# MAE3145: Solution for Homework 4

**Problem 1** We observed the position and the velocity of a spacecraft orbiting the Earth as follows:

$$\vec{r}_0 = [6000, 6000, 6000]\,\mathrm{km}, \quad \vec{v}_0 = [-5, -5, 0]\,\mathrm{km/s}.$$

Assume that $\mu = 398600\,\mathrm{km}^3/\mathrm{s}^2$.

(a) Using the Matlab code shown in the class, find the orbital elements $(h, e, \theta, \Omega, \omega, i)$.

**Sol:** We use the Matlab function `rv2oe.m` as follows:

```
r_vec=[6000 6000 6000]';
v_vec=[-5 -5 0]';
[h,e,theta,Omega,omega,i]=rv2oe(r_vec,v_vec)
```

These commands yield:

```
h =   4.2426e+04 (km^2/s)
e =    0.8351
theta =   -2.3146 (rad)
Omega =    0.7854 (rad)
omega =    2.9301 (rad)
i =    1.5708 (rad)
```

(b) Write a Matlab function `oe2rv.m` that computes the position and the velocity vector for given orbital elements.

**Sol:**

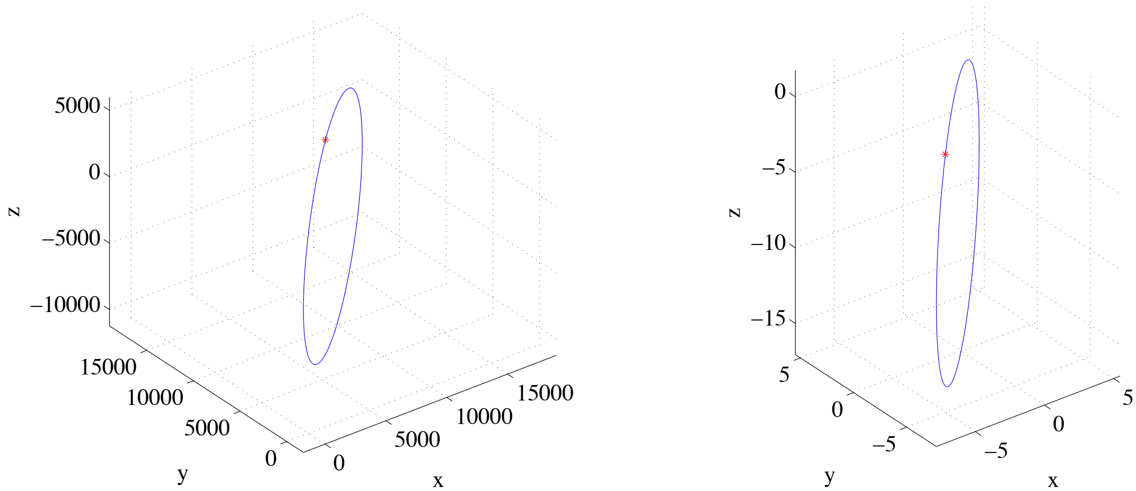
```
function [r_vec, v_vec]=oe2rv(h,e,theta,Omega,omega,i)
xhat=[1;0;0];
yhat=[0;1;0];
zhat=[0;0;1];
mu=398600;

Nhat=cos(Omega)*xhat+sin(Omega)*yhat;
hhat=sin(i)*sin(Omega)*xhat-sin(i)*cos(Omega)*yhat+cos(i)*zhat;
Nthat=-sin(Omega)*cos(i)*xhat+cos(Omega)*cos(i)*yhat+sin(i)*zhat;

urhat=cos(theta+omega)*Nhat+sin(theta+omega)*Nthat;
uthat=-sin(theta+omega)*Nhat+cos(theta+omega)*Nthat;

r=h^2/mu*1/(1+e*cos(theta));
E=-1/2*mu^2/h^2*(1-e^2);
v=sqrt(2*(E+mu/r));
gam=atan2(e*sin(theta),1+e*cos(theta));

r_vec=r*urhat;
v_vec=v*cos(gam)*uthat+v*sin(gam)*urhat;
```




(a) Position $\vec{r}$

(b) Velocity $\vec{v}$

(c) Evaluate the function `oe2rv.m` for varying `theta=linspace(0,2*pi,200)`. The other five orbital elements $(h, e, \Omega, \omega, i)$ are fixed at your solution of (a). Plot the position and the velocity vector in a three-dimensional space.

**Sol:** Matlab code is given as follows.

```
[h,e,theta,Omega,omega,i]=rv2oe(r_vec,v_vec);

theta=linspace(0,2*pi,200);
for k=1:200
     [r_vec_theta(:,k),v_vec_theta(:,k)]=oe2rv(h,e,theta(k),Omega,omega,i);
end
plot3(r_vec_theta(1,:),r_vec_theta(2,:),r_vec_theta(3,:));
hold on;
plot3(r_vec(1),r_vec(2),r_vec(3),'r*');
figure;
plot3(v_vec_theta(1,:),v_vec_theta(2,:),v_vec_theta(3,:));
hold on;
plot3(v_vec(1),v_vec(2),v_vec(3),'r*');
```

These commands generate the following figures in the next page.

(d) Check that $\vec{r}_0$ and $\vec{v}_0$ are on your curves at (c).

**Sol:** In the above figures, $\vec{r}_0$ and $\vec{v}_0$ are denoted by red stars, which are on the curves generated at (c).

**Problem 2** A satellite satisfies the following condition at the current time.

- $\vec{r} = [-6634.2, -1261.8, -5230.9]\,\mathrm{km}, \quad \vec{e} = [-0.40907, -0.48751, -0.63640]$
- It is flying toward its periapsis.

(a) What is the type of orbit.

**Sol:** Since $e = \|\vec{e}\| = 0.9$, it is an elliptic orbit.

(b) Find the direction of the specific angular momentum $\hat{h} = \frac{\vec{h}}{h}$.

**Sol:** The vectors $\vec{r}$ and $\vec{e}$ are on the orbital plane, and the vector $\hat{h}$ is normal to the orbital plane. Therefore, $\hat{h}$ can be obtained by the cross product of $\vec{r}$ and $\vec{e}$. Since it is flying toward its periapsis, we have $180^\circ < \theta < 360^\circ$. These imply

$$\hat{h} = \frac{\vec{r} \times \vec{e}}{\|\vec{r} \times \vec{e}\|} = [-0.4545\;\; -0.5417,\; 0.7071].$$

(c) Find the inclination $i$.

**Sol:** The inclination is given by $i = \cos^{-1}(\hat{h} \cdot \hat{z}) = 0.7854\,\mathrm{rad} = 45^\circ$.

(d) Find the direction of the node vector $\hat{N} = \frac{\vec{N}}{N}$. **Sol:** The direction of the node vector can be written as

$$\hat{N} = \frac{\hat{z} \times \vec{h}}{\|\hat{z} \times \vec{h}\|} = \frac{\hat{z} \times \hat{h}}{\|\hat{z} \times \hat{h}\|} = [0.7661, -0.6428, 0].$$

(e)-(g) **Sol:** Similarly, we have

$$\begin{aligned}
\Omega &= \tan^{-1}\left(\frac{\hat{y} \cdot \hat{N}}{\hat{x} \cdot \hat{N}}\right) = -0.6981\,\mathrm{rad} = -40^\circ, \\
\omega &= \tan^{-1}\left(\frac{\hat{h} \cdot (\hat{N} \times \vec{e})}{(\hat{N} \cdot \vec{e})}\right) = -1.5708\,\mathrm{rad} = -90^\circ, \\
\theta &= \tan^{-1}\left(\frac{\hat{h} \cdot (\vec{e} \times \vec{r})}{(\vec{e} \cdot \vec{r})}\right) = -0.5236\,\mathrm{rad} = -30^\circ.
\end{aligned}$$