

Constants

$$\begin{aligned}\omega_{\oplus} &= 15^{\circ} \text{ sidereal hr}^{-1} \\ \omega_{\oplus} &= 0.000\,072\,921\,151\,467 \text{ rad solar sec}^{-1} \\ \mu_{\oplus} &= 398\,600.5 \text{ km}^3 \text{ s}^{-2} \\ R_{\oplus} &= 6378.137 \text{ km} \\ e_{\oplus} &= 0.081\,819\,190\,842\,6 \\ J_2 &= 1.082\,63 \times 10^{-3} \\ \rho_o &= 1.225 \text{ kg m}^{-3}\end{aligned}$$

Satellites - Basics

$$\begin{aligned}0 &= \ddot{\bar{r}} + \frac{\mu}{r^3} \bar{r} \quad r = \frac{a(1-e^2)}{1+e\cos\nu} \\ h &= rv\cos\gamma = \sqrt{\mu a(1-e^2)} \quad \bar{h} = \bar{r} \times \bar{v} \\ a &= \frac{r_a + r_p}{2} \quad e = \frac{2c}{2a} = \frac{r_a - r_p}{r_a + r_p} \\ r_p &= a(1-e) \quad r_a = a(1+e) \\ p &= a(1-e^2) = \frac{h^2}{\mu} \quad \bar{e} = \frac{\bar{v} \times \bar{h}}{\mu} - \frac{\bar{r}}{r} \\ P &= 2\pi\sqrt{\frac{a^3}{\mu}} \quad \varepsilon = \frac{v^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a}\end{aligned}$$

Time

$$\begin{aligned}\theta_g &= \theta_{g_0} + 1.00273790935 \times 2\pi * D \\ D &= \text{Day Num} - 1 + \frac{\text{HR}}{24} + \frac{\text{MIN}}{1440} \\ &+ \frac{\text{SEC}}{86400} \quad \text{where D is elapsed days} \\ \text{LST} &= \theta_g + \lambda_E \\ \text{EST} &= \text{UT} - 5 \quad \text{EDT} = \text{UT} - 4\end{aligned}$$

Preliminary Orbit Determination

$$\begin{aligned}x &= \left| \frac{a_e}{\sqrt{1-e^2\sin^2 L}} + H \right| \cos L \\ z &= \left| \frac{a_e(1-e^2)}{\sqrt{1-e^2\sin^2 L}} + H \right| \sin L \\ \bar{r}_s &= x \cos \text{LST} \hat{i} + x \sin \text{LST} \hat{j} + z \hat{k} \\ \rho_s &= -\rho \cos \alpha \cos \beta \\ \rho_e &= \rho \sin \alpha \cos \beta \\ \rho_z &= \rho \sin \beta \\ \dot{\rho}_s &= -\dot{\rho} \cos \alpha \cos \beta + \rho \dot{\alpha} \sin \alpha \cos \beta + \rho \dot{\beta} \cos \alpha \sin \beta \\ \dot{\rho}_e &= \dot{\rho} \sin \alpha \cos \beta + \rho \dot{\alpha} \cos \alpha \cos \beta - \rho \dot{\beta} \sin \alpha \sin \beta \\ \dot{\rho}_z &= \dot{\rho} \sin \beta + \rho \dot{\beta} \cos \beta \\ [\text{IJK}] &= \text{ROT}_3(\text{LST}) \text{ROT}_2(\text{COLAT}) [\text{SEZ}]\end{aligned}$$

$$\begin{aligned}\bar{r} &= \bar{\rho} + \bar{r}_s \\ \bar{v} &= \dot{\bar{\rho}} + \bar{\omega}_{\oplus} \times \bar{r} \\ \bar{v}_2 &= -\Delta t_{32} \left(\frac{1}{\Delta t_{21} \Delta t_{31}} + \frac{\mu}{12r_1^3} \right) \bar{r}_1 \\ &+ (\Delta t_{32} - \Delta t_{21}) \left(\frac{1}{\Delta t_{21} \Delta t_{32}} + \frac{\mu}{12r_2^3} \right) \bar{r}_2 \\ &+ \Delta t_{21} \left(\frac{1}{\Delta t_{32} \Delta t_{31}} + \frac{\mu}{12r_3^3} \right) \bar{r}_3\end{aligned}$$

Transfers

$$\begin{aligned}\Delta V_{OTB}^2 &= V_1^2 + V_2^2 - 2V_1 V_2 \cos \Delta\phi \\ \tan \phi &= \frac{e \sin \nu}{1 + e \cos \nu} \quad \Delta V_s = 2V_i \sin \frac{\theta}{2} \\ \Delta V_{COMB}^2 &= V_1^2 + V_2^2 - 2V_1 V_2 \cos \Delta i\end{aligned}$$

Rendezvous

$$\begin{aligned}\text{TOF} &= \pi \sqrt{\frac{a^3}{\mu}} \quad \omega = \sqrt{\frac{\mu}{r_{\text{circ}}^3}} \\ \alpha_{\text{lead}} &= \omega_t \times \text{TOF} \quad \phi_f = \pi - \alpha_{\text{lead}} \\ \text{Wait time}_{\text{coplanar}} &= \frac{\phi_f - \phi_i \pm 2\pi n}{\omega_t - \omega_i} \\ \text{Wait time}_{\text{noncoplanar}} &= \frac{\phi_f - \phi_i \pm 2\pi n}{\omega_t} = \frac{\alpha_i - \alpha_f + 2\pi n}{\omega_t}\end{aligned}$$

Keplers Problem

$$\begin{aligned}n &= \sqrt{\frac{\mu}{a^3}} \quad M_f = M_i + n \times \text{TOF} - 2k\pi \\ M &= E - e \sin E \\ \cos E &= \frac{e + \cos \nu}{1 + e \cos \nu} \quad \cos \nu = \frac{\cos E - e}{1 - e \cos E} \\ E_{n+1} &= E_n + \frac{M - E_n + e \sin E_n}{1 - e \cos E_n}\end{aligned}$$

COEs to RV

$$\begin{aligned}\bar{r} &= r [\cos \nu \hat{p} + \sin \nu \hat{q}] \\ \bar{v} &= \sqrt{\frac{\mu}{p}} [-\sin \nu \hat{p} + (e + \cos \nu) \hat{q}] \\ [\text{IJK}] &= \text{ROT}_3(\Omega) \text{ROT}_1(i) \text{ROT}_3(\omega) [PQW]\end{aligned}$$

Perturbations

$$\bar{n} = n_0 \left[1 + \frac{3}{2} J_2 \left(\frac{R_{\oplus}}{p_0} \right)^2 \sqrt{1 - e_0^2} \left(1 - \frac{3}{2} \sin^2 i_0 \right) \right]$$

$$n = n_0 + \dot{n}_0 \Delta t$$

$$\dot{e}_{drag} = \frac{-2(1 - e_0)\dot{n}_0}{3\bar{n}}$$

$$e = e_0 + \dot{e}_{drag} \Delta t$$

$$\dot{\Omega}_{J_2} = \left[-\frac{3}{2} J_2 \left(\frac{R_{\oplus}}{p_0} \right)^2 \cos i_0 \right] \bar{n}$$

$$\Omega = \Omega_0 + \dot{\Omega}_{J_2} \Delta t$$

$$\dot{\omega}_{J_2} = \left[\frac{3}{2} J_2 \left(\frac{R_{\oplus}}{p_0} \right)^2 \left(2 - \frac{2}{5} \sin^2 i_0 \right) \right] \bar{n}$$

$$\omega = \omega_0 + \dot{\omega}_{J_2} \Delta t$$

$$M = M_0 + n_0 \Delta t + \frac{\dot{n}_0}{2} \Delta t^2$$

$$\bar{a}_{drag} = -\frac{1}{2} \rho \frac{C_D A}{m} v \bar{v}$$

Entry - $p_0 = 1.225 \text{ kg m}^{-3}$

$$\dot{r} = v \sin \phi_E$$

$$\dot{v} = \frac{\rho_0}{2\Delta} v^2 \exp(-\beta h) \quad \beta = \frac{1}{7.315 \text{ km}}$$

$$\Delta = \frac{m}{C_D A}$$

$$v = v_e \exp \left(\frac{\rho_0}{2\Delta\beta \sin \phi_E} \exp(-\beta h) \right)$$

$$h_{max_g} = \frac{1}{\beta} \ln \left(\frac{-\rho_0}{\Delta\beta \sin \phi_E} \right)$$

$$max_g = \frac{-\beta v_E^2 \sin \phi_E}{2g_0 \exp(1)}$$

$$v_{max_g} = v_E \exp(-0.5) \approx 0.61 v_E$$

Proximity Operations

$$x(t) = \frac{\dot{x}_0}{\omega} \sin \omega t - \left(3x_0 + \frac{2\dot{y}_0}{\omega} \right) \cos \omega t + \left(4x_0 + \frac{2\dot{y}_0}{\omega} \right)$$

$$\dot{x}(t) = \dot{x}_0 \cos \omega t + (3\omega x_0 + 2\dot{y}_0) \sin \omega t$$

$$y(t) = \left(6x_0 + \frac{4\dot{y}_0}{\omega} \right) \sin \omega t + \frac{2\dot{x}_0}{\omega} \cos \omega t - (6\omega x_0 + 3\dot{y}_0) t + \left(y_0 - \frac{2\dot{x}_0}{\omega} \right)$$

$$\dot{y}(t) = (6\omega x_0 + 4\dot{y}_0) \cos \omega t - 2\dot{x}_0 \sin \omega t - (6\omega x_0 + 3\dot{y}_0)$$

$$z(t) = z_0 \cos \omega t + \frac{\dot{z}_0}{\omega} \sin \omega t$$

$$\dot{z}(t) = -z_0 \omega \sin \omega t + \dot{z}_0 \cos \omega t$$

Attitude Kinematics

$$ROT_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha \\ 0 & \sin \alpha & \cos \alpha \end{bmatrix}$$

$$ROT_2 = \begin{bmatrix} \cos \beta & 0 & \sin \beta \\ 0 & 1 & 0 \\ -\sin \beta & 0 & \cos \beta \end{bmatrix}$$

$$ROT_3 = \begin{bmatrix} \cos \gamma & -\sin \gamma & 0 \\ \sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Note: Assuming column vectors.