PIPES AND CISTERNS

IMPORTANT FACTS AND FORMULAE

1. Inlet : A pipe connected with a tank or a cistern or a reservoir, that fills it, is known as an inlet.

Outlet : A pipe connected with a tank or a cistern or a reservoir, emptying it, is known as an outlet.

2.   (i) If a pipe can fill a tank in x hours, then :

part filled in 1 hour $= \dfrac{1}{x}$.

(ii) If a pipe can empty a full tank in y hours, then :

part emptied in 1 hour $= \dfrac{1}{y}$.

(iii) If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours (where y > x), then on opening both the pipes, the net part filled in 1 hour $= \left(\dfrac{1}{x} - \dfrac{1}{y} \right)$.

(iv) If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours (where x > y), then on opening both the pipes, the net part emptied in 1 hour $= \left(\dfrac{1}{y} - \dfrac{1}{x} \right)$.

SOLVED EXAMPLES

Ex. 1. *Two pipes can fill a tank in 10 hours and 12 hours respectively while a third pipe empties the full tank in 20 hours. If all three pipes operate simultaneously, in how much time will the tank be filled?*

Sol.   Net part filled in 1 hour $= \left(\dfrac{1}{10} + \dfrac{1}{12} - \dfrac{1}{20} \right) = \dfrac{8}{60} = \dfrac{2}{15}$.

∴   The tank will be full in $\dfrac{15}{2}$ hrs = 7 hrs 30 min.

Ex. 2. *A cistern has two taps which fill it in 12 minutes and 15 minutes respectively. There is also a waste pipe in the cistern. When all the three are opened, the empty cistern is full in 20 minutes. How long will the waste pipe take to empty the full cistern?*

Sol.   Work done by the waste pipe in 1 minute

$$= \dfrac{1}{20} - \left(\dfrac{1}{12} + \dfrac{1}{15} \right) = - \dfrac{1}{10} \qquad \text{[– ve sign means emptying]}$$

∴   Waste pipe will empty the full cistern in 10 minutes.

Ex. 3. *Two pipes can fill a cistern in 14 hours and 16 hours respectively. The pipes are opened simultaneously and it is found that due to leakage in the bottom it took 32 minutes more to fill the cistern. When the cistern is full, in what time will the leak empty it?*

Sol.   Work done by the two pipes in 1 hour $= \left(\dfrac{1}{14} + \dfrac{1}{16} \right) = \dfrac{15}{112}$.

∴   Time taken by these pipes to fill the tank $= \dfrac{112}{15}$ hrs $= $ 7 hrs 28 min.

Due to leakage, time taken = 7 hrs 28 min + 32 min = 8 hrs

$\therefore$ Work done by (two pipes + leak) in 1 hour $= \dfrac{1}{8}$.

        Work done by the leak in 1 hour $= \left(\dfrac{15}{112} - \dfrac{1}{8}\right) = \dfrac{1}{112}$.

Ex. 4. *Two pipes A and B can fill a tank in 24 min. and 32 min. respectively. If both the pipes are opened simultaneously, after how much time B should be closed so that the tank is full in 18 minutes?*

Sol.   Let B be closed after x minutes. Then,

       part filled by (A + B) in x min. + part filled by A in (18 – x) min. = 1

$\therefore$ $x\left(\dfrac{1}{24} + \dfrac{1}{32}\right) + (18-x) \times \dfrac{1}{24} = 1$ $\Leftrightarrow$ $\dfrac{7x}{96} + \dfrac{18-x}{24} = 1$

$\Leftrightarrow$ $7x + 4(18 - x) = 96$ $\Leftrightarrow$ $x = 8$.

       Hence, B must be closed after 8 minutes.

Exercise – 1

(OBJECTIVE TYPE QUESTIONS)

Directions : Mark ($\sqrt{}$) against the correct answer :

1.   Two pipes A and B can fill a tank in 20 and 30 minutes respectively. If both the pipes are used together, then how long will it take to fill the tank?

      (a) 12 min   (b) 15 min   (c) 25 min   (d) 50 min

2.   A cistern can be filled by a tap in 4 hours while it can be emptied by another tap in 9 hours. If both the taps are opened simultaneously then after how much time will the cistern get filled?

      (a) 4.5 hrs   (b) 5 hrs   (c) 6.5 hrs   (d) 7.2 hrs

3.   A tap can fill a tank in 6 hours. After half the tank is filled, three more similar taps are opened. What is the total time taken to fill the tank completely?

      (a) 3 hrs 15 min   (b) 3 hrs 45 min   (c) 4 hrs   (d) 4 hrs 15 min

4.   A water tank is two-fifth full. Pipe A can fill a tank in 10 minutes and pipe B can empty it in 6 minutes. If both the pipes are open, how long will it take to empty or fill the tank completely?

      (a) 6 min. to empty   (b) 6 min. to fill   (c) 9 min. to empty

      (d) 9 min. to fill   (e) None of these

5.   Pipe A can fill a tank in 5 hours, pipe B in to 10 hours and pipe C in 30 hours. If all the pipes are open, in how many hours will the tank be filled?

      (a) 2   (b) 2.5   (c) 3   (d) 3.5

6.   Pipes A and B can fill a tank in 5 and 6 hours respectively. Pipe C can empty it in 12 hours. If all the three pipes are opened together, then the tank will be filled in:

(a) $1\frac{13}{17}hours$ (b) $2\frac{8}{11}hours$ (c) $3\frac{9}{17}hours$ (d) $4\frac{1}{2}hours$

7. Three pipes A, B and C can fill a tank from empty to full in 30 minutes, 20 minutes and 10 minutes respectively. When the tank is empty, all the three pipes are opened. A, B and C discharge chemical solutions P, Q and R respectively. What is the proportion of solution R in the liquid in the tank after 3 minutes?

(a) $\frac{5}{11}$ (b) $\frac{6}{11}$ (c) $\frac{7}{11}$ (d) $\frac{8}{11}$

8. Two pipes A and B can separately fill a cistern in 60 minutes and 75 minutes respectively. There is a third pipe in the bottom of the cistern to empty it. If all the three pipes are simultaneously opened, then the cistern is full in 50 minutes. In how much time the third pipe alone can empty the cistern?

(a) 90 min   (b) 100 min   (c) 110 min   (d) 120 min

9. A pump can fill a tank with water in 2 hours. Because of a leak, it took $2\frac{1}{3}$ hours to fill the tank. The leak can drain all the water of the tank in:

(a) $4\frac{1}{3}hrs$ (b) 7 hrs   (c) 8 hrs   (d) 14 hrs

10. Two pipes A and B together can fill a cistern in 4 hours. Had they been opened separately, then B would have taken 6 hours more than A to fill the cistern. How much time will be taken by A to fill the cistern separately?

(a) 1 hr   (b) 2 hrs   (c) 6 hrs   (d) 8 hrs.

11. One pipe can fill a tank three times as fast as another pipe. If together the two pipes can fill the tank in 36 minutes, then the slower pipe alone will be able to fill the tank in:

(a) 81 min   (b) 108 min   (c) 144 min   (d) 192 min

12. A tank is filled in 5 hours by three pipes A, B and C. The pipe C is twice as fast as B and B is twice as fast as A. How much time will pipe A alone take to fill the tank?

(a) 20 hrs   (b) 25 hrs   (c) 35 hrs

(c) Cannot be determined   (e) None of these

13. A tank is filled by three pipes with uniform flow. The first two pipes operating simultaneously fill the tank in the same time during which the tank is filled by the third pipe alone. The second pipe fills the tank 5 hours faster than the first pipe and 4 hours slower than the third pipe. The time required by the first pipe is:

(a) 6 hrs   (b) 10 hrs   (c) 15 hrs   (d) 30 hrs

14. Two pipes A and B can fill a tank in 15 minutes and 20 minutes respectively. Both the pipes are opened together but after 4 minutes, pipe A is turned off. What is the total time required to fill the tank?

(a) 10 min 20 sec   (b) 11 min 45 sec   (c) 12 min 30 sec   (d) 14 min 40 sec

15. A large tanker can be filled by two pipes A and B in 60 minutes and 40 minutes respectively. How many minutes will it take to fill the tanker from empty state if B is used for half the time and A and B fill it together for the other half?

(a) 15 min   (b) 20 min   (c) 27.5 min   (d) 30 min

16. Three taps A, B and C can fill a tank in 12, 15 and 20 hours respectively. If A is open all the time and B and C are open for one hour each alternately, the tank will be full in:

(a) 6 hrs   (b) $6\frac{2}{3} hrs$ (c) 5   (d) $7\frac{1}{2} hrs$

17. Two pipes can fill a tank in 20 and 24 minutes respectively and a waste pipe can empty 3 gallons per minute. All the three pipes working together can fill the tank in 15 minutes. The capacity of the tank is:

(a) 60 gallons   (b) 100 gallons   (c) 120 gallons   (d) 180 gallons

18. Two pipes A and B can fill a cistern in $37\frac{1}{2}$ minutes and 45 minutes respectively. Both pipes are opened. The cistern will be filled in just half an hour, if the pipe B is turned off after:

(a) 5 min   (b) 9 min   (c) 10 min   (d) 15 min

19. Three pipes A, B and C and fill a tank in 6 hours. After working at it together for 2 hours, C is closed and A and B can fill the remaining part in 7 hours. The number of hours taken by C alone to fill the tank is:

(a) 10   (b) 12   (c) 14   (d) 16