PHYS 250 Homework 2

Due: 10 February 2025

Year	Population (Billions)
1960	3.015470894
1970	3.694683794
1980	4.447606236
1990	5.327803110
2000	6.171702993
2010	7.021732148
2020	7.887001292
2050	9.664378587

Problem 1. (15 points) The world population for a number of years is provided in the table at the right.† We will use this data to explore some aspects of interpolation and extrapolation with linear and quadratic polynomials. The Lagrange interpolating polynomial can easily be constructed "by hand" and there are some advantages to doing this, once. Alternatively you may use `scipy.interpolate.lagrange` if you prefer. Notice that the table includes a population for the year 2050. This value is based on a model.[*Note*: I have included all the digits in the population values given in the data base. Does it make sense that the population of the entire world can be determined to within a single person? Does it even make sense to say we know the population of the entire world to within 10,000 people? On what day is the population determined for the year? For our purposes we can truncate the values to a few significant digits, if desired.]

(i) Using the data from 1980 and 2000 linearly interpolate to find an estimate for the population in 1990. Calculate the fractional error between your estimate and the reported value.

(ii) Using your linear polynomial from the previous part extrapolate it to estimate the populations in 1960 and 2020. Again calculate the fractional errors between your estimates and the reported values.

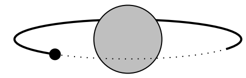
(iii) Again using your linear polynomial from the previous part extrapolate it to estimate the populations in 2050 and 2100. We will comment on these values below.

(iv) Repeat all the previous parts now using a quadratic polynomial and the data from 1970, 1980, and 2000.

(v) In general, extrapolation is dangerous, it can easily give nonsensical results. Based on our results, do we expect linear or quadratic interpolation to give better estimates in 2050 and 2100? Keep in mind that the 2050 value listed in the table is *not* a true value, it is just a different estimate. Explain your reasoning. [*Hint*: What is the behavior of a quadratic at large values of its argument? Is this a good or bad feature for extrapolating the population?]

†This is a subset of the data from worldometers.info. The numbers seem to change every year I look at it, even for years from decades ago.

Problem 2. (15 points) Observations of the angular distance between a satellite and the center of the planet it is orbiting, as measured from the center of the planet, are made at the same time every night for nine nights. For the first four nights it is observed on the left side of the planet, for the next two nights it is behind the planet so is not seen, and for the last three nights it is observed on the right of the planet. This is represented schematically in the figure at the right. Observations are made while the planet travels along the solid path. When the satellite is either in front of or behind the planet no observations can be made (the part of the dotted line intersecting the planet and the part of the path where the solid line goes behind the planet). Also notice that a complete orbit of the satellite is not observed (i.e., there are no observations on the part of the path represented by the dotted line). The angular distance between the planet and the satellite is given in the table. This data is available online and can be read directly from the web as described in PreLab02.



Observation Time (day)	**Distance (arcsecond)**
1	-3.4083885
2	-5.6100975
3	-5.8256509
4	-3.9787359
7	5.3127362
8	5.9562532
9	4.4910645

(i) Estimate the position of the satellite on the days it is obscured by the planet. Briefly describe how you have made these estimates. This means, give enough, general, information so that someone else in the class could repeat your procedure. You do not need to give detailed code.

(ii) Estimate the minimum and maximum angular diameter of the planet. Justify your estimates.

(iii) Next we want to estimate the period of the satellite's orbit. There are many ways to do this from the data given. Briefly break the problem down and give a high level description of each step. Approach it as giving instructions to a fellow student in the class of the steps they should follow. You do not have to give explicit details (you do not need to provide code). Your steps should be in terms of generic algorithms, not explicit function names, so that they could be implemented in any programming environment, not just the one we are using. Here we want to focus on the general concepts, not the implementation details.

(iv) Now implement the steps you outlined in the previous part and obtain an estimate of the period of the orbit. Clearly describe your implementation and provide your estimate of the period.