

1. RATIO AND PROPORTION

IMPORTANT FACTS AND FORMULAE

1. RATIO : The ratio of two quantities a and b in the same units, is the fraction $\frac{a}{b}$ and we write it as $a : b$.

In the ratio $a : b$, we call a as the **first term** or **antecedent** and b , the **second term** or **consequent**.

Ex. The ratio $5 : 9$ represents $\frac{5}{9}$ with antecedent = 5, consequent = 9.

Rule : The multiplication or division of each term of a ratio by the same non-zero number does not affect the ratio.

Ex. $4 : 5 = 8 : 10 = 12 : 15$ etc. Also, $4 : 6 = 2 : 3$.

2. PROPORTION : The equality of two ratios is called proportion.

If $a : b = c : d$, we write, $a : b :: c : d$ and we say that a, b, c, d are in proportion. Here a and d are called **extremes**, while b and c are called **mean terms**.

Product of means = Product of extremes.

Thus, $a : b :: c : d \Leftrightarrow (b \times c) = (a \times d)$.

3. (i) Fourth Proportional : If $a : b = c : d$, then d is called the fourth proportional to a, b, c .

(ii) Third Proportional : If $a : b = b : c$, then c is called the third proportional to a and b .

(iii) Mean Proportional : Mean proportional between a and b is \sqrt{ab} .

4. (i) COMPARISON OF RATIOS :

We say that $(a:b) > (c:d) \Leftrightarrow \frac{a}{b} > \frac{c}{d}$.

(ii) COMPOUNDED RATIO :

The compounded ratio of the ratios $(a : b), (c : d), (e : f)$ is $(ace : bdf)$.

5. (i) Duplicate ratio of $(a : b)$ is $(a^2 : b^2)$.

(ii) Sub-duplicate ratio of $(a : b)$ is $(\sqrt{a} : \sqrt{b})$.

(iii) Triplicate ratio of $(a : b)$ is $(a^3 : b^3)$.

(iv) Sub-triplicate ratio of $(a : b)$ is $\left(\sqrt[3]{a} : \sqrt[3]{b} \right)$.

(v) If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$. **(componendo and dividendo)**

6. VARIATION :

(i) We say that x is directly proportional to y , if $x = ky$ for some constant k and we write, $x \propto y$.

(ii) We say that x is inversely proportional to y , if $xy = k$ for some constant k and we write, $x \propto \frac{1}{y}$.

SOLVED PROBLEMS

Ex. 1. If $a : b = 5 : 9$ and $b : c = 4 : 7$, find $a : b : c$.

Sol. $a : b = 5 : 9$ and $b : c = 4 : 7 = \left(4 \times \frac{9}{4}\right) : \left(7 \times \frac{9}{4}\right) = 9 : \frac{63}{4}$

$$\Rightarrow a : b : c = 5 : 9 : \frac{63}{4} = 20 : 36 : 63.$$

Ex. 2. Divide Rs. 672 in the ratio 5 : 3.

Sol. Sum of ratio terms $= (5 + 3) = 8$.

$$\therefore \text{First part} = \text{Rs. } \left(672 \times \frac{5}{8}\right) = \text{Rs. } 420; \text{ Second part} = \text{Rs. } \left(672 \times \frac{3}{8}\right) = \text{Rs. } 252.$$

Ex. 3. Divide Rs. 1162 among A, B, C in the ratio 35 : 28 : 20.

Sol. Sum of ratio terms $= (35 + 28 + 20) = 83$.

$$\text{A's share} = \text{Rs. } \left(1162 \times \frac{35}{83}\right) = \text{Rs. } 490; \text{ B's share} = \text{Rs. } \left(1162 \times \frac{28}{83}\right) = \text{Rs. } 392;$$

$$\text{C's share} = \text{Rs. } \left(1162 \times \frac{20}{83}\right) = \text{Rs. } 280.$$

Exercise – 1

(OBJECTIVE TYPE QUESTIONS)

Directions : *Mark (✓) against the correct answer :*

1. If $A : B : C = 2 : 3 : 4$, then $\frac{A}{B} : \frac{B}{C} : \frac{C}{A}$ is equal to :
(a) 4 : 9 : 16 (b) 8 : 9 : 12 (c) 8 : 9 : 16 (d) 8 : 9 : 24
2. If $A : B = 2 : 3$, $B : C = 4 : 5$ and $C : D = 6 : 7$, then $A : B : C : D$ is:
(a) 16 : 22 : 30 : 35 (b) 16 : 24 : 15 : 35 (c) 16 : 24 : 30 : 35 (d) 18 : 24 : 30 : 35
3. If $0.75 : x :: 5 : 8$, then x is equal to :
(a) 1.12 (b) 1.20 (c) 1.25 (d) 1.30
4. If $x : y = 5 : 2$, then $(8x + 9y) : (8x + 2y)$ is:
(a) 22 : 29 (b) 26 : 61 (c) 29 : 22 (d) 61 : 26
5. The salaries of A, B, C are in the ratio 2 : 3 : 5. If the increments of 15%, 10% and 20% are allowed respectively in their salaries, then what will be the new ratio of their salaries?
(a) 3 : 3 : 10 (b) 10 : 11 : 20 (c) 23 : 33 : 60 (d) Cannot be determined
6. If Rs. 782 be divided into three parts, proportional to $\frac{1}{2} : \frac{2}{3} : \frac{3}{4}$, then the first part is:
(a) Rs. 182 (b) Rs. 190 (c) Rs. 196 (d) Rs. 204
7. Two numbers are in the ratio 3 : 5. If 9 is subtracted from each, the new numbers are in the ratio 12 : 23. The smaller number is:
(a) 27 (b) 33 (c) 49 (d) 55
8. Two numbers are in the ratio 1 : 2. If 7 is added to both, their ratio changes to 3 : 5. The greatest number is:
(a) 24 (b) 26 (c) 28 (d) 32
9. In a bag, there are coins to 25 p, 10 p and 5 p in the ratio of 1 : 2 : 3. If there are Rs. 30 in all, how many 5 p coins are there?
(a) 50 (b) 100 (c) 150 (d) 200

10. Salaries of Ravi and Sumit are in the ratio 2 : 3. If the salary of each is increased by Rs. 4000, the new ratio becomes 40 : 57. What is Sumit's present salary?
 (a) Rs. 17,000 (b) Rs. 20,000 (c) Rs. 25,500 (d) None of these
11. If Rs. 510 be divided among A, B, C in such a way that A gets $\frac{2}{3}$ of what B gets and B gets $\frac{1}{4}$ of what C gets, then their shares are respectively:
 (a) Rs. 120, Rs. 240, Rs. 150 (b) Rs. 60, Rs. 90, Rs. 360
 (c) Rs. 150, Rs. 300, Rs. 60 (d) None of these
12. The sum of three numbers is 98. If the ratio of the first to the second is 2 : 3 and that of the second to the third is 5 : 8, then the second number is:
 (a) 20 (b) 30 (c) 48 (d) 58
13. A fraction which bears the same ratio to $\frac{1}{27}$ that $\frac{3}{11}$ does to $\frac{5}{9}$, is equal to:
 (a) $\frac{1}{55}$ (b) $\frac{1}{11}$ (c) $\frac{3}{11}$ (d) 55
14. A sum of Rs. 1300 is divided amongst P, Q, R and S such that

$$\frac{P's\ share}{Q's\ share} = \frac{Q's\ share}{R's\ share} = \frac{R's\ share}{S's\ share} = \frac{2}{3}. Then, P's\ share\ is :$$

 (a) Rs. 140 (b) Rs. 160 (c) Rs. 240 (d) Rs. 320
15. A and B together have Rs. 1210. If $\frac{4}{15}$ of A's amount is equal to $\frac{2}{5}$ of B's amount, how much amount does B have?
 (a) Rs. 460 (b) Rs. 484 (c) Rs. 550 (d) Rs. 664
16. Two numbers are respectively 20% and 50% more than a third number. The ratio of the two numbers is:
 (a) 2 : 5 (b) 3 : 5 (c) 4 : 5 (d) 6 : 7
17. Seats for Mathematics, Physics and Biology in a school are in the ratio 5 : 7 : 8. There is a proposal to increase these seats by 40%, 50% and 75% respectively. What will be the ratio of increased seats?
 (a) 2 : 3 : 4 (b) 6 : 7 : 8 (c) 6 : 8 : 9 (d) None of these
18. The ratio of the number of boys and girls in a college is 7 : 8. If the percentage increase in the number of boys and girls be 20% and 10% respectively, what will be the new ratio?

- (a) 8 : 9 (b) 17 : 18 (c) 21 : 22 (d) Cannot be determined
- 19.** A sum of money is to be distributed among A, B, C, D in the proportion of 5 : 2 : 4 : 3. If C gets Rs. 1000 more than D, what is B's share?
- (a) Rs. 500 (b) Rs. 1500 (c) Rs. 2000 (d) None of these
- 20.** If 40% of a number is equal to two-third of another number, what is the ratio of first number to the second number?
- (a) 2 : 5 (b) 3 : 7 (c) 5 : 3 (d) 7 : 3
- 21.** Ratio of the earnings of A and B is 4 : 7. If the earnings of A increase by 50% and those of B decrease by 25%, the new ratio of their earnings becomes 8 : 7. What are A's earnings?
- (a) Rs. 21,000 (b) Rs. 26,000 (c) Rs. 28,000 (d) Data inadequate
- 22.** The fourth proportional to 5, 8, 15 is :
- (a) 18 (b) 24 (c) 19 (d) 20 (e) 21
- 23.** x varies inversely as square of y . Given that $y = 2$ for $x = 1$. The value of x for $y = 6$ will be equal to:
- (a) 3 (b) 9 (c) $\frac{1}{3}$ (d) $\frac{1}{9}$
- 24.** If 10% of $x = 20\%$ of y , then $x : y$ is equal to:
- (a) 1 : 2 (b) 2 : 1 (c) 5 : 1 (d) 10 : 1

2. PARTNERSHIP

IMPORTANT FACTS AND FORMULAE

1. Partnership: When two or more than two persons run a business jointly, they are called *partners* and the deal is known as *partnership*.

2. Ratio of Division of Gains:

(i) When investments of all the partners are for the same time, the gain or loss is distributed among the partners in the ratio of their investments.

Suppose A and B invest Rs. x and Rs. y respectively for a year in a business, then at the end of the year:

(A's share of profit) : (B's share of profit) = $x : y$.

(ii) When investments are for different time periods, then equivalent capitals are calculated for a unit of time by taking (capital \times number of units of time). Now, gain or loss is divided in the ratio of these capitals.

Suppose A invests Rs. x for p months and B invests Rs. y for q months, then (A's share of profit) : (B's share of profit) = $xp : yq$.

3. Working and Sleeping Partners: A partner who manages the business is known as a *working partner* and the one who simply invests the money is a *sleeping partner*.

SOLVED EXAMPLES

Ex.1. A, B and C started a business by investing Rs. 1,20,000, Rs. 1,35,000 and Rs. 1,50,000 respectively. Find the share of each, out of an annual profit of Rs. 56,700.

Sol. Ratio of shares of A, B and C = Ratio of their investments
 $= 120000 : 135000 : 150000 = 8 : 9 : 10$.

$$\therefore \text{A's share} = \text{Rs.} \left(56700 \times \frac{8}{27} \right) = \text{Rs. } 16800.$$

$$\text{B's share} = \text{Rs.} \left(56700 \times \frac{9}{27} \right) = \text{Rs. } 18900.$$

$$\text{C's share} = \text{Rs.} \left(56700 \times \frac{10}{27} \right) = \text{Rs. } 21000.$$

Ex.2. A, B and C enter into partnership. A invests 3 times as much as B invests and B invests two-third of what C invests. At the end of the year, the profit earned is Rs. 6600. What is the share of B?

Sol. Let C's capital = Rs. x . Then, B's capital = Rs. $\frac{2}{3}x$.

$$\text{A's capital} = \text{Rs.} \left(3 \times \frac{2}{3}x \right) = \text{Rs. } 2x.$$

$$\therefore \text{Ratio of their capitals} = 2x : \frac{2}{3}x : x = 6 : 2 : 3.$$

$$\text{Hence, B's share} = \text{Rs.} \left(6600 \times \frac{2}{11} \right) = \text{Rs. } 1200.$$

Ex.6. A invested Rs. 76,000 in a business. After few months, B joined him with Rs. 57,000. At the end of the year, the total profit was divided between them in the ratio 2 : 1. After how many months did B join?

Sol. Suppose B joined after x months. Then, B's money was invested for (12 - x) months.

$$\therefore \frac{76000 \times 12}{57000 \times (12 - x)} = \frac{2}{1} \Leftrightarrow 912000 = 114000 (12 - x)$$

$$\Leftrightarrow 114 (12 - x) = 912 \Leftrightarrow (12 - x) = 8 \Leftrightarrow x = 4.$$

Hence, B joined after 4 months.

Exercise – 1

(OBJECTIVE TYPE QUESTIONS)

Directions : Mark (✓) against the correct answer:

- P and Q started a business investing Rs. 85,000 and Rs. 15,000 respectively. In what ratio the profit earned after 2 years be divided between P and Q respectively?
(a) 3 : 4 (b) 3 : 5 (c) 15 : 23 (d) 17 : 23 (e) None of these
- A, B, C enter into a partnership investing Rs. 35,000 and Rs. 45,000 and Rs. 55,000 respectively. The respective shares of A, B, C in an annual profit of Rs. 40,500 are:
(a) Rs. 10,500, Rs. 13,500, Rs. 16,500 (b) Rs. 11,500, Rs. 13,000, Rs. 16,000
(c) Rs. 11,000, Rs. 14,000, Rs. 15,500 (d) Rs. 11,500, Rs. 12,500, Rs. 16,500.
- Kamal started a business by investing Rs. 9000. After five months, Sameer joined with a capital of Rs. 8000. If at the end of the year, they earn a profit of Rs. 6970, then what will be the share of Sameer in the profit?
(a) Rs. 1883.78 (b) Rs. 2380 (c) Rs. 3690 (d) Rs. 3864
- Simran started a software business by investing Rs. 50,000. After six months, Nanda joined her with a capital of Rs. 80,000. After 3 years, they earned a profit of Rs. 24,500. What was Simran's share in the profit?
(a) Rs. 9423 (b) 10,250 (c) Rs. 12,500 (d) Rs. 14,000 (e) None of these
- A and B started a business in partnership investing Rs. 20,000 and Rs. 15,000 respectively. After six months, C joined them with Rs. 20,000. What will be B's share in the total profit of Rs. 25,000 earned at the end of 2 years from the starting of the business?
(a) Rs. 7500 (b) Rs. 9000 (c) Rs. 9500 (d) Rs. 10,000 (e) None of these
- Aman started a business investing Rs. 70,000. Rakhi joined him after six months with an amount of Rs. 1,05,000 and Sagar joined them with Rs. 1.4 lakhs after another six months. The amount of

- profit earned should be distributed in what ratio among Aman, Rakhi and Sagar respectively, 3 years after Aman started the business?
- (a) 7 : 6 : 10 (b) 12 : 15 : 16 (c) 42 : 45 : 56
(d) Cannot be determined (e) None of these
7. Arun, Kamal and Vinay invested Rs. 8000, Rs. 4000 and Rs. 8000 respectively in a business. Arun left after six months. If after eight months, there was a gain of Rs. 4005, then what will be the share of Kamal?
- (a) Rs. 890 (b) Rs. 1335 (c) Rs. 1602 (d) Rs. 1780
8. A, B and C enter into a partnership. They invest Rs. 40,000, Rs. 80,000 and Rs. 1,20,000 respectively. At the end of the first year, B withdraws Rs. 40,000, while at the end of the second year, C withdraw Rs. 80,000. In what ratio will the profit be shared at the end of 3 years?
- (a) 2 : 3 : 5 (b) 3 : 4 : 7 (c) 4 : 5 : 9 (d) None of these
9. Shekhar started a business investing Rs. 25,000 in 1999. In 2000, he invested an additional amount of Rs. 10,000 and Rajeev joined him with an amount of Rs. 35,000. In 2001, Shekhar invested another additional amount of Rs. 10,000 and Jatin joined them with an amount of Rs. 35,000. What will be Rajeev's share in the profit of Rs. 1,50,000 earned at the end of 3 years from the start of the business in 1999?
- (a) Rs. 45,000 (b) Rs. 50,000 (c) Rs. 70,000 (d) Rs. 75,000 (e) None of these
10. A and B start a business with investments of Rs. 5000 and 4500 respectively. After 4 months, A takes out half of his capital. After two more months, B takes out one-third of his capital while C joins them with a capital of Rs. 7000. At end of a year, they earn a profit of Rs. 5080. Find the share of each member in the profit.
- (a) A – Rs. 1400, B – Rs. 1900, C – Rs. 1780
(b) A – Rs. 1600, B – Rs. 1800, C – Rs. 1680
(c) A – Rs. 1800, B – Rs. 1500, C – Rs. 1780
(d) A – Rs. 1680, B – Rs. 1600, C – Rs. 1800
(e) None of these
11. A, B and C rent a pasture. A puts 10 oxen for 7 months, B puts 12 oxen for 5 months and C puts 15 oxen for 3 months for grazing. If the rent of the pasture is Rs. 175, how much must C pay as his share of rent?
- (a) Rs. 45 (b) Rs. 50 (c) Rs. 55 (d) Rs. 60

12. In a business, A and C invested amounts in the ratio 2 : 1, whereas the ratio between amounts invested by A and B was 3 : 2. If Rs. 1,57,300 was their profit, how much amount did B receive?
 (a) Rs. 24,200 (b) Rs. 36,300 (c) Rs. 48,400 (d) Rs. 72,600
13. A and B started a partnership business investing some amount in the ratio of 3 : 5. C joined them after six months with an amount equal to that of B. In what proportion should the profit at the end of one year be distributed among A, B and C?
 (a) 3 : 5 : 2 (b) 3 : 5 : 5 (c) 6 : 10 : 5
 (d) Data inadequate (e) None of these
14. A, B and C enter into a partnership and their shares are in the ratio $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$. After 2 months, A withdraws half of his capital and after 10 months, a profit of Rs. 378 is divided among them. What is B's share?
 (a) Rs. 129 (b) Rs. 144 (c) Rs. 156 (d) Rs. 168
15. A, B and C enter into a partnership in the ratio $\frac{7}{2} : \frac{4}{3} : \frac{6}{5}$. After 4 months, A increases his share by 50%. If the total profit at the end of one year be Rs. 21,600, then B's share in the profit is:
 (a) Rs. 2100 (b) Rs. 2400 (c) Rs. 3600 (d) Rs. 4000
16. A and B entered into partnership with capitals in the ratio 4 : 5. After 3 months, A withdrew $\frac{1}{4}$ of his capital and B withdrew $\frac{1}{5}$ of his capital. The gain at the end of 10 months was Rs. 760. A's share in this profit is:
 (a) Rs. 330 (b) Rs. 360 (c) Rs. 380 (d) Rs. 430
17. A, B and C jointly thought of engaging themselves in a business venture. It was agreed that A would invest Rs. 6500 for 6 months, B, Rs. 8400 for 5 months and C, Rs. 10,000 for 3 months. A wants to be the working member for which he was to receive 5% of the profits. The profit earned was Rs. 7400. Calculate the share of B in the profit.
 (a) Rs. 1900 (b) Rs. 2660 (c) Rs. 2800 (d) Rs. 2840
18. A began a business with Rs. 85,000. He was joined afterwards by B with Rs. 42,500. For how much period does B join, if the profits at the end of the year are divided in the ratio of 3 : 1?
 (a) 4 months (b) 5 months (c) 6 months (d) 8 months

- 19.** A starts business with Rs. 3500 and after 5 months, B joins with A as his partner. After a year, the profit is divided in the ratio 2 : 3. What is B's contribution in the capital?
 (a) Rs. 7500 (b) Rs. 8000 (c) Rs. 8500 (d) Rs. 9000
- 20.** Two friends P and Q started a business investing in the ratio of 5 : 6. R joined them after six months investing an amount equal to that of Q's. At the end of the year, 20% profit was earned which was equal to Rs. 98,000. What was the amount invested by R?
 (a) Rs. 1,05,000 (b) Rs. 1,75,000 (c) Rs. 2,10,000
 (d) Data inadequate (e) None of these
- 21.** Three partners shared the profit in a business in the ratio 5 : 7 : 8. They had partnered for 14 months, 8 months and 7 months respectively. What was the ratio of their investments?
 (a) 5 : 7 : 8 (b) 28 : 49 : 64 (c) 38 : 28 : 21 (d) None of these
- 22.** A and B are partners in a business. A contributes $\frac{1}{4}$ of the capital for 15 months and B received $\frac{2}{3}$ of the profit. For how long B's money was used?
 (a) 6 months (b) 9 months (c) 10 months (d) 1 year

3. TIME AND WORK

IMPORTANT FACTS AND FORMULAE

1. If A can do a piece of work in n days, then A's day's work $= \frac{1}{n}$.
2. If A's 1 day's work $= \frac{1}{n}$, then A can finish the work in n days.
3. If A is thrice as good a workman as B, then:
Ratio of work done by A and B = 3 : 1.
Ratio of times taken by A and B to finish a work = 1 : 3.

SOLVED EXAMPLES

Ex. 1. *A and B together can complete a piece of work in 4 days. If A alone can complete the same work in 12 days, in how many days can B alone complete that work?*

Sol. (A + B)'s 1 day's work $= \frac{1}{4}$, A's 1 day's work $= \frac{1}{12}$.

\therefore B's 1 day's work $= \left(\frac{1}{4} - \frac{1}{12} \right) = \frac{1}{6}$.

Hence, B alone can complete the work in 6 days.

Ex. 2. *A is twice as good a workman as B and together they finish a piece of work in 18 days. In how many days will A alone finish the work?*

Sol. (A's 1 day's work) : (B's 1 day's work) = 2 : 1.

(A + B)'s 1 day's work $= \frac{1}{18}$.

Divide $\frac{1}{18}$ in the ratio 2 : 1.

\therefore A's 1 day's work $= \left(\frac{1}{18} \times \frac{2}{3} \right) = \frac{1}{27}$.

Hence, A alone can finish the work in 27 days.

Ex. 3. *A and B undertake to do a piece of work for Rs. 600. A alone can do it in 6 days while B alone can do it in 8 days. With the help of C, they finish it in 3 days. Find the share of each.*

Sol. C's 1 day's work $= \frac{1}{3} - \left(\frac{1}{6} + \frac{1}{8} \right) = \frac{1}{24}$.

\therefore A : B : C = Ratio of their 1 day's work $= \frac{1}{6} : \frac{1}{8} : \frac{1}{24} = 4 : 3 : 1$.

\therefore A's share = Rs. $\left(600 \times \frac{4}{8} \right)$ = Rs. 300, B's share = Rs. $\left(600 \times \frac{3}{8} \right)$ = Rs. 225.

C's share = Rs. $[600 - (300 + 225)]$ = Rs. 75.

Ex. 3. *A and B working separately can do a piece of work in 9 and 12 days respectively. If they work for a day alternately, A beginning, in how many days, the work will be completed?*

Sol. $(A + B)\text{'s } 2 \text{ day's work} = \left(\frac{1}{9} + \frac{1}{12}\right) = \frac{7}{36}.$

Work done in 5 pairs of days $= \left(5 \times \frac{7}{36}\right) = \frac{35}{36}.$

Remaining work $= \left(1 - \frac{35}{36}\right) = \frac{1}{36}.$

On 11th day, it is A's turn. $\frac{1}{9}$ work is done by him in 1 day.

$\frac{1}{36}$ work is done by him in $\left(6 \times \frac{1}{36}\right) = \frac{1}{6}$ day.

\therefore Total time taken $= \left(10 + \frac{1}{6}\right)$ days $= 10\frac{1}{6}$ days.

Ex. 4. *2 men and 3 boys can do a piece of work in 10 days while 3 men and 2 boys can do the same work in 8 days. In how many days can 2 men and 1 boy do the work?*

Sol. Let 1 man's 1 day's work = x and 1 boy's 1 day's work = y.

Then, $2x + 3y = \frac{1}{10}$ and $3x + 2y = \frac{1}{8}.$

Solving, we get : $x = \frac{7}{200}$ and $y = \frac{1}{100}.$

\therefore (2 men + 1 boy)'s 1 day's work $= \left(2 \times \frac{7}{200} + 1 \times \frac{1}{100}\right) = \frac{16}{200} = \frac{2}{25}.$

So, 2 men and 1 boy together can finish the work in $\frac{25}{2} = 12\frac{1}{2}$ days.

Exercise – 1

1. A does a work in 10 days and B does the same work in 15 days. In how many days they together will do the same work?
(a) 5 days (b) 6 days (c) 8 days (d) 9 days
2. A can finish a work in 18 days and B can do the same work in half the time taken by A. Then, working together, what part of the same work they can finish in a day?
(a) $\frac{1}{6}$ (b) $\frac{1}{9}$ (c) $\frac{2}{5}$ (d) $\frac{2}{7}$
3. A tyre has two punctures. The first puncture alone would have made the tyre flat in 9 minutes and the second alone would have done it in 6 minutes. If air leaks out at a constant rate, how long does it take both the punctures together to make it flat?
(a) $1\frac{1}{2}$ min *utes* (b) $3\frac{1}{2}$ min *utes* (c) $3\frac{3}{5}$ min *utes* (d) $4\frac{1}{4}$ min *utes*
4. A, B and C can complete a piece of work in 24, 6 and 12 days respectively. Working together, they will complete the same work in:
(a) $\frac{1}{24}$ *day* (b) $\frac{7}{24}$ *day* (c) $3\frac{3}{7}$ *day* (d) 4 days
5. A man can do a job in 15 days. His father takes 20 days and his son finishes it in 25 days. How long will they take to complete the job if they all work together?
(a) Less than 6 days (b) Exactly 6 days
(c) Approximately 6.4 days (d) More than 10 days
6. A man can do a piece of work in 5 days, but with the help of his son, he can do it in 3 days. In what time can the son do it alone?
(a) $6\frac{1}{2}$ *days* (b) 7 days (c) $7\frac{1}{2}$ *days* (d) 8 days
7. A can lay railway track between two given stations in 16 days and B can do the same job in 12 days. With the help of C, they did the job in 4 days only. Then, C alone can do the job in:
(a) $9\frac{1}{5}$ *days* (b) $9\frac{2}{5}$ *days* (c) $9\frac{3}{5}$ *days* (d) 10 days

8. A takes twice as much time as B or thrice as much time to finish a piece of work. Working together, they can finish the work in 2 days. B can do the work alone in:
 (a) 4 days (b) 6 days (c) 8 days (d) 12 days
9. Ronald and Elan are working on an assignment. Ronald takes 6 hours to type 32 pages on a computer, while Elan takes 5 hours to type 40 pages. How much time will they take, working together on two different computers to type an assignment of 110 pages?
 (a) 7 hours 30 minutes (b) 8 hours
 (c) 8 hours 15 minutes (d) 8 hours 25 minutes
10. P can complete a work in 12 days working 8 hours a day. Q can complete the same work in 8 days working 10 hours a day. If both P and Q work together, working 8 hours a day, in how many days can they complete the work?
 (a) $5\frac{5}{11}$ (b) $5\frac{6}{11}$ (c) $6\frac{5}{11}$ (d) $6\frac{6}{11}$
11. A and B can do a work in 12 days, B and C in 15 days, C and A in 20 days. If A, B and C work together, they will complete the work in:
 (a) 5 days (b) $7\frac{5}{6}$ days (c) 10 days (d) $15\frac{2}{3}$ days
12. A and B can do a work in 8 days, B and C can do the same work in 12 days. A, B and C together can finish it in 6 days. A and C together will do it in:
 (a) 4 days (b) 6 days (c) 8 days (d) 12 days
13. A can do a piece of work in 4 hours; B and C together can do it in 3 hours, while A and C together can do it in 2 hours. How long will B alone take to do it?
 (a) 8 hours (b) 10 hours (c) 12 hours (d) 24 hours
14. A can do a certain work in the same time in which B and C together can do it. If A and B together could do it in 10 days and C alone in 50 days, then B alone could do it in:
 (a) 15 days (b) 20 days (c) 25 days (d) 30 days
15. A works twice as fast as B. If B can complete a work in 12 days independently, the number of days in which A and B can together finish the work is:
 (a) 4 days (b) 6 days (c) 8 days (d) 18 days

16. A is thrice as good a workman as B and therefore is able to finish a job in 60 days less than B. Working together, they can do it in:
- (a) 20 days (b) $22\frac{1}{2}$ days (c) 25 days (d) 30 days
17. A and B can do a job together in 7 days. A is $1\frac{3}{4}$ times as efficient as B. The same job can be done by A alone in:
- (a) $9\frac{1}{3}$ days (b) 11 days (c) $12\frac{1}{4}$ days (d) $16\frac{1}{3}$ days
18. Sakshi can do a piece of work in 20 days. Tanya is 25% more efficient than Sakshi. The number of days taken by Tanya to do the same piece of work is:
- (a) 15 (b) 16 (c) 18 (d) 25
19. A is 30% more efficient than B. How much time will they, working together, take to complete a job which A alone could have done in 23 days?
- (a) 11 days (b) 13 days (c) $20\frac{3}{17}$ days (d) None of these
20. A can do a work in 15 days and B in 20 days. If they work on it together for 4 days, then the fraction of the work that is left is:
- (a) $\frac{1}{4}$ (b) $\frac{1}{10}$ (c) $\frac{7}{15}$ (d) $\frac{8}{15}$
21. A can finish a work in 18 days and B can do the same work in 15 days. B worked for 10 days and left the job. In how many days, A alone can finish the remaining work?
- (a) 5 (b) $5\frac{1}{2}$ (c) 6 (d) 8
22. A and B can complete a work in 15 days and 10 days respectively. They started doing the work together but after 2 days B had to leave and A alone completed the remaining work. The whole work was completed in:
- (a) 8 days (b) 10 days (c) 12 days (d) 15 days
23. A can finish a work in 24 days, B in 9 days and c in 12 days. B and C start the work but are forced to leave after 3 days. The remaining work was done by A in:
- (a) 5 days (b) 6 days (c) 10 days (d) $10\frac{1}{2}$ days

24. A machine P can print one lakh books in 8 hours, machine Q can print the same number of books in 10 hours while machine R can print them in 12 hours. All the machines are started at 9 a.m. while machine P is closed at 11 a.m. and the remaining two machines complete the work. Approximately at what time will the work be finished?
 (a) 11:30 a.m. (b) 12 noon (c) 12:30 p.m. (d) 1 p.m.
25. X and Y can do a piece of work in 20 days and 12 days respectively. X started the work alone and then after 4 days Y joined him till the completion of the work. How long did the work last?
 (a) 6 days (b) 10 days (c) 15 days (d) 20 days
26. A and B can together finish a work in 30 days. They worked together for 20 days and then B left. After another 20 days, A finished the remaining work. In how many days A alone can finish the job?
 (a) 40 (b) 50 (c) 54 (d) 60
27. X can do a piece of work in 40 days. He works at it for 8 days and then Y finished it in 16 days. How long will they together take to complete the work?
 (a) $13\frac{1}{3}$ days (b) 15 days (c) 20 days (d) 56 days
28. A does $\frac{4}{5}$ of a work in 20 days. He then calls in B and they together finish the remaining work in 3 days. How long B alone would take to do the whole work?
 (a) 23 days (b) 37 days (c) $37\frac{1}{2}$ days (d) 40 days
29. A and B together can do a piece of work in 30 days. A having worked for 16 days, B finishes the remaining work alone in 44 days. In how many days shall B finish the whole work alone?
 (a) 30 days (b) 40 days (c) 60 days (d) 70 days
30. A and B can do a piece of work in 45 days and 40 days respectively. They began to do the work together but A leaves after some days and then B completed the remaining work in 23 days. The number of days after which A left the work was:
 (a) 6 (b) 8 (c) 9 (d) 12

4. PIPES AND CISTERNS

IMPORTANT FACTS AND FORMULAE

1. Inlet : A pipe connected with a tank or a cistern or a reservoir, that fills it, is known as an inlet.

Outlet : A pipe connected with a tank or a cistern or a reservoir, emptying it, is known as an outlet.

2. (i) If a pipe can fill a tank in x hours, then :

$$\text{part filled in 1 hour} = \frac{1}{x}.$$

(ii) If a pipe can empty a full tank in y hours, then :

$$\text{part emptied in 1 hour} = \frac{1}{y}.$$

(iii) If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours (where $y > x$), then on opening both the pipes, the net part filled in 1 hour $= \left(\frac{1}{x} - \frac{1}{y} \right)$.

(iv) If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours (where $x > y$), then on opening both the pipes, the net part emptied in 1 hour $= \left(\frac{1}{y} - \frac{1}{x} \right)$.

SOLVED EXAMPLES

Ex. 1. Two pipes can fill a tank in 10 hours and 12 hours respectively while a third pipe empties the full tank in 20 hours. If all three pipes operate simultaneously, in how much time will the tank be filled?

Sol. Net part filled in 1 hour $= \left(\frac{1}{10} + \frac{1}{12} - \frac{1}{20} \right) = \frac{8}{60} = \frac{2}{15}$.

\therefore The tank will be full in $\frac{15}{2}$ hrs = 7 hrs 30 min.

Ex. 2. A cistern has two taps which fill it in 12 minutes and 15 minutes respectively. There is also a waste pipe in the cistern. When all the three are opened, the empty cistern is full in 20 minutes. How long will the waste pipe take to empty the full cistern?

Sol. Work done by the waste pipe in 1 minute

$$= \frac{1}{20} - \left(\frac{1}{12} + \frac{1}{15} \right) = -\frac{1}{10} \quad [- \text{ve sign means emptying}]$$

\therefore Waste pipe will empty the full cistern in 10 minutes.

Ex. 3. Two pipes can fill a cistern in 14 hours and 16 hours respectively. The pipes are opened simultaneously and it is found that due to leakage in the bottom it took 32 minutes more to fill the cistern. When the cistern is full, in what time will the leak empty it?

Sol. Work done by the two pipes in 1 hour $= \left(\frac{1}{14} + \frac{1}{16} \right) = \frac{15}{112}$.

\therefore Time taken by these pipes to fill the tank $= \frac{112}{15}$ hrs = 7 hrs 28 min.

Due to leakage, time taken = 7 hrs 28 min + 32 min = 8 hrs

\therefore Work done by (two pipes + leak) in 1 hour $= \frac{1}{8}$.

Work done by the leak in 1 hour $= \left(\frac{15}{112} - \frac{1}{8} \right) = \frac{1}{112}$.

Ex. 4. Two pipes A and B can fill a tank in 24 min. and 32 min. respectively. If both the pipes are opened simultaneously, after how much time B should be closed so that the tank is full in 18 minutes?

Sol. Let B be closed after x minutes. Then,
part filled by (A + B) in x min. + part filled by A in (18 - x) min. = 1

$$\therefore x \left(\frac{1}{24} + \frac{1}{32} \right) + (18 - x) \times \frac{1}{24} = 1 \quad \Leftrightarrow \quad \frac{7x}{96} + \frac{18 - x}{24} = 1$$

$$\Leftrightarrow 7x + 4(18 - x) = 96 \quad \Leftrightarrow \quad x = 8.$$

Hence, B must be closed after 8 minutes.

Exercise – 1

(OBJECTIVE TYPE QUESTIONS)

Directions : Mark (✓) against the correct answer :

- Two pipes A and B can fill a tank in 20 and 30 minutes respectively. If both the pipes are used together, then how long will it take to fill the tank?
(a) 12 min (b) 15 min (c) 25 min (d) 50 min
- A cistern can be filled by a tap in 4 hours while it can be emptied by another tap in 9 hours. If both the taps are opened simultaneously then after how much time will the cistern get filled?
(a) 4.5 hrs (b) 5 hrs (c) 6.5 hrs (d) 7.2 hrs
- A tap can fill a tank in 6 hours. After half the tank is filled, three more similar taps are opened. What is the total time taken to fill the tank completely?
(a) 3 hrs 15 min (b) 3 hrs 45 min (c) 4 hrs (d) 4 hrs 15 min
- A water tank is two-fifth full. Pipe A can fill a tank in 10 minutes and pipe B can empty it in 6 minutes. If both the pipes are open, how long will it take to empty or fill the tank completely?
(a) 6 min. to empty (b) 6 min. to fill (c) 9 min. to empty
(d) 9 min. to fill (e) None of these

5. Pipe A can fill a tank in 5 hours, pipe B in 10 hours and pipe C in 30 hours. If all the pipes are open, in how many hours will the tank be filled?
 (a) 2 (b) 2.5 (c) 3 (d) 3.5
6. Pipes A and B can fill a tank in 5 and 6 hours respectively. Pipe C can empty it in 12 hours. If all the three pipes are opened together, then the tank will be filled in:
 (a) $1\frac{13}{17}$ hours (b) $2\frac{8}{11}$ hours (c) $3\frac{9}{17}$ hours (d) $4\frac{1}{2}$ hours
7. Three pipes A, B and C can fill a tank from empty to full in 30 minutes, 20 minutes and 10 minutes respectively. When the tank is empty, all the three pipes are opened. A, B and C discharge chemical solutions P, Q and R respectively. What is the proportion of solution R in the liquid in the tank after 3 minutes?
 (a) $\frac{5}{11}$ (b) $\frac{6}{11}$ (c) $\frac{7}{11}$ (d) $\frac{8}{11}$
8. Two pipes A and B can separately fill a cistern in 60 minutes and 75 minutes respectively. There is a third pipe in the bottom of the cistern to empty it. If all the three pipes are simultaneously opened, then the cistern is full in 50 minutes. In how much time the third pipe alone can empty the cistern?
 (a) 90 min (b) 100 min (c) 110 min (d) 120 min
9. A pump can fill a tank with water in 2 hours. Because of a leak, it took $2\frac{1}{3}$ hours to fill the tank. The leak can drain all the water of the tank in:
 (a) $4\frac{1}{3}$ hrs (b) 7 hrs (c) 8 hrs (d) 14 hrs
10. Two pipes A and B together can fill a cistern in 4 hours. Had they been opened separately, then B would have taken 6 hours more than A to fill the cistern. How much time will be taken by A to fill the cistern separately?
 (a) 1 hr (b) 2 hrs (c) 6 hrs (d) 8 hrs.
11. One pipe can fill a tank three times as fast as another pipe. If together the two pipes can fill the tank in 36 minutes, then the slower pipe alone will be able to fill the tank in:
 (a) 81 min (b) 108 min (c) 144 min (d) 192 min

12. A tank is filled in 5 hours by three pipes A, B and C. The pipe C is twice as fast as B and B is twice as fast as A. How much time will pipe A alone take to fill the tank?
 (a) 20 hrs (b) 25 hrs (c) 35 hrs
 (c) Cannot be determined (e) None of these
13. A tank is filled by three pipes with uniform flow. The first two pipes operating simultaneously fill the tank in the same time during which the tank is filled by the third pipe alone. The second pipe fills the tank 5 hours faster than the first pipe and 4 hours slower than the third pipe. The time required by the first pipe is:
 (a) 6 hrs (b) 10 hrs (c) 15 hrs (d) 30 hrs
14. Two pipes A and B can fill a tank in 15 minutes and 20 minutes respectively. Both the pipes are opened together but after 4 minutes, pipe A is turned off. What is the total time required to fill the tank?
 (a) 10 min 20 sec (b) 11 min 45 sec (c) 12 min 30 sec (d) 14 min 40 sec
15. A large tanker can be filled by two pipes A and B in 60 minutes and 40 minutes respectively. How many minutes will it take to fill the tanker from empty state if B is used for half the time and A and B fill it together for the other half?
 (a) 15 min (b) 20 min (c) 27.5 min (d) 30 min
16. Three taps A, B and C can fill a tank in 12, 15 and 20 hours respectively. If A is open all the time and B and C are open for one hour each alternately, the tank will be full in:
 (a) 6 hrs (b) $6\frac{2}{3}$ hrs (c) 5 (d) $7\frac{1}{2}$ hrs
17. Two pipes can fill a tank in 20 and 24 minutes respectively and a waste pipe can empty 3 gallons per minute. All the three pipes working together can fill the tank in 15 minutes. The capacity of the tank is:
 (a) 60 gallons (b) 100 gallons (c) 120 gallons (d) 180 gallons
18. Two pipes A and B can fill a cistern in $37\frac{1}{2}$ minutes and 45 minutes respectively. Both pipes are opened. The cistern will be filled in just half an hour, if the pipe B is turned off after:
 (a) 5 min (b) 9 min (c) 10 min (d) 15 min

- 19.** Three pipes A, B and C and fill a tank in 6 hours. After working at it together for 2 hours, C is closed and A and B can fill the remaining part in 7 hours. The number of hours taken by C alone to fill the tank is:
- (a) 10 (b) 12 (c) 14 (d) 16

5. TIME AND DISTANCE

IMPORTANT FACTS AND FORMULAE

1. $\text{Speed} = \left(\frac{\text{Distance}}{\text{Time}} \right), \text{Time} = \left(\frac{\text{Distance}}{\text{Speed}} \right), \text{Distance} = (\text{Speed} \times \text{Time})$
2. $x \text{ km / hr} = \left(x \times \frac{5}{18} \right) \text{ m/sec}$
3. $x \text{ m / sec} = \left(x \times \frac{18}{5} \right) \text{ km / hr}$
4. If the ratio of the speeds of A and B is $a : b$, then the ratio of the time taken by them to cover the same distance is $\frac{1}{a} : \frac{1}{b}$ or $b : a$.
5. Suppose a man covers a certain distance at $x \text{ km/hr}$ and an equal distance at $y \text{ km / hr}$. Then, the average speed during the whole journey is $\left(\frac{2xy}{x+y} \right) \text{ km / hr}$.

SOLVED EXAMPLES

Ex. 1. A cyclist covers a distance of 750 m in 2 min 30 sec. What is the speed in km / hr of the cyclist?

Sol. $\text{Speed} = \left(\frac{750}{150} \right) \text{ m / sec} = 5 \text{ m / sec} = \left(5 \times \frac{18}{5} \right) \text{ km/hr} = 18 \text{ km / hr}.$

Ex. 2. While covering a distance of 24 km, a man noticed that after walking for 1 hour and 40 minutes, the distance covered by him was $\frac{5}{7}$ of the remaining distance. What was his in metres per second?

Sol. Let the speed be $x \text{ km / hr}$.

Then, distance covered in 1 hr. 40 min. i.e., $1\frac{2}{3} \text{ hrs} = \frac{5x}{3} \text{ km}.$

Remaining distance $= \left(24 - \frac{5x}{3} \right) \text{ km}.$

$$\therefore \frac{5x}{3} = \frac{5}{7} \left(24 - \frac{5x}{3} \right) \Leftrightarrow \frac{5x}{3} = \frac{5}{7} \left(\frac{72-5x}{3} \right) \Leftrightarrow 7x = 72 - 5x$$

$$\Leftrightarrow 12x = 72 \Leftrightarrow x = 6$$

Hence, speed $= 6 \text{ km / hr} = \left(6 \times \frac{5}{18} \right) \text{ m / sec} = \frac{5}{3} \text{ m / sec} = 1\frac{2}{3} \text{ m / sec}.$

Ex. 3. A man travelled from the village to the post-office at the rate of 25 kmph and walked back at the rate of 4 kmph. If the whole journey took 5 hours 48 minutes, find the distance of the post-office from the village.

Sol. Average speed $= \left(\frac{2xy}{x+y} \right) \text{ km/hr} = \left(\frac{2 \times 25 \times 4}{25+4} \right) \text{ km/hr} = \frac{200}{29} \text{ km/hr}.$

Distance travelled in 5 hours 48 minutes i.e., $5\frac{4}{5} \text{ hrs} = \left(\frac{200}{29} \times \frac{29}{5} \right) \text{ km} = 40 \text{ km}.$

\therefore Distance of the post-office from the village $= \left(\frac{40}{2} \right) = 20 \text{ km}.$

Ex. 4. Walking at $\frac{5}{6}$ of its usual speed, a train is 10 minutes too late. Find its usual time to cover the journey.

Sol. New speed $= \frac{5}{6}$ of the usual speed

\therefore New time taken $= \frac{6}{5}$ of the usual time

So, $\left(\frac{6}{5} \text{ of the usual time} \right) - (\text{usual time}) = 10 \text{ min}.$

$\Rightarrow \frac{1}{5} \text{ of the usual time} = 10 \text{ min} \Rightarrow \text{usual time} = 50 \text{ min}.$

Ex. 5. A and B are the two stations 390 km apart. A train starts from A at 10 a.m. and travels towards B at 65 kmph. Another train starts from B at 11 a.m. and travels towards A at 35 kmph. At what time do they meet?

Sol. Suppose they meet x hours after 10 a.m. Then,
(Distance moved by first in x hrs) + [Distance moved by second in (x - 1) hrs] = 390.

$\therefore 65x + 35(x - 1) = 390 \Rightarrow 100x = 425 \Rightarrow x = 4\frac{1}{4}.$

So, they meet 4 hrs. 15 min. after 10 a.m. i.e., at 2.15 p.m.

Ex. 6. A thief is spotted by a policeman from a distance of 100 meters. When the policeman starts the chase, the thief also starts running. If the speed of the thief be 8 km / hr and that of the policeman 10 km / hr, how far the thief will have run before he is overtaken?

Sol. Relative speed of the policeman = (10 - 8) km / hr = 2 km / hr.

Time taken by policeman to cover 100 m $= \left(\frac{100}{1000} \times \frac{1}{2} \right) \text{ hr} = \frac{1}{20} \text{ hr}.$

In $\frac{1}{20}$ hrs, the thief covers a distance of $\left(8 \times \frac{1}{20} \right) \text{ km} = \frac{2}{5} \text{ km} = 400 \text{ m}.$

Exercise – 1

(OBJECTIVE TYPE QUESTIONS)

Directions : *Mark (✓) against the correct answer:*

1. A car moves at the speed of 80 km / hr. What is the speed of the car in metres per second?
(a) 8 m / sec (b) $20\frac{1}{9}$ m/sec (c) $22\frac{2}{9}$ m/sec (d) None of these
2. An athlete runs 200 metres race in 24 seconds. His speed is:
(a) 20 km / hr (b) 24 km / hr (c) 28.5 km / hr (d) 30 km / hr
3. A person crosses a 600 m long street in 5 minutes. What is his speed in km per hour?
(a) 3.6 (b) 7.2 (c) 8.4 (d) 10
4. A man walking at the rate of 5 km / hr crosses a bridge in 15 minutes. The length of the bridge (in metres) is:
(a) 600 (b) 750 (c) 1000 (d) 1250
5. How long will a boy take to run round a square field of side 35 metres, if he runs at the rate of 9 km / hr?
(a) 50 sec (b) 52 sec (c) 54 sec (d) 56 sec
6. A car is running at a speed of 108 kmph. What distance will it cover in 15 seconds?
(a) 45 metres (b) 55 metres (c) 450 metres
(d) Cannot be determined (e) None of these
7. One of the two buses completes a journey of 300 km in $7\frac{1}{2}$ hours and the other a journey of 450 km in 9 hours. The ratio of their average speeds is:
(a) 2 : 3 (b) 3 : 4 (c) 4 : 5 (d) 8 : 9
8. A truck covers a distance of 550 metres in 1 minute whereas a bus covers a distance of 33 kms in 45 minutes. The ratio of their speeds is:
(a) 3 : 4 (b) 4 : 3 (c) 3 : 5 (d) 50 : 3
9. The ratio between the speeds of two trains is 7 : 8. If the second train runs 400 kms in 4 hours, then the speed of the first train is:
(a) 70 km / hr (b) 75 km / hr (c) 84 km / hr (d) 87.5 km / hr

10. A train travels at an average of 50 miles per hour for $2\frac{1}{2}$ hours and then travels at a speed of 70 miles per hour for $1\frac{1}{2}$ hours. How far did the train travel in the entire 4 hours?
- (a) 120 miles (b) 150 miles (c) 200 miles (d) 230 miles
11. Sound is said to travel in air at about 1100 feet per second. A man hears the axe striking the tree, $\frac{11}{5}$ seconds after he sees it strike the tree. How far is the man from the wood chopper?
- (a) 2197 ft (b) 2420 ft (c) 2500 ft (d) 2629 ft
12. An express train travelled at an average speed of 100 km / hr, stopping for 3 minutes after every 75 km. How long did it take to reach its destination 600 km from the starting point?
- (a) 6 hrs 21 min (b) 6 hrs 24 min (c) 6 hrs 27 min (d) 6 hrs 30 min
13. A motor car starts with the speed of 70 km / hr with its speed increasing every two hours by 10 kmph. In how many hours will it cover 345 kms?
- (a) $2\frac{1}{4}$ hrs (b) 4 hrs 5 min (c) $4\frac{1}{2}$ hrs
- (d) Cannot be determined (e) None of these
14. The speed of a car increases by 2 kms after every one hour. If the distance travelled in the first one hour was 35 kms, what was the total distance travelled in 12 hours?
- (a) 456 kms (b) 482 kms (c) 552 kms (c) 556 kms (d) None of these
15. A train covers a distance of 10 km in 12 minutes. If its speed is decreased by 5 km / hr, the time taken by it to cover the same distance will be:
- (a) 10 min (b) 11 min 20 sec (c) 13 min (d) 13 min 20 sec
16. Anna left for city A from city B at 5.20 a.m. She travelled at the speed of 80 km / hr for 2 hours 15 minutes. After that the speed was reduced to 60 km / hr. If the distance between two cities is 350 kms, at what time did Anna reach city A?
- (a) 9.20 a.m. (b) 9.25 a.m. (c) 9.35 a.m. (c) 10.05 a.m. (e) None of these
17. An aeroplane covers a certain distance at a speed of 240 kmph in 5 hours. To cover the same distance in $1\frac{2}{3}$ hours, it must travel at a speed of:
- (a) 300 kmph (b) 360 kmph (c) 600 kmph (d) 720 kmph

18. A salesman travels a distance of 50 km in 2 hours and 30 minutes. How much faster, in kilometers per hour, on an average, must he travel to make such a trip in $\frac{5}{6}$ hour less time?
- (a) 10 (b) 20 (c) 30 (d) None of these
19. A person has to cover a distance of 6 km in 45 minutes. If he covers one-half of the distance in two-thirds of the total time; to cover the remaining distance in the remaining time, his speed (in km / hr) must be:
- (a) 6 (b) 8 (c) 12 (d) 15
20. A can complete a journey in 10 hours. He travels first half of the journey at the rate of 21 km / hr and second half at the rate of 24 km / hr. Find the total journey in km.
- (a) 220 km (b) 224 km (c) 230 km (d) 234 km
21. A person travels equal distances with speeds of 3 km / hr, 4 km / hr and 5 km / hr and takes a total time of 47 minutes. The total distance (in km) is:
- (a) 2 (b) 3 (c) 4 (d) 5
22. A farmer travelled a distance of 61 km in 9 hours. He travelled partly on foot @ 4 km / hr and partly on bicycle @ 9 km / hr. The distance travelled on foot is:
- (a) 14 km (b) 15 km (c) 16 km (d) 17 km
23. A is faster than B. A and B each walk 24 km. The sum of their speeds is 7 km / hr and the sum of times taken by them is 14 hours. Then, A's speed is equal to:
- (a) 3 km / hr (b) 4 km / hr (c) 5 km / hr (d) 7 km / hr
24. A person travels from P to Q at a speed of 40 kmph and returns by increasing his speed by 50%. What is his average speed for both the trips?
- (a) 36 kmph (b) 45 kmph (c) 48 kmph (d) 50 kmph
25. A boy goes to his school from his house at a speed of 3 km / hr and returns at a speed of 2 km / hr. If he takes 5 hours in going and coming, the distance between his house and school is:
- (a) 5 km (b) 5.5 km (c) 6 km (d) 6.5 km
26. A man on tour travels first 160 km at 64 km / hr and the next 160 km at 80 km / hr. The average speed for the first 320 km of the tour is:
- (a) 35.55 km / hr (b) 36 km / hr (c) 71.11 km / hr (d) 71 km / hr

- 27.** A man travels 600 km by train at 80 km / hr, 800 km by ship at 40 km / hr, 500 km by aeroplane at 400 km / hr and 100 km by car at 50 km / hr. What is the average speed for the entire distance?
- (a) 60 km / hr (b) $60\frac{5}{123} \text{ km/hr}$ (c) 62 km / hr (d) $65\frac{5}{123} \text{ km/hr}$
- 28.** A car travels the first one-third of a certain distance with a speed of 10 km / hr, the next one-third distance with a speed of 20 km / hr, and the last one-third distance with a speed of 60 km / hr. The average speed of the car the whole journey is:
- (a) 18 km / hr (b) 24 km / hr (c) 30 km / hr (d) 36 km / hr
- 29.** A car travelling with $\frac{5}{7}$ of its actual speed covers 42 km in 1 hr 40 min 48 sec. Find the actual speed of the car.
- (a) $17\frac{6}{7} \text{ km/hr}$ (b) 25 km / hr (c) 30 km / hr (d) 35 km / hr
- 30.** A man can reach a certain place in 30 hours. If he reduces his speed by $\frac{1}{15}$ *th*, he goes 10 km less in that time. Find his speed.
- (a) 4 km / hr (b) 5 km / hr (c) $5\frac{1}{2} \text{ km/hr}$ (d) 6 km / hr

6. PROBLEMS ON TRAINS

IMPORTANT FACTS AND FORMULAE

1. $a \text{ km/hr} = \left(a \times \frac{5}{18}\right) \text{ m/s}.$
2. $a \text{ m/s} = \left(a \times \frac{18}{5}\right) \text{ km/hr} = \left(a \times \frac{18}{5}\right) \text{ km/hr}.$
3. Time taken by a train of length l metres to pass a pole or a standing man or a signal post is equal to the time taken by the train to cover l metres.
4. Time taken by a train of length l metres to pass a stationary object of length b metres is the time taken by the train to cover $(l + b)$ metres.
5. Suppose two trains or two bodies are moving in the same direction at $u \text{ m/s}$ and $v \text{ m/s}$, where $u > v$, then their relative speed $= (u - v) \text{ m/s}$.
6. Suppose two trains or two bodies are moving in opposite direction at $u \text{ m/s}$ and $v \text{ m/s}$, then their relative speed is $= (u + v) \text{ m/s}$.
7. If two trains of length a metres and b metres are moving in opposite directions at $u \text{ m/s}$ and $v \text{ m/s}$, then time taken by the trains to cross each other $= \frac{(a+b)}{(u+v)} \text{ sec}.$
8. If two trains of length a metres and b metres are moving in the same direction at $u \text{ m/s}$ and $v \text{ m/s}$, then the time taken by the faster train to cross the slower train $= \frac{(a+b)}{(u-v)} \text{ sec}.$
9. If two trains (or bodies) start at the same time from points A and B towards each other and after crossing they take a and b sec in reaching B and A respectively, then **(A's speed) : (B's speed) = $(\sqrt{a} : \sqrt{b})$.**

SOLVED EXAMPLES

Ex. 1. A train is moving at a speed of 132 km/hr. If the length of the train is 110 metres, how long will it take to cross a railway platform 165 metres long?

Sol. Speed of train $= \left(132 \times \frac{5}{18}\right) \text{ m/sec} = \left(\frac{110}{3}\right) \text{ m/sec}.$

Distance covered in passing the platform $= (110 + 165) \text{ m} = 275 \text{ m}.$

\therefore Time taken $= \left(275 \times \frac{3}{110}\right) \text{ sec} = \frac{15}{2} \text{ sec} = 7 \frac{1}{2} \text{ sec}.$

Ex. 2. A train 150 m long is running with a speed of 68 kmph. In what time will it pass a man who is running at 8 kmph in the same direction in which the train is going?

Sol. Speed of the train relative to man $= (68 - 8) \text{ kmph}$

$$= \left(60 \times \frac{5}{18}\right) \text{ m/sec} = \left(\frac{50}{3}\right) \text{ m/sec}.$$

Time taken by the train to cross the man

$$= \text{Time taken by it to cover 150 m at } \left(\frac{50}{3}\right) \text{ m/sec} = \left(150 \times \frac{3}{50}\right) \text{ sec} = 9 \text{ sec}.$$

Ex. 3. Two trains 137 metres and 163 metres in length are running towards each other on parallel lines, one at the rate of 42 kmph and another at 48 kmph. In what time will they be clear of each other from the moment they meet?

Sol. Relative speed of the trains = (42 + 48) kmph = 90 kmph

$$= \left(90 \times \frac{5}{18}\right) \text{ m/sec} = 25 \text{ m/sec}.$$

Time taken by the trains to pass each other

$$= \text{Time taken to cover } (137 + 163) \text{ m at } 25 \text{ m/sec} = \left(\frac{300}{25}\right) \text{ sec} = 12 \text{ sec}.$$

Ex. 4. A train 100 metres long takes 6 seconds to cross a man walking at 5 kmph in a direction opposite to that of the train. Find the speed of the train.

Sol. Let the speed of the train be x kmph.

$$\text{Speed of the train relative to man} = (x + 5) \text{ kmph} = (x + 5) \times \frac{5}{18} \text{ m/sec}.$$

$$\therefore \frac{100}{(x + 5) \times \frac{5}{18}} = 6 \Leftrightarrow 30(x + 5) = 1800 \Leftrightarrow x = 55.$$

\therefore Speed of the train is 55 kmph.

Ex. 5. A man sitting in a train which is travelling at 50 kmph observes that a goods train, travelling in opposite direction, takes 9 seconds to pass him. If the goods train is 280 m long, find its speed.

$$\text{Sol. Relative speed} = \left(\frac{280}{9}\right) \text{ m/sec} = \left(\frac{280}{9} \times \frac{18}{5}\right) \text{ kmph} = 112 \text{ kmph}.$$

\therefore Speed of goods train = (112 – 50) kmph = 62 kmph.

Exercise – 1

(OBJECTIVE TYPE QUESTIONS)

Directions : *Mark (✓) against the correct answer:*

1. In what time will a train 100 metres long cross an electric pole, if its speed be 144 km / hr?
(a) 2.5 seconds (b) 4.25 seconds (c) 5 seconds (d) 12.5 seconds
2. A train 280 m long, running with a speed of 63 km / hr will pass a tree in:
(a) 15 sec (b) 16 sec (c) 18 sec (d) 20 sec
3. How long does a train 110 metres long running at the speed of 72 km / hr take to cross a bridge 132 metres in length?
(a) 9.8 sec (b) 12.1 sec (c) 12.42 sec (d) 14.3 sec
4. A train 360 m long is running at a speed of 45 km / hr. In what time will it pass a bridge 140 m long?
(a) 40 sec (b) 42 sec (c) 45 sec (d) 48 sec
5. A train running at the speed of 60 km / hr crosses a pole in 9 seconds. What is the length of the train?
(a) 120 metres (b) 180 metres (c) 324 metres
(d) Cannot be determined (e) None of these
6. A train covers a distance of 12 km in 10 minutes. If it takes 6 seconds to pass a telegraph post, then the length of the train is:
(a) 90 m (b) 100 m (c) 120 m (d) 140 m
7. A train 240 m long passed a pole in 24 seconds. How long will it take to pass a platform 650 m long?
(a) 65 sec (b) 89 sec (c) 100 sec (d) 150 sec
8. The length of the bridge, which a train 130 metres long and travelling at 45 km / hr can cross in 30 seconds, is:
(a) 200 m (b) 225 m (c) 245 m (d) 250 m
9. A train 800 metres long is running at a speed of 78 km / hr. If it crosses a tunnel in 1 minute, then the length of the tunnel (in metres) is:
(a) 130 (b) 360 (c) 500 (d) 540

10. A goods train runs at the speed of 72 kmph and crosses a 250 m long platform in 26 seconds. What is the length of the goods train?
(a) 230 m (b) 240 m (c) 260 m (d) 270 m
11. A train passes a station platform in 36 seconds and a man standing on the platform in 20 seconds. If the speed of the train is 54 km / hr, what is the length of the platform?
(a) 120 m (b) 240 m (c) 300 m (d) None of these
12. A 300 metre long train crosses a platform in 39 seconds while it crosses a signal pole in 18 seconds. What is the length of the platform?
(a) 320 m (b) 350 m (c) 650 m
(d) Data inadequate (e) None of these
13. A train speeds past a pole in 15 seconds and a platform 100 m long in 25 seconds. Its length is:
(a) 50 m (b) 150 m (c) 200 m (d) Data inadequate
14. A train moves past a telegraph post and a bridge 264 m long in 8 seconds and 20 seconds respectively. What is the speed of the train?
(a) 69.5 km / hr (b) 70 km / hr (c) 79 km / hr (d) 79.2 km / hr
15. How many seconds will a 500 metre long train take to cross a man walking with a speed of 3 km / hr in the direction of the moving train if the speed of the train is 63 km / hr?
(a) 25 (b) 30 (c) 40 (d) 45
16. A jogger running at 9 kmph alongside a railway track is 240 metres ahead of the engine of a 120 metre long train running at 45 kmph in the same direction. In how much time will the train pass the jogger?
(a) 3.6 sec (b) 18 sec (c) 36 sec (d) 72 sec
17. A train 110 metres long is running with a speed of 60 kmph. In what time will it pass a man who is running at 6 kmph in the direction opposite to that in which the train is going?
(a) 5 sec (b) 6 sec (c) 7 sec (d) 10 sec
18. Two trains 140 m and 160 m long run at the speed of 60 km / hr and 40 km / hr respectively in opposite directions on parallel tracks. The time (in seconds) which they take to cross each other, is:
(a) 9 (b) 9.6 (c) 10 (d) 10.8

19. Two trains are moving in opposite directions @ 60 km / hr and 90 km / hr. Their lengths are 1.10 km and 0.9 km respectively. The time taken by the slower train to cross the faster train in seconds is:
- (a) 36 (b) 45 (c) 48 (d) 49
20. A train 125 m long passes a man, running at 5 kmph in the same direction in which the train is going, in 10 seconds. The speed of the train is:
- (a) 45 km / hr (b) 50 km / hr (c) 54 km / hr (d) 55 km / hr
21. Two goods train each 500 m long, are running in opposite directions on parallel tracks. Their speeds are 45 km / hr and 30 km / hr respectively. Find the time taken by the slower train to pass the driver of the faster one.
- (a) 12 sec (b) 24 sec (c) 48 sec (d) 60 sec
22. Two trains of equal length are running on parallel lines in the same direction at 46 km / hr and 36 km / hr. The faster train passes the slower train in 36 seconds. The length of each train is:
- (a) 50 m (b) 72 m (c) 80 m (d) 82 m
23. A 270 metres long train running at the speed of 120 kmph crosses another train running in opposite direction at the speed of 80 kmph in 9 seconds. What is the length of the other train?
- (a) 230 m (b) 240 m (c) 260 m (c) 320 m (d) None of these
24. Two trains are running in opposite directions with the same speed. If the length of each train is 120 metres and they cross each other, in 12 seconds, then the speed of each train (in km / hr) is:
- (a) 10 (b) 18 (c) 36 (d) 72
25. Two trains of equal lengths take 10 seconds and 15 seconds respectively to cross a telegraph post. If the length of each train be 120 metres, in what time (in seconds) will they cross each other travelling in opposite direction?
- (a) 10 (b) 12 (c) 15 (d) 20
26. Two trains are running at 40 km / hr and 20 km / hr respectively in the same direction. Fast train completely passes a man sitting in the slower train in 5 seconds. What is the length of the fast train?
- (a) 23 m (b) $23\frac{2}{9} m$ (c) 27 m (d) $27\frac{7}{9} m$

27. Two trains, each 100 m long, moving in opposite directions, cross each other in 8 seconds. If one is moving twice as fast the other, then the speed of the faster train is:
(a) 30 km / hr (b) 45 km / hr (c) 60 km / hr (d) 75 km / hr
28. Two trains running in opposite directions cross a man standing on the platform in 27 seconds and 17 seconds respectively and they cross each other in 23 seconds. The ratio of their speeds is:
(a) 1 : 3 (b) 3 : 2 (c) 3 : 4 (d) None of these
29. Two trains, one from Howrah to Patna and the other from Patna to Howrah, start simultaneously. After they meet, the trains reach their destinations after 9 hours and 16 hours respectively. The ratio of their speeds is :
(a) 2 : 3 (b) 4 : 3 (c) 6 : 7 (d) 9 : 16

7. BOATS AND STREAMS

IMPORTANT FACTS AND FORMULAE

1. In water, the direction along the stream is called **downstream**. And, the direction against the stream is called **upstream**.
2. If the speed of a boat in still water is u km / hr and the speed of the stream is v km / hr, then:
$$\text{Speed downstream} = (u + v) \text{ km / hr}$$
$$\text{Speed upstream} = (u - v) \text{ km / hr}.$$
3. If the speed downstream is s km / hr and the speed upstream is b km / hr, then:

$$\text{Speed in still water} = \frac{1}{2} (s + b) \text{ km / hr}$$

$$\text{Rate of stream} = \frac{1}{2} (s - b) \text{ km / hr}$$

SOLVED EXAMPLES

Ex. 1. *A man can row upstream at 7 kmph and downstream at 10 kmph. Find man's rate in still and the rate of current.*

Sol. Rate in still water $= \frac{1}{2} (10 + 7) \text{ km / hr} = 8.5 \text{ km / hr}.$

$$\text{Rate of current} = \frac{1}{2} (10 - 7) \text{ km / hr} = 1.5 \text{ km / hr}.$$

Ex. 2. *A man can row 18 kmph in still water. It takes him thrice as long to row up as to row down the river. Find the rate of stream.*

Sol. Let man's rate upstream be x kmph. Then, his rate downstream $= 3x$ kmph.

$$\therefore \text{Rate in still water} = \frac{1}{2} (3x + x) \text{ kmph} = 2x \text{ kmph}.$$

$$\text{So, } 2x = 18 \text{ or } x = 9.$$

$$\therefore \text{Rate upstream} = 9 \text{ km / hr, Rate downstream} = 27 \text{ km / hr}.$$

$$\text{Hence, rate of stream} = \frac{1}{2} (27 - 9) \text{ km / hr} = 9 \text{ km / hr}.$$

Ex. 3. *There is a road beside a river. Two friends started from a place A, moved to a temple situated at another place B and then returned to A again. One of them moves on a cycle at a speed of 12 km / hr, while the other sails on a boat at a speed of 10 km / hr. If the river flows at the speed of 4 km / hr, which of the two friends will return to place A first?*

Sol. Clearly, the cyclist moves both ways at a speed of 12 km / hr.

So, average speed of the cyclist $= 12 \text{ km / hr}.$

The boat sailor moves downstream @ $(10 + 4)$ i.e., 14 km / hr and upstream @ $(10 - 4)$ i.e., 6 km / hr.

$$\begin{aligned} \text{So, average speed of the boat sailor} &= \left(\frac{2 \times 14 \times 6}{14 + 6} \right) \text{ km / hr} \\ &= \frac{42}{5} \text{ km / hr} = 8.4 \text{ km / hr}. \end{aligned}$$

Since the average speed of the cyclist is greater, he will return to A first.

Ex. 4. A man can row $7\frac{1}{2}$ kmph in still water. If in a river running at 1.5 km an hour, it takes him 50 minutes to row to a place and back, how far off is the place?

Sol. Speed downstream = $(7.5 + 1.5)$ kmph = 9 kmph;
Speed upstream = $(7.5 - 1.5)$ kmph = 6 kmph.
Let the required distance be x km. Then,

$$\frac{x}{9} + \frac{x}{6} = \frac{50}{60} \quad \Leftrightarrow \quad 2x + 3x = \left(\frac{5}{6} \times 18\right) \Leftrightarrow 5x = 15 \Leftrightarrow x = 3.$$

Hence, the required distance is 3 km.

Exercise – 1

- In one hour, a boat goes 11 km along the stream and 5 km against the stream. The speed of the boat in still water (in km / hr) is:
(a) 3 (b) 5 (c) 8 (d) 9
- A boat running downstream covers a distance of 16 km in 2 hours while for covering the same distance upstream, it takes 4 hours. What is the speed of the boat in still water?
(a) 4 km / hr (b) 6 km / hr (c) 8 km / hr (d) Data inadequate
- A boatman goes 2 km against the current of the stream of 1 hour and goes 1 km along the current in 10 minutes. How long will it take to go 5 km in stationary water?
(a) 40 minutes (b) 1 hour (c) 1 hr 15 min (d) 1 hr 30 min
- A man can row three-quarters of a kilometer against the stream in $11\frac{1}{4}$ minutes. The speed (in km / hr) of the man in still water is:
(a) 2 (b) 3 (c) 4 (d) 5
- A man takes twice as long to row a distance against the stream as to row the same distance in favour of the stream. The ratio of the speed of the boat (in still water) and the stream is:
(a) 2 : 1 (b) 3 : 1 (c) 3 : 2 (d) 4 : 3
- A boat running upstream takes 8 hours 48 minutes to cover a certain distance, while it takes 4 hours to cover the same distance running downstream. What is the ratio between the speed of the boat and speed of the water current respectively?
(a) 2 : 1 (b) 3 : 2 (c) 8 : 3 (d) Cannot be determined
(e) None of these

7. A man's speed with the current is 15 km / hr and the speed of the current is 2.5 km / hr. The man's speed against the current is:
 (a) 8.5 km / hr (b) 9 km / hr (c) 10 km / hr (d) 12.5 km / hr
8. A boat can travel with a speed of 13 km / hr in still water. If the speed of the stream is 4 km / hr, find the time taken by the boat to go 68 km downstream.
 (a) 2 hours (b) 3 hours (c) 4 hours (d) 5 hours
9. A man can row at 5 kmph in still water. If the velocity of current is 1 kmph and it takes him 1 hour to row to a place and come back, how far is the place?
 (a) 2.4 km (b) 2.5 km (c) 3 km (d) 3.6 km
10. A boat covers a certain distance downstream in 1 hour, while it comes back in $1\frac{1}{2}$ hours. If the speed of the stream be 3 kmph, what is the speed of the boat in still water?
 (a) 12 kmph (b) 13 kmph (c) 14 kmph (d) 15 kmph (e) None of these
11. A motorboat, whose speed is 15 km / hr in still water goes 30 km downstream and comes back in a total of 4 hours 30 minutes. The speed of the stream (in km / hr) is:
 (a) 4 (b) 5 (c) 6 (d) 10
12. A boat takes 90 minutes less to travel 36 miles downstream than to travel the same distance upstream. If the speed of the boat in still water is 10 mph, the speed of the stream is:
 (a) 2 mph (b) 2.5 mph (c) 3 mph (d) 4 mph
13. At his usual rowing rate, Rahul can travel 12 miles downstream in a certain river in 6 hours less than it takes him to travel the same distance upstream. But if he could double his usual rowing rate for his 24-mile round trip, the downstream 12 miles would then take only one hour less than the upstream 12 miles. What is the speed of the current in miles per hour?
 (a) $1\frac{1}{3}$ (b) $1\frac{2}{3}$ (c) $2\frac{1}{3}$ (d) $2\frac{2}{3}$

Answers:

Chapter – 1

Exercise – 1

1. (d), 2. (c), 3. (b), 4. (c), 5. (c), 6. (d), 7. (b), 8. (c), 9. (c), 10. (d), 11. (b), 12. (b), 13. (a), 14. (b), 15. (b), 16. (c), 17. (a), 18. (c), 19. (c), 20. (c), 21. (d), 22. (b), 23. (d), 24. (d).

Chapter – 2

Exercise – 1

1. (e), 2. (a), 3. (b), 4. (e), 5. (a), 6. (b), 7. (a), 8. (b), 9. (b), 10. (b), 11. (a), 12. (c), 13. (c), 14. (b), 15. (d), 16. (a), 17. (b), 18. (d), 19. (d), 20. (c), 21. (d), 22. (c).

Chapter – 3

Exercise – 1

1. (b), 2. (a), 3. (c), 4. (c), 5. (c), 6. (c), 7. (c), 8. (b), 9. (c), 10. (a), 11. (c), 12. (c), 13. (c), 14. (c), 15. (a), 16. (b), 17. (b), 18. (b), 19. (b), 20. (d), 21. (c), 22. (c), 23. (c), 24. (d), 25. (b), 26. (d), 27. (a), 28. (c), 29. (c), 30. (b).

Chapter – 4

Exercise – 1

1. (a), 2. (d), 3. (b), 4. (a), 5. (c), 6. (c), 7. (b), 8. (b), 9. (d), 10. (c), 11. (c), 12. (c), 13. (c), 14. (d), 15. (d), 16. (c), 17. (c), 18. (b), 19. (c).

Chapter – 5

Exercise – 1

1. (c), 2. (d), 3. (b), 4. (d), 5. (d), 6. (c), 7. (c), 8. (a), 9. (a), 10. (d), 11. (b), 12. (a), 13. (c), 14. (c), 15. (d), 16. (e), 17. (d), 18. (a), 19. (c), 20. (b), 21. (b), 22. (c), 23. (b), 24. (c), 25. (c), 26. (c), 27. (d), 28. (a), 29. (d), 30. (c)

Chapter – 6

Exercise – 1

1. (a), 2. (b), 3. (b), 4. (a), 5. (e), 6. (c), 7. (b), 8. (c), 9. (c), 10. (d), 11. (b), 12. (b), 13. (b), 14. (d), 15. (b), 16. (c), 17. (b), 18. (d), 19. (c), 20. (b), 21. (c), 22. (a), 23. (a), 24. (c), 25. (b), 26. (d), 27. (c), 28. (b), 29. (b),

Chapter – 7

Exercise – 1

1. (a), 2. (c), 3. (c), 4. (d), 5. (b), 6. (c), 7. (c), 8. (c), 9. (a), 10. (d), 11. (b), 12. (a), 13. (d)