1)
$$h(t) \Leftrightarrow H(f)$$

$$h(at) \Leftrightarrow \frac{1}{|a|} H\left(\frac{f}{a}\right)$$

$$H(f) = \int_{-\infty}^{\infty} h(t)e^{-2\pi i ft} dt \in \partial e f$$
 in the interval of FT

which is the same as the definition, except for an a frequency component

$$\left(\frac{1}{|a|}H\left(\frac{f}{a}\right) = \frac{1}{|a|}\int_{a}^{\infty}h(at)e^{-2\pi i ft}dt\right)$$

ii)
$$t \rightarrow t - t_0$$

$$e^{-2\pi i f t} \cdot \int_{-\infty}^{\infty} h(u) e^{-2\pi i f t} du$$

$$e^{-2\pi i f t} \cdot \int_{-\infty}^{\infty} h(t - t_{\circ}) e^{-2\pi i f(t - t_{\circ})} dt$$

$$+ (f)$$

$$h(t-t_o) \Leftrightarrow e^{-2\pi i f t_o} H(f)$$

[h(t)]* e 2 mift dt

so then we have

for the equation to look how we need it to, we need $H^*(f) = \int h(t)e^{-2\pi i(-f)t} dt$

$$iv)h(-t)=h(t)$$

$$\int_{h(t)}^{\infty} h(t)e^{-2\pi i \int t} dt = \int_{h(-t)}^{\infty} h(-t)e^{-2\pi i \int (-t)} dt$$

$$\lim_{t \to \infty} H(f)$$

$$\lim_{t \to \infty} h(t)e^{-2\pi i \int t} dt$$

$$\lim_{t \to \infty} h(t)e^{-2\pi i \int (-t)} dt$$

$$\lim_{t \to \infty} h(t)e^{-2\pi i \int (-t)} dt$$

$$\lim_{t \to \infty} h(t)e^{-2\pi i \int (-t)} dt$$

Requestant han the process of the state of the process of the substitute of the su		LAD VIANA FRIS
In (f) = H(-f) (g * h)_k \(\Delta \) Gn Hn The fourier transform of (g * h)_k is giron by \[\begin{align*}	H (f)	we know this is pavivalent b
H(f): H(f) 2) (g * h)_k \$\iff G_n H_n\$ (g*h)_k \$\equiv G_n H_n\$ (g*h)_k \$\equiv \frac{1}{k':0} g_{k-k}\$, hk, \[\begin{align*} \left \frac{\text{N}}{\text{K}} e^{2\pi \text{c} (n-n')^{k/N}} & = \text{S}_{nh'} \\ \begin{align*} \left \frac{\text{N}}{\text{K}} e^{2\pi \text{c} (n-n')^{k/N}} & = \text{S}_{nh'} \\ \end{align*} \[\begin{align*} \left \frac{\text{N}}{\text{K}} e^{2\pi \text{c} \text{K} n/N} & = \text{S}_{nh'} \\ \text{and using inverse fourier transforms, we can And that} \\ \hk_k & = \frac{1}{N} \sum_{n=0}^{\text{N}} \text{H}_n e^{2\text{r} \text{k} n/N} & \text{gk-k} \cdot \frac{2}{N} \text{E}_n^2 \text{G}_n e^{2\text{r} \text{k} (n-n')/N} \\ \text{this allows us to write the FT of (g+h)_k as} \\ \begin{align*} \text{N} & \frac{1}{N} & \text{N} & \text{H}_n e^{2\pi \text{t} \text{k} n/N} & \text{E}_n^2 \text{G}_n e^{2\pi \text{t} \text{k} (n-n')/N} \\ \text{N} & \text{N} & \text{N} & \text{N} & \text{E}_n \text{G}_n e^{2\pi \text{t} \text{k} (n-n')/N} \\ \text{N} & \text{N} & \text{N} & \text{E}_n \text{Using the completiness relation emerga in that expression, and can use this to rewrite it as \begin{align*} \text{N} & \text{N} & \text{E}_n & \text{N} & \text{E}_n & \text{G}_n, \text{G}_n e^{2\pi \text{r} \text{k} n/N} \\ \text{E}_n & \text{N} & \text{E}_n & \text{G}_n, \text{G}_n e^{2\pi \text{r} \text{k} n/N} \\ \text{E}_n & \text{N} & \text{E}_n & \text{S}_n & \text{C}_n & \text{E}_n & \text{E}_n & \text{E}_n \te		n(t) - substituting that
$(H(f):H(f))$ $2)(g*h)_{k} \Leftrightarrow G_{n}H_{n}$ $(g*h)_{k} \stackrel{?}{=} \sum_{k=0}^{N-1} g_{k-k}, h_{k},$ $\frac{1}{N} \sum_{k=0}^{N-1} e^{2\pi i (n-n')^{k}/N} = S_{nh'}$ $\frac{1}{N} \sum_{k=0}^{N-1} e^{2\pi i (n-n')^{k}/N} = S_{nh'}$ $\frac{1}{N} \sum_{k=0}^{N-1} e^{2\pi i (n-n')^{k}/N} = S_{nh'}$ $\frac{1}{N} \sum_{k=0}^{N-1} g_{k-k'} e^{-2\pi i k n/N}$ $\frac{1}{N} \sum_{k=0}^{N-1} g_{k-k'} e^{-2\pi i k n/N}$ $\frac{1}{N} \sum_{k=0}^{N-1} g_{k-k'} e^{-2\pi i k n/N}$ $\frac{1}{N} \sum_{k=0}^{N-1} g_{k-k'} e^{2\pi i k n/N}$ $\frac{1}{N$		in yields what we
2) $(g * h)_k \Leftrightarrow G_n H_n$ $(g * h)_k \Rightarrow G_n H_n$ $(g * h)_k \Rightarrow \sum_{k=0}^{N-1} g_{k-k}, h_k, h_k$ $\frac{1}{N} \sum_{k=0}^{N-1} e^{2\pi i (n-n')^k/N} = S_{nn'}$ The fourier transform of $(g * h)_k$ is given by $\sum_{k=0}^{N-1} h_k, \sum_{k=0}^{N-1} g_{k-k}, e^{-2\pi i k n/N}$ and using inverse fourier transforms, we can find that $h_k = \frac{1}{N} \sum_{k=0}^{N-1} H_n e^{2\pi i k n/N}$ $g_{k-k} = \frac{1}{N} \sum_{n=0}^{N-1} G_n e^{2\pi i k (n-n')/N}$ This allows us to write the FT of $(g * h)_k$ as $\sum_{k=0}^{N-1} \frac{1}{N} \sum_{n=0}^{N-1} H_n e^{2\pi i k n/N} \sum_{k=0}^{N-1} G_n \frac{e^{2\pi i k (n-n')/N} e^{-2\pi i k n/N}}{\sum_{n=0}^{N-1} h_n}$ we see the discrete completness relation emerge in that expression, and can use this to rewrite it as $\sum_{k=0}^{N-1} \frac{1}{N} \sum_{n=0}^{N-1} H_n e^{2\pi i k n/N} \sum_{k=0}^{N-1} S_{nn'}, G_n e^{-2\pi i k n/N}$ $\sum_{k=0}^{N-1} \frac{1}{N} \sum_{n=0}^{N-1} H_n e^{2\pi i k n/N} \sum_{k=0}^{N-1} S_{nn'}, G_n e^{-2\pi i k n/N}$ Cancelling the exponential terms reduces this to		found for H(-J) in (iii)
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r -, ,	W =1	רא		
< \	< 11	<		1
2 N	2 H	2	D	(J.,
k'-0	N=0	k=0	nn	и

the knoniker delta tells us that

so the above expression goes to 0 in the case that n' + n. we can now simplify the expression to

the sum and in term cancel, and we find that

3) hk sampled at N values; Hn is its DFT

the discrete FT of hk, is given by

the DFT of gk, is given by

we can rewrite this as

which simplifies to

the arannest of the sum is how idential to that man expression for H

the argument of the sum is now identical to that in our expression for H_n . we know also that $g_{1c} = 0$ for all $k' \ge N$, so those terms contribute nothing to the sum; only the terms from 0 to N-1 contribute. We can rewrite the sum to reflect this:

so then

we begin by taking the OFT of gk, (and thereby of hak,):

since we are only summing over even values we can rewrite this with that in mind for our N and K values (where K=2K')

we can do the same thing but for the odd values:

combining the even and odd expressions for 6m gives us Hm, since it includes both the even and odd values:

substituting in m+ 1/2 for m in Hm, we find

$$H_{m+\frac{N}{2}} = \sum_{k=0}^{N-1} h_k e^{-2\pi i \left(m+\frac{N}{2}\right)k/N} = \sum_{k=0}^{N-1} h_k e^{-2\pi i mk/N} e^{-\pi i k}$$

(-1)-K via euler's 1denthy

$H_{m} + H_{m} \cdot \frac{N}{a} = G_{m,even} + G_{m,odd} + G_{m,even} - G_{m,odd}$ $H_{m} + H_{m} \cdot \frac{N}{a} = 2 G_{m,even}$	$H_{m,\frac{N}{2}} = \sum_{k=0}^{N-1} h_k e^{-2\pi i mk/N} \leftarrow \text{even case}$ $H_{m,\frac{N}{2}} = \sum_{k=0}^{N-1} - h_k e^{-2\pi i mk/N} \leftarrow \text{odd case}$ So we find that $H_{m,\frac{N}{2}}$; $G_{m,\text{even}} = G_{m,\text{even}}$ and $H_{m,\frac{N}{2},\text{odd}} = -G_{m,\text{odd}}$. we add $H_{m,\text{and}} + H_{m,\frac{N}{2}} = G_{m,\text{even}} + G_{m,\text{odd}} + G_{m,\text{even}} = G_{m,\text{odd}}$ $H_{m,\frac{N}{2}} = G_{m,\text{even}} + G_{m,\text{odd}} + G_{m,\text{even}} = G_{m,\text{odd}}$ $H_{m,\frac{N}{2}} = 2G_{m,\text{even}}$ however, we only want the even valves, so $G_{m,\text{even}}$ is simply $G_{m,\text{even}}$ $G_{m,\text{even}} = G_{m,\text{even}}$ $G_{m,\text{even}} = $	' 'm+ ½	K-0 K	K=0 K	(-1)-K via eu	ler's Identity
Hm + $\frac{N}{2} = \sum_{k=0}^{N-1} - h_k e^{-2\pi i m k/N} \leftarrow odd cese$ So we find that $H_{m+\frac{N}{2}}$, even $= G_{m,even}$ and $H_{m+\frac{N}{2},odd} = -G_{m,odd}$. we nadd H_{m} and $H_{m+\frac{N}{2}} = G_{m,even} + G_{m,odd} + G_{m,even} = G_{m,odd}$ Hm $+ H_{m+\frac{N}{2}} = G_{m,even} + G_{m,odd} + G_{m,even} = G_{m,odd}$ Hm $+ H_{m+\frac{N}{2}} = 2G_{m,even}$ however, we only want He even valves, so $G_{m,even}$ is simply G_{m} . an then G_{md} that $2G_{m} = H_{m} + H_{m+\frac{N}{2}}$ or,	Hm + $\frac{N}{2} = \sum_{k=0}^{N-1} - h_k e^{-2\pi i m k/N} \leftarrow odd$ (25e) So we find that $H_{m+\frac{N}{2}}$, even $= G_{m,even}$ and $H_{m+\frac{N}{2},odd} = -G_{m,odd}$. we add H_m and $H_{m+\frac{N}{2}} = G_{m,even} + G_{m,odd} + G_{m,even} = G_{m,odd}$ Hm $+ H_{m+\frac{N}{2}} = G_{m,even} + G_{m,odd} + G_{m,even} = G_{m,odd}$ Hm $+ H_{m+\frac{N}{2}} = 2G_{m,even}$ however, we only want the even valves, so $G_{m,even}$ is simply G_{m} . an then $G_{n,d} = G_{m+\frac{N}{2}} = G_{m+\frac{N}{2}}$ or,					-1 respective
So we find that $H_{m+\frac{N}{2}}$, even $\stackrel{\cdot}{=} G_{m,even}$ and $H_{m+\frac{N}{2},odd} \stackrel{\cdot}{=} -G_{m,odd}$. we nadd H_m and $H_{m+\frac{N}{2}} = G_{m,even} + G_{m,odd} + G_{m,even} - G_{m,odd}$ $H_m + H_m \cdot \frac{N}{2} = 2G_m$, even however, we only want the even values, so G_m , even is simply G_m . can then find that $2G_m = H_m + H_{m+\frac{N}{2}}$ or,	So we find that $H_{m+\frac{N}{2}}$, even = $G_{m,even}$ and $H_{m+\frac{N}{2},odd}$ = $-G_{m,odd}$. we add H_m and $H_{m+\frac{N}{2}}$ = $G_{m,even}$ + $G_{m,odd}$ + $G_{m,even}$ - $G_{m,odd}$ $H_m + H_{m+\frac{N}{2}}$ = $2G_{m,even}$ however, we only want He even values, so $G_{m,even}$ is simply G_{m} . can then G_{nd} that $2G_m = H_m + H_{m+\frac{N}{2}}$ or,				£ 80	
$H_{m} + H_{m+\frac{N}{2}} = G_{m,even} + G_{m,odd} + G_{m,even} - G_{m,odd}$ $H_{m} + H_{m+\frac{N}{2}} = 2G_{m,even}$ $however, we only want the even values, so G_{m,even} is simply G_{m}. an Hhen find Hhat an H_{m+\frac{N}{2}} = G_{m,even} + G_{m,odd} + G_{m,even} - G_{m,odd} G_{m,even} = G_{m,odd} + G_{m,odd} + G_{m,even} - G_{m,odd} G_{m,even} = G_{m,odd} + G_{m,odd} + G_{m,odd} G_{m,odd} = G_{m,odd} G_{m$	$H_m + H_{m+\frac{N}{2}} = G_{m,even} + G_{m,odd} + G_{m,even} - G_{m,odd}$ $H_m + H_m \cdot \frac{N}{2} = 2G_{m,even}$ however, we only want the even valves, so $G_{m,even}$ is simply $G_{m,even}$ and $G_{m,even}$ is $G_{m,even}$ and G_{m	Hm + 29	N = 5 - h R e-2"	rim K/N ← odd	(ase	
$H_{m} + H_{m+\frac{N}{2}} = G_{m,even} + G_{m,odd} + G_{m,even} - G_{m,odd}$ $H_{m} + H_{m+\frac{N}{2}} = 2G_{m,even}$ $however, we only want the even valves, so G_{m,even} is simply G_{m}. C_{m} = H_{m} + H_{m+\frac{N}{2}} C_{m} = H_{m} + H_{m+\frac{N}{2}}$	$H_m + H_m + \frac{N}{a} = G_{m,even} + G_{m,odd} + G_{m,even} - G_{m,odd}$ $H_m + H_m + \frac{N}{a} = 2G_{m,even}$ Nowever, we only want the even valves, so $G_{m,even}$ is simply $G_{m,even}$ and $G_{m,even}$ is $G_{m,even}$ and G_{m	so we Av	nd that Hm+ 1/2, er and Hm+ 1/2;	en - Gm, even an	10 Hm - N , 000 = -	-6 _{m,od} . we n
however, we only want the even values, so G_{m} , even is simply G_{m} . an then find that $2G_{m} = H_{m} + H_{m} \cdot \frac{N}{2}$ or,	however, we only want the even values, so Gm, even is simply Gm. can then find that 2 Gm = Hm + Hmr N 2 or,					
$2G_{m} = H_{m} + H_{mr} \frac{N}{2}$ or,	$2G_{m} = H_{m} + H_{mr} \frac{N}{2}$ or,					
or,	or,	however, u can then	we only want H find that	e even valves,	so Gm, even 1s	simply 6m.
		26m =	- Hm + Hm+ Ng			
$\left(G_{m} = \frac{1}{2}\left(H_{m} + H_{m} + \frac{v}{2}\right)\right)$	$\left(G_{m} = \frac{1}{2}\left(H_{m} + H_{m} + \frac{y}{2}\right)\right)$					
		(Gm = =	12 (Hm + Hm + 1/2))			