MAE3145: Solution for Homework 6

Problem 1
```
clear all;
close all;
mu=398600;

rA=7000;
thetaA=210*pi/180;
vA=sqrt(mu/rA);
h1=rA*vA;
e1=0;
rB=6378;
thetaB=0*pi/180;

% (a)
vA1_vec=mu/h1*[-sin(thetaA) (e1+cos(thetaA))]

% (b)
e2=(rB-rA)/(rA*cos(thetaA)-rB*cos(thetaB))
h2=sqrt(mu*rA*rB)*sqrt((cos(thetaA)-cos(thetaB))/(rA*cos(thetaA)-rB*cos(thetaB)))

% (c)
vA2_vec=mu/h2*[-sin(thetaA) (e2+cos(thetaA))]

% (d)
delvA_vec=vA2_vec-vA1_vec

% (e)
vB3_vec=mu/h2*[-sin(thetaB) (e2+cos(thetaB))]

>> prob1
vA1_vec =
    3.7730   -6.5351
e2 =
    0.0500
h2 =
   5.1666e+04
vA2_vec =
    3.8575   -6.2956
delvA_vec =
    0.0844    0.2395
vB3_vec =
         0    8.1007
```

Problem 2
```
clear all;
close all;
mu=398600;
```

```
ra=13000;
rp=8000;

% (a) Position and Velocity at P
e1=(ra-rp)/(ra+rp);
h1=sqrt(rp*mu*(1+e1));

rP_vec=[rp 0]
vP1_vec=[0 h1/rp]

% (b) Position and Velocity at D

thetaD=pi/2;
rD=h1^2/mu/(1+e1*cos(thetaD));

rD_vec=rD*[cos(thetaD) sin(thetaD)]
vD1_vec=mu/h1*[-sin(thetaD) (e1+cos(thetaD))]

% (c) Time at D
ED=2*atan(sqrt((1-e1)/(1+e1))*tan(thetaD/2));
MeD=ED-e1*sin(ED);
a1=1/2*(ra+rp);
T1=2*pi/sqrt(mu)*a1^(3/2);
tD=MeD/2/pi*T1

% Time at C
thetaC=30*pi/180;
EC=2*atan(sqrt((1-e1)/(1+e1))*tan(thetaC/2));
MeC=EC-e1*sin(EC);
tC=MeC/2/pi*T1

tPD=tD-tC

% (d) Lambert Problem
[vP2_vec vD2_vec a e h]=LambertProb(rP_vec,rD_vec,tPD,mu)

% (e)
delvP=vP2_vec-vP1_vec
delvD=vD1_vec-vD2_vec

% (f)
delV=norm(delvP)+norm(delvD)

>> prob2
rP_vec =
        8000           0
vP1_vec =
```

```
          0    7.8542
rD_vec =
   1.0e+03 *
    0.0000    9.9048
vD1_vec =
   -6.3438    1.5104
tD =
   1.8731e+03
tC =
  542.7857
tPD =
   1.3304e+03
vP2_vec =
   -2.5294    9.5746
vD2_vec =
   -7.7333    4.3707
delvP =
   -2.5294    1.7204
delvD =
    1.3896   -2.8603
delV =
    6.2390
```

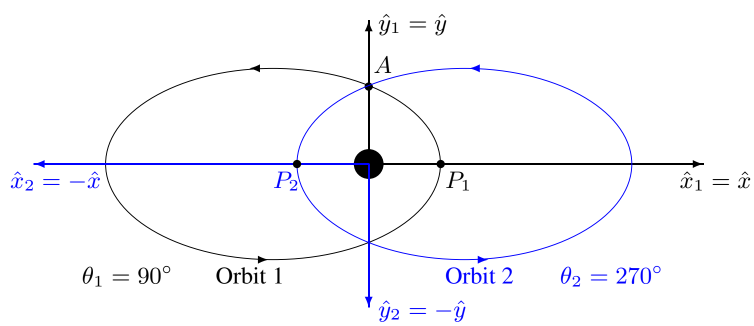
Problem 3 Consider the following two elliptic orbits that have the same eccentricity e and specific angular momentum h. The gravitational parameter is given by μ.

(a) Assume that $\vec{h}_1 = \vec{h}_2 = h\hat{z}$, i.e. in both orbits, the spacecraft rotates counter-clockwise. Find the magnitude of the required velocity change at A on Orbit 1 to transfer the spacecraft to Orbit 2.

Solution: The velocity vector of the i-th spacecraft can be written as

$$\vec{v}_i = \frac{\mu}{h}[-\sin\theta_i \hat{x}_i + (e + \cos\theta_i)\hat{y}_i],$$

where θ_i is measured along the direction of movement from the periapsis, $\hat{x}_i$ points toward the periapsis, and $\hat{y}_i$ points toward $\hat{h}_i \times \hat{x}_i$, or $\theta_i = 90°$. The unit-vectors $\hat{x}_i, \hat{y}_i$ and the true anomaly θ_i for both spacecraft are illustrated as follows:

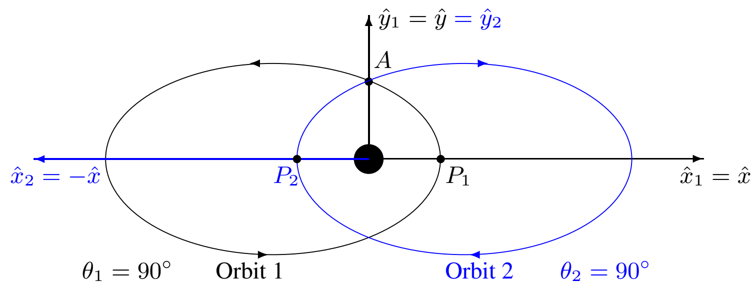


Therefore, the velocity vectors are given by

$$\vec{v}_1 = \frac{\mu}{h}[-\sin\theta_1 \hat{x}_1 + (e + \cos\theta_1)\hat{y}_1] = \frac{\mu}{h}[-\sin 90^\circ \hat{x} + (e + \cos 90^\circ)\hat{y}] = \frac{\mu}{h}[-\hat{x} + e\hat{y}],$$
$$\vec{v}_2 = \frac{\mu}{h}[-\sin\theta_2 \hat{x}_2 + (e + \cos\theta_2)\hat{y}_2] = \frac{\mu}{h}[-\sin 270^\circ(-\hat{x}) + (e + \cos 270^\circ))(-\hat{y})] = \frac{\mu}{h}[-\hat{x} - e\hat{y}],$$
$$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1 = -\frac{2\mu e}{h}\hat{y}.$$

(b) Assume that $\vec{h}_1 = -\vec{h}_2 = h\hat{z}$, i.e. the spacecraft rotates counter-clockwise on Orbit 1, and it rotates clockwise on Orbit 2. Find the magnitude of the required velocity change at A on Orbit 1 to transfer the spacecraft to Orbit 2.

Solution: Similarly, we have



Therefore, the velocity vectors are given by

$$\vec{v}_1 = \frac{\mu}{h}[-\sin\theta_1 \hat{x}_1 + (e + \cos\theta_1)\hat{y}_1] = \frac{\mu}{h}[-\sin 90^\circ \hat{x} + (e + \cos 90^\circ)\hat{y}] = \frac{\mu}{h}[-\hat{x} + e\hat{y}],$$
$$\vec{v}_2 = \frac{\mu}{h}[-\sin\theta_2 \hat{x}_2 + (e + \cos\theta_2)\hat{y}_2] = \frac{\mu}{h}[-\sin 90^\circ(-\hat{x}) + (e + \cos 90^\circ)\hat{y}] = \frac{\mu}{h}[\hat{x} + e\hat{y}],$$
$$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1 = \frac{2\mu}{h}\hat{x}.$$