NCERT Notes

On Mathematics and Science (Class 10)

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Contents

Part I

Mathematics

Chapter 1

Real Numbers

- **Theorem 1.1** (Euclid's Division Lemma). *Given two positive integers* a *and* b*, there exist unique whole numbers satisfying* $a = bq + r, 0 \leqslant r < b$*.*

- *Euclid's division algorithm*: In order to compute the HCF of two positive integers, say a and b, with $a > b$, we take the following steps:

  - Apply Euclid's division algorithm to a and b. and obtain whole numbers q_1 and r_1 such that $a = bq_1 + r_1, 0 \leqslant r_1 < b$.
  - If $r_1 = 0$, b is the HCF of a and b.
  - If $r_1 \neq 0$, apply Euclid's division lemma to b and r_1 and obtain whole numbers q_2 and r_2 such that $b = r_1 q_2 + r_2$.
  - If $r_2 = 0$, then r_1 is the HCF of a and b.
  - If $r_2 \neq 0$, then apply Euclid's division lemma to r_1 and r_2 and continue the above process until the remainder r_n is zero. The divisor at this stage, i.e. r_{n-1}, or the non-zero remainder at the previous stage, is the HCF of a and b.

- **Theorem 1.2** (The Fundamental Theorem of Arithmetic). *Every composite number can be expressed as a product of primes, and this factorisation is unique apart from the order in which the prime factors occur.*

- Every composite number can be uniquely expressed as a product of powers of primes in ascending or descending order.

- The HCF of two or more numbers is the product of the smallest powers of each of the numbers' common prime factors.

- The LCM of two or more numbers is the product of the greatest powers of each of the numbers' prime factors.

- For any two positive integers a and b, $ab = \mathrm{HCF}(a, b) \times \mathrm{LCM}(a, b)$

- **Theorem 1.3.** *Let* a *be a positive integer and* p *be a prime number. If* p *divides* a^2*, then* p *divides* a*.*

- If p is a prime number, then $\sqrt{p}$ is an irrational number.

  Proof. Assume, to the contrary, that $\sqrt{p}$ is a rational number and express it thus:
  $$\sqrt{p} = \frac{a}{b}$$
  where a and b are co-prime integers and $q \neq 0$. Next, isolate a and square the equation:
  $$b\sqrt{p} = a \tag{1.1}$$
  $$pb^2 = a^2 \tag{1.2}$$
  Since p divides a^2, p divides a (as per Theorem 1.3). Therefore,
  $$a = pc$$
  for some integer c. Substitute for a in (1.2):
  $$pb^2 = (pc)^2$$
  $$pb^2 = p^2c^2$$
  $$b^2 = pc^2$$
  Since p divides b^2, p divides b (as per Theorem 1.3). p is a common factor of a and b. This contradicts the fact that a and b are co-prime. Our assumption that $\sqrt{p}$ is rational is incorrect. Hence, $\sqrt{p}$ is irrational. QED

Chapter 2

Polynomials

- A polynomial in variable x is of the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

where x is a variable, n is a positive integer and $a_0, a_1, a_2, \ldots, a_n$ are constants.

- The exponent of the highest degree term in a polynomial is known as its degree.

- The names and forms of various degree polynomials:

Degree	Name of the polynomial	Form of the polynomial
0	Constant polynomial	$f(x) = a$, a is a constant.
1	Linear polynomial	$f(x) = ax + b, a \neq 0$
2	Quadratic polynomial	$f(x) = ax^2 + bx + c, a \neq 0$
3	Cubic polynomial	$f(x) = ax^3 + bx^2 + cx + d, a \neq 0$
4	Biquadratic polynomial	$f(x) = ax^4 + bx^3 + cx^2 + dx + e, a \neq 0$

- If $f(x)$ is a polynomial and α is any real number, then the real number obtained by replacing x by α in $f(x)$ is known as the value of $f(x)$ at $x = \alpha$ and is denoted by $f(\alpha)$.

- A real number α is a zero of a polynomial $f(x)$, if $f(\alpha) = 0$.

- A polynomial of degree n can have at most n real zeroes.

- Geometrically, the zeroes of a polynomial $f(x)$ are the x-coordinates of the points where the graph $y = f(x)$ interesects the x-axis.

- If α and β are the zeroes of a quadratic polynomial $f(x) = ax^2 + bx + c$, then

$$\alpha + \beta = -\frac{b}{a} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} \qquad \alpha\beta = \frac{c}{a} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

- $f(x) = x^2 - (\alpha + \beta)x + \alpha\beta$ is a quadratic polynomial whose zeroes are α and β.

Chapter 3

Pair of Linear Equations in Two Variables

- A pair of linear equations in two variables x and y can be represented algebraically as follows:

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

where $a_1, a_2, b_1, b_2, c_1, c_2$ are real numbers such that $a_1^2 + b_1^2 \neq 0, a_2^2 + b_2^2 \neq 0$.

- Graphically, or geometrically, a pair of linear equations

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

in two variables represents a pair of straight lines which are

  - interesecting, if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
  - parallel, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
  - coincident, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

- To solve a pair of linear equations in two variables graphically, we first draw the lines represented by them.

  - If the pair of lines interesect at a point, then the pair of equations is said to be *consistent* and the coordinates of the point provide a unique solution.
  - If the pair of lines are parallel, then the pair of equations is said to be *inconsistent*; it has no solution.
  - If the pair of lines are coincident, then the pair of equations is said to be *dependent* (*consistent*); it has infinitely many solutions.

- To solve a pair of linear equations in two variables algebraically, we can use:
  - Substitution Method
  - Elimination Method
- If $\begin{matrix} a_1x + b_1y + c_1 = 0 \\ a_2x + b_2y + c_2 = 0 \end{matrix}$ is a pair of linear equations in two variables x and y such that
  - $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, then the pair of equations is consistent.
  - $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, then the pair of linear solutions is inconsistent.
  - $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, then the pair of linear equations is dependent and consistent.

Part II

Science

Chapter 1

Chemical Reactions and Equations

1.1 Introduction

- A chemical reaction is a process in which new substances with new properties are found.
- Chemical reactions inolve the breaking and making of bonds between atoms to produce new substances.
- The substances which take part in a chemical reaction are called reactants.
- The new substances produced as a result of a chemical reaction are called products.
- The properties of the products are completely different from the properties of the reactants.
- When a magnesium ribbon is heated, it burns in air (combining with oxygen) with a dazzling white flame to form a white powder called magnesium oxide. This is an example of a chemical reaction.
- A magnesium ribbon usually has a coating of magnesium oxide on its surface which is formed by the slow action of oxygen of air on it. So the magnesium ribbon is cleaned by rubbing with sandpaper before burning it in air. This is done to remove the protective layer of magnesium oxide from the surface of the ribbon so that it may readily combine with the oxygen of air on heating.
- The important characteristics of chemical reactions are:
  - Evolution of a gas,
  - Formation of a precipitate,

- Change in colour,
- Change in temperature, and
- Change in state

  Any one of these characteristics can tell us whether a chemical reaction has taken place or not.

1.2 Chemical Equations

- The representation of a chemical reaction with the help of symbols and formulae of the substances involved in it is known as a chemical equation.

- In a chemical equation, the reactants are written in the left hand side and the products – in the right hand side. They separated by an arrow sign pointing towards the right hand side ($\longrightarrow$). Each individual reactant or product is separated from other reactants or products with plus signs (+).

1.2.1 Balanced and Unbalanced Chemical Equations

- A balanced chemical equation has an equal number of atoms of different elements in the reactants and products.

- An unbalanced or skeletal chemical equation has an unequal number of atoms of one or more elements in the reactants and products.

- It is essential to balance chemical equations in order to satisfy the law of conservation of mass in chemical reactions, i.e. matter can neither be created nor destroyed in a chemical reaction.

- In order to balance a chemical equation, symbols and formulae are multiplied by figures like 2, 3 and 4.

- To make a chemical equation more informative, the physical states of the reactants and products are mentioned along with their symbols and formulae. The gaseous, liquid, aqueous and solid states of reactants and products are represented by the notations (g), (l), (aq) and (s) respectively. The word aqueous (aq) is written if the reactant or product is present as a solution in water.

- Reaction conditions such as temperature, pressure, catalyst, etc. are indicated above and/or below the arrow in the equation.

- Some examples of balanced chemical equations:

$$
\begin{aligned}
2\,Mg_{(s)} + O_{2(g)} &\longrightarrow 2\,MgO_{(s)}\\
Zn_{(s)} + H_2SO_{4(aq)} &\longrightarrow ZnSO_{4(aq)} + H_{2(g)}\\
3\,Fe_{(s)} + 4\,H_2O_{(g)} &\longrightarrow Fe_3O_{4(s)} + 4\,H_{2(g)}\\
CO_{(g)} + {}_2H_{2(g)} &\xrightarrow{340atm} CH_3OH_{(l)}\\
6\,CO_{2(aq)} + 12\,H_2O_{(l)} &\xrightarrow[\text{Chlorophyll}]{\text{Sunlight}} C_6H_{12}O_{6(aq)} + 6\,O_{2(aq)} + 6\,H_2O_{(l)}\\
H_{2(g)} + Cl_{2(g)} &\longrightarrow 2\,HCl_{(g)}\\
3\,BaCl_{2(aq)} + Al_2(SO_4)_{3(aq)} &\longrightarrow 3\,BaSO_{4(s)} + 2\,AlCl_{3(aq)}\\
2\,Na_{(s)} + 2\,H_2O_{(l)} &\longrightarrow 2\,NaOH_{(aq)} + H_{2(g)}\\
NaOH_{(aq)} + HCl_{(aq)} &\longrightarrow NaCl_{(aq)} + H_2O_{(l)}
\end{aligned}
$$

1.3 Types of Chemical Reactions