

NCERT Notes

ON MATHEMATICS AND SCIENCE (CLASS 10)

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Part I

Mathematics

Chapter 1

Real Numbers

- **Theorem 1.1** (Euclid's Division Lemma). *Given two positive integers a and b , there exist unique whole numbers satisfying $a = bq + r, 0 \leq r < b$.*
- *Euclid's division algorithm:* In order to compute the HCF of two positive integers, say a and b , with $a > b$, we take the following steps:
 - Apply Euclid's division algorithm to a and b . and obtain whole numbers q_1 and r_1 such that $a = bq_1 + r_1, 0 \leq r_1 < b$.
 - If $r_1 = 0$, b is the HCF of a and b .
 - If $r_1 \neq 0$, apply Euclid's division lemma to b and r_1 and obtain whole numbers q_2 and r_2 such that $b = r_1q_2 + r_2$.
 - If $r_2 = 0$, then r_1 is the HCF of a and b .
 - If $r_2 \neq 0$, then apply Euclid's division lemma to r_1 and r_2 and continue the above process until the remainder r_n is zero. The divisor at this stage, i.e. r_{n-1} , or the non-zero remainder at the previous stage, is the HCF of a and b .
- **Theorem 1.2** (The Fundamental Theorem of Arithmetic). *Every composite number can be expressed as a product of primes, and this factorisation is unique apart from the order in which the prime factors occur.*
- Every composite number can be uniquely expressed as a product of powers of primes in ascending or descending order.
- The HCF of two or more numbers is the product of the smallest powers of each of the numbers' common prime factors.
- The LCM of two or more numbers is the product of the greatest powers of each of the numbers' prime factors.
- For any two positive integers a and b , $ab = \text{HCF}(a, b) \times \text{LCM}(a, b)$

- **Theorem 1.3.** *Let a be a positive integer and p be a prime number. If p divides a^2 , then p divides a .*
- If p is a prime number, then \sqrt{p} is an irrational number.

Proof. Assume, to the contrary, that \sqrt{p} is a rational number and express it thus:

$$\sqrt{p} = \frac{a}{b}$$

where a and b are co-prime integers and $b \neq 0$. Next, isolate a and square the equation:

$$b\sqrt{p} = a \tag{1.1}$$

$$pb^2 = a^2 \tag{1.2}$$

Since p divides a^2 , p divides a (as per Theorem 1.3). Therefore,

$$a = pc$$

for some integer c . Substitute for a in (1.2):

$$pb^2 = (pc)^2$$

$$pb^2 = p^2c^2$$

$$b^2 = pc^2$$

Since p divides b^2 , p divides b (as per Theorem 1.3). p is a common factor of a and b . This contradicts the fact that a and b are co-prime. Our assumption that \sqrt{p} is rational is incorrect. Hence, \sqrt{p} is irrational.

QED

Chapter 2

Polynomials

- A polynomial in variable x is of the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

where x is a variable, n is a positive integer and $a_0, a_1, a_2, \dots, a_n$ are constants.

- The exponent of the highest degree term in a polynomial is known as its degree.
- The names and forms of various degree polynomials:

Degree	Name of the polynomial	Form of the polynomial
0	Constant polynomial	$f(x) = a, a$ is a constant.
1	Linear polynomial	$f(x) = ax + b, a \neq 0$
2	Quadratic polynomial	$f(x) = ax^2 + bx + c, a \neq 0$
3	Cubic polynomial	$f(x) = ax^3 + bx^2 + cx + d, a \neq 0$
4	Biquadratic polynomial	$f(x) = ax^4 + bx^3 + cx^2 + dx + e, a \neq 0$

Part II

Science

Chapter 1

Chemical Reactions and Equations