2017 MIDTERM SOLUTION

PROBLEM 1

A $\mathcal{E} = -\frac{\mu}{2a}$

$a_{ISS} = 6378.137 + 422$ km

$a_{GPS} = 6378.137 + 20200$ km

$\mathcal{E}_{ISS} < \mathcal{E}_{GPS} \rightarrow \boxed{\text{FALSE}}$ $a \uparrow \rightarrow \mathcal{E} \uparrow$

B $\mathbb{P} = 2\pi\sqrt{\frac{a^3}{\mu}}$ $a \uparrow \rightarrow \mathbb{P} \uparrow$

$P_{ISS} < P_{GPS} \rightarrow \boxed{\text{FALSE}}$

C NEWTON → UNIVERSAL LAW OF GRAVITATION

D KEPLER → FIRST LAW

E PTOLEMY / COPERNICUS

F

1. THE ORBIT OF EACH PLANET IS AN ELLIPSE WITH THE SUN AT A FOCUS
2. THE LINE JOINING THE PLANET TO THE SUN SWEEPS OUT EQUAL AREAS IN EQUAL TIMES
3. THE SQUARE OF THE PERIOD OF A PLANET IS PROPORTIONAL TO THE CUBE OF ITS MEAN DISTANCE FROM THE SUN.

2017 MIDTERM SOLUTION

$$\ddot{\bar{r}} + \frac{\mu}{r^3}\bar{r} = 0$$

$$r = \frac{a(1-e^2)}{1+e\cos\nu}$$

c

$r_a = 25000$ km

$r_p = 8000$ km

$v_a = 2.8$ km/sec

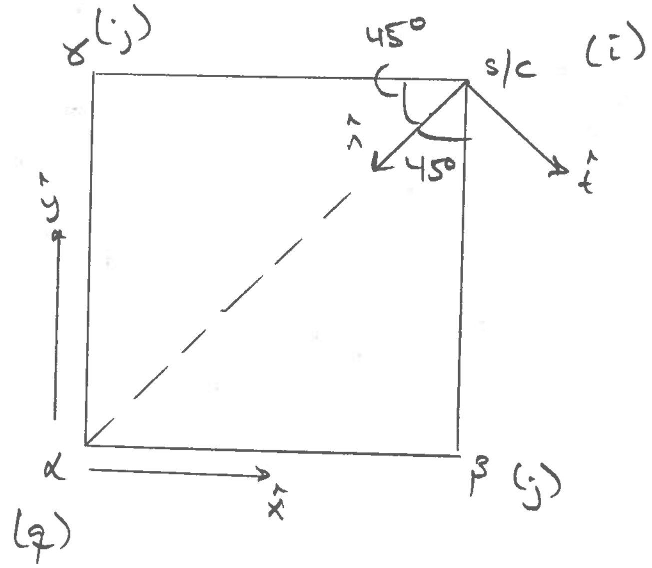
$v_p = 8.7$ km/sec

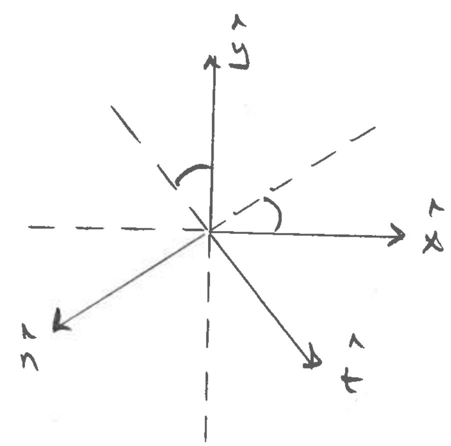
$$\varepsilon = -\frac{\mu}{2a} = \frac{v^2}{2} - \frac{\mu}{r}$$

$$v = \sqrt{2\left(\frac{\mu}{r} - \frac{\mu}{2a}\right)}$$

$v_p > v_a \rightarrow \boxed{\text{FALSE}}$

2017 MIDTERM SOLUTION





REFERENCE FRAME

$\hat{r}_{\alpha s} = -\hat{n}$

$\hat{r}_{s\beta} = -\hat{y}$

$\hat{r}_{\alpha\beta} = \hat{x}$

$\hat{r}_{s\gamma} = -\hat{x}$

$\hat{r}_{\alpha\gamma} = \hat{y}$

__A__ RELATIVE N-BODY EOMS.

$$\ddot{\bar{r}}_{qi} + \frac{\mu_i + \mu_q}{r_{qi}^2}\hat{r}_{qi} = \sum_{\substack{j=1 \\ j\neq i}}^{n} \mu_j \left(\frac{\hat{r}_{ij}}{r_{ij}^2} - \frac{\hat{r}_{qj}}{r_{qj}^2} \right)$$

MOTION OF S/C WRT ALPHA.

$$\ddot{\bar{r}}_{\alpha s} = \underset{\text{DOMINANT}}{\underbrace{-\frac{(\overset{0}{\cancel{\mu_s}} + \mu_\alpha)}{r_{\alpha s}^2}\hat{r}_{\alpha s}}} + \mu_\beta \left(\underset{\text{DIRECT}}{\underbrace{\frac{\hat{r}_{s\beta}}{r_{s\beta}^2}}} - \underset{\text{INDIRECT}}{\underbrace{\frac{\hat{r}_{\alpha\beta}}{r_{\alpha\beta}^2}}} \right) + \mu_\gamma \left(\underset{\text{DIRECT.}}{\underbrace{\frac{\hat{r}_{s\gamma}}{r_{s\gamma}^2}}} - \underset{\text{INDIRECT.}}{\underbrace{\frac{\hat{r}_{\alpha\gamma}}{r_{\alpha\gamma}^2}}} \right)$$ 4

__B__ DOMINANT ACCEL : $\frac{-2\mu}{(\sqrt{2}d)^2}(-\hat{n}) = \frac{\mu}{d^2}\hat{n} = \bar{A}_D$

DIRECT : $\frac{\mu}{d^2}(-\hat{y}) + \frac{\mu}{d^2}(-\hat{x}) = \bar{A}_{DIRECT}$ $\begin{cases} \text{MAG} : \frac{\sqrt{2}\mu}{d^2} \\ \text{DIRECTION} : +\hat{n} \end{cases}$

INDIRECT ACCEL : $-\frac{\mu}{d^2}(\hat{x}) - \frac{\mu}{d^2}(\hat{y}) = \bar{A}_{INDIRECT} \begin{cases} \text{MAG: } \frac{\sqrt{2}\,\mu}{d^2} \\ \text{DIR: } +\hat{n} \end{cases}$

4

C TOTAL ACCELERATION:

$$\frac{\mu}{d^2}\hat{n} - \frac{2\mu}{d^2}\hat{x} - \frac{2\mu}{d^2}\hat{y}$$

$$= \Big(\underbrace{\frac{\mu}{d^2}}_{\text{DOMINANT}} + \underbrace{\frac{2\sqrt{2}\,\mu}{d^2}}_{\text{PERTURBING}}\Big)\hat{n} \quad \begin{cases} \text{MAG: } 3.828\,\frac{\mu}{d^2} \\ \text{DIRECTION: } +\hat{n} \end{cases}$$

4

D NET PERTURBING ACCEL IN $+\hat{n}$ DIRECTION

$\Longrightarrow$ TOWARDS ALPHA

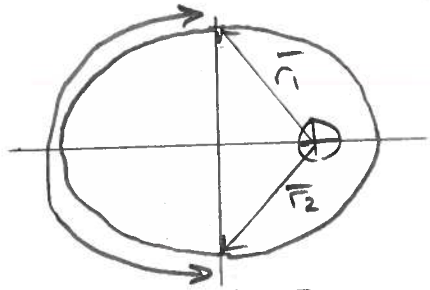
PERTURBING ACCEL > DOMINANT ACCEL. 4

$$\frac{\text{PERT}}{\text{DOM}} = 3.828\,!$$

A MODEL COMPRISED OF ONLY TWO BODIES (ALPHA + S/C) IS **NOT** REASONABLE. NEGLECTING THE PERTURBING ACCELERATION IS NOT A VALID ASSUMPTION. 4

2017 MIDTERM SOLUTION

A



$E_1 = 90°$ $E_2 = 270°$

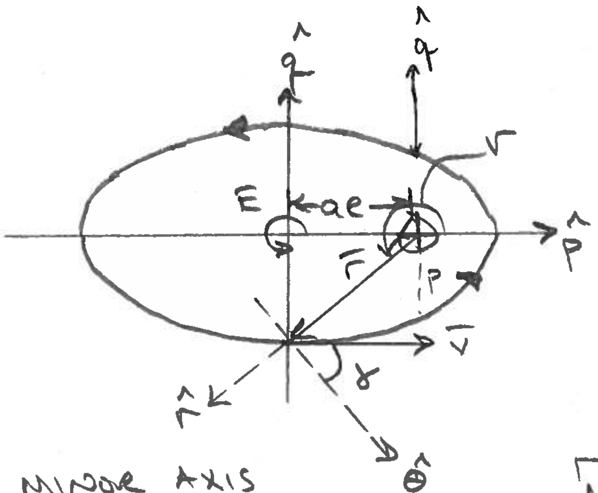
$n(t-T) = E - e\sin E$ +5

$$\left.\begin{aligned} t_1 - T &= \frac{1}{n}\left(\frac{\pi}{2} - e\right) \\ t_2 - T &= \frac{1}{n}\left(\frac{3\pi}{2} + e\right) \end{aligned}\right\} (t_2 - t_1) = \frac{1}{n}(\pi + 2e)$$ +5

$$P = \frac{2\pi}{n} \Rightarrow \boxed{\frac{t_2 - t_1}{P} = \frac{\pi + 2e}{2\pi}}$$ +5

B $e = 0.75 \Rightarrow \dfrac{t_2 - t_1}{P} = 0.7387 \Rightarrow \boxed{73.9\%}$ +5

2017 MIDTERM SOLUTION



$\bar{r} = -4R\,\hat{p} - 4\sqrt{3}\,\hat{q} \rightarrow r = 8R$

$|\bar{v}| = 3 \ \frac{rad}{sec} \qquad \bar{v} = v\,\hat{p}$

MINOR AXIS

$\boxed{r = a = 8R} \qquad \frac{a}{R} = 8$

$v = \sqrt{\frac{\mu}{R}}$

$\boxed{E = 270^\circ = -90^\circ}$

$ae = 4R \Rightarrow \boxed{e = 1/2}$

$\boxed{b = 4\sqrt{3}\,R} \qquad \frac{b}{R} = 4\sqrt{3}$

$p = a(1-e^2) = \boxed{6R = p} \qquad \frac{p}{R} = 6$

$$\mathcal{E} = \frac{v^2}{2} - \frac{\mu}{\underset{\uparrow \; v^2 = 9}{r}} \Rightarrow \mathcal{E} = -\frac{v^2}{2} \Rightarrow \boxed{\mathcal{E} = -4.5 \ \frac{km^2}{sec^2}}$$

$h^2 = \mu p$ BUT $v^2 = \frac{\mu}{r} = \frac{\mu}{8R} \Rightarrow h^2 = 432 R^2$

$$\boxed{\frac{h}{R} = 12\sqrt{3} \ \frac{km}{sec^2}}$$

$\boxed{\nu = 180^\circ + 60^\circ = 240^\circ}$

$\tan\gamma = \frac{e}{\sqrt{1-e^2}}$ @ MINOR AXIS

$\boxed{\gamma = -30^\circ \text{ BELOW LOCAL HORIZON}}$

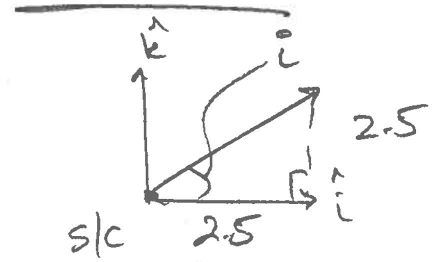
PROBLEM 5

$$\left.\begin{aligned}\bar{r} &= -26560\,\hat{j} \text{ km} \\ \bar{V} &= 2.5\,\hat{i} + 2.5\,\hat{k} \text{ km/sec}\end{aligned}\right\} \text{ ECI FRAME}$$

IN EQ. PLANE $\Rightarrow$ AT A NODE

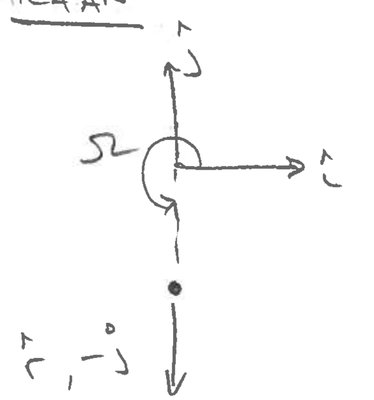
$\bar{V} \cdot \hat{k} > 0 \Rightarrow$ AT ASCENDING NODE !

INCLINATION



$\boxed{i = 45^\circ}$

RAAN



$\boxed{\Omega = 270^\circ}$

ECCENTRICITY

$$\bar{e} = \frac{\bar{V} \times \bar{h}}{\mu} - \frac{\bar{r}}{r} \Rightarrow$$

$$\bar{h} = \bar{r} \times \bar{V}$$

$$= (-26560 \cdot 2.5)(\hat{j} \times \hat{i}) + (-26560 \cdot 2.5)(\hat{j} \times \hat{k})$$

$$\boxed{\bar{h} = 66400\,\hat{i} - 66400\,\hat{k}}$$

$$\bar{e} = \frac{\bar{v} \times \bar{h}}{\mu} - \frac{\bar{r}}{r}$$

$$\bar{v} \times \bar{h} = (2.5\hat{i} + 2.5\hat{k}) \times (66400\hat{i} - 66400\hat{k})$$

$$= -166000\hat{j} - 166000\hat{j} = -332000\hat{j}$$

$$\boxed{\bar{e} = -0.8329\hat{j} + 1\hat{j} = 0.167\hat{j}}$$

$\bar{e}$ IN OPPOSITE DIRECTION OF $\bar{r}$!

$$\boxed{\begin{aligned} \omega &= 180^\circ \\ \nu &= 180^\circ \end{aligned}} \qquad \boxed{e = 0.167}$$

$$\mathcal{E} = -\frac{\mu}{2a} = \frac{v^2}{2} - \frac{\mu}{r}$$

$$\boxed{a = 22757.5 \text{ km}}$$