HOMEWORK 3 SOLUTION

PROBLEM 1

$$\frac{d}{dt}(\bar{v} \times \bar{h}) = -\frac{\mu}{r^3}\bar{r} \times \bar{h} \quad \leftarrow \text{PROVE THIS}$$

$$\bar{v} = \frac{d}{dt}\bar{r}, \quad \frac{d}{dt}\bar{v} = \frac{d^2}{dt^2}\bar{r} = \ddot{\bar{r}}$$

ALREADY KNOW $\frac{d}{dt}\bar{h} = 0$

$$\frac{d}{dt}(\bar{v} \times \bar{h}) = \frac{d}{dt}\bar{v} \times \bar{h} + \bar{v} \times \cancel{\frac{d}{dt}\bar{h}}$$

$$= \ddot{\bar{r}} \times \bar{h} \qquad \leftarrow \quad \ddot{\bar{r}} = -\frac{\mu}{r^3}\bar{r}$$

$$\boxed{\frac{d}{dt}(\bar{v} \times \bar{h}) = -\frac{\mu}{r^3}\bar{r} \times \bar{h}} \quad \text{VII}$$

PART B

SHOW $\frac{1}{r^3}\bar{r} \times \bar{h} = \underline{\frac{\bar{r}\dot{r} - \dot{\bar{r}}r}{r^2}}$

$\bar{h} = \bar{r} \times \bar{v}$

$$\frac{1}{r^3}\bar{r} \times (\bar{r} \times \bar{v}) = \frac{1}{r^3}\left((\bar{r}\cdot\bar{v})\bar{r} - (\bar{r}\cdot\bar{r})\bar{v}\right) \quad \cancel{\bar{v} = \dot{\bar{r}}}$$

$$\bar{r}\cdot\bar{v} = \bar{r}\cdot\dot{\bar{r}} = r\dot{r}$$

$$\bar{a} \times (\bar{b} \times \bar{c}) = (\bar{a}\cdot\bar{c})\bar{b} - (\bar{a}\cdot\bar{b})\bar{c}$$

$$\frac{1}{r^3}\left(r\dot{r}\bar{r} - r^2\dot{\bar{r}}\right) = \boxed{\frac{\bar{r}\dot{r} - r\dot{\bar{r}}}{r^2} = \frac{1}{r^3}\bar{r} \times \bar{h}}$$

PART C SHOW $\frac{d}{dt}\left(\frac{\bar{r}}{r}\right) = -\frac{\bar{r}\dot{r} - \dot{\bar{r}} r}{r^2}$

$$\frac{d}{dt}\left(\frac{\bar{r}}{r}\right) = \frac{d}{dt}\bar{r} \cdot \frac{1}{r} + \bar{r} \cdot \frac{d}{dt}\left(\frac{1}{r}\right)$$

$$= \dot{\bar{r}} \cdot \frac{1}{r} + \bar{r} \cdot -1\frac{1}{r^2} \cdot \dot{r}$$

$$= \frac{\dot{\bar{r}}}{r} - \frac{\dot{r}\bar{r}}{r^2} = \frac{r\dot{\bar{r}} - \dot{r}\bar{r}}{r^2} = \boxed{-\frac{\left(\dot{r}\bar{r} - r\dot{\bar{r}}\right)}{r^2} = \frac{d}{dt}\left(\frac{\bar{r}}{r}\right)}$$

PART D SHOW $\frac{d}{dt}\bar{e} = 0$ $\bar{e} = \frac{\bar{v} \times \bar{h}}{\mu} - \frac{\bar{r}}{r}$

$\frac{d}{dt}\bar{e} = \frac{1}{\mu}\frac{d}{dt}\left(\bar{v} \times \bar{h}\right) - \frac{d}{dt}\left(\frac{\bar{r}}{r}\right)$ FROM ABOVE

$$= \frac{1}{\mu}\left(-\frac{\mu}{r^3}\bar{r} \times \bar{h}\right) + \frac{\bar{r}\dot{r} - \dot{\bar{r}} r}{r^2}$$

$$= -\frac{\bar{r}\dot{r} - \dot{\bar{r}} r}{r^2} + \frac{\bar{r}\dot{r} - \dot{\bar{r}} r}{r^2}$$

$\boxed{\frac{d}{dt}\bar{e} = \bar{0}}$ FIXED VECTOR.

HOMEWORK 3 SOLUTION

PROBLEM 2

$r_p = 7000$ km

$r_a = 70{,}000$ km

$\mu = 398600.5 \frac{km^3}{sec^2}$

$R_\oplus = 6378.137$ km

FIND: $e, \bar{P}, \mathcal{E}, \theta$ @ $r = 1000$ km, V_R, V_θ @ $r = 1000$ km

1. $e = \frac{r_a - r_p}{r_a + r_p} = \boxed{0.8181 = e}$

$a = \frac{r_p + r_a}{2} = \boxed{38500 \text{ km} = a}$ $\quad \boxed{p = 12727.27 \text{ km}}$

2. $\bar{P} = 2\pi\sqrt{\frac{a^3}{\mu}} = \boxed{75179.95 \text{ sec} = \bar{P}}$

3. $\mathcal{E} = -\frac{\mu}{2a} = \boxed{-5.176 \frac{km^2}{sec^2} = \mathcal{E}}$

4. $r = \frac{a(1-e^2)}{1 + e\cos\theta} = 1000 + R_\oplus = 7378.137$ km

$\cos\theta = \frac{a(1-e^2)}{re} - \frac{1}{e} \Rightarrow \boxed{\theta = \pm 27.6117^\circ}$

5. $V_R = \sqrt{\frac{\mu}{p}}\, e \sin\nu$

$V_\theta = \sqrt{\frac{\mu}{p}}\,(1 + e\cos\nu)$

$\theta = +27.6117^\circ$

$V_R = 2.12$ km/sec

$V_\theta = 9.6536$ km/sec

ASCENDING

$\theta = -27.6117^\circ$

$V_R = -2.122$ km/sec

$V_\theta = 9.653$ km/sec

DESCENDING

HOMEWORK 3

PROBLEM 3

$$\varepsilon = -\frac{\mu}{2a} = \frac{v^2}{2} - \frac{\mu}{r} \qquad h = rv \qquad @ \quad r = r_p \text{ PERIAPSIS}$$

$$2\varepsilon r_p^2 + 2\mu r_p - h^2 = 0 = 2\left(\frac{v^2}{2} - \frac{\mu}{r}\right) r_p^2 + 2\mu r_p - (rv)^2$$

$$= \left(v^2 - \frac{2\mu}{r_p}\right) r_p^2 + 2\mu r_p - r_p^2 v^2$$

$$= 0 \longrightarrow \text{SATISFIED}$$

PART B

SOLVE FOR r_p GIVEN ε, h

$$2\varepsilon r_p^2 + 2\mu_\oplus r_p - h^2 = 0 \qquad \text{USE np.roots}$$

AST	ε (km^2/sec^2)	h (km^2/sec)	r_p (km)
1	1	1×10^5	~~-4.10772~~, 12172 km
→ 2	100	1×10^5	~~-9339~~, 5353.56 km
IMPACTS → 3	0	7×10^4	6146.5 km
4	0	8×10^4	8028.08
5	10	8×10^4	~~-46710.7~~, 6850.67

PART C

FOR AST 2, 3 $r_p < R_\oplus \Rightarrow$ NEED VELOCITY AT $R_\oplus = r$

$$\varepsilon = \frac{v^2}{2} - \frac{\mu}{R_\oplus} \longrightarrow v = \sqrt{2\left(\varepsilon + \frac{\mu}{R_\oplus}\right)}$$

AST	v (km/sec)
2	18.027
3	11.179

PART D

FOR OTHER ASTEROIDS WE FIND v @ $r = r_p$

$$v = \sqrt{2\left(\varepsilon + \frac{\mu}{r_p}\right)}$$

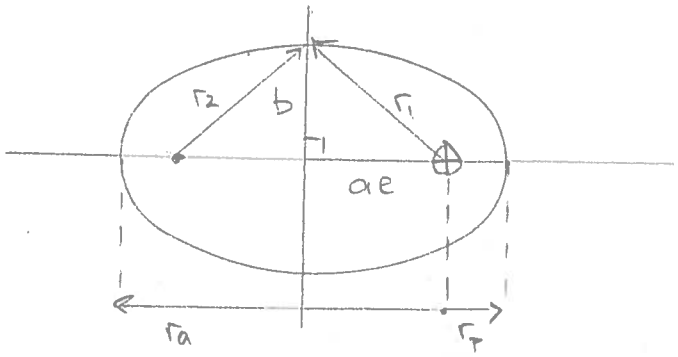
AST	v (km/sec)
1	8.215
4	9.965
5	11.677

PROBLEM 4 HOMEWORK 3 – SOLUTION

GIVEN: ELLIPTICAL ORBIT AT END OF SEMI-MINOR AXIS

FIND: PROVE $r=a$, $V=\sqrt{\frac{\mu}{r}}$, $\theta = \cos^{-1}(-e)$

SOLUTION: SKETCH AN ELLIPTICAL ORBIT



- FROM OUR DEFINITION OF AN ELLIPSE WE KNOW $r_a + r_p = 2a$
- AN ELLIPSE IS DEFINED AS THE LOCUS OF ALL POINTS WHERE THE SUM OF THE DIST. FROM EACH FOCI IS A CONSTANT

$\Rightarrow$ THEREFORE AT THE ENDS OF THE SEMI-MINOR AXIS

$r_1 + r_2 = 2a$ AND FROM THE RIGHT TRIANGLE

$(ae)^2 + b^2 = r_1 = r_2 \Big\} \Longrightarrow \boxed{r_1 = r_2 = a}$

TO FIND THE VELOCITY WE USE THE SPECIFIC MECHANICAL ENERGY

$$\mathcal{E} = -\frac{\mu}{2a} = \frac{V^2}{2} - \frac{\mu}{r} \qquad r=a$$

$$-\frac{\mu}{2a} + \frac{\mu}{a} = \frac{V^2}{2} \Rightarrow V^2 = \frac{\mu}{a} \Rightarrow \boxed{V = \sqrt{\frac{\mu}{r}}}$$

FINALLY THE CONIC EQUATION GIVES TRUE ANOMALY

$$r = \frac{p}{1+e\cos\theta} \qquad p = a(1-e^2) \qquad r=a$$

$$1 + e\cos\theta = (1-e^2) \Rightarrow \cos\theta = -e \Rightarrow \boxed{\theta = \cos(-e)}$$

PART B GIVEN CONIC EQUATION PROVE THAT $\dot{r}$ HAS A MAXIMUM OF $\pm e\sqrt{\frac{\mu}{p}}$ AT THE ENDS OF THE SEMI-LATUS RECTUM

$$r = \frac{p}{1+e\cos\theta}$$

$$\dot{r} = \frac{p e \dot{\theta} \sin\theta}{(1+e\cos\theta)^2}$$

ALSO KNOWN

$$h = r^2\dot{\theta} \Rightarrow \dot{\theta} = \frac{h}{r^2}$$

$$r^2 = \frac{p^2}{(1+e\cos\theta)^2}$$

USE $\dot{\theta} = h/r^2$

$$\dot{r} = \frac{p\,e\,h\,(1+e\cos\theta)^2 \sin\theta}{p^{\not 2}(1+e\cos\theta)^2} = \boxed{\frac{e\,h\sin\theta}{p} = \dot{r}}$$

TO PROVE $\dot{r}$ POSSES A MAXIMUM WE FIND $\ddot{r}$ AND FIND THE VALUE OF θ WHICH MAKES $\ddot{r} = 0$

$$\ddot{r} = \frac{e\,h\dot{\theta}\cos\theta}{p} = 0$$

$$\cos\theta = 0 \Rightarrow \theta = 90^\circ, 270^\circ \text{ or } \pi/2, 3\pi/2$$

MAXIMUM OCCURS AT ENDS OF SEMI-LATUS RECTUM

PLUG $\theta = 90^\circ$ OR 270° INTO $\dot{r}$

$$\dot{r} = \pm\frac{e\,h}{p} \quad \leftarrow \quad h = \sqrt{\mu p}$$

$$= \pm\frac{e\sqrt{\mu p}}{p} = \boxed{\pm e\sqrt{\frac{\mu}{p}} = \dot{r}}$$

HOMEWORK 3

PROBLEM 5

GIVEN : POSITION + VELOCITY OF CASSINI IN SATURN INERTIAL FRAME

$$\bar{r} = [-7546026.6144396 \quad -3717105.21901527 \quad -1515557.34280287] \text{ km}$$

$$\bar{v} = [0.89506649 \quad -0.33312074 \quad 0.21519571] \frac{\text{km}}{\text{sec}^2}$$

A. FIND TYPE OF ORBIT - CAN EITHER FIND ECCENTRICITY OR SPECIFIC MECH. ENERGY TO CLASSIFY ORBIT.

$$\mathcal{E} = \frac{v^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a} \qquad p = a(1-e^2)$$

$|\bar{r}| = 8547297.9903$ km

$|\bar{v}| = 0.9789906$ km/sec

$\mu = 37931300.0 \frac{\text{km}^3}{\text{sec}^2}$

$$\boxed{\mathcal{E} = -3.9586 \text{ km}^2/\text{sec}^2 \qquad a = 4790998.7508 \text{ km}}$$

USE $\bar{h} = \bar{r} \times \bar{v}$ TO GET p THEN e TO VERIFY

$|\bar{h}| = 5990724.035 \text{ km}^2/\text{sec}^2$

$p = \frac{h^2}{\mu} = 946151.976$ km

$p = a(1-e^2) \rightarrow \boxed{e = 0.895}$

$\mathcal{E} < 0$, $e < 1$ } ELLIPTICAL ORBIT FOR CASSINI

PART B

WE HAVE KNOWN RELATIONSHIPS FOR ALL OF THESE ORBITAL PARAMETERS → PLUG IN VALUES

$$T = 2\pi\sqrt{\frac{a^3}{\mu}} = 10698431.027 \text{ sec} = 2971.786 \text{ hr}$$

$$\bar{h} = \bar{r} \times \bar{v} = [-1304768.67 \quad 2673479.4 \quad 5840794.316] \text{ km}^2/\text{sec}$$

$$p = a(1-e^2) = 946151.976 \text{ km}$$

$$\varepsilon = \frac{v^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a} = -3.958 \text{ km}^2/\text{sec}^2$$

$$r = \frac{p}{1+e\cos\theta} \Rightarrow \theta = 186.921^\circ \leftarrow \text{ALWAYS DOUBLE VALUED}$$

$$r_p = a(1-e) = 499069.57 \text{ km}$$

$$r_a = a(1+e) = 9,082,927.93 \text{ km}$$

INVERSE TRIG FCNS ARE ALWAYS DOUBLE VALUED

WE CAN CHECK THE RADIAL VELOCITY OR FPA TO VERIFY

IF $0 < \theta < 180^\circ$ (ASCENDING) OR $180 < \theta < 360^\circ$ (DESC.)

RADIAL VELOCITY $v_R = \sqrt{\frac{\mu}{p}}\, e \sin\theta \rightarrow -0.683 \text{ km/sec}$

FPA $\cos\gamma = \frac{h}{rv} \rightarrow -44.28^\circ$

} ALL SHOULD BE CONSISTENT.

SPACECRAFT IS PAST APOAPSIS AND DESCENDING BACK TOWARDS SATURN.

PART C

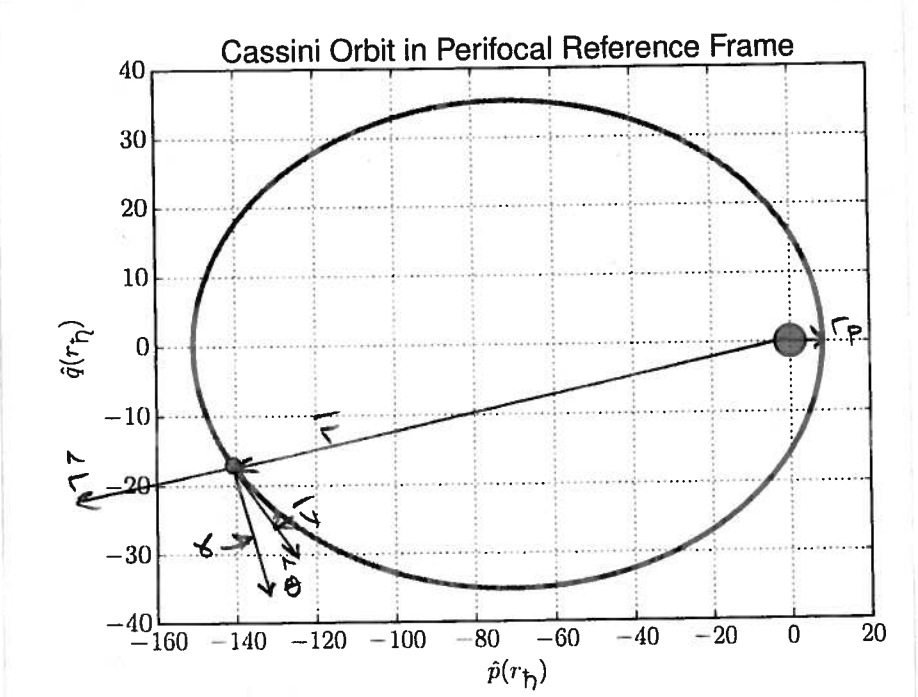
CIRCULAR ORBIT VELOCITY AT $r \rightarrow V_c = \sqrt{\frac{\mu}{r}}$

ESCAPE SPEED AT $r \rightarrow V_{esc} = \sqrt{2}\, V_c$

$V_c = 2.16066$ km/sec

$V_{esc} = 2.979$ km/sec

OUR VELOCITY IS LESS THAN V_{esc}. THIS MAKES SENSE SINCE WE'RE IN A CAPTURED ORBIT.



PART D

WE'LL PASS THROUGH THE E RING DURING PERIJANSIS PASSAGE AND THE PHOEBE RING DURING APOAPSIS PASSAGE.

PROBLEM 6

GIVEN: CAPSULE IN CIRCULAR ORBIT AT $h = 120$ km

FIND: ESCAPE SPEED + CIRCULAR VELOCITY

WE'RE IN EARTH ORBIT WITH $a = R_\oplus + 120\,\text{km} = 6498.137$ km

VELOCITY IN CIRCULAR ORBIT IS $V_c = \sqrt{\frac{\mu}{r}} = \sqrt{\frac{398600.5}{6498.137}} = 7.83 \frac{\text{km}}{\text{sec}}$

TO ESCAPE EARTH WE NEED $\boxed{V_{esc} = \sqrt{2}\, V_c = 11.076 \frac{\text{km}}{\text{sec}}}$

AT THIS ALTITUDE $\rightarrow$ ΔV OF $\boxed{\Delta V = V_{esc} - V_c = 3.24 \frac{\text{km}}{\text{sec}}}$

PART B

NOW INSTEAD WE'RE ON AN ELLIPTICAL ORBIT.

NEED TO FIND ESCAPE SPEED AT CURRENT ALTITUDE + VELOCITY AT CURRENT ALTITUDE.

$a = 6498.137$ km
$e = 0.3$
$\nu = 145°$

$$r = \frac{a(1-e^2)}{1 + e\cos\nu} = 7839.934 \text{ km}$$

$V = V_R \hat{r} + V_\theta \hat{\theta}$ $\qquad V_R = \sqrt{\frac{\mu}{p}}\, e \sin\nu$ $\qquad V_\theta = \sqrt{\frac{\mu}{p}}\,(1 + e\cos\nu)$

$V_R = 1.412$ km/sec
$V_\theta = 6.192$ km/sec

$$|V| = 6.3516 \text{ km/sec}$$

AT THIS LOCATION IN THE ELLIPTICAL ORBIT WE HAVE A LOWER VELOCITY MAGNITUDE.

$V_{ESC} = \sqrt{2}\sqrt{\frac{\mu}{r}} = 10.083 \frac{\text{km}}{\text{sec}}$ $\qquad \boxed{\Delta V = 3.732 \frac{\text{km}}{\text{sec}}}$

IT IS INTERESTING TO NOTE THAT WHILE THE ALTITUDE IS HIGHER ON THE ELLIPTICAL ORBIT, WHICH DECREASES THE ESCAPE SPEED BY $\approx$ 1 KM/SEC, THE VELOCITY MAGNITUDE IS MUCH LOWER ON THE ELLIPTICAL ORBIT AS COMPARED TO THE CIRCULAR ORBIT.

$\Delta V_{CIRC} = 3.24$ KM/SEC

$\Delta V_{ELP} = 3.732$ KM/SEC

PROBLEM 7

GIVEN: POSITION & VELOCITY VECTORS IN LVLH FRAME

$$\bar{r} = r_1 \hat{r} + r_2 \hat{\theta} + r_3 \hat{h} \quad \text{km}$$

$$\bar{v} = v_1 \hat{r} + v_2 \hat{\theta} + v_3 \hat{h} \quad \text{km/sec}$$

FOR EARTH ORBITING SATELLITES

FIND: $P, \mathcal{E}, a, p, e, v, r_p, r_a, \bar{h}, h, \gamma, \bar{r}_{PQW}, \bar{v}_{PQW}$

HAND CALCULATIONS FOR FIRST SATELLITE SHOWN.

1. $\bar{h} = \bar{r} \times \bar{v}$ ← $\begin{cases} \bar{r} = 6781.675\,\hat{r} \quad \text{km} \\ \bar{v} = -0.002574\,\hat{r} + 7.667057\,\hat{\theta} \quad \frac{\text{km}}{\text{sec}} \end{cases}$

$$\bar{h} = 51995.4936 \frac{\text{km}^2}{\text{sec}}\,\hat{h}$$

$$|\bar{h}| = 51995.4936 \frac{\text{km}^2}{\text{sec}}$$

2. $h = \sqrt{\mu p}$ $\quad \mu = 398600.5 \frac{\text{km}^3}{\text{sec}^2}$

$$p = 6782.558 \text{ km}$$

3. $\mathcal{E} = -\frac{\mu}{2a} = \frac{v^2}{2} - \frac{\mu}{r}$ → $\mathcal{E} = -29.384 \frac{\text{km}^2}{\text{sec}^2}$

$$a = 6782.5597 \text{ km}$$

4. $p = a(1-e^2)$ → $e = \sqrt{1 - \frac{p}{a}}$ → $e = 0.00036011$

VERY CIRCULAR!

5. $P = 2\pi\sqrt{\frac{a^3}{\mu}} = 5559.4937$ SEC

6. $r_p = a(1-e) = 6780.117$ km
   $r_a = a(1+e) = 6785.002$ km
   } ONLY 5 km DIFF!

7. $r = \frac{p}{1+e\cos\theta} \Rightarrow \cos\theta = \frac{p}{re} - \frac{1}{e}$

$\theta = +68.787°$ OR $\boxed{291.213° = \theta}$ DESCENDING

SINCE $\bar{r}\cdot\bar{v} = -17.45 \frac{km^2}{sec} < 0 \rightarrow 180° \le \nu \le 360°$

8. FLIGHT PATH ANGLE $\cos\gamma = \frac{h}{rv}$

$\gamma = \mp 0.01923°$ SINCE $\nu \ge 180°$

WE KNOW WE'RE DESCENDING $\therefore \gamma < 0 \rightarrow \boxed{\gamma = -0.0192°}$

9. PERIFOCAL REF FRAME

$\bar{r}_{PQW} = R_{LVLH2}^{PQW}\ \bar{r}_{LVLH}$; $R_{LVLH2}^{PQW} = \begin{bmatrix} C\nu & -S\nu & 0 \\ S\nu & C\nu & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\bar{v}_{PQW} = R_{LVLH2}^{PQW}\ \bar{v}_{LVLH}$

$\bar{r}_{PQW} = 2453.84\,\hat{p} - 6322.16\,\hat{q}$ km

$\bar{v}_{PQW} = 7.1466\,\hat{p} + 2.776\,\hat{q}$ $\frac{km}{sec}$

PLOTS + OTHER SOLUTIONS FOLLOW

ISS (ZARYA) 25544

Satellite State
Position and Velocity in LVLH frame
r_hat: 6781.6752240256 km rd_hat: -0.00257359868086831 km/sec
t_hat: 0 km td_hat: 7.66705746939915 km/sec
h_hat: 0 km hd_hat: 0 km/sec

Position and Velocity in EPH/PQW frame
e_hat: 2453.84042760766 km ed_hat: 7.14662603444703 km/sec
p_hat: -6322.16624267355 km pd_hat: 2.77660101315318 km/sec
h_hat: 0 km hd_hat: 0 km/sec

Position and Velocity in IJK frame
i_hat: -4226.54578373763 km id_hat: -1.66958104755974 km/sec
j_hat: 3746.04288415422 km jd_hat: -6.15442824782474 km/sec
k_hat: -3754.27653379581 km kd_hat: -4.25667580753874 km/sec

RAD_MAG : 6781.6752240256 km = 4.53326990820498e-05 AU
VEL_MAG : 7.66705790133865 km/sec

Orbital Elements
sma: 6782.55976540987 km raan: 286.709516148979 deg
ecc: 0.000360113803137797 arg_p: 293.694918634304 deg
inc: 51.643 deg nu: 291.21286881209 deg

lliptic Orbital Parameters
P : 6782.55888583428 km = 4.53386059964241e-05 AU
ANG MOM : 51995.4936814046 km^2/sec
PERIOD : 5559.06039988679 sec = 1.544183444413 hr
ENGERGY : -29.3842231979148 km^2/sec^2
RAD_PER : 6780.11727201773 km = 4.53222848160721e-05 AU
RAD_APO : 6785.002258802 km = 4.53549389359755e-05 AU

VEL_CIRC : 7.66655800402987 km/sec
VEL_ESC : 10.8421503060191 km/sec
TRUE_ANOM : 291.21286881209 deg
FPA : -0.0192324549052291 deg
ECC_ANOM : 291.232102521087 deg
MEAN_ANOM : 291.251334975467 deg
MEAN_MOT : 0.064759145269825 deg/sec

T_PAST_PER: 4497.45489632301 sec = 1.24929302675639 hr

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Satellite State
Position and Velocity in LVLH frame
r_hat: 41655.9406370472 km rd_hat: -1.10362387718295 km/sec
t_hat: 0 km td_hat: 1.70618367470225 km/sec
h_hat: 0 km hd_hat: 0 km/sec

osition and Velocity in EPH/PQW frame
e_hat: -40083.6505071477 km ed_hat: 1.52630293742109 km/sec
p_hat: -11336.5935085473 km pd_hat: -1.34143495456491 km/sec
h_hat: 0 km hd_hat: 0 km/sec

Position and Velocity in IJK frame
i_hat: 20947.9270507434 km id_hat: 0.282206960660129 km/sec
j_hat: -3532.56555381949 km jd_hat: 1.53920864269926 km/sec
k_hat: 35831.8674261041 km kd_hat: -1.2962424077709 km/sec

RAD_MAG : 41655.9406370472 km = 0.000278452765651323 AU
VEL_MAG : 2.03200600247854 km/sec

Orbital Elements
sma: 26557.9592688127 km raan: 233.615066201033 deg
ecc: 0.723070008007509 arg_p: 267.50733653601 deg
inc: 62.1154 deg nu: 195.792143409379 deg

Elliptic Orbital Parameters
P : 12672.6551438737 km = 8.471146777607e-05 AU
ANG MOM : 71072.685869296 km^2/sec
PERIOD : 43072.7895880465 sec = 11.9646637744574 hr
ENGERGY : -7.504351067894 km^2/sec^2
RAD_PER : 7354.69544764921 km = 4.91631027076068e-05 AU
RAD_APO : 45761.2230899762 km = 0.000305894883997855 AU

EL_CIRC : 3.09335986670617 km/sec
VEL_ESC : 4.37467147679649 km/sec
TRUE_ANOM : 195.792143409379 deg
FPA : -32.8963105020739 deg
ECC_ANOM : 218.166387518816 deg
MEAN_ANOM : 243.767238186524 deg
MEAN_MOT : 0.00835794485203036 deg/sec

T_PAST_PER: 29165.9304412982 sec = 8.10164734480505 hr

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Satellite State
Position and Velocity in LVLH frame
r_hat: 6894.474714852 km rd_hat: -0.184823165597574 km/sec
t_hat: 0 km td_hat: 8.03804198929817 km/sec
h_hat: 0 km hd_hat: 0 km/sec

Position and Velocity in EPH/PQW frame
e_hat: 6735.40457630325 km ed_hat: 1.53612139043265 km/sec
p_hat: -1472.44924776604 km pd_hat: 7.89205991475719 km/sec
h_hat: 0 km hd_hat: 0 km/sec

Position and Velocity in IJK frame
i_hat: -3480.88641788832 km id_hat: 3.1579047087912 km/sec

```
j_hat:          3324.75769062584 km jd_hat:        -5.06887034337584 km/sec
_hat:         -4935.90899816017 km kd_hat:        -5.38316541790249 km/sec

RAD_MAG    :        6894.474714852 km = 4.60867171093546e-05 AU
VEL_MAG    :      8.04016657938516 km/sec

Orbital Elements
sma:            7818.04303506166 km raan:           308.652932973563 deg
ecc:           0.120318073004321    arg_p:          238.547138893386 deg
inc:                    82.5924 deg nu:             347.668385754978 deg

Elliptic Orbital Parameters
P          :     7704.86561437729 km = 5.15038457056728e-05 AU
ANG MOM    :     55418.0772521349 km^2/sec
PERIOD     :      6879.5183572263 sec =      1.91097732145175 hr
ENGERGY    :    -25.4923449648711 km^2/sec^2
RAD_PER    :     6877.39116241819 km = 4.59725206142186e-05 AU
RAD_APO    :     8758.69490770513 km = 5.85482594618836e-05 AU

VEL_CIRC   :     7.60358364700502 km/sec
VEL_ESC    :     10.7530911162328 km/sec
TRUE_ANOM  :     347.668385754978 deg
FPA        :    -1.31720158962751 deg
ECC_ANOM   :     349.063666563664 deg
MEAN_ANOM  :     350.371529619475 deg
MEAN_MOT   :   0.0523292447678191 deg/sec

_PAST_PER:     6695.52047185178 sec =      1.85986679773661 hr

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Satellite State
Position and Velocity in LVLH frame
r_hat:          7491.57882277646 km rd_hat:       -0.859794393583042 km/sec
t_hat:                         0 km td_hat:         7.40806686127668 km/sec
h_hat:                         0 km hd_hat:                        0 km/sec

Position and Velocity in EPH/PQW frame
e_hat:          1903.17116142025 km ed_hat:         6.94660960199078 km/sec
p_hat:         -7245.80518563748 km pd_hat:         2.71354308200775 km/sec
h_hat:                         0 km hd_hat:                        0 km/sec

Position and Velocity in IJK frame
i_hat:          6124.28008566007 km id_hat:         3.52484103409163 km/sec
j_hat:         -1026.93439118335 km jd_hat:        0.576387361207735 km/sec
k_hat:         -4190.74604891124 km kd_hat:         6.54690570533946 km/sec

RAD_MAG    :      7491.57882277646 km = 5.00781115585166e-05 AU
VEL_MAG    :       7.4577946485796 km/sec

Orbital Elements
sma:            7847.35498258531 km raan:           355.510753316539 deg
```

```
ecc:          0.123771802770756    arg_p:          40.9437455503057 deg
inc:                  82.5981 deg nu:           284.716790945332 deg

Elliptic Orbital Parameters
P          :     7727.13774840676 km = 5.16527257266882e-05 AU
ANG MOM    :     55498.1168156525 km^2/sec
PERIOD     :     6918.24434802876 sec =      1.9217345411191 hr
ENGERGY    :    -25.3971243103292 km^2/sec^2
RAD_PER    :     6876.07370940865 km = 4.59637139847556e-05 AU
RAD_APO    :     8818.63625576197 km = 5.89489426270706e-05 AU

VEL_CIRC   :     7.29427685384374 km/sec
VEL_ESC    :      10.31566525441 km/sec
TRUE_ANOM  :     284.716790945332 deg
FPA        :    -6.62023799015209 deg
ECC_ANOM   :     291.487344445206 deg
MEAN_ANOM  :     298.086069384961 deg
MEAN_MOT   :   0.0520363233632498 deg/sec

T_PAST_PER:      5728.42295763504 sec =     1.59122859934307 hr

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Satellite State
Position and Velocity in LVLH frame
r_hat:          35855.749929422 km rd_hat:         -1.4798439485762 km/sec
t_hat:                        0 km td_hat:         1.89093029718632 km/sec
h_hat:                        0 km hd_hat:                        0 km/sec

Position and Velocity in EPH/PQW frame
e_hat:         -33616.0444097058 km ed_hat:         2.04524065255232 km/sec
p_hat:         -12473.8270489027 km pd_hat:        -1.25799291496498 km/sec
h_hat:                        0 km hd_hat:                        0 km/sec

Position and Velocity in IJK frame
i_hat:         -14897.7814818406 km id_hat:       -0.985368544751844 km/sec
j_hat:          31600.5520160964 km jd_hat:         -2.1839607648322 km/sec
k_hat:          8068.21059458428 km kd_hat:        0.157859778417242 km/sec

RAD_MAG    :       35855.749929422 km =  0.00023968088535901 AU
VEL_MAG    :     2.40115711709058 km/sec

Orbital Elements
sma:            24204.5402657568 km raan:           76.0923325377783 deg
ecc:          0.723554925788048    arg_p:          200.562070774206 deg
inc:                  20.0924 deg nu:           200.358261497512 deg

Elliptic Orbital Parameters
P          :     11532.6954112696 km = 7.70913075919439e-05 AU
ANG MOM    :     67800.7238698805 km^2/sec
PERIOD     :     37476.2567870086 sec =     10.4100713297246 hr
ENGERGY    :     -8.2340027041108 km^2/sec^2
```

```
RAD_PER   :     6691.22593003332 km = 4.47280828933816e-05 AU
RAD_APO   :     41717.8546014802 km = 0.000278866634942598 AU

VEL_CIRC  :     3.33418362640515 km/sec
VEL_ESC   :     4.71524770390447 km/sec
TRUE_ANOM :     200.358261497512 deg
FPA       :    -38.0467692156495 deg
ECC_ANOM  :     228.296400947532 deg
MEAN_ANOM :     259.247781493705 deg
MEAN_MOT  :  0.00960608211343018 deg/sec

T_PAST_PER:     26987.8789742233 sec =     7.49663304839537 hr

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Satellite State
Position and Velocity in LVLH frame
r_hat:         38208.4419642608 km rd_hat:      -0.925820823681464 km/sec
t_hat:                        0 km td_hat:        2.1677438597932 km/sec
h_hat:                        0 km hd_hat:                      0 km/sec

Position and Velocity in EPH/PQW frame
e_hat:        -36062.6899404331 km ed_hat:        1.59005081485913 km/sec
p_hat:        -12624.0814159476 km pd_hat:       -1.74011380242763 km/sec
h_hat:                        0 km hd_hat:                      0 km/sec

Position and Velocity in IJK frame
i_hat:         36885.7872896271 km id_hat:      -0.717511549504995 km/sec
j_hat:        -6558.17081380056 km jd_hat:        2.13912046254742 km/sec
k_hat:         7504.27404468207 km kd_hat:       0.682347757577603 km/sec

RAD_MAG   :     38208.4419642608 km = 0.000255407660311346 AU
VEL_MAG   :     2.35717153368891 km/sec

Orbital Elements
sma:           26038.2501524082 km raan:          326.104510912452 deg
ecc:          0.582258639176563    arg_p:          186.934792800011 deg
inc:                   26.3856 deg nu:            199.293075022529 deg

Elliptic Orbital Parameters
P         :     17210.6291944979 km = 0.000115045950818002 AU
ANG MOM   :     82826.1154602911 km^2/sec
PERIOD    :     41814.6668797485 sec =     11.6151852443746 hr
ENGERGY   :    -7.65413377755598 km^2/sec^2
RAD_PER   :     10877.2540521281 km = 7.27099527027228e-05 AU
RAD_APO   :     41199.2462526884 km = 0.000275399952236542 AU

VEL_CIRC  :     3.22990132932457 km/sec
VEL_ESC   :      4.5677702650577 km/sec
TRUE_ANOM :     199.293075022529 deg
FPA       :    -23.1268232863326 deg
ECC_ANOM  :      216.60837102698 deg
```
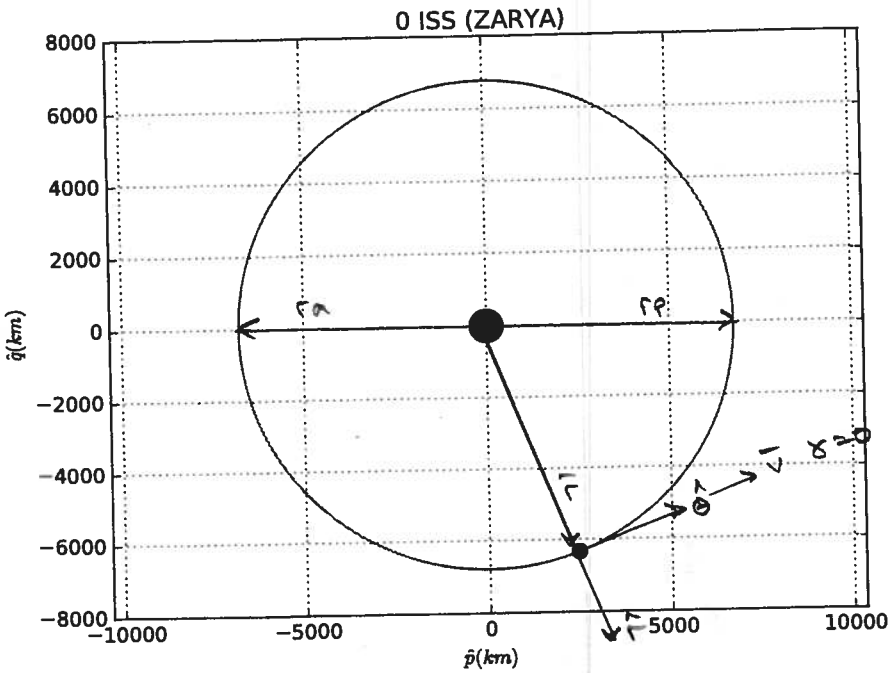

```
MEAN_ANOM :       236.50291959035 deg
MEAN_MOT  :  0.00860941929862304 deg/sec

T_PAST_PER:     27470.2522187734 sec =     7.63062561632594 hr
```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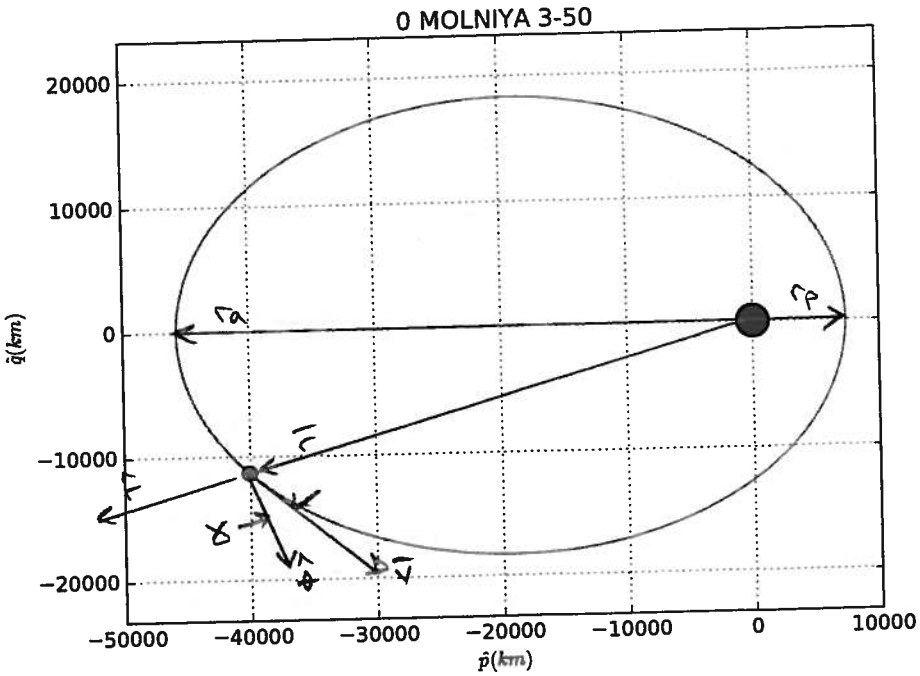
6000
4000
2000
0
−2000
−4000
−6000
−8000
−10000
−5000
0
5000
10000
$\hat{q}(km)$
$\hat{p}(km)$


0 MOLNIYA 3-50
20000
10000
0
−10000
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−50000
−40000
−30000
−20000
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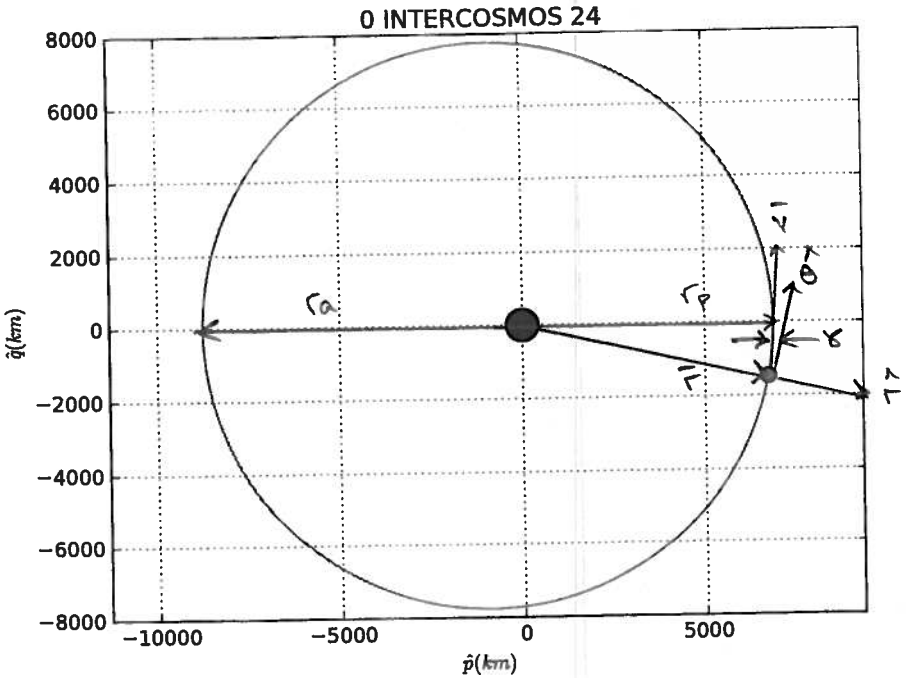
0
10000
$\hat{q}(km)$
$\hat{p}(km)$


0 INTERCOSMOS 24
8000
6000
4000
2000
0
−2000
−4000
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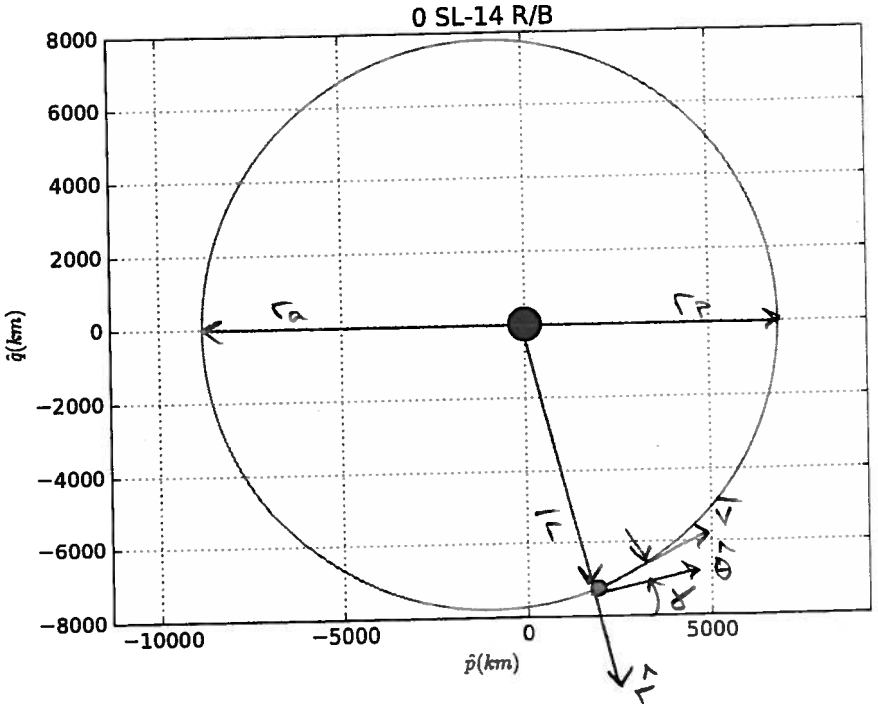
0
5000
$\hat{q}(km)$
$\hat{p}(km)$


0 SL-14 R/B
8000
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−8000
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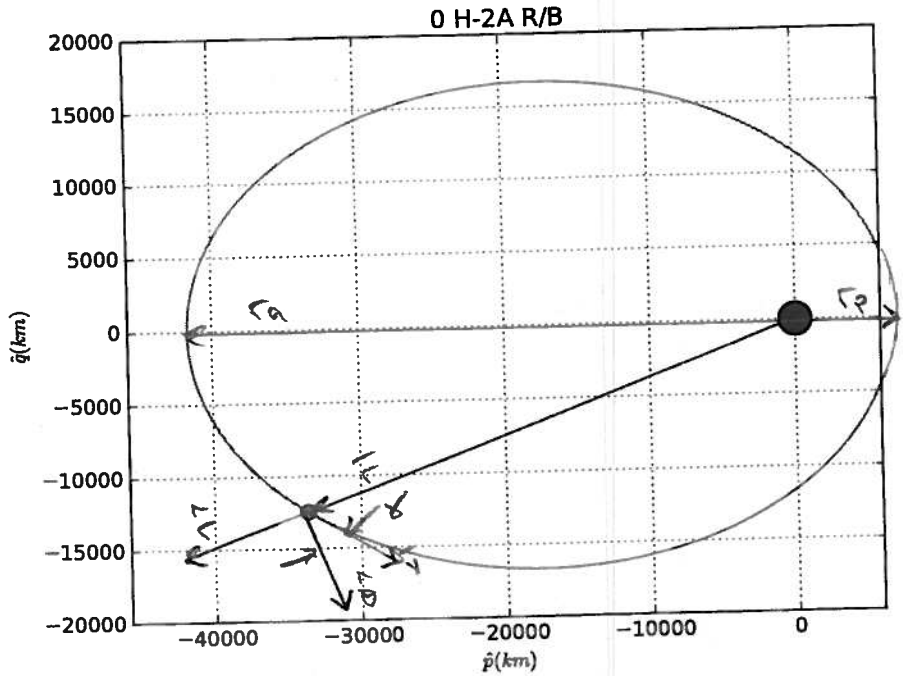
0
5000
$\hat{q}(km)$
$\hat{p}(km)$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15000
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5000
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−40000
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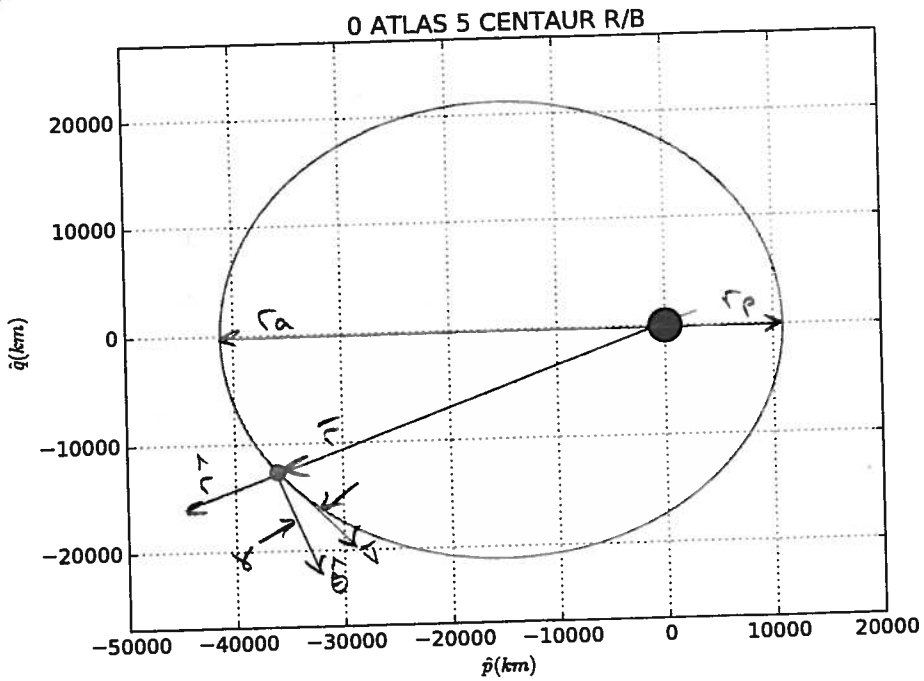
0
$\hat{q}(km)$
$\hat{p}(km)$


0 ATLAS 5 CENTAUR R/B
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−50000
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10000
20000
$\hat{q}(km)$
$\hat{p}(km)$