Collapse of Fuzzy Dark Matter in Simulations

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Dark matter builds the backbone for galaxy formation in modern cosmological simulations. While the current dark matter paradigm is able to capture the large-scale structure of the Universe, it predicts more small-scale structure than is actually observed. This issue may be resolved with the theory of fuzzy dark matter, which is made of ultra-light, wavelike particles. However, its behavior has yet to be fully understood in present theoretical models. Using 3D numerical simulations, we performed a comprehensive analysis of spherical collapse in fuzzy dark matter halos. Then, we made use of a semi-analytic treatment to predict how likely it is for the halo to collapse, and thus explore small-scale structure formation in the Universe.

Why study fuzzy dark matter?

Since its discovery by Vera Rubin, the theory of dark matter has eluded physicists and astronomers as one of the greatest questions in cosmology: *What is the dark matter particle made of? What is its mass? And how does it interact with other matter in order to form the large-scale structure of the Universe?*

Modern cosmological simulations work to capture the behavior of dark matter through numerical models. These N-body simulations are extremely powerful tools: they can broadly simulate any dynamical system of particles under the influence of physical forces. In these simulations, dark matter builds the backbone for the formation of galaxies, which are expected to form at the centers of dark matter clumps called *halos* (Vogelsberger et al., 2019). Due to the force of gravity, these dark matter halos would (1) grow as they pulled surrounding gas into their cores...until (2) they collapsed, and (3) stabilized into the first galaxies. Scientists are interested in studying this process of halo collapse.

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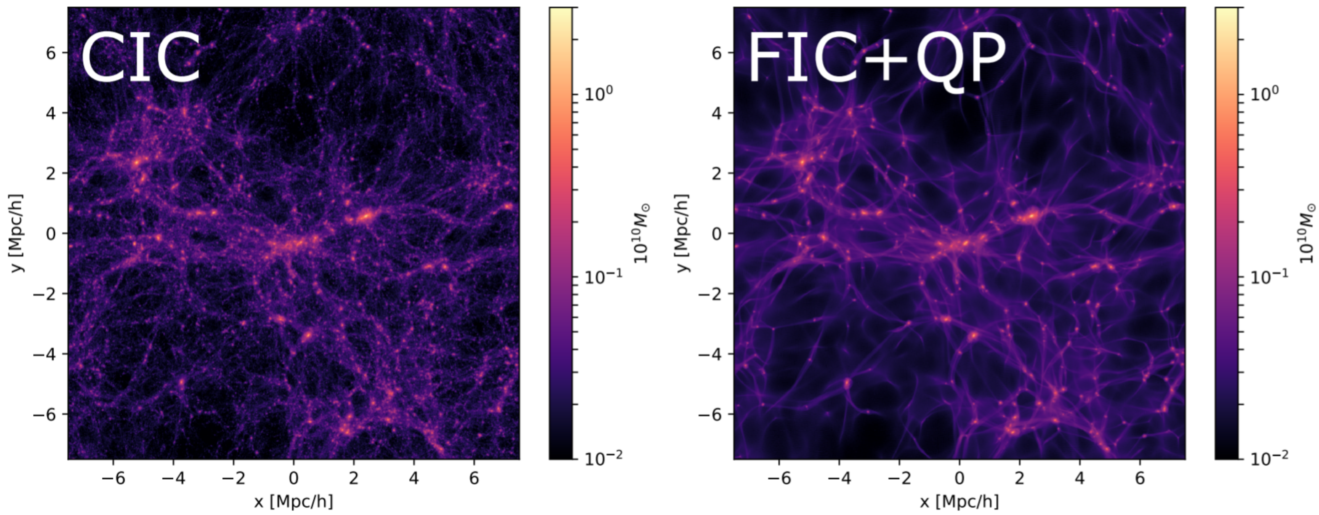
The most prevalent model in cosmological simulations is the *cold dark matter* cosmology, where dark matter is made of cold, slow-moving particles. The cold dark matter model has been successful in explaining observations with respect to the large-scale structure formation of the Universe. Despite its overall success, however, the cold dark matter model fails in two small-scale problems. It predicts the existence of more dwarf galaxies than observed in the real Universe, giving rise to the "missing satellites" crisis (Klypin et al., 1999; Moore et al., 1999; Hu et al., 2000). Moreover, it leads to discrepancies in the density-profiles of these galaxies which cause their cores to appear 'cuspy' rather than flat (de Blok, 2010). Though these problems are indeed small-scale, they are indicative of some 'missing' component in the cold dark matter model and represent a gap in our current understanding of dark matter.

These tensions lead physicists to hypothesize the existence of *fuzzy dark matter*. This type of dark matter is composed of ultra-light particles called 'axions' (Hu et al., 2000; Marsh, 2016). Compared to cold dark matter, these particles behave in a quantum fashion according to Schrödinger's equation, giving it wave-like properties that manifest on galactic scales. Like the cold dark matter model, it is able to reproduce the large-scale structure of Universe, as desired. Most interestingly, the fuzzy dark matter model suppresses small-scale structure and thus alleviates the 'missing satellites' problem (Schive et al., 2014; Marsh & Silk, 2013). Moreover, it predicts flat cores in the center of dark matter halos, as preferred by observations. Thus, the fuzzy dark matter model demonstrates several promising characteristics toward understanding dark matter as it behaves in the real Universe.

Nevertheless, reproducing the effects of fuzzy dark matter on sub-galactic scales meets it challenges in the need for sufficiently high resolutions in cosmological simulations (Hu et al., 2000; Schive et al., 2014). Standard N-body methods are not adequate, as in the cold dark matter case; instead, solving the Schrödinger equa-



The above is adapted from Nori & Baldi (2018). It shows a comparison of large-scale structures that resulted from a cosmological simulation, using standard cold dark matter initial conditions (left) and fuzzy dark matter initial conditions, with quantum pressure (right). The high intensity points represent overdensities, where halos are expected to collapse and galaxies eventually form. Although both models produce very similar large-scale structures, as seen in these images, fuzzy dark matter behaves very differently on smaller scales. Due to quantum pressure, the growth of overdensities is suppressed on small scales. This leads to less cosmic structure in the fuzzy case, where there are fewer bright points of overdensity than in the cold case.

tion results in a complex wave function which oscillates rapidly in *time* and has interference in *space*, requiring high resolutions in both dimensions. Nevertheless, several works have been able to solve the Schrödinger equation and thus test the spherical collapse of fuzzy dark matter halos in their cosmological code (see for example Schwabe et al. 2020).

One notable area of active research, however, is in modeling the quantum effect which distinguishes fuzzy dark matter from cold dark matter. Arising from the uncertainty principle, quantum pressure in fuzzy dark matter can lead to a minimum size for a collapsing halo. While new studies are being done to understand how quantum pressure affects the collapse of dark matter halos (Sreenath, 2019), they are often constrained to simple one-dimensional problems in hydrodynamical simulations. This approach does not work well in the case of *shell-crossing*, when the inner shells of a halo come close to each other and experience repulsion. Solving the three-dimensional Schrödinger-Poisson equation can thus provide more insight to the behavior of fuzzy dark matter halos during their formation.

Purpose of this work

In this way, a more comprehensive analysis of how quantum pressure will affect the collapse of dark matter halos, and thus structure formation in the Universe, is needed. In this work, we use a three-dimensional treatment of fuzzy dark matter to study the effect of quantum pressure in the formation of dark matter overdensities. Not only would we like to determine *how* dark matter halos collapse in this model, but also *when*:

- *How do the dark matter particles evolve?* We will look at the evolution of the velocities and densities of dark matter particles over time.

- *When does the dark matter halo collapse?* We would like to determine when the fuzzy dark matter halo forms given the initial amplitude and size in the early Universe.

- *How does the halo stabilize?* Since the fuzzy dark matter model involves an additional quantum pressure term, this can potentially alter the way in which the halo reaches a state at which it neither expands nor collapses.

We can then predict the halo abundance in the fuzzy dark matter model, particularly on the low-mass end, in a *halo mass function* (see Press–Schechter theory in Bond et al. 1991). Ultimately, we would like to compare our results with those of the cold dark matter paradigm in order to assess the potential of fuzzy dark matter as an alternative theory for dark matter. This can allow us to have a greater understanding of how dark matter overdensities may have developed, and thus shaped the eventual formation of galaxies.

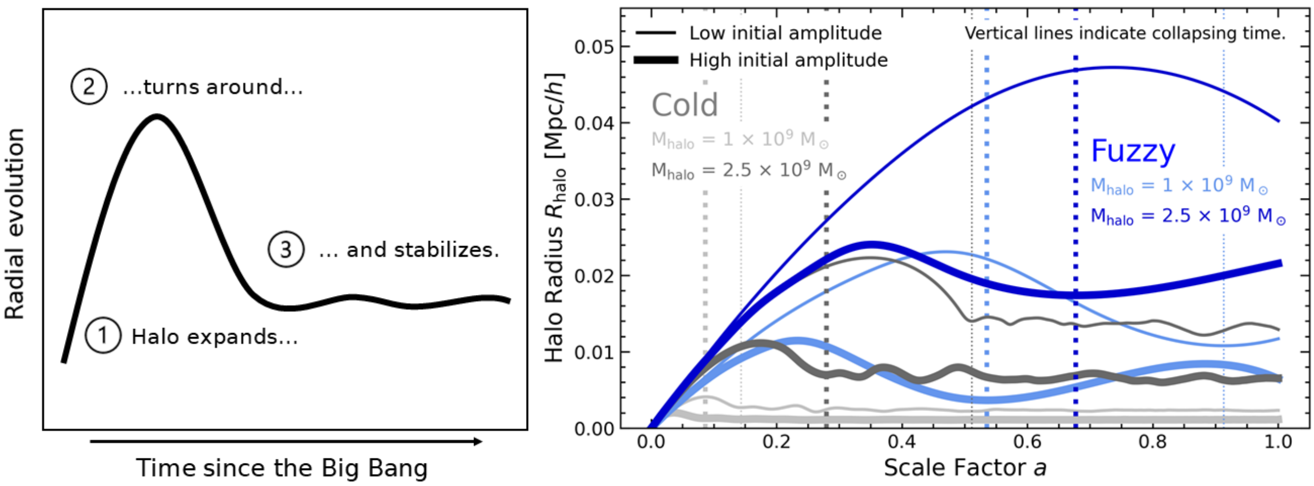
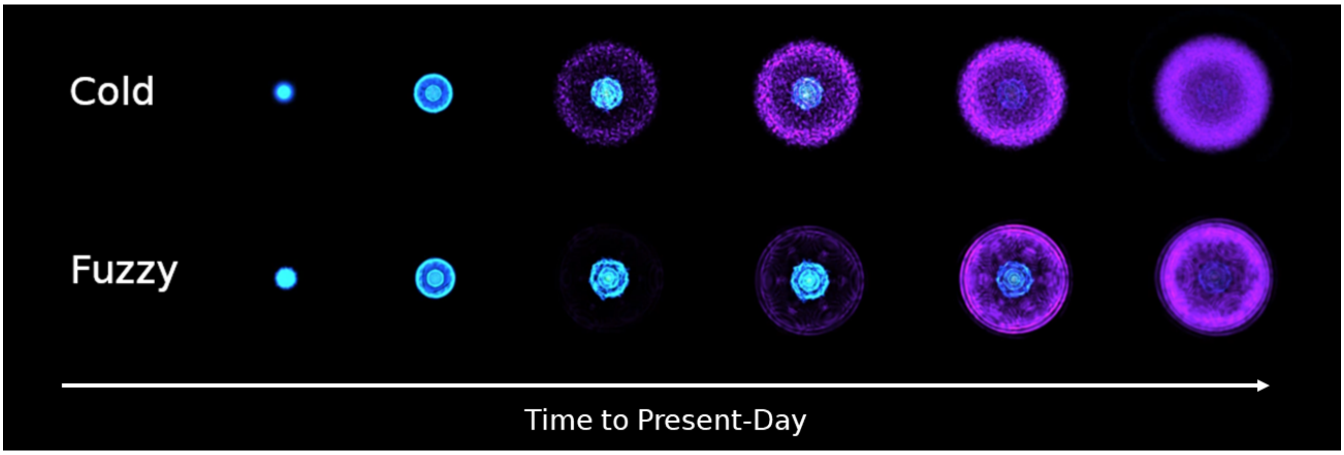


Figure 1: *Radial evolution of simulated halos over time. Left: a schematic for interpreting the basic "story" of the halo from the evolution curves. Right: shown for both the cold dark matter (gray) and fuzzy dark matter (blue) cases, using different sets of initial conditions; i.e. low vs. high mass (light vs. darker shades) and low vs. high initial amplitudes (thin vs. thick). We additionally indicate the time of collapse (vertical dotted). Due to suppression from quantum pressure, the fuzzy dark matter halos collapse at later times and stabilize at larger radii.*



Evolution of both a simulated cold (top) and fuzzy (bottom) dark matter halo over time. The initial overdensity grows before is starts to collapse and form a stable halo. This process is delayed in the fuzzy dark matter case, where quantum pressure suppresses halo formation on small scales.

How to simulate fuzzy dark matter

We ran 3-D numerical simulations to model the evolution of a dark matter halo over time, in both the cold and fuzzy cases. Unlike previous studies, we focused on a *single* halo so that we could render it with a large number of particles in a small volume to get high resolutions in our results.

Setting initial conditions

The very first step is to set-up appropriate initial conditions for the halo that we are simulating. This means setting its shape and growth at a very early time in the Universe.

Consider a smooth field of particles at the mean density of the Universe. We can perturb the density field at the position of the halo so that it has higher central concentration of dark matter—called the overdensity. The *overdensity* of a dark matter halo is how dense it is compared to the mean density of the Universe. The *initial amplitude* of this overdensity is what sets its growth, and causes the halo to collapse. Meanwhile, the size of the halo is determined by the *initial mass* of the particles inside of the region of the overdensity.

While we can extract the positions of the particles from their density distribution, we need to use the following continuity equation (Binney & Tremaine, 2008) to get the radial velocities of the particles:

$$\frac{\partial\,\text{overdensity}}{\partial\,\text{time}} = -\nabla\,\text{velocity}.$$

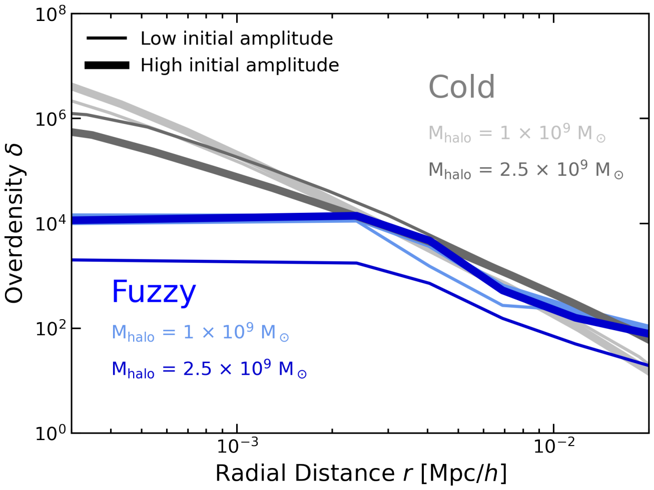


Figure 2: *Overdensity of the simulated halos with respect to the distance from the halo centers, at present-day. The profiles are shown for each set of initial conditions, like in Figure 1. While the cold dark matter profiles keep increasing toward the center of the halos, the fuzzy dark matter profiles flatten at lower central overdensities. In this way, the fuzzy halos collapse less.*

Thus, we can calculate the initial position and velocity vectors of the N-body particles in the simulation.

In the current iteration of our work, we present four sets of simulations for the cold and fuzzy cases each, specifically for halos with two different masses, and with both low and high initial overdensity amplitudes. The table of initial parameter values is shown below.

Halo mass [M$_\odot$]	Initial amplitude
1×10^9	0.04
1×10^9	0.08
2.5×10^9	0.01
2.5×10^9	0.02

Set of initial conditions for running the simulations of cold and fuzzy dark matter halos.

This results in a total of eight simulations to compare.

Running the simulations

We evolve each halo using the cold dark matter paradigm as a basis for comparison. Here, we use the simulation suite called GADGET (Springel et al., 2021). This gravitational N-body code is widely-used in cosmological simulations that are based on the cold dark matter paradigm.

Then, we use the exact same set of initial conditions to run a spectral code for fuzzy dark matter, developed in Du et al. (2018). It numerically solves the three-dimensional Schrödinger-Poisson equation for the wavefunction that describes the evolution of fuzzy dark matter.

We run each simulation starting at approximately 10 billion years after the Big Bang (redshift $z = 100$), to the present-day.

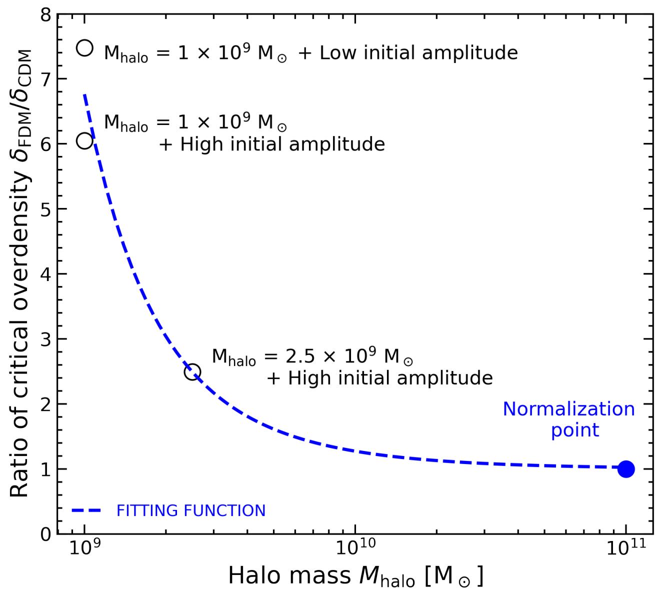


Figure 3: *Ratio of critical overdensities in the fuzzy model to the cold dark matter case, as a function of the halo masses in each simulation set (black circles). Notice that we include an additional "normalization point" (blue dot) at the very high-mass end, where we estimate the ratio to be one. This is because, at very large scales, we expect cold dark matter and fuzzy dark matter to behave very similarly. By fitting a function (blue dashed line) to these critical overdensities, we can predict how likely it is for a fuzzy dark matter halo to collapse.*

Results of the simulations

Now, we can look at how the different simulated halos evolve in the fuzzy model, versus in the cold dark matter case.

Basic "story" of the halo

One way to observe the collapsing process is through tracing the radius of the halo, since it provides a sens of how the overdensity grows or becomes smaller over time. This is shown in Figure 1. The typical "story" of the cold dark matter halos can be summarized three steps:

1. At first, the halo expands along with the Universe.
2. Due to gravity, the halo 'turns around' at a maximum radius and begins to shrink and collapse.

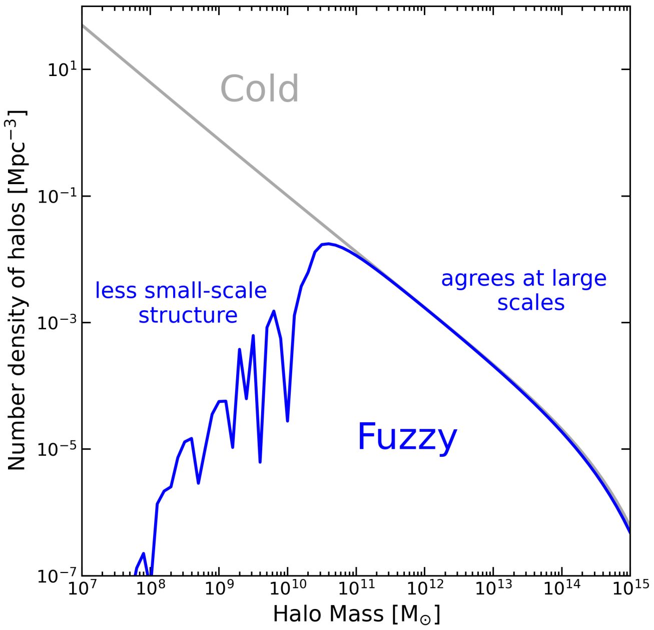


Figure 4: *Halo mass function from the cold (gray) and fuzzy (blue) dark matter simulations. This function predicts the number density of halos at different masses. Bot models agree at the high-mass, large-scale end. However, the fuzzy model has less small-scale structure than the cold dark matter model.*

3. Eventually, the halo stabilizes at an equilibrium radius.

While the fuzzy dark matter halos roughly follow this path, they do so with several differences. Not only do they collapse at later times, but also stabilize at larger radii than their cold dark matter counterparts. In some cases, the halo expands again. For the high mass-low initial amplitude run, the fuzzy dark matter halo does not even finish collapsing before the present day.

In this way, the collapsing process is delayed and suppressed in the fuzzy model, compared to the cold dark matter case.

Fuzzy dark matter halos "collapse less"

We can also see the suppression of collapse in the density profiles of the fuzzy dark matter halos.

The final overdensity of the different simulated halos are shown in Figure 2, with respect to the distance from the halo centers. While the cold dark matter profiles continue to increase toward the center of the halo, the density of the fuzzy dark matter halos flatten out. Once again, we see that halos cannot collapse to the same extent in the fuzzy dark matter case as in the cold model.

This suppression is likely due to the additional influence of quantum pressure in fuzzy dark matter, which causes there to be a minimum requirement for halo collapse to occur. But what is that requirement?

What does it take to collapse?

In other words, we would like to know the critical overdensity needed for a fuzzy dark matter halo to collapse.

As demonstrated in Figure 1, we can identify the time of collapse for each of the simulated halos by tracing their radial evolutions. Then, we can calculate the overdensity of each halo at that point in time. Figure 3 compares the critical overdensities from the fuzzy and cold models as a ratio, for each simulation set at different mass halos. At low masses, the fuzzy halos need to attain a much higher density than the cold dark matter halos in order to collapse; meanwhile, the threshold for collapse is similar between both models at the high-mass end.

These results are in line with the prediction that suppression from quantum pressure is greater at smaller scales. However, we can expect fuzzy dark matter and cold dark matter to behave similarly at large scales. But what do these critical overdensities *really* tell us about small-scale structure formation in the Universe?

Small-scale structure in the fuzzy Universe

To be able to predict the numbers of satellite galaxies in the fuzzy model, compared to the cold dark matter case, we need to determine a *halo mass function*. This function describes the number density of halos we can expect at different masses.

Using the relation between the critical overdensity of collapse and halo mass from Figure 3, we can statistically determine the halo mass function. This is accomplished using the semi-analytic model called *Galacticus* (Benson, 2012). We show the resulting halo mass functions for the cold and fuzzy dark matter models in Figure 4. Again, both models agree at the high-mass, large-scale end. Meanwhile, whereas the cold dark matter model predicts an abundance of low-mass halos, the fuzzy model has less small-scale structure.

What did we learn?

In this work, we ran 3-D simulations of spherical collapse in the fuzzy and cold dark matter models. With our results, we were able to determine the resulting halo mass functions, and thus demonstrate the ability of the fuzzy model to cause less small-scale structure formation to occur. We find that fuzzy dark matter is able to suppress halo collapse by:

1. delaying collapsing-times,
2. leading to less compact core-formation in terms of larger stabilizing radii and less concentrated central densities,
3. and requiring a higher critical overdensity for collapse to even occur.

In this way, the fuzzy dark matter model can indeed address the small-scale problems of the prevalent cold dark matter paradigm. In terms of the "missing satellites" crisis, the halo mass function shows that the fuzzy model leads to fewer low-mass galaxies. Moreover, its flatter overdensity profiles can potentially resolve the "core-cusp" problem.

But there is still a lot to explore!

While the overdensity profiles and halo mass function are promising in terms of solving the problems of cold dark matter, it would be insightful to compare our simulated results with observations of the real Universe. This would allow us to test how well the fuzzy dark matter model can capture galaxy formation, compared to the current paradigm.

Moreover, although we altered the initial density and and mass of the simulated halos in this work, it is also possible to test different masses of the fuzzy dark matter particle. Comparing the results of these simulations with observations might help us constrain the mass of dark matter!

To investigate deeper, we can also analyze the actual quantum pressure energy that is influencing the collapse of the fuzzy dark matter halos. Moreover, running more simulations at different halo masses can improve the precision of our halo mass function.

Beyond the scope of this work, it would also be exciting to explore what happens to the fuzzy dark matter halos after they collapse. In other words, how does this alternative model affect the actual galaxies that form?

Looking forward, understanding the spherical collapse of different dark matter models is an important step to figuring out galaxy-formation in the Universe.

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