# MODULAR REPRESENTATION THEORY AND LIMITS OF $F$-INVARIANTS

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This talk is based on my arXiv preprint titled "Analysis in Hilbert-Kunz theory".

In this talk, we prove an inequality conjectured by Watanabe and Yoshida, which states that the Hilbert-Kunz multiplicity of the Fermat hypersurface of degree 2 in characteristic $p$ is greater than or equal to its limit when $p$ goes to infinity. We point out that equality holds if and only if the dimension of the hypersurface is at most 3 and establish a similar inequality on $F$-signature. To prove the above results, we introduce a numerical invariant for local rings of characteristic $p$ called $h$-function. It is a real function of one variable that recovers both the Hilbert-Kunz multiplicity and the $F$-signature of hypersurface rings.

We begin by developing the representation theory of $k$-objects, that is, finitely generated $k[T]$-modules annihilated by a power of $T$. We show that the representation theory of $k$-objects describes the modular representation theory of cyclic $p$-groups. We then derive an integral formula for the $h$-function of hypersurfaces defined by polynomials of the form $f(x) + g(y)$ using limit representation theory. Finally, we express Hilbert-Kunz multiplicity in terms of integrals and show that comparison between certain functions leads to inequality between Hilbert-Kunz multiplicities. The inequality for $F$-signature is proved in a similar manner.