A NOTE ON MODULES WITH MINIMAL MULTIPLICITY

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In this talk, we discuss modules of minimal multiplicity, a notion originally introduced and studied by Puthenpurakal [3] in 2003, whose various properties have been further investigated recently in [2]. While the classical theory of modules with minimal multiplicity is formulated for Cohen-Macaulay modules over Noetherian local rings of arbitrary dimension, our study begins with the aim of extending the notion of Arf rings to the module-theoretic framework. For this reason, we focus on modules over Cohen-Macaulay rings of dimension one.

Let $(R, \mathfrak{m})$ be a one-dimensional Cohen-Macaulay local ring and I an $\mathfrak{m}$-primary ideal of R. A finitely generated torsion-free R-module $M \neq (0)$ is said to have *minimal multiplicity with respect to I* if it satisfies the equality $e_R(I, M) = \ell_R(IM/I^2 M)$, where $e_R(I, -)$ denotes the Hilbert-Samuel multiplicity with respect to I, and $\ell_R(-)$ stands for the length as an R-module. We then investigate the relation between modules with minimal multiplicity and *I-Ulrich module* in the sense of [1], that is, modules M satisfying the equality $e_R(I, M) = \ell_R(M/IM)$. Moreover, we examine modules of minimal multiplicity with respect to trace or reflexive ideals, and as a consequence, we establish new characterizations of almost Gorenstein rings in terms of modules of minimal multiplicity associated to their canonical ideals.

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