MAE3145: Homework 1

Due date: September 21, 2016

Problem 1 The motion of two point masses acting under their mutual gravity is described, with respect to an inertial frame, by the following set of ordinary differential equations.

$$m_1\ddot{R}_1 = G\frac{m_1 m_2}{r^2}\hat{u}_r, \tag{1}$$

$$m_2\ddot{R}_2 = -G\frac{m_1 m_2}{r^2}\hat{u}_r, \tag{2}$$

where $r = R_2 - R_1$, $r = \|r\|$, $\hat{u}_r = \frac{r}{r}$. Suppose that the units are normalized such that $m_1 = 2\,\mathrm{kg}$, $m_2 = 1\,\mathrm{kg}$, $G = 1\,\mathrm{m}^3/\mathrm{kgs}^2$.

The initial conditions are given by

$$R_1(0) = [0,0,0]^T\,(\mathrm{m}), \qquad V_1(0) = [0,0,0]^T\,(\mathrm{m/s}),$$
$$R_2(0) = [1,0,0]^T\,(\mathrm{m}), \qquad V_2(0) = [1,1,0]^T\,(\mathrm{m/s}).$$

We wish to compute the resulting trajectories of m_1 and m_2 using Matlab.

First, we rewrite the equations of motion as the standard form of $\dot{x} = f(t,x)$. Let the state vector be $x = [R_1^T,\, V_1^T,\, R_2^T,\, V_2^T] \in \mathbb{R}^{12}$. The equations of motion can be rewritten as

$$\begin{bmatrix} \dot{R}_1 \\ \dot{V}_1 \\ \dot{R}_2 \\ \dot{V}_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ G\frac{m_2}{r^2}\hat{u}_r \\ V_2 \\ -G\frac{m_1}{r^2}\hat{u}_r \end{bmatrix}. \tag{3}$$

(a) Write a Matlab m-file function, namely `eomTBI.m` that returns $\dot{x}$ for given (t,x). The first few lines and the last line of `eomTBI.m` are given as follows:

```
function X_dot = eomTBI(t,X)
R1=X(1:3);
V1=X(4:6);
.
.
.
X_dot = [R1_dot; V1_dot; R2_dot; V2_dot];
```

Upload your `eomTBI.m` file to Blackboard.

(You may verify the Matlab function `eomTBI.m` by checking its output when $t = 0$. Type the following Matlab commands

```
R10=[0 0 0]';
R20=[1 0 0]';
V10=[0 0 0]';
V20=[1 1 0]';
X0=[R10; V10; R20; V20];
eomTBI(0,X0)
```

And, check that the results are given by

```
ans =
     0
     0
     0
     1
     0
     0
     1
     1
     0
    -2
     0
     0
```

If the output of `eomTBI` is different from above, go back to part (b) and fix your code. You don't have to submit anything for this verification.)

(b) Write a Matlab script `m`-file, entitled `simTBI.m` to obtain $R_1(t), R_2(t)$ using `ode45`, and plot the trajectories of m_1, m_2 together on a single xy plane (The x axis is for the x-component of R_1, R_2, and the y axis is for their y components. There is no need to plot the z-components, as they are identical to zero). The simulation time is $0 \leq t \leq 10$ seconds.

Upload your `simTBI.m` and the plot saved as `R1R2.PNG`.

(c) The position of the mass center of two point masses is given by

$$R_G = \frac{m_1 R_1 + m_2 R_2}{m_1 + m_2}.$$

Compute the trajectory of the mass center using the results of (d) and plot it on the xy plane.

Upload your plot saved as `RG.PNG`.

(d) The relative position of m_2 from m_1 is given by

$$r = R_2 - R_1.$$

Compute the trajectory of the relative motion, and plot it on the xy plane.

Upload your plot saved as `r.PNG`.