -0.48751, -0.63640]$
- It is flying toward its periapsis.

(a) What is the type of orbit.

(b) Find the direction of the specific angular momentum $\hat{h} = \frac{\vec{h}}{h}$.

(c) Find the inclination i.

(d) Find the direction of the node vector $\hat{N} = \frac{\vec{N}}{N}$.

(e) Find the longitude of the ascending node Ω.

(f) Find the argument of periapsis ω.

(g) Find the true anomaly θ.