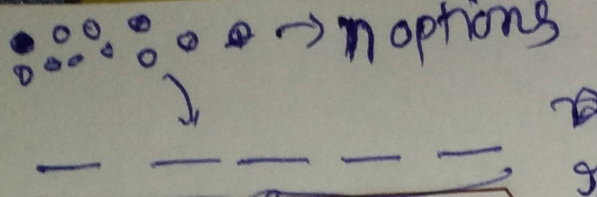


MAY मई 2019

कृभको KRIBHCO

Combinatorics

29 बुधवार Wednesday

(1) Permutation : ORDER MATTERS

→ with ~~replacement~~ repetations = n^r

lock pad of four digits (0 to 9)

$\underline{10} \times \underline{10} = 10^2 = 100$ ways

30 गुरूवार Thursday

→ without repetations = $n!/(n-r)!$

3 winners (GSB) of race of 3 people (A,B,C)

$\underline{3} \times \underline{2} \times \underline{1} = 6$ ways

A B C
A C B
B A C
B C A
C A B
C B A

31 शुक्रवार Friday

If order matters it is multiplication

$n \times n \times n \times \ldots r$ times

$n \times (n-1) \times (n-2) \times \ldots \times 1$ r terms

01 शनिवार Saturday

② Combination : ORDER DOES NOT MATTER

↳ with repetitions : $\frac{(r+n-1)!}{r!\,(n-1)!}$

Buy two bikes from 3 models H, B, T

HH, BB, TT, HB, BT, HT } 6 ways

02 रविवार Sunday

↳ without repetitions : ${}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$

Buy two distinct mode bike from H, B, T

HB BT HT } 3 ways

03 सोमवार Monday

कृभको KRIBHCO

04 मंगलवार Tuesday

Probability

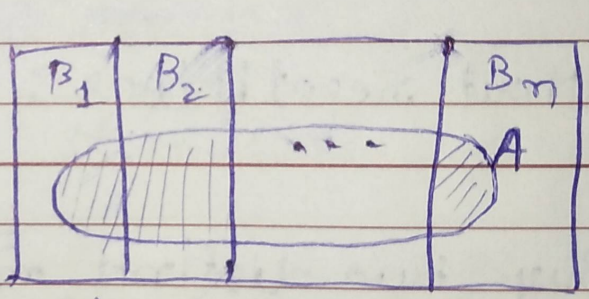
$P = \frac{\text{no. of favourable events}}{\text{Total events}}$

This naive definition is true if all events have same probability.

Eles use

$P(A \cap B) = P(A) \cdot P(B)$ for independent events.

=> Law of total prob. LOTP



05 बुधवार Wednesday

$B_1, B_2, \ldots B_n$ are partition of sample space.

$P(A) = P(A \cap B_1) + P(A \cap B_2) + \ldots + P(A \cap B_n)$

$= P(A|B_1) \cdot P(B_1) + P(A|B_2) P(B_2) + \ldots + P(A|B_n) P(B_n)$

=> Bayes Rule $P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$

$\frac{P(A|B)}{P(A^c|B)} = \frac{P(B|A)}{P(B|A^c)} \frac{P(A)}{P(A^c)}$

odds form of Bayes rule

$P(A|B) = \frac{P(B|A) P(A)}{P(B|A) P(A) + P(B|A^c) P(A^c)}$

53 Questions

JUNE जून 2019

कृभको KRIBHCO

08 शनिवार Saturday

(1) P(strings are not identical)
= 1 - P(strings are identical)
= 1 - (1/2 1/2 1/2 ... n times)
= 1 - 1/2^n

H T T H H H T (S1)
1/2 1/2 1/2 1/2 1/2 1/2 1/2 (S2)
prob of getting H = 1/2

(2) set = { ---------- 2n }
n(H) n(T)

09 रविवार Sunday

M-1: 2 2 2 2 ... 2n times
(2 ways to fill blank - H/T)
Total events = $2^{2n} = 4^n$

10 सोमवार Monday

$\therefore P = \frac{\text{fav.}}{\text{total}} = \frac{{}^{2n}C_n}{4^n}$

M-2 ${}^{2n}C_n (P(H))^n (P(T))^{2n-n}$
$= {}^{2n}C_n (1/2)^n (1/2)^n$
$= {}^{2n}C_n \left(\frac{1}{2^n}\right)\left(\frac{1}{2^n}\right)$
$= \frac{{}^{2n}C_n}{4^n}$

11 मंगलवार Tuesday

③ $P(A \cup B) \leq P(A) + P(B) - P(A \cap B)$; $P(A \cap B) \geq 0$

④ $P(A) = 1$ $\quad P(A|B) = ?$

$P(B) = 1/2$ $\quad P(B|A) = ?$

$P(A \cap B) = 1/2$

$P(A) = 1$ so A is S

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/2}{1/2} = 1$

$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{1/2}{1} = 1/2$

⑤

	P : 1/3	Q : 2/3	
Red	2	3	= 5
Blue	3	1	= 4
	= 5	= 4	

12 बुधवार Wednesday

A: Ball is picked up from P $\quad 1/3$

B: Red Ball is picked up

C: A^c : Ball is picked up from Q $\quad 2/3$

~~$P(B \cap A) = P(A|B) P(B)$~~

It is given that Red ball is picked up

$\therefore P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)} = \frac{P(B|A) P(A)}{P(B|A) P(A) + P(B|C) P(C)}$

$= \frac{2/5 \cdot 1/3}{2/5 \cdot 1/3 + 3/4 \cdot 2/3} = \frac{4}{19}$

13 गुरूवार Thursday

⑥ 1, 2, 3, 4, 5, 6

1, 2, 3, 4, 5, 6

$P(\text{sum} \geq 6) = P(1,5) + P(1,6) + P(2,4) + P(2,5) + P(2,6) + P(3,3) + P(3,4) + P(3,5) + P(3,6) + P(6)$

$= \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \ldots + 9 \text{ times} + \frac{1}{6}$

$= \frac{9}{36} + \frac{1}{6}$

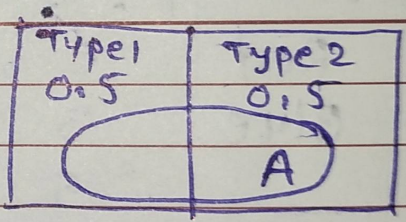
$= \frac{5}{12}$

14 शुक्रवार Friday

⑦ A = Bulb lasts ≥ 100hr

T_1 = Bulb is Type 1

T_2 = Bulb is Type 2



$P(A) = P(A|T_1)P(T_1) + P(A|T_2) \cdot P(T_2)$

$= (0.7)(0.5) + (0.4)(0.5)$

$= 0.55$

⑧ P: P applies for a job

Q: Q applies for a job

$P(P) = \frac{1}{4} \quad \therefore P(P^c) = \frac{3}{4}$

$P(P|Q) = \frac{1}{2} \quad \therefore P(P^c|Q) = \frac{1}{2}$

$P(Q|P) = \frac{1}{3} \quad \therefore P(Q^c|P) = \frac{2}{3}$

$P(P^c|Q^c) = (?)$ OR we can find $P(P|Q^c)$

Not easy

$$P(P^c|Q^c) = \frac{P(Q^c|P^c)P(P^c)}{P(Q^c)}$$

given

$$P(P|Q^c) = \frac{P(Q^c|P)P(P^c)}{P(Q^c)}$$

Now $P(P|Q) = \frac{P(Q|P)\cdot P(P)}{P(Q)}$

$\therefore \frac{1}{2} = \frac{1/3 \cdot 1/4}{P(Q)}$

$\therefore P(Q) = \frac{1}{6}$ $\therefore P(Q^c) = \frac{5}{6}$

$\therefore P(P^c|Q^c) = 1 - P(P|Q^c)$

$= 1 - \frac{P(Q^c|P)\,P(P^c)}{P(Q^c)}$

$= 1 - \frac{1/2 \cdot 3/4}{5/6}$

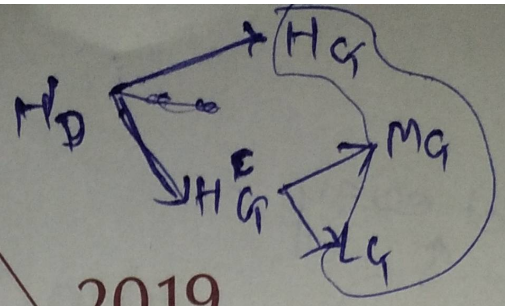
$= 4/5$

(9) A|B A B

	H_D	M_D	L_D
H_G	0.4	0.48	0.12
M_G	0.1	0.65	0.25
L_G	0.01	0.5	0.49

$P(H_G) = 0.2$
$P(M_G) = 0.5$
$P(L_G) = 0.3$

$P(H_G|H_D) = (?)$



कृभको KRIBHCO

JUNE जून 2019

18 मंगलवार Tuesday

$$P(H_G|H_D) = \frac{P(H_D|H_G)\,P(H_G)}{P(H_D)}$$

$$= \frac{P(H_D|H_G)\,P(H_G)}{P(H_D|H_G)P[H_G) + P(H_D|M_G)P(M_G) + P(H_D|L_G)P(L_G)}$$

$$= \frac{(0.4)(0.2)}{(0.4)(0.2)+(0.1)(0.5)+(0.01)(0.3)}$$

$$= 0.6015$$

(10) $P(A) = 0.8 \quad P(B) = 0.5 \quad P(C) = 0.3$

19 बुधवार Wednesday

all are independent events

(A) prob that all occurred

$= 0.8 \times 0.5 \times 0.3$

$= 0.120 = 12\ \%$

(B) prob none of them occurred

$= 0.2 \times 0.5 \times 0.7$

$= 0.070 = 7\ \%$

(C) At least 1 occured $= 1 - P(\text{none occured})$

$= 1 - 0.07$

$= 0.93$

$= 93\ \%$

20 गुरूवार Thursday

(11) Total events (numbers) = 999 − 100 + 1
= 900

Fav events (permutations)

(1,2,3,4,5,6,8,9) 8 × 9 × 9 = 648
(0,1,2,3,4,5,6,8,9)
सौ दहाई इकाई

∴ P(num without 7) = $\frac{648}{900} = \frac{4\times2\times9\times9}{9\times25\times4}$

$= \frac{18}{25}$

(12) ⇒ Total ways of drawing balls = 25 × 24
OR $^{25}P_2 = 25\times24$

21 शुक्रवार Friday

⇒ Total ways of drawing one white & one black ball = $\frac{10}{w} \times \frac{15}{b}$
2! ways to arrange wb, bw

$P(wb, bw) = \frac{2\times10\times15}{25\times24}$

$= \frac{1}{2}$

OR $^{2}C_1 \; ^{10}C_1 \; ^{15}C_1$
↑ shuffle ↑ 1 out of 10 W, 1 out of 15 B

AT LEAST = 100% − None

JUNE जून 2019

कृभको KRIBHCO

22 शनिवार Saturday

(13) M1 at least = 1 − (no dice = 6)

1 dice = 6

$= 1 - \left(\frac{5}{6} \times \frac{5}{6}\right)$

$= \frac{11}{36}$

M2 at least = $\frac{1}{1\text{dice}=6} \times \frac{5}{5} = 5$

$\frac{5}{5} \times \frac{1}{1} = 5$

$\frac{1}{1} \times \frac{1}{1} = 1$

11 ways

$\therefore \frac{11}{36}$

$d_1/d_2 = 6$

$d_2/d_1 = 1, 2, 3, 4, 5$

$d_1 = 6$ $d_2 = 6$

23 रविवार Sunday

(14) choose two ACE from cards (52)

$\frac{{}^4C_2}{{}^{52}C_2} = \frac{2 \times 4 \times 3}{2 \times 52 \times 51} = \frac{4}{52} \times \frac{3}{51}$

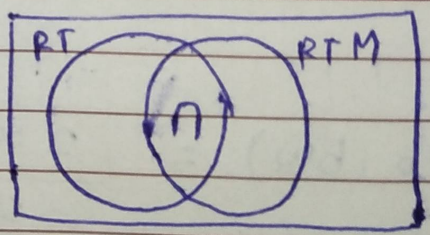
24 सोमवार Monday

(15) P(RT) = 0.5

P(RTM) = 0.6

P(RT ∪ RTM) = 0.7

P(RT ∩ RTM) = (?)



P(RT ∪ RTM) = P(RT) + P(RTM) − P(RT ∩ RTM)

0.7 = 0.5 + 0.6 − P(RT ∩ RTM)

∴ P(RT ∩ RTM) = 0.4

(16) Fav perm. = $\underbrace{3}_{\text{odd}} \times \underbrace{3}_{\text{even}} \times \underbrace{3}_{\text{even}}$ ← 3 even, 3 odd

shuffle = $\frac{3!}{2!} = 3$ ← group of even

$\therefore P(\text{exact 1 odd}) = \frac{\text{fav}}{\text{total}} = \frac{3 \times 3 \times 3 \times 3}{6 \times 6 \times 6}$

$= \frac{3}{8}$

(17) $P(E_1) = 1/2$

$P(E_2) = 1/3$

$P(E_1 \cap E_2) = 1/5$

(A) $P(E_1 \cup E_2) = 2/3$

(B) $P(E_1 \cap E_2) = P(E_1) P(E_2)$

(C) $P(E_1 \cap E_2) \neq P(E_1) P(E_2)$

(D) $P(E_1 | E_2) = 4/5$

26 बुधवार Wednesday

→ $P(E_1 \cup E_2) = 1/2 + 1/3 - 1/5 = 19/30$ (A)

→ $P(E_1 | E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)} = \frac{1/5}{1/3} = \frac{3}{5}$ (D)

→ $P(E_1) \cdot P(E_2) = 1/2 \cdot 1/3 = 1/6 \neq P(E_1 \cap E_2)$ (B)

option : C E_1 & E_2 are not independent events

(18) $P(E_1) = P(E_2) = x$

$P(E_1 \cup E_2) = 1$

E_1 & E_2 are independent

$x = ?$

27 गुरूवार Thursday

$P(E_1 \cap E_2) = P(E_1) \cdot P(E_2) = x \cdot x = x^2$

Now $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$\therefore 1 = x + x - x^2$

$\therefore x^2 - 2x + 1 = 0$

$\therefore (x-1)^2 = 0$

$\therefore \boxed{x = 1}$

$\therefore P(E_1) = P(E_2) = 1$

(19)

1	1	1	1	1	1	1	
M	T	(W)	T	F	S	S	Shuffle day of acc
$^7C_0$	$^7C_0$	$^7C_1$	$^7C_0$	$^7C_0$	$^7C_0$	$^7C_0$	$^7C_1$ = 7 ways
No acc out of 7		seven out of 7 acc		~~negative~~ out of 7			

28 शुक्रवार Friday

Total outcomes = $\underline{7}\ \underline{7}\ \underline{7}, \underline{7}\ \underline{7}\ \underline{7}\ \underline{7}$

$= 7^7$

$\therefore P = \frac{7 \times (1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1)}{7^7} = \frac{1}{7^6}$

(20) P(at least one head or one tail) = 1 − P(all heads or tails)

$= 1 - (1/16 + 1/16)$

$= 14/16$

$= 7/8$

Out of all 16 combinations only HHHH and TTTT don't have H/T both

(21) Theory ⇨ $F(t) = \int_0^t f_1(x)\, f_2(t-x)\, dx$

29 शनिवार Saturday

Sequential modules are running (one after another)

Prob Prog run 4 units

$\therefore F(4) = f_1(0)\cdot f_2(4) + f_1(1) * f_2(3) + f_1(2) f_2(2) + f_1(3) * f_2(1) + f_1(4) f_2(0)$

} ∞ flood values also

$\therefore F(t) = \int_0^t f(x)\, f(t-x)\, dx$

30 रविवार Sunday

(22) Binomial

$P(2H, 2T) = {}^4C_2 (P(H))^2 (P(T))^2$

$= \left(\frac{4\times 3}{2}\right)\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2$

01 सोमवार Monday

$= 3/8$

(23) $P(d \text{ differ}, (n-d) \text{ same}) = {}^nC_d \left(P(d)\right)^d \left(P(d^c)\right)^{n-d}$

$= {}^nC_d \left(\tfrac{1}{2}\right)^d \left(\tfrac{1}{2}\right)^{n-d}$

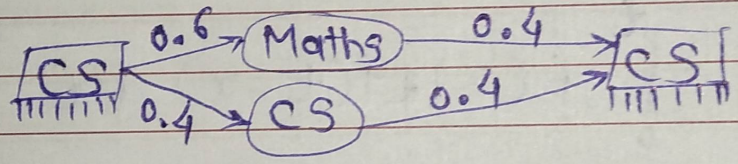
$= \frac{{}^nC_d}{2^n}$

02 मंगलवार Tuesday

(24) Out of 10 even numbers any one can come at ~~first~~ earlier position than any other

so it is $\frac{1}{10}$ chance 2 is at earliest position.

(25) Law of Total probability



03 बुधवार Wednesday

$$P(CS^{wed}) = P(CS^{wed} | M^{tue}) P(M^{tue}) + P(CS^{wed} | CS^{tue}) P(CS^{tue})$$

$$= (0.4)(0.6) + (0.4)(0.4)$$

$$= 0.4$$

Notice that

$$P(M^{tue}) = P(M^{tue} | CS^{mon}) P(CS^{mon}) + P(M^{tue} | M^{mon}) P(M^{mon})$$

($P(CS^{mon})$ given = 1; $P(M^{mon})$ = 0)

$$P(M^{tue}) = P(M^{tue} | CS^{mon}) = 0.6$$

Same logic applied of $P(CS^{tue})$

(26)

2	4	6	→ x
1	3	5	→ 0.9x

$1.9x = 1$

$\therefore x = \frac{1}{1.9}$

Now we are given $X > 3$ and getting even number is 75% probable.

$X > 3 \Rightarrow$ 4, 5, 6

4, 6, 5

75% 25%

$P(X \text{ is even} \mid X > 3) = P(X = \{4,6\} \mid X > 3) = 0.75$

$P(X \text{ is odd} \mid X > 3) = P(X = \{5\} \mid X > 3) = 0.25$

$P(X > 3) = ? = P(4) + P(5) + P(6)$

Here $P(2) = P(4) = P(6) = x/3$

and $\frac{P(5)}{P(4)+P(5)} = \frac{25\%}{75\%} = \frac{1}{3}$

$\therefore P(5) = \frac{x/3 + x/3}{x/3} = \frac{2x}{9}$

$\therefore P(x > 3) = \frac{x}{3} + \frac{2x}{9} + \frac{x}{3} = \frac{8x}{9} = \frac{8}{9} \times \frac{1}{1.9}$

$= 0.468$

कृभको KRIBHCO

06 शनिवार *Saturday* (27)

Computer → P Faulty → q ✓ → Declared faulty = Pq
1-q ✗ → okay
1-P No faulty → q ✓ → okay
1-q ✗ → faulty = (1-P)(1-q)

F ✓ NF ✗
Pq + (1-P)(1-q)

TEST RESULT
ACCURACY = q

07 रविवार *Sunday* (28) Background $N = a^m b^n c^p \ldots$

N → number; a, b, c → prime factors

No of possible factors for N
$= (m+1)(n+1)(p+1)\ldots$

Eg. $14 = 2^1 \times 7^1 = (2\times2) = 4$
1, 2, 7, 14

08 सोमवार *Monday*

$100 = 2^2 \times 5^2 = (3\times3) = 9$
1, 2, 4, 5, 10, 20, 25, 50, 100

ANS ⇒ $10^{99} = 2^{99} \times 5^{99}$ = 100 × 100 factors

$10^{99} = 10^{96} \times 10^3$
$= 10^{96} \times \underbrace{2^3 \times 5^3}_{4\times4 \text{ factors}}$

$$P = \frac{4\times4}{100\times100} = \frac{1}{625}$$

H	H
H	T
T	H
T	T

at least one is head is given
so total 3 posibilities are there
and only 1 is HH. So (1/3)

(29) A : ~~First~~ coin is H (one of the) => 3/4
B : ~~second~~ coin is H (one of the) => 3/4
A = B

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/4}{3/4} = \frac{1}{3}$$

09 मंगलवार Tuesday

(30) Total ways = $^5P_2 = 5 \times 4 = 20$

first one card = second card + 1

2	1
3	2
4	3
5	4

} 4 ways

$\therefore P = 4/20 = 1/5$

10 बुधवार Wednesday

(31) Total ways = $6 \times 6 \times 6 \times 6 = 1296$

fixed
(4) 6 6 6
(5) 6 6 5
(6) 4 6 6

any dice can not be less than 4
$\therefore$ 3, (21 from 3 dice Possi)

This is same as 1st comb

$\therefore$ Sets = {4, 6, 6, 6} → $4!/3!$ perm = 4
{5, 6, 6, 5} → $4!/2!2!$ perm = 6

$\therefore P = \frac{10}{1296} = \frac{x}{1296}$ $\therefore x = 10$

कृभको KRIBHCO

11 गुरूवार Thursday

(32) A : Number is divisible by 2
B : " " " by 3
C : " " " by 5

$P(A \cup B \cup C)' = 1 - P(A \cup B \cup C)$
$= 1 - \{A + B + C - A \cap B - B \cap C - A \cap C + A \cap B \cap C\}$

$P(A) = 50/100$ $P(A \cap B) = 16/100$ (×6)

$P(B) = 33/100$ $P(B \cap C) = 6/100$ (×15)

$P(C) = 20/100$ $P(A \cap C) = 10/100$ (×10)

$P(A \cap B \cap C) = 3/100$ (×30)

12 शुक्रवार Friday

$P(A \cup B \cup C)' = 1 - \frac{1}{100}(50 + 33 + 20 - 16 - 6 - 10 + 3)$

$= 1 - 74/100$

$= 0.26$

Ans

(33)

A	B
x	1-x

S

$P(A \cup B) = 1 = P(S)$

$A \cup B = S$

(34)

13 शनिवार Saturday

$P(A) = x \quad \therefore P(B) = 1-x$

find max $P(A) \cdot P(B)$

$\therefore f(x) = x(1-x) = x - x^2$

$\therefore f'(x) = 1 - 2x$

$\therefore f''(x) = -2 < 0$, max when $f' = 0$

$\therefore f'(x) = 1 - 2x = 0$

$\therefore x = \frac{1}{2}$

14 रविवार Sunday

at $x = 1/2$ $f(x)$ is maximum

$\therefore P(A) = P(B) = 1/2$

(34) $f(x) = \begin{cases} 1/x^2 & \text{for } x \in [a, 1] \\ 0 & \text{otherwise} \end{cases}$ $a = ?$

15 सोमवार Monday

Now $\int_a^1 f(x)\,dx = 1$

$\therefore \int_a^1 x^{-2}\,dx = 1$

$\therefore \left[-x^{-1}\right]_a^1 = 1$

$\therefore -1 + a^{-1} = 1$

$\therefore -1 + \frac{1}{a} = 1$

$\therefore \boxed{a = 0.5}$

$\int x^n\,dx = \frac{x^{n+1}}{n+1}$

$x^{-2} = \frac{x^{-2+1}}{-2+1} = \frac{x^{-1}}{-1} = -x^{-1}$

16 मंगलवार Tuesday

(35) HH → } o/p N, stop } X
HT →
TH → o/p Y, stop } ✓
TT → repeat

Probability of getting Y = 1st try + 2nd try + 3rd try + ---

$= \frac{1}{4} + \frac{1}{4}\frac{1}{4} + \frac{1}{4}\frac{1}{4}\frac{1}{4} + \cdots$

got TH TT, TH TT, TT, TH. ---

$a = \frac{1}{4}$ $r = \frac{1/4 \cdot 1/4}{1/4} = \frac{1}{4}$

17 बुधवार Wednesday

$S_\infty = \frac{a}{1-r} = \frac{1/4}{1-1/4} = \frac{1}{3} = 0.\bar{3}$

(36) Trial 1 tie × Trial 2 tie × Trial 3 one win

P, Q are throwing dice.

$= \frac{6}{36} \times \frac{6}{36} \times \frac{30}{36}$

$= \frac{1}{6} \times \frac{1}{6} \times \frac{5}{6}$

$= 0.023$

18 गुरूवार Thursday

(37) $P(a < x \leq b) = \int_a^b f(x)\,dx$

probability of RV x

(38) RV $X = [x_1, x_2, \ldots, x_n]$ μ_x σ_x M_x

RV $Y = \{y_i \mid y_i = ax_i + b, \forall x_i\}$ μ_y σ_y M_y

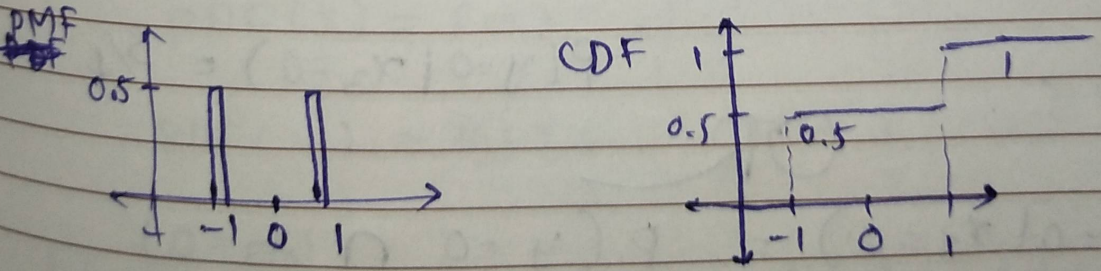
$\mu_y = a\mu_x + b$

$\sigma_y \neq a\sigma_x + b$ ∵ σ is non-linear

Index of Median and Mode doesn't change.

19 शुक्रवार Friday

(39) $X = \{0, -1, 1\}$ $P(-1) = 0.5$ $P(1) = 0.5$



$F(-1) = F(x \leq -1) = 0.5$

$F(0) = F(x \leq 0) = 0.5$

$F(1.2) = F(x \leq 1.2) = 1$

$F(1) = F(x \leq 1) = 1$

20 शनिवार Saturday

(40) $P(x_1) = P(x_2) = P(x_3) = 1/3$

$y = x_1 \cdot x_2 \oplus x_3$

x_1	x_2	x_3	$x_1 x_2$	y
1	1	1	1	0
1	1	0	1	1
1	0	1	0	1
1	0	0	0	0
0	1	1	0	1
0	1	0	0	0
0	0	1	0	1
0	0	0	0	0

21 रविवार Sunday

$P(Y=0 \mid x_3=0) = ?$

$x_3 = 0$ is given $n(S) = 4$

for given S $n(y) = 3$

$\therefore P(y=0 \mid x_3=0) = 3/4$

22 सोमवार Monday

OR

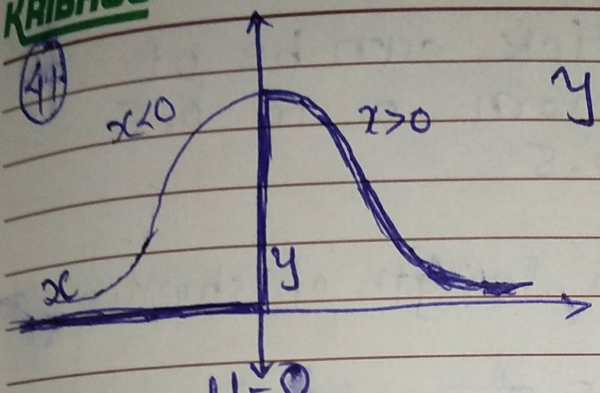
$$P(y=0 \mid x_3=0) = \frac{P(y=0 \cap x_3=0)}{P(x_3=0)}$$

$$= \frac{3/8}{4/8}$$

$$= 3/4$$

23 मंगलवार Tuesday

(41)



$y = \max(x, 0)$

Median of Y = (?)

= 0 because it's middle value of y

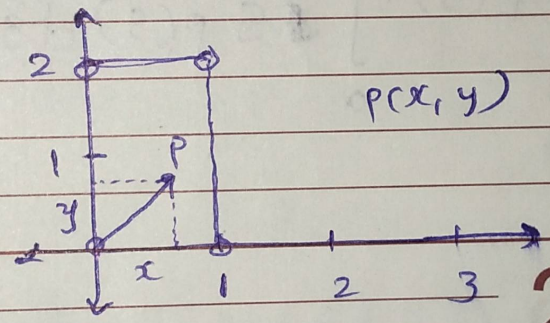
(42) p is random point

$p = \sqrt{x^2 + y^2}$

$p^2 = x^2 + y^2$

$E[p^2] = E[x^2 + y^2]$

$= E[x^2] + E[y^2]$



24 बुधवार Wednesday

⇒ distribution is uniform

$\therefore PDF(x) = f(x) = \frac{1}{1-0} = 1$

$PDF(y) = f(y) = \frac{1}{2-0} = \frac{1}{2}$

$E[x^2] = \int_0^1 x^2 f(x)\,dx = \int_0^1 x^2 \cdot 1\,dx = \left[\frac{x^3}{3}\right]_0^1 = \frac{1}{3}$

$E[y^2] = \int_0^2 y^2 f(y)\,dy = \int_0^2 y^2 \cdot \frac{1}{2}\,dy = \frac{1}{2}\left[\frac{y^3}{3}\right]_0^2 = \frac{4}{3}$

$\therefore E[p^2] = \frac{1}{3} + \frac{4}{3} = \frac{5}{3}$

25 गुरूवार Thursday

(43) shorter stick can be of length from 0 to 0.5

$\therefore a = 0 \quad , b = 0.5$

→ let's say S is length of shorter stick

$$P(S) = \frac{1}{b-a} = \frac{1}{0.5-0} = 2$$

$$\rightarrow E(S) = \int_0^{0.5} S\, p(S)\, dS = \int_0^{0.5} S \cdot 2 \cdot dx$$

$$= 2\left[\frac{S^2}{2}\right]_0^{0.5}$$

$$= 0.25$$

26 शुक्रवार Friday

(44) THEORY : ans option C

$$E(X) = x_1 P_1 + x_2 P_2 + \cdots + x_n P_n$$

$$5 = x_1 P_1 + x_2 P_2 + \cdots + x_n P_n$$

here $P_1 + P_2 + P_3 + \cdots + P_n = 1$

So to be $E(X) = 5$ at least one $x_i \geq 5$

$\therefore \forall x_i = 4 \not< 5 \quad \therefore 4(\Sigma P_i) = 4(1) = 4 \neq 5$

27 शनिवार Saturday

(45) Method 1 :

25% → correct

75% → incorrect

Marks = $150 \times 0.25 \times 1 - 150 \times 0.75 \times 0.25$

$= 9.375$

total marks $= 1000 \times 9.375$

$= 9375$

28 रविवार Sunday

Method 2 : prob of correct $P_1 = 1/4$

prob of in-correct $P_2 = 3/4$

$E(X) = \sum_{i=1}^{2} x_i P_i = x_1 P_1 + x_2 P_2$ ← one que

$= 1 \cdot 1/4 + (-0.25)\frac{3}{4}$

$= 0.0625$

29 सोमवार Monday

∴ total marks $= 0.0625 \times 150 \times 1000$

$= 9375$

(46) $X = \{x_1, x_2, \ldots, x_n\}$ $x_i = 2^i$

$= \{2, 2^2, 2^3, \ldots, 2^n\}$

Sample $S \subseteq X$ drawn with

prob$(x_i) = P_i = \frac{1}{2}$

30 मंगलवार Tuesday

z = value of smallest number
$P(z)$ = probability of selecting z

let's say i^{th} term is smallest in S

$\therefore z = 2^i$

$P(z) = P(\text{not selecting } (i-1) \text{ terms}) \cdot P(\text{selecting } i^{th} \text{ term})$

$= \left(\frac{1}{2}\right)^{i-1} \cdot \left(\frac{1}{2}\right)$

$= \left(\frac{1}{2}\right)^i$

31 बुधवार Wednesday

$\therefore E(z) = \int \sum_{i=1}^{n} z\, p(z)\, dz$

$= \int \sum_{i=1}^{n} 2^i \left(\frac{1}{2}\right)^i$

$= \int \sum_{i=1}^{n} 2^i \left(\frac{1}{2^i}\right)$

$= \sum_{i=1}^{n} 1$

$E[z] = n$

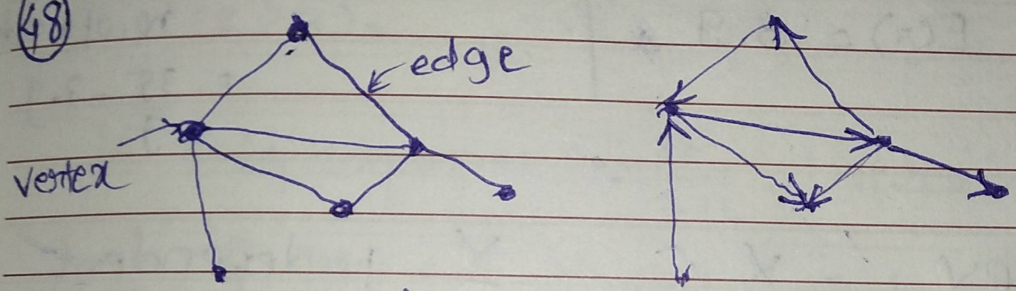
01 गुरूवार Thursday

(47) σ^2 or var $= E[x^2] - \mu^2$

$R = E[x^2] - (E[x])^2$

$\therefore \boxed{R \geq 0}$

(48)



undirected directed

02 शुक्रवार Friday

$\Rightarrow$ prob (edge between two vertices) $= \frac{1}{2}$

x = 3 vertices form triangle

$= {}^8C_3 = \frac{8 \times 7 \times 6}{3 \times 2 \times 1} = 56$

$P(x) = P(\text{edge between } V_1 \& V_2) \times P(\text{edge } V_2V_3) \times P(\text{edge } V_1V_3)$

$= \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$

$E(x) = x \cdot p(x) = 56 \times \frac{1}{8} = 7$

$x_1p_1 + x_2p_2 + \cdots$

03 शनिवार Saturday

(49) $E(x) = x_1p_1 + x_2p_2 + x_3p_3 + \dots + x_9p_9$

$= (3+5+5+3+5+4+3+4+3)\frac{1}{9}$

$= \frac{35}{9}$

$E(x) = 3.9$

OR avg length = $\frac{\text{total length}}{\text{no. of words}}$

$= \frac{35}{9} = 3.9$

04 रविवार Sunday

(50) RV X Y independent

expo(λ_1) expo(λ_2)

mean $E[X] = \frac{1}{\lambda_1} = \alpha$ $E[Y] = \frac{1}{\lambda_2} = \beta$

$\therefore \lambda_1 = \frac{1}{\alpha}$ $\lambda_2 = \frac{1}{\beta}$

05 सोमवार Monday

$\therefore f(x) = \lambda_1 e^{-\lambda_1 x}$ $f(y) = \lambda_2 e^{-\lambda_2 y}$

$= \frac{1}{\alpha} e^{-\frac{x}{\alpha}}$ $= \frac{1}{\beta} e^{-\frac{y}{\beta}}$

$F_x(a) = P(x \leq a) = 1 - e^{-\lambda_1 a}$

$\therefore P(x > a) = e^{-\lambda_1 a} = e^{-a/\alpha}$

$F_y(a) = P(y \leq a) = 1 - e^{-\lambda_2 a}$

$P(y > a) = e^{-\lambda_2 a} = e^{-a/\beta}$

$$P(Z > a) = P(X > a) \cdot P(Y > a)$$

$$= e^{-a/\alpha} \cdot e^{-a/\beta}$$

$$= e^{-a(1/\alpha + 1/\beta)}$$

$$P(Z > a) = e^{-a\left(\frac{\alpha+\beta}{\alpha\beta}\right)} \quad \text{——— (1)}$$

if Z is exponentially distributed

$$f_Z(a) = P(Z \le a) = 1 - e^{-\lambda_3 a}$$

$$P(Z > a) = e^{-\lambda_3 a} \quad \text{(2)}$$

From eqn 1 & 2 $\lambda_3 = \frac{\alpha+\beta}{\alpha \cdot \beta}$

$$\therefore \text{mean} = E(Z) = \frac{1}{\lambda_3} = \frac{\alpha \cdot \beta}{\alpha+\beta}$$

(51) $X \sim N(1, 4)$

$Y \sim N(-1, \sigma^2)$

$$Z = \frac{X-1}{\sqrt{4}} \quad \therefore X = 2Z + 1$$

$$Z = \frac{Y-(-1)}{\sigma} \quad Y = \sigma Z - 1$$

कृभको KRIBHCO

08 गुरूवार Thursday

$P(X \leq -1) = P(Y \geq 2)$

$P(2Z + 1 \leq -1) = P(\sigma Z - 1 \geq 2)$

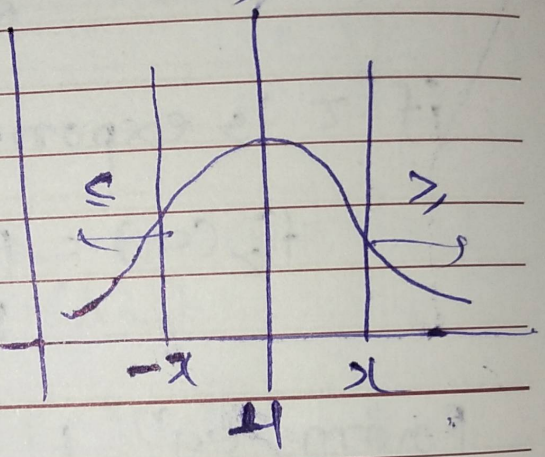
$P(2Z \leq -2) = P(\sigma Z \geq 3)$

$P(Z \leq -1) = P(Z \geq 3/\sigma)$

$\therefore -(-x) = x$

$\therefore -(-1) = 3/\sigma$

$\therefore \boxed{\sigma = 3}$



09 शुक्रवार Friday

(S2) mean = $F(x) = \lambda = 3$ cal min

$P(X < 3) = P(X \leq 2) = F_X(2) = e^{-\lambda} \sum_{i=0}^{2} \frac{\lambda^i}{i!}$

$F_X(2) = e^{-\lambda} \sum_{i=0}^{2} \frac{\lambda^i}{i!} = e^{-3} \sum_{i=0}^{2} \frac{3^i}{i!}$

$= e^{-3}\left(\frac{3^0}{0!} + \frac{3^1}{1!} + \frac{3^2}{2!}\right)$

$= \frac{1}{e^3}\left(1 + 3 + \frac{9}{2}\right)$

$= \frac{17}{2e^3}$

10 शनिवार Saturday

(53) mean = E[X] = μ = 5 = λ
variance = mean = 5 = λ
poisson

Now var = $E[X^2] - E[X]^2$
$5 = E[X^2] - 5^2$
$E[X^2] = 30$

$E[(X+2)^2] = E[X^2 + 4X + 4]$
$= E[X^2] + 4E[X] + 4$
$= 30 + 4(5) + 4$
$= 30 + 20 + 4$
$= 54$

11 रविवार Sunday

There are two 42 NO. Questions

(42) $g_x(z) = \sum_{n=0}^{N} p_j z^j$

$g_x(z) = p_0 z^0 + p_1 z^1 + p_2 z^2 + \dots + p_n z^n$

12 सोमवार Monday

$g'_x(z) = p_0 0 + p_1 + 2p_2 z + 3p_3 z^3 + \dots$
if we take $z = 1$

$g'_x(1) = 1 \cdot p_1 + 2p_2 + 3p_3 + \dots$
$= \sum_{x=1}^{N} x \cdot p(x)$ ← Expectation

So if we put $z = 1$ and take derivation of polynomial we get $E[X]$

13 मंगलवार Tuesday

Now $g_y(z) = (1-\beta+\beta z)^N$

$E[Y] = g'_y(1) = N(1-\beta+\beta(1))^{N-1} \cdot (0-0+\beta)$

$= N(1)^{N-1} \cdot (\beta)$

$= N \cdot \beta$

Verification

① $N=1$

$\therefore g_y(z) = (1-\beta+\beta z)^1$

$g'_y(z) = 0-0+\beta = 1 \cdot \beta = N\beta$

14 बुधवार Wednesday

Y	0	1
$P(Y)$	$1-\beta$	β

$g_y = p_0 + p_1 z + p_2 z^2 + \dots$

$= (1-\beta) + \beta z$

$E[Y] = \sum Y P(Y)$

$= 0(1-\beta) + 1(\beta)$

$= \beta$

$= 1 \cdot \beta$

$E[Y] = N \cdot \beta$

② $N=2$ $g_y(z) = (1-\beta+\beta z)^2 = (K+\beta z)^2$ $K = 1-\beta$

$= K^2 + 2K\beta z + \beta^2 z^2$

Y	0	1	2
$P(Y)$	K^2	$2K\beta$	β^2

$E(Y) = \sum Y P(Y) = 0(K^2) + 1(2K\beta) + \beta^2$

$= 2(1-\beta)\beta + \beta^2$

$= 2\beta = N \cdot \beta$ $\because N=2$

Data and visualization

कृभको KRIBHCO

2019 अगस्त AUGUST

#51 Questions (50)

15 गुरूवार Thursday

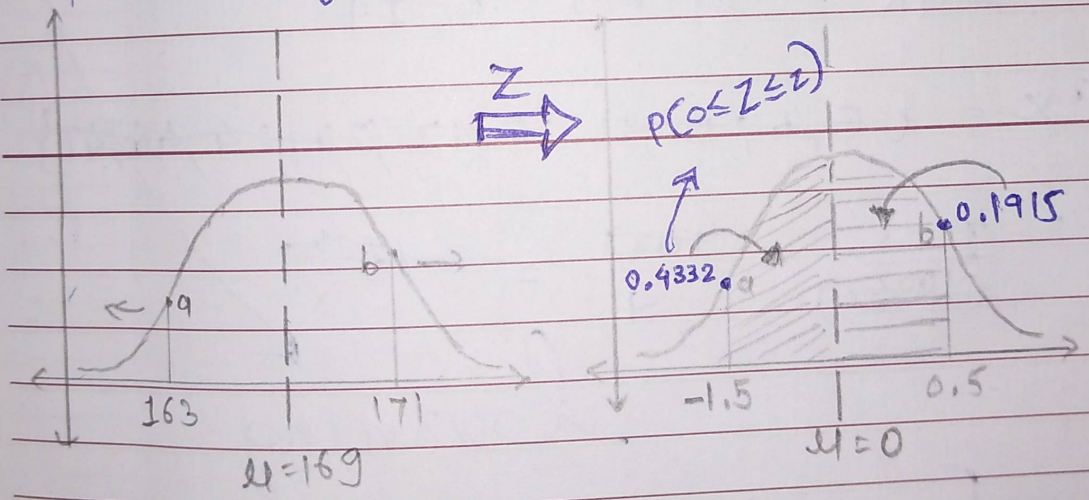
① no_of_students = 400

height ~ $N(\mu, \sigma^2)$ = $N(169, 4)$ cm

$z = \frac{x - \mu}{\sigma}$

$P(x \leq 163 \text{ or } x \geq 171)$

HINT: we can convert / find z value for Normal/gaussian distribution and find probability value using z-table.



16 शुक्रवार Friday

for $X = 163$ $z = \frac{163 - 169}{4} = -1.5$

$X = 171$ $z = \frac{171 - 169}{4} = 0.5$

$P(x \leq 163 \text{ or } x \geq 171) = P(z \leq -1.5 \text{ or } z \geq 0.5)$

$= (0.5 - 0.4332) + (0.5 - 0.1915)$

$= 0.3753$

$\therefore n(x \leq 163 \text{ or } x \geq 171)$

$= 0.3753 \times 400 \approx 150$

$$\text{percentile}(P) = \left(1 + \frac{n-1}{(100/P)}\right)^{th} \text{term}$$ (n: number of terms)

20 मंगलवार Tuesday

(3) IQR for [1, 27]

$IQR = Q_3 - Q_1 = per(75) - per(25)$

$per(75) = 1 + \frac{27-1}{(100/75)} = 20.5^{th}$ term $= 20.5$

$per(25) = 1 + \frac{27-1}{(100/25)} = 7.5^{th}$ term $= 7.5$

$IQR = 20.5 - 7.5 = 13$

21 बुधवार Wednesday

(4) $X = \{5, 6, 10, 11, 15, 17, 20, 24, 46, 47\}$

$Q_1 = P(25) = 1 + \frac{(10-1)\times 25}{100} = 3.25^{th}$ term

$Q_1 = x_3 + 0.25(x_4 - x_3)$
$= 10 + 0.25(11 - 10)$
$= 10.25$

$Q_3 = P(75) = 1 + \frac{(10-1)\times 75}{100} = 7.75^{th}$ term

$Q_3 = x_7 + 0.75(x_8 - x_7)$
$= 20 + 0.75(24 - 20)$
$= 23$

$IQR = Q_3 - Q_1$
$= 23 - 10.25$
$= 12.75$

22 गुरूवार Thursday

Lower limit (whisker) $= Q_1 - 1.5 \cdot IQR$
$= 10.25 - 1.5(12.75)$
$= -8.875$

Upper limit (whisker) $= Q_3 + 1.5 \cdot IQR$
$= 23 + 1.5(12.75)$
$= 42.125$

{46, 47} are greater than U.L. so they are outliars.

(9) $X = \{1, 2, 3, 4, 5\}$ $\lambda = 2$
$Y = (?)$

23 शुक्रवार Friday

For box-cox transformation

$$y_i = \begin{cases} \frac{x_i^{\lambda} - 1}{\lambda} & \lambda \neq 0 \\ \ln(x_i) & \lambda = 0 \end{cases}$$

$\therefore y_i = \frac{x_i^2 - 1}{2}$ $\because \lambda = 2$

$X = [1, 2, 3, 4, 5]$

$Y = \left[\frac{1^2-1}{2}, \frac{2^2-1}{2}, \frac{3^2-1}{2}, \frac{4^2-1}{2}, \frac{5^2-1}{2}\right]$

$Y = [0, 1.5, 4, 7.5, 12]$

$\therefore \text{mean } Y = E[X] = \frac{0+1.5+4+7.5+12}{5} = 5$

24 शनिवार Saturday

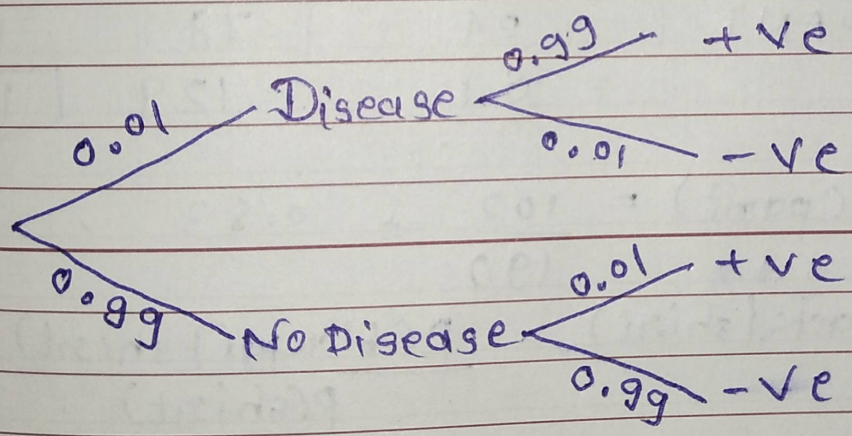
(10) array = [1, 2, 3, 5, 10, 11, 12, 13, 14, 15, 21, 23, 25, 26, 27, 29, 30, 31, 35, 51]

bins = 5 ∴ range (bin width) $= \frac{51-1}{5} = 10$

bin; (Range)	no. of points
$1 \leq x < 11$	5
$11 \leq x < 21$	5
$21 \leq x < 31$	7
$31 \leq x < 41$	2 ← Ans
$41 \leq x \leq 51$	1

25 रविवार Sunday

(11)



26 सोमवार Monday

A: person is positive (has disease) B: Test is positive

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B|A) \cdot P(A) + P(B|A^c) \cdot P(A^c)}$$

$$= \frac{(0.99)(0.01)}{(0.99)(0.01) + (0.01)(0.99)}$$

$$= 0.5$$

OR 50%

27 मंगलवार Tuesday

(12) There are four possibility Nadal wins

Nadal	Federer	prob
4	0	$(0.6)^4$
4	1	$^4C_1 \cdot 0.6^4 \cdot 0.4$
4	2	$^5C_2 \cdot 0.6^4 \cdot 0.4^2$
4	3	$^6C_3 \cdot 0.6^4 \cdot 0.4^3$

0.710208

greater than 60% ← 71%

(26)

	NOT BLACK	BLACK	
SHIRT	37	51	= 88
PANT	24	78	= 102
	= 61	= 129	190

28 बुधवार Wednesday

i) $P(\text{pant}) = \frac{102}{190} = 0.53$

ii) $P(\text{black} | \text{shirt}) = \frac{P(\text{black} \cap \text{shirt})}{P(\text{shirt})}$

$= \frac{51/190}{88/190}$

$= 0.58$

iii) $P(\text{shirt} | \text{not black}) = \frac{P(\text{shirt} \cap \text{not black})}{P(\text{not black})}$

$= \frac{37/190}{61/190}$

$= 0.60$

29 गुरूवार Thursday

iv) $P(\text{pant} \mid \text{Black}) = \frac{P(\text{pant} \cap \text{Black})}{P(\text{black})}$

$= \frac{78/190}{129/190}$

$= 0.60$

(47)

$v_i \rightarrow$	v_1	v_2	v_3	v_4	v_5
λ_i	1.5	1	0.8	1.8	1.75

v_i sorted	v_4	v_5	v_1	v_2	v_3	total
degr	1.8	1.75	1.5	1	0.8	6.85
cummu-lative	1.8	3.55	5.05	6.05	6.85	3.425 50%
%	26.3	51.8	73.7	88.3	100	

30 शुक्रवार Friday