MAE3145: Homework 5

Due date: $2\,458\,078.2395$ JD

Problem 1. Observations at a certain instant indicate the following data for an Earth satellite:

$$r_c = 4R_\oplus \quad v_c = 4.54\,\mathrm{km\,s^{-1}} \quad \gamma_c = -40°$$

(a) Determine the following orbital characteristics of the original orbit:

$$a \quad e \quad \mathbb{P} \quad \mathcal{E} \quad r_p \quad r_a \quad \nu_c \quad E_c \quad (t_c - T)$$

(b) In exactly $8.5\,\mathrm{h}$, a maneuver will be implemented. What are the orbital characteristics in $8.5\,\mathrm{h}$ at the maneuver point? In other words, find the orbital properties, r_m, v_m^-, γ_m^- immediately before applying the maneuver.

(c) The maneuver is defined as $\|\Delta v\| = 1200\,\mathrm{m\,s^{-1}}$ (a very large maneuver) directed such that $\alpha = 30°$ with respect to the original velocity vector at the maneuver point. Determine the following properties immediately after the impulsive maneuver:

$$a^+ \quad e^+ \quad \mathbb{P}^+ \quad \mathcal{E}^+ \quad r_p^+ \quad r_a^+ \quad \nu_c^+ \quad E_c^+ \quad (t_c - T)^+ \quad \Delta\omega$$

Ensure you draw a proper vector diagram!

(d) Create a plot with both the old and new orbit. Mark the appropriate quantities on the plot.

Problem 2. A vehicle has been successfully launched into an orbit such that $e = 0.5$ and $a = 6.0R_\oplus$. Currently at $t = t_0$ the vehicle is located at perigee. A single in-plane maneuver is employed to circularize the orbit at geosynchronous **altitude** $r = 6.6R_\oplus$.

(a) Determine $\vec{r}_1^-, \vec{v}_1^-, \gamma_1^-$ at the maneuver point. These are the conditions on the orbit prior to the maneuver. The maneuver occurs at what value of ν_1^-? What is the "wait time" until the maneuver, i.e. how long to go from perigee to the required maneuver point?

(b) Compute the required maneuver $\|\Delta\vec{v}\|, \alpha$. Ensure you include a proper vector diagram. What are the conditions on the orbit immediately following the maneuver, i.e. find $\vec{r}_1^+, \vec{v}_1^+, \gamma_1^+$?

(c) Plot the old and new orbits (together). On the plot, mark the following:

$$\vec{r}_0 \quad \vec{r}_1 \quad \vec{v}_1^- \quad \nu_1^- \quad \text{local horizon} \quad \gamma_1^1 \quad \vec{v}_1^+ \quad \gamma_1^+ \quad \Delta\vec{v} \quad \alpha$$

Problem 3. A vehicle is currently in Earth orbit such that $e = 0.75$ and $a = 4.5R_\oplus$. A single in-plane maneuver will be used to raise perigee and lower apogee. New values are specified as $r_p = 2.0R_\oplus$ and $r_a = 6.0R_\oplus$. It is also required that perigee advance by $35°$, i.e. $\Delta\omega = +35°$.

(a) At what location ν in the original orbit should the maneuver be implemented? Determine $\vec{r}_1^-, \vec{v}_1^-, \gamma_1^-$ at the maneuver point.

(b) Determine the maneuver $(\|\Delta\vec{v}\|, \alpha, \beta)$ to accomplish the objective. If there are two possibilities, **ALWAYS** choose the one with lowest cost. Do not forget the vector diagrams! Can you deduce the lowest cost option from the vector diagrams? What are the values of $\vec{v}_1^+, \gamma^+$?

(c) Define the maneuver in the VNC reference frame, i.e. write down the vector $\Delta\vec{v} = a\hat{v} + b\hat{n} + c\hat{c}$. Create a plot with the old and new orbit as well as the properties:

$$\vec{r}_0 \quad \vec{r}_1 \quad \vec{v}_1^- \quad \nu_1^- \quad \text{local horizon} \quad \gamma_1^+ \quad \vec{v}_1^+ \quad \gamma_1^+ \quad \Delta\vec{v} \quad \alpha$$

(d) Determine the position and velocity at $\nu = 250°$ in the new orbit and mark this location on the plot. Determine the amount of time to go from the maneuver to $\nu = 250°$ in the new orbit.

Problem 4. As part of of an interplanetary mission, a spacecraft is in the following orbit around Mars (relative to a Mars-centered inertial coordinate frame):

$$a = 5R_{\mars} \quad e = 0.5 \quad i = 30° \quad \Omega = 45° \quad \omega = -60°$$

At $\nu = 120°$, the following maneuver is implemented

$$\Delta\vec{v} = 0.1\hat{x} - 0.25\hat{y} + 0.2\hat{z}\,\mathrm{km\,s}^{-1}.$$

(a) Transform the $\Delta\vec{v}$ into the $\hat{r}, \hat{\theta}, \hat{h}$ components corresponding to the original orbit. How much of the $\Delta\vec{v}$ is out of the plane? Define this out of plane component as Δv_h and find the percentage as compared to the total $\Delta\vec{v}$.

(b) Define $\Delta\vec{v}_{r\theta}$ as the projection of $\Delta\vec{v}$ in the orbital plane. Determine $\|\Delta\vec{v}_{r\theta}\|$, β, ϕ. Determine α between the velocity vector in the original orbit and $\Delta\vec{v}_{r\theta}$. Sketch this in-plane projection of components of the vector diagram. Add the unit vectors $\hat{r}, \hat{\theta}$ and $\hat{v}, \hat{c}$ to the sketch.

(c) To apply the maneuver, all position and velocities must be written in the same set of unit vectors, such as the inertial unit vectors $\hat{x}, \hat{y}, \hat{z}$. Determine the new $\vec{r}^+, \vec{v}^+$ immediately after the maneuver.

(d) Determine the orbital elements of the new orbit. Compare $\hat{r}^-, \hat{\theta}^-, \hat{h}^-$ (pre-maneuver in the original orbit) and $\hat{r}^+, \hat{\theta}^+, \hat{h}^+$ (post-maneuver in the new orbit).

Problem 5. In class, the optimality of the Hohmann transfer was discussed. Investigate it further by considering a transfer between circular, coplanar Earth orbits. The initial and final orbital radii are $r_1 = 1.25R_\oplus$ and $r_2 = 6.6R_\oplus$, respectively.

(a) For the Hohmann transfer, determine $\|\Delta\vec{v}\|$, α for each manuever, the total $\|\Delta\vec{v}\|$, and the time of flight. Remember to draw a vector diagram!

(b) Assume that this is a rendezvous problem, i.e. the vehicle should arrive at the final circular orbit to meet another spacecraft. What is the phase angle that is required between the two vehicles at departure from the lower orbit? Mark the pahse angle in a sketch of the transfer problem. What is the time before the same angular seperation is repeated?