PERCENTAGE

IMPORTANT FACTS AND FORMULAE

I. Concept of Percentage : By a certain ***percent***, we mean that many hundredths.
Thus, x percent means x hundredths, written as x%.

To express x% as a fraction : We have, $x\% = \frac{x}{100}$.

Thus, $20\% = \frac{20}{100} = \frac{1}{5}; 48\% = \frac{48}{100} = \frac{12}{25}$, *etc.*

To express $\frac{a}{b}$ ***as a percent :*** *We have,* $\frac{a}{b} = \left(\frac{a}{b} \times 100\right)\%$.

Thus, $\frac{1}{4} = \left(\frac{1}{4} \times 100\right)\% = 25\%; 0.6 = \frac{6}{10} = \frac{3}{5} = \left(\frac{3}{5} \times 100\right)\% = 60\%$.

II. If the price of a commodity increases by R%, then the reduction in consumption so as not to increase the expenditure is

$$\left[\frac{R}{(100+R)} \times 100\right]\%$$

If the price of a commodity decreases by R%, then the increase in consumption so as not to decrease the expenditure is

$$\left[\frac{R}{(100-R)} \times 100\right]\%$$

III. Results on Population : Let the population of a town be P now and suppose it increase at the rate of R% per annum, then :

1. Population after *n* years = $P\left(1+\frac{R}{100}\right)^n$.
1. Population *n* years ago = $\frac{P}{\left(1+\frac{R}{100}\right)^n}$.

IV. Results on Depreciation : Let the present value of a machine be P. Suppose it depreciates at the rate of R% per annum. Then :

1. Value of the machine after *n* years = $P\left(1-\frac{R}{100}\right)^n$.
2. Value of the machine *n* years ago = $\frac{P}{\left(1-\frac{R}{100}\right)^n}$.

V. If A is R% more than B, then B is less than A by

$$\left[\frac{R}{(100+R)} \times 100\right]\%$$

If A is R% less than B, then B is more than A by

$$\left[\frac{R}{(100-R)}\times 100\right]\%$$

SOLVED EXAMPLES

Ex. 1. Express each of the following as a fraction :

(i) 56% ***(ii) 4%*** ***(iii) 0.6%*** ***(iv) 0.08%***

Sol. *(i)* $56\%=\frac{56}{100}=\frac{14}{25}.$ *(ii)* $4\%=\frac{4}{100}=\frac{1}{25}.$

(iii) $0.6\%=\frac{0.6}{100}=\frac{6}{1000}=\frac{3}{500}.$ *(iv)* $0.08\%=\frac{0.08}{100}=\frac{8}{10000}=\frac{1}{1250}.$

Ex. 2. Express each of the following as a decimal :

(i) 6% ***(ii) 28%*** ***(iii) 0.2%*** ***(iv) 0.04%***

Sol. *(i)* $6\%=\frac{6}{100}=0.06.$ *(ii)* $28\%=\frac{28}{100}=0.28.$

(iii) $0.2\%=\frac{0.2}{100}=0.002.$ *(iv)* $0.04\%=\frac{0.04}{100}=0.0004.$

Ex. 3. Express each of the following as rate percent :

(i) $\frac{23}{36}$ ***(ii)*** $6\frac{3}{4}$ ***(iii) 0.004***

Sol. *(i)* $\frac{23}{36}=\left(\frac{23}{36}\times 100\right)\%=\left(\frac{575}{9}\right)\%=63\frac{8}{9}\%.$

(ii) $0.004=\frac{4}{1000}=\left(\frac{4}{1000}\times 100\right)\%=0.4\%.$

(iii) $6\frac{3}{4}=\frac{27}{4}=\left(\frac{27}{4}\times 100\right)\%=675\%.$

Ex. 4. Evaluate :

(i) 28% of 450 + 45% of 280

(ii) $16\frac{2}{3}\%\,of\ 600\,gm-33\frac{1}{3}\%\,of\ 180\,gm$

Sol. *(i)* 28% of 450 + 45% of 280 $=\left(\frac{28}{100}\times 450+\frac{45}{100}\times 280\right)=(126+126)=252.$

(ii) $16\frac{2}{3}\%\,of\,600\,gm-33\frac{1}{3}\%\,of\,180\,gm$

$=\left[\left(\frac{50}{3}\times\frac{1}{100}\times 600\right)-\left(\frac{100}{3}\times\frac{1}{100}\times 180\right)\right]$ gm = (100 – 60) gm = 40 gm.

Ex. 5. (i) 2 is what percent of 50?

(ii) $\frac{1}{2}$ ***is what percent of*** $\frac{1}{3}$***?***

(iii) What percent of 7 is 84?

(iv) What percent of 2 metric tonnes is 40 quintals?

(v) What percent of 6.5 litres is 130 ml?

Sol. *(i)* Required percentage = $\left(\frac{2}{50}\times 100\right)\% = 4\%.$

(ii) Required percentage = $\left(\frac{1}{2}\times\frac{3}{1}\times 100\right)\% = 150\%.$

(iii) Required percentage = $\left(\frac{84}{7}\times 100\right)\% = 1200\%.$

(iv) 1 metric tonne = 10 quintals.

$\therefore$ Required percentage = $\left(\frac{40}{2\times 10}\times 100\right)\% = 200\%.$

(v) Required percentage = $\left(\frac{130}{6.5\times 1000}\times 100\right)\% = 2\%.$

Ex. 6. Find the missing figures :

(i) ? % of 25 = 2.125 ***(ii) 9% of ? = 63*** ***(iii) 0.25% of ? = 0.04***

Sol. *(i)* Let x% of 25 = 2.125. Then, $\frac{x}{100}\times 25 = 2.125 \Leftrightarrow x = (2.125\times 4) = 8.5.$

(ii) Let 9% of x = 6.3. Then, $\frac{9}{100}x = 6.3 \Leftrightarrow x = \left(\frac{6.3\times 100}{9}\right) = 70.$

(iii) Let 0.25% of x = 0.04. Then, $\frac{0.25}{100}x = 0.04 \Leftrightarrow x = \left(\frac{0.04\times 100}{0.25}\right) = 16.$

Ex. 7. An inspector rejects 0.08% of the meters as defective. How many will he examine to reject 2?

Sol. Let the number of meters to be examined be x.

Then, 0.08% of x = 2 $\Leftrightarrow \left(\frac{8}{100}\times\frac{1}{100}\times x\right) = 2 \Leftrightarrow x = \left(\frac{2\times 100\times 100}{8}\right) = 2500.$

Ex. 8. Sixty-five percent of a number is 21 less than four-fifth of that number. What is the number?

Sol. Let the number be x.

Then, $\frac{4}{5}x - (65\%\ of\ x) = 21 \Leftrightarrow \frac{4}{5}x\frac{65}{100}x = 21 \Leftrightarrow 15x = 2100 \Leftrightarrow x = 140.$

Ex. 9. If 50% of (x - y) = 30% of (x + y), then what percent of x is y?

Sol. $50\%\ of\ (x-y) = 30\%\ of\ (x-y) \Leftrightarrow \frac{50}{100}(x-y) = \frac{30}{100}(x+y)$

$\Leftrightarrow 5(x-y) = 3(x+y) \Leftrightarrow 2x = 8y \Leftrightarrow x = 4y.$

$\therefore$ Required percentage = $\left(\frac{y}{x}\times 100\right)\% = \left(\frac{y}{4y}\times 100\right)\% = 25\%.$

Ex. 10. 10% of the inhabitants of a village having died of cholera, a panic set in, during which 25% of the remaining inhabitants left the village. The population is then reduced to 4050. Find the number of original inhabitants.

Sol. Let the total number of original inhabitants be x.

Then, (100 – 25)% of (100 – 10)% of x = 4050

$$\Leftrightarrow \quad \left(\frac{75}{100}\times\frac{90}{100}\times x\right) = 4050 \;\Leftrightarrow\; \frac{27}{40}x = 4050 \;\Leftrightarrow\; x = \left(\frac{4050\times 40}{27}\right) = 6000.$$

$\therefore$ Number of original inhabitants = 6000.

Ex. 11. The value of a machine depreciates at the rate of 10% per annum. If its present value is Rs. 1,62,000, what will be its worth after 2 years? What was the value of the machine 2 years ago?

Sol. Value of the machine after 2 years

$$= Rs.\left[162000\times\left(1-\frac{10}{100}\right)^2\right] = Rs.\left(162000\times\frac{9}{10}\times\frac{9}{10}\right) = Rs.131220.$$

Value of the machine 2 years ago

$$= Rs.\left[\frac{162000}{\left(1-\frac{10}{100}\right)^2}\right] = Rs.\left(162000\times\frac{10}{9}\times\frac{10}{9}\right) = Rs.200000.$$

Ex. 12. If A earns $33\frac{1}{3}\%$ ***more than B, how much percent does B earn less than A?***

Sol. Required percentage $= \left[\frac{\left(\frac{100}{3}\right)}{\left(100+\frac{100}{3}\right)}\times 100\right]\% = \left(\frac{100}{400}\times 100\right)\% = 25\%.$

Ex. 13. If A's salary is 20% less than B's salary, by how much percent is B's salary more than A's?

Sol. Required percentage = $\left[\frac{20}{(100-20)}\times 100\right]\% = 25\%.$

Ex. 14. How many kg of pure salt must be added to 30 kg of 2% solution of salt and water to increase it to a 10% solution?

Sol. Amount of salt in 30 kg solution = $\left(\frac{2}{100}\times 30\right)$ kg = 0.6 kg.

Let x kg of pure salt be added.

Then, $\frac{0.6+x}{30+x} = \frac{10}{100} \Leftrightarrow 60+100x = 300+10x \Leftrightarrow 90x = 240 \Leftrightarrow x = \frac{8}{3} = 2\frac{2}{3}.$

Ex. 15. Due to a reduction of $6\frac{1}{4}$ ***% in the price of sugar, a man is able to buy 1 kg more for Rs. 120. Find the original and reduced rate of sugar.***

Sol. Let original rate be Rs. x per kg.

Reduced rate = Rs. $\left[\left(100-\frac{25}{4}\right)\times\frac{1}{100}x\right]=Rs.\frac{15x}{16}\ per\ kg.$

$\therefore$ $\frac{120}{\frac{15x}{16}}-\frac{120}{x}=1\Leftrightarrow\frac{128}{x}-\frac{120}{x}=1\Leftrightarrow x=8.$

So, original rate = Rs. 8 per kg.

Reduced rate = Rs. $\left(\frac{15}{16}\times 8\right)$ per kg = Rs. 7.50 per kg.

Ex. 16. In an examination, 35% of total students failed in Hindi, 45% failed in English and 20% in both. Find the percentage of those who passed in both the subjects.

Sol. Let A and B be the sets of students who failed in Hindi and English respectively.

Then, n (A) = 35, n (B) = 45, n (A $\cap$ B) = 20.

So, n (A $\cup$ B) = n (A) + n (B) – n (A $\cap$ B) = (35 + 45 – 20) = 60.

$\therefore$ Percentage failed in Hindi or English or both = 60%.

Hence, percentage passed = (100 – 60)% = 40%.

EXERCISE – 1

(OBJECTIVE TYPE QUESTIONS)

Directions : ***Mark (√) against the correct answer :***

1. The ratio 5 : 4 expressed as a percent equals :

(a) 12.5% (b) 40% (c) 80% (d) 125%

2. 3.5 can be expressed in terms of percentage as :

(a) 0.35% (b) 3.5% (c) 35% (d) 350%

3. Half of 1 percent written as a decimal is :

(a) 0.005 (b) 0.05 (c) 0.02 (d) 0.2

4. What is 15 percent of Rs. 34?

(a) Rs. 3.40 (b) Rs. 3.75 (c) Rs. 4.50 (d) Rs. 5.10

5. 63% of $3\frac{4}{7}is$:

(a) 2.25 (b) 2.40 (c) 2.50 (d) 2.75

6. 860% of 50 + 50% of 860 = ?

(a) 430 (b) 516 (c) 860 (d) 960

7. 45% of 750 – 25% of 480 = ?

(a) 216 (b) 217.50 (c) 236.50 (d) 245

8. 40% of 1640 + ? = 35% of 980 + 150% of 850

(a) 372 (b) 842 (c) 962 (d) 1052

9. 60% of 264 is the same as :

(a) 10% of 44 (b) 15% of 1056 (c) 30% of 132 (d) None of these

10. 0.01 is what percent of 0.1?

(a) $\frac{1}{100}$ (b) $\frac{1}{10}$ (c) 10 (d) 100

11. What percent of Rs. 2650 is Rs 1987.50 ?

(a) 60% (b) 75% (c) 80% (d) 90%

12. What percent of a day is 3 hours?

(a) $12\frac{1}{2}\%$ (b) $16\frac{2}{3}\%$ (c) $18\frac{2}{3}\%$ (d) $22\frac{1}{2}\%$

13. It costs Rs. 1 to photocopy a sheet of paper. However, 2% discount is allowed on all photocopies done after first 1000 sheets. How, much will it cost to copy 5000 sheets of paper?

(a) Rs. 3920 (b) Rs. 3980 (c) Rs. 4900 (d) Rs. 4920

14. How many litres of pure acid are there in 8 litres of a 20% solution?

(a) 1.4 (b) 1.5 (c) 1.6 (d) 2.4

15. Rajeev buys goods worth Rs. 6650. He gets a rebate of 6% on it. After getting the rebate, he pays sales tax @ 10%. Find the amount he will have to pay for the goods.

(a) Rs. 6876.10 (b) Rs. 6999.20 (c) Rs. 6654 (d) Rs. 7000

16. Which one of the following shows the best percentage?

(a) $\frac{384}{540}$ (b) $\frac{425}{500}$ (c) $\frac{570}{700}$ (d) $\frac{480}{660}$

17. 0.15% of $33\frac{1}{3}\%$ of Rs. 10,000 is :

(a) Rs. 0.05 (b) Rs. 5 (c) Rs. 105 (d) Rs. 150

18. ?% of 360 = 129.6

(a) 36 (b) 64 (c) 72 (d) 77

19. ?% of 932 + 30 = 309.6

(a) 25 (b) 30 (c) 35 (d) 40

20. 45% of 1500 + 35% of 1700 = ?% of 3175

(a) 30 (b) 35 (c) 45 (d) None of these

21. 65% of ? = 20% of 422.50

(a) 84.5 (b) 130 (c) 139.425 (d) 200

22. If Rs. 2800 is $\frac{2}{7}$ percent of the value of a house, the worth of the house (in Rs.) is :

(a) 8, 00, 000 (b) 9,80,000 (c) 10,00,000 (d) 12,00,000

23. If 35% of a number is 175, then what percent of 175 is that number?

(a) 35% (b) 65% (c) 280% (d) None of these

24. Two-fifth of one-third of three-seventh of a number is 15. What is 40 percent of that number?

(a) 72 (b) 84 (c) 136

(d) 140 (e) None of these

25. The difference between a number and its two-fifth is 510. What is 10% of that number?

(a) 12.75 (b) 85 (c) 204 (d) None of these

26. If 35% of a number is 12 less than 50% of that number, then the number is :

(a) 40 (b) 50 (c) 60 (d) 80

27. The number which exceeds 16% of it by 42 is :

(a) 50 (b) 52 (c) 58 (d) 60

28. What percentage of numbers from 1 to 70 have squares that end in the digit 1?

(a) 1 (b) 14 (c) 20 (d) 21

29. If a number x is 10% less than another number y and y is 10% more than 125, then x is equal to:

(a) 123.75 (b) 140.55 (c) 143 (d) 150

30. If 75% of a number is added to 75, then the result is the number itself. The number is :

(a) 50 (b) 60 (c) 300 (d) 400

31. A number, when 35 is subtracted from it, reduces to its 80 percent. What is four-fifth of that number?

(a) 70 (b) 90 (c) 120 (d) 140

32. The sum of two numbers is 2490. if 6.5% of one number is equal to 8.5% of the other, then the numbers are :

(a) 989, 1501 (b) 1011, 1479 (c) 1401, 1089 (d) 1411, 1079

33. The sum of two numbers is $\frac{28}{25}$ of the first number. The second number is what percent of the first?

(a) 12% (b) 14% (c) 16% (d) 18%

34. If 25% of a number is subtracted from a second number, the second number reduces to its five-sixth. What is the ratio of the first number to the second number?

(a) 1 : 3 (b) 2 : 3 (c) 3 : 2 (d) Data inadequate

35. The difference of two numbers is 20% of the larger number. If the smaller number is 20, then the larger number is :

(a) 25 (b) 45 (c) 50 (d) 80

36. When any number is divided by 12, then dividend becomes $\frac{1}{4}$ th of the other number. By how much percent first number is greater than the second number?

(a) 150 (b) 200 (c) 300 (d) Data inadequate

37. If one number is 80% of the other and 4 times the sum of their squares is 656, then the numbers are :

(a) 4, 5 (b) 8, 10 (c) 16, 20 (d) None of these

38. Two numbers A and B are such that the sum of 5% of A and 4% of B is two-third of the sum of 6% of A and 8% of B. Find the ratio of A : B.

(a) 2 : 3 (b) 1 : 1 (c) 3 : 4 (d) 4 : 3

39. Three candidates contested an election and received 1136, 7636 and 11628 votes respectively. What percentage of the total votes did the winning candidate get?

(a) 57% (b) 60% (c) 65% (d) 90%

40. The population of a town increased from, 1,75,000 to 2,62,500 in a decade. The average percent increase of population per year is :

(a) 4.37% (b) 5% (c) 6% (d) 8.75%