Coupled Asteroid Trajectory Design: From Orbit to Landing

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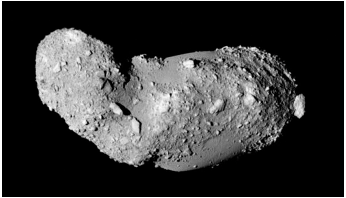
Asteroid Missions

- Science - insight into the early formation of the solar system
- Mining - vast quantities of useful materials
- Impact - high risk from hazardous near-Earth asteroids



Asteroid Mining

- Useful materials can be extracted from asteroids to support:
  - Propulsion, construction, life support, agriculture, and precious/strategic metals
- Commercialization of near-Earth asteroids is feasible

Element	Price ($/kg)	Sales ($M/yr)
Phosphorous (P)	0.08	2167
Gallium (Ga)	300.00	1544
Germanium (Ge)	745.00	6145
Platinum (Pt)	12 394.00	1705
Gold (Au)	12 346.00	49
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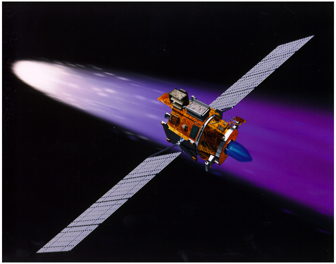
Low-thrust vehicles

- Low-thrust orbital transfers increase mission oportunities
  - Electric propulsion is increasing in capability
  - Offers much higher specific impulse than chemical engines
  - Requires much longer operating periods for maneuvers
  - Enables long duration missions with frequent thrusting



Ideal Rockets

Spacecraft Autonomy

- Autonomous control of space vehicles is critical
  - Avoid extensive planning and interaction by operators
  - Ability to operate safely with system uncertainty
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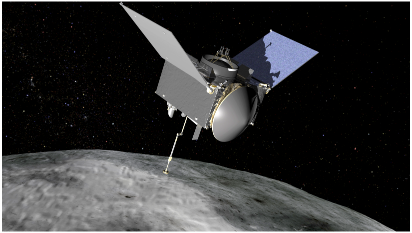

Why send spacecraft to asteroids?

- Some properties only available at the asteroid:
  - High fidelity gravitational model
  - Surface samples or return missions
- Gain experience for future missions
  - Weak gravitational field allows for less costly manuevers
  - Asteroid tours for future deep-space human missions
- Avoiding future impacts
  - Local spacecraft can aid in ground based tracking
  - Mitigation: Gravity tractors, kinetic impactors, solar sails

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Proposed research

Thesis Statement

The application of geometric mechanics enables the innovative design of complex trajectories around asteroids, including orbital transfers, hovering, and landing.

- Geometric mechanics enables novel capabilities
  - Poincaré sections allow for insight into dynamics
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Asteroid mission design

- Reachability set on Poincaré section allows for systematic transfer design
  - Transfer design on lower dimensional subspace
  - Simple method to incorporate effects of low-thrust
  - Avoids the issue of determining initial conditions
- Coupled dynamics play a critical role when near the surface
  - Irregular asteroid shape causes complex gravity field
  - Requires a global representation of kinematics
  - Autonomous obstacle avoidance for robust landing scenarios

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Challenges for Optimal Transfer Design

- Optimization in astrodynamics
  - Orbital dynamics are nonlinear and chaotic
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  - Intuition required by designer to enable convergence
- Transfers using low-thrust propulsion
  - Requires long periods of thrusting/coasting
  - Small perturbations require accurate numerical integration
  - Difficult to capture the long-term effects accurately
- Direct Optimal Control
  - Reformulate problem as parameter optimization
  - Allows for use of nonlinear programming methods
  - High dimensional problem and computationally intensive
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Dynamic System Modelling

- Astrodynamics - motion of objects in space ▸ Intro to Astro
- Attitude coupling is dependent on ratio $\epsilon = \frac{l}{R}$
  - Typically ignored for Earth based missions
  - Force depends on attitude and oment depends on position
  - Vastly different time scales

$$m\dot{v} = mv \times \Omega + \sum F(b, R)$$

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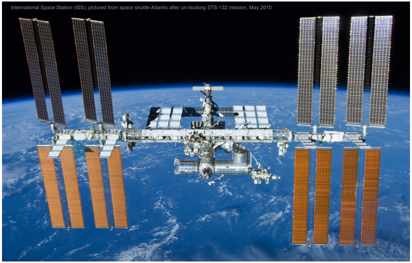
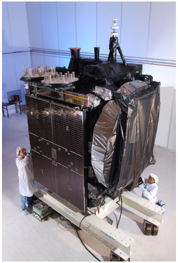
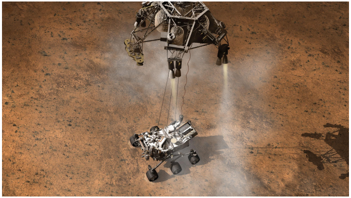
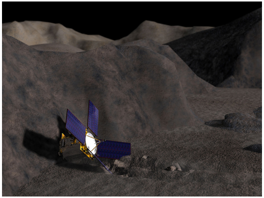




Planetary Landing

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  - Extensive human planning and analysis

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- Unable to handle failures
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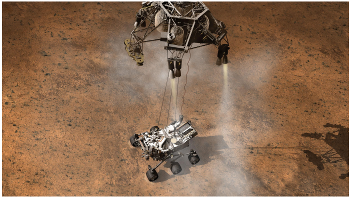

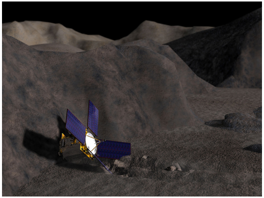

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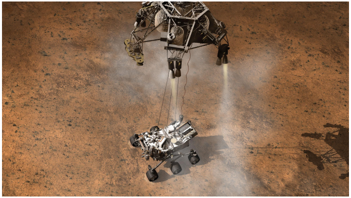

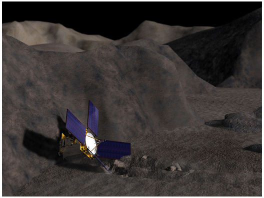

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- Intersection of a periodic orbit with a lower dimensional subspace
  - Poincaré section - discrete map between intersections
- Useful for investigating the stability and structure
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  - Used for initial and target periodic orbits
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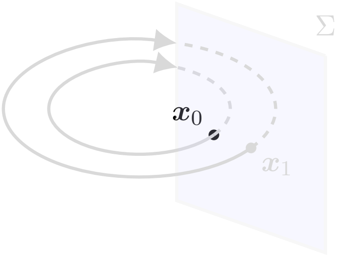

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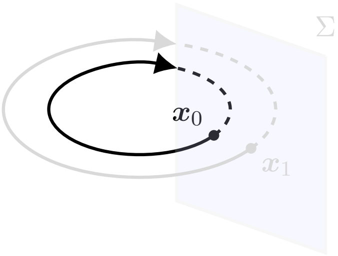

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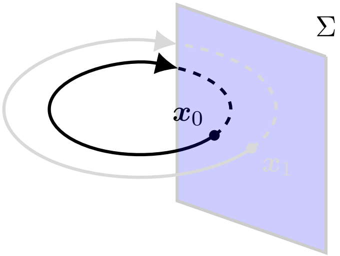

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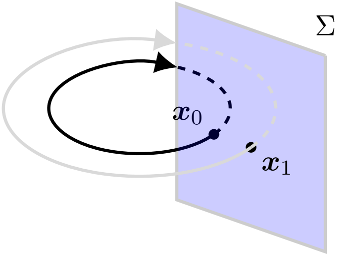

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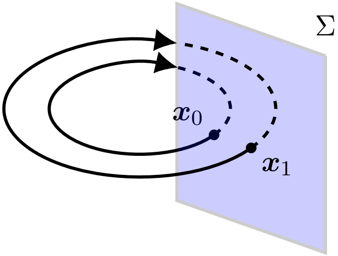

Reachability Set

- Set of states achievable from a given initial condition over fixed t_f s.t. maximum control constraint

$$R(\boldsymbol{x}_0, \mathcal{U}, t_f) = \{\boldsymbol{x}_f \subseteq \mathcal{X} | \exists \boldsymbol{u} \in \mathcal{U}, \boldsymbol{x}(t_f) = \boldsymbol{x}_f\}$$

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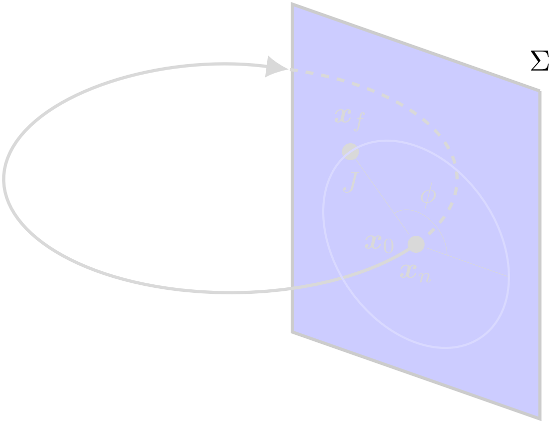

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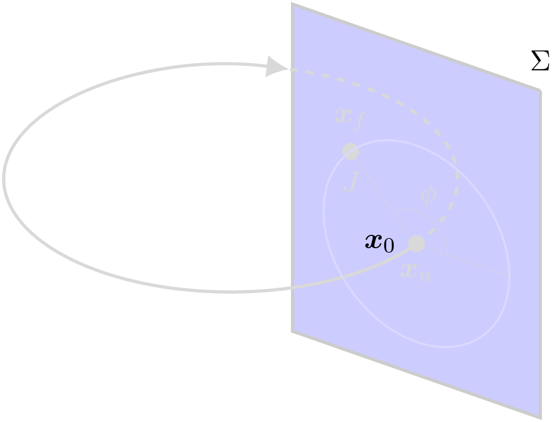

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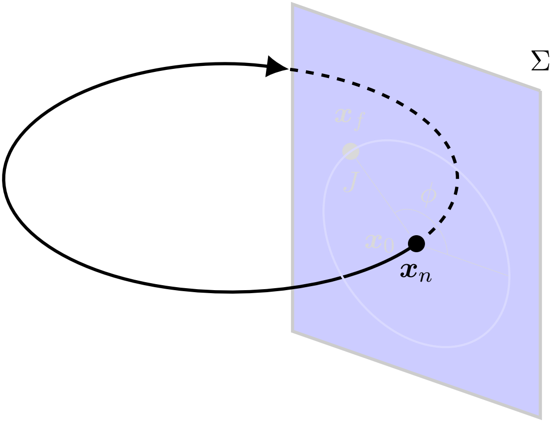

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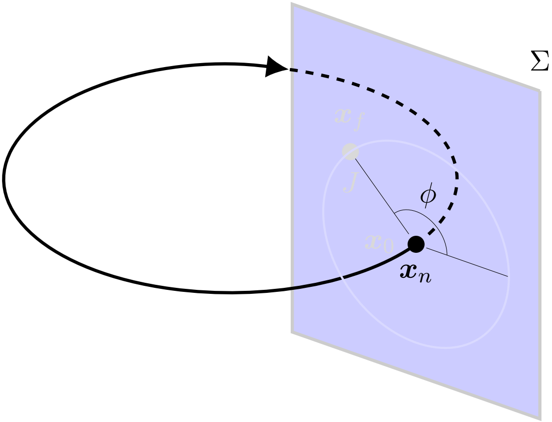

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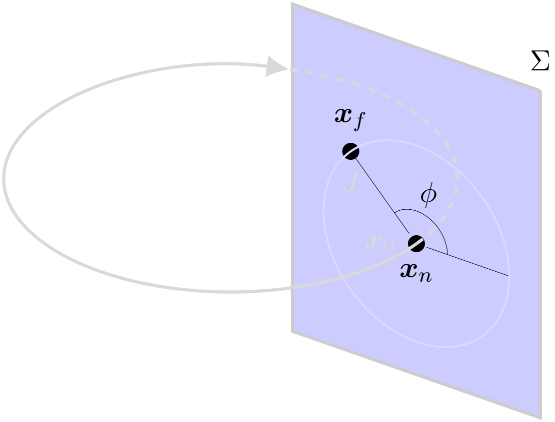

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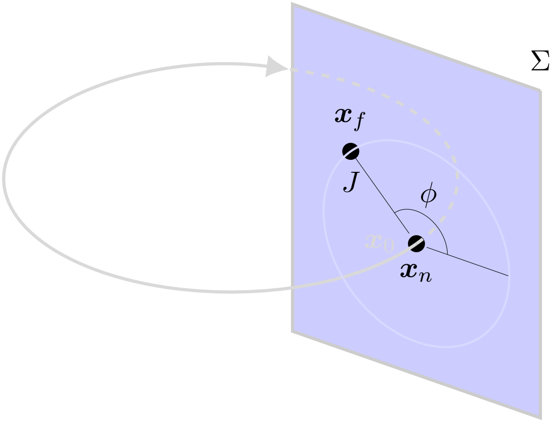

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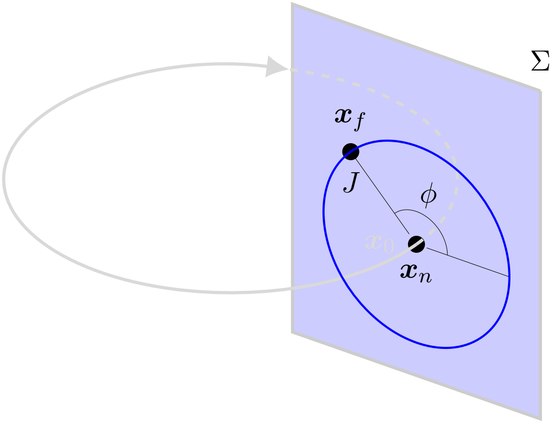

Optimal Control Problem

- Reachability defined as distance between controlled and uncontrolled states

$$J = -\frac{1}{2}\left(\boldsymbol{x}(t_f) - \boldsymbol{x}_n(t_f)\right)^T Q \left(\boldsymbol{x}(t_f) - \boldsymbol{x}_n(t_f)\right)$$

- Terminal constraints - $\boldsymbol{m}_i(\boldsymbol{x}_f) = 0$ ensures Σ intersection
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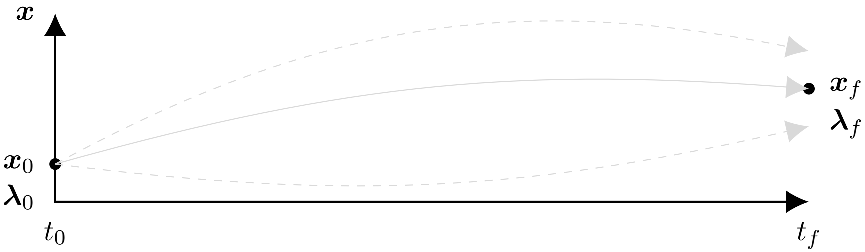

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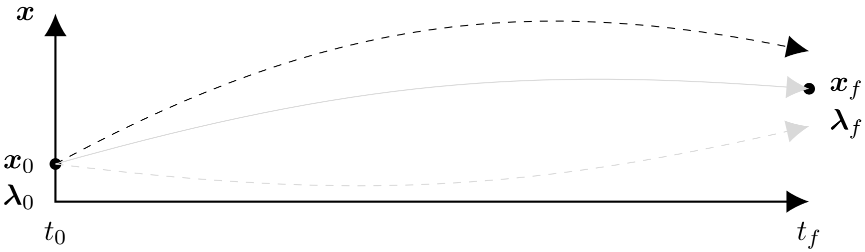

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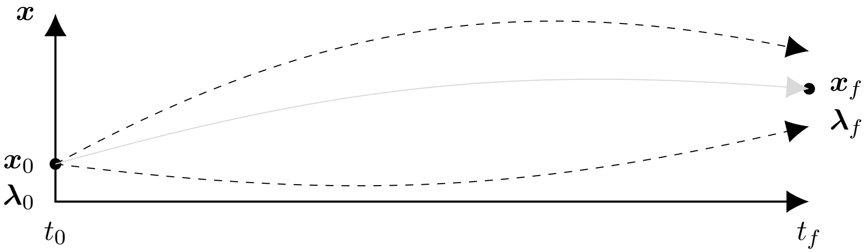

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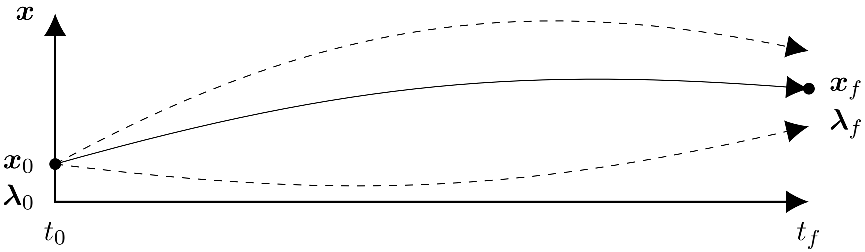

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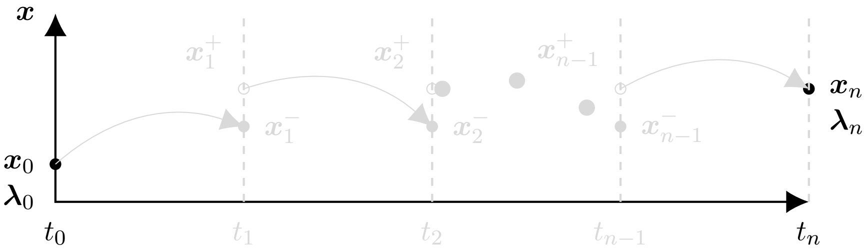

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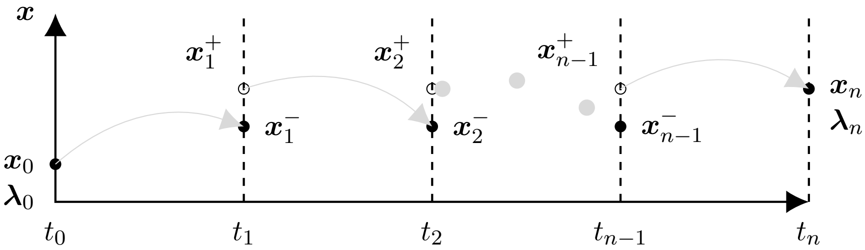

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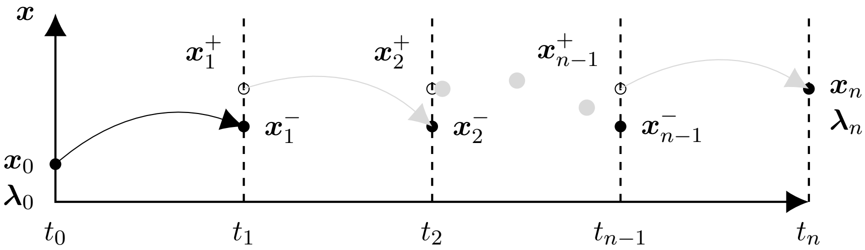

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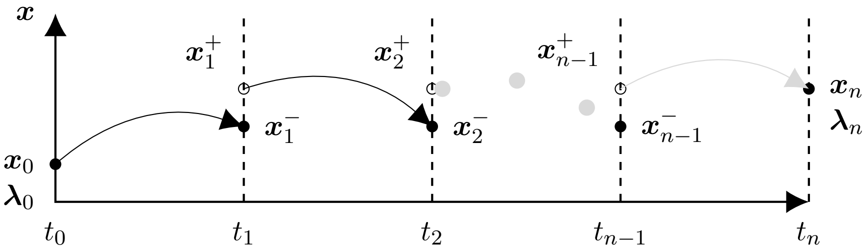

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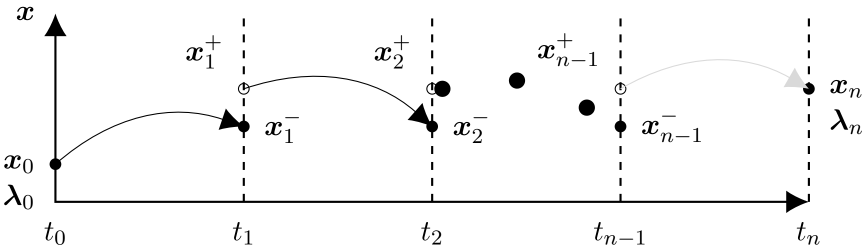

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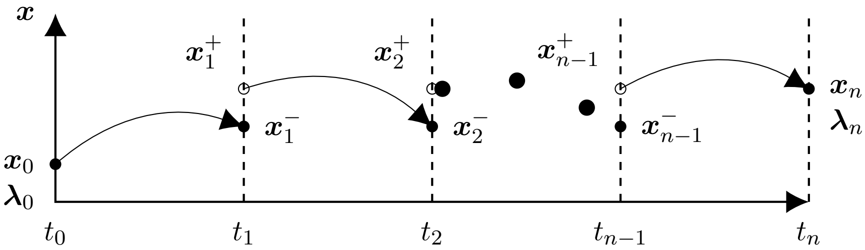

Solving the Optimal Control Problem

- Shooting method to solve the necessary conditions
- Approximate the reachable set via ϕ_i
  - Parameterize a direction on Σ
- From the reachable set we chose the state which minimizes d
- Compute another reachable set if target is not feasible

$$d = \|\boldsymbol{x}_f - \boldsymbol{x}_t\|$$

Solving the Optimal Control Problem

- Shooting method to solve the necessary conditions
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- Numerical simulations in two different environments
  - Planar Circular Restricted Three Body Problem
  - Restricted Two Body Problem
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  - Planar vs. Three Dimensional
  - Gravitational Potential
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- Transfer from L_1 orbit to periodic orbit near the Moon
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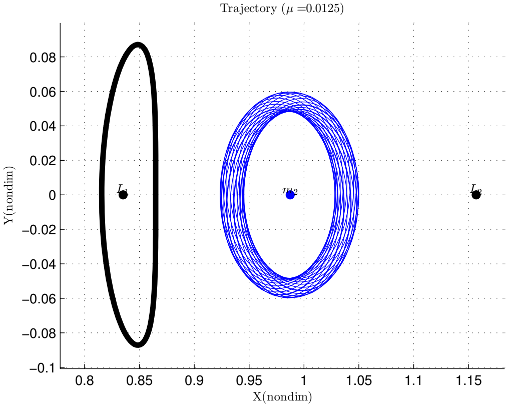
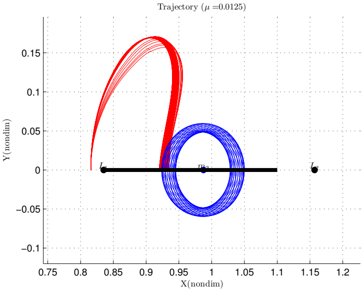


Reachable Set Transfer

- Approximate the reachable set on the Poincaré section
  - Generate many optimal solutions
- Intersection point used to generate a transfer
  - Shorter time of flight than uncontrolled dynamics



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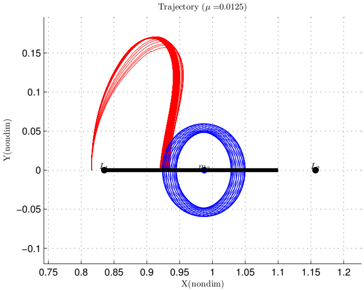

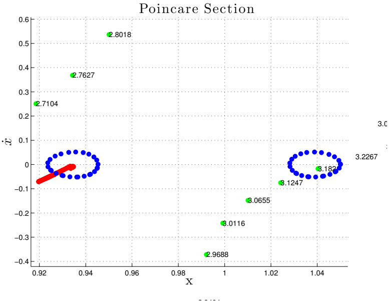

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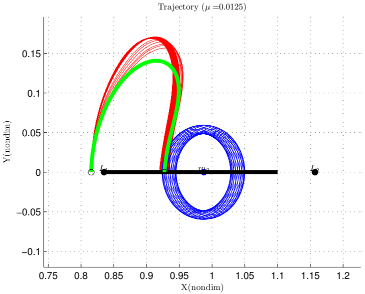
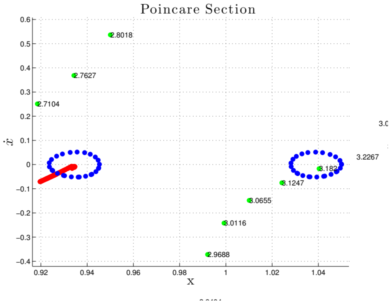
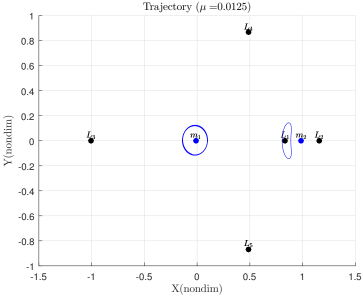
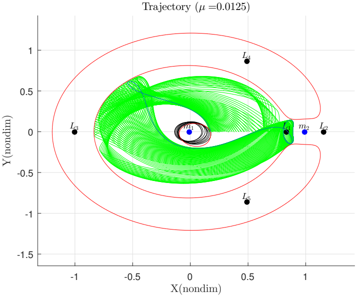




Geostationary transfer

- Transfer from geostationary orbit to a L_1 periodic orbit
- Multiple iterations of reachable set required for transfer





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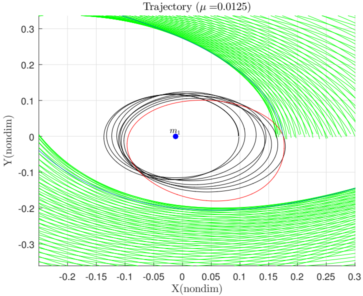
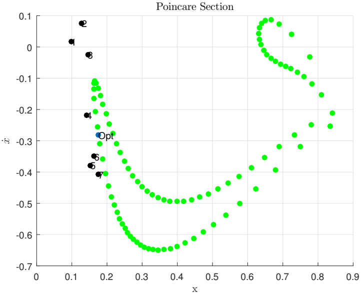
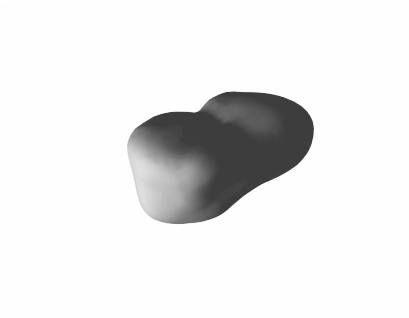
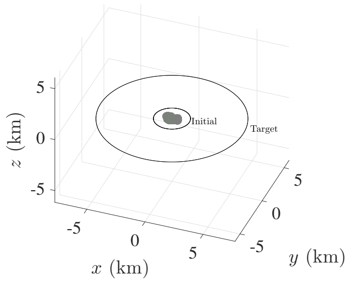




Transfer Objective

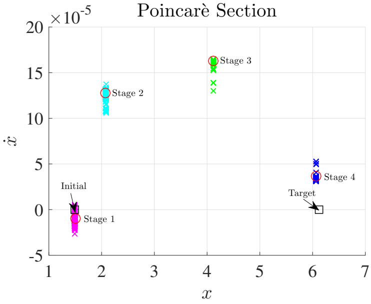
- Goal is to transfer between two equatorial periodic orbits
- Typical scenario during study of an asteroid ▸ Polyhedron

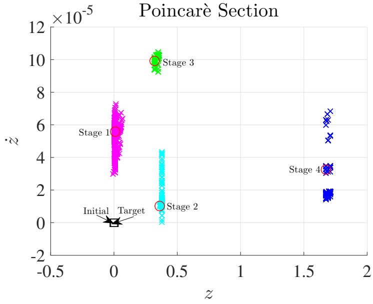




Simulation

- Generate the reachability set through discretization of ϕ_i
- Visualize $\Sigma \in \mathbb{R}^4$ through the use of two 2-D sections
- Control input allows for depature from natural dynamics





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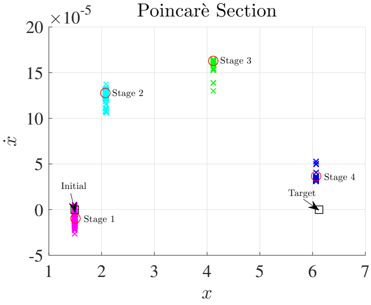
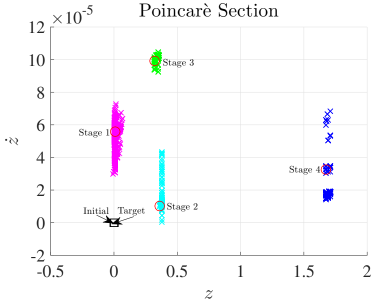
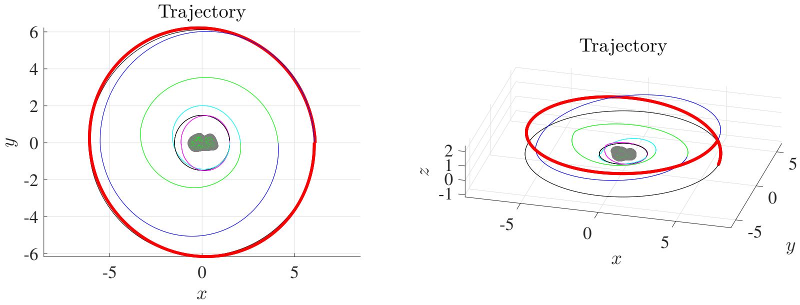




Transfer Simulation

- Four iterations of the reachable state to meet the target set
- Final transfer is computed with a fixed terminal state



Complete transfer

- We visualize the trajectory in body and inertial frames

Spacecraft Orientation

- Attitude Representation: rotation matrix from body to inertial frame ▸ Attitude Kinematics

$$\mathsf{SO}(3) = \{R \in \mathbb{R}^{3\times 3} \,|\, R^T R = I,\ \det[R] = 1\}.$$

- Rigid body attitude dynamics:

$$J\dot{\Omega} + \Omega \times J\Omega = u + W(R, \Omega)\Delta, \quad \dot{R} = R\hat{\Omega}.$$

- Sensor and obstacles defined by unit vectors in $\mathbb{R}^3$
  - Body fixed sensor: $r \in \mathsf{S}^2$
  - Inertially fixed hazard: $v \in \mathsf{S}^2$

- Hard cone constraint: $r^T R^T v \leq \cos\theta$

Control Objective

Nonlinear Control Design

Design control input u that stabilizes system from initial attitude R_0 to desired attitude R_d while avoiding obstacles

- Avoid drawbacks of other approaches
  - Geometric control - analysis is conducted directly on $\mathsf{SO}(3)$
  - Barrier function - allows for arbitrary amount of constraints
  - Efficient - real time feedback control
  - Stability - Lyapunov analysis gives rigourous stability proof
  - Adaptive - handles system uncertainties

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Configuration Error Function

- Error function quantifies "distance" to desired attitude

$$\Psi(R, R_d) = A(R, R_d)B(R).$$

- Combination of attractive and repulsive terms

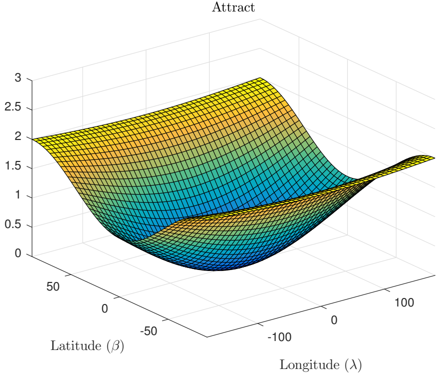
$$A(R, R_d) = \frac{1}{2}\mathrm{tr}\Big[G\left(I - R_d^T R\right)\Big].$$

$$B_i(R) = 1 - \frac{1}{\alpha_i}\ln\left(-\frac{r^T R^T v_i - \cos\theta_i}{1 + \cos\theta_i}\right).$$

Configuration Error Function

- Attractive well at the desired attitude

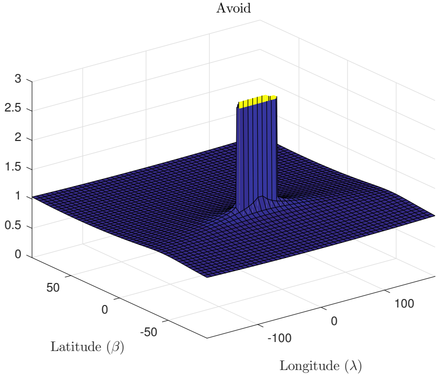
$$A(R, R_d) = \frac{1}{2}\text{tr}\Big[G\left(I - R_d^T R\right)\Big].$$



Configuration Error Function

- Define a barrier around obstacles

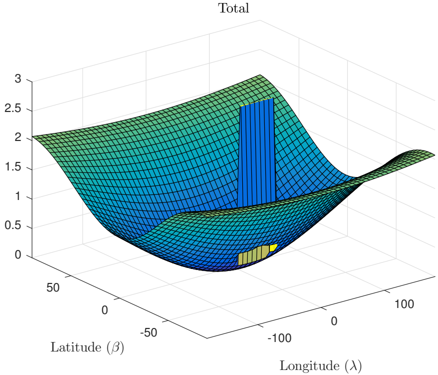
$$B_i(R) = 1 - \frac{1}{\alpha_i} \ln\left(-\frac{r^T R^T v_i - \cos\theta_i}{1 + \cos\theta_i} \right).$$



Configuration Error Function

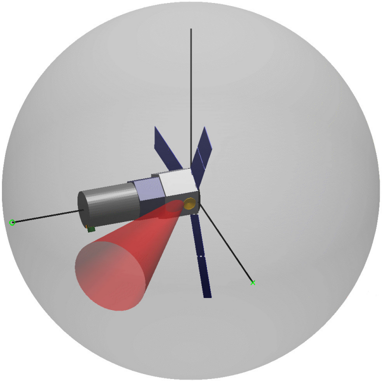
- Configuration error: $\Psi : \mathsf{Q} \times \mathsf{Q} \to \mathbb{R}$ with control chosen to follow slope of Ψ to minimum at R_d

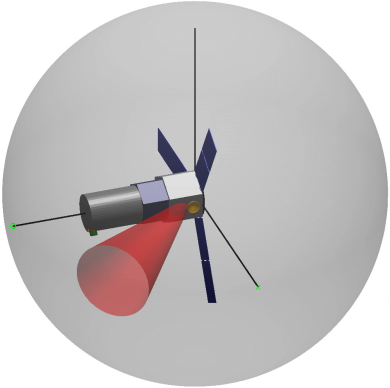
$$\Psi(R, R_d) = A(R, R_d)B(R).$$



Numerical Simulation

- Simulate a S/C completing a yaw rotation
- Single obstacle in the path of sensor

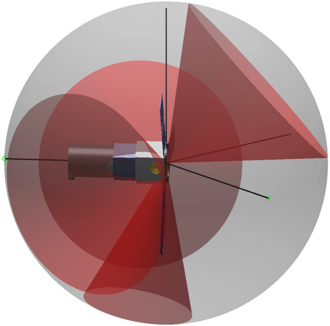




Multiple obstacles

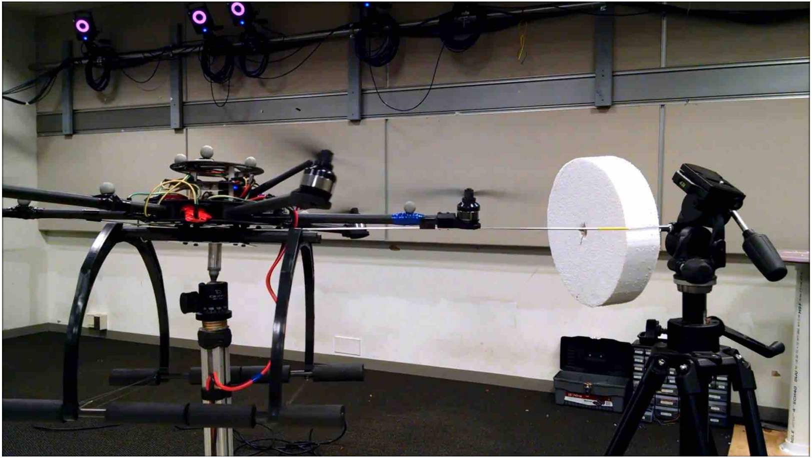
- Easily handle multiple arbitrary constraints

$$\Psi = A(R)\left[1 + \sum_i C_i(R)\right] \qquad C_i = B(R) - 1$$



Hexrotor Experiment

- Attached to spherical joint to allow only attitude dynamics



Research Objectives

- Coupled motion
  - Equations of motion of a rigid body near an asteroid
- Stability properties of coupled dynamics
  - Existence/Behavior of equilibrium solutions
  - Deviations over extended time spans
  - Differences in reachability set computations
- Landing trajectories
  - Obstacle avoidance when near irregular surface features
  - Translation/Rotational constraints
  - Avoid use of expensive optimization methods

End-to-End Asteroid Transfers

Complete transfer trajectories from arrival to landing

- Geometrically exact representation
- Obstacle/Constraint avoidance
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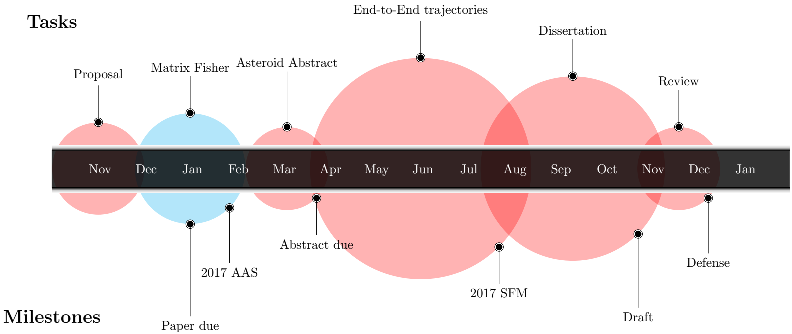

Conclusions

1. Systematic method of designing low-thrust orbital transfers via reachability sets
   - Alleviates the need for selecting accurate initial guesses
   - Gives insight into the possible motion of the spacecraft
2. Constrained attitude control allows for simple method of satisfying attitude constraints
   - Completely avoids singularities and ambiguities
   - Geometrically exact and computationally efficient
3. Asteroid landing requires the coupled approach
   - Global mission scenarios are possible
   - Dynamics derived in a consistent and geometrically exact framework
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Publications

Journals

- S. Kulumani and T. Lee. "Systematic Design of Optimal Low-Thrust Transfers for the Three-Body Problem". In: *Acta Astronautica* (under review)
- S. Kulumani and T. Lee. "Constrained Geometric Attitude Control on $\mathsf{SO}(3)$". In: *International Journal of Control, Automation, and Systems* (under review)

Conferences

- S. Kulumani and T. Lee. "Systematic Design of Optimal Low-Thrust Transfers for the Three-Body Problem". In: *Proceedings of the AAS/AIAA Astrodynamics Specialist Conference, Vail, Colorado*. 757. Aug. 2015. arXiv: `1510.02695 [math.OC]`
- S. Kulumani, C. Poole, and T. Lee. "Geometric Adaptive Control of Attitude Dynamics on $\mathsf{SO}(3)$ with State Inequality Constraints". In: *2016 American Control Conference (ACC)*. July 2016, pp. 4936–4941. arXiv: `1602.04286 [math.OC]`
- S. Kulumani et al. "Estimation of Information-Theoretic Quantities for Particle Clouds". In: *Proceedings of the AIAA/AAS Astrodynamics Specialists Conference, Long Beach, California*. Sept. 2016
- S. Kulumani and T. Lee. "Low-Thrust Trajectory Design Using Reachability Sets near Asteroid 4769 Castalia". In: *Proceedings of the AIAA/AAS Astrodynamics Specialists Conference, Long Beach, California*. Sept. 2016.

Thank you

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School of Engineering & Applied Science

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Astrodynamics

Newton's Law of Universal Gravitation

Any two bodies attract one another with a force proportional to the product of their masses and invesely proportional to the square of the distance between them

$$\boldsymbol{F} = -\frac{Gm_1m_2}{r^2}\frac{\boldsymbol{r}}{r}$$

Dynamic Challenges

Astrodynamics

N-Body

Gravitational attraction of n bodies acting on the particle of interest m_i

$$m_i \ddot{\boldsymbol{r}}_i = -G \sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{m_i m_j}{r_{ji}^3} \boldsymbol{r}_{ji}$$

Motion of $\bar{r}_j(t)$ is not known - Not solvable in general

▸ Dynamic Challenges

Gravitational Modeling

Centrobaric Body

$\boldsymbol{F} = -\frac{Gm_1 m_2}{R^2}\boldsymbol{a}_1$ for all particles outside of body

- Only applies to spherically symmetric bodies
- Gravity Model requires accurate tracking of SC
  - Spherical Harmonic
  - Mass concentration
  - Polyhedron Potential

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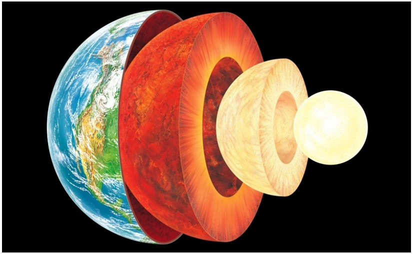

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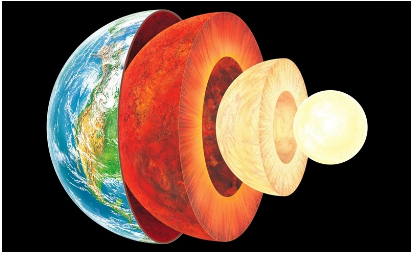

Polyhedron Gravitation Model

- Potential is a function of only the shape model
- Globally valid, closed-form expression of potential
- Exact potential assumes a constant density
- Accuracy solely dependent on shape model

Asteroid Transfer

Polyhedron Gravitation Model

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$$U(\boldsymbol{r}) = \frac{1}{2} G\sigma \sum_{e \in \text{edges}} \boldsymbol{r}_e \cdot \boldsymbol{E}_e \cdot \boldsymbol{r}_e \cdot L_e - \frac{1}{2} G\sigma \sum_{f \in \text{faces}} \boldsymbol{r}_f \cdot \boldsymbol{F}_f \cdot \boldsymbol{r}_f \cdot \omega_f$$

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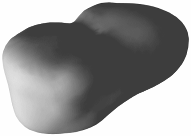

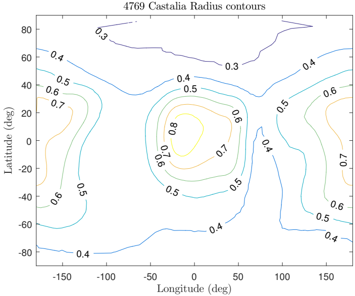

Astrodynamics

Integrals of Motion

Require $6n$ integrals of motion but know 10

- Linear Momentum - system CM constant speed - 6 constants
- Angular Momentum - system angular moment - 3 constants
- Total Energy - Conservative system - 1 constant

Even two-body problem is not solvable

Astrodynamics

Relative Motion

Really care about relative motion of m_i wrt m_q

$$\ddot{\boldsymbol{r}}_{qi} + G\frac{m_i + m_q}{r_{qi}^3}\boldsymbol{r}_{qi} = G\sum_{\substack{j=1\\ j\neq i,q}}^{n} m_j\left(\frac{\boldsymbol{r}_{ij}}{r_{ij}^3} - \frac{\boldsymbol{r}_{qj}}{r_{qj}^3}\right)\boldsymbol{r}_{ji}$$

Gravity Expansion

Force due to gravity on body B from particle P

$$\boldsymbol{F} = -Gm_P \int_{\mathcal{B}} \boldsymbol{r} \left(r^2\right)^{-\frac{3}{2}} \, dm$$

Gravity Expansion

Binomial Expansion

$$\boldsymbol{F} = -\frac{G m_P m_B}{R^2}\left(\boldsymbol{a}_1 + \sum_{i=2}^{\infty} \boldsymbol{f}^{(i)}\right)$$

$$\boldsymbol{f}^{(2)} = \frac{1}{m_B R^2}\left\{\frac{3}{2}\left[\mathrm{tr}[J_B] - 5\boldsymbol{a}_1 \cdot J_B \cdot \boldsymbol{a}_1\right]\boldsymbol{a}_1 + 3 J_B \cdot \boldsymbol{a}_1\right\}$$

Gravity Expansion

Gravity Moment

$$\boldsymbol{M} = \frac{3Gm_B}{R^3}\boldsymbol{a}_1 \times J \cdot \boldsymbol{a}_1 + \frac{Gm_B m_P}{R}\sum_{i=3}^{\infty}\boldsymbol{a}_1 \times \boldsymbol{f}^{(i)}$$

Solar Radiation Pressure

Constant Area Approximation

Momentum transfer from solar photons striking spacecraft

$$\boldsymbol{a}_{SRP} = -\frac{(1+\rho)\,P_0 A_S}{M_S}\frac{\boldsymbol{d}-\boldsymbol{r}}{\|\boldsymbol{d}-\boldsymbol{r}\|^3}$$

P_0 is a solar flux constant $1 \times 10^8\,\mathrm{kg\,km^3\,s^{-2}\,m^{-2}}$
$B_S = \frac{M_S}{A_S}$ is a mass to area ratio - $20 - 40\,\mathrm{kg\,m^{-2}}$
ρ is total reflectance or albedo of body
$\boldsymbol{d}, \boldsymbol{r}$ defined in small body frame to Sun and S/C respectively

- Large bodies, i.e. 433 Eros, SRP may be neglected
- Small bodies, $< 1 - 5$km, SRP is crucial

Lagrange's Equations

Conservative System

All applied forces $\boldsymbol{F}_i$ are derivable from a potential function $V(x_1, x_2, \ldots, x_{3N})$: $\boldsymbol{F}_i = -\frac{\partial V}{\partial \boldsymbol{x}_i}$

Hamilton's Principle

The actual path in configuration space followed by a holonomic system during the fixed interval t_0 to t_1 is such that the action integral:

$$S = \int_{t_0}^{t_1} L(q, \dot{q}) dt$$

is stationary with respect to path variations which vanish at the end-points.

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Variation of action integral

$$\begin{aligned}
\delta S &= \int_{t_0}^{t_1} \frac{\partial L}{\partial q}\delta q + \frac{\partial L}{\partial \dot{q}}\delta \dot{q}\, dt \\
&= \int_{t_0}^{t_1} \frac{\partial L}{\partial q}\delta q - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)\delta q\, dt - \left[\frac{\partial L}{\partial \dot{q}}\delta q\right]\Big|_0^T \\
&= \int_{t_0}^{t_1} \frac{\partial L}{\partial q} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)\, dt\,,
\end{aligned}$$

Jacobi Integral

Total Mechanical Energy

1. Generalized force from potential function: $Q_i = -\frac{\partial V}{\partial q_i}$
2. Work is path independent: $W = \sum_{i=1}^{n} \int_{A_i}^{B_i} Q_i \, dq_i$

If no other forces do work, then total mechanical energy is conserved: $E(q, \dot{q}) = T + V$

Jacobi Integral

More general constant

1. Standard Form of Lagrange's equation applies
2. Lagrangian is not explicit fcn of time
3. Constraints may be expressed: $\sum_{i=1}^{n} a_{ji} dq_i = 0$

$$h = \sum_{i=1}^{n} \frac{\partial L}{\partial \dot{q}_i} \dot{q}_i - L$$

Variational Principle

- Variational Integrators
  - Structure-preserving integrators for Hamiltonian systems
  - Obtained by discretizing variational principle

Continuous Time	Discrete Time
Configuration Space $(q, \dot{q}) \in TQ$	Configuration Space $(q_k, q_{k+1}) \in Q \times Q$
Lagrangian $L(q, \dot{q})$	Lagrangian $L_d(q_k, q_{k+1})$
Action Integral $S = \int_0^T L(q, \dot{q})\, dt$	Action Sum $S_d = \sum_{k=0}^{N-1} L_d(q_k, q_{k+1})$
Stationary Action $\delta S = 0$	Stationary Action $\delta S_d = 0$
Equation of Motion $\ddot{q} = f(q, \dot{q})$	Equation of Motion $q_{k+2} = f_d(q_k, q_{k+1})$

Variational Principle

- Variational Integrators
  - Structure-preserving integrators for Hamiltonian systems
  - Obtained by discretizing variational principle

Continuous Time	Discrete Time
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Dynamics of Rigid Body

Newton's Law

$$F_x = m\left(\dot{v}_x + v_z\omega_y - v_y\omega_z\right)$$
$$F_y = m\left(\dot{v}_y + v_x\omega_z - v_z\omega_x\right)$$
$$F_z = m\left(\dot{v}_z + v_y\omega_x - v_x\omega_y\right)$$

Euler's Law

$$M_x = I_{xx}\dot{\omega}_x + \left(I_{zz} - I_{yy}\right)\omega_y\omega_z$$
$$M_y = I_{yy}\dot{\omega}_y + \left(I_{xx} - I_{zz}\right)\omega_z\omega_x$$
$$M_z = I_{zz}\dot{\omega}_z + \left(I_{yy} - I_{xx}\right)\omega_x\omega_y$$

Spacecraft Propulsion

Ideal Rocket Equation

Amount of propellant required for a given velocity change

$$\Delta V = -g_0 I_{sp} \ln\left(\frac{m_f}{m_i}\right)$$

- High exhaust speeds make electric propulsion attractive
- Much higher efficiency than chemical propulsion

Low-Thrust Vehicles

Attitude Kinematics

- Many ways to represent the orientation of a rigid body
  - Conceptual simplicity
  - Computational considerations
  - Legacy hardware/code
  - Mathmatical simplicity/convience

- Euler axis and angle (4)
- Rotation Matrix (9)
- Euler angles (3)
- Quaternion (4)
- Rodriguez Parameters (4)
- Modified Rodriguez parameters (4)

Minimal Representations

All minimal attitude representations have kinematic singularities and are not suitable for large rotations.

Attitude Control

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  - Singularities exist for large angle slews: requires switching between 24 sequences
  - Complicated trigonometric functions
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  - Two anti-podal quaternions for the same attitude
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  - Globally and uniquely characterize attitude: $R \in \mathsf{SO}(3)$
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Constrained Attitude Control

- Constrained attitude control : reorient vehicle while avoiding pointing at obstacles
  - Exclusion zones for payloads e.g infrared telescope
  - UAVs manuevering in congested locations
  - Laser/Radio emitters on spacecraft
- Previous approaches have several issues
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  - Ad-hoc path planning: difficult to generalize to arbitrary obstacles
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