MAE3145: Final Exam

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Last Name	First Name	Student ID

Prob. 1 (20)	Prob. 2 (20)	Prob. 3 (20)	Prob. 4 (20)	Prob. 5 (20)	Total (100)

Problem 1. Your spacecraft is currently in a circular orbit about Planet X with $\Omega = 90°$ and $i = 30°$ relative to an inertial reference frame defined by $\hat{x}, \hat{y}, \hat{z}$. At the descending node, the following maneuver is implemented:

$$\bar{\Delta} v = \frac{1}{\sqrt{2}} \hat{V} - \hat{C} + \sqrt{\frac{3}{2}} \hat{N} \mathrm{km\,s^{-1}}.$$

Express the $\Delta \bar{v}$ in terms of :

(a) Inertial reference frame: $\hat{x}, \hat{y}, \hat{z}$

(b) $\|\Delta \bar{v}\|, \alpha, \beta$ relative to the $\hat{V}, \hat{N}, \hat{C}$ reference frame

Problem 2. Assume that a spacecraft is moving in an orbit about the Earth characterized by $e = 0.5$ and $a = 4R_{\oplus}$. To meet some scientific objective, an in-plane maneuver is planned to adjust the orbit. The new orbit has exactly the same eccentricity and semi-major axis but periapsis will advance by $90°$.

(a) The maneuver can be implemented at one of two values of true anomaly. Determine these two options.

(b) Assume that the maneuver will be implemented at $\theta = 45°$. Determine the required maneuver in terms of $\|\Delta\bar{v}\|$ and α.

Problem 3. The following problems should be easy. Do not spend large amounts of time here.

(a) Draw a picture to explain why Greenwich Sidereal Time is approximately 100° at 0000 (midnight) UTC on 1 January each year. Recall that the Winter Solstice is in the third week of December annually.

(b) Given the total energy of a circular orbit, show that the orbital speed is constant and given by $v_{circ} = \sqrt{\frac{\mu}{r}}$.

(c) The Molniya orbit is a highly eccentric orbit specifically designed such that the spacecraft spends the majority of time in the vicinity of apogee.

   (a) How would you determine the velocity of the Molniya orbit at perigee? Assume you are given the semi-major axis, a, and eccentricity, e. Express your solution in terms of a and e.

Problem 4. To prepare for anti-satellite avoidance, US Space Command will require every satellite operator to generate a plot of total ΔV versus time of flight (TOF) for their satellite to increase its altitude by 25 km. Provide an algorithm to generate this plot assuming that the satellites are initially in a circular orbit and the final orbit is circular and in the same inclination as the initial orbit. Furthermore, assume that the first maneuver will be a tangential burn, while the second maneuver will be non-tangential (a so called **One Tangent Burn**).

(a) Write your algorithm as neatly and legibly as possible. Furthermore, your algorithm should be in a logical sequence. State any assumptions you make in your algorithm.

GIVEN: r_i (initial radius of circular orbit) and $\Delta\nu$ (change in true true anomaly along the transfer orbit) from 0° to 180°.

FIND: Total ΔV and TOF (time of flight)

(b) Draw a vector diagram showing all three velocity vectors involved in the second burn and label them correctly. Include the change in flight path angle $\Delta\gamma$ and firing angle α.

Problem 5. A radar tracking site is located at the following location: (assume a perfectly spherical Earth)

- Latitude: 90° North
- Altitude: 0 km
- Greenwich Sidereal Time: 180°

A satellite is in a circular polar orbit with $a = 9020.5\,\text{km}, \Omega = 90°, \theta = 45°$.
Determine the following:

(a) **Range-Vector** from the site to the satellite in the Earth Centered Inertial Reference frame.

(b) **Elevation angle** and **Range** from the site to the satellite

(c) Draw a sketch of the orbit, Earth, observation site, and the relative vector $\vec{\rho}$ from the site to the satellite. On your sketch, ensure you label the following vectors, $\vec{r}_{site}$, $\vec{r}_{sat}$, and $\vec{\rho}$.

Problem 6. Develop an algorithm to determine the **PERIOD OF THE PHASING ORBIT** for a non-coplanar rendezvous problem to deploy a satellite from an inclined circular low Earth orbit to a higher, circular equatorial orbit at the **first opportunity**. The first few steps have been outlined for you; complete the remaining algorithm.

GIVEN:

Interceptor satellite COEs: $a_{int}, i, \Omega, \theta$

Target satellite COEs: a_{tgt}, θ

FIND:

Period of phasing orbit : $\mathbb{P}_{\text{phasing}}$

1. Calculate the angular speed (mean motion) for both interceptor and target.

$$\omega_{int} = \sqrt{\frac{\mu}{a_{int}^3}} \qquad \omega_{tgt} = \sqrt{\frac{\mu}{a_{tgt}^3}}$$

2. Calculate the TOF for the Hohmann transfer (complete the equation below).

3. Calculate ...