# Rover Models

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## 1 Model

The input channels for the controller will be denoted $u_1 \in [-\pi, \pi]$ and $u_0 \in [-\pi, \pi]$ for steering and throttle, respectively

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \\ \dot{v_x} \\ \dot{\delta} \end{bmatrix} = \begin{bmatrix} v_x(\cos\psi - \frac{l_r}{l}\sin\psi\tan\delta) \\ v_x(\sin\psi + \frac{l_r}{l}\cos\psi\tan\delta) \\ \frac{v_x}{l}\tan\delta \\ \frac{F_{rx} - m_o \frac{\tan\delta}{\cos^2\delta}\dot{\delta}v_x}{m + m_o\tan^2\delta} \\ k_{st}(\delta_{des} - \delta) \end{bmatrix} \tag{1}$$

**where:**

$$\delta_{des} = f(u_1)$$
$$F_{rx} = f(u_0, v_x)$$

## 2 Vehicle Parameters

| Param | Value | Unit |
|---|---|---|
| $m$ | 7.780 | $kg$ |
| $m_o$ | 2.972 | $kg$ |
| $I_z$ | 0.2120 | $kg\,m^2$ |
| $l$ | 0.3302 | $m$ |
| $l_r$ | 0.12 | $m$ |

$m_o = \frac{m l_r^2 + I_z}{l^2}$

## 3 Steering Input

The map from wheel angle to input can be approximated by a line:

$$\delta_{des} = 0.224314009055080 u_1 - 0.008867066788855 \tag{2}$$

The control gain on the wheel angle is modeled as proportional:

$$k_{st} = 4.300730919846748 \tag{3}$$

## 4 Throttle Input

**4.1 Throttle Channel** The channel inputs can be mapped to a velocity setpoints with the following approximation:

$$v_{des} = -10.445339156721717 u_0 - 3.584452482313747 \tag{4}$$

This map seems to be a good fit for input values in the interval $u_0 \in [-0.525, -0.4] \mapsto v_{des} \in [1.899, 0.594]$ m/s. Note that it is not good for commands greater than -0.4. There is a deadzone in the motor for the rover. And this approximation is not the steady state equlibrium of the model for driving force (see below)

**4.2 Driving Force (acceleration)** This was fit with throttle inputs greater than -0.4 when the vehicle was driving in a straight line

$$\begin{aligned} &F_{rx} = c_{m1} + c_{m2}\, u_0 + c_{m3}\, v_x + c_{m4}\, v_x\, u_0 + c_{m5}\, v_x^2 + c_{m6}\, u_0^2 + c_{m7}\, v_x\, u_0^2 + c_{m8}\omega^2 \\ &\text{where: } \omega = \dot{\psi} \end{aligned} \tag{5}$$

| Const | Value | Unit |
|---|---|---|
| $c_{m1}$ | -12.5810995587748 | $N$ |
| $c_{m2}$ | -33.0170773577599 | $N$ |
| $c_{m3}$ | 4.33920832891501 | $\frac{N\,s}{m}$ |
| $c_{m4}$ | 20.3041178298046 | $\frac{N\,s}{m}$ |
| $c_{m5}$ | 0.156420898500981 | $\frac{N\,s^2}{m^2}$ |
| $c_{m6}$ | 4.20678380627274 | $N$ |
| $c_{m7}$ | 10.2828808092518 | $\frac{N\,s}{m}$ |
| $c_{m8}$ | -0.610920415224012 | $\frac{rad^2}{s^2}$ |

### 4.3 Driving Force (braking)

For throttle inputs of $u_0 \geq 0.0$

$$F_{rx} = c_{b1} + c_{b2} v_x + c_{b3} v_x^2 \tag{6}$$

| Const | Value | Unit |
|---|---|---|
| $c_{b1}$ | -4.11177295309464 | $N$ |
| $c_{b2}$ | -15.1817204116634 | $\frac{N\,s}{m}$ |
| $c_{b3}$ | 5.22364002070909 | $\frac{N\,s^2}{m^2}$ |

### 4.4 Driving Force (coasting)

This was fit with a throttle input of $-0.35 \geq u_0 > 0.0$

$$F_{rx} = c_{c1} + c_{c2} u_0 + c_{c3} v_x + c_{c4} v_x^2 \tag{7}$$

| Const | Value | Unit |
|---|---|---|
| $c_{c1}$ | -5.55660998280113 | $N$ |
| $c_{c2}$ | -13.8953541919073 | $N$ |
| $c_{c3}$ | -2.47286920126272 | $\frac{N\,s}{m}$ |
| $c_{c4}$ | 0.480990612787014 | $\frac{N\,s^2}{m^2}$ |