NCERT Notes

On Mathematics and Science (Class 10)

Santhosh Kumaran

Contents

Part I

Mathematics

Chapter 1

Real Numbers

- **Theorem 1.1** (Euclid's Division Lemma). *Given two positive integers a and b, there exist unique whole numbers satisfying $a = bq + r, 0 \leq r < b$.*

- *Euclid's division algorithm*: In order to compute the HCF of two positive integers, say a and b, with $a > b$, we take the following steps:

  - Apply Euclid's division algorithm to a and b. and obtain whole numbers q_1 and r_1 such that $a = bq_1 + r_1, 0 \leq r_1 < b$.
  - If $r_1 = 0$, b is the HCF of a and b.
  - If $r_1 \neq 0$, apply Euclid's division lemma to b and r_1 and obtain whole numbers q_2 and r_2 such that $b = r_1 q_2 + r_2$.
  - If $r_2 = 0$, then r_1 is the HCF of a and b.
  - If $r_2 \neq 0$, then apply Euclid's division lemma to r_1 and r_2 and continue the above process until the remainder r_n is zero. The divisor at this stage, i.e. r_{n-1}, or the non-zero remainder at the previous stage, is the HCF of a and b.

- **Theorem 1.2** (The Fundamental Theorem of Arithmetic). *Every composite number can be expressed as a product of primes, and this factorisation is unique apart from the order in which the prime factors occur.*

- Every composite number can be uniquely expressed as a product of powers of primes in ascending or descending order.

- The HCF of two or more numbers is the product of the smallest powers of each of the numbers' common prime factors.

- The LCM of two or more numbers is the product of the greatest powers of each of the numbers' prime factors.

- For any two positive integers a and b, $ab = \text{HCF}(a, b) \times \text{LCM}(a, b)$

- **Theorem 1.3.** *Let a be a positive integer and p be a prime number. If p divides a^2, then p divides a.*

- If p is a prime number, then $\sqrt{p}$ is an irrational number.

  Proof. Assume, to the contrary, that $\sqrt{p}$ is a rational number and express it thus:
  $$\sqrt{p} = \frac{a}{b}$$
  where a and b are co-prime integers and $q \neq 0$. Next, isolate a and square the equation:
  $$b\sqrt{p} = a \tag{1.1}$$
  $$pb^2 = a^2 \tag{1.2}$$
  Since p divides a^2, p divides a (as per Theorem 1.3). Therefore,
  $$a = pc$$
  for some integer c. Substitute for a in (1.2):
  $$pb^2 = (pc)^2$$
  $$pb^2 = p^2c^2$$
  $$b^2 = pc^2$$
  Since p divides b^2, p divides b (as per Theorem 1.3). p is a common factor of a and b. This contradicts the fact that a and b are co-prime. Our assumption that $\sqrt{p}$ is rational is incorrect. Hence, $\sqrt{p}$ is irrational. QED

Chapter 2

Polynomials

- A polynomial in variable x is of the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

  where x is a variable, n is a positive integer and $a_0, a_1, a_2, \ldots, a_n$ are constants.

- The exponent of the highest degree term in a polynomial is known as its degree.

- The names and forms of various degree polynomials:

Degree	Name of the polynomial	Form of the polynomial
0	Constant polynomial	$f(x) = a$, a is a constant.
1	Linear polynomial	$f(x) = ax + b, a \neq 0$
2	Quadratic polynomial	$f(x) = ax^2 + bx + c, a \neq 0$
3	Cubic polynomial	$f(x) = ax^3 + bx^2 + cx + d, a \neq 0$
4	Biquadratic polynomial	$f(x) = ax^4 + bx^3 + cx^2 + dx + e, a \neq 0$

- If $f(x)$ is a polynomial and α is any real number, then the real number obtained by replacing x by α in $f(x)$ is known as the value of $f(x)$ at $x = \alpha$ and is denoted by $f(\alpha)$.

- A real number α is a zero of a polynomial $f(x)$, if $f(\alpha) = 0$.

- A polynomial of degree n can have at most n real zeroes.

- Geometrically, the zeroes of a polynomial $f(x)$ are the x-coordinates of the points where the graph $y = f(x)$ interesects the x-axis.

- If α and β are the zeroes of a quadratic polynomial $f(x) = ax^2 + bx + c$, then

$$\alpha + \beta = -\frac{b}{a} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} \qquad \alpha\beta = \frac{c}{a} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

- $f(x) = x^2 - (\alpha + \beta)x + \alpha\beta$ is a quadratic polynomial whose zeroes are α and β.

Chapter 3

Pair of Linear Equations in Two Variables

- A pair of linear equations in two variables x and y can be represented algebraically as follows:

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

  where $a_1, a_2, b_1, b_2, c_1, c_2$ are real numbers such that $a_1^2 + b_1^2 \neq 0, a_2^2 + b_2^2 \neq 0$.

- Graphically, or geometrically, a pair of linear equations

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

  in two variables represents a pair of straight lines which are

  – interesecting, if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

  – parallel, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

  – coincident, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

- To solve a pair of linear equations in two variables graphically, we first draw the lines represented by them.

  – If the pair of lines interesect at a point, then the pair of equations is said to be *consistent* and the coordinates of the point provide a unique solution.

  – If the pair of lines are parallel, then the pair of equations is said to be *inconsistent*; it has no solution.

  – If the pair of lines are coincident, then the pair of equations is said to be *dependent* (*consistent*); it has infinitely many solutions.

- To solve a pair of linear equations in two variables algebraically, we can use:
  - Substitution Method
  - Elimination Method
- If $\begin{matrix} a_1x + b_1y + c_1 = 0 \\ a_2x + b_2y + c_2 = 0 \end{matrix}$ is a pair of linear equations in two variables x and y such that
  - $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, then the pair of equations is consistent.
  - $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, then the pair of linear solutions is inconsistent.
  - $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, then the pair of linear equations is dependent and consistent.

Part II

Science

Chapter 1

Chemical Reactions and Equations