hw 6

Saturday, March 22, 2025 10:07 PM

1)
$$R = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

i) $R^T R$:

$$\begin{pmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \cos^2(\theta) + \sin^2(\theta) + 0 & -\sin(\theta)\cos(\theta) + \sin(\theta)\cos(\theta) + 0 & 0+0+0 \\ -\sin(\theta)\cos(\theta) + \sin(\theta)\cos(\theta) + 0 & \sin^2(\theta) + \cos^2(\theta) + 0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

RR^T:

$$\begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \cos^2(\theta) + \sin^2(\theta) + 0 & \sin(\theta)\cos(\theta) - \sin(\theta)\cos(\theta) + 0 & 0+0+0 \\ \sin(\theta)\cos(\theta) - \sin(\theta)\cos(\theta) + 0 & \sin^2(\theta) + \cos^2(\theta) + 0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$R^T R = RR^T = I$ ✓

ii) for some sine or cosine of θ, the inverse rotation is given by $-\theta$

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so $\varphi = -\theta$

$$R(-\theta) = \begin{pmatrix} \cos(-\theta) & -\sin(-\theta) & 0 \\ \sin(-\theta) & \cos(-\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

making use of sine being an odd function and cosine being even, we get:

$$R(-\theta) = \begin{pmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

so $R(-\theta) = R^T = R^{-1}$ ✓

2) $$\partial x_1 \partial x_2 \partial x_3 = \left\| \frac{\partial(x_1, x_2, x_3)}{\partial(x_1', x_2', x_3')} \right\| \partial x_1' \partial x_2' \partial x_3'$$

$$J_{i,j} = \frac{\partial x_i}{\partial x_j'}$$

i) to convert from cartesian to spherical coordinates, we use

$x = r\sin(\theta)\cos(\varphi)$ $\quad y = r\sin(\theta)\sin(\varphi)$ $\quad z = r\cos(\theta)$

$$J = \begin{bmatrix} \sin(\theta)\cos(\varphi) & r\cos(\theta)\cos(\varphi) & -r\sin(\theta)\sin(\varphi) \\ \sin(\theta)\sin(\varphi) & r\cos(\theta)\sin(\varphi) & r\sin(\theta)\cos(\varphi) \\ \cos(\theta) & -r\sin(\theta) & 0 \end{bmatrix}$$

ii) $$\sin(\theta)\cos(\varphi) \begin{vmatrix} r\cos(\theta)\sin(\varphi) & r\sin(\theta)\cos(\varphi) \\ -r\sin(\theta) & 0 \end{vmatrix}$$

$$= \sin(\theta)\cos(\varphi)\left[0 + r^2\sin^2(\theta)\cos(\varphi)\right]$$

$$= r^2\sin^3(\theta)\cos^2(\varphi) \quad 1$$

$$r\cos(\theta)\cos(\varphi) \begin{vmatrix} \sin(\theta)\sin(\varphi) & r\sin(\theta)\cos(\varphi) \\ \cos(\theta) & 0 \end{vmatrix}$$

$$r\cos(\theta)\cos(\varphi)\begin{vmatrix} & \\ \cos(\theta) & 0 \end{vmatrix}$$

$$= r\cos(\theta)\cos(\varphi)\left[0 - r\sin(\theta)\cos(\theta)\cos(\varphi)\right]$$

$$= -r^2\cos^2(\theta)\cos^2(\varphi)\sin(\theta) \quad (2)$$

$$-r\sin(\theta)\sin(\varphi)\begin{vmatrix} \sin(\theta)\sin(\varphi) & r\cos(\theta)\sin(\varphi) \\ \cos(\theta) & -r\sin(\theta) \end{vmatrix}$$

$$= -r\sin(\theta)\sin(\varphi)\left[-r\sin^2(\theta)\sin(\varphi) - r\cos^2(\theta)\sin(\varphi)\right]$$

$$= r^2\sin^3(\theta)\sin^2(\varphi) + r^2\cos^2(\theta)\sin^2(\varphi)\sin(\theta)$$

$$= r^2\sin^2(\varphi)\left(\sin^3(\theta) + \cos^2(\theta)\sin(\theta)\right)$$

$$= r^2\sin^2(\varphi)\sin(\theta) \quad (3)$$

$(1) - (2) + (3)$

$$r^2\sin^3(\theta)\cos^2(\varphi) + r^2\cos^2(\theta)\cos^2(\varphi)\sin(\theta) + r^2\sin^2(\varphi)\sin(\theta)$$

$$r^2\sin(\theta)\left[\sin^2(\theta)\cos^2(\varphi) + \cos^2(\theta)\cos^2(\varphi) + \sin^2(\varphi)\right]$$

$$r^2\sin(\theta)\left[\cos^2(\varphi)\left(\sin^2(\theta) + \cos^2(\theta)\right) + \sin^2(\varphi)\right]$$

$$r^2\sin(\theta)\left[\cos^2(\varphi) + \sin^2(\varphi)\right]$$

$$r^2\sin(\theta)$$

$$\partial x\,\partial y\,\partial z = r^2\sin(\theta)\,\partial r\,\partial\theta\,\partial\varphi$$

3) $f(y) = 4y + 12$

i) $0 = 4y + 12$

$$\frac{-12}{4} = \frac{4y}{4}$$

$$y = -3$$

ii) $$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$f'(y) = 4$

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$$y_1 = y_0 - \frac{4y_0 + 12}{4}$$

$$y_1 = y_0 - y_0 + 3$$

$$y_1 = 3$$

iii) For any linear function, the derivative will simply be the constant multiple of the independent variable. Dividing the original function by that multiple will yield simply the independent variable and any other constant term in the function also divided by the constant multiple, which is what you end up with when algebraically solving a linear equation. Alternatively, we consider that a linear polynomial is exactly defined by two parameters, and with the two pieces of information we need to use the Newton-Rhapson method, we can exactly define the linear polynomial, and thus we can use it in one step to exactly solve an equation.