MAE3145: Midterm Exam

October 28, 2016

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Last Name First Name Student ID

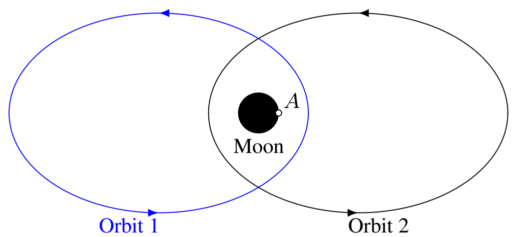
Prob. 1 (15)	Prob. 2 (12)	Prob. 3 (16)	Prob. 4 (16)	Prob. 5 (15)	Total (74)

Problem 1 ***(15pt)*** Mark whether each statement written in *italic font* is True or False.

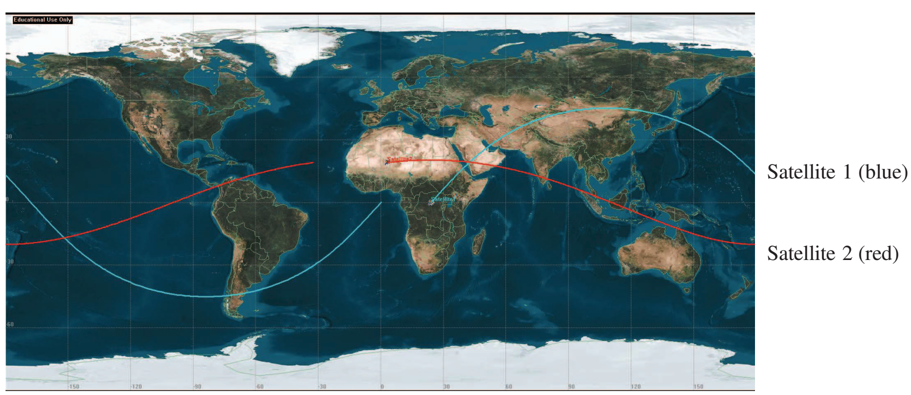
(a) International space station (ISS) is on a circular orbit at the altitude of $422\,\mathrm{km}$, and GPS satellites are on circular orbits at the altitude of $20200\,\mathrm{km}$. *The specific orbital energy of ISS is greater than GPS satellites, i.e.* $\mathcal{E}_{ISS} > \mathcal{E}_{GPS}$, [True, False]

(b) *The orbital period of ISS is greater than GPS satellites, i.e.* $T_{ISS} > T_{GPS}$, [True, False]

(c) The Aitken basin is the largest crater on the far side of the Moon. The following two lunar orbits, namely Orbit 1 and Orbit 2 are proposed to generate a topographic map of the Aitken basin, which is denoted by A below. The size and the shape of two orbits are identical, i.e., $a_1 = a_2$, $e_1 = e_2$, and $T_1 = T_2$. Assume that the Moon is not rotating: A is stationary with respect to both orbits. Then, *spacecraft on Orbit 1 can take images of* A *for a longer time period per each revolution than another spacecraft on Orbit 2.* [True, False]

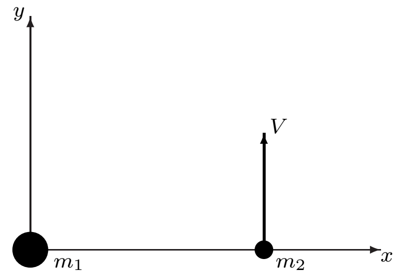


(d) Ground track of a satellite is the projection of the orbit of the satellite onto the surface of the Earth. Ground tracks for two satellites are illustrated as follows. *The inclination of Satellite 1 is greater than Satellite 2, i.e.* $i_1 > i_2$. [True, False]



(e) Since the Earth is rotating, ground track depends on the spin rate of the Earth. *Assuming the ground tracks at (d) are illustrated for one revolution of each satellite, the orbital period of Satellite 1 is greater than Satellite 2, i.e.,* $T_1 > T_2$. [True, False]

Problem 2 ***(15pt)*** **(Two-body problem with respect to the inertial frame)** We consider a planar motion of two masses acting under their mutual gravitational potential. A mass m_1 is initially at rest with respect to an inertial frame. Another mass m_2 is moving with a velocity V as follows:



More explicitly, the initial position vector and the initial velocity vector of m_1, m_2 in the inertial frame are given by

$$\vec{R}_1(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \text{ m}, \quad \dot{\vec{R}}_1(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \text{ m/s}, \qquad \vec{R}_2(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{ m}, \quad \dot{\vec{R}}_2(0) = \begin{bmatrix} 0 \\ V \end{bmatrix} \text{ m/s}.$$

Assume that $m_1 = 2\,\mathrm{kg}$, $m_2 = 1\,\mathrm{kg}$, $\underline{\mu = 1\,\mathrm{m}^3/\mathrm{s}^2}$.

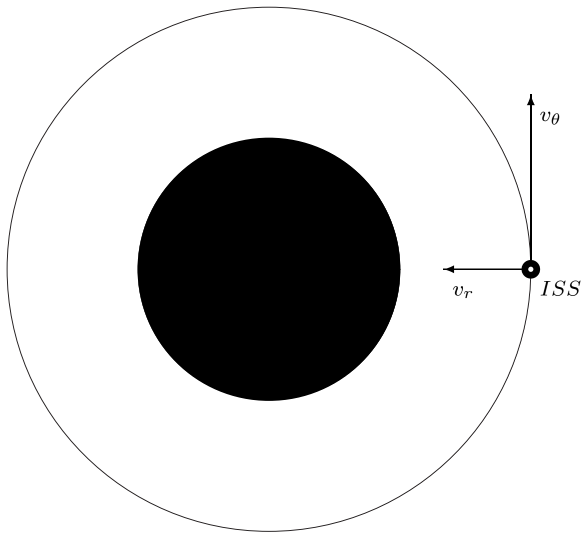
(a) Suppose that $V = 1\,\mathrm{m/s}$. What is the location of the mass center $\vec{R}_G$ at $t = 3$ seconds (specify the units).

(b) Suppose that $V = 1\,\mathrm{m/s}$. What is the type of the orbit for the relative motion $\vec{r} = \vec{R}_2 - \vec{R}_1$.
(Hint: compute $\mathcal{E}$ and h, then use the following equation to determine e, $\quad \mathcal{E} = -\frac{1}{2}\frac{\mu^2}{h^2}(1 - e^2)$.)

(c) What is the minimum value of V that makes the distance between two masses approaches infinity, i.e. a parabolic orbit (specify the units).

Problem 3 ***(16pt)*** **(Properties of orbit in 2D)** International Space Station is on a circular orbit at the altitude of $h = 422\,\mathrm{km}$. A bullet is fired from ISS toward the center of the Earth at the velocity of $v_r = -0.3\,\mathrm{km/s}$. We wish to determine whether the bullet hits the surface of the Earth or not. Assume that

$$R_E = 6378\,\mathrm{km}, \quad \mu = 398{,}600\,\mathrm{km}^3/\mathrm{s}^2.$$

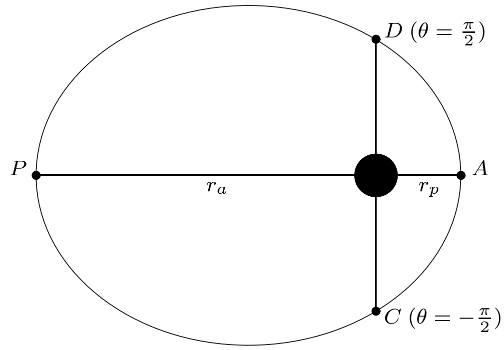


(a) Show that the specific energy of the bullet is given by $\mathcal{E} = -29.2638\,\mathrm{km}^2/\mathrm{s}^2$.
(Hint: $\vec{v} = v_r\hat{u}_u + v_\theta\hat{u}_\theta$)

(b) Show that the eccentricity of the bullet is given by $e = 0.0392$.

(c) Determine whether the bullet hits the surface or the Earth or not.

Problem 4 ***(16pt)*** **(Orbital position as a function of time)** Consider a spacecraft in an elliptic orbit around the Earth.



We observe that the maximum distance r_a, and the minimum distance r_p to the center of the Earth are given by

$$r_a = 32000\,\text{km}, \qquad r_p = 8000\,\text{km}.$$

Assume that the gravitational parameter of the Earth is given by $\mu = 398{,}600\ \text{km}^3/\text{s}^2$.

(a) Find the eccentricity e and the semi-major axis a (specify the units).

(b) Find the specific energy $\mathcal{E}$ and the angular momentum h (specify the units).

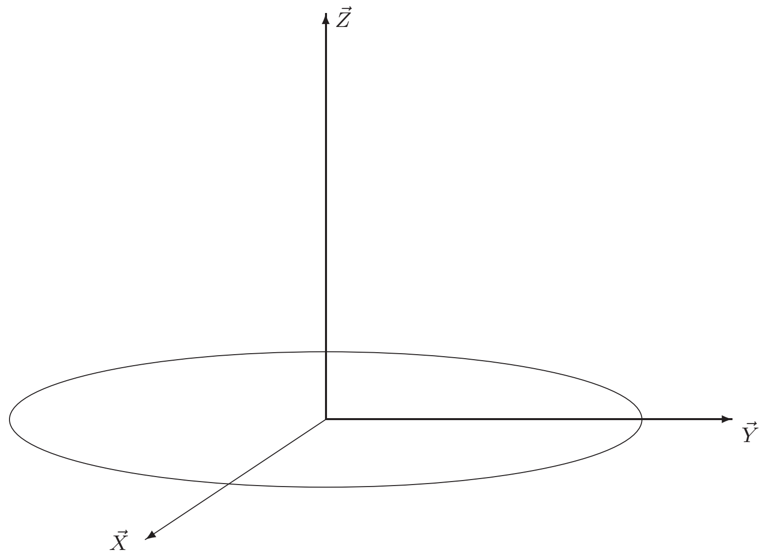
(c) Show the time required for the spacecraft to move from A to P through D, namely t_{ADP} is 3.9095 hours.

(d) Show the time required for the spacecraft to move from C to D through A, namely t_{CAD} is 1.1133 hours.

Problem 5 ***(15pt)*** **(Geometry of orbit in 3D)** The orbital elements for a spacecraft orbiting around the Earth are given as follows:

$$(e = 1.2, \quad \theta = 90^\circ, \quad i = 5^\circ, \quad \Omega = 180^\circ, \quad \omega = 90^\circ).$$

The following figure illustrates the geocentric equatorial frame and the Earth equatorial plane.



Sketch the orbit of this spacecraft according to the following steps.

(a) Draw the node vector $\vec{N}$, and specify the angle between $\vec{N}$ and $\vec{X}$.

(b) Draw the direction of the angular momentum vector $\vec{h}$. Specify the angle between the orbital plane and the equatorial plane.

(c) Draw the eccentricity vector $\vec{e}$, and specify the angle between $\vec{N}$ and $\vec{e}$.

(d) Sketch the orbit. Mark the periapsis by P.

(e) Mark the location of the spacecraft on the orbit by S.