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* sem 1
  - Preparatory session
  - S1 : Essentials of AI
  - S2 : Data Analysis and Visualization
  - S3 : Machine learning.

① Intro and Learning Method

② $\boxed{2.1 \text{ to } 2.5}$: Intro to Geometry and Application in Real world.
- ↳ We can seperate datapoints using line, circle, ellipse, etc. in 2D
- ↳ We need plane, sphere, cubes etc in 3D.
- ↳ We are trying solve ML classification problem here.

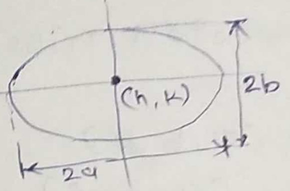
$\boxed{2.6 \text{ to } 2.11}$: Equation of line, plane, circle, ellipse, etc.

(i) Line : $y = mx + c$ or $ax + by + c = 0$ or $w_1x_1 + w_2x_2 + w_0 = 0$

(ii) plane : $\pi \Rightarrow ax + by + cz + d = 0$ or $w_1x_1 + w_2x_2 + w_3x_3 + w_0 = 0$

(iii) Circle : $(x-a)^2 + (y-b)^2 - r^2 = 0$ where (a,b) is center
r is radius.

(iv) ellipse :



$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} - 1 = 0$$

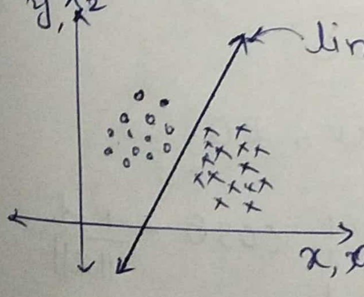
We can write equation in n-dimension as well,

n-dimension hyper plane $\Rightarrow w_1x_1 + w_2x_2 + \dots + w_nx_n + w_0 = 0$

sometimes it is written as b intercept → bias

Keep in mind that dimension is nothing but number of features (properties)

Q : find a line, plane, hyperplane, circle, ellipsoid, ... to separate datapoints.



line : $f(x,y) = ax + by + c$

or

$F(x_1, x_2) = w_1x_1 + w_2x_2 + w_0$

or

$f(x_1, x_2) = [w_1 \; w_2]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + w_0$

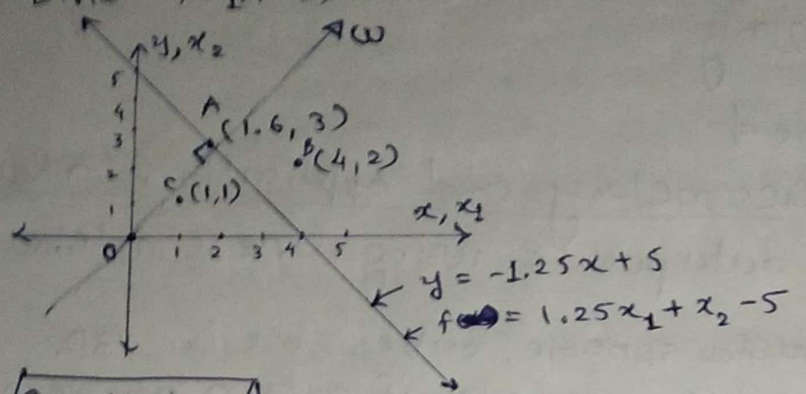
find line which separates the categories and we can classify unseen datapoint in future using $f(x_1, x_2, \dots x_n)$ which is line, plane, sphere, ...

Generalized $\underbrace{f(x_1, x_2, \dots x_n)}_{f^n \text{ of features}} = w^T \cdot x + w_0$

weight ↑ (w^T), features ↑ (x), bias ↑ (w_0)

* To classify using line, plane, hyper plane we need to check where the point is i.e. which side.
* To classify using circle, ellipse, sphere, ellipsoid etc. we need to check whether the point is inside or outside.

⇒ Line $f(x_1, x_2) = w_1 x_1 + w_2 x_2 + w_0$



⇒ A (1.6, 3)
$f(1.6, 3) = (1.25)(1.6) + 3 - 5 = 0$

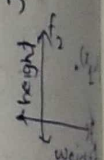
⇒ B (4, 2)
$f(4, 2) = (1.25)(4) + 2 - 5 = 2$

⇒ C (1, 1)
$f(1, 1) = (1.25)(1) + 1 - 5 = -2.75$

Conclusion

$f(x) = \begin{cases} 0, & \text{point is on line (OR plane, hyperplane)} \\ <0, & \text{point is on negative half} \\ >0, & \text{point is on positive half} \end{cases}$

$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ → ALERT x_1, x_2 in point (data point) is different then axis (features) $x_1, x_2, \ldots$ dimension.



⇒ w is normal or perpendicular vector passing through origin (0, 0)

line $f = ax + by + c$ $\quad w = (a, b)$
plane $f = ax + by + cz + d$ $\quad w = (a, b, c)$
Genlly $f(x_1, x_2, \ldots, x_n) = w_1 x_1 + w_2 x_2 + \ldots + w_n x_n + w_0$

$w = (w_1, w_2, \ldots, w_n)$

$\|w\| = \sqrt{w_1^2 + w_2^2 + \ldots + w_n^2}$

⇒ Plane 3D $f(x_1, x_2, x_3) = w_1 x_1 + w_2 x_2 + w_3 x_3 + w_0$

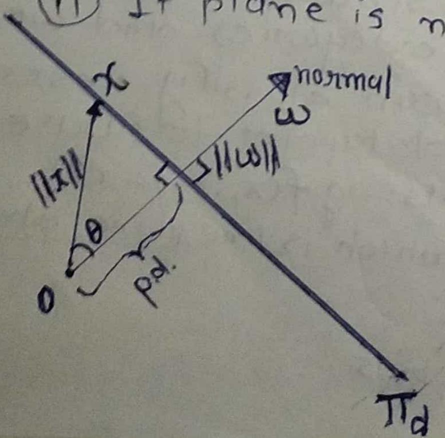
Hyperplane $f(X) = w^T . x + w_0$ OR $w^T x + w_0 = 0$

(i) If plane is passing through origin $x = (0, 0, \ldots, 0)$

$f(0, 0, \ldots) = w^T . (0, 0, \ldots 0) + w_0$

$\boxed{f(x) = w^T x}$ or $\boxed{w_0 = 0}$ ⇒ $w^T x = 0$

(ii) If plane is not passing through origin $\boxed{w_0 \neq 0}$



$w^T . x + w_0 = 0$

$\|w\| \|x\| . \cos\theta + w_0 = 0$

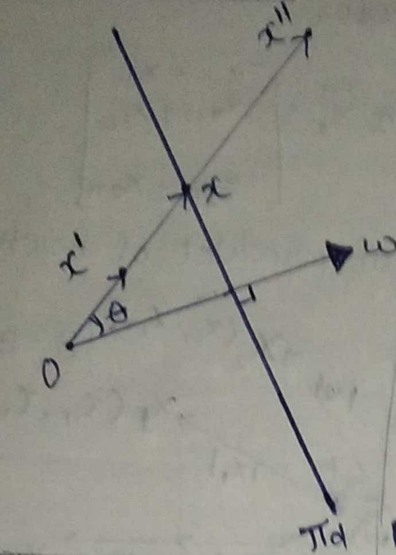
$\boxed{p.d. = \frac{-w_0}{\|w\|}}$

perpendicular distance from origin

$\cos\theta = \frac{p.d.}{\|x\|}$

unit vector $\hat{w} = \frac{w}{\|w\|}$

* ⇒ HyperPlane



$$\|x'\| < \|x\| < \|x''\|$$

$$\|w\|\,\|x'\|\cos\theta + w_0 < \|w\|\,\|x\|\cos\theta + w_0 < \|w\|\,\|x''\|\cos\theta + w_0$$

$$wx' + w_0 < 0 < wx'' + w_0$$

$\because f(x) = 0$ x is on the plane

Hence proved

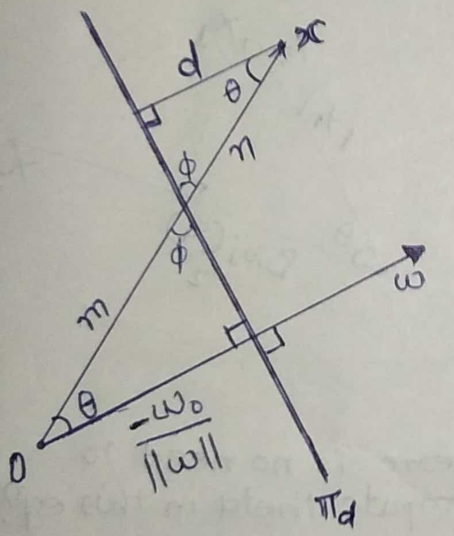
$f(x) < 0$ is x is in negative half

$f(x) > 0$ if x is in positive half

NOTE: positive half is in the direction of w and negative half is in opposite dirn of w.

Now, we have identify how to determine location of data point and generalized equation of n dimensional plane.

Let's calculate distance ($\perp$ distance) of point from plane. So we are assured and confident about category.



$$\|x\| = m + n$$

$$\therefore \|x\| = \frac{-w_0}{\|w\|\cdot\cos\theta} + \frac{d}{\cos\theta}$$

$$\therefore \|x\|\cdot\cos\theta + \frac{w_0}{\|w\|} = d$$

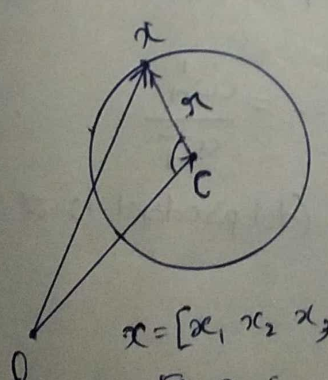
$$\therefore d = \frac{\|w\|\cdot\|x\|\cdot\cos\theta + w_0}{\|w\|}$$

$$\therefore \boxed{d = \frac{w\cdot x + w_0}{\|w\|}} \quad \text{OR} \quad \frac{f(x)}{\|w\|}$$

$f(x)$ ← evaluate function at point x

$\|w\|$ ← normal $\sqrt{w_1^2 + w_2^2 + \dots + w_n^2}$

* Using Vector for Circle, sphere, hyper-sphere,



$x = [x_1\ x_2\ x_3 \dots x_n]$

$c = [c_1\ c_2\ c_3 \dots c_n]$

$$\vec{oc} + \vec{cx} = \vec{ox}$$

$$\vec{cx} = \vec{ox} - \vec{oc}$$

$$\|\vec{cx}\| = \|\vec{ox} - \vec{oc}\|$$

$$r = \|x - c\|$$

$$r = \left\| \begin{bmatrix} x_1 - c_1 \\ x_2 - c_2 \\ \vdots \\ x_n - c_n \end{bmatrix} \right\|$$

$$r = \sqrt{(x_1 - c_1)^2 + (x_2 - c_2)^2 + \dots + (x_n - c_n)^2}$$

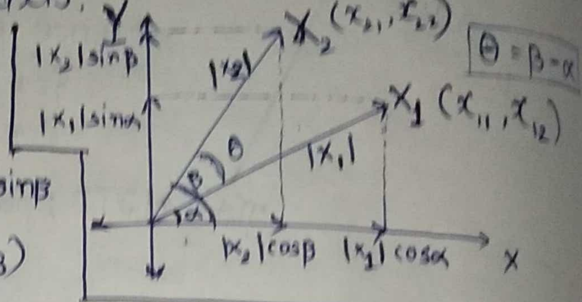
$$\boxed{r^2 = (x_1 - c_1)^2 + (x_2 - c_2)^2 + \dots + (x_n - c_n)^2}$$

n dimensional hyper-sphere.

Vector :- In Machine learning context every data point can be represented as n-dimensional vector.

* given $\vec{x_1} = \begin{bmatrix} x_{11} \\ x_{12} \\ \vdots \\ x_{1n} \end{bmatrix}$ and $\vec{x_2} = \begin{bmatrix} x_{21} \\ x_{22} \\ \vdots \\ x_{2n} \end{bmatrix}$ $\quad \vec{x_1} \pm \vec{x_2} = \begin{bmatrix} x_{11} \pm x_{21} \\ x_{12} \pm x_{22} \\ \vdots \\ x_{1n} \pm x_{2n} \end{bmatrix}$

* Dot product $\vec{x_1} \cdot \vec{x_2}$ is defined as sum of product of each vector's magnitude in X and Y axis.



$\theta = \beta - \alpha$

Now $\vec{x_1} \cdot \vec{x_2} = x_{11} \cdot x_{21} + x_{12} \cdot x_{22}$

OR

$x_1.\text{dot}(x_2)$

$= |x_1| \cdot \cos\alpha \cdot |x_2| \cos\beta + |x_1| \sin\alpha |x_2| \sin\beta$

$= |x_1||x_2| (\cos\alpha \cdot \cos\beta + \sin\alpha \sin\beta)$

$= |x_1||x_2| \cos(\beta - \alpha)$

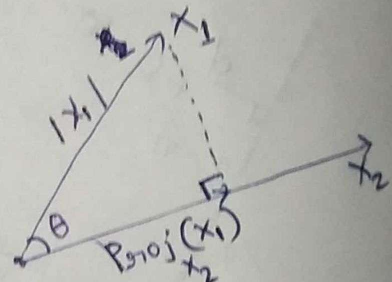
$$\boxed{\vec{x_1} \cdot \vec{x_2} = |x_1| \cdot |x_2| \cos\theta = [x_{11} \ x_{12}] \cdot \begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix}}$$

$\cos(90^\circ) = 0$

$\therefore$ when $\theta = 90^\circ$ dot product of vectors are zero. (orthogonal)

* Projection of a vector on another vector :-

Projection of $\vec{x_1}$ on to $\vec{x_2}$:-



$\text{Proj}_{x_2}(\vec{x_1}) = |\vec{x_1}| \cdot \cos\theta \cdot \hat{x_2}$

OR

$= |\vec{x_1}| \cdot \cos\theta \cdot \dfrac{\vec{x_2}}{|\vec{x_2}|}$

$= \dfrac{|\vec{x_1}| \cdot |\vec{x_2}| \cdot \cos\theta \cdot \vec{x_2}}{|\vec{x_2}|^2}$

$$\boxed{\text{Proj}_{x_2}(\vec{x_1}) = \frac{\vec{x_1} \cdot \vec{x_2}}{|x_2|^2} \cdot \vec{x_2}}$$ $\longrightarrow$ There is no need to compute theta in this eq^n.

* Angle bet^n two planes.

Plane 1 $\pi_d^1 = w^{1T} \cdot x + w_0^1$, normal $w^1 = [w_1^1, w_2^1, \ldots, w_d^1]$

Plane 2 $\pi_d^2 = w^{2T} x + w_0^2$, normal $w^2 = [w_1^2, w_2^2, \ldots, w_d^2]$

They are $\Rightarrow$

- Parallel if $\dfrac{w_1^1}{w_1^2} = \dfrac{w_2^1}{w_2^2} = \dfrac{w_3^1}{w_3^2} = \ldots = \dfrac{w_d^1}{w_d^2}$
- Perpendicular if $w^1 \cdot w^2 = 0$ (dot product is zero)
- otherwise $\theta = \cos^{-1}\left(\dfrac{w^1 \cdot w^2}{|w^1||w^2|}\right)$