HOMEWORK 5 SOLUTION

PROBLEM 1

$r = 4 R_\oplus \quad V = 4.54 \frac{km}{sec} \quad \gamma = -40^\circ$

SEMI-MAJOR AXIS $\quad -\frac{\mu}{2a} = \frac{V^2}{2} - \frac{\mu}{r} \rightarrow \boxed{a = 37477.36 \text{ km}}$

ANG. MOM $\quad h = rV\cos\gamma \rightarrow \boxed{h = 88728.64 \frac{km^2}{sec}}$

SEMI-PARAMETER $\quad p = \frac{h^2}{\mu} \rightarrow \boxed{p = 19751.06 \text{ km}}$

ECCENTRICITY $\quad p = a(1-e^2) \rightarrow \boxed{e = 0.68774}$

RADII APOAPSIS/PERIAPSIS

$$\left.\begin{aligned} r_p &= a(1-e) \\ r_a &= a(1+e) \end{aligned}\right\} \rightarrow \boxed{\begin{aligned} r_p &= 11702.66 \text{ km} \\ r_a &= 63252.06 \text{ km} \end{aligned}}$$

SPECIFIC MECH. ENERGY $\quad \mathcal{E} = -\frac{\mu}{2a} \rightarrow \boxed{\mathcal{E} = -5.318 \frac{km^2}{sec^2}}$

PERIOD $\quad P = 2\pi\sqrt{\frac{a^3}{\mu}} \rightarrow \boxed{P = 72204.5 \text{ sec} = 20.1 \text{ hrs}}$

TRUE ANOMALY $\quad r = \frac{p}{1+e\cos\theta} \rightarrow \theta = \pm 109.16^\circ$

$\gamma < 0 \rightarrow \theta > 180 \rightarrow \boxed{\theta = -109.16^\circ = 250.8^\circ}$

ECCENTRIC ANOMALY $\quad r = a(1-\cos E) \rightarrow E = \pm 62.34^\circ$

$\gamma < 0 \rightarrow E > 180 \rightarrow \boxed{E = -62^\circ = 297.6^\circ}$

TIME PAST PERIAPSIS

$$n(t-T_p) = E - e\sin E$$

$$\rightarrow \boxed{(t-t_p) = 6670.1 \text{ sec} = 18.53 \text{ hr}}$$

TIME UNTIL PERIAPSIS

$$\boxed{P-(t-T) = 5503 \text{ sec} = 1.5 \text{ hr}}$$

PART B

IN 8.5 hr S/C WILL BE PAST PERIAPSIS AND IN A NEW LOCATION.

$$V_1 \rightarrow E_1 \rightarrow M_1 \rightarrow \Delta t \rightarrow M_2 \rightarrow E_2 \rightarrow r_2$$

$$M_2 = M_1 + \Delta t =$$

$$\boxed{E_2 = 146.738^\circ} \leftarrow \text{USE NEWTON'S METHOD}$$

FIND CONDITIONS PRIOR TO MANEUVER

$$r_m^- = a(1-e\cos E_2) = \boxed{59029.6 \text{ km}}$$

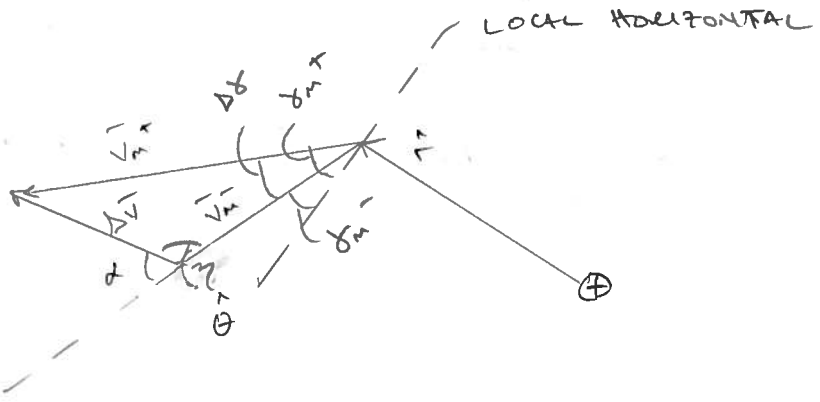
$$-\frac{\mu}{2a} = \frac{v^2}{2} - \frac{\mu}{r} \rightarrow \boxed{v_m^- = 1.69 \frac{\text{km}}{\text{sec}}}$$

$$h = r_m^- v_m^- \cos\gamma_m^- \rightarrow \gamma_m^- = \pm 27.45^\circ$$

$$0 < E < 180^\circ \rightarrow \boxed{\gamma_m^- = +27.45^\circ}$$

PART C $|\Delta \bar{V}| = 1200 \frac{m}{sec}$ $\alpha = +30^\circ$

VECTOR DIAGRAM



$r_M = 59029.63$ km ← SAME POSITION FOR MINUSCULE MAN.

$\eta = 180 - \alpha = 150^\circ$

COSINE LAW $V_M^{+2} = V_M^{-2} + \Delta V^2 - 2 V_M^- \Delta V \cos\eta$

$$\boxed{V_M^+ = 2.798 \frac{km}{sec}}$$

SINE LAW $\frac{\sin\Delta\gamma}{\Delta V} = \frac{\sin\eta}{V_M^+} \rightarrow \boxed{\Delta\gamma = 12.4^\circ}$

NEW FPA $\gamma_M^+ = \gamma_M^- + \Delta\gamma \rightarrow \boxed{\gamma_M^+ = 39.84^\circ}$

CAN FIND THE NEW ORBITAL ELEMENTS USING THE SAME PROCESS AS PART A.

NEW ORBITAL ELEMENTS

$a^+ = 70237.5$ km

$e^+ = 0.65$

$r_{p^+} = 24427.1$ km

$r_{a^+} = 116047.95$ km

$\mathcal{E} = -2.837$ km^2/sec^2

$P = 185252.8$ sec $= 51.45$ hr

$\Theta^+ = 119.0°$ ← ASCENDING

$E^+ = 75.83°$

$(t - T) = 20380.1$ sec $= 5.66$ hr

$\Delta W = \Theta^- - \Theta^+ = 46.34°$ ← PERIAPSIS ADVANCED

Initial Orbital Elements

Satellite State
Position and Velocity in LVLH frame
r_hat: 25512.548 km rd_hat: -2.91825574797689 km/sec
t_hat: 0 km td_hat: 3.47784177176016 km/sec
h_hat: 0 km hd_hat: 0 km/sec

Position and Velocity in EPH/PQW frame
e_hat: -8377.48621900577 km ed_hat: 4.24325558076028 km/sec
p_hat: -24097.8801993593 km pd_hat: 1.61442933457824 km/sec
h_hat: 0 km hd_hat: 0 km/sec

Position and Velocity in IJK frame
i_hat: -8377.48621900577 km id_hat: 4.24325558076028 km/sec
j_hat: -24097.8801993593 km jd_hat: 1.61442933457824 km/sec
k_hat: 0 km kd_hat: 0 km/sec

RAD_MAG : 25512.548 km = 0.000170540850615052 AU
VEL_MAG : 4.54 km/sec

Orbital Elements
sma: 37477.2179812508 km raan: 0 deg
ecc: 0.687739800828592 arg_p: 0 deg
inc: 0 deg nu: 250.830293500232 deg

Elliptic Orbital Parameters
P : 19751.0172962967 km = 0.00013202739648831 AU
ANG MOM : 88728.6051384361 km^2/sec
PERIOD : 72204.1177713576 sec = 20.0566993809327 hr
ENGERGY : -5.31790406907221 km^2/sec^2
RAD_PER : 11702.6435512156 km = 7.82273407449962e-05 AU
RAD_APO : 63251.792411286 km = 0.000422812118991307 AU

VEL_CIRC : 3.95268314807451 km/sec
VEL_ESC : 5.58993811577055 km/sec
TRUE_ANOM : 250.830293500232 deg
FPA : -40 deg
ECC_ANOM : 297.658742127688 deg
MEAN_ANOM : 332.560493858385 deg
MEAN_MOT : 0.00498586522641243 deg/sec

T_PAST_PER: 66700.6585129213 sec = 18.5279606980337 hr

Orbital Elements just before burn

Satellite State
Position and Velocity in LVLH frame
r_hat: 59029.5126720355 km rd_hat: 0.780986341147941 km/sec
t_hat: 0 km td_hat: 1.50312277913265 km/sec
h_hat: 0 km hd_hat: 0 km/sec

```
Position and Velocity in EPH/PQW frame
e_hat:        -57112.4360236472 km ed_hat:        -1.13558403949721 km/sec
p_hat:         14921.5621750141 km pd_hat:        -1.25688768131695 km/sec
h_hat:                        0 km hd_hat:                        0 km/sec

Position and Velocity in IJK frame
i_hat:        -57112.4360236472 km id_hat:        -1.13558403949721 km/sec
j_hat:         14921.5621750141 km jd_hat:        -1.25688768131695 km/sec
k_hat:                        0 km kd_hat:                        0 km/sec

RAD_MAG    :      59029.5126720355 km = 0.000394587922087631 AU
VEL_MAG    :      1.69390606416268 km/sec

Orbital Elements
sma:            37477.2179812508 km raan:                         0 deg
ecc:          0.687739800828592    arg_p:                         0 deg
inc:                          0 deg nu:            165.357838322619 deg

Elliptic Orbital Parameters
P          :      19751.0172962967 km =  0.00013202739648831 AU
ANG MOM    :      88728.6051384361 km^2/sec
PERIOD     :      72204.1177713576 sec =     20.0566993809327 hr
ENGERGY    :     -5.31790406907221 km^2/sec^2
RAD_PER    :      11702.6435512156 km = 7.82273407449962e-05 AU
RAD_APO    :       63251.792411286 km = 0.000422812118991307 AU

VEL_CIRC   :      2.59856940376349 km/sec
VEL_ESC    :       3.6749320935701 km/sec
TRUE_ANOM  :      165.357838322619 deg
FPA        :      27.4553002598493 deg
ECC_ANOM   :      146.739358595831 deg
MEAN_ANOM  :      125.127969786605 deg
MEAN_MOT   :   0.00498586522641243 deg/sec

T_PAST_PER:      25096.5407415637 sec =     6.97126131710103 hr

Orbital Elements after burn

Satellite State
Position and Velocity in LVLH frame
r_hat:         59029.5126720355 km rd_hat:          1.7925527541048 km/sec
t_hat:                        0 km td_hat:        2.14867168678999 km/sec
h_hat:                        0 km hd_hat:                        0 km/sec

Position and Velocity in EPH/PQW frame
e_hat:        -28626.1967189395 km ed_hat:        -2.74840014691788 km/sec
p_hat:           51623.87265313 km pd_hat:       0.525672927097673 km/sec
h_hat:                        0 km hd_hat:                        0 km/sec

Position and Velocity in IJK frame
i_hat:        -57112.4360236472 km id_hat:        -2.27748098485728 km/sec
j_hat:         14921.5621750141 km jd_hat:        -1.62576620627789 km/sec
```

```
k_hat:                          0 km kd_hat:                          0 km/sec

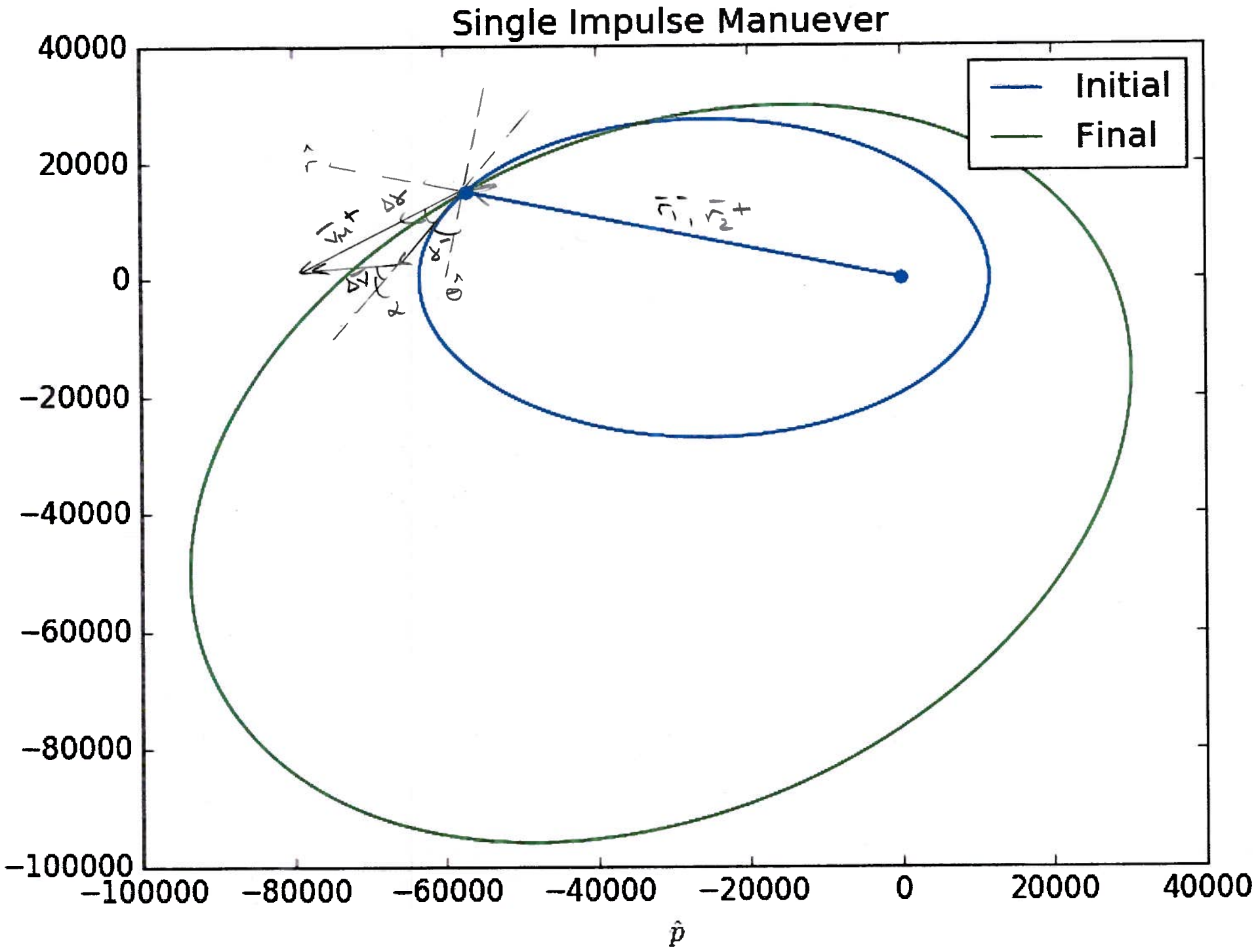
RAD_MAG    :      59029.5126720355 km = 0.000394587922087631 AU
VEL_MAG    :      2.79822004028662 km/sec

Orbital Elements
sma:           70236.8535102774 km raan:                             0 deg
ecc:          0.652216801878363    arg_p:          46.3488253549185 deg
inc:                          0 deg nu:            119.009012967701 deg

Elliptic Orbital Parameters
P          :       40359.026198068 km = 0.000269783428053273 AU
ANG MOM    :      126835.042563414 km^2/sec
PERIOD     :      185250.129533377 sec =      51.4583693148269 hr
ENGERGY    :     -2.83754524924493 km^2/sec^2
RAD_PER    :      24427.1975398052 km = 0.000163285730871738 AU
RAD_APO    :       116046.50948075 km = 0.000775723006488994 AU

VEL_CIRC   :      2.59856940376349 km/sec
VEL_ESC    :       3.6749320935701 km/sec
TRUE_ANOM  :      119.009012967701 deg
FPA        :      39.8369211046226 deg
ECC_ANOM   :      75.8388382620978 deg
MEAN_ANOM  :      39.6051676089685 deg
MEAN_MOT   :  0.00194331847921941 deg/sec

T_PAST_PER:       20380.1734159792 sec =      5.66115928221645 hr
```


Single Impulse Manuever
Initial
Final
40000
20000
0
−20000
−40000
−60000
−80000
−100000
−100000
−80000
−60000
−40000
−20000
0
20000
40000
$\hat{p}$

HOMEWORK 5 SOLUTION

PROBLEM 2

S/C CURRENTLY IN $e = 0.5$ $a = 6.0 R_\oplus$

AT $t = t_0$ S/C AT PERIAPSIS OF EARTH

SINGLE MANEUVER TO CIRCULARIZE AT $r = 7.6 R_\oplus$

PART A

MANEUVER OCCURS AT $r_m^- = 7.6 R_\oplus$ ← SINGLE IMPULSE

TRUE ANOMALY $r = \dfrac{p}{1 + e\cos\theta} \rightarrow \theta = \pm 144.66^\circ$

TWO POSSIBLE LOCATIONS FOR MANEUVER - SAME MAGNITUDE FOR r^-, v^- BUT DIFFERENT DIRECTIONS DUE TO DIFF. FPA

CHOOSE $\theta_1^- = +144.6^\circ$ → ASCENDING MANEUVER.

DEFINE CONDITIONS AT MANEUVER POINT

$$\boxed{\bar{r}_1^- = r\hat{r} = 7.6 R_\oplus \hat{r}}$$

$$\bar{v}_1^- = v_r \hat{r} + v_\theta \hat{\theta} = \frac{\mu}{h}\sin\theta\,\hat{r} + \frac{\mu}{h}(1 + e\cos\theta)\hat{\theta}$$

$h = \sqrt{\mu p}$ →

$$\bar{v}_1^- = 1.077\,\hat{r} + 2.206\,\hat{\theta}\ \frac{km}{sec} \leftarrow \theta_1^- > 0^\circ$$

$$\bar{v}_1^- = -1.077\,\hat{r} + 2.206\,\hat{\theta}\ \frac{km}{sec} \leftarrow \theta_1^- < 0^\circ$$

SHOULD GIVE $v_r < 0$ AND v_r MATCHES SIGN OF θ

FLIGHT PATH ANGLE $h = r_1^- v_1^- \cos\gamma_1^-$

$\gamma_1^- = \pm 26.0301^\circ \Rightarrow$ SIGN CHOSEN TO MATCH θ_1^-

$$\boxed{\gamma_1^- = +26.0301^\circ}$$

WAIT TIME UNTIL MANEUVER FOUND USING KEPLER'S EQ.

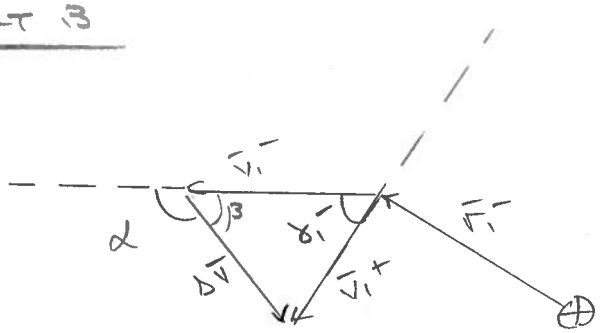
$r = a(1 - e\cos E_1^-) \rightarrow \boxed{E_1^- = 122.23^\circ}$ $E>0$ SINCE $\theta > 0$

TIME FROM PERIAPSIS TO MANEUVER POINT

$$n(t - T) = E_1^- - e\sin E_1^-$$

$$\boxed{(t - T) = 5.63 \text{ hr}}$$ SHOULD BE LESS THAN TP - TIME UNTIL MANEUVER.

PART 3



FOR $\theta_1^- > 0^\circ$

SIMILIARY DIAGRAM FOR $\theta_1^- < 0^\circ$

$\bar{v}_1^+$ IS ALONG LOCAL HORIZON $\hat{\theta}$ DIRECTION - CIRCULAR ORBIT

$\Rightarrow \Delta\gamma = \gamma_1^- = 26.03^\circ$

CIRCULAR ORBIT VELOCITY $\boxed{v_1^+ = \sqrt{\frac{\mu}{r}} = 2.867 \frac{km}{sec}}$

COSINE LAW $\Delta v^2 = v_1^{+2} + v_1^{-2} - 2v_1^+ v_1^- \cos\Delta\gamma$

$$\boxed{|\Delta\bar{v}_1| = 1.264 \frac{km}{sec}}$$

SINE LAW $\quad \frac{\sin \Delta\alpha}{\Delta V} = \frac{\sin \beta}{V_{1^+}} \quad \Rightarrow \beta = -84.5°,\ -95.49°$

$V_{1^+} > V_{1^-} \Rightarrow \beta > 90°$

$\alpha + 180° - \beta = \boxed{-84.505° = \alpha}$ ← DIRECTED TOWARDS EARTH SO CHOSEN AS NEGATIVE

AFTER THE MANEUVER

$\gamma_{1^+} = 0 \qquad \bar{r}_{1^+} = 7.6 R_\oplus \hat{r} \qquad \bar{V}_{1^+} = 2.861 \hat{\theta}\ \frac{km}{sec.}$

Initial Orbit prior to manuever

Satellite State
Position and Velocity in LVLH frame
r_hat: 48473.8412 km rd_hat: 1.07764498955317 km/sec
t_hat: 0 km td_hat: 2.20655451938311 km/sec
h_hat: 0 km hd_hat: 0 km/sec

Position and Velocity in EPH/PQW frame
e_hat: -39544.4494 km ed_hat: -2.15528997910635 km/sec
p_hat: 28034.796277727 km pd_hat: -1.17682907700433 km/sec
h_hat: 0 km hd_hat: 0 km/sec

Position and Velocity in IJK frame
i_hat: -39544.4494 km id_hat: -2.15528997910635 km/sec
j_hat: 28034.796277727 km jd_hat: -1.17682907700433 km/sec
k_hat: 0 km kd_hat: 0 km/sec

RAD_MAG : 48473.8412 km = 0.000324027616168599 AU
VEL_MAG : 2.45564687414927 km/sec

Orbital Elements
sma: 38268.822 km raan: 0 deg
ecc: 0.5 arg_p: 0 deg
inc: 0 deg nu: 144.665498208177 deg

Elliptic Orbital Parameters
P : 28701.6165 km = 0.000191858456941934 AU
ANG MOM : 106960.173371719 km^2/sec
PERIOD : 74503.8283888311 sec = 20.6955078857864 hr
ENGERGY : -5.2079013563574 km^2/sec^2
RAD_PER : 19134.411 km = 0.000127905637961289 AU
RAD_APO : 57403.233 km = 0.000383716913883867 AU

VEL_CIRC : 2.86757774813813 km/sec
VEL_ESC : 4.05536734257624 km/sec
TRUE_ANOM : 144.665498208177 deg
FPA : 26.0301234715598 deg
ECC_ANOM : 122.230952635502 deg
MEAN_ANOM : 97.997554599094 deg
MEAN_MOT : 0.00483196646112172 deg/sec

T_PAST_PER: 20281.0916399333 sec = 5.63363656664815 hr

Delta V: 1.2642272076105507 km/sec
Alpha: 84.50533326476899 deg, Beta: 95.49466673523101 deg
Final Orbit after manuever

Satellite State
Position and Velocity in LVLH frame
r_hat: 48473.8412 km rd_hat: 0 km/sec
t_hat: 0 km td_hat: 2.86757774813813 km/sec

```
h_hat:                          0 km hd_hat:                          0 km/sec

Position and Velocity in EPH/PQW frame
e_hat:                -39544.4494 km ed_hat:        -1.65846064577188 km/sec
p_hat:           28034.796277727 km pd_hat:        -2.33933974190216 km/sec
h_hat:                          0 km hd_hat:                          0 km/sec

Position and Velocity in IJK frame
i_hat:                -39544.4494 km id_hat:        -1.65846064577188 km/sec
j_hat:           28034.796277727 km jd_hat:        -2.33933974190216 km/sec
k_hat:                          0 km kd_hat:                          0 km/sec

RAD_MAG    :            48473.8412 km = 0.000324027616168599 AU
VEL_MAG    :      2.86757774813813 km/sec

Orbital Elements
sma:                  48473.8412 km raan:                           0 deg
ecc:                           0    arg_p:                           0 deg
inc:                           0 deg nu:            144.665498208177 deg

Elliptic Orbital Parameters
P          :            48473.8412 km = 0.000324027616168599 AU
ANG MOM    :      139002.508391901 km^2/sec
PERIOD     :      106211.636984611 sec =     29.5032324957254 hr
ENGERGY    :     -4.11150107080848 km^2/sec^2
RAD_PER    :            48473.8412 km = 0.000324027616168599 AU
RAD_APO    :            48473.8412 km = 0.000324027616168599 AU

VEL_CIRC   :      2.86757774813813 km/sec
VEL_ESC    :      4.05536734257624 km/sec
TRUE_ANOM  :      144.665498208177 deg
FPA        :                     0 deg
ECC_ANOM   :      144.665498208177 deg
MEAN_ANOM  :      144.665498208177 deg
MEAN_MOT   :   0.00338945910467569 deg/sec

T_PAST_PER:       42680.9982774579 sec =     11.8558328548494 hr
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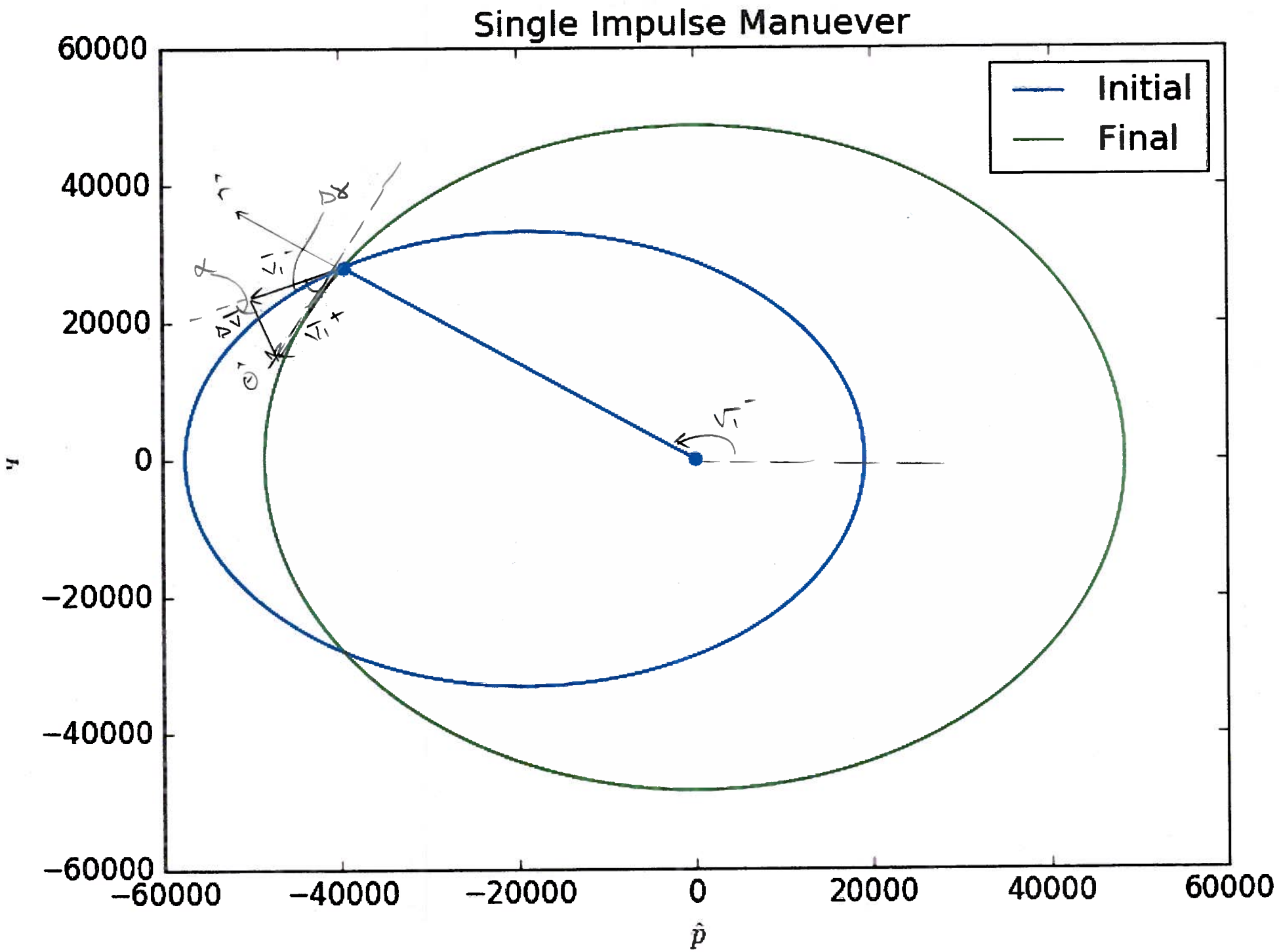

Single Impulse Manuever
Initial
Final
60000
40000
20000
0
−20000
−40000
−60000
−60000
−40000
−20000
0
20000
40000
60000
$\hat{p}$
$\hat{r}$
$\Delta\alpha$
α
$\vec{v}_1^-$
$\Delta\vec{v}_1$
$\vec{v}_1^+$
$\hat{\theta}'$
$\vec{v}_1^-$

HOMEWORK 5

PROBLEM 3

S/C IN EARTH ORBIT $e = 0.75$ $a = 4.5\ R_\oplus$

SINGLE MANEUVER TO RAISE PERIAPSIS + LOWER APOAPSIS

$\rightarrow$ $r_p = 2\ R_\oplus$ $r_a = 6.0 R_\oplus$ $\Delta w = +35^\circ$

PART A

ONLY HAVE A SINGLE MANEUVER — MUST FIND LOCATIONS WHERE ORBITS INTERSECT.

SINCE $\Delta w = +35^\circ \rightarrow$ $\boxed{\theta_f = \theta_i - 35^\circ}$

USE CONIC EQUATION TO FIND INTERSECTION

$$\frac{p_0}{1 + e_0 \cos\theta_0} = \frac{p_f}{1 + e_f \cos\theta_f}$$

$\leftarrow$ $a_0 = 4.5 R_\oplus$, $e_0 = 0.75$, $a_f = 4.0 R_\oplus$, $e_f = 0.5$

SOLVE NUMERICALLY — ONE APPROACH IS TO USE NEWTON'S METHOD.

$$\theta_{in} = \theta_{i0} - \frac{f(\theta_{i0})}{f'(\theta_{i0})}$$

$$f(\theta_{i0}) = \frac{p_f}{1 + e_f \cos(\theta_{i0} - \Delta w)} - \frac{p_0}{1 + e_0 \cos\theta_{i0}}$$

$$f'(\theta_{i0}) = \frac{p_f e_f \sin(\theta_{i0} - \Delta w)}{(1 + e_f \cos(\theta_{i0} - \Delta w))^2} - \frac{p_0 e_0 \sin(\theta_{i0})}{(1 + e_0 \cos\theta_{i0})^2}$$

TWO POSSIBLE SOLUTIONS

$\boxed{\theta_{i0} = 110.342^\circ,\ 206.93^\circ}$ $\leftarrow$ $0^\circ < \theta_{i0} < 180^\circ$ OR $180^\circ < \theta_{i0} < 360^\circ$

FIND CONDITIONS ON INITIAL ORBIT AT BOTH θ

$\theta_1 = 110.34$

$$r = \frac{p_1}{1+e\cos\theta_1} = 2.66 R_\oplus$$

$$\bar{r}_1^- = 2.66 \hat{r} R_\oplus$$

$$\bar{v}_1^- = 3.96 \hat{r} + 4.16 \hat{\theta} \ \frac{km}{sec}$$

$$v_r = \frac{\mu_e}{h} \sin\theta_1$$

$$v_\theta = \frac{\mu}{h}(1+e\cos\theta_1)$$

$$h = r_1 v_1^- \cos\gamma_1 \rightarrow \gamma_1^- = 43.56^\circ$$

$\theta_1 = 206.93^\circ$

$$\bar{r}_1^- = 37894 \hat{r} \ km$$

$$\bar{v}_1^- = -1.9142 \hat{r} + 1.8664 \hat{\theta} \ \frac{km}{sec}$$

$$\gamma_1^- = -45.71^\circ$$

CONDITIONS IN FINAL ORBIT AFTER BURN

$\theta_2^+ = 75.34^\circ$

$$r_2 = r_1 = 16895.37 \hat{r} \ km$$

$$\bar{v}_2^+ = 2.207 \hat{r} + 5.14 \hat{\theta} \ km/sec$$

$$\gamma_2^+ = 23.23^\circ$$

$\theta_2^+ = 171.93^\circ$

$$r_2 = r_1 = 37894 \hat{r} \ km$$

$$v_2^+ = 0.32 \hat{r} + 2.304 \hat{\theta} \ km/sec$$

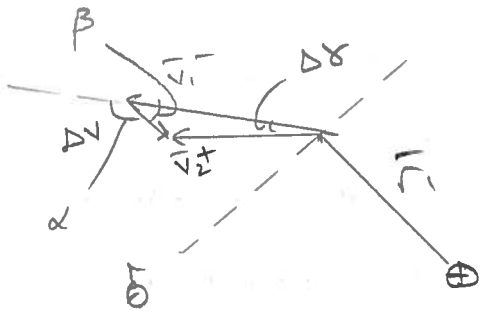
$$\gamma_2^+ = 7.9075^\circ$$

BURN A

BURN B

BURN A

$\boxed{\Delta\gamma = -20.33^\circ}$



$V_1^- = 5.74$ km/sec

$V_2^+ = 5.59$ km/sec

$\Delta V^2 = V_1^{-2} + V_2^{+2} - 2 V_1^- V_2^+ \cos\Delta\gamma$

$\boxed{|\Delta V| = 2.0 \text{ km/sec}}$

$\frac{\sin\Delta\gamma}{\Delta V} = \frac{\sin\beta}{V_2^+}$

$\beta = 75.5^\circ \Rightarrow \boxed{\alpha = -104.46^\circ}$

SIGN CHECK!

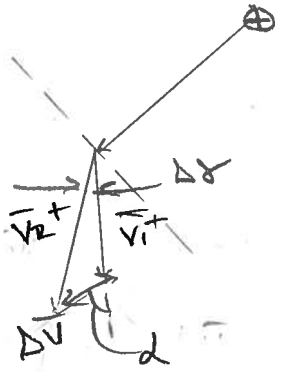
TOWARDS EARTH.

BURN B

$\boxed{\Delta\gamma = 53.6^\circ}$

$\boxed{|\Delta V| = 2.2767 \frac{\text{km}}{\text{sec}}}$

$\beta = 55.367^\circ \quad \boxed{\alpha = -124.63^\circ}$



BURN A IS $\approx$ 200 m/sec CHEAPER

IF WE HAD DRAWN VECTOR DIAGRAMS TO SCALE WE COULD HAVE GRAPHICALLY DETERMINED THE BEST BURN.

$\phi = \gamma_1 + \alpha$

$\boxed{\Delta\bar{V} = -0.5015 \hat{v} - 1.944 \hat{c} \ \frac{\text{km}}{\text{sec}}}$

$= \Delta V (c\beta c\alpha \hat{v} + s\beta \hat{n} + c\beta s\alpha \hat{c})$

$= \Delta V (c\beta s\phi \hat{r} + c\beta c\phi \hat{\theta} + s\beta \hat{h})$

B $\leftarrow$ OUT OF PLANE ANGLE DIFFERENT THAN IN PLANE B FROM ABOVE

PART D

FIND $\bar{r}_2, \bar{v}_2$ @ $\nu = 250^\circ$

$$r_2 = \frac{p}{1 + e\cos\nu} \Rightarrow \boxed{\bar{r}_2 = 3.61\,8\, R_\oplus \hat{r}}$$

$$\boxed{\bar{v}_2 = -2.144\,\hat{r} + 3.78\,\hat{\theta}\ \text{km/sec}}$$

TIME TO TRAVEL FROM MANEUVER TO $\nu = 250$

$$(t_2 - T) - (t_M - T) = \frac{1}{n}\left((E_2 - e_f \sin E_2) - (E_M - e_f \sin E_M)\right)$$

$E_2 = 4.904$ rad $\qquad E_M = 0.83$ rad

$$(t_2 - t_M) = 3.18 \times 10^4 \text{ sec} = \boxed{8.83\ \text{hr}}$$

Initial Orbit prior to BURN A

Satellite State
Position and Velocity in LVLH frame

r_hat:	16985.3694503186 km	rd_hat:	3.96206004579555 km/sec
t_hat:	0 km	td_hat:	4.16520258446032 km/sec
h_hat:	0 km	hd_hat:	0 km/sec

Position and Velocity in EPH/PQW frame

e_hat:	-5904.54964209152 km	ed_hat:	-5.2827467277274 km/sec
p_hat:	15926.05000896 km	pd_hat:	2.26702875737601 km/sec
h_hat:	0 km	hd_hat:	0 km/sec

Position and Velocity in IJK frame

i_hat:	-5904.54964209152 km	id_hat:	-5.2827467277274 km/sec
j_hat:	15926.05000896 km	jd_hat:	2.26702875737601 km/sec
k_hat:	0 km	kd_hat:	0 km/sec

RAD_MAG : 16985.3694503186 km = 0.00011354018242585 AU
VEL_MAG : 5.74863743647869 km/sec

Orbital Elements

sma:	28701.6165 km	raan:	0 deg
ecc:	0.75	arg_p:	0 deg
inc:	0 deg	nu:	110.34215621947 deg

Elliptic Orbital Parameters

P	:	12556.95721875 km = 8.3938074912096e-05 AU
ANG MOM	:	70747.5047324806 km^2/sec
PERIOD	:	48391.6560479429 sec = 13.4421266799842 hr
ENGERGY	:	-6.9438684751432 km^2/sec^2
RAD_PER	:	7175.404125 km = 4.79646142354834e-05 AU
RAD_APO	:	50227.828875 km = 0.00033752299648384 AU
VEL_CIRC	:	4.84430435286485 km/sec
VEL_ESC	:	6.8508809160845 km/sec
TRUE_ANOM	:	110.34215621947 deg
FPA	:	43.5681787781843 deg
ECC_ANOM	:	57.0246512717616 deg
MEAN_ANOM	:	20.9753721152261 deg
MEAN_MOT	:	0.00743929903211698 deg/sec

T_PAST_PER: 2819.53609132676 sec = 0.783204469812988 hr

Final Orbit after BURN A

Satellite State
Position and Velocity in LVLH frame

r_hat:	16985.3694503186 km	rd_hat:	2.20781046650545 km/sec
t_hat:	0 km	td_hat:	5.14163769642392 km/sec
h_hat:	0 km	hd_hat:	0 km/sec

```
Position and Velocity in EPH/PQW frame
e_hat:          4298.08309936272 km ed_hat:        -4.41562093301091 km/sec
p_hat:           16432.566964257 km pd_hat:          3.4370273541615 km/sec
h_hat:                         0 km hd_hat:                        0 km/sec

Position and Velocity in IJK frame
i_hat:          4298.08309936272 km id_hat:        -4.41562093301091 km/sec
j_hat:           16432.566964257 km jd_hat:          3.4370273541615 km/sec
k_hat:                         0 km kd_hat:                        0 km/sec

RAD_MAG    :      16985.3694503186 km =  0.00011354018242585 AU
VEL_MAG    :      5.59561124965794 km/sec

Orbital Elements
sma:                    25512.548 km raan:                         0 deg
ecc:                          0.5    arg_p:                         0 deg
inc:                            0 deg nu:            75.3421562194696 deg

Elliptic Orbital Parameters
P          :             19134.411 km = 0.000127905637961289 AU
ANG MOM    :      87332.6158534456 km^2/sec
PERIOD     :      40554.7474303377 sec =     11.2652076195383 hr
ENGERGY    :      -7.8118520345361 km^2/sec^2
RAD_PER    :             12756.274 km = 8.52704253075261e-05 AU
RAD_APO    :             38268.822 km = 0.000255811275922578 AU

VEL_CIRC   :      4.84430435286485 km/sec
VEL_ESC    :       6.8508809160845 km/sec
TRUE_ANOM  :      75.3421562194696 deg
FPA        :      23.2386031010182 deg
ECC_ANOM   :      48.0509596772919 deg
MEAN_ANOM  :      26.7443877268577 deg
MEAN_MOT   :  0.00887688921299249 deg/sec

T_PAST_PER:      3012.81080400482 sec =      0.83689189000134 hr

Delta V: 2.0076894964896463 km/sec
Alpha: 104.46737280529219 deg, Beta: 75.53262719470781 deg
Delta V : -0.5015783563342162 Vhat -1.9440257886077323 Chat 0.0 Nhat km/sec
Initial Orbit prior to BURN B

Satellite State
Position and Velocity in LVLH frame
r_hat:          37894.1834765912 km rd_hat:        -1.91421578684265 km/sec
t_hat:                         0 km td_hat:         1.86697530443384 km/sec
h_hat:                         0 km hd_hat:                        0 km/sec

Position and Velocity in EPH/PQW frame
e_hat:         -33782.9683437883 km ed_hat:          2.5522877157902 km/sec
p_hat:         -17166.2515197744 km pd_hat:       -0.797274282659297 km/sec
h_hat:                         0 km hd_hat:                        0 km/sec
```

```
Position and Velocity in IJK frame
i_hat:        -33782.9683437883 km id_hat:          2.5522877157902 km/sec
j_hat:        -17166.2515197744 km jd_hat:       -0.797274282659297 km/sec
k_hat:                        0 km kd_hat:                        0 km/sec

RAD_MAG   :     37894.1834765912 km = 0.000253306972061776 AU
VEL_MAG   :     2.67391452106522 km/sec

Orbital Elements
sma:                  28701.6165 km raan:                         0 deg
ecc:                        0.75    arg_p:                        0 deg
inc:                           0 deg nu:            206.936640510268 deg

Elliptic Orbital Parameters
P         :       12556.95721875 km =  8.3938074912096e-05 AU
ANG MOM   :     70747.5047324806 km^2/sec
PERIOD    :     48391.6560479429 sec =     13.4421266799842 hr
ENGERGY   :     -6.9438684751432 km^2/sec^2
RAD_PER   :          7175.404125 km = 4.79646142354834e-05 AU
RAD_APO   :         50227.828875 km = 0.000335752299648384 AU

VEL_CIRC  :     3.24326654904048 km/sec
VEL_ESC   :     4.58667154004403 km/sec
TRUE_ANOM :     206.936640510268 deg
FPA       :    -45.7157901795123 deg
ECC_ANOM  :     244.720103775369 deg
MEAN_ANOM :     283.576630833836 deg
MEAN_MOT  :  0.00743929903211698 deg/sec

T_PAST_PER:     38118.7299515152 sec =     10.5885360976431 hr

Final Orbit after BURN B

Satellite State
Position and Velocity in LVLH frame
r_hat:        37894.1834765912 km rd_hat:       0.320103368351105 km/sec
t_hat:                        0 km td_hat:         2.3046443501651 km/sec
h_hat:                        0 km hd_hat:                       0 km/sec

Position and Velocity in EPH/PQW frame
e_hat:       -37519.5449531824 km ed_hat:       -0.64020673670221 km/sec
p_hat:        5315.34456678806 km pd_hat:       -2.23695933835614 km/sec
h_hat:                        0 km hd_hat:                       0 km/sec

Position and Velocity in IJK frame
i_hat:       -37519.5449531824 km id_hat:       -0.64020673670221 km/sec
j_hat:        5315.34456678806 km jd_hat:       -2.23695933835614 km/sec
k_hat:                        0 km kd_hat:                       0 km/sec

RAD_MAG   :     37894.1834765912 km = 0.000253306972061776 AU
VEL_MAG   :     2.32676852032548 km/sec
```

```
Orbital Elements
sma:                25512.548 km raan:                              0 deg
ecc:                      0.5    arg_p:                            0 deg
inc:                        0 deg nu:                 171.936640510268 deg

Elliptic Orbital Parameters
P          :          19134.411 km = 0.00012790563796129 AU
ANG MOM    :    87332.6158534456 km^2/sec
PERIOD     :    40554.7474303377 sec =     11.2652076195383 hr
ENGERGY    :    -7.8118520345361 km^2/sec^2
RAD_PER    :          12756.274 km = 8.52704253075261e-05 AU
RAD_APO    :          38268.822 km = 0.00025581127592258 AU

VEL_CIRC   :    3.24326654904048 km/sec
VEL_ESC    :    4.58667154004403 km/sec
TRUE_ANOM  :    171.936640510268 deg
FPA        :    7.90750112767181 deg
ECC_ANOM   :    166.079635889963 deg
MEAN_ANOM  :    159.187725958912 deg
MEAN_MOT   :  0.00887688921299249 deg/sec

T_PAST_PER:     17932.8278340931 sec =     4.98134106502585 hr

Delta V: 2.2767820011711795 km/sec
Alpha: 124.63275391546344 deg, Beta: 55.36724608453655 deg
Delta V : -1.140998468913161 Vhat -1.6519502438792373 Chat 0.0 Nhat km/sec
Final orbit at nu=250

Satellite State
Position and Velocity in LVLH frame
r_hat:       23081.5964656903 km rd_hat:       -2.14445625400785 km/sec
t_hat:                      0 km td_hat:        3.78364711397937 km/sec
h_hat:                      0 km hd_hat:                       0 km/sec

Position and Velocity in EPH/PQW frame
e_hat:       -7894.37093138065 km ed_hat:        4.2889125080157 km/sec
p_hat:       -21689.6058747673 km pd_hat:      0.721046189272395 km/sec
h_hat:                      0 km hd_hat:                       0 km/sec

Position and Velocity in IJK frame
i_hat:       -7894.37093138065 km id_hat:        4.2889125080157 km/sec
j_hat:       -21689.6058747673 km jd_hat:      0.721046189272395 km/sec
k_hat:                      0 km kd_hat:                       0 km/sec

RAD_MAG    :     23081.5964656903 km =  0.00015429094321791 AU
VEL_MAG    :     4.34910083907902 km/sec

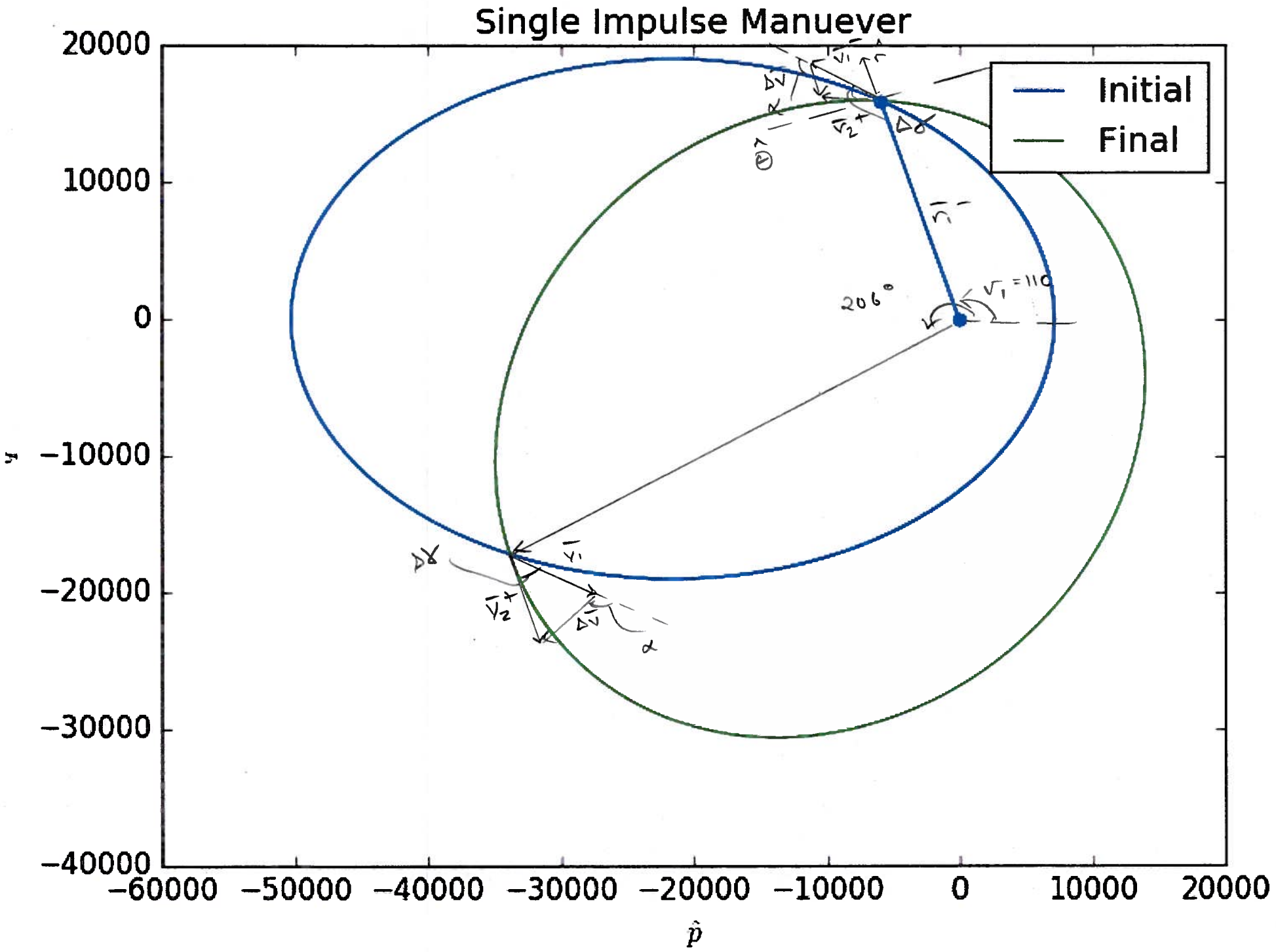
Orbital Elements
sma:                25512.548 km raan:                              0 deg
ecc:                      0.5    arg_p:                            0 deg
inc:                        0 deg nu:                           250 deg
```

```
Elliptic Orbital Parameters
P          :             19134.411 km = 0.000127905637961289 AU
ANG MOM    :      87332.6158534456 km^2/sec
PERIOD     :      40554.7474303377 sec =     11.2652076195383 hr
ENGERGY    :      -7.8118520345361 km^2/sec^2
RAD_PER    :             12756.274 km = 8.52704253075261e-05 AU
RAD_APO    :             38268.822 km = 0.000255811275922578 AU

VEL_CIRC   :      4.15562162483244 km/sec
VEL_ESC    :      5.87693646192895 km/sec
TRUE_ANOM  :                   250 deg
FPA        :     -29.5432472802594 deg
ECC_ANOM   :       280.98599752434 deg
MEAN_ANOM  :       309.10887991712 deg
MEAN_MOT   :  0.00887688921299249 deg/sec

T_PAST_PER:      34821.7570930928 sec =      9.67271030363689 hr

TOF from 75.3421562194696 to 250 : [ 31808.94628909] sec = [ 8.83581841] hr
```


Single Impulse Manuever
Initial
Final
20000
10000
0
−10000
−20000
−30000
−40000
−60000
−50000
−40000
−30000
−20000
−10000
0
10000
20000
206°
$\hat{p}$

HOMEWORK 5 SOLUTION

PROBLEM 4

MARS ORBIT -

$a = 5 R_\oplus$ $\quad \Omega = 45^\circ$

$e = 0.5$ $\quad \omega = -60^\circ$

$i = 30^\circ$ $\quad \nu = 120^\circ$

$$\Delta\bar{V} = 0.1\hat{x} - 0.25\hat{y} + 0.2\hat{z} \ \frac{km}{sec}$$

PART A — ROTATE $\Delta\bar{V}$ FROM MCI TO LVLH

$$\begin{bmatrix} \hat{r} \\ \hat{\theta} \\ \hat{h} \end{bmatrix} = ROT3(-\nu)\,ROT3(-\omega)\,ROT3(-i)\,ROT3(-\Omega) \begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{bmatrix}$$

$$\boxed{\Delta\bar{V} = -0.15\hat{r} + 0.034\hat{\theta} + 0.296\hat{h} \ km/sec}$$

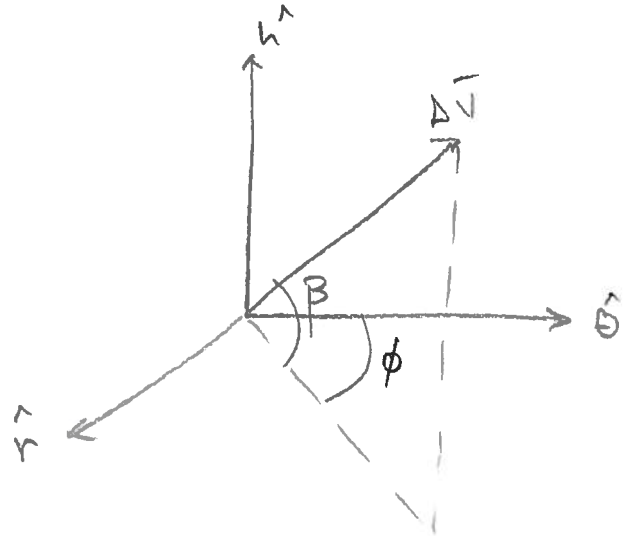
OUT OF PLANE COMPONENT IS $\Delta V_h = 0.29649$

$$\frac{\Delta V_h}{|\Delta\bar{V}|} = 98.53\% \text{ OF TOTAL } \Delta\bar{V}$$

PART B

$\Delta\bar{V}_{r\theta}$ IS PROJECTION OF $\Delta\bar{V}$ INTO ORBITAL PLANE

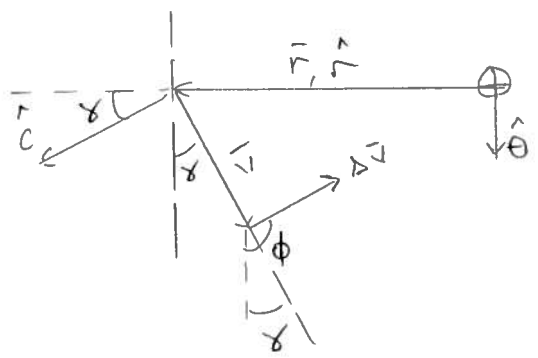
$$\Delta\bar{V}_{r\theta} = -0.15205\hat{r} + 0.034691\hat{\theta} \qquad \boxed{|\Delta\bar{V}_{r\theta}| = 0.1559 \ km/sec}$$



$$\tan\beta = \frac{\Delta V_h}{\Delta V_{r\theta}} \Rightarrow \beta = 62.29^\circ$$

$$\tan\phi = \frac{\Delta V_r}{\Delta V_\theta} \Rightarrow \phi = -77.14^\circ$$

α IS THE ANGLE BTWN THE VELOCITY VECTOR AND $\bar{V}_{R\Theta}$



$\Rightarrow \alpha = \phi - |\gamma|$

$(\cos\gamma)\, r\, v = h \rightarrow \gamma = -30^\circ$

$r_1 = 5 R_\sigma$

$V_1 = 1.58$ km/sec

$\Rightarrow \boxed{\alpha = -107.14^\circ}$

IMMEDIATELY AFTER THE MANEUVER

$\bar{r}_1^+ = r_1 \hat{r} = -0.883\, R_m \hat{x} + 4.414\, R_m \hat{y}$

$\bar{V}_1^- = V_{1r}\hat{r} + V_{1\Theta}\hat{\Theta} = 0.793\,\hat{r} + 1.375\,\hat{\Theta}$ km/sec

TRANSFORM TO MARS INERTIAL

$\bar{V}_I = -1.40\,\hat{x} + 0.28\,\hat{y} - 0.687\,\hat{z}$ km/sec

VELOCITY AFTER MANEUVER

$\bar{V}_1^+ = \bar{V}_I + \Delta\bar{V}$

$= -1.303\,\hat{x} + 0.031\,\hat{y} + 0.887\,\hat{z}$ km/sec.

PART D

FIND NEW ORBITAL ELEMENTS.

SEMI-MAJOR AXIS

$-\frac{\mu}{2a} = \frac{v^2}{2} - \frac{\mu}{r} \rightarrow \boxed{a^+ = 4.932 R_m}$

ECCENTRICITY

$h = \sqrt{\mu p} = \sqrt{\mu a (1-e^2)} \Rightarrow \boxed{e^+ = 0.407}$

$= |\bar{r}_1^+ \times \bar{V}_1^+|$

FROM THE TRANSFORMATION BETWN LVLH + MCI

$${}^{MCI}R^{LVLH} = \left[\begin{array}{c|c|c} \hat{r} & \hat{\theta} & \hat{h} \end{array}\right]$$ COLUMNS ARE LVLH FRAME EXPRESSED IN MCI FRAME

$$\hat{h} = 0.535\,\hat{x} - 0.282\,\hat{y} + 0.795\,\hat{z}$$

R_{33}

$$\cos i = R_{33} \rightarrow \boxed{i = +37.26^\circ}$$

$$\hat{r} = -0.176\,\hat{x} + 0.883\,\hat{y} + 0.483\,\hat{z} = \frac{\bar{r}}{|r|}$$

R_{31}

$$\hat{\theta} = \hat{h} \times \hat{r} = -0.825\,\hat{x} - 0.372\,\hat{y} + 0.425\,\hat{z}$$

R_{32}

$$\left.\begin{array}{l} \sin\Omega \sin i = 0.535 \\ -\cos\Omega \sin i = -0.283 \end{array}\right\} \begin{array}{l} \Omega^T = \underline{62.146^\circ}, 117.85^\circ \\ \Omega^+ = \underline{62.146^\circ}, -62.146^\circ \end{array}$$

$$\boxed{\Omega^+ = 62.146^\circ}$$

CONIC EQUATION

$$r = \frac{p}{1 + e\cos\nu} \rightarrow \boxed{\nu^+ = 115.723^\circ}$$

$$\sin i \cos\Theta = 0.423 \rightarrow \Theta_1^+ = 45.66^\circ, 134.33^\circ$$

$$\sin i \sin\Theta = 0.433 \rightarrow \Theta_1^+ = 45.66^\circ, -45.66^\circ$$

USE $\dot{r} > 0$ TO CHECK SIGN

$$\Theta_1^+ = 45.662^\circ = \omega + \nu$$

$$\boxed{\omega = -70.061^\circ} \text{ or } 289.9^\circ$$

WE CAN EXAMINE THE COLUMNS OF THE ROTATION MATRIX TO FIND $\hat{r}, \hat{\theta}, \hat{h}$ AT EACH ORBIT.

BEFORE MANEUVER

$$\hat{r}^- = -0.176777\,\hat{x} + 0.883883\,\hat{y} + 0.433013\,\hat{z}$$

$$\hat{\theta}^- = -0.918559\,\hat{x} - 0.306186\,\hat{y} + 0.25\,\hat{z}$$

$$\hat{h}^- = 0.353553\,\hat{x} - 0.353553\,\hat{y} + 0.866025\,\hat{z}$$

AFTER MANEUVER

$$\hat{r}^+ = -0.176777\,\hat{x} + 0.883883\,\hat{y} + 0.433013\,\hat{z}$$

$$\hat{\theta}^+ = -0.825972\,\hat{x} - 0.372479\,\hat{y} + 0.423119\,\hat{z}$$

$$\hat{h}^+ = 0.535276\,\hat{x} - 0.282859\,\hat{y} + 0.795909\,\hat{z}$$

$\hat{r}$ REMAINS UNCHANGED

$\hat{\theta}$ RISES OUT OF THE PLANE AND ROTATES W/IN THE INERTIAL PLANE

$\hat{h}$ MOVES INTO THE INERTIAL PLANE AND ROTATES TOWARDS $\hat{x}$

SINCE $\hat{h}$ IS NORMAL TO THE PLANE IT TELLS US THAT THE NEW ORBIT INCLINATION IS LARGER.

Initial Mars Orbit

Satellite State
Position and Velocity in LVLH frame

r_hat:	16985 km	rd_hat:	0.793968282983193 km/sec
t_hat:	0 km	td_hat:	1.37519340572511 km/sec
h_hat:	0 km	hd_hat:	0 km/sec

Position and Velocity in EPH/PQW frame

e_hat:	-8492.49999999999 km	ed_hat:	-1.58793656596639 km/sec
p_hat:	14709.4414832787 km	pd_hat:	4.071390352929 88e-16 km/sec
h_hat:	0 km	hd_hat:	0 km/sec

Position and Velocity in IJK frame

i_hat:	-3002.55216961337 km	id_hat:	-1.40355089236114 km/sec
j_hat:	15012.7608480669 km	jd_hat:	0.280710178472228 km/sec
k_hat:	7354.72074163934 km	kd_hat:	0.687596702862557 km/sec

RAD_MAG : 16985 km = 0.000113537712802996 AU
VEL_MAG : 1.58793656596639 km/sec

Orbital Elements

sma:	16985 km	raan:	45 deg
ecc:	0.5	arg_p:	-60 deg
inc:	30 deg	nu:	120 deg

Elliptic Orbital Parameters

P	:	12738.75 km = 8.51532846022473e-05 AU
ANG MOM	:	23357.6599962411 km^2/sec
PERIOD	:	67206.6534203762 sec = 18.6685148389934 hr
ENGERGY	:	-1.26077126876656 km^2/sec^2
RAD_PER	:	8492.5 km = 5.67688564014982e-05 AU
RAD_APO	:	25477.5 km = 0.000170306569204495 AU
VEL_CIRC	:	1.58793656596639 km/sec
VEL_ESC	:	2.24568142777782 km/sec
TRUE_ANOM	:	120 deg
FPA	:	30 deg
ECC_ANOM	:	90 deg
MEAN_ANOM	:	61.3521102434588 deg
MEAN_MOT	:	0.00535661250305394 deg/sec

T_PAST_PER: 11453.5278048357 sec = 3.18153550134325 hr

Delta V : -0.15204599827201593 rhat 0.034690689107605074 that 0.2969487674645336 hhat km/sec
Percentage out of plane : 88.53301731806333 %
Delta V inplane magnitude : 0.15204599827201593 km/sec
Beta (out of plane) : 62.88629553971101 deg
Phi (angle from theta_hat) : -77.14744821643559 deg
Alpha (angle from Vhat) : -107.14744821643559 deg
Final Orbit

```
Satellite State
Position and Velocity in LVLH frame
r_hat:                    16985 km rd_hat:        0.641922284711177 km/sec
t_hat:                        0 km td_hat:         1.44081634199539 km/sec
h_hat:                        0 km hd_hat:                        0 km/sec

Position and Velocity in EPH/PQW frame
e_hat:         -7372.01633611597 km ed_hat:        -1.57664264038573 km/sec
p_hat:          15301.7515383056 km pd_hat:      -0.0470524758893704 km/sec
h_hat:                        0 km hd_hat:                        0 km/sec

Position and Velocity in IJK frame
i_hat:         -3002.55216961337 km id_hat:        -1.30355089236114 km/sec
j_hat:          15012.7608480669 km jd_hat:       0.0307101784722282 km/sec
k_hat:          7354.72074163935 km kd_hat:        0.887596702862557 km/sec

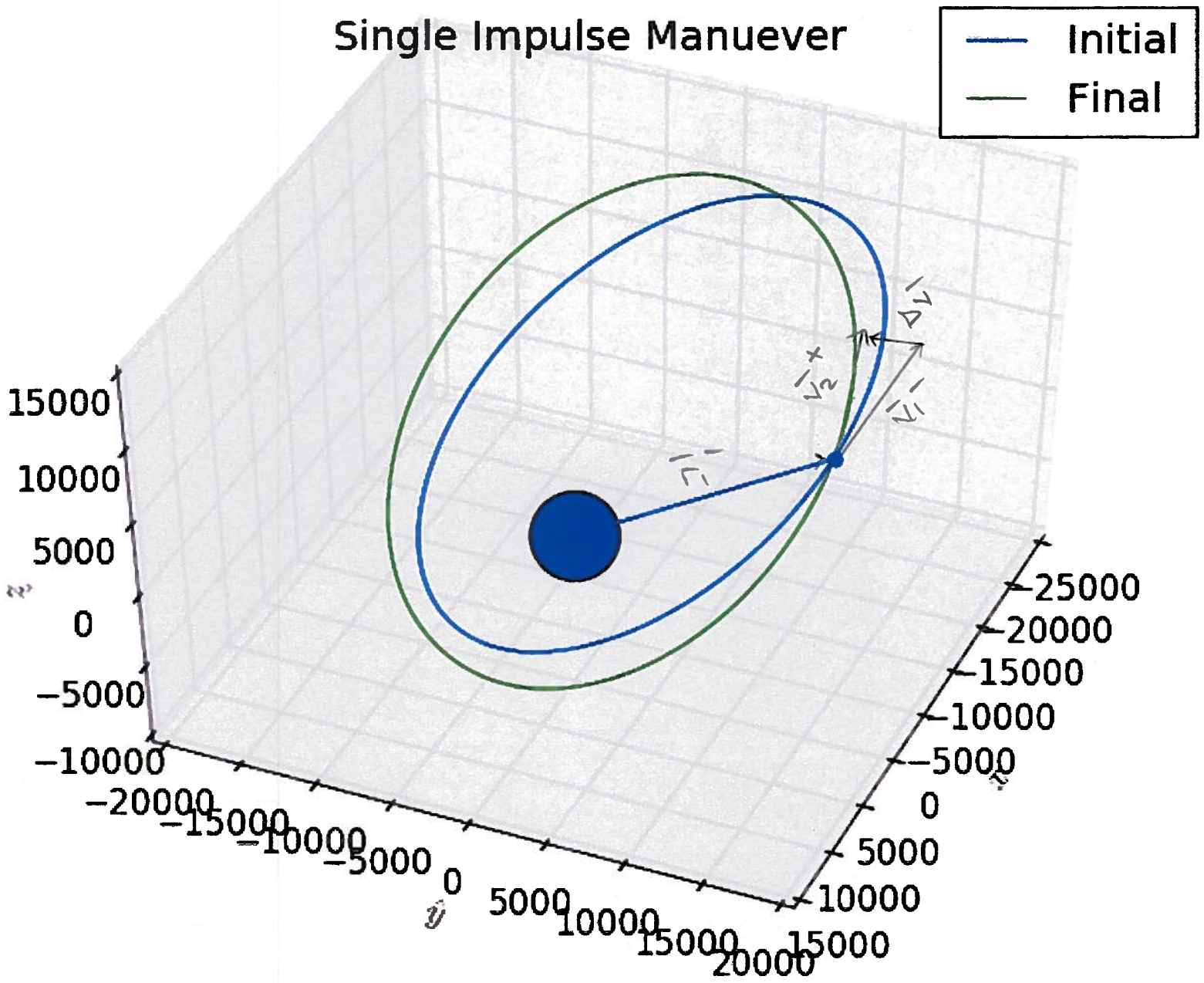
RAD_MAG   :                 16985 km = 0.000113537712802996 AU
VEL_MAG   :      1.57734458853156 km/sec

Orbital Elements
sma:              16762.129680208 km raan:           62.1464270483275 deg
ecc:           0.407145074139394    arg_p:            289.9384734766 deg
inc:              37.2588316405192 deg nu:           115.723650175731 deg

Elliptic Orbital Parameters
P         :      13983.5198622752 km = 9.34740572327349e-05 AU
ANG MOM   :      24472.2655687917 km^2/sec
PERIOD    :      65888.2143277418 sec =       18.302281757706 hr
ENGERGY   :     -1.27753456204822 km^2/sec^2
RAD_PER   :      9937.51114882559 km =   6.642815936367e-05 AU
RAD_APO   :      23586.7482115904 km = 0.000157667674089146 AU

VEL_CIRC  :      1.58793656596639 km/sec
VEL_ESC   :      2.24568142777782 km/sec
TRUE_ANOM :      115.723650175731 deg
FPA       :      24.0142530757261 deg
ECC_ANOM  :      91.8714305347672 deg
MEAN_ANOM :        68.55617857169 deg
MEAN_MOT  :  0.00546379961383207 deg/sec

T_PAST_PER:      12547.3449645068 sec =     3.48537360125189 hr
```


Single Impulse Manuever
Initial
Final
Δv_1
v_2^+
v_1^-
r_1
15000
10000
5000
0
−5000
−10000
−20000
−15000
−10000
−5000
0
5000
10000
15000
20000
25000
20000
15000
10000
5000
0
−5000

HOMEWORK 5 SOLUTION

PROBLEM 5

HOHMANN TRANSFER DOWN $r_1 = 1.25 R_\oplus$ $r_2 = 6.6 R_\oplus$

FIND: ΔV, α, TOF, PHASE ANGLE

INITIAL ORBIT

$a_1 = r_1 = 7972.7$ km $\qquad V_1 = \sqrt{\frac{\mu}{r_1}} = 7.07$ km/sec

$e_1 = 0$

$\gamma_1 = 0^\circ$

TRANSFER ORBIT

$a_T = \frac{1}{2}(r_p + r_a) = 3.925 R_\oplus = 25034.18$ km

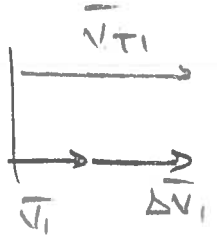
$r_p = a_T(1 - e_T) \Rightarrow \boxed{0.6815 = e_T}$

FIRST BURN

$V_{T1} = \sqrt{2\left(\mathcal{E} + \frac{\mu}{r_1}\right)} \qquad V_{T1} = 7.07 \frac{km}{sec}$

$\gamma_{T1} = 0^\circ \rightarrow$ PERIAPSIS

$\boxed{\Delta V_1 = 2.10 \frac{km}{sec}}$

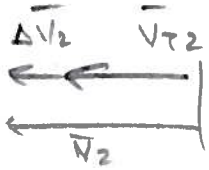


$\alpha = 0^\circ$ TANGENTIAL

SECOND BURN

$V_{T2} = \sqrt{2\left(\mathcal{E} + \frac{\mu}{r_2}\right)} \qquad V_{T2} = 3.077 \frac{km}{sec}$

$\gamma_{T2} = 0^\circ$



$\alpha = 0^\circ$

FINAL ORBIT

$V_2 = \sqrt{\frac{\mu}{r_2}} = 3.077 \frac{km}{sec}$

$\gamma_2 = 0^\circ$

$\boxed{\Delta V = 1.34 \text{ km/sec}}$

TOTAL $\Delta V = \Delta V_1 + \Delta V_2 = 3.438$ km/sec

TOF $\frac{P}{2} = \pi\sqrt{\frac{a_T^3}{\mu}} = 19709$ sec $= \boxed{5.48 \text{ hrs}}$

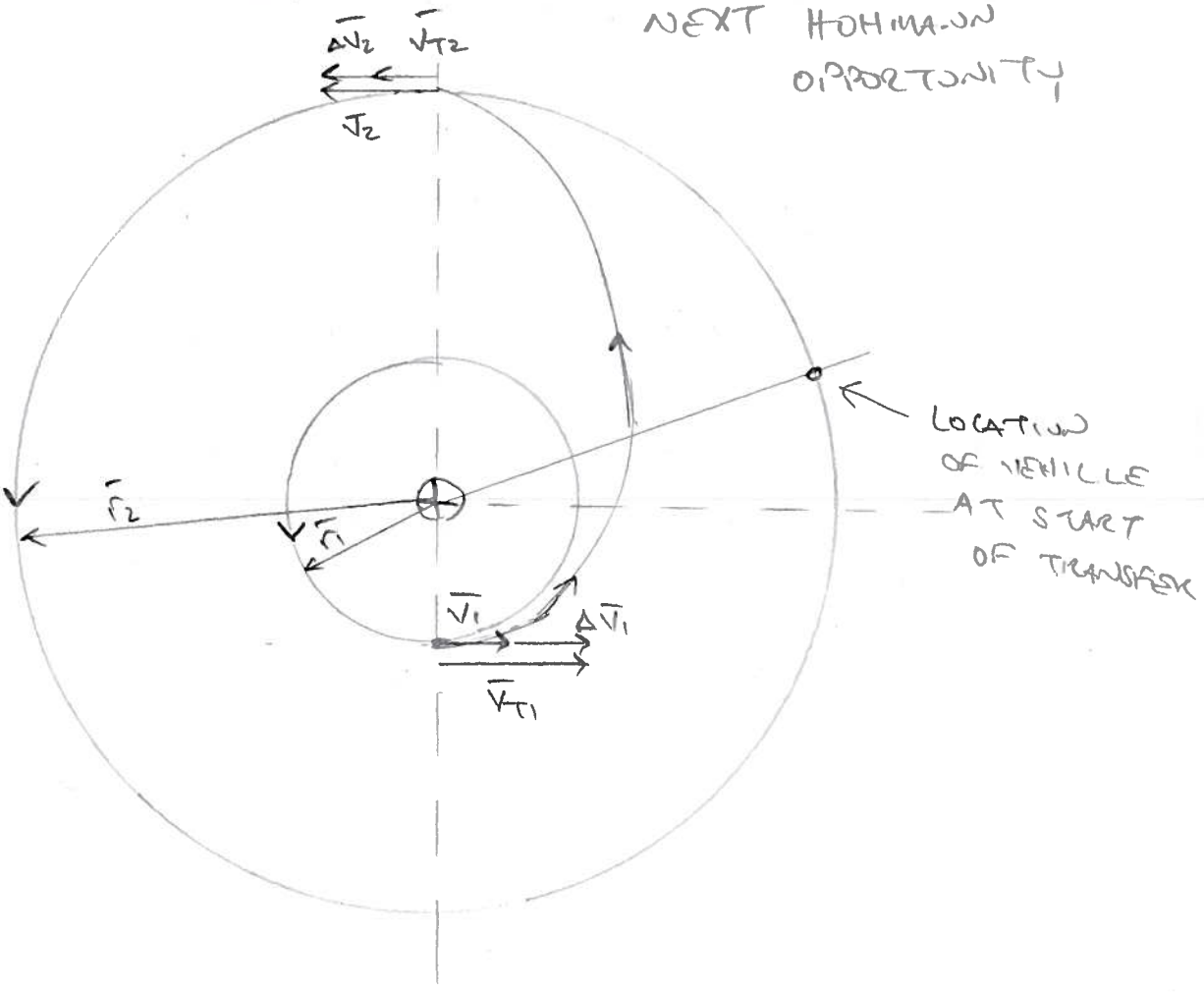
$n_2 = \sqrt{\frac{\mu}{a_2^3}} = 0.004179\ ^\circ/\text{sec}$

$180 - \phi = n_2 \text{ TOF} \rightarrow \boxed{\phi = 97.45^\circ}$ PHASE ANGLE

SYNODIC PERIOD

$S = \left|\frac{2\pi}{n_2 - n_1}\right| \rightarrow 7721 \text{ sec} = 2.14 \text{ hr}$

TIME TO WAIT FOR NEXT HOHMANN OPPORTUNITY



DV1 : 2.098165382653767 km/sec
DV2 : 1.340617999664432 km/sec
TOF : 19709.754279320045 sec = 5.474931744255568 hr
Phase Angle : 97.45018707236412 deg
Synodic Period : 7721.0128861138755 sec = 2.1447258016982986 hr