Orbital Position as a Function of Time

Time since periapsis passage: t (for any type of orbit)

$$\frac{\mu^2}{h^3}t = \int_0^\theta \frac{d\theta}{(1+e\cos\theta)^2}. \tag{1}$$

Circular orbit: $\theta \sim t$

$$t = \frac{\theta}{2\pi}T, \tag{2}$$

where $T = \frac{2\pi}{\sqrt{\mu}}r^{3/2}$.

Elliptic orbit: $\theta \sim E \sim M_e \sim t$

$$\tan\frac{E}{2} = \sqrt{\frac{1-e}{1+e}}\tan\frac{\theta}{2}, \tag{3}$$

$$M_e = E - e\sin E, \tag{4}$$

$$t = \frac{M_e}{2\pi}T, \tag{5}$$

where $T = \frac{2\pi}{\sqrt{\mu}}a^{3/2}$.

Parabolic orbit: $\theta \sim M_p \sim t$

$$M_p = \frac{1}{2}\tan\frac{\theta}{2} + \frac{1}{6}\tan^3\frac{\theta}{2}, \tag{6}$$

$$\tan\frac{\theta}{2} = \left[3M_p + \sqrt{(3M_p)^2+1}\right]^{1/3} - \left[3M_p + \sqrt{(3M_p)^2+1}\right]^{-1/3}, \tag{7}$$

$$t = M_p \Big/ \left(\frac{\mu^2}{h^3}\right). \tag{8}$$

Hyperbolic orbit: $\theta \sim F \sim M_h \sim t$

$$\tanh\frac{F}{2} = \sqrt{\frac{e-1}{e+1}}\tan\frac{\theta}{2}, \tag{9}$$

$$M_h = e\sinh F - F, \tag{10}$$

$$t = M_h \Big/ \left(\frac{\mu^2}{h^3}(e^2-1)^{3/2}\right). \tag{11}$$