

Question 4.1

Factorial:

Pre-condition:

$P(\text{input}): \text{input} \geq 0$

Post-condition:

$Q(\text{input}, \text{output}): \text{input} \leq \text{output}$

Loop invariant condition:

$\text{Product} == (\text{factor}-1)!$

Termination ordering:

$n - (\text{factor}-1)$

Revert Immutable:

For this question I will use a function `size()` which determines the number of elements in the list

Pre-condition:

$P(\text{input}[]): \text{size}(\text{input}) > 0$

Post-condition:

$Q(\text{input}[], \text{rev}[]): \text{size}(\text{input}) == \text{size}(\text{rev})$

Loop invariant:

$(\text{size}(\text{rev}) + \text{size}(\text{rest})) == \text{size}(x)$

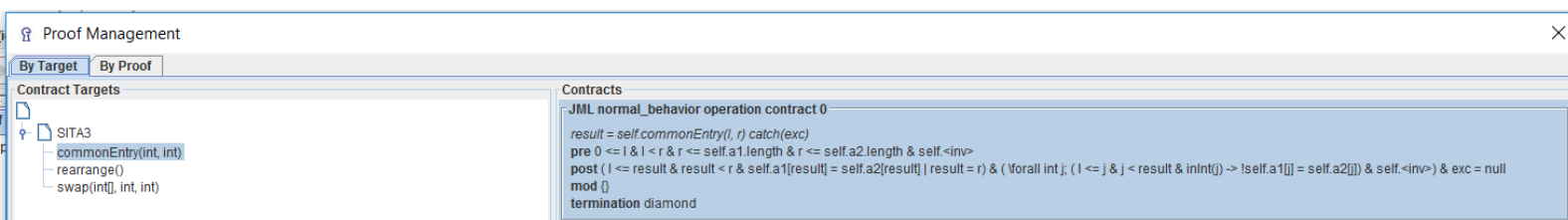
Termination ordering:

$\text{size}(\text{rest}) == 0$

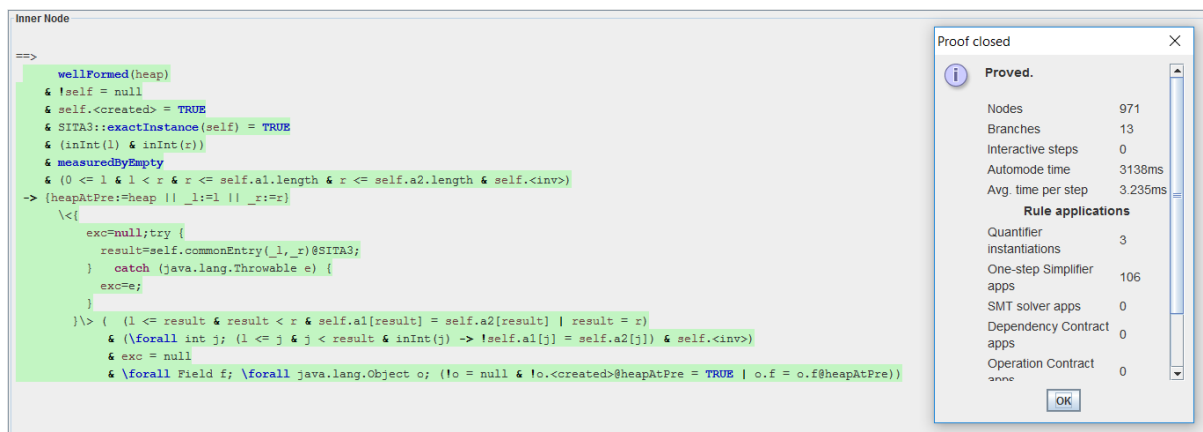
Question 4.2

I chose the function “search in two arrays”:

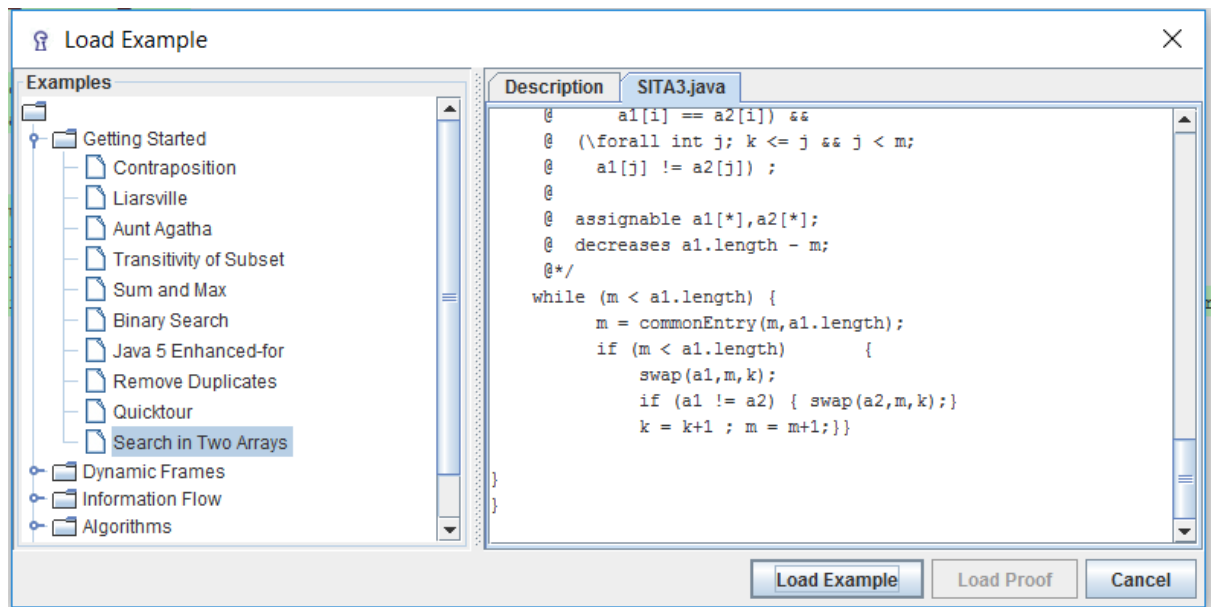
Pre/Post conditions:



Proving (w/ proof closed):



While loop:



Question 4.3

Take the if/else definition from 11.1.2. The lecture notes defines soundness as “a set of rules is sound if all provable formulas are theorems”. Two cases arise:

Case 1: C holds true and $[t]$ is executed to reach F . Then we know from 9.4 “Grammar and intuitions” that F will be true pre- and post- evaluation.

Case 2: The same goes for $\neg C$. If it holds true and $[t']$ is executed to reach F , then again from 9.4 we know that F will be true for pre- and post- evaluation.

If the statement is provable for both cases, which it is, we know that it is sound.

Take the while loop definition from 11.1.3. Much like the if/else statement, we again arrive at two distinct cases:

Case 1: C holds := If C holds then we execute $[t]$ by evaluating the loop invariant (which always holds true) and the condition C and reach F .

Case 2: $\neg C$ holds := If C does not hold then we evaluate the condition C and the loop invariant and simple do not execute $[t]$ to reach F .

In either case we reach a definitive true state and no side effects can be seen. Hence the statement is sound.