Summary of Orbital Properties

Any Type of Orbit

$$\vec{h} = \vec{r} \times \vec{v}, \qquad \vec{r} = r\hat{u}_r \qquad \vec{v} = v_r\hat{u}_r + v_\theta\hat{u}_\theta,$$

$$h = rv\cos\gamma = rv_\theta = r^2\dot{\theta}, \qquad r = \frac{h^2/\mu}{1 + e\cos\theta}, \qquad v_r = \frac{\mu}{h}e\sin\theta = \dot{r},$$

$$\vec{e} = \frac{\vec{v} \times \vec{h}}{\mu} - \frac{\vec{r}}{r}, \qquad r_p = \frac{h^2/\mu}{1 + e}, \qquad v_\theta = \frac{\mu}{h}(1 + e\cos\theta) = r\dot{\theta},$$

$$\mathcal{E} = \frac{1}{2}v^2 - \frac{\mu}{r} = -\frac{1}{2}\frac{\mu^2}{h^2}(1 - e^2), \; \tan\gamma = \frac{v_r}{v_\theta} = \frac{e\sin\theta}{1 + e\cos\theta}.$$

Circular Orbits: ($e = 0$)

$$v = \sqrt{\frac{\mu}{r}}, \qquad \mathcal{E} = -\frac{\mu}{2r}, \qquad T = \frac{2\pi}{\sqrt{\mu}}r^{3/2}.$$

Elliptic Orbits: ($0 < e < 1$)

$$r = \frac{a(1 - e^2)}{1 + e\cos\theta}, \qquad a = \frac{h^2/\mu}{1 - e^2} = \frac{1}{2}(r_a + r_p), \qquad T = \frac{2\pi}{\sqrt{\mu}}a^{3/2},$$

$$r_p = \frac{h^2/\mu}{1 + e} = a(1 - e), \qquad b = a\sqrt{1 - e^2}, \qquad e = \frac{r_a - r_p}{r_a + r_p},$$

$$r_a = \frac{h^2/\mu}{1 - e} = a(1 + e), \qquad \mathcal{E} = -\frac{\mu}{2a}, \qquad h = \sqrt{\mu a(1 - e^2)}.$$

Parabolic Orbits: ($e = 1$)

$$v = \sqrt{\frac{2\mu}{r}}, \qquad \mathcal{E} = 0.$$

Hyperbolic Orbits: ($e > 1$)

$$r = \frac{a(e^2 - 1)}{1 + e\cos\theta}, \qquad a = \frac{h^2/\mu}{e^2 - 1}, \qquad \theta_\infty = \cos^{-1}(-1/e),$$

$$r_p = a(e - 1), \qquad b = a\sqrt{e^2 - 1}, \qquad \beta = \cos^{-1}(1/e),$$

$$\mathcal{E} = \frac{\mu}{2a}, \qquad h = \sqrt{\mu a(e^2 - 1)}.$$