Constants

$$\omega_\oplus = 15\,^\circ \text{ sidereal hr}^{-1}$$
$$\omega_\oplus = 0.000\,072\,921\,151\,467\,\text{rad solar sec}^{-1}$$
$$\mu_\oplus = 398\,600.5\,\text{km}^3\,\text{s}^{-2}$$
$$R_\oplus = 6378.137\,\text{km}$$
$$e_\oplus = 0.081\,819\,190\,842\,6$$
$$J_2 = 1.082\,63 \times 10^{-3}$$
$$\rho_o = 1.225\,\text{kg}\,\text{m}^{-3}$$

Satellites - Basics

$$0 = \ddot{\bar{r}} + \frac{\mu}{r^3}\bar{r} \qquad r = \frac{a\left(1-e^2\right)}{1+e\cos\nu}$$
$$h = rv\cos\phi = \sqrt{\mu a\left(1-e^2\right)} \qquad \bar{h} = \bar{r}\times\bar{v}$$
$$a = \frac{r_a + r_p}{2} \qquad e = \frac{2c}{2a} = \frac{r_a - r_p}{r_a + r_p}$$
$$r_p = a\left(1-e\right) \qquad r_a = a\left(1+e\right)$$
$$p = a\left(1-e^2\right) = \frac{h^2}{\mu} \qquad \bar{e} = \frac{\bar{v}\times\bar{h}}{\mu} - \frac{\bar{r}}{r}$$
$$P = 2\pi\sqrt{\frac{a^3}{\mu}} \qquad \varepsilon = \frac{v^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a}$$

Time

$$\theta_g = \theta_{g_0} + 1.00273790935 \times 2\pi * D$$
$$D = \text{Day Num} - 1 + \frac{\text{HR}}{24} + \frac{\text{MIN}}{1440} + \frac{\text{SEC}}{86400} \quad \text{where D is elapsed days}$$
$$\text{LST} = \theta_g + \lambda_E$$
$$\text{EST} = \text{UT} - 5 \qquad \text{EDT} = UT - 4$$

Preliminary Orbit Determination

$$x = \left|\frac{a_e}{\sqrt{1-e^2\sin^2 L}} + H\right|\cos L$$
$$z = \left|\frac{a_e\left(1-e^2\right)}{\sqrt{1-e^2\sin^2 L}} + H\right|\sin L$$
$$\bar{r}_s = x\cos\text{LST}\hat{\imath} + x\sin\text{LST}\hat{\jmath} + z\hat{k}$$
$$\rho_s = -\rho\cos\alpha\cos\beta$$
$$\rho_e = \rho\sin\alpha\cos\beta$$
$$\rho_z = \rho\sin\beta$$
$$\dot{\rho}_s = -\dot{\rho}\cos\alpha\cos\beta + \rho\dot{\alpha}\sin\alpha\cos\beta + \rho\dot{\beta}\cos\alpha\sin\beta$$
$$\dot{\rho}_e = -\dot{\rho}\sin\alpha\cos\beta + \rho\dot{\alpha}\cos\alpha\cos\beta - \rho\dot{\beta}\sin\alpha\sin\beta$$
$$\dot{\rho}_z = \dot{\rho}\sin\beta + \rho\dot{\beta}\cos\beta$$
$$[\text{IJK}] = ROT_3(-\text{LST})ROT_2(-\text{COLAT})\,[\text{SEZ}]$$

$$\bar{r} = \bar{\rho} + \bar{r}_s$$
$$\bar{v} = \dot{\bar{\rho}} + \bar{\omega}_\oplus \times \bar{r}$$
$$\bar{v}_2 = -\Delta t_{32}\left(\frac{1}{\Delta t_{21}\Delta t_{31}} + \frac{\mu}{12r_1^3}\right)\bar{r}_1 + \left(\Delta t_{32} - \Delta t_{21}\right)\left(\frac{1}{\Delta t_{21}\Delta t_{32}} + \frac{\mu}{12r_2^3}\right)\bar{r}_2 + \Delta t_{21}\left(\frac{1}{\Delta t_{32}\Delta t_{31}} + \frac{\mu}{12r_3^3}\right)\bar{r}_3$$

Transfers

$$\Delta V_{OTB}^2 = V_1^2 + V_2^2 - 2V_1V_2\cos\Delta\phi$$
$$\tan\phi = \frac{e\sin\nu}{1+e\cos\nu} \qquad \Delta V_s = 2V_i\sin\frac{\theta}{2}$$
$$\Delta V_{COMB}^2 = V_1^2 + V_2^2 - 2V_1V_2\cos\Delta i$$

Rendezvous

$$\text{TOF} = \pi\sqrt{\frac{a^3}{\mu}} \qquad \omega = \sqrt{\frac{\mu}{r_{circ}^3}}$$
$$\alpha_{lead} = \omega_t \times \text{TOF} \qquad \phi_f = \pi - \alpha_{lead}$$
$$\text{Wait time}_{coplanar} = \frac{\phi_f - \phi_i \pm 2\pi n}{\omega_t - \omega_i}$$
$$\text{Wait time}_{noncoplanar} = \frac{\phi_f - \phi_i \pm 2\pi n}{\omega_t} = \frac{\alpha_i - \alpha_f + 2\pi n}{\omega_t}$$

Keplers Problem

$$n = \sqrt{\frac{\mu}{a^3}} \qquad M_f = M_i + n \times \text{TOF} - 2k\pi$$
$$M = E - e\sin E$$
$$\cos E = \frac{e+\cos\nu}{1+e\cos\nu} \qquad \cos\nu = \frac{\cos E - e}{1 - e\cos E}$$
$$E_{n+1} = E_n + \frac{M - E_n + e\sin E_n}{1 - e\cos E_n}$$

COEs to RV

$$\bar{r} = r\left[\cos\nu\hat{p} + \sin nu\hat{q}\right]$$
$$\bar{v} = \sqrt{\frac{\mu}{p}}\left[-\sin\nu\hat{p} + (e+\cos\nu)\hat{q}\right]$$
$$[\text{IJK}] = ROT_3(-\Omega)ROT_1(-i)ROT_3(-\omega)\,[PQW]$$

Perturbations

$$\bar{n} = n_0 \left[1 + \frac{3}{2} J_2 \left(\frac{R_\oplus}{p_0}\right)^2 \sqrt{1 - e_0^2} \left(1 - \frac{3}{2}\sin^2 i_0\right)\right]$$

$$n = n_0 + \dot{n}_0 \Delta t$$

$$\dot{e}_{drag} = \frac{-2(1 - e_0)\dot{n}_0}{3\bar{n}}$$

$$e = e_0 + \dot{e}_{drag}\Delta t$$

$$\dot{\Omega}_{J_2} = \left[-\frac{3}{2} J_2 \left(\frac{R_\oplus}{p_0}\right)^2 \cos i_0\right] \bar{n}$$

$$\Omega = \Omega_0 + \dot{\Omega}_{J_2}\Delta t$$

$$\dot{\omega}_{J_2} = \left[\frac{3}{2} J_2 \left(\frac{R_\oplus}{p_0}\right)^2 \left(2 - \frac{2}{5}\sin^2 i_0\right)\right] \bar{n}$$

$$\omega = \omega_0 + \dot{\omega}_{J_2}\Delta t$$

$$M = M_0 + n_0 \Delta t + \frac{\dot{n}_0}{2}\Delta t^2$$

$$\bar{a}_{drag} = -\frac{1}{2}\rho \frac{C_D A}{m} v\bar{v}$$

Entry - $p_0 = 1.225\,\mathrm{kg\,m^{-3}}$

$$\dot{r} = v \sin\phi_E$$

$$\dot{v} = \frac{\rho_0}{2\Delta} v^2 \exp -\beta h \qquad \beta = \frac{1}{7.315\,\mathrm{km}}$$

$$\Delta = \frac{m}{C_D A}$$

$$v = v_e \exp\left(\frac{\rho_0}{2\Delta\beta\sin\phi_E}\exp(-\beta h)\right)$$

$$h_{max_g} = \frac{1}{\beta}\ln\left(\frac{-\rho_0}{\Delta\beta\sin\phi_E}\right)$$

$$max_g = \frac{-\beta v_E^2 \sin\phi_E}{2 g_0 \exp(1)}$$

$$v_{max_g} = v_E \exp(-0.5) \approx 0.61 v_E$$

Proximity Operations

$$x(t) = \frac{\dot{x}_0}{\omega}\sin\omega t - \left(3x_0 + \frac{2\dot{y}_0}{\omega}\right)\cos\omega t + \left(4x_0 + \frac{2\dot{y}_0}{\omega}\right)$$

$$\dot{x}(t) = \dot{x}_0 \cos\omega t + (3\omega x_0 + 2\dot{y}_0)\sin\omega t$$

$$y(t) = \left(6x_0 + \frac{4\dot{y}_0}{\omega}\right)\sin\omega t + \frac{2\dot{x}_0}{\omega}\cos\omega t - (6\omega x_0 + 3\dot{y}_0)\,t + \left(y_0 - \frac{2\dot{x}_0}{\omega}\right)$$

$$\dot{y}(t) = (6\omega x_0 + 4\dot{y}_0)\cos\omega t - 2\dot{x}_0 \sin\omega t - (6\omega x_0 + 3\dot{y}_0)$$

$$z(t) = z_0 \cos\omega t + \frac{\dot{z}_0}{\omega}\sin\omega t$$

$$\dot{z}(t) = -z_0\omega\sin\omega t + \dot{z}_0\cos\omega t$$

Attitude Kinematics

$$ROT_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$ROT_2 = \begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix}$$

$$ROT_3 = \begin{bmatrix} \cos\gamma & \sin\gamma & 0 \\ -\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$